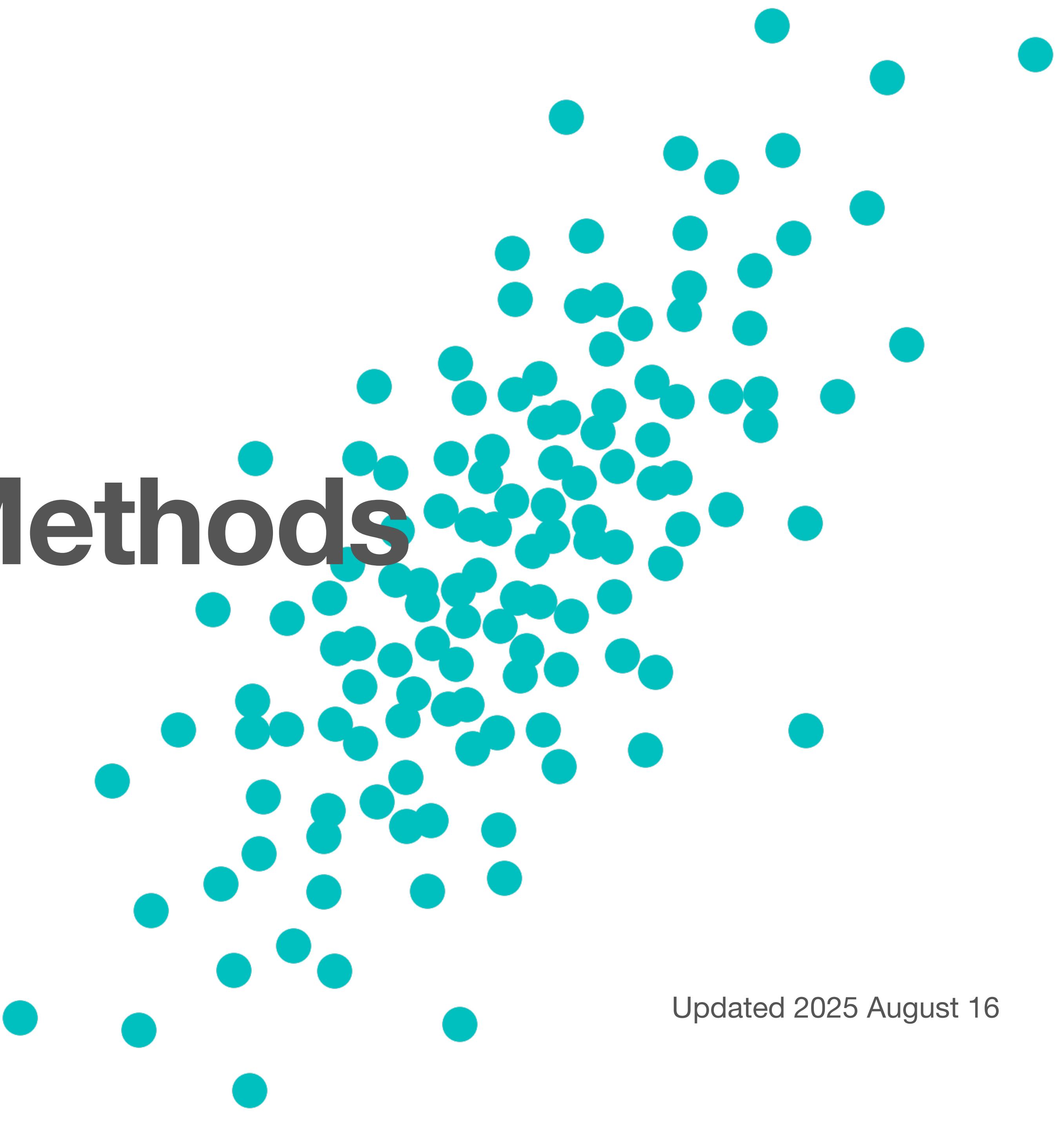


Monte Carlo Methods

MATH 565

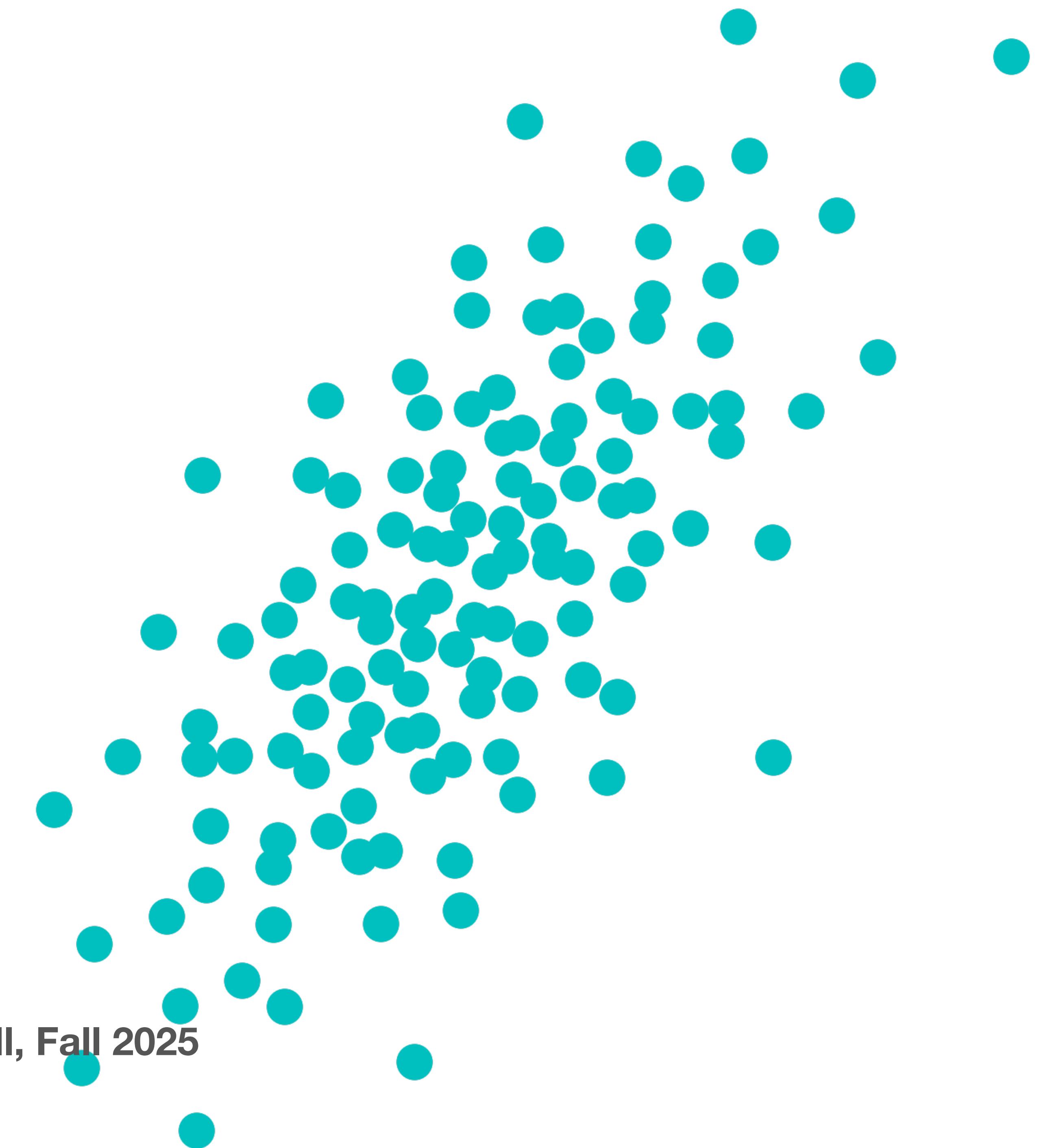
Fred Hickernell, Fall 2025

Updated 2025 August 16

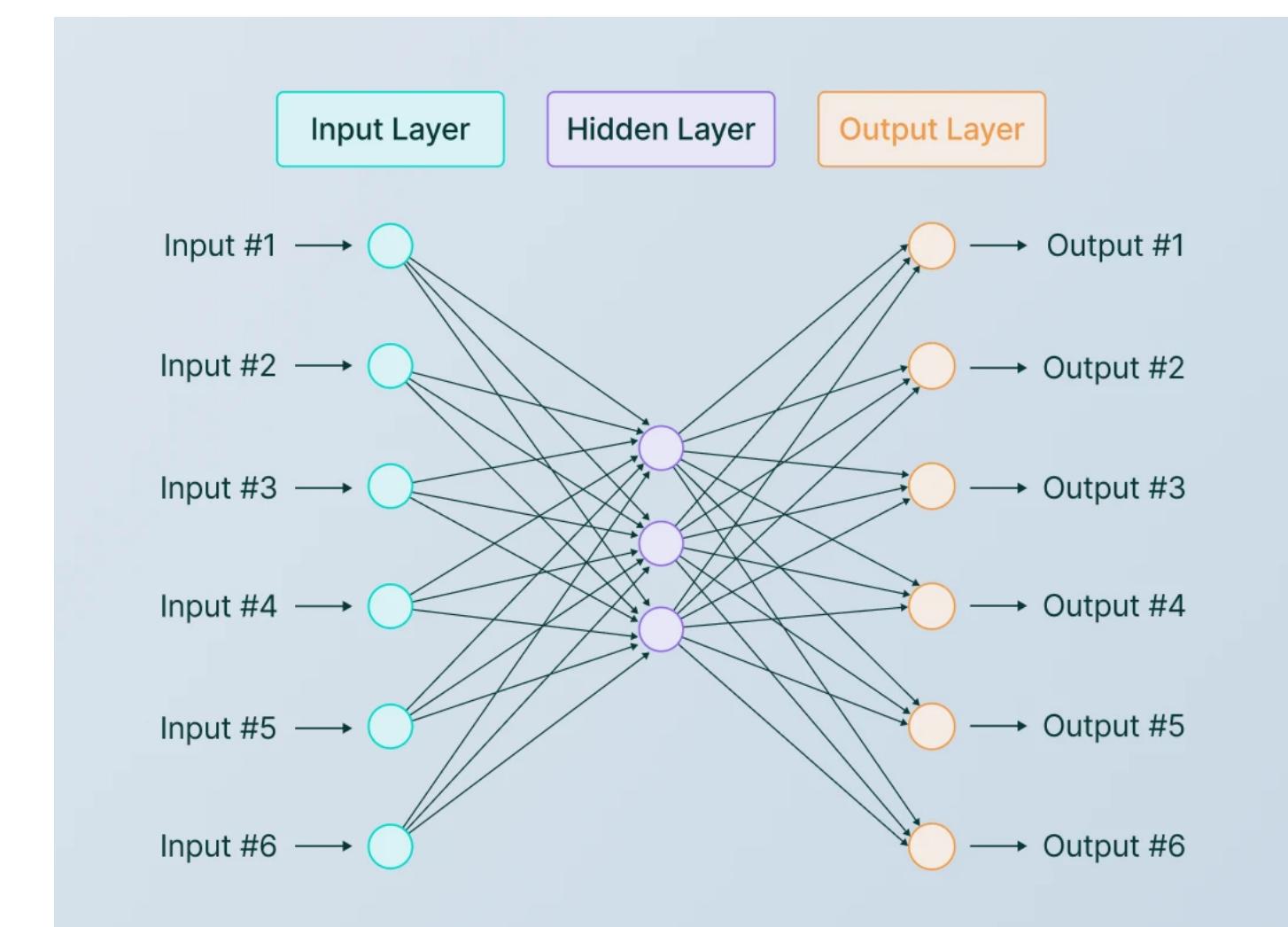
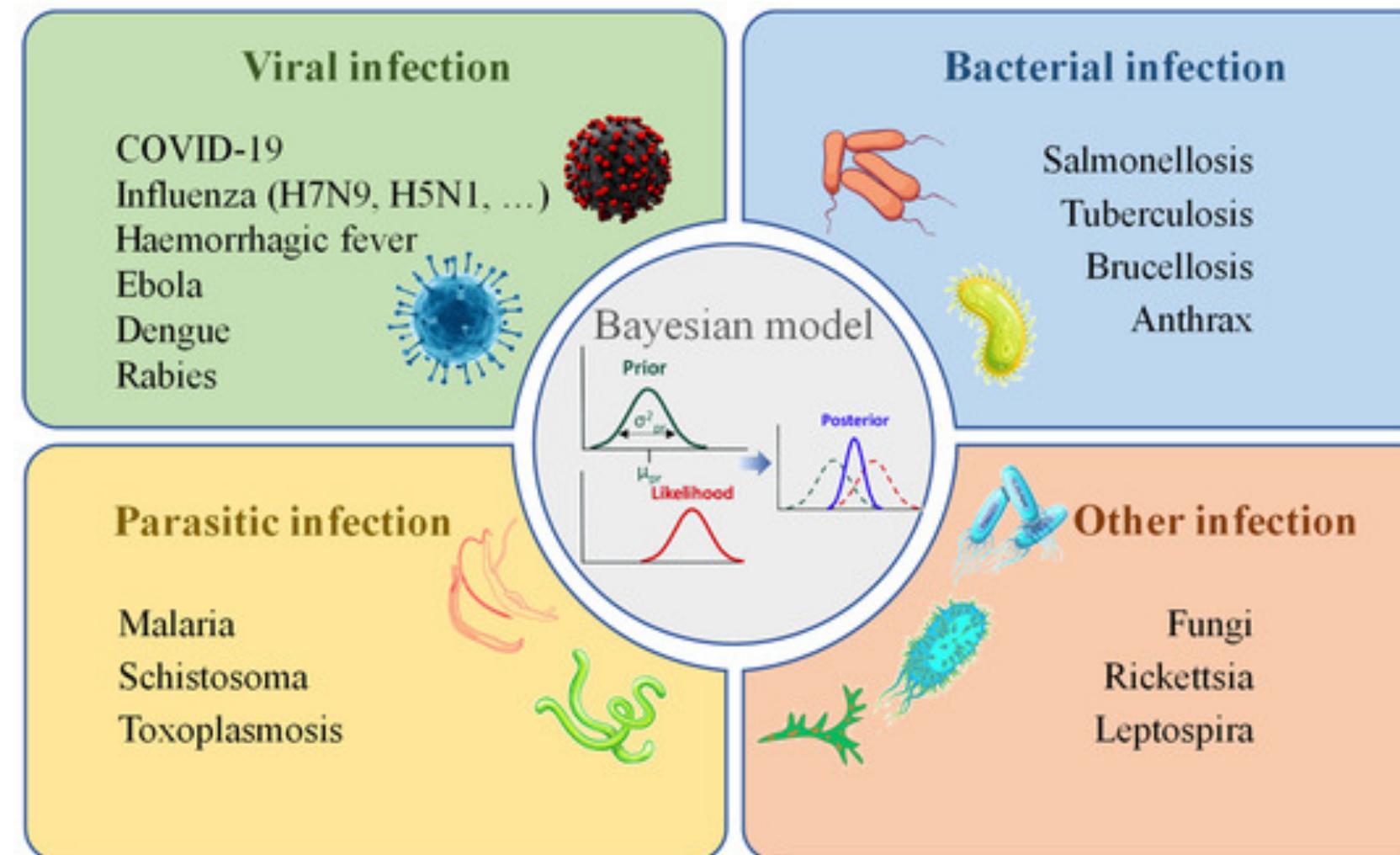
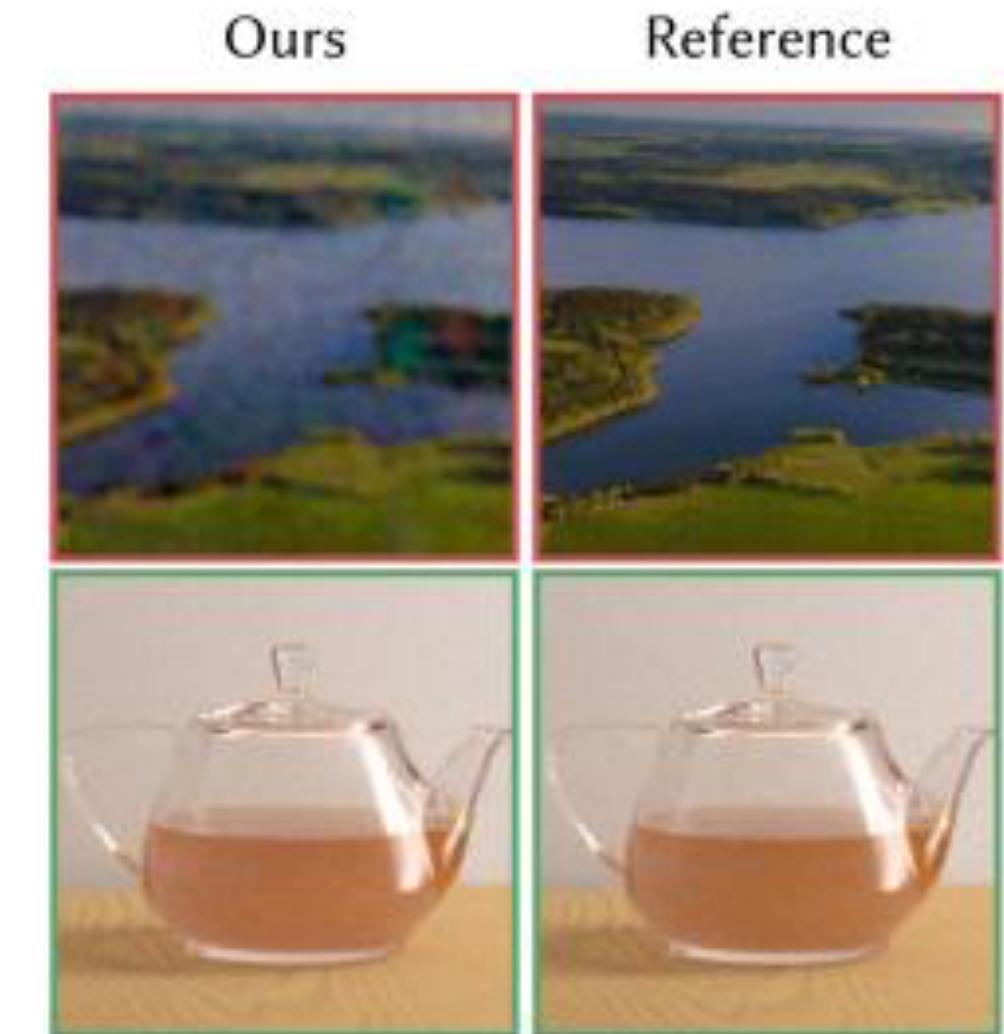
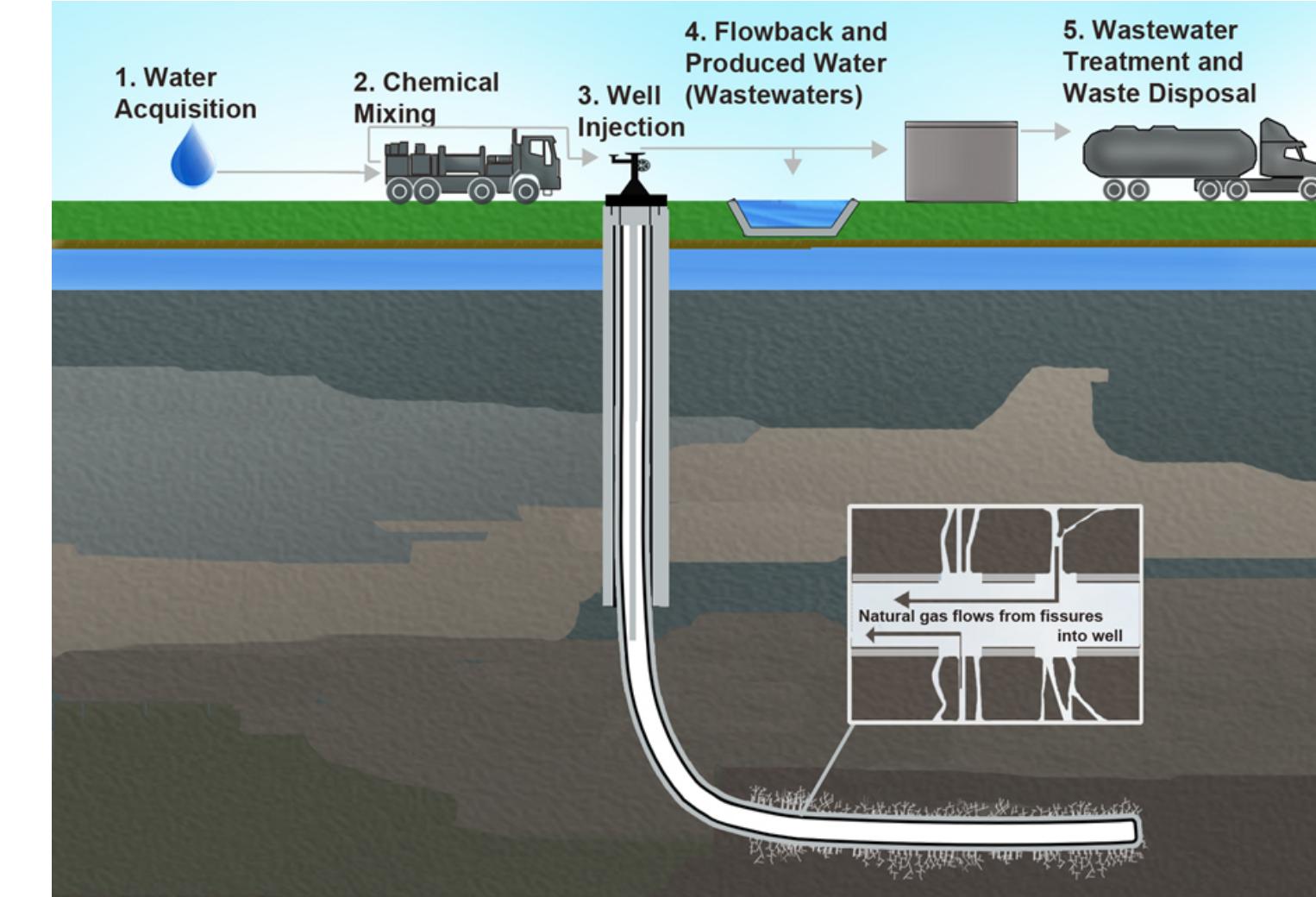


Introduction

MATH 565 Monte Carlo Methods, Fred Hickernell, Fall 2025



Monte Carlo Helps With Uncertainty





Why is there uncertainty?

- Finance – **market forces**, often modeled by stochastic processes driven by Brownian motion
- Engineering – **variability of system parameters**, sometimes modeled by Gaussian processes
- Image rendering – **can only trace some rays**, must rely on a finite sample
- Bayesian inference – the **posterior probability distribution** is a combination of a prior and what is learned from data
- Neural networks – many parameters need to be tuned, but one cannot search in **all possible directions**
- Queues – **arrival times and service times** of customers



How is this expressed quantitatively?

Y = random variable denoting **quantity of interest** = $\left\{ \begin{array}{l} \text{option payoff} \\ \text{fluid pressure} \\ \text{pixel intensity} \\ \text{statistical model parameter} \\ \text{neural network parameter} \\ \text{service time} \end{array} \right.$

= $f(\mathbf{X})$, where

\mathbf{X} = multivariate random variable with a **simpler distribution**

Our goal is to estimate the **mean**, **variance**, **quantile**, or **probability distribution** of Y



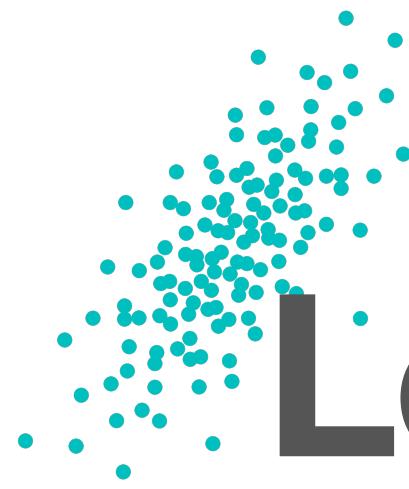
Are we there yet?

You are visiting your friend and it will require

- A **5** minute walk to the ‘L’ station
- Waiting for the train, which arrives every **20** minutes
- Traveling **35** minutes by ‘L’
- Catching a taxi at the ‘L’ destination
 - There is a **20%** chance that the car is waiting for you
 - Otherwise the average wait time is **10** minutes
- A **12** minute taxi ride

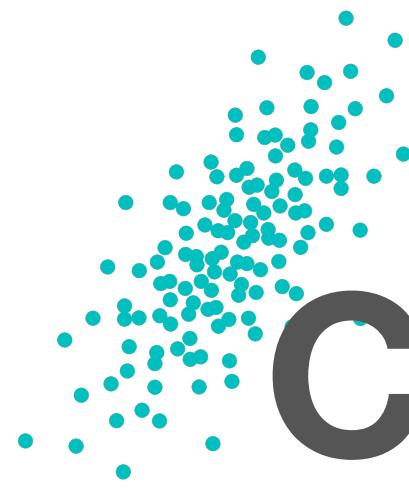
How long should you plan for the trip to take?

Let’s look at this Jupyter Notebook [AreWeThereYet](#) on the [class website](#)



Let me know you better

- Go to menti.com
- Use code 6222 607



Class website and repository

Website

Git repository



Why should you attend in synchronously?

- You will better keep pace?
- You will get real time answers to questions?
- You can influence the pace and direction of the course?
- You can help your peers learn and benefit from them—partake a community