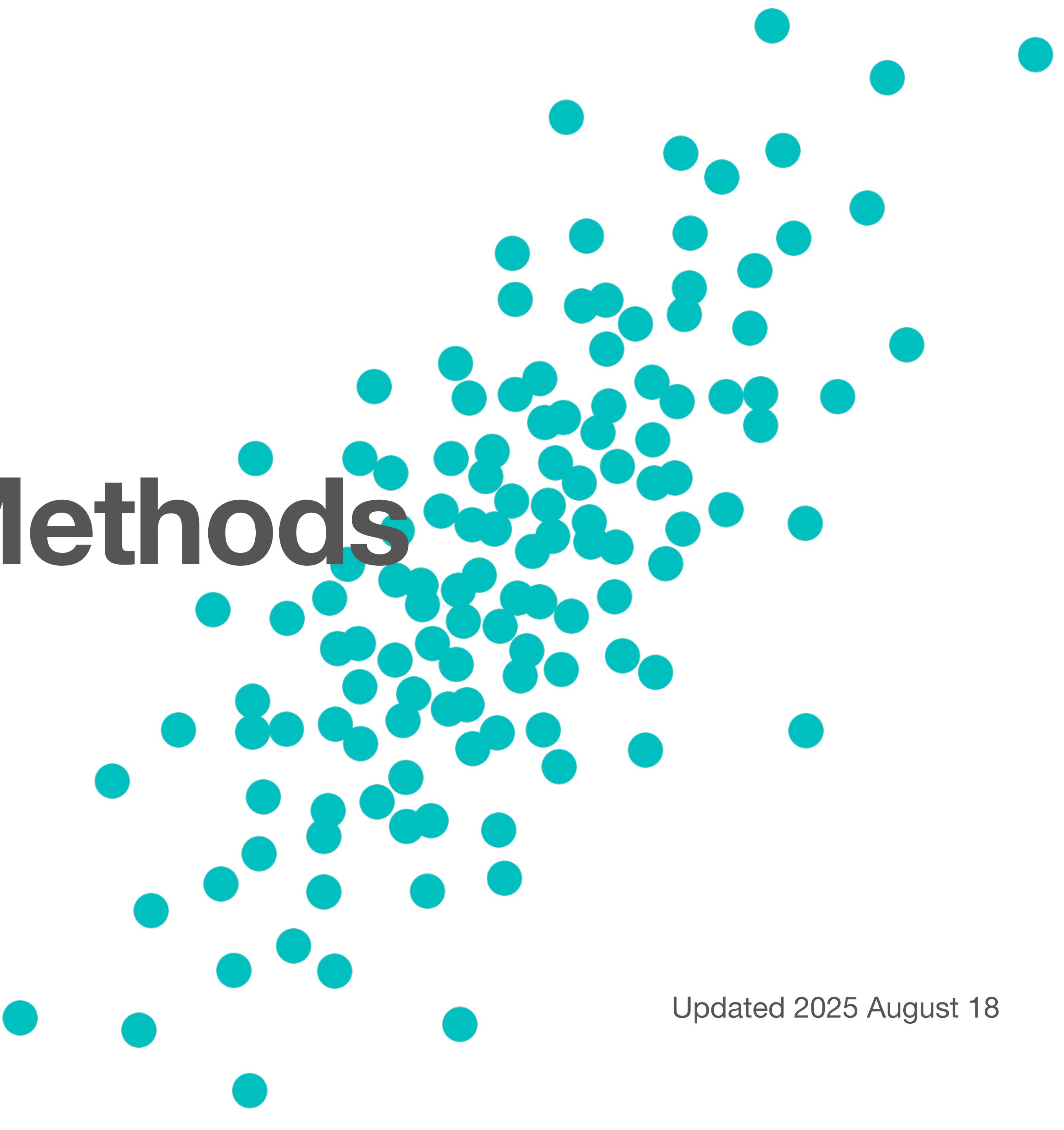


# Monte Carlo Methods

## MATH 565

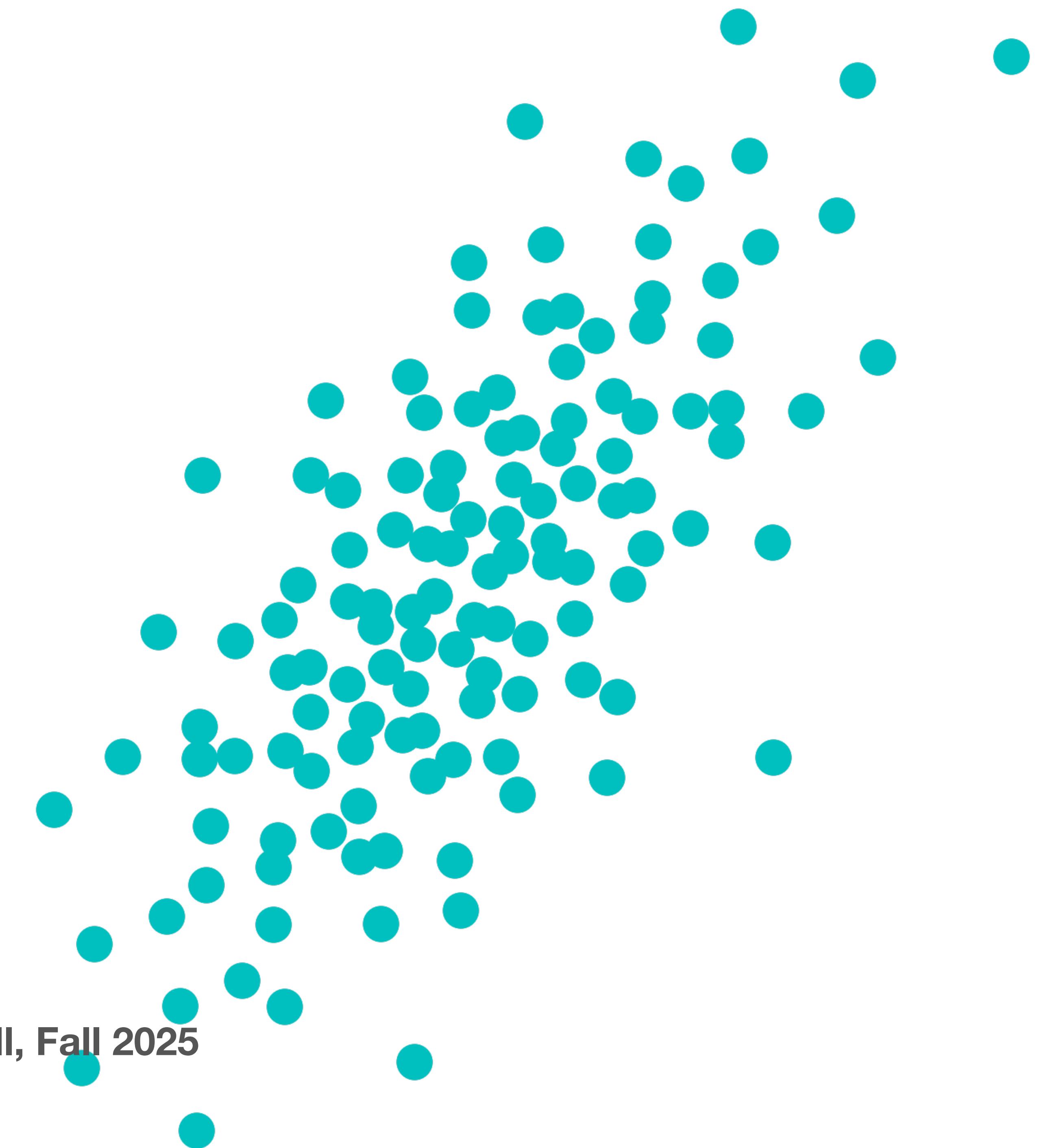
Fred Hickernell, Fall 2025

Updated 2025 August 18

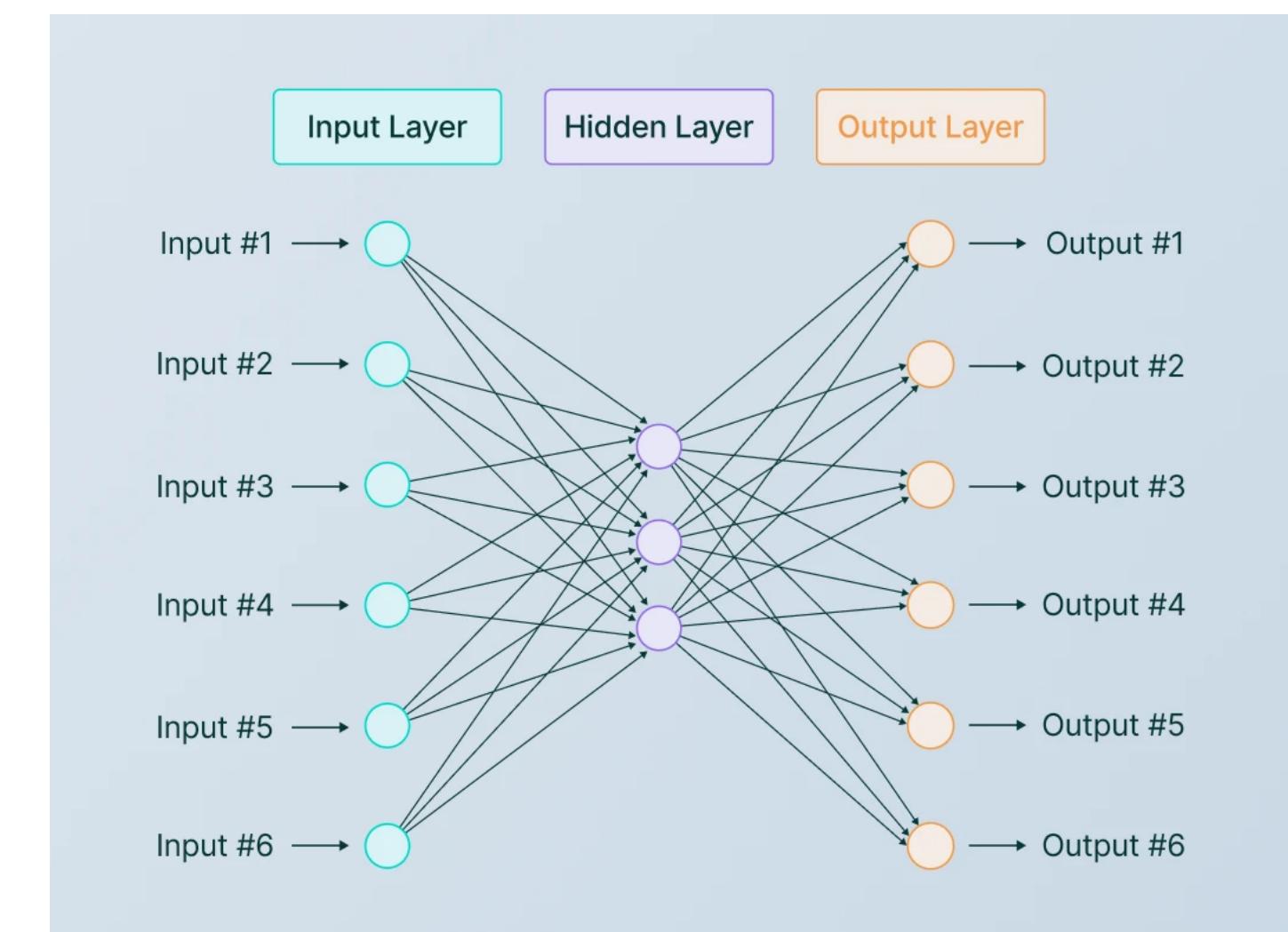
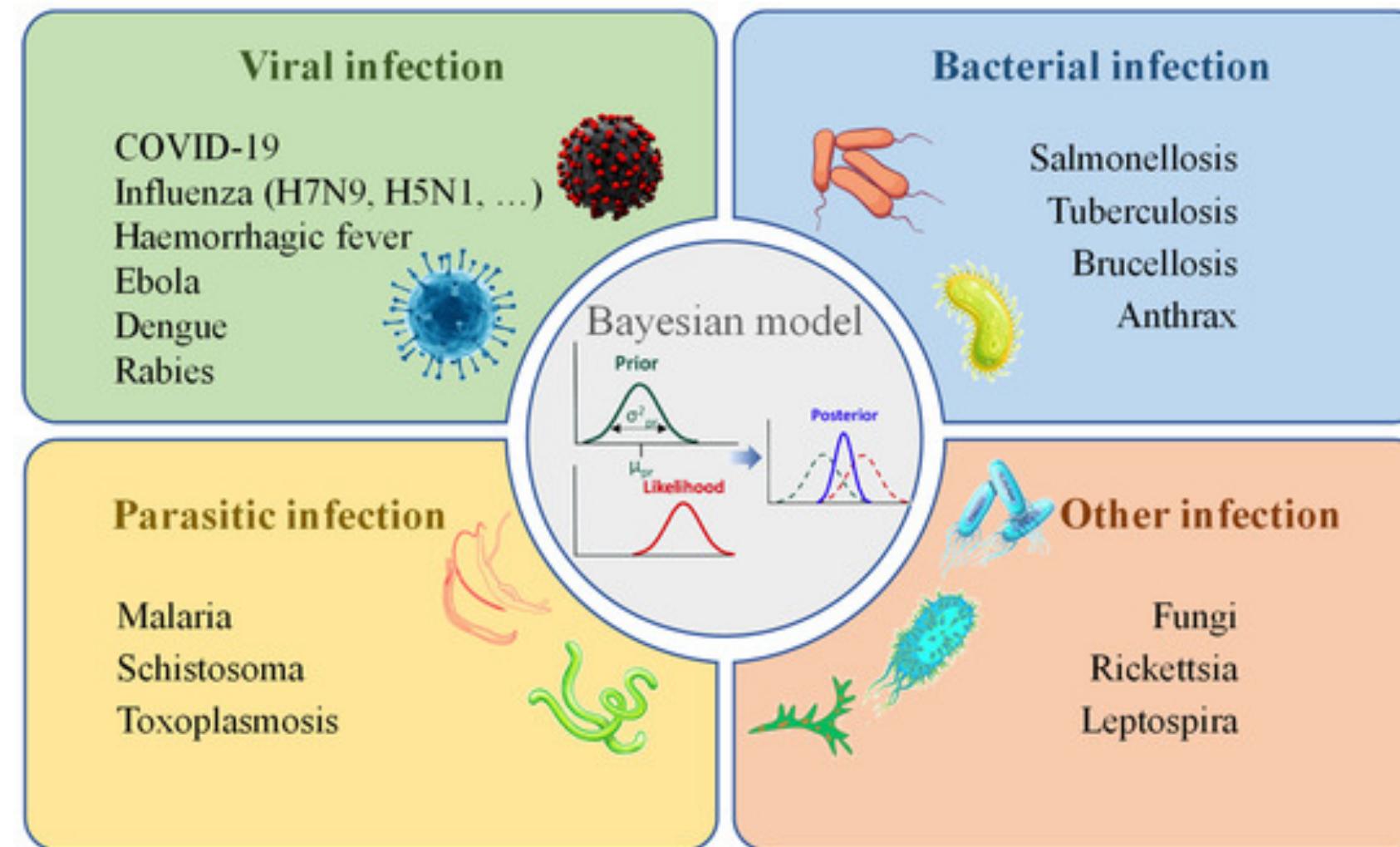
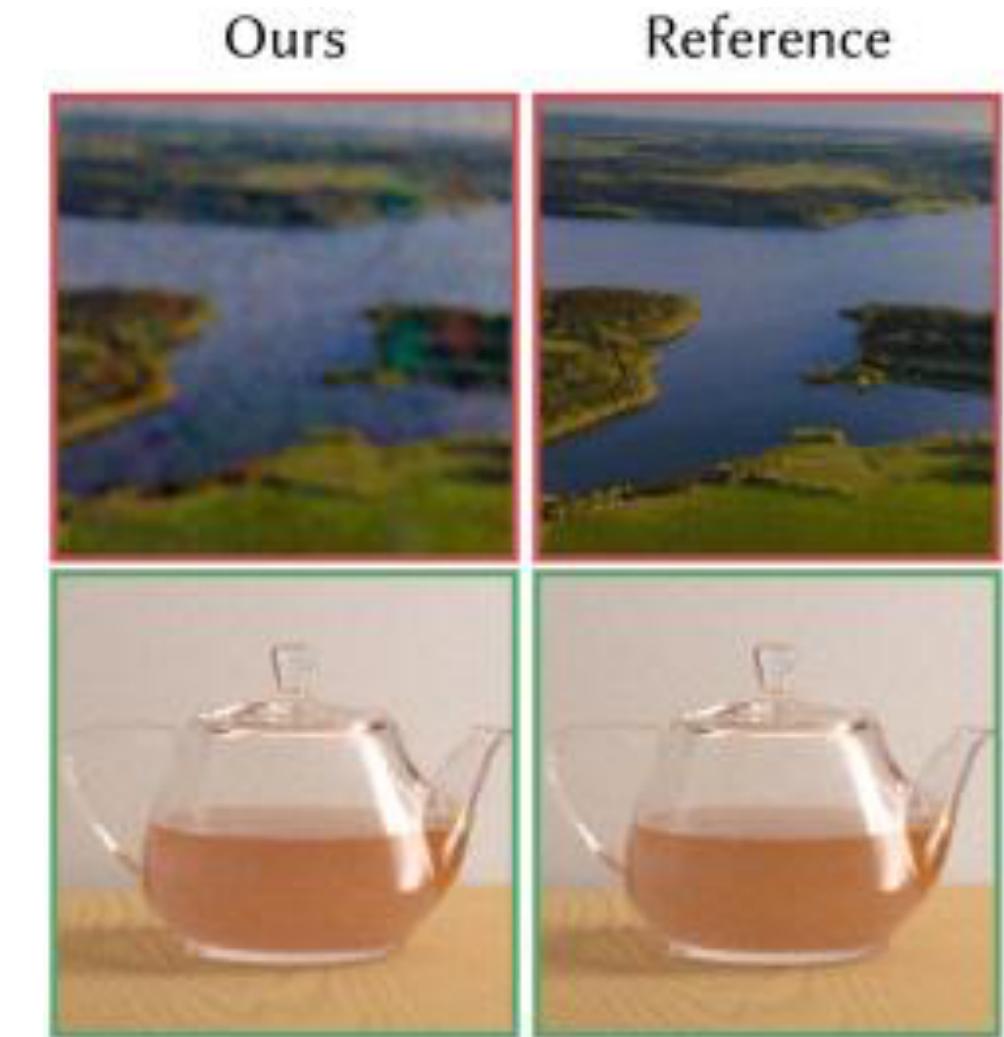
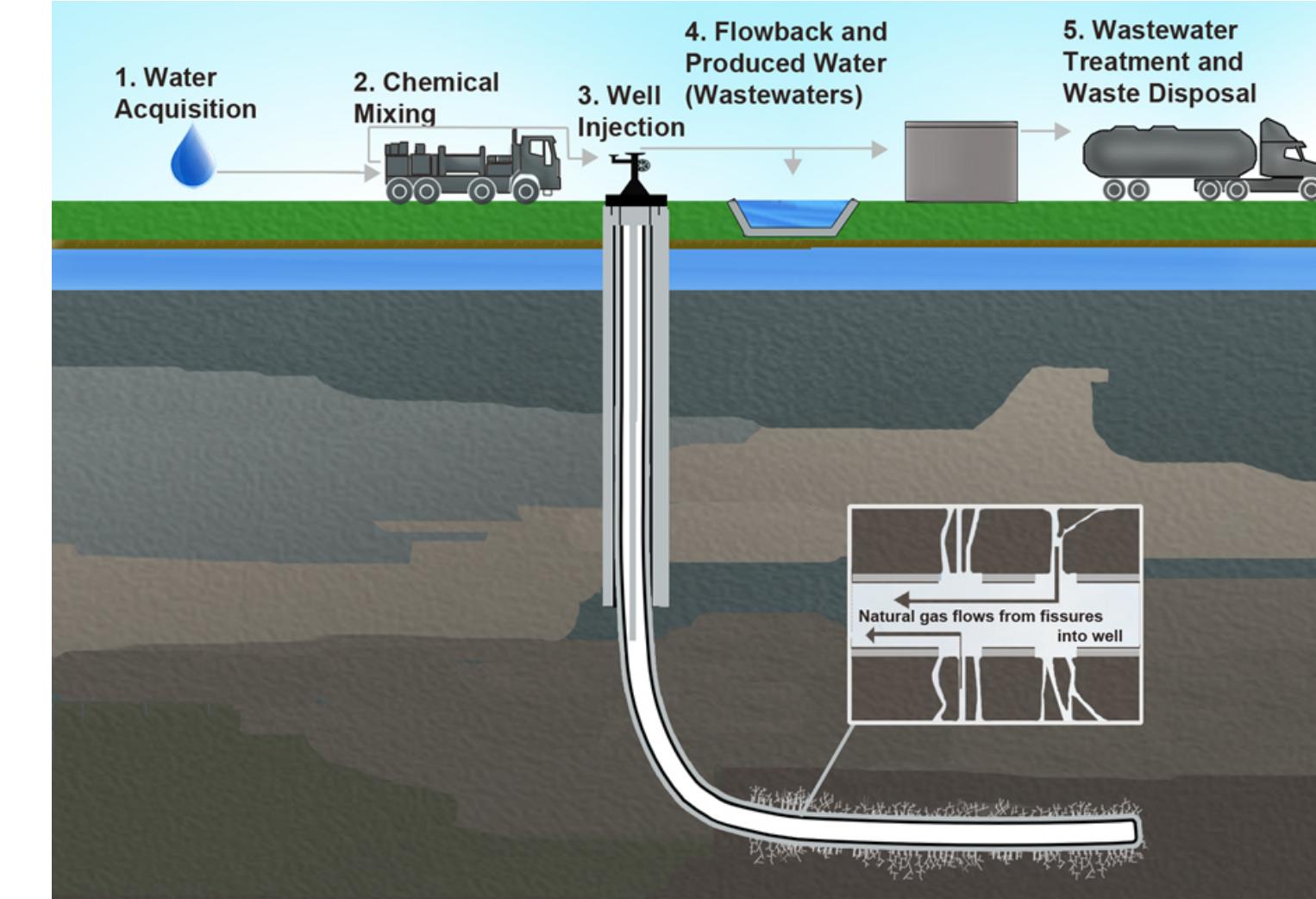


# Introduction

MATH 565 Monte Carlo Methods, Fred Hickernell, Fall 2025



# Monte Carlo Helps With Uncertainty





# Why is there uncertainty?

- Finance – **market forces**, often modeled by stochastic processes driven by Brownian motion
- Engineering – **variability of system parameters**, sometimes modeled by Gaussian processes
- Image rendering – **can only trace some rays**, must rely on a finite sample
- Bayesian inference – the **posterior probability distribution** is a combination of a prior and what is learned from data
- Neural networks – many parameters need to be tuned, but one cannot search in **all possible directions**
- Queues – **arrival times and service times** of customers



# How is this expressed quantitatively?

$Y$  = random variable denoting **quantity of interest** =  $\left\{ \begin{array}{l} \text{option payoff} \\ \text{fluid pressure} \\ \text{pixel intensity} \\ \text{statistical model parameter} \\ \text{neural network parameter} \\ \text{service time} \end{array} \right.$

=  $f(\mathbf{X})$ , where

$\mathbf{X}$  = multivariate random variable with a **simpler distribution**

Our goal is to estimate the **mean**, **variance**, **quantile**, or **probability distribution** of  $Y$



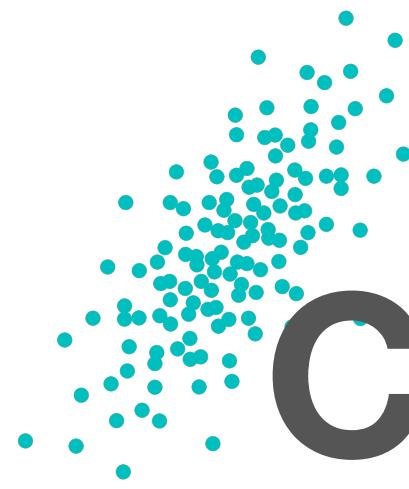
# Are we there yet?

You are visiting your friend and it will require

- A **5** minute walk to the ‘L’ station
- Waiting for the train, which arrives every **20** minutes
- Traveling **35** minutes by ‘L’
- Catching a taxi at the ‘L’ destination
  - There is a **20%** chance that the car is waiting for you
  - Otherwise the average wait time is **10** minutes
- A **12** minute taxi ride

How long should you plan for the trip to take?

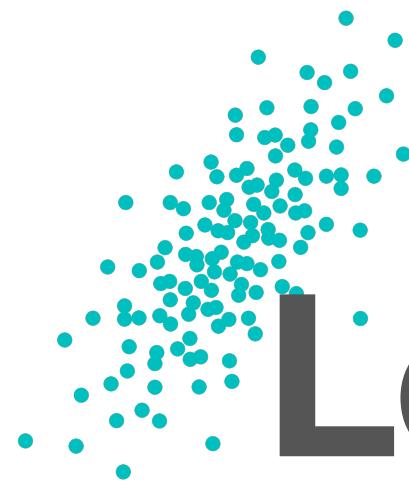
Let’s look at this Jupyter Notebook [AreWeThereYet](#) on the [class website](#)



# Class website and repository

Website

Git repository



# Let me know you better

- Go to [menti.com](https://menti.com)
- Use code 6222 607



# Why should you attend in synchronously?

- You will better keep pace?
- You will get real time answers to questions?
- You can influence the pace and direction of the course?
- You can help your peers learn and benefit from them—partake in a leaning community
- To help me know you, in case you want a reference