ILLINOIS TECH | College of Computing

More Efficient Portfolio Allocation with Quasi-Monte Carlo Methods using QMCPy

Larysa Matiukha and Dr. Sou-Cheng T. Choi

Illinois Institute of Technology

Problem

Portfolio allocation is a strategic process that aims to balance risk and return by distributing funds among diverse assets with varying risks and returns [5]. The goal is to create a diversified portfolio that can mitigate risk and enhance returns over time, while reducing the impact of market changes on individual assets. In this study for portfolio allocation, utilizing the open-source QMCPy library [2, 3], we apply quasi-Monte Carlo methods in the distribution of funds among individual stocks in a portfolio.

Existing Approach

Monte Carlo (MC) methods play a crucial role in portfolio allocation, enabling investors to model and predict a range of possible portfolio outcomes by randomly generating various scenarios and weights for each asset in the portfolio.

- To simulate diverse portfolio scenarios, generate weights randomly using independent identically distributed (i.i.d.) points in the hypercube $[0,1]^d$, where d is the number of assets in a portfolio.
- For realistic allocations, normalize the non-negative asset weights in a portfolio by one-norm to ensure the weights sum up to 1. Hence the transformed weights are in the probability simplex, $\{x = (x_1, \ldots, x_d) \in \mathbb{R}^d \mid x_i \geq 0, \sum_{i=1}^d x_i = 1\} \subset \mathbb{R}^{d-1}$.
- Use a common risk-adjusted portfolio performance measure, the Sharpe Ratio [7, 8], defined as $(R_P - R_F)/S_P$
  - R_F is risk-free rates, which we assumed to be zero in this study
  - $R_P = 252\sum_{i=1}^d R_i w_i$ is annualized expected portfolio return with R_i being the average of daily (log) returns of asset i in a given time period and w_i being the weights of asset i
  - $S_P = \sqrt{252 \sum_{i=1}^d \sum_{j=1}^d \sigma_{ij} w_i w_j}$ is annualized expected portfolio volatility, with σ_{ij} being the covariance of daily returns of assets i and j in the same time period [4].

Our hypothesis is that sub-optimal portfolio performance over time may result from i.i.d. points clustering or gaps.

Our Approach

We aim to investigate if utilizing low-discrepancy sequences such as lattice point sets, which tend to be more evenly distributed than i.i.d. points, could result in improved portfolio performance.

- We generate weights for diverse asset allocations via MC simulation with i.i.d. points and QMC simulation with lattice and Sobol points from the QMCPy Python library
- Compute Sharpe ratios for all portfolios
- Classify our portfolios into three different risk levels, namely low, medium, and high, that represent 1/3, 2/3, and 100th quantiles of annualized portfolio volatilities
- Select the portfolio weights associated with the maximum Sharpe ratio in each class

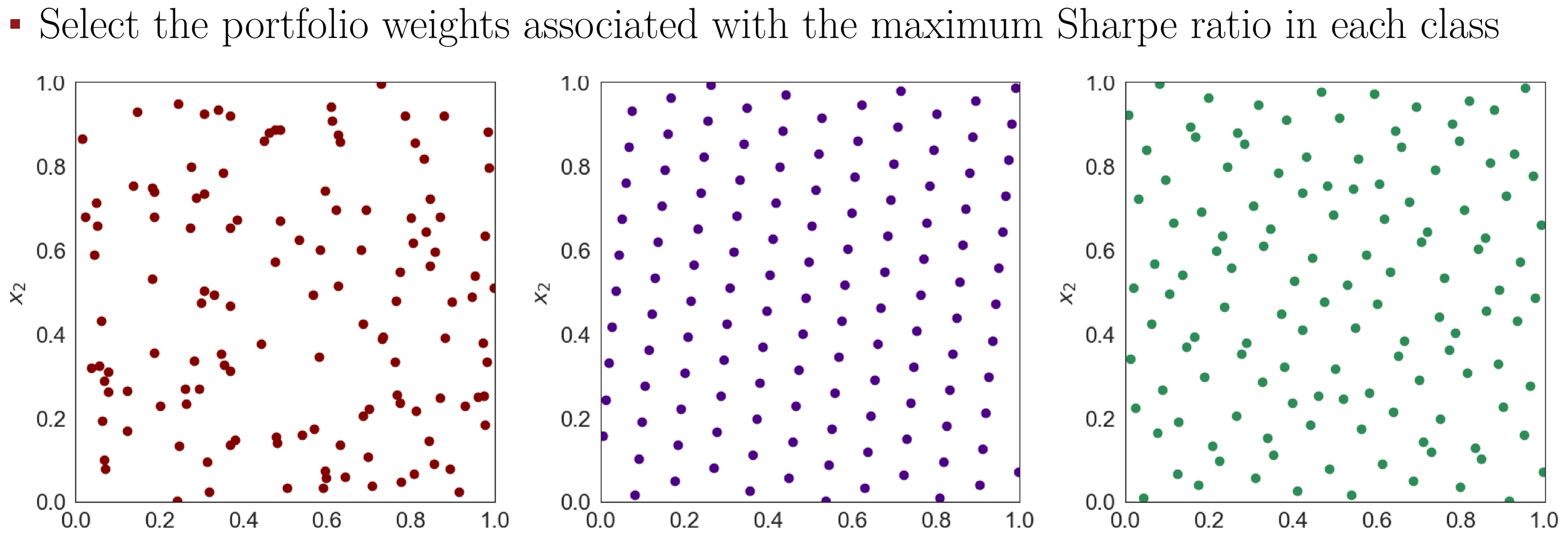


Figure 1. i.i.d. (left) vs. Low Discrepancy Sampling Points with lattice (middle) and Sobol (right)

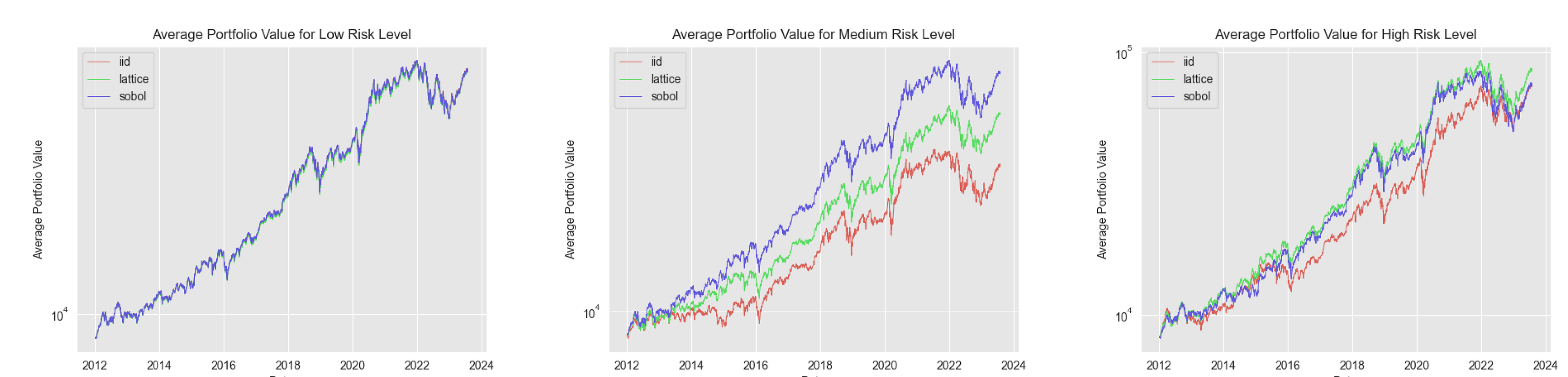
Data

We collected historical data from Yahoo Finance, spanning the period from January 1, 2012 to August 4, 2023. This dataset comprises the daily adjusted closing prices of selected publicly traded stocks.

Results

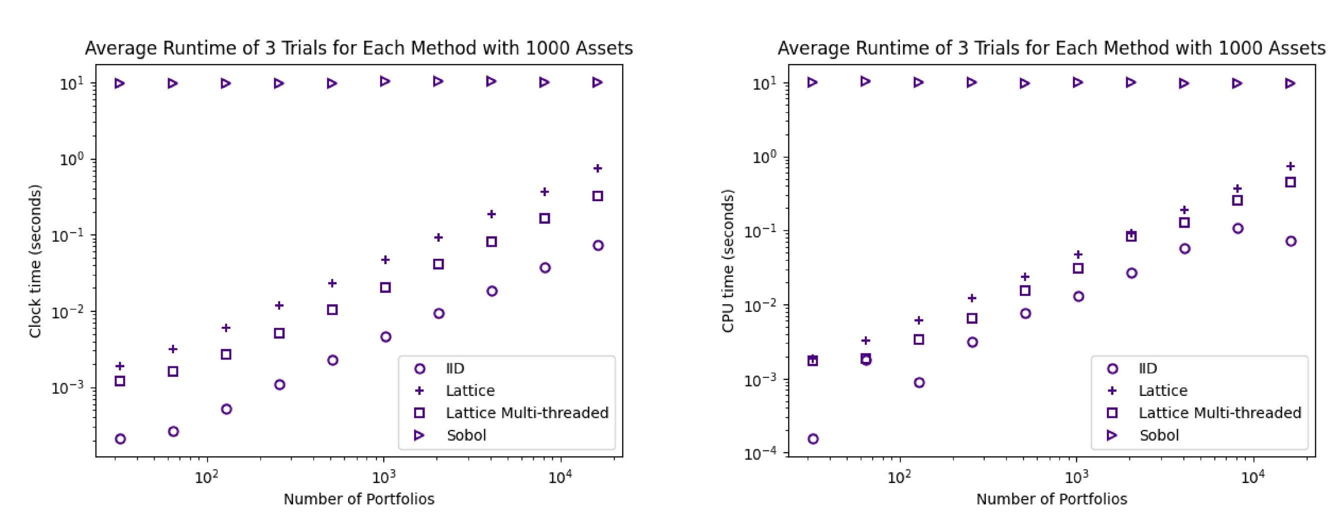
Sampling Method	Number of Assets	Number of Portfolios	Low-risk Sharpe	Medium-risk Sharpe	High-risk Sharpe
iid	20	8192	0.871553	0.871553	0.871553
lattice	20	8192	0.85494	0.915332	0.916822
sobol	20	8192	0.874393	0.897874	0.897874
iid	20	16384	0.886243	0.886243	0.886243
lattice	20	16384	0.870524	0.915332	0.916822
sobol	20	16384	0.875062	0.897874	0.897874
iid	20	32768	0.886243	0.886243	0.886243
lattice	20	32768	0.877995	0.915332	0.948492
sobol	20	32768	0.878877	0.897874	0.897874

Figure 2. Averaged Maximum Sharpe ratios comparison for selected 20 stocks over 50 simulations.



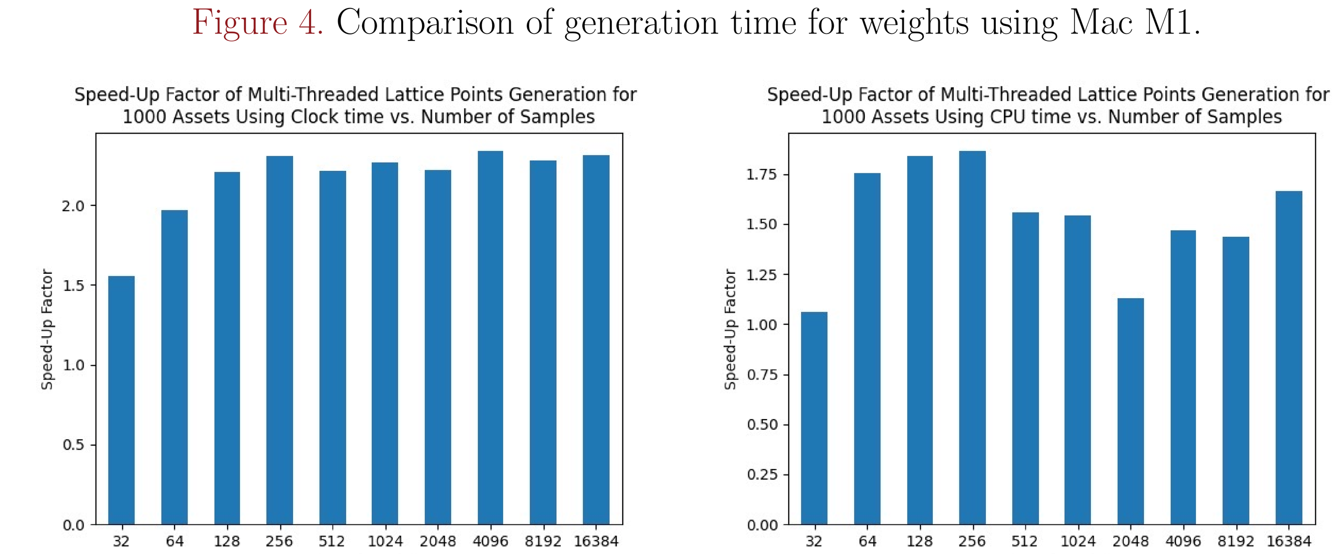
(a) Low-risk portfolios (b) Medium-risk portfolios (c) High-risk portfolios

Figure 3. In-sample backtesting (a method used to retrospectively evaluate the effectiveness of a strategy by simulating its performance using historical data) averaged portfolio values over 100 simulations for portfolio consisting of Apple, Amazon, Cisco Systems, and IBM stocks using the allocations obtained from each sampling method.



(a) Clock time (b) CPU time

Figure 4. Comparison of generation time for weights using Mac M1.



(a) Clock time (b) CPU time

Figure 5. We have implemented multi-threaded lattice point generation in QMCPy. Using Mac M1, the speed up factors are 1.5–2.4 in clock time (lower left) and 1.02–1.8 in CPU time (lower right).

Conclusions

Sharpe ratio values and backtesting results suggest that QMC methods can potentially enhance portfolio performance especially for medium and high risk strategies with a computational trade-off.

We have integrated a multi-threaded lattice point generation feature within QMCPy that produces significant speedup.

While our findings present promising insights, certain critical assumptions have been made. As we continue, our focus will be on refining our methodologies to further validate and strengthen our conclusions.

Future Work

1. Include risk-free rates in computation of Sharpe ratios
2. Implement out-sample backtesting and rebalancing
3. Compare to the performance of some benchmark index such as S&P 500
4. Consider downside risk as opposed to total volatility
5. Utilize other low-discrepancy distributions (e.g., Halton)
6. Investigate alternative performance measures (e.g., Sortino ratios)
7. Implement more optimal simplex transformation for lattice and Sobol sequences [6, 1]
8. Explore ways to reduce generation time of lattice and Sobol points in QMCPy by parallel processing or other techniques.

Acknowledgements

A special thanks to Dr. Fred Hickernell, Aleksei Sorokin, and QMCPy team for helpful discussion.

References

[1] Kinjal Basu and Art B Owen. Transformations and hardy–krause variation. SIAM Journal on Numerical Analysis, 54(3):1946–1966, 2016.

[2] S.-C. T. Choi, F. J. Hickernell, R. Jagadeeswaran, M. McCourt, and A. Sorokin. QMCPy: A quasi-Monte Carlo Python library (versions 1–1.4), 2023.

[3] Sou-Cheng T. Choi, Fred J. Hickernell, Rathinavel Jagadeeswaran, Michael J. McCourt, and Aleksei G. Sorokin. Quasi-Monte Carlo Software. In Alexander Keller, editor, Monte Carlo and Quasi-Monte Carlo Methods, pages 23–47, Cham, 2022. Springer International Publishing.

[4] John C. Hull. Options, Futures and Other Derivatives. Pearson Education, 9th edition, 2018. Global edition.

[5] Harry M. Markowitz. Portfolio selection. The Journal of Finance, 7(1):77–91, 1952.

[6] Tim Pillards and Ronald Cools. Transforming low-discrepancy sequences from a cube to a simplex. Journal of Computational and Applied Mathematics, 174(1):29–42, 2005.

[7] William F. Sharpe. Mutual fund performance. The Journal of Business, 39(1):119–138, 1966.

[8] William F Sharpe. The Sharpe ratio. Streetwise–the Best of the Journal of Portfolio Management, 3:169–85, 1998.

lmatiukha@hawk.iit.edu