The Semantic Web Lab Logic Exercises 3
Description Logic

Description Logic (DL) describes a subset of the predicate calculus (First Order Logic, FOL). Check the slides for the syntactic equivalences and the semantics. For now focus on DL being composed of *concepts*, *roles* (also called properties) and *individuals*, as opposed to the *predicates* and *objects* in FOL.

1 Description Logic Translation from FOL

1.1 Value restriction and quantification

One big difference between DL and FOL syntax is the way quantification is represented. In DL, there can be *value restriction* on concepts and roles. In DL, roles have a domain (first argument) that is quantified over the entire universe of things, so the universal quantification in DL statements is often dropped where it would be present in FOL. With this in mind, convert the following from FOL statements to DL statements:

1. $P(x) \wedge \forall y.R(x,y) \rightarrow C(y)$
2. $\exists y.R(x,y) \wedge (A(y) \vee B(y))$

1.2 Cardinality

While the existential and universal operators work well in FOL, cardinality is verbose to define in the syntax. In DL, on the other hand there are simple ways of notating this by using the 'atleast' and 'at most' restriction symbols followed by a number in front of the concept. With concepts consisting of roles one must specify this restriction like the quantifiers above, and again it applies only to the relation's range, not its domain. Translate the following into DL:

1. At least 3 women.
2. Someone with at least one male child.
3. A woman with at most two daughters.

2 Translate English into description Logic: Subsumption/Inclusion

A key concept in DL ontologies is the concept hierarchy. This hierarchy is an ordering on the concepts defined by the notion of subsumption (or inclusion). Example: a mother is a parent. [$Mother \sqsubseteq Parent$]. This can be read in DL terms as the concept $Mother$ being fully included in the concept $Parent$.

1. A human is a bipedal animal.
2. Having a Masters degree implies having a Bachelors degree. (Masters graduates are a subclass of bachelors graduates.).

3 Knowledge bases: A-boxes and T-boxes

DL knowledge bases can be split into two sections: concept definitions are normally defined in the *T-box* (think of 'T' standing for 'Terminology'), while assertions about the individuals and roles are found in the *A-box* ('A' for 'Assertion'). The type of logical sentence is not qualitatively different in the way we reason with them, but the distinction is useful for efficiency and resources. We would want to re-use T-boxes in a variety instantiations of situations in the same domain, while the A-box contains different assertions about the state of that particular situation.

T-box concept definitions: Translate English into description Logic
Define concepts for the below using complex concepts and/or roles (with the value restrictions as above where necessary). For example, a woman is a female person $Woman \equiv Person \sqcap Female$; a male is a person who is not female: $Male \equiv Person \sqcap \neg Female$; A father is a man with a human child: [$Father \equiv Man \sqcap \exists hasChild.Human$.

1. A mother
2. A parent
3. A grandmother
4. A dog with spots
5. A large dog with a dark spot

A-box assertions
Translate the following into assertions.

1. Rover is a mother
2. Rover is a grandmother
3. Rover is a dog with spots and a parent

Statements and queries
We may want to check the satisfiability of two statements in an ontology. These statements take the same form as the A-box assertions and T-box definitions. Try to answer and give reasons for the following query of satisfiability:

1. Is [$\forall Child.Male \sqcap \exists Child.\neg Male$] satisfiable?

4 Types of classification

Suppose that we have classes $A, B_1, B_2, \ldots, B_k$ with $B_i \sqsubseteq A$ for all i. Say that the classification is *exhaustive* if we have

$$A \equiv B_1 \sqcup B_2 \sqcup \ldots \sqcup B_k,$$

and that it is *disjoint* if we have

$$B_i \sqcap B_j \sqsubseteq \perp \quad \text{for all } i,\ j \text{ with } i \neq j.$$

1. Give examples of real-life classifications which are
   (a) exhaustive but not disjoint
   (b) disjoint but not exhaustive
   (c) both disjoint and exhaustive.
2. What is wrong with this classification (due to Borges): "Animals are divided into
   (a) belonging to the Emperor
   (b) embalmed
   (c) tame
   (d) sucking pigs
   (e) sirens
   (f) fabulous
   (g) stray dogs
   (h) included in the present classification
   (i) frenzied
   (j) innumerable
   (k) drawn with a very fine camel hair brush
   (l) et cetera
   (m) having just broken the water pitcher
   (n) that from a long way off look like flies."

5 Good and bad definitions

Say whether these definitions are good, or not (that is, do they define a concept, and only one concept). If they are good, give a model of them.

1.
$$\begin{aligned} A &\equiv B \sqcup C \\ C &\equiv \neg A \end{aligned}$$

2.
$$\begin{aligned} A &\equiv B \sqcup C \\ C &\equiv A \end{aligned}$$