The Semantic Web Logic Exercises 3: Description Logic Solutions

1 Description Logic Translation from FOL

1.1 Value restriction and quantification

1. $P(x) \sqcap \forall R.\top \sqsubseteq C$

2. $\exists R.(A \sqcup B)$

1.2 Cardinality

1. At least 3 women. [$\geq 3 Woman$]

2. Someone with at least one male child. [$Person \sqcap \geq 1 hasChild.Male$] alternatively just [$\geq 1 hasChild.Male$]

3. A woman with at most two daughters. [$Woman \sqcap \leq 2 hasChild.Female$]

2 Translate English into description Logic: Subsumption/Inclusion

1. A human is a bipedal animal. [$Human \sqsubseteq Animal \sqcap Biped$]

2. Having a Masters degree implies having a Bachelors degree. (Masters graduates are a subclass of bachelors graduates.). [$\exists hasDegree.Masters \sqsubseteq \exists hasDegree.Bachelors$]

3. (extra one not in lab) A mother is a parent. [$Mother \sqsubseteq Parent$] (Here 'is a' translates as 'included in' rather than the identity relation. You must use your intuition as to which is best.")

3 Knowledge bases: A-boxes and T-boxes

T-box concept definitions: Translate English into description Logic

1. A mother [$Mother \equiv Female \sqcap \exists hasChild.T$]

2. A parent [$Parent \equiv Father \sqcup Mother$], alternatively [$Parent \equiv \exists hasChild.T$]

3. A grandmother [$Mother \sqcap \exists hasChild.Parent$]

4. A dog with spots [$DWS \equiv Dog \sqcap \geq 2 has.Spot$]

5. A large dog with a dark spot [$LDWDS \equiv LargeDog \sqcap 1 has.(Spot \sqcap Dark)$]
   Note while we could use *Large* here, it is normally used as a non-intersective adjective (it should clearly not the same *Large* concept when talking about large ants, so defining the concept *LargeDog* elsewhere gets us round the problem. We could define $LargeDog \equiv Dog \sqcap \geq 120 hasHeight.cm$ for example.

A-box assertions

1. Rover is a mother [$Mother(Rover)$] or [$Rover : Mother$]

2. Rover is a grandmother [$Grandmother(Rover)$] or [$Rover : Grandmother$]

3. Rover is a dog with spots and a parent [$Rover \sqsupseteq Dog \sqcap hasSpots \sqcap Parent$] or [$Rover : dog \sqcap hasSpots \sqcap Parent$]
   Note that the assertion (or query) can be translated into exactly the same notation as the T-box representation, but is often changed to one of the above syntactic sugars to distinguish it from domain knowledge.

Statements and queries

1. Is [$\forall Child.Male \sqcap \exists Child.\neg Male$] satisfiable? [No- we can construct a contradiction for this concept where all the children are Male and at least one child is not male.]

4 Classifications

1. exhaustive but not disjoint- e.g. Nationalities. Everyone has one (exhaustive), though people can have more than one, e.g. in dual citizenship, so these classes can have intersections (hence not disjoint). Dog Breeds- you can list them but arguably you can always have a new one out of two previous ones in cross breeds.

2. disjoint but not exhaustive- e.g. Different sentences of the English Language. While they are all different (disjoint), you can always generate a new sentence through recursion "John likes the cat" → "John likes the cat who sat on the mat" → "John likes the cat who sat on the mat who purred... " etc. ad infinitum. Natural (and some artificial) languages are not exhaustible because of the *discrete infinity* permitted by their syntax.

3. both disjoint and exhaustive- e.g. Cards in a standard pack of playing cards. You can account for all of them if you treat each one as a class (exhaustive), and they are all different entities (disjoint).

4. By ‘divided into’ Borges seems to mean that the class of animals should be composed of disjoint subclasses and that the definition should be exhaustive, however the classes (j) “innumerable” and (l) “et cetera...” suggest that the definition is not exhaustive and (h) “included in the present classification” is self-referential to the class and so is generally not good as a definition, and it also violates the disjointness property expressed at the beginning of the statement.

5 Definitions

1.

$$A \equiv B \sqcup C$$
$$C \equiv \neg A$$

These are not good. We can use the fact that A and C are defined concepts. We can check for contradiction by substituting the second definition (of C)’s right-hand side into the first definition’s right-hand side to get: $A \equiv B \sqcup \neg A$.
Anything with a biconditional with a formula on one-side and its negation on the other is invalid if either is in a disjunction. Without doing the full model enumeration or proof tree it is clear that something can not be in A and not in A.

2.

$$A \equiv B \sqcup C$$
$$C \equiv A$$

These are not a good pair of definitions either, because while you can find a model that works truth conditionally you will get a cycle. If you assume B is bottom (false), you will get a tautology $A \equiv A$, which is not a good definition as it doesn’t define anything.
You can always assume definitions with their own names in the definiens (right-hand side) are bad in terms of failing to define at least and only one concept.