Data Semantics - Logic Exercises 2 - Solutions

1. Translations into predicate logic:
   (a) Some dogs are hairy = $\exists x(dog(x) \wedge hairy(x))$
   (b) All poodles are dogs = $\forall x(poodle(x) \rightarrow dog(x))$
   (c) Some hairy things are dogs = $\exists x(hairy(x) \wedge dog(x))$
   (d) Not every hairy thing is a dog = $\neg\forall x(hairy(x) \rightarrow dog(x))$
   (e) There is no dog which does not bark = $\neg\exists x(dog(x) \wedge \neg barking(x))$
   (f) Every student has read a Java textbook = $\forall x\exists(student(x) \rightarrow (JavaTextbook(y) \wedge read(x,y)))$
   (g) John lost his laptop in the library or the cafeteria = $\exists y(laptop(y) \wedge owns(John, y) \wedge$ $(lost\_in(john, y, cafeteria) \vee lost\_in(john, y, library)) \wedge$ $\neg(lost\_in(john, y, cafeteria) \wedge lost\_in(john, y, library)))$

2. Translation into English
   (a) John opened a bank account at Barclays.
   (b) There is a lecturer every student knows.
   (c) Everything is an advisor and for every advisor there is a student advised by him/her.
   (d) A student who is clever, or a student who works hard, will pass the examination
   (e) Some clever students will not pass the examination

3. (a) VALID iff treating the word ‘are’, in set-theoretic terms, as denoting ‘is a subset of’. i.e. all members of A are also members of B and all members of B (and therefore all As) are members of C.
   (b) INVALID - ‘some As are B’ does not guarantee C-ness for all As- there could an A that is not a B and therefore this A would not be a C.
   (c) VALID - similar to (a), again iff we assume the meaning of the English word ‘are’ indicates the subset relation. That is to say if every member of B is contained in C, the A’s that are in B will also, by the second premise, be in C.
   (d) INVALID, we can provide a counterexample as nothing in the stated premises restricts the negation of the stated conclusion- there could be some members of A that are not in C.

4. (a) ‘a is best’ = $\forall y(better(a, y))$
   or, more strictly:
   ‘a is best’ = $\forall y(a \neq y \rightarrow better(a, y))$
   (to ensure a and y are not the same individual)
   (b) This should lead to a contradiction if we interpret ‘better’ as being transitive (applicable in one direction) and assume we’re discussing the same domain of objects, in which both a and b are contained. The contradiction can be proven by showing that a model cannot support these two logical sentences. One simple, though long winded-way, of doing this is to ‘unpack’ the definitions of ‘a is best’ and ‘b is best’ into two longer conjunctions of predicates. ‘a is best’ in domain $\{a, b, c, ..\}$ could be unpacked to $better(a, b) \wedge better(a, c) \wedge$ $...etc.$. When similarly unpacking ‘b is best’ in the same domain we get $better(b, a) \wedge$ $better(b, c) \wedge ...etc.$. Consequently get a contradiction in the first conjunct of these two long sentences- $better(a, b)$ and $better(b, a)$ should not be permitted to both be true in our semantics. Note there are more efficient ways to do this using logical inference rules.

(c) 'y is x's favourite' = $\forall z(prefers(x, y, z))$
Note that x and y are unbound.

(d) 'Everyone is someone's favourite' = $\forall y \exists x \forall z(prefers(x, y, z))$
or 'Everyone is someone's favourite' = $\exists x \forall y \forall z(prefers(x, y, z))$
This can be seen as an ambiguity (two different readings). Note scope matters. In the first reading there are many potentially different 'someone's x as the existential quantifier is inside the scope of the universal one, however in the second reading there is potentially only one person x such that x is the someone who favours everyone.
(More accurate answer:)
While this can be construed as ambiguous if we characterise the definition of 'favourite' in the way above, note that we don't want anyone to have more than one favourite for a good definition, so if we make this more accurate $\forall y \forall z \exists x(x \neq y \neq z \rightarrow (prefers(x, y, z))$, we can rule out the second definition. Ambiguity can now be entertained in that it could also be the case that there is only one person who favours each y (a unique favourite per person)- we could capture this second reading by introducing another variable w to rule out two or more people having the same favourite: $\forall y \forall z \exists x \forall w((x \neq y \neq z) \wedge (x \neq w) \wedge \neg prefers(w, y, z) \rightarrow (prefers(x, y, z))$

5. $\forall x \forall y(farmer(x) \wedge donkey(y) \wedge owns(x, y) \rightarrow beat(x, y))$
is the closest we can get to the appropriate logical form of the sentence, however it does provide a problem for our conversion process from English to logical forms, as the indefinite article 'a' does not translate to $\exists$ here as you'd expect. You may have initially tried something like:
$\forall x \exists y(farmer(x) \wedge donkey(y) \wedge owns(x, y) \rightarrow beat(x, y))$ (INCORRECT)
However this fails to give the correct truth conditions: Imagine a situation where there is a farmer who owns both a donkey and a pig, and does not beat his donkey. The formula will inappropriately still be true in that situation, because to preserve its truth, for each farmer we need to find at least one object that either is not a donkey that is owned by this farmer OR find an object that is beaten by the farmer (this is because we can use the right-hand side of the logical equivalence $P \rightarrow Q \equiv \neg P \vee Q$, rather than the left-side). Hence, if the object y denotes the pig, the sentence will not be contradicted, and remain true in that situation, which is clearly not what we want as the farmer is not beating his donkey! Donkey sentences are notoriously difficult for semanticists!

6. (b) can be seen as a clear contradiction (and a case of Russell's paradox) as the truth conditions of its conjuncts cannot be satisfied in any model. (j) can be seen as contradictory for similar reasons if you assume 'It is raining' to be a statement that is asserted to be believed to be true by the same speaker who asserts belief in its falsity. (d) can be seen as contradictory if we take the predicate 'lower than' as transitive. (h) can be seen as contradictory if you assert in your model the predicate $know(a, b)$ where b must be a truthful sentence.

   Despite these apparent contradictions, you could in fact argue that any of the sentences in this exercise are not contradictory if you define your semantics in obscure enough ways (for example defining models that do not conform to geometry and mathematics for (a), (f), (g), (h), other real world knowledge ((c), (d), (g), (h)) or altering the *arity* of predicates (i.e. which arguments they can take, for instance asserting the predicate $know(a, b)$ where b needn't be constrained to a truthful sentence in (h).

7. (a) Somebody lives in London = $\exists x(person(x) \wedge livesIn(x, London))$- contingent
   (b) Nobody lives in London = $\neg \exists x(person(x) \wedge livesIn(x, London))$ - contingent
   (c) At least two people live in London = $\exists x \exists y(person(x) \wedge liveIn(x, London) \wedge person(y) \wedge liveIn(y, London))$ - contingent
   (d) At least 10,000 people live in London = $\exists x_1 \exists x_2 ... \exists x_{10,000}(person(x_1) \wedge liveIn(x_1, London) \wedge person(x_2) \wedge liveIn(x_2, London) ... \wedge person(x_{10,000}) \wedge liveIn(x_{10,000}, London))$ - contingent, notice the paucity of first-order logic on its own for expressing cardinality!
   (e) Some concert pianists are French = $\exists x(concertPianist(x) \wedge french(x))$ - contingent
   (f) Every black dog is a dog = $\forall x(black(x) \wedge dog(x) \rightarrow dog(x))$ - tautology

(g) Every fake Picasso is a Picasso = $\forall x(looksLike(x, PicassoPainting) \wedge \neg Is(x, PicassoPainting) \rightarrow Is(x, PicassoPainting))$ - contradiction, if 'fake Picasso' is translated in this way.

(h) Every suspected criminal is suspected = $\forall x(\exists y(person(x) \wedge crime(y) \wedge suspectedOf(x, y) \wedge \neg provenGuiltyOf(x, y) \rightarrow suspectedOf(x, y)))$ - tautology, if 'suspected criminal' is translated in this way.

(i) Every suspected criminal is a criminal = $\forall x(\exists y(person(x) \wedge crime(y) \wedge suspectedOf(x, y) \wedge \neg provenGuiltyOf(x, y) \rightarrow provenGuiltyOf(x, y)))$ - contradiction, if 'suspected criminal' and 'criminal' are translated in this way.

(j) All ravens are black = $\forall x(raven(x) \rightarrow black(x))$ - contingent

(k) If two people are brothers then they are siblings = $\forall x \forall y(male(x) \wedge male(y) \wedge haveSameParents(x, y) \rightarrow haveSameParents(x, y))$ - tautology, if 'brothers' is interpreted in this way, assuming the two people are each others' brothers and we're not just asserting they're a brother to someone or other.