Definite Descriptions

Upper and Lower Bounds

- We can set upper and lower bounds on the number of objects in the domain which satisfy an open sentence.
  - At least two, at least three, at least four, . . .
  - At most one, at most two, at most three, . . .
- These bounds can be specified by the use of multiple quantifiers and identity symbols.

Lower Bounds

- To say there are at least *n* objects satisfying an open sentence, one needs to use *n* existential quantifiers and state (if necessary) non-identities for all the distinct variables they contain.
  - At least one P: $\exists x.P(x)$
  - At least two Ps: $\exists x, y(P(x) \wedge P(y) \wedge x \neq y)$
  - At least three Ps: $\exists x,y,z[P(x) \wedge P(y) \wedge P(z) \wedge x \neq y \wedge y \neq z \wedge \neq z]$

Upper Bounds

- To say there are at most *n* objects satisfying an open sentence, one needs to use *n+1* universal quantifiers and state identity for at least one pair of distinct variables the quantifiers contain.
  - At most one P: $\forall x,y((P(x) \wedge P(y)) \longrightarrow x = y)$
  - At most two Ps: $\forall x,y,z[(P(x) \wedge P(y) \wedge P(z)) \longrightarrow x = y \vee y = z \vee x = z]$
  - At most three Ps: $\forall x,y,z,w[(P(x) \wedge P(y) \wedge P(z) \wedge Pw) \longrightarrow$

    $x = y \vee x = z \vee x = w \vee y = z \vee y = w \vee z = w]$

Exact Quantities

- To say that there are exactly *n* objects satisfying an open sentence, one must state that there are at least *n* and at most *n* such objects.
- It is possible to combine these specifications in a single sentence.
  - Exactly one P: $\exists x, \forall y((P(x) \wedge P(y)) \longrightarrow x = y)$
  - Exactly two Ps: $\exists x,y[(P(x) \wedge P(y) \wedge x \neq y) \wedge \forall z.P(z) \longrightarrow (x = z \vee y = z)]$