Data Semantics - Logic Exercises 1

Logic allows us to reason about knowledge automatically. Humans do this naturally but it is a knowledge hungry task. We take intuitions from philosophy and semantics that human knowledge can be encoded as sentences of a logical language. This language may be one consisting of "flat" propositions or statements such as P^{592} = "John is dead", that is statement or *propositional logic.*

1. **Definitions**
   Define what satisfiability, validity, unsatisfiability and falsifiability mean for a formula P in propositional logic.

2. **Propositional Logic Syntax**
   Translate the following into simple propositional logic sentences using symbols to represent propositions and use the logical connectives where possible. Indicate what the propositions in your sentences stand for (their extension) in English. Be careful about natural language problems. "John ate and slept" should not have $P =$"slept" as one of its propositions, but $P =$"John slept" is fine. The first is an example.

   (a) (Example) John went to Paris and Clara went to Nancy
   $P \wedge Q$
   where $P =$ "John went to Paris" and $Q =$ "Clara went to Nancy"

   (b) John is in the library or out for lunch

   (c) John does not dance

   (d) John or Clara dance

   (e) If Bert dances then Ernie sings

   (f) If Ernie sings then Bert dances

   (g) If Ernie sings then Tim Berners-Lee will not come to the party

   (h) If Ernie sings, Bert dances, and vice-versa

3. **Proof with Truth Table**
   Prove De Morgans equivalence $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$ by filling out and comparing the truth table for each formula.

4. **Meaning as Truth Conditions**
   The above exercise is one in translating natural language concepts into the syntax of propositional logic, but what about finding out the *meaning* of these utterances within a knowledge base? The notion of meaning in logic can be restricted to one of the truth conditions of a series of logical statements (an argument). An argument cannot be sound if its premises are *all* true and its conclusion false; a necessary condition of sound reasoning is that from truths only truths follow. In making a logical argument that we claim to be valid, we assert that the premises entail the conclusion, i.e. assert that the conclusion holds.

   This exercise concerns checking the validity of logical arguments. First follow the example (a). Here we create three propositional logic statements from English sentences, then use a truth-table to prove whether the conclusion follows given the premises, or a logically valid argument (i.e. to check whenever both premises are true, the conclusion is true). If the conclusion is sound, this means the whole argument statement is either satisfiable (*contingent*) (true in at least one model, that is in one row of the table) or a *tautology* (true in every model/row). On the other hand if it is not true in any model/row, it is a *contradiction.*

(a) (Example- just follow the steps)
If Tim Berners-Lee works, there will be an informatics revolution
There will not be an informatics revolution
Therefore, Tim Berners-Lee does not work

(Step 1: translation into propositional logic premises and conclusion)
premise 1: $P \to Q$
premise 2: $\neg Q$
conclusion: $\neg P$

(Step 2: create a truth table with all possible truth values (models) of the component propositions, and calculate the truth value of the premises and conclusion for each model)

		premise 1	premise 2	conclusion	formula
P	Q	$P \to Q$	$\neg Q$	$\neg P$	$((P \to Q) \wedge \neg Q) \to \neg P$
T	T	T	F	F	T
T	F	F	T	F	T
F	T	T	F	T	T
F	F	**T**	**T**	**T**	T

(Step 3: determine whether the conclusion is sound and what kind of overall formula we have)
We are only interested in the model (table row) where the premises are both true to check the soundness of the reasoning (*validity* of the argument). We check the validity of the argument in the table. It indeed shows whenever both premises are true (in the bottom row), the conclusion is true, so the reasoning is sound (argument is valid). The overall formula $((P \to Q) \wedge \neg Q) \to \neg P$ is not a contradiction, and as it is in fact true in every model, it can be called a tautology (see the last column of the table).

(b) Follow the steps above to (i) determine whether the conclusion is sound given the premises in the below statements, and (ii) state whether the overall formula is a contradiction, tautology or contingent. If it is contingent, state the truth conditions under which the conclusion can be true (this will emerge from the truth table):

If Tim Berners-Lee works, there will be an informatics revolution
Tim Berners-Lee doesn't work
Therefore, there will not be an informatics revolution