Data Semantics - Exercises 1 - Solutions

1. **Definitions**

   Satisfiable: There is some model that makes the formula true.
   Falsifiable: There is a model in which the formula is false.
   Unsatisfiable/Contradiction: There is no model that satisfies the formula.
   Valid/Tautology: The formula is true in all models.

2. **Propositional Logic**

   (a) (example)

   (b) $P \vee Q$
   where P = "John is in the library" and Q = "John is out for lunch"

   (c) $\neg P$
   where P = "John dances"

   (d) $P \vee Q$
   where P = "John dances" and Q = "Clara dances"

   (e) $P \to Q$
   where P = "Bert dances" and Q = Ernie sings

   (f) $Q \to P$
   where P = "Bert dances" and Q = Ernie sings (if keeping the assignment the same as above)

   (g) $Q \to R$
   where Q = Ernie sings and R = "Tim Berners-Lee will not come to the party" (if keeping the assignment the same as above)

   (h) $Q \leftrightarrow P$ or $P \leftrightarrow Q$ (they are equivalent)
   where P = "Bert dances" and Q = Ernie sings (if keeping the assignment the same as above)

3. **Proof with Truth Table**

   De Morgans equivalence $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg(P \vee Q)$	$\neg P \wedge \neg Q$
1	1	0	0	1	0	0
1	0	0	1	1	0	0
0	1	1	0	1	0	0
0	0	1	1	0	1	1

   As you can see on the table, the values on columns 6 ($\neg(P \vee Q)$) and 7 ($\neg P \wedge \neg Q$) proves that De Morgans equivalence is correct.

4. **Meaning as Truth Conditions**

(a) If Tim Berners-Lee works, there will be an informatics revolution
Tim Berners-Lee doesn't work
Therefore, there will not be an informatics revolution

(Step 1: translation into propositional logic premises and conclusion)
premise 1: $P \to Q$
premise 2: $\neg P$
conclusion: $\neg Q$

(Step 2: truth table)

		premise 1	premise 2	conclusion	formula
P	Q	$P \to Q$	$\neg P$	$\neg Q$	$((P \to Q) \land \neg P) \to \neg Q$
T	T	T	F	F	T
T	F	F	F	T	T
F	T	**T**	**T**	**F**	F
F	F	T	T	T	T

(Step 3: determine whether the conclusion is sound and what kind of overall formula we have)
(i) As the table shows, when they are both true in the third row, the conclusion is false, so the reasoning is not sound. (ii) The overall formula $((P \to Q) \land \neg P) \to \neg Q$ is not a contradiction however, as it is true in the final row in three cases. It is *contingent* on either Q being false or P and Q both being true.