

# Introduction to Cryptography

ECE568 – Lecture 7  
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# Outline

## Introduction to Cryptography

- Why is it useful?
- Four important properties

## Basic Ciphers

- Kerckhoffs' principle
- Substitution ciphers
- Poly-alphabetic and periodic ciphers
- One-Time Pad and Vernam ciphers

## Stream Ciphers versus Block Ciphers



# Introduction to Cryptography

Confidentiality,  
Integrity, Authentication,  
Non-Repudiation

# Cryptography

Cryptography literally means “secret writing”

- Cryptography is an old field: existed far before computers, possibly over 2000 years
- Capable of much more than keeping data secret; its main use is in protecting **stored** or **transmitted** data. Indispensable tool in security

Cryptography is a huge field

- We will focus on the concepts and key attributes of commonly used cryptographic algorithms, without dealing with analysis of strength, etc.

# Uses of Cryptography

Cryptography helps establish four properties for data:

## Confidentiality

- Secrecy of the data
- This is provided by algorithms called **ciphers**

## Integrity

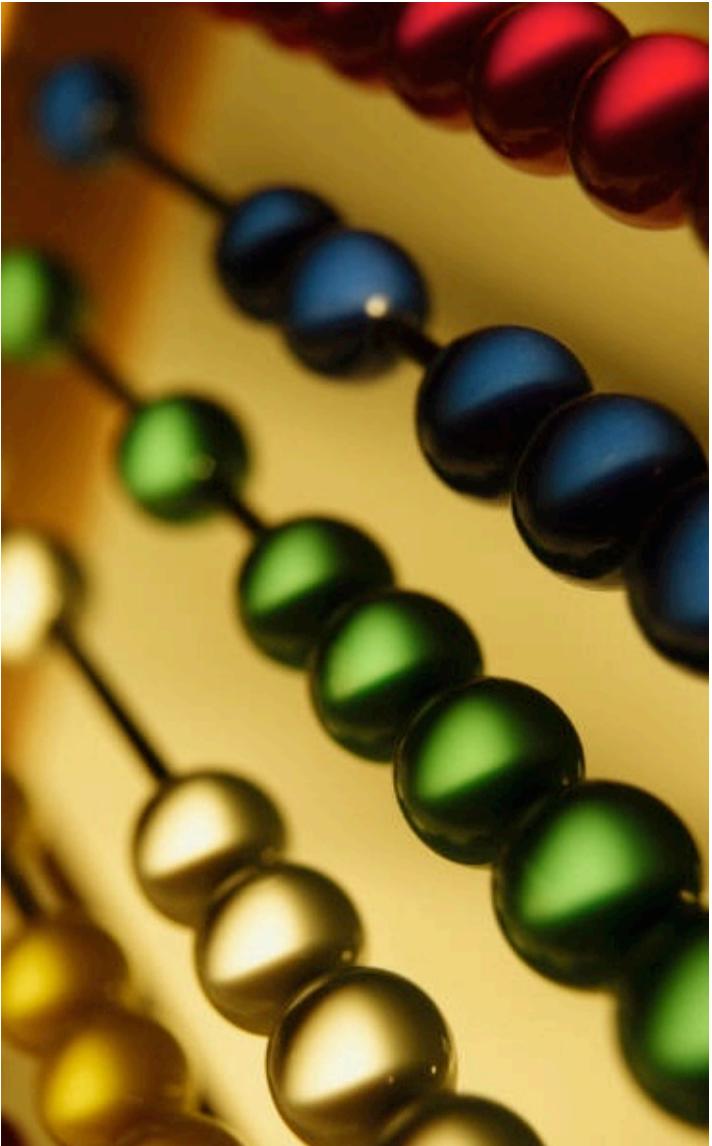
- The trustworthiness of the data
- Provided by **hashes**

## Authentication

- Allows a principal (user or machine) to prove their identity, or the origin of a piece of data
- Provided by **signatures** and **Message Authentication Codes** (MACs)

## Non-repudiation

- Prevents a principal from denying they performed an action
- Achieved with the help of a **trusted third party**

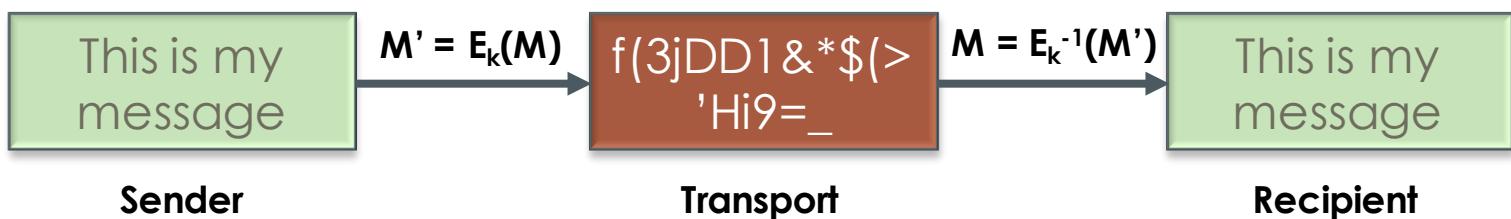


## Basic Ciphers

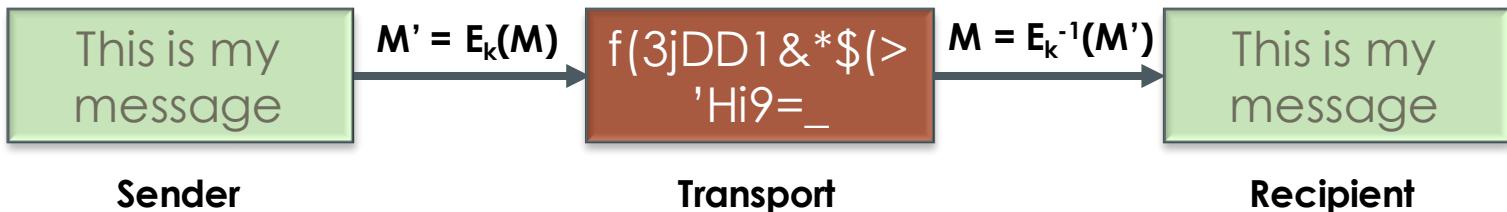
Kerckhoffs' principle,  
substitution ciphers,  
polyalphabetic ciphers,  
Vernam ciphers

# Ciphers

A **cipher** is an algorithm that obfuscates information so that it seems random to anyone who does not possess special information called a **key**



# Ciphers



Ciphers are based on a class of functions called **trapdoor one-way** functions:

- A **one-way** function is a function that is easy to compute, but whose inverse is difficult to compute
- The **trapdoor** means that, given special information (the key), the inverse becomes easy to compute

# Ciphers

**Interesting fact:** It has never been proven (or disproven) that one-way functions exist.

- Ciphers are based on functions that are believed to be one-way because no one has ever shown an easy way of computing their inverse
- A formal proof that a one-way function exists would also prove that  $P \neq NP$

Whichever one-way function we choose, the **function itself** must not be the critical secret...

# Kerckhoff's Principle

The security of any given encryption system must depend only on the secrecy of the key, **K**, and not on the secrecy of the algorithm

- Algorithms are hard to change: compiled into software, wired into circuits
- Need to be able to rely on using an algorithm for a long time
- Algorithms are easily disclosed: must be distributed in every program / device that might exchange information

# Mifare Hack



Mifare RFID smartcards are used in a variety of critical applications (access control, dozens of major public transit systems, etc.)

- Trusted for audit, fare payment, etc.
- In 2008, a team of Dutch researchers reverse-engineered the proprietary encryption used for transmitting data, and hacked the cards

<http://www.ru.nl/ds/research/rfid/>

# Mifare Hack

“The security of Mifare Classic is terrible. This is not an exaggeration; it's kindergarten cryptography. Anyone with any security experience would be embarrassed to put his name to the design. [Mifare] attempted to deal with this embarrassment by keeping the design secret.”

- Bruce Schneier

Manufacturer sued to keep the research secret; Dutch court ruled that the potential “damage to [Mifare] is not the result of the publication of the article, but of the production and sale of a chip that appears to have shortcomings.”

# Ciphers

Two good, well-researched candidates for one-way functions are **factoring** and **discrete log**:

- **Factoring**

Suppose  $z = (x \cdot y)$

Given **z**, find **x** and **y**.

- **Discrete log**

Suppose  $z = (x^y \text{ mod } m)$

Given **z**, **x** and **m**, find  $y = (\log_x z) \text{ mod } m$

# Caesar (Shift) Cipher

When Caesar mounted his campaign against the Gauls (modern France), he wanted to communicate with his troops securely

- He contrived a very simple cipher:
  - Take each letter, and replace it with the letter shifted 3 letters to the right in the alphabet
  - If there are no more letters, wrap around to the beginning of the alphabet
  - This is similar to modern day **rot13** used for simple obfuscation
  - Decryption is just the inverse
- This type of cipher is also called a **shift** cipher

# Substitution Ciphers

The shift cipher is an instance of a class of ciphers called **substitution** ciphers

- Each plaintext letter is replaced with exactly one ciphertext letter
- The key is the mapping between plaintext letters and ciphertext letters

**Example:** Assume a five letter alphabet  $\{ABCDE\}$  and a shift of 2:  $\{DEABC\}$

$$A=D, B=E, C=A, D=B, E=C$$

# Attacks on Ciphers

When attacking a cipher, may be several goals:

- Get the plaintext corresponding to a ciphertext
- Get the key (and possibly the algorithm if it's not public)

## Brute-force attack

- Try all possible keys on some ciphertext until output is an intelligible plaintext

## Cryptanalysis

- Aim is to do better than brute-force attack
- Relies on nature or characteristics of the algorithm
- Some knowledge of plaintext characteristics
- May use samples of plaintext-ciphertext pairs



# Brute-Force Attack

Key Size	Number of Possible Keys	Time Required at $10^6$ tests/sec	Time Required at $10^{12}$ tests/sec
32 bits	$2^{32} = 4.3 \cdot 10^9$	36 minutes	2.2 ms
56 bits	$2^{56} = 7.2 \cdot 10^{16}$	1142 years	10 hours
128 bits	$2^{128} = 3.4 \cdot 10^{38}$	$5.4 \cdot 10^{24}$ years	$5.4 \cdot 10^{18}$ years
168 bits	$2^{168} = 3.7 \cdot 10^{50}$	$5.9 \cdot 10^{36}$ years	$5.9 \cdot 10^{30}$ years
26 characters (permutation)	$26! = 4 \times 10^{26}$	$6.4 \times 10^{12}$ years	$6.4 \cdot 10^6$ years

# Cryptanalysis

When analyzing ciphers, cryptographers categorize the strength of a cipher against several types of attacks (ranked from hardest to easiest):

## Ciphertext-Only

- Adversary only has ciphertexts (encrypted messages)

## Known-Plaintext

- Adversary has some number of plaintext and ciphertext pairs
- The more pairs it takes to break cipher, the stronger it is

## Chosen-Plaintext / Chosen-Ciphertext

- Adversary can pick a plaintext and get corresponding ciphertext or vice-versa
- Adversary can **adaptively** select plaintexts or ciphertexts that help break the cipher

How strong is a substitution cipher?

# Breaking Substitution Ciphers

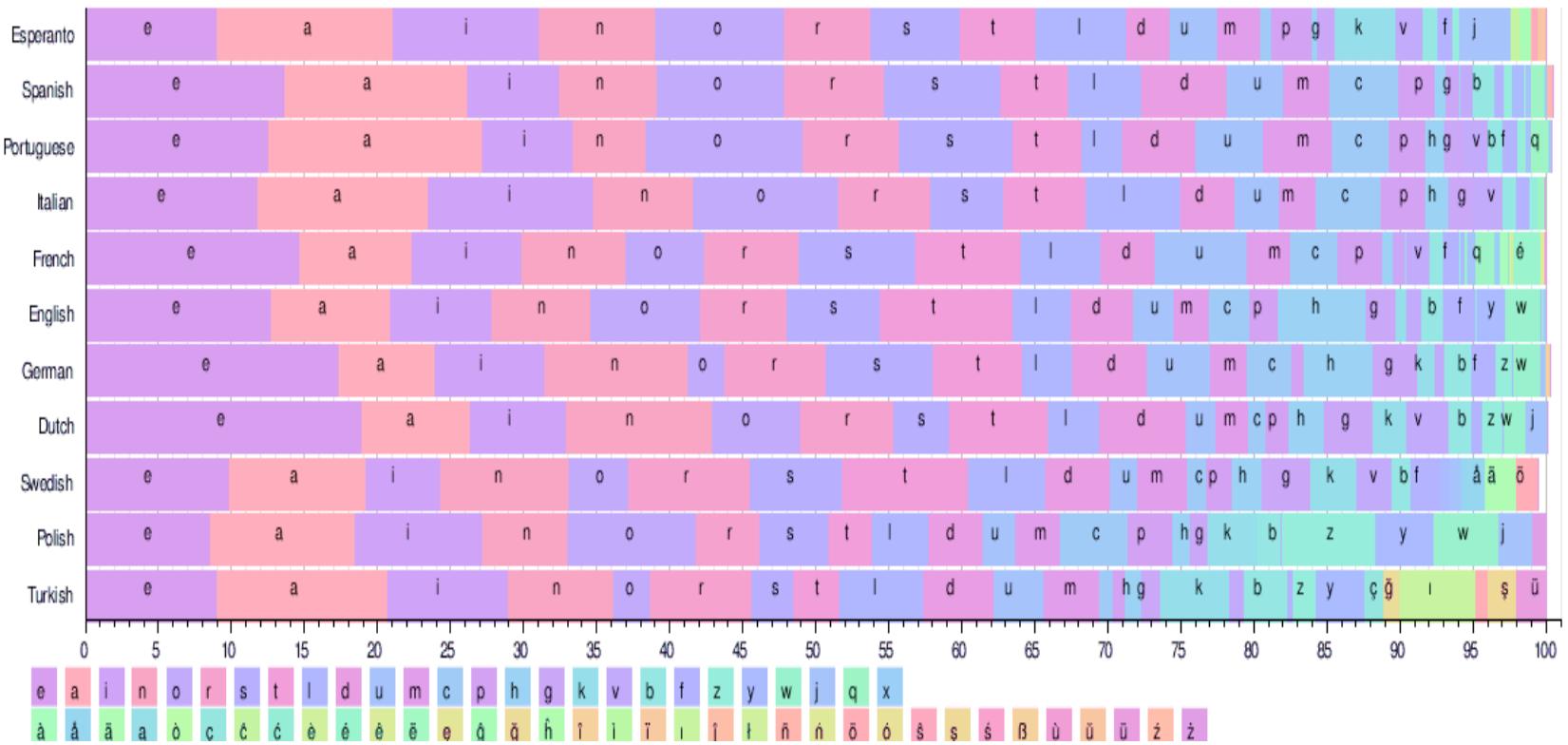
These ciphers are very easy to break with analysis:

- The weakness is that every letter in the plaintext alphabet always gets encrypted to the **same** letter in the ciphertext alphabet

If the attacker knows the original message is in English:

- Probability distribution: **E=13%**, **T=9.3%**, **A=7.3%**, ...
- Most common digraphs: **TH**, **HE**, **IN**, ...
- Attacker can perform **frequency analysis** on the ciphertext to identify and decode common letters, then match against common English words to recover the plaintext and eventually the key
- These attacks are easy enough to be done by hand

# Breaking Substitution Ciphers



# Improving Substitution Ciphers

Substitution ciphers do not hide frequency information because every plaintext letter always encrypted to the **same** ciphertext letter

- One way to improve the cipher is to use a **Polyalphabetic Cipher**
  - Instead of having one mapping, have a set of  $n$  mappings, and change the mapping with every character
  - When mappings are used up, then repeat with the first one

# Polyalphabetic Ciphers

**Alphabet:**

ABCDE

**Key #1:**

BEDAC

**Key #2:**

ACBDE

**Key #3:**

DACBE

$$E("BED") = EEB$$

$$E("ABACADA") = BCDDABB$$

# Polyalphabetic Ciphers

Because of this repetition, these polyalphabetic ciphers are sometimes called periodic ciphers

- If the attacker knows, or can somehow guess the period, an attack is possible
- For small  $n$  (number of keys), only incrementally harder than a plain substitution cipher, but for large  $n$ , much more difficult
- The attack requires more ciphertext examples, but becomes easier if the adversary has some plaintext/ciphertext pairs

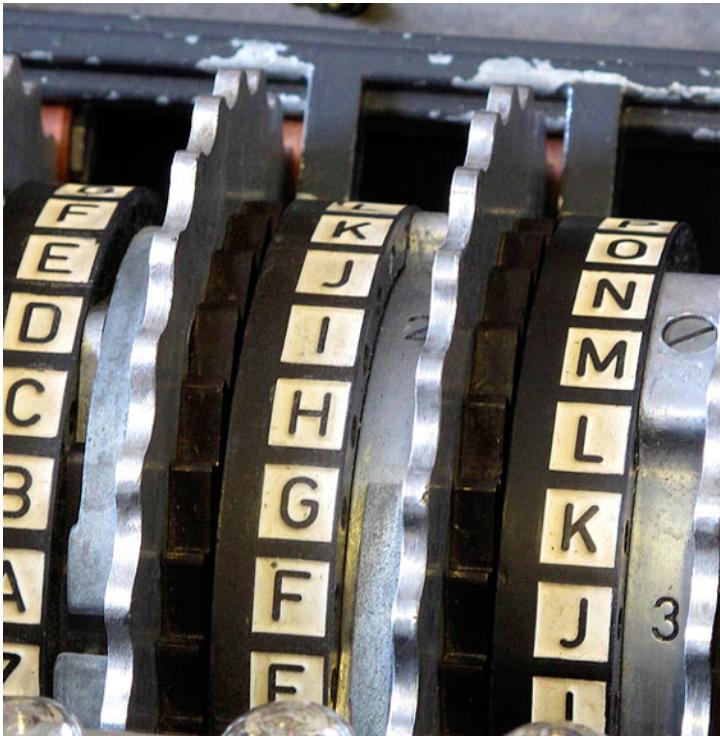
# Enigma Machine

Famous early large-scale mechanization of secrecy

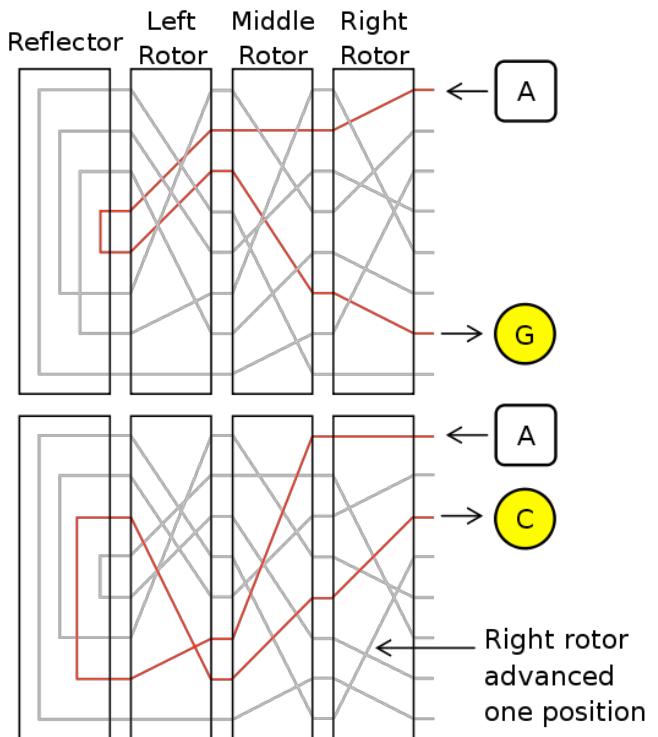
- Series of rotors produce a polyalphabetic cipher with very large  $n$
- Permutation mapping of alphabet that changes with each keystroke



# Enigma Machine



Source: Wikimedia Commons



# One-Time Pad or Vernam Cipher

A **One-Time Pad** (OTP), also called the **Vernam Cipher** after its creator, is a special type of polyalphabetic cipher that never repeats

- A random substitution is used for every character
- Think about it as using an infinite number of keys

# One-Time Pad

- The cipher requires the key to be the same length as the message to be encrypted
- The ciphertext is created by computing the bitwise XOR of the plaintext and the key at the binary level
  - A plaintext bit is flipped when key bit is 1
  - A plaintext bit remains the same when key bit is 0

# One-Time Pad

XOR

	0	1
0	0	1
1	1	0

**Plaintext:** 0101

**Key:** 1001

**Encrypted:** 1100

# Properties of One-Time Pad

If the key is well chosen (i.e., random), the ciphertext is the plaintext with randomly flipped bits

- For a message containing  $n$  bits of information, OTP adds exactly  $n$  bits of randomness, creating a completely random ciphertext
- **Theoretically unbreakable**

# Disadvantages of One-Time Pad

- Key length = message length
  - Key overhead of 100% is generally not acceptable
- Each key can be used **once** (hence the name)
  - Key must be sent separately, for every message sent (serious problem for distribution infrastructure)
  - Synchronization problem if messages are lost or reordered
  - If any key is used to encrypt more than one message, security is reduced significantly
  - Impractical for most applications
- Cipher is **malleable**
  - Bit flip in ciphertext, flips only one bit in plaintext
  - Requires combining with integrity check to avoid tampering
- Need a good source of randomness for the key

# Strength of One Time Pad

How strong is OTP against the following attacks?

- **Ciphertext-only**

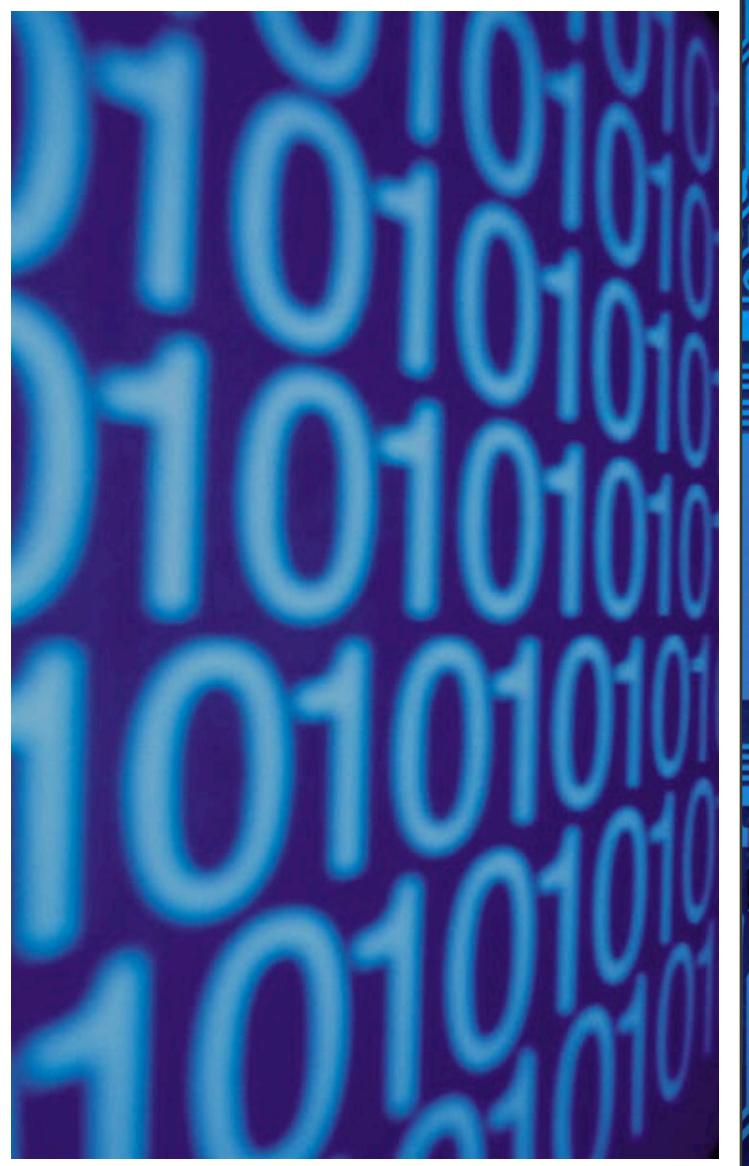
- Proven to be information theoretically secure
- If used properly, impossible to break

- **Known-Plaintext**

- Very weak
- Just XOR CT with PT to reveal the key
- Only need one pair
- Of course, key is not supposed to repeat

- **Chosen-Ciphertext/Plaintext**

- Since it's weak against a Known-CT/PT attack, it is weak against this attack



# Stream Ciphers vs. Block Ciphers

# Practical Ciphers

- Fixed length keys that are much shorter than the message
  - Do not depend on message length
- Efficient for encryption and decryption
- Ciphertexts should be computationally difficult to decrypt without the key
  - Note: “computationally difficult” is a moving target, as computers are getting more and more powerful
- Two type of ciphers
  - Symmetric key
  - Public (assymmetric) key

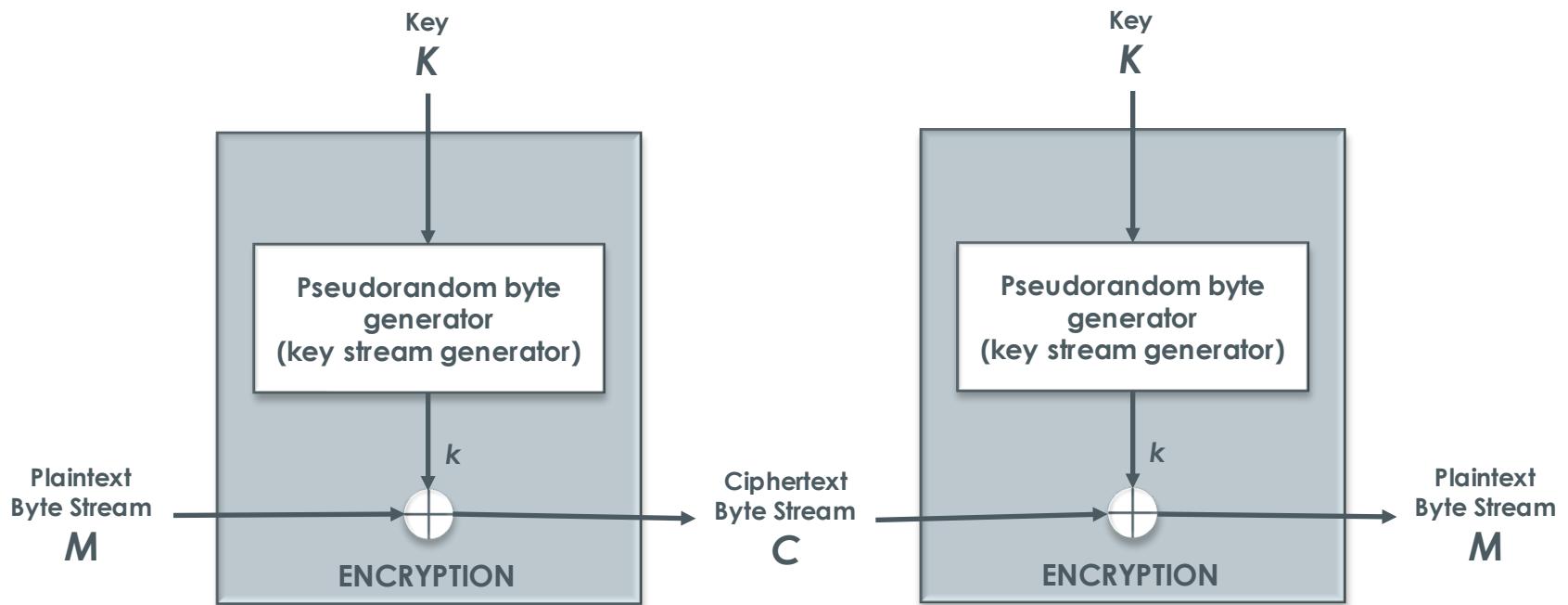
# Symmetric Key Ciphers

- Symmetric key ciphers use the same key to encrypt and decrypt data
- There are two types of symmetric key ciphers
  - Stream Ciphers
  - Block Ciphers

# Stream Ciphers

- These are similar to OTP's
  - Instead of truly random key bits, a key is used to generate a pseudo-random sequence of bits
  - The bits are then XOR'ed with the plaintext
- Plaintext is encrypted a bit at a time, making it useful for streaming applications
  - e.g., voice or video
- These ciphers suffer from synchronization problems, if any bits are lost, the entire stream may be corrupted

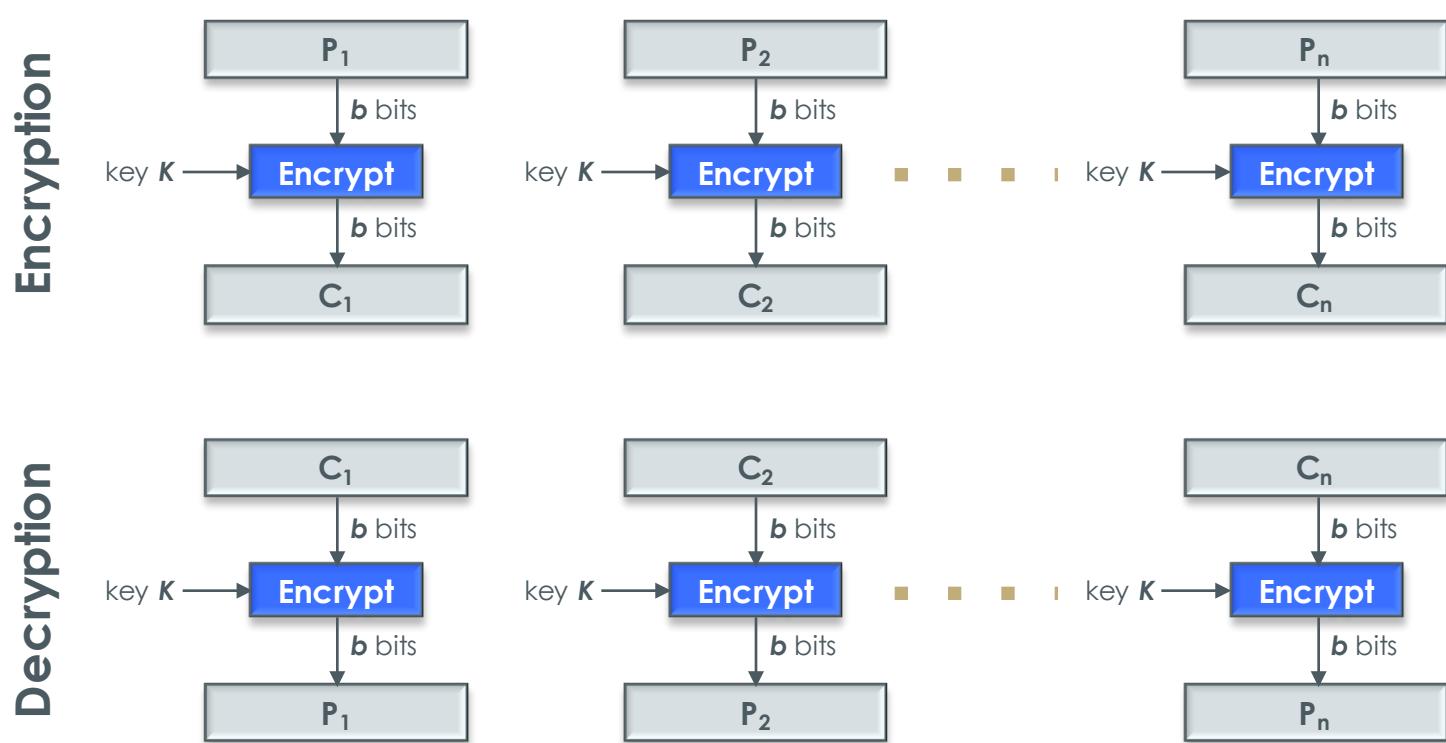
# Stream Ciphers



# Block Ciphers

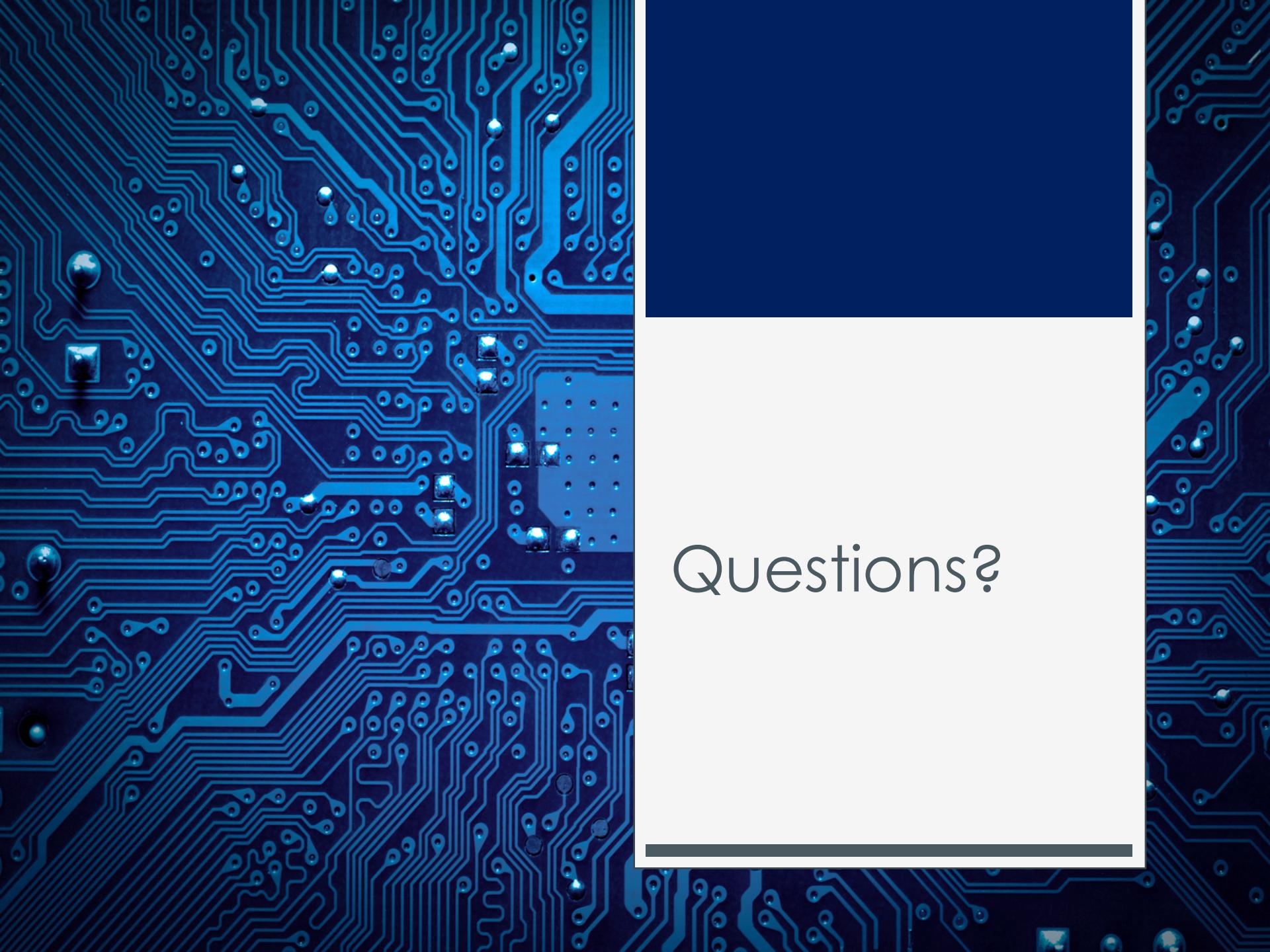
- Encrypt a block of plaintext at a time
- Usually 64 bits or a multiple
- Plaintext is divided into blocks and each is encrypted separately
- The last block might need to be “padded” to make it a full block length

# Block Ciphers



# Stream Ciphers vs. Block Ciphers

- Stream ciphers are generally simple and very fast
- Block ciphers are more common than stream ciphers
  - Unfortunately, the reasons are not entirely logical
  - In the past, most stream ciphers were proprietary, so they could not be analyzed, and therefore people could not be confident of their security
  - In contrast, there are many publicly available and well-studied block ciphers



Questions?