

## User Manual for Running LINVER Stochastic Simulations in Matlab

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LINVER stochastic simulations provide a way to gauge uncertainty about future economic outcomes. To run such simulations, users specify several key parameters that have an important influence on the model's dynamics, such as how monetary policy responds to changes in economic conditions. Conditional on these assumptions, many different possible future paths for the economy are generated by simulating the model repeatedly, with each path subject to randomly drawn shocks. The simulated paths are then used to generate estimates of means, standard deviations, and other moments that summarize the range of future possibilities for such factors as the unemployment rate, inflation, and interest rates. Importantly, the distributional statistics computed by the routines included in this package are intended to gauge uncertainty about future outcomes over the longer term after the effects of initial conditions have worn off, and not uncertainty about the near-term economic outlook.<sup>1</sup>

The Matlab directory of the LINVER package contains various programs and files that run stochastic simulations of the model in Matlab under different assumptions for monetary and fiscal policy, the manner in which agents form their expectations for the future, and other aspects of the economy's dynamics. The random shocks applied in these simulations are constructed to be consistent with previous disruptions to the economy as manifested in the historical residuals of the main behavioral equations of the FRB/US model. Nevertheless, the uncertainty estimates produced by LINVER stochastic simulations may differ from the variability seen historically for several reasons. First, users can condition the simulations on a monetary policy that differs materially from that employed in the past. Second, the degree to which the ELB is assumed to bind in the simulations may be more pronounced than that seen on average over the past few decades because of declines over time in the trend rate of inflation and the equilibrium real interest rate. Third, the historical equation residuals from which shocks are drawn may not account for all the historical variation in economic conditions because they exclude important low-frequency influences, such as secular changes in the size of the government sector, the composition of aggregate income, and the openness of the economy.

The programs and files can run stochastic simulations of LINVER in its basic linear form but their primary function is to provide efficient routines for solving LINVER when policy-related nonlinearities are present, such as the effective lower bound (ELB) constraint on the nominal federal funds rate. The user must install the Dynare freeware package to be able to run these routines, as they take as their main input the decision-rule matrices created when Dynare processes LINVER. Dynare documentation is available at [www.dynare.org/manual/](http://www.dynare.org/manual/) and the package can be downloaded at [www.dynare.org/download/](http://www.dynare.org/download/).

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<sup>1</sup> With suitable modifications to the programs in the Matlab directory, stochastic simulations can be used to estimate confidence intervals and other statistics for conditions over, say, the next 10 years, as is done by Chung et al (2019) and Arias et al (2020). Such a modified routine would require: a detailed medium-term forecast for real activity, inflation, interest rates, and other factors; explicit assumptions about the *unconstrained* monetary policy rule, the ELB constraint, and any threshold conditions for liftoff that underly the projected medium-term path of the federal funds rate; and any effects of quantitative easing on the outlook and any trade-off between those effects and the threshold conditions. Although medium-term "consensus" economic projections can be constructed from surveys, identifying the various policy-related factors underlying those projections poses a major challenge.

The programs require users to set a parameter that determines how agents in LINVER form their expectations of the future. Under one setting, all agents have forward-looking model-consistent (MC) expectations, while under another all expectations are based on the average historical dynamics of the economy as manifested in the predictions of estimated limited-information VAR models. Other allowed settings assume that financial market participants (along with wage and price setters, if desired) have MC expectations, and that all others have VAR-based expectations. The routines for solving LINVER when policy-related nonlinearities are present can be used with any of these expectational assumptions, but their applicability is broader and their power more pronounced when at least some agents have MC expectations. As a result, this document contains only occasional references to LINVER simulations run with purely VAR expectations.

In the MC case, computing the adjustments needed to prevent ELB violations is challenging because they depend on the paths of inflation, resource utilization and interest rates expected by agents, where those paths in turn depend on the adjustments. Because of this simultaneity, iterative procedures are used to solve for the current and future adjustments to the expected path of the federal funds rate that, at each point in time, ensures its consistency with the ELB. The methodology used to compute these adjustments exploits the parsimonious procedure for imposing expected shocks on a linear model developed by Bodenstein, Guerrieri, and Gust (2013); see the **appendix** for details.

In addition to the ELB constraint, the solution routines are able to handle two other sources of nonlinearity related to monetary policy. The first concerns the rule used to set the federal funds rate in simulations, which users must select from a list of options drawn from the economic literature. Most of the available options, such as the Taylor rule, respond linearly to economic conditions and thus require no special treatment beyond that needed to impose the ELB constraint. However, the list includes two rules — the asymmetric average inflation targeting rule discussed in Arias *et al* (2020) and the asymmetric unemployment-response rule discussed in Chung *et al* (2019) — that are inherently nonlinear. The second occurs when users elect to impose unemployment and inflation “threshold” conditions that must be satisfied before the federal funds rate is allowed to lift off from the ELB. In both cases, additive adjustments beyond those required to impose the ELB constraint are made as needed to the projected path of the federal funds rate to ensure consistency with a nonlinear rule’s prescriptions and any threshold conditions.

As discussed in the **dynamic instabilities** section, LINVER can become dynamically unstable when the ELB constraint binds severely under some monetary policy rules, to the point that stochastic simulations cannot be used to reliably estimate the volatility of the economy without adjustments to the model. The simulation routines provide users with several ways to eliminate this problem. One of these is the application of asymmetric “emergency” fiscal shocks to prevent resource utilization from falling to extremely low levels when the ELB constrains monetary policy. This stabilizing mechanism is discussed in the **extreme case fiscal stabilization (ECFS)** section. Other options include modifying the default equations of LINVER by increasing the countercyclicality of fiscal policy or reducing the countercyclicality of the term premiums embedded in long-term interest rates.

The simulation routines are reasonably fast. Using a run-of-the-mill laptop, it takes about seven minutes to simulate 5000 stochastic outcomes, each of 200 quarters length, when some or all agents are assumed to have MC expectations, the threshold options are not employed, and a frequently binding ELB constraint is imposed on the prescriptions of a linear monetary policy rule for 15 years into the future. Runtimes roughly double when the simulations are run using one of the nonlinear policy rules without thresholds. But even in the worst case, in which threshold conditions are combined with asymmetric average inflation targeting, 5000 stochastic simulations can be completed in roughly an hour. In addition, the simulation routines are always able to impose the ELB constraint and other nonlinearities reliably on LINVER solutions, provided ECFS or one of the other stabilizing options is used.

## Overview of the stochastic-simulation procedure

The program *stochsims* is the parent process used to generate stochastic simulations of LINVER conditional on user-supplied assumptions about monetary policy and other factors. This program runs the following sequence of Matlab commands and scripts:

1. The first part of the *stochsims* script contains the user-specified values of required parameters and overrides to the default settings of other parameters. In particular, the user must specify how agents' expectations are formed, the interest rate rule used by monetary policymakers, and whether the ELB constraint is imposed. If the last is "yes", then the user must also specify the value of the ELB. More detailed information is presented below in the **parameters** section.
2. The script *make\_parameters* verifies the validity of the user-specified parameter values and sets any other parameters to their default values.
3. The script *make\_runmod* constructs a Dynare *mod* file that is consistent with the set of parameter values and calls Dynare to parse it. To do this, the script first loads a text file that contains the base specification of LINVER for the selected expectations option. The base specification is written in the Dynare model format and includes endogenous and exogenous variable declarations, equations, and coefficients stored as internal parameters. The script then edits the text file so that it conforms with all other parameter settings, such as how far into the future additive adjustments are applied to the expected path of the federal funds rate to prevent it from violating the ELB constraint. Finally, the edited model file is parsed by calling Dynare. The user must specify the location of the Dynare code with either the Matlab "set path" dialog or the "addpath" command.
4. The script *make\_matrices* retrieves the Dynare-generated "decision rule" matrices and constructs other matrices and namelists used in the stochastic simulation calculations. The latter includes matrices used to project the future paths of the federal funds rate and other series conditional on the state of the economy in the prior quarter, the shocks to behavioral equations hitting the economy in the current quarter, and any anticipated adjustments to the expected path of the federal funds rate implied by the nonlinear constraints. The script also constructs the matrix of derivatives of the federal funds rate projected at time  $t+j$  with respect to an adjustment to the federal funds rate at time  $t+k$ .

5. The script *make\_shocks* reads historical FRB/US behavioral equation residuals from a CSV text file and then generates a matrix of random shocks consistent with those residuals for application during the stochastic simulations. As discussed in the **sampling methods** section, the user has several options for how to generate these shocks, including the method used to sample from the set of historical residuals.
6. The script *stochloop* simulates *nreplic* separate outcomes for the economy, each of *nsimqtrs* quarters length, subject to random economic shocks, where *nreplic* and *nsimqtrs* are parameters that can be specified by the user.
  - a. If the ELB is imposed or a nonlinear rule is used, and if thresholds are not imposed, *stochloop* calls the script *addscal* to compute any required adjustments to the future path of the federal funds rate expected at each point in time, conditional on the state of the economy at that time. If ECFS is used, the *addscal* script also computes emergency fiscal shocks to prevent the output gap from falling to excessively low levels.
  - b. If thresholds are imposed, *stochloop* calls the script *addscal\_thresh* to search for the appropriate liftoff point from the ELB along the projected future path of the federal funds rate at each point in time. During its execution, *addscal\_thresh* calls *addscal* as needed to impose the ELB constraint, the prescriptions of the nonlinear policy rule (if one is selected), emergence fiscal shocks (if ECFS is used, and a trial liftoff date.

The **appendix** provides information on the methodology used by the *stochloop*, *addscal*, and *addscal\_thresh* scripts to compute the constraint-consistent additive adjustments.

7. The script *summarize\_results* reports results from the stochastic simulations, including means, standard deviations, and other distributional statistics for selected variables, as well as information about the model, the calculation of constraint-consistent adjustments, and convergence.
8. Finally, the script *save\_output* stores stochastic simulation results for later examination of processing, if this option has been selected by the user.

## Parameters

Prior to running the stochastic simulations, the user must define the following parameters at the top of the *stochsims* script:

- *expvers* — specifies how expectations for future economic conditions are formed in the model. The allowed settings are:
  - “var” — all expectations are based on the predictions of a small VAR model.
  - “mcap” — financial market expectations are model consistent (i.e., “rational”), while other expectations are based on the predictions of a small VAR model.

- “mcapwp” — financial market expectations and the expectations influencing wage and price formation are model consistent, while other expectations are based on the predictions of a small VAR model.
- “mceall” — all expectations are model consistent.

Agents with model-consistent expectations (MCE) form their expectations with a full understanding of the future implications of any shock once it hits, as predicted by LINVER subject to any nonlinear constraints. However, MCE agents do not have perfect foresight and so model-consistent expectations are generated assuming no future shocks.

- *mprule* — specifies the monetary policy rule used to set the federal funds rate in simulations, abstracting from any constraints imposed by the ELB or threshold conditions for liftoff. Allowed options are “tay”, “intay”, “fpt”, “infpt”, “ait”, “rw”, “kr”, “adur”, and “aaait”; see the **monetary policy rules** section for the definitions of these rules.
- *elb\_imposed* — specifies whether the ELB constraint is imposed on the expected path of the federal funds rate in simulations must be set to either “yes” or “no”.
- *elb* — the value of the effective lower bound in the simulations. If *elb\_imposed* = “yes”, the user must specify this parameter; otherwise, the user can omit this command. Because the baseline values of all variables in LINVER equal zero, setting *elb* equal to -2 in simulations would correspond to a real-world value of the equilibrium real federal funds rate ( $R^*$ ) equal to zero if the real-world trend inflation rate equals 2 percent; setting *elb* equal to -4 would correspond to  $R^*=2$  in the real world.

In addition to the mandatory parameter settings listed above, the user has the option of overriding the default settings of other parameters related to the model’s structure or the manner in which the simulations are run. The following is the list of the optional parameters, together with their allowed settings and their default values:

- *elbqtrs* — the number of quarters, from  $t$  to  $t+elbqtrs-1$ , that the ELB constraint is imposed on the path of the federal funds rate expected by MCE agents at time  $t$ ; the default is 61. (For agents with VAR-based expectations, the ELB only constrains their expectations in the current quarter.) The default value turns out to be sufficiently large that typically almost all projected paths of the federal funds rate are unconstrained by the end of the interval. In addition, setting *elbqtrs* to a higher value slows convergence and can lead to solution difficulties.
- *asymqtrs* — the number of quarters, from  $t$  to  $t+asymqtrs-1$ , that the path of the federal funds rate expected by MCE agents at time  $t$  is constrained to equal the prescriptions of either nonlinear policy rule, if selected. The default setting is *elbqtrs* if the ELB is imposed, and 61 otherwise. For quarters beyond *asymqtrs-1*, the path of the federal funds rate expected by MC agents conforms to the prescriptions of the inertial Taylor rule.<sup>2</sup> If

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<sup>2</sup> Beyond the current quarter, agents with VAR-based expectations always expect the future path of the federal funds rate to conform with the average historical “reaction function” embedded in the estimated VAR model.

the ELB constraint is imposed on one of the nonlinear rules, *asymqtrs* must be no greater than *elbqtrs*.

- *uthresh\_imposed* and *uthresh* — parameters governing whether labor slack, defined as the difference between the unemployment rate and the natural rate, must fall below a certain level before liftoff from the ELB is allowed. The first parameter determines whether the threshold condition is imposed during the simulations (the default is “no”) and the second parameter determines the value of the threshold that must be crossed (the default is zero). Thresholds are allowed only if the user also imposes the ELB constraint. See the **thresholds** section for further details.
- *pithresh\_imposed*, *pithresh*, and *pithresh\_var* — parameters governing whether inflation must rise above a specified value before liftoff from the ELB is allowed. The first parameter determines whether the threshold condition is imposed during the simulations (the default is “no”), the second parameter determines the value of the threshold that must be crossed (the default is zero), and the third parameter defines the inflation measure used to define the threshold (the default is “picx4”). Available settings for *pithresh\_var* are “pic4”, “pic12”, “pic20”, “picx4”, “picx12”, and “picx20”, where the “pic” and “picx” prefixes denote total and core PCE inflation respectively, and the suffix denotes the number of quarters over which inflation is averaged. Thresholds are allowed only if the user also imposes the ELB constraint. See the **thresholds** section for further details.
- *maxfgq* — a parameter that governs how many quarters into the future MC agents expect monetary policymakers to remain committed to the threshold strategy; its default value is *elbqtrs*. See the **thresholds** section for further details.
- *ecfs\_option* and *ecfs\_floor* — parameters governing whether extreme-case fiscal stabilization is employed to prevent simulated economic downturns from becoming excessively severe. When the ECFS option is used, an additional fiscal shock is applied to federal government spending to prevent the project output gap from falling below a specified floor. The first parameter determines whether the option is used and the second specifies the floor imposed on the output gap. If the ELB is imposed, the default settings are “yes” and -15, respectively. See the **ECFS** section for further details.
- *ctp\_option* — a parameter that governs what cyclical effects are included in the equations for the Treasury bond term premiums RG5P, RG10P, and RG30P used in stochastic simulations. If equal to 0 (the default setting), no modifications are made to the standard equations; as a result, LINVER can become dynamically unstable in circumstances in which the ELB is expected to bind for long periods. If the user instead sets this parameter equal to 1, the cyclical effects embedded in the standard equations are suppressed and the model is always stable. The model is also always stable if the user sets *ctp\_option* equal to 2; in this case, the standard term premium equations are replaced by ones estimated using an alternative specification whose goodness of fit is about the same as that of the standard equations. See the **dynamic instabilities** section for further details.

- *tax\_gamma* — a parameter that governs the countercyclicality of the trend personal tax rate. Its default value equals .00075, the estimated historical value, when *expvers* is set to “var”, “mcap”, or “mcapwp”. If *expvers* is set to “mceall”, however, *tax\_gamma* is raised by default to .00130 to ensure dynamic stability of the model under all policy rules using the standard term premium equations, provided the ECFS is used. Because other methods are available to ensure model stability, users have the option of overriding these defaults to facilitate comparison of stochastic simulation results across different expectational versions of the model. See the **dynamic instabilities** section for further details.
- *add\_track\_names* — a vector of variable names, e.g. [“eco\_l”, “reqp”], with the names written in lower case and corresponding to endogenous variables in the LINVER model. If this vector is defined, its values are added to the default list of variables for which simulation results are tracked. By default, results are tracked for the output gap, the unemployment rate, the unemployment gap, private employment, total and core PCE inflation measured on a 4-quarter basis, the federal funds rate, and the 10-year Treasury yield. If the ECFS option is used, the default list also includes the variable “fiscal”.
- *nreplic* and *nsimqtrs* — respectively, the parameters that determine how many stochastic outcomes will be simulated and the length in quarters of each outcome; their default settings are 5000 and 200. To remove the effects of initial conditions, the first 100 quarters of simulated data are discarded prior to the calculation of means, standard deviations, and other long-run distributional statistics for tracked variables. (If *nsimqtrs*<101, no long-run statistics are reported.) Setting these parameters to higher values does little to increase the precision of the estimated means and standard deviations, although it can increase the precision of probability estimates of tail events.
- *residuals\_file* — the name of an Excel spreadsheet holding historical residuals of the main behavioral equations of the FRB/US model, plus a recession-period indicator; the default is ‘hist\_residuals’. See the **sampling methods** section for further details on the required format of the spreadsheet.
- *draw\_method* — a parameter defining the sampling method used to construct stochastic shocks from the set of historical behavior equation residuals. Available options are “boot” (conventional bootstrapping), “state” (state-contingent bootstrapping), and “mvnorm” (random draws from an estimated multivariate normal distribution). The default is “state”. See the **sampling methods** section for further details.
- *res\_drop* — a list of behavioral equation shocks to be dropped from the standard set used in stochastic simulations, e.g. [“reqp\_aerr”, “lur\_aerr”]. If *res\_drop* is not defined, all variables included in the historical-residuals spreadsheet are used.
- *alt\_range* — a vector holding the start and end dates of an alternative range for sampling from the set of historical residuals, e.g. [“1983Q1”, “2017Q4”], where “Q” must be written in upper case. If *alt\_range* is not defined, the default is the full date range included in the historical-residuals spreadsheet. There are restrictions on the allowed values of *alt\_range*; see the **sampling methods** section for further details.

- *rescale\_wpshocks* — a parameter indicating whether the variances of the pre-1983 residuals for the wage-price equations are reduced to match that observed from 1983 on; the default is “yes”. See the **sampling methods** section for further details.
- *save\_option* and *save\_name* — parameters that determine whether simulation results will be saved, and if so, which results will be stored and under what name in the current Matlab directory. The defaults are “no” and “stochsims\_results”, respectively. If *save\_option* equals “limited”, only summary statistics are saved; if it equals “full”, matrices holding the simulations results for all tracked variables are saved as well. Under the latter option, the resulting save file can be quite large.

## Sampling methods

The stochastic-simulation procedure applies random shocks to the major behavioral equations of LINVER, where the stocks are derived from the historical residuals of the FRB/US model. Unless otherwise specified, the historical residuals are stored in the Matlab directory as a CSV text file under the name *hist\_residuals.txt*. The format of this file is as follows:

- The first column contains the observation dates of the residuals, the subsequent columns the residuals of the individual equations, and the final column a recession indicator.
- The column headers are in the first row, starting with “obs”, followed by the names of the residuals, and ending with “rescind” in the final column. All column headers must be in lower case.
- All residual names must match the names of the corresponding behavioral equation shocks in LINVER, e.g., *ebfi\_1\_aerr*.
- The observation dates in the first column should take the form e.g., 2002Q2 with an upper-case “Q”.
- The recession indicator reported in the final column has values equal to 0 for quarters when the economy was not in recession, 1 for the quarters of the recession that began in 1970Q1, 2 for the recession that began in 1974Q1, and so on for subsequent recessions. The Great Recession is indexed by the number 7.

As noted earlier, the parameter *draw\_method* provides users with three options for how random shocks are generated using the historical residuals — “mvnorm”, “boot”, and “state”:

1. Under the mvnorm option, shocks are drawn randomly from a multivariate normal distribution with mean zero and a variance-covariance matrix estimated using the historical sample of residuals.
2. Under the boot option, shocks are constructed by randomly sampling quarters of the matrix of (demeaned) historical residuals. Unlike the first option, this form of bootstrapping preserves any deviations from normality in the multivariate characteristics of the historical residuals. As is the case with the mvnorm option, the boot option assumes that the equation residuals are uncorrelated over time.



3. Under the state option (the default), the shocks hitting the economy at each point in time depend on the state of the simulated economy, where the latter is determined randomly using a Markov-switching model with three states — normal, mild slump, and severe slump. In the normal state, shocks are bootstrapped by random sampling of the historical non-recessionary quarters of the matrix of historical residuals. In the mild slump state, shocks equal the residuals that occurred in one of the recessions that occurred between 1970 and 2001, with the sequence of shocks matching that seen historically. In the severe slump state, shocks equal the sequence of residuals that occurred during the Great Recession. The transition probabilities and steady-state frequencies of the three states are calibrated to match that seen since 1970. See González-Astudillo and Vilán (2019) for further details.

In the simulations, shocks are applied to all the exogenous variables with entries in the CSV file unless *res\_drop* is defined, in which case no shocks will be applied to the equations for those variables. The shocks applied in the simulations are based on the full range of residuals in the file unless *alt\_range* is defined, in which case an alternative sample period is used. Under any of the sampling options, the alternative sample period cannot begin earlier or end later than the file's range. In addition, if random shocks are drawn using the state method, the alternative sampling range should begin no later than 1970Q1 and end no earlier than 2009Q4. Finally, the historical residuals are demeaned prior to sampling.

The volatility and persistence of inflation has declined appreciably over the past 50 years. To a substantial degree, this secular increase in price stability reflects better monetary policy that in turn has caused long-run inflation expectations to become quite stable over time. By construction, the stochastic simulations replicate much of the greater stability seen in recent years, because all the allowed monetary policy rules are designed to stabilize inflation over time at a fixed target (abstracting from the ELB constraint), and because no shocks are applied to the policy rule in the stochastic simulations. However, the secular decline in the volatility of inflation over the past 50 years also partly manifests itself in LINVER as a marked reduction in the variances of the historical wage-price equation residuals starting in the early 1980s. To incorporate this additional source of greater inflation stability in the stochastic simulations, by default the pre-1983 residuals of the wage-price equations are rescaled prior to sampling so that their variances are equal to that seen from 1983. However, the user can turn off this rescaling by setting the *rescale\_wpshocks* parameter equal to “no”.

### Monetary policy rules

As noted above, the user must specify a monetary policy rule to be used to set the federal funds rate path in the simulations, abstracting from the ELB constraint and threshold conditions. There are nine available options: *tay*, a Taylor-type rule first discussed by Taylor (1999) and referred to as the balanced-approach rule by Yellen (2012); *intay*, an inertial version of the *tay* rule; *fpt*, the flexible price-level targeting rule proposed by Bernanke, Kiley, and Roberts (2019); *infpt*, an inertial version of the *fpt* rule; *ait*, the average inflation targeting rule proposed by Arias *et al* (2020); *rw*, the make-up rule proposed by Reifschneider and Williams (2000); *kr*, the change rule analyzed by Kiley and Roberts (2017); *adur*, a nonlinear rule discussed by Chung *et al* (2019) that responds asymmetrically to movements in the unemployment rate; and *aait*, an asymmetric

form of average inflation targeting analyzed by Arias *et al* (2020). Omitting terms that are held constant in the stochastic simulations such as the target rate of inflation, the target price level, and the long-run equilibrium real federal funds rate, the specifications of these rules are:

- (tay)  $RULE_t = 1.5PICX4_t + 1.0XGAP2_t$
- (intay)  $RULE_t = 0.85RFF_{t-1} + 0.15[1.5PICX4_t + 1.0XGAP2_t]$
- (fpt)  $RULE_t = 1.5PICX4_t + 1.0XGAP2_t + PCNIA\_L_t$
- (infpt)  $RULE_t = 0.85RFF_{t-1} + 0.15[1.5PICX4_t + 1.0XGAP2_t + PCNIA\_L_t]$
- (ait)  $RULE_t = 0.85RFF_{t-1} + 0.15[PICX4_t + 1.0XGAP2_t + 8.0PIC32_t]$
- (rw)  $RULE_t = 1.5PICX4_t + 1.0XGAP2_t + RWTERM_{t-1}$   
 $RWTERM_t = RWTERM_{t-1} + 1.5PICX4_t + 1.0XGAP2_t - RFF_t$
- (kr)  $RULE_t = RULE_{t-1} + 0.4PICX4_t + 0.4XGAP2_t$
- (adur)  $RULE_t = 0.85RFF_{t-1} + 0.15[1.5PICX4_t + 1.0XGAP2_t] + \varepsilon_t$   
 $\varepsilon_t = -0.85DLUR2_t$  if  $DLUR2_t > 0$  and  $UCOND_t > 0$ , 0 otherwise
- (aaait)  $RULE_t = 0.85RFF_{t-1} + 0.15[1.5PICX4_t + 1.0XGAP2_t] + \varepsilon_t$   
 $\varepsilon_t = 0.15[8PIC32_t - 0.5PICX4_t]$  if  $PIC32_t > 0$ , 0 otherwise

In these equations, *RULE* denotes the unconstrained prescriptions of the chosen rule, *RFF* is the federal funds rate, *PICX4* is the four-quarter average rate of core PCE inflation, *XGAP2* is the GDP output gap, *PCNIA\_L* is the log level of PCE prices, *DLUR2* is the two-quarter change in the unemployment rate, *UCOND* is the unemployment gap (the difference between the actual and natural rates of unemployment), and *PIC32* is the 8-year average annual rate of headline PCE inflation. *RWTERM* in the Reifschneider-Williams make-up rule equals the cumulative difference between the actual path of the federal funds rate and the unconstrained prescriptions of the inertial Taylor rule, which can be greater than (but not less than) zero as a result of the ELB constraint.

As discussed in the **appendix**, the non-linear ELB and threshold constraints are imposed on the LINVER solution by applying additive adjustments to the model's federal funds rate equation,  $RFF_t = RULE_t + ERADD_t$ . If one of the nonlinear rules is selected, these additive adjustments also include the asymmetric  $\varepsilon$  component of the rule, which being nonlinear cannot be directly incorporated into a linear model. In this case, the monetary policy rule that is embedded in the model is the linear component of these rules (that is, the inertial Taylor rule).

Users interested in exploring the effects of a linear policy rule other than one of the nine listed above can easily do so, provided the new rule depends on variables already included in LINVER. In this case, the user simply replaces the specification for one of the standard linear rules in *make\_runmod* with that of the non-standard rule while retaining the name of the standard rule.

With somewhat more difficulty, users can also simulate the effects of a linear rule that responds to variables not in the standard version of LINVER, although doing so would require also adding new variable definitions and equations to the Dynare model text files. Finally, simulating the effects of a nonlinear rule other than the two included in the package would require coding changes to a number of the scripts.<sup>3</sup>

## Thresholds

If the user opts to impose threshold conditions on both labor utilization and inflation, any liftoff of the federal funds rate from the ELB prescribed by the unconstrained monetary policy rule is delayed until both threshold conditions are deemed to have been persistently satisfied without the ELB binding. Specifically, liftoff is not allowed until the following conditions are all met in both the liftoff quarter and the next three quarters: the unemployment gap is below its specified threshold level (*uthresh*), the selected measure of inflation is above its threshold level (*pithresh*), and the unconstrained policy rule prescribes setting the federal funds rate above the ELB. This forward-average definition of threshold satisfaction prevents liftoff from occurring in situations where conditions are expected to be only momentarily back to acceptable levels for initiating policy tightening.<sup>4</sup> The **appendix** provides further details.

In the solution procedure, MC agents are assumed to believe that monetary policymakers will be willing to delay liftoff for up to *maxfgl* quarters into the future to satisfy the threshold conditions. Unless otherwise specified, *maxfgl* equals *elbqtr*, where the latter equals 61 by default. Thus, under the default settings, agents regard policymakers' forward guidance about the state-contingent timing of liftoff as fully credible for 15 years into the future. By setting *maxfgl* to a smaller value, however, users can simulate the effects of a shorter horizon for credible forward guidance.

In addition to potentially delaying how quickly the federal funds rate lifts off from the ELB, the threshold procedure is also assumed to speed up how quickly the federal funds rate falls to the ELB in the first place. Specifically, the threshold solution routine immediately forces the federal funds to *elb* whenever the path of the federal funds rate conditional on no thresholds is projected to become constrained by the ELB at some point in the future. Alternatively put, the solution routine's implementation of thresholds incorporates the strategy advocated by Reifschneider and Williams (2000) of immediately driving to the ELB whenever conditions indicate that

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<sup>3</sup> Currently, the stochastic simulation routines do not allow policymakers to influence term premiums and thus long-term interest rates directly using large-scale asset purchases whenever the federal funds rate becomes constrained at the ELB. However, the specification of QE effects employed by Kiley (2018) and Bernanke (2020) would provide a relatively simple way to do this, and future releases of the LINVER package may include this option. A more difficult task would be to modify the routines to use the more sophisticated term-structure model developed by Li and Wei (2013) to control for QE effects in LINVER stochastic simulations, as Chung *et al* (2019) have done. Among other things, their approach requires a detailed accounting of the actual and expected evolution of the overall size, average maturity, and holdings of individual CUSIPS in the Federal Reserve's portfolio, as well as an explicit strategy for how purchases, sales, and reinvestment are adjusted in response to changing economic conditions.

<sup>4</sup> If a threshold is set for labor utilization but not inflation, or vice-versa, only the operative threshold condition must be satisfied for liftoff to occur. Also, because baseline values in LINVER equal zero, the default settings for *uthresh* and *pithresh* (zero) imply that liftoff does not occur until conditions in the operation areas are back to normal.

conventional monetary policy will soon be constrained, rather than holding back as might be prescribed by an inertial policy rule.

## Dynamic instabilities

For the stochastic simulation routines to work properly, the model must be dynamically stable in response to adverse shocks that cause the ELB to bind persistently. Without special adjustments, however, this may not always be the case for some monetary policy rules (including the Taylor rule) when some or all agents have MC expectations. In such circumstances, the projected path of the economy may begin to oscillate wildly as the solution routines attempt to impose the ELB constraint. These oscillations, in turn, inhibit convergence and may cause the simulated volatility of the economy to explode.

Users have several options to ensure the stability of LINVER under all policy rules and expectational assumptions. The default way to do this is to use the ECFS option described below, which applies “emergency” fiscal stimulus as necessary to prevent the projected output gap from falling to excessively low levels. This asymmetric fiscal mechanism helps to compensate for the inability of monetary policy to provide additional accommodation once the ELB is hit, thereby suppressing oscillatory behavior. By itself, ECFS is sufficient to ensure stability under all the monetary policy options except when all agents have MC expectations. In the latter case, stability also requires that *tax\_gamma* be set by default at .00130 rather than .00075, the default value used in the other expectational cases. This parameter determines the coefficient on the output gap in the trend personal tax rate equation, and thus the (symmetric) countercyclicality of fiscal policy in LINVER. By boosting this coefficient, fiscal policy thus is made more generally activist under full MC expectations than it has been on average historically.

An alternative way to ensure stability that does not involve fiscal policy is to override the Treasury term premium equations using the *ctp\_option*. In the standard version of the model, bond term premiums move inversely with the average cyclical state of the economy expected over the life of the bond. As a result, term premiums and thus long-term interest rates can sometimes spiral upwards in response to adverse shocks when short-term interest rates are trapped at their lower bound, fueling the oscillatory behavior. One way to short-circuit this unstable dynamic is to suppress the cyclical components of the term premiums altogether by setting *ctp\_option*=1; this approach has been used in a number of studies reporting results from FRB/US stochastic simulations. Because the countercyclicality of term premiums is well established empirically, however, users may instead prefer to set *ctp\_option*=2. Doing so causes the standard term premium equations to be overridden with ones estimated using a specification that also causes term premiums to vary inversely with the cyclical state of the economy, but in a more stable way.<sup>5</sup> If either approach is used, the model is stable under all policy rules and expectational assumptions even if ECFS is not used and *tax\_gamma* is set to .00075 under all expectational assumptions.

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<sup>5</sup> The alternative specification has term premiums responding to both the current and lagged output gap. Because the coefficients on the current and lagged gap are roughly equal but opposite in sign, the level of resource utilization has no long-run effect on term premiums in the alternative specification, in contrast to the standard term premium equations. The historical goodness of fit of the standard and alternative specifications is about the same.

## Extreme case fiscal stabilization

When the equilibrium level of interest rates is assumed to be low and the ELB constraint is thus likely to bind frequently (e.g.,  $elb \geq -3$ ), LINVER stochastic simulations occasionally generate economic downturns that are more severe than the Great Depression in the absence of a countercyclical fiscal response beyond that embedded in the model's estimated tax and government spending equations. Such extreme outcomes can occur even when the model is dynamically stable under the chosen monetary policy rule, expectational assumption, and other parameter settings. Simulations with such extreme tail events can be a useful way to illustrate the potentially dire implications of the current low level of the equilibrium real interest rate. But it seems likely that more realistic estimates of uncertainty would be obtained if the stochastic simulations allowed for a special fiscal response once resource utilization falls below some threshold. The overall response of fiscal policy to the severe Covid-driven recession, which was much more aggressive than the standard model equations would have predicted, may illustrate such behavior.

As noted above, the model can also become dynamically unstable under some monetary policy rules when the ELB binds persistently, resulting in unstable oscillations. Such oscillatory behavior can be greatly damped or eliminated altogether by the application of emergency fiscal stimulus whenever the projected output gap falls to extremely low levels in response to adverse shocks that leave the federal funds rate trapped at the ELB.

For these reasons, the stochastic simulation routines include an extreme-case fiscal stabilization (ECFS) option. Under this option, the equations  $fiscal_t = .97fiscal_{t-1} + fiscal\_aerr_t$  and  $fiscalav_t = .9fiscalav_{t-1} + fiscal_t$  are added to the model, with  $fiscal\_aerr_t$  defined as an exogenous shock. Government outlays on goods and services are then linked to  $fiscal_t$  and government transfer payments to  $fiscalav_t$  by adding new RHS terms to these equations. The dynamics of these modified equations are such that a fiscal shock which increases  $fiscal_t$  by 1 translates into a direct fiscal impetus to GDP that initially is slightly greater than 1 percent and thereafter gradually fades away, abstracting from accompanying endogenous movements in real activity, inflation, and interest rates.

Using this mechanism, an emergency fiscal shock is applied at time  $t$  whenever the minimum value of the output gap along its projected path from  $t$  to  $t+7$  falls below the value specified by the parameter  $ecfs\_floor$ . The size of the shock is determined iteratively to be whatever is required to raise the minimum projected output gap to within plus-or-minus 1 percentage of the specified floor. Using ECFS is the default option if the ELB is imposed but is not allowed if the ELB is not imposed because of potential convergence problems.

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## Appendix: Methodology for Imposing Nonlinear Constraints on LINVER Solutions

In the absence of any nonlinearities associated with the ELB constraint, thresholds, or a monetary policy rule that responds asymmetrically to economic conditions, the simulated response of the economy to stochastic shocks is easily computed using the Dynare-generated decision rule representation of LINVER,  $y_t = Ay_{t-1} + Bx_t$ . Here,  $y_t$  denotes the solution values for the model's endogenous variables at time  $t$  conditional on outcomes in the prior quarter ( $y_{t-1}$ ) and the values of the model's exogenous variables in the current period ( $x_t$ ), where  $A$  and  $B$  are fixed matrices that define the dynamics of the model. In the absence of any nonlinear constraints,  $x_t$  includes only the random shocks to the model's behavioral equations that occur at time  $t$ .

The problem becomes more complicated when nonlinear constraints are imposed on the linear model's solution. In this case, the decision rule is still used to compute the current state of the economy, save that  $x_t$  now includes agents' expectations at time  $t$  for the additive adjustments to the future path of the federal funds rate implied by the ELB constraint and any other nonlinearities. As described below, the procedures used to compute these adjustments depend on which nonlinear constraints are operative.

### ELB constraint alone

The first nonlinearity to consider is when the monetary policy rule is linear and no threshold conditions are imposed on the timing of liftoff. In this case, imposing the ELB constraint on agents' expectations for  $M=elbqtrs-1$  quarters into the future implies that the predicted  $t+j$  value of the federal funds rate ( $RFF$ ), conditional on information at time  $t$ , must equal the predicted  $t+j$  value of the unconstrained rule's prescriptions made at time  $t$  plus a projected non-negative additive adjustment consistent with the effective lower bound. That is:

$$\begin{aligned} RFF_{t+j|t} &= RULE_{t+j|t} + \varepsilon_{t+j|t}, \quad j \leq M, \text{ with} \\ \varepsilon_{t+j|t} &= ELB - RULE_{t+j|t} \text{ if } RULE_{t+j|t} < ELB, \quad 0 \text{ otherwise} \end{aligned}$$

Following the approach suggested by Bodenstein, Guerrieri, and Gust (2013), the nonlinear condition for  $\varepsilon_{t+j|t}$  is parsimoniously imposed on the simulations of LINVER by including the following equations in the model:

$$\begin{aligned} RFF_t &= RULE_t + ERADD_t \\ ERADD_t &= ERADD1_{t-1} + E0_t \\ ERADD1_t &= ERADD2_{t-1} + E1_t \\ ERADD2_t &= ERADD3_{t-1} + E2_t \\ &\dots \\ ERADDM_t &= EM_t \end{aligned}$$

Here,  $RULE_t$  is a linear policy rule such as the Taylor rule, and  $E0, E1, \dots, EM$  are defined as exogenous variables in the model file. The procedure then solves for any additive adjustments to the path of the federal funds rate expected at time  $t$ ,  $E_t = \{E0_t, E1_t, \dots, EM_t\}$ , that are required to satisfy the ELB constraint for  $M$  quarters into the future.

Specifically, the *stochloop* routine at the start of each new quarter draws a new set of time  $t$  behavioral equation shocks and sets the values of the additive adjustments projected in the prior quarter,  $\{ERADD_{t-1}, ERADD1_{t-1}, \dots, ERADDM_{t-1}\}$ , equal to zero. The script then computes an initial forecast of the future path of the federal funds rate from  $t$  to  $t+M$  that is conditioned on prior quarter economic conditions, the new shocks, and no expected additive adjustments. If this initial no-adjustments projection for *RFF* does not violate the ELB constraint within the  $t$  to  $t+M$  evaluation window, the simulation routine moves on to the next quarter and a new set of shocks. Otherwise, *stochloop* calls the *addscal* script to solve for the vector of non-negative adjustments that prevent the ELB constraint from being violated along the projected path of the federal funds rate made at time  $t$ . To do this, *addscal* uses an iterative search procedure that on each iteration:

- Computes the adjustments needed to force the federal funds rate to the ELB in those quarters that have been provisionally identified as requiring adjustments to satisfy the ELB constraint; this is done using OLS. (On the first iteration, these quarters are those that violate the ELB constraint along the initial projected path of *RFF*.)
- Updates the projected path of *RFF* conditional on the new adjustments and evaluates a quadratic loss function  $L$  that penalizes the sum from  $t$  to  $t+M$  of three factors each expressed as squares: violations of the ELB constraint; positive adjustments in quarters in which *RFF* is greater than the ELB; and negative adjustments. If  $L < .0001$ , the process is deemed converged, which as a practical matter ensures that the adjustments are all non-negative and the minimum value of the federal funds rate along its projected path is no more than 1 basis point below the ELB.
- If  $L > .0001$ , the list of quarters requiring adjustments is updated by adding any new quarters in which the projected value of *RFF* is less than the ELB and dropping any quarters in which the adjustments are less than zero, and a new iteration is started.

Although this procedure almost always finds the ELB-consistent solution in a few iterations, on rare occasions it fails to converge. When this happens, *addscal* continues the iterative search using Matlab's LSQNONNEG command in place of OLS to estimate the required non-negative values of *ERADD*S.

### **Nonlinear rule constraints**

If the user selects one of the nonlinear policy rules, *addscal* is also used to compute the additive adjustments needed to make the projected path of the unconstrained federal funds rate conform to the prescriptions of the unconstrained nonlinear rule. In this case, the adjustments implied by the nonlinear rule by itself are not restricted in sign because either one may prescribe pushing the path of *RFF* above or below the path prescribed by the linear *RULE* equation embedded in the model, whose specification matches the inertial Taylor rule. In addition to the nonlinear rule adjustments, additional adjustments may be needed to prevent ELB violations if the ELB constraint is imposed.



As when the only nonlinearity is the ELB constraint, in this case *stochloop* starts each new quarter by drawing a new set of behavioral equation shocks and computing new projected paths of the federal funds rate and the elements of the nonlinear rule conditioned on no additive adjustments. These projections are then used to compute  $\varepsilon_{t+j}$ , the difference between the nonlinear rule and the inertial Taylor rule along the projected path:

$$\begin{aligned} \text{(ADUR)} \quad \varepsilon_{t+j} &= -0.85DLUR2_{t+j}S_{t+j} \\ S_{t+j} &= \{1 \text{ if } DLUR2_{t+j} > 0 \text{ and } UCOND_{t+j} > 0, 0 \text{ otherwise}\} \end{aligned}$$

$$\begin{aligned} \text{(AAIT)} \quad \varepsilon_{t+j} &= 0.15[8PIC32_{t+j} - 0.5PICX4_{t+j}]S_{t+j} \\ S_{t+j} &= \{1 \text{ if } PIC32_{t+j} < 0, 0 \text{ otherwise}\} \end{aligned}$$

If  $\varepsilon_{t+j}$  deviates from zero in any quarter from  $t$  to  $t+asymqtrs-1$ , or if ELB violations are projected from  $t$  to  $t+elbqtrs-1$ , *addscal* is called to find the required additive adjustments. Otherwise, the routine moves on to a new quarter and a new set of shocks.

To determine the correct set of  $\varepsilon_{t+j}$  adjustments, the search procedure embedded in *addscal* computes a trial value of  $\varepsilon_{t+j}$ ,  $j=1$  to  $M$  on each iteration prior to recomputing any additional adjustments that may be needed to prevent projected ELB violations. To facilitate convergence, the change in the trial value of  $\varepsilon_{t+j}$  from the previous iteration is damped.<sup>6</sup> Convergence is judged to have occurred when the maximum absolute change in  $\varepsilon_{t+j}$  from the previous iteration is less than 2 basis points and, if the ELB is imposed, the convergence test for those adjustments is also satisfied.<sup>7</sup>

### Threshold constraints

When threshold conditions for liftoff are imposed on top of the ELB constraint, the solution procedure becomes somewhat more complicated. At the start of each new quarter, *stochloop* again draws a new set of behavioral equation shocks and computes initial projected paths for the federal funds rate, the unemployment gap, and inflation, conditional on no additive adjustments. The program then checks to see if one of the following conditions holds:

- (1) There are ELB violations along the initial projected path of *RFF*.
- (2) Monetary policy in the prior quarter was constrained ( $RFF_{t-1} = elb$ ) and the threshold conditions, which were not fully satisfied at time  $t-1$ , remain unsatisfied in the first quarter of the initial projection path.

<sup>6</sup> Convergence is also facilitated by approximating the discontinuous switching component of the ADUR rule,  $S$ , by the continuous function  $S_t = [.5 + .5 \tanh(15DLUR2_t)][.5 + .5 \tanh(15UCOND_t)]$ . The first part of this function evolves smoothly from 1 to 0 as  $DLUR2$  moves from just above zero to just below, while the second part does the same as  $UCOND$  moves from just below zero to just above. For the same reason, the switching component of the AAIT rule is approximated by  $S_t = [.5 + .5 \tanh(-25PIC32_t - 2.5)]$ . This approximation implies that when the shortfall of headline inflation from target over the previous eight years averages 0.2 percentage or more, monetary policy focuses on returning average inflation to baseline ( $S_t > 0.99$ ), but when there is no shortfall monetary policy focuses instead on current core inflation ( $S_t < 0.01$ ).

<sup>7</sup> When the ECFS option is used, *addscal* also updates the trial value of the fiscal shock on each iteration and checks for its convergence as well.

- (3) There are no projected ELB violations and the ELB was not binding in the prior quarter, but the policy rule is nonlinear.

If none of these conditions holds, the program moves on to the next quarter and a new set of shocks. If the third condition holds, *addscal* is called to compute the adjustments needed to ensure that the projected path of the federal funds rate is consistent with the prescriptions of the nonlinear rule. If the first or second conditions hold, however, *addscal\_thresh* is called to find the additive adjustments needed to enforce the ELB constraint along the projected path of the federal funds rate from  $t$  to  $t+elbqtrs-1$  and to delay liftoff until either the threshold conditions are satisfied without the ELB binding or the expiration date for the threshold commitment,  $t+maxfgl$ , is reached. (Unless otherwise specified, *maxfgl* equals *elbqtrs*.)

The solution algorithm used by *addscal\_thresh* has three key features. First, whenever the current outlook, conditioned on no thresholds, shows monetary policy becoming constrained by the ELB in the future, the federal funds rate is immediately lowered to the ELB even if the monetary policy rule by itself would prescribe a more gradual pace of easing. Second, the threshold strategy involves holding the federal funds rate at the ELB continuously from  $t$  to  $t+lfqtr$  along its projected path, where  $t+lfqtr$  is the earliest point at which the threshold conditions are deemed “persistently” satisfied. Finally, “persistent” satisfaction is operationally defined as a projected unemployment gap running continuously below its threshold setting, and inflation running continuously above its threshold setting, from  $t+lfqtr+1$  to  $t+lfqtr+4$  without the ELB binding. This forward-average definition serves to prevent premature liftoff in response to a brief improvement in economic conditions.

Using this algorithm, *addscal\_thresh* iteratively searches for the correct value of *lfqtr* by gradually narrowing the range in which it must lie. Specifically, on each iteration it:

- Provisionally sets *lfqtr* equal to the midpoint of its feasible range [*lfqtr\_min*, *lfqtr\_max*] as computed on the previous iteration, based on the projections of the federal funds rate, unemployment, and inflation generated during the previous iterations. (On the first iteration, the feasible range equals [1, *maxfgl*].)
- Calls *addscal* to compute the additive adjustments that force *RFF* to the ELB from  $t$  to  $t+lfqtr$ , prevent ELB violations beyond that point, and ensure that unconstrained projected path of the federal funds rate is consistent with the prescriptions of the nonlinear rule (if one is selected).
- Computes the projected paths of an unemployment state, an inflation state, and an ELB-binding state. The unemployment state equals 1 at time  $t+j$  if the projected unemployment gap is less than *uthresh* in quarters  $t+j+1$  through  $t+j+4$ ; otherwise, it is less than 1. The inflation state equals 1 at time  $t+j$  if projected inflation is greater than *pithresh* in quarters  $t+j+1$  through  $t+j+4$ ; otherwise, it is less than 1. Finally, the ELB-binding state equals 1 at time  $t+j$  if the federal funds rate is greater than the ELB in quarters  $t+j+1$  through  $t+j+4$ ; otherwise, it is less than 1. If the sum of the three states,

*combstate*, equals 3 in quarter  $t+j$ , then liftoff in quarter  $t+j+1$  is treated as a potentially valid.<sup>8</sup>

- Tests to see if  $t+lfqtr$  is the first quarter along the projected path where *combstate* equals 3, or if all quarters from  $t$  to  $t+maxfgl$  have been forced to the ELB without satisfaction of the threshold conditions.<sup>9</sup> If either condition holds, the search process has converged and the program stops iterating.<sup>10</sup> Otherwise, the feasible range for *lfqtr* is narrowed. Specifically, if *combstate* first equals 3 in some earlier quarter, indicating that the trial value of *lfqtr* overshoot, then *lfqtr\_max* is reduced to *lfqtr*. Otherwise, the trial value undershot and *lfqtr\_min* is increased to *lfqtr*. After updating the feasible range, a new iteration is started.

When *elbqtrs* and *maxfgl* equal their default values (61), the *lfqtr* iterative search procedure usually requires six to seven iterations to complete. However, these “outer-loop” iterations require repeated calls to *addscalc*, which employs its own “inner-loop” iterations to compute the adjustments needed to impose the ELB and the asymmetric components of the nonlinear rules. As a result, imposing thresholds significantly increases runtimes.

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<sup>8</sup> If a threshold condition is imposed for unemployment but not inflation, or vice-versa, the state for the one not imposed always equals 1.

<sup>9</sup> Sluggish adjustment of unemployment and inflation to shocks sometimes results in both variables temporarily satisfying their threshold conditions along the initial portion of the projected paths only to violate them thereafter. To prevent premature liftoff in such circumstances, if the ELB was not binding in the prior  $t-1$  quarter, conditions in quarters  $t$  to  $t+2$  along the current projected path are ignored in determining whether  $t+lfqtr$  is the first point along the projected path in which *combstate* equals 3.

<sup>10</sup> The procedure also stops iterating whenever a trial value of *lfqtr* is repeated. This situation indicates that the appropriate liftoff point is “fuzzy” — a situation that arises when the difference between *lfqtr\_max* and *lfqtr\_min* has shrunk to 2 or less and no value within the feasible range results in the threshold conditions first being satisfied at time  $t+lfqtr$ .