

Chapter 1

Photons and Electrons

Learning goals:

- (1) Explain how we can measure objects too small for us to see
- (2) Describe the experiments that led to the concept of the photon
- (3) Discuss how the properties of an electron are determined
- (4) Analyze what behavior can be described using classical ideas and where new quantum ideas are needed

Material to review:

- (a) The driven harmonic oscillator
- (b) Resonance
- (c) Forces on a charged particle in electric and magnetic fields
- (d) Solving first-order inhomogeneous differential equations
- (e) Complex numbers and complex exponentials

Welcome! You are about to enter the world of quantum mechanics. You may have heard a lot about what quantum mechanics is from earlier readings or videos. Many of those sources will have said things like “quantum mechanics is completely different from classical mechanics” or “it is so bizarre, you will not be able to reason about it,” or even “nobody understands quantum mechanics.” Here, we will develop how quantum mechanics unfolds using a different approach — emphasizing classical ideas and showing how they are modified to encompass the full quantum world. This approach enables you to build upon the strengths you already possess from classical mechanics and electromagnetism, and then add quantum reasoning on top. You will find this to be different from how everyone else does it. The standard approach is challenging to follow as it brings in ideas that are stated without any foundation and delivered with a “trust us” attitude because this is the way the world is. We will not do that. We will instead show you how you can develop quantum thinking out of classical thinking, which will help you reason about the phenomena more easily. Along the way, we will challenge your reasoning, but we will always keep the discussion grounded in what you already know.

1.1 What are photons?

Our discussion of quantum mechanics begins with the photon. You might be thinking that this is just a particle of light, but it is so much more. In this section, we develop precisely what a photon is, which will give you an anchor into one of the most important cornerstones of the quantum world. Imagine yourself as a physicist at the dawn of the 20th century. Our story starts with the photoelectric effect. This is a curious effect, where shining light onto the surface of a metal emits electrons from the metal. How can this be measured when we cannot see the electrons? Fortunately, electrons are charged (which was known at this time, as we discuss in Sec. 1.2). Thus, by having a second metal plate that can collect the emitted electrons, we can measure their current flow and determine how many are emitted. To describe this experiment further, we need to recall some ideas from electromagnetism. The experimental setup is shown schematically in Fig. 1.1.

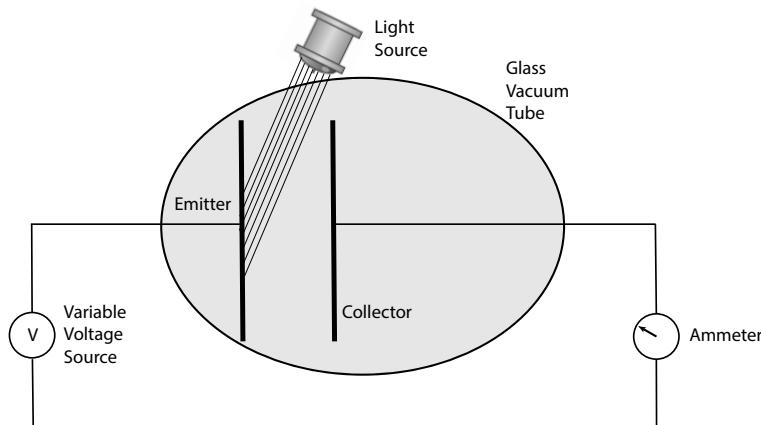


Fig. 1.1: Schematic of a photoelectric effect. The light source shines onto the metallic plate on the left, which emits electrons in random directions towards the right. By making the voltage large and positive, all emitted electrons are captured, while by making the voltage negative, the electrons are repelled, and eventually they do not have enough kinetic energy to reach the collector plate. This is called the stopping voltage. The electrons collected by the collector plate will create a current that moves through the ammeter, allowing them to be detected. Note that everything needs to be in a high vacuum for the experiment to work (grey ellipsoid).

To begin, recall that a charged particle feels a force proportional to its charge and the strength of the electric field at its location. Furthermore, an electric field is found from the negative gradient of the electric potential. Finally, the field between two parallel metallic plates is nearly uniform and nearly perpendicular to the plates; it

is directed from the high (positive) voltage collector to the grounded emitter plate (zero voltage).

Using these ideas, we immediately see that by making the voltage on the collector plate large and positive, we will eventually collect all the emitted electrons (note that the emitted electrons can be emitted in random directions towards the right of the emitting plate). What happens if we make it negative? Now, the field pushes electrons away. This reduces the current all the way to zero. The last electrons to make it to the collector are the ones emitted perpendicular to the emitter, with maximal momentum directed toward the collector. In terms of energies, they are the electrons whose initial kinetic energy upon emission was equal to $e\Delta V$, proportional to the change in voltage between the plates. We have $\Delta V = V_c - V_e = V_c$, the voltage at the collector, because the emitter is grounded. The voltage at which the current first vanishes is called the stopping voltage. It changes with different metals used in the experiment.

Now, here is where things begin to get weird. Philipp Lenard was an expert in making vacuum tubes with electrodes going through the glass, yet still able to maintain a high vacuum inside. For these experiments, the better the vacuum, the better they worked. Lenard examined the photoelectric effect carefully and found that as he increased the intensity of the light, the saturation current rose while the stopping voltage remained unchanged [1]. These results were very odd in the classical world. They are our next puzzle to tackle. An image of Philipp Lenard from 1905 is shown in Fig. 1.2, where you will also see a schematic of the experimental results.

But, before we tackle this puzzle, it is important to recognize that the people involved in science are not always the best role models. Lenard, who won the Nobel prize in 1905, was a “Nazi” even before the Nazi party was created. His antisemitism was more important to him than his science. Now, when we look back, we have to make a decision about how to discuss both the science and the scientist, when some important scientists led evil lives. Here, we will try to present all the facts as we know them. Some excellent scientists are awful role models. Lenard is our first, but far from our last.

Back to our science puzzle. You might ask why this is puzzling? The naive classical reasoning about the photoelectric effect goes as follows. If we use no model for a metal except that it is composed of electrons, and something else to make it electrically neutral, then the simplest assumption is that all of the energy from the light will be absorbed by the electrons and divided among them, causing them to be emitted. Since the energy in a classical light wave is proportional to the square of the amplitude of the wave, we expect that as the amplitude of the light is increased to make it brighter, the same number of electrons will be emitted, and that they will have absorbed more energy during the emission. Hence, we would predict that increasing the intensity of light will keep the saturation current the same, but make the stopping voltage more negative. This was not seen in the experiment, so something else is going on.

Lenard thought that the electrons already had their energy pre-stored in the metal, similar to a compressed spring, and the light acted simply to liberate the electrons, which each emerged with the same energy they had pre-stored. The energy available

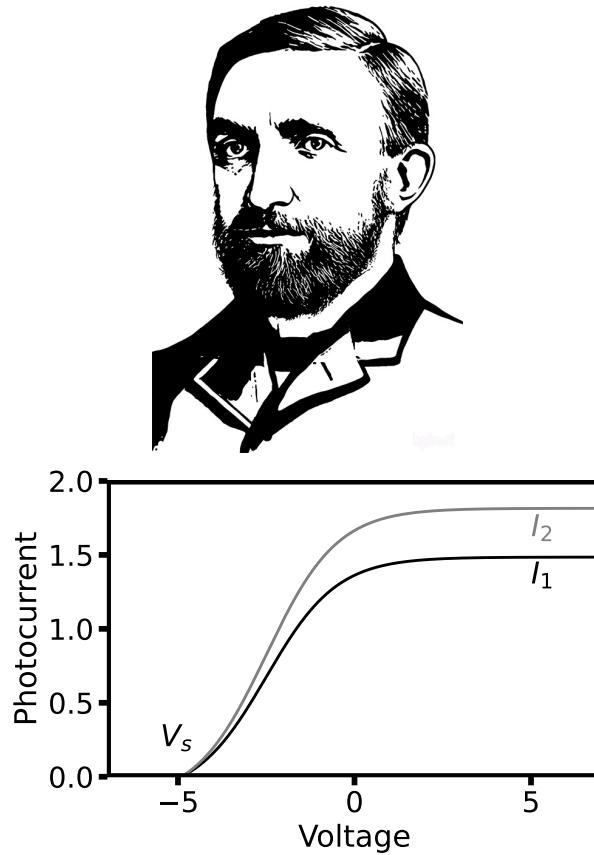


Fig. 1.2: Top: Image of Lenard from 1905, when he won the Nobel prize. Bottom: schematic of Lenard's experimental results when the intensity of light is changed. Note how the stopping voltage stays the same, while the saturation photocurrent increases for a higher intensity ($I_2 > I_1$). [The Lenard image is public domain and converted to look like a stencil by the authors.]

per electron would depend on the material, as the stopping voltage does. Then, the light acts simply to trigger the emission. One could even imagine that the saturation current increases with the light intensity because the higher the energy in the light, the more electrons that could be triggered to be emitted.

It seems like our analysis is becoming more challenging. If we think carefully, it seems like we have to come up with a model for how the electrons are held in the metal in order to be able to describe the photoelectric effect properly, and none of the previous theories really does this. How do we start? Well, we know that nearly all potentials look like a harmonic oscillator near their minimum, so we can start by modeling them as harmonic oscillator traps. Next, because electrons do not come

out of metals without light shining on them, there must be a minimum curvature to the trap, or a minimum harmonic oscillator frequency that holds the electron in the metal. Finally, because the metals are opaque and can easily carry current, it seems reasonable to assume that there is a wide range of available trap frequencies in the metal as opposed to just one or a few.

Indeed, just such a model for materials was already being put forward by the father of quantum mechanics, Max Planck. At the dawn of the 20th century, Max Planck determined a mathematical formula for the distribution of the energies of photons due to black-body radiation [2]. Max Planck is shown in Fig. 1.3 in 1938. The theory for black-body radiation is quite complicated and requires one to understand statistical mechanics, so we will not present it here. We will keep in mind his cornerstone idea though. A black body has oscillators in it and the energy of the oscillator comes in discrete packets given by $\hbar\omega$, where \hbar is the reduced Planck constant and ω is the angular frequency of the oscillator. In electromagnetism, we learn that light can be produced by oscillating charges, and Planck postulated that materials have charges in them that can oscillate at any frequency. When they do, the energy of the oscillator can only change in discrete jumps. Using this reasoning, weird as it may sound, is enough to derive the precise formula for black-body radiation, which was a critically important formula of the time.



Fig. 1.3: Max Planck in 1938. Max Planck was a thoughtful and helpful person; he was a strong supporter of all good science. But, his life was full of tragedy and difficulty. He brought forth the idea that materials are composed of oscillating charged particles that can react to light. They carried energy in discrete lumps, which ushered in the quantum age. [Image is public domain and is converted to look like a stencil by the authors.]

Our theory is similar in spirit to Planck's, but we have a minimum frequency for the oscillator in the metal due to the fact that it does not emit any electrons

in the absence of light. This model, consistent with Planck, was developed further by Hendrik Lorentz. Lorentz is shown in Fig. 1.4. His theory for metals was that they consisted of electrons trapped in harmonic potentials. First, this was a single frequency, but then quickly developed to many frequencies, or even a continuum of frequencies [3]. If we use this model for a metal, then the electron is excited resonantly. Resonant excitation is similar to how we must precisely time the pushes when pushing a child on a swing to get them to swing higher and higher.



Fig. 1.4: Hendrik Lorentz in 1902. Lorentz was the physicist who popularized Maxwell's work. Maxwell's electromagnetic theory was difficult to follow. Lorentz translated it into a much clearer formalism that was more easily understood. He was the leading physicist of his time. He had a theory that metals had electrons trapped in harmonic oscillator potentials. [Public domain and converted to a stencil by the authors]

The driven harmonic oscillator is often studied in an introductory class. In our case, we will assume there is no damping, which will be an important assumption for the model. Atoms often showed behavior that indicated they had no damping in them, even though classically, one might have thought there always is some damping present. Work by others on metals even tried to estimate those damping rate, and the estimates were for very fast rates. Nevertheless, we will ignore damping in our model as it is necessary to get behavior similar to the photoelectric effect experiments.

The equation of motion for an electron in a harmonic trap and driven by an oscillating force is

$$m\ddot{x} = -m\omega_0^2 x + F \cos(\omega t), \quad (1.1)$$

where each dot signifies a derivative with respect to time, m is the mass of the electron, ω_0 is the angular frequency of the trap potential, F is amplitude of the applied force, and ω is its angular frequency. The trick to solving this equation in the steady state is to assume that the position x oscillates at the frequency ω for long times. If you have studied differential equations, this would correspond to the particular solution. Using as an ansatz that $x(t) = x_0 \cos(\omega t)$ yields

$$x_0 = \frac{F}{m(\omega_0^2 - \omega^2)}, \quad (1.2)$$

which is the largest distance the electron travels during the motion. Now, because metals screen out an electric field, it is only electrons close to the surface that can be emitted. Since an atom's size is on the order of one Å, a simple estimate is that the electron must move about ten times this distance or 1 nm before it will be released from the metal. How close to resonance do we need to be in order to release the electron by this resonant excitation? It was not known at the time, but we now know that the mass of the electron is $mc^2 = 511,000$ ev. Furthermore, the light needed to emit the electrons is typically in the near ultraviolet. This light oscillates at an angular frequency of approximately 5×10^{16} rad/s. Let us use this to estimate the maximal distance due to the electric field of the light, where $F = |eE|$, for an electric field of 100 V/m.

We have

$$x_0 \approx \frac{|e|Ec^2}{mc^2(\omega_0 + \omega)|\omega_0 - \omega|} \approx \frac{100 \frac{\text{eV}}{\text{m}} (3 \times 10^8 \frac{\text{m}}{\text{s}})^2}{511000 \text{ eV } 10^{17} \frac{1}{\text{s}} 5 \times 10^{16} \frac{1}{\text{s}} \frac{\Delta\omega}{\omega}} \approx 4 \times 10^{-19} \frac{\omega}{\Delta\omega} \text{ m}, \quad (1.3)$$

which is $4 \times 10^{-10} \omega / \Delta\omega$ nm. Note how we used the electron volt (eV) to measure the energy, and we summarize the detuning from resonance via $\Delta\omega/\omega = (\omega_0 - \omega)/\omega$ (we also drop the dimensionless radian unit). To get the system driven to a displacement of 1 nm requires the light to be exceedingly close to resonance ($\Delta\omega/\omega \approx 10^{-10}$), so we assume it must be precisely on resonance for this to happen. This requires us to discuss the resonant driving problem next.

Using a complex number representation, where we take the real part of everything at the end, we have that

$$m\ddot{x} = -m\omega_0^2 x + Fe^{-i\omega_0 t}. \quad (1.4)$$

One can immediately see that trying the same ansatz as before now fails, as $x(t) = x_0 \exp(i\omega_0 t)$ does not solve the differential equation. Instead, our ansatz needs an additional power of time, so it becomes $x(t) = v_0 t \exp(i\omega_0 t)$, where v_0 is a velocity. Substituting into the differential equation and going through some algebra, we ultimately find that

$$v_0 = \frac{eE}{2m\omega_0} i. \quad (1.5)$$

Note that because the displacement has an amplitude proportional to t , if we wait long enough, it will be as large as we want it to be. This implies that the resonant drive must always emit an electron. It is also why we insisted there be no damping, as

damping will again introduce a maximal displacement we can reach. Now, we turn the logic backwards and determine the time it takes to reach a 1 nm displacement. Plugging in numbers, we find that

$$v_0 \approx \frac{100 \frac{\text{eV}}{\text{m}} (3 \times 10^8 \frac{\text{m}}{\text{s}})^2}{511000 \text{ eV} 10^{17} \frac{1}{\text{s}}} \approx 0.0002 \frac{\text{m}}{\text{s}}. \quad (1.6)$$

This will take a time of about 5×10^{-6} s to be emitted and requires about 10^{11} oscillations (which is why we cannot have any damping). Now, if we make the light dimmer, say $E = 1 \text{ V/m}$, we would wait about half a millisecond. Though challenging, these are timescales that could be measured, even in the early 1900s.

Summarizing, if we describe a metal as a continuum of Lorentz oscillators with a minimal frequency, then we would predict the following three behaviors: (i) electrons would not be emitted until the angular frequency of the light was larger than the minimal frequency of the traps; (ii) assuming the metal can replenish emitted electrons at some rate (since they conduct), we anticipate the saturation photocurrent to increase with the amplitude of the light until we reach a steady state where we pull out as much from the metal as is internally replenished and the current cannot increase further; and (iii) there will be a time delay before photoemission starts that gets longer for dimmer light and should be measurable. In addition, because the electron is emitted once the position reaches a threshold, we would expect all electrons to be emitted with essentially the same kinetic energy, independent of the frequency or amplitude of the light. This last observation assumes the resonant excitation takes place over many oscillations so that the energy added during a single period of oscillation is small. This is the case for the light sources available in the early 1900s, but is not true for powerful lasers available now.

Since physicists had only broadband light sources at the time (Lenard used carbon and zinc arc lamps), it is not possible to check how the electron kinetic energy might depend on the frequency of light. They did observe the saturation photocurrent to increase with the amplitude of light, so that is consistent with the classical model. The key experimental issue is that they always saw the electrons being emitted immediately after the light was turned on. They also knew there was some form of light-frequency threshold, as they could use a more optical light source than an ultraviolet one and see that electrons would not be photo-emitted for some metals with the optical source.

Note that this behavior and discussion is quite different from the standard discussion, which will (wrongly) say that the time delays are extremely long (as you can see in the problems) and that classical physics cannot have a threshold. But, now you know that it can.

So, how do we describe all of this behavior? In 1905, Albert Einstein (see Fig. 1.5) thought he had the answer. He described the process of photoemission using a particle picture for the light. This agreed with all of the known experimental results, as we shall soon see, and it made an extraordinary prediction, which was verified ten years later. This was the final nail in the coffin for the classical theory. Let us now explore how this works.

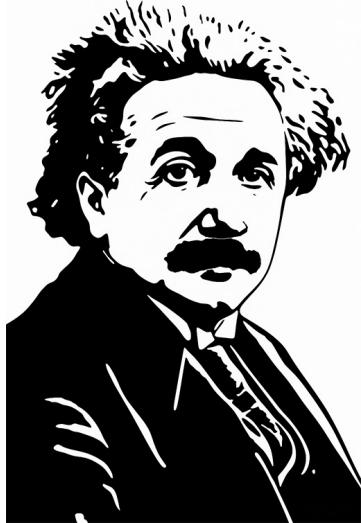


Fig. 1.5: Albert Einstein in the 1920s. Einstein devised the model for the photoelectric effect that treated the photons as quantum particles, each carrying an energy of $\hbar\omega$ and won the Nobel prize for this work in 1921. [Figure is public domain. Converted to look like a stencil by the authors.]

In Einstein's model, each photon carries energy $\hbar\omega$ and the electron is emitted from the metal if a single photon has enough energy to knock it out [4]. Since the electron needs a minimal energy to be emitted, this provides a minimal frequency of light for the photoelectric effect, just as in the classical resonant drive picture. It further predicts that increasing the intensity of the light means more particles, so more electrons will be emitted. Furthermore, we expect the electrons to be emitted instantly upon turning on the light, because the first photon that hits the metal can photo-emit. Finally, he made a bold new prediction, that is different from the threshold emission of the resonant drive model (where every electron has about the same kinetic energy, always, because they are all released near threshold). Using energy conservation, we define the work function to be the minimum energy needed to emit an electron, essentially with vanishing kinetic energy. Then, energy conservation says that the emitted electron kinetic energy is given by the photon energy minus the workfunction, or $KE = \hbar\omega - W$. Because the kinetic energy is given by the electric charge times the work function, we then find that $|eV_s| = \hbar\omega - W$, or

$$|V_s| = \frac{\hbar}{e}\omega - \frac{W}{e}. \quad (1.7)$$

The stopping voltage plotted versus the photon energy is given by an intercept that depends on the workfunction and has a *universal* dependence on the photon frequency with a slope of \hbar/e . This is the same for all metals! The key question is

which workfunction, as we have the emitter and collector plates. While you might have thought, from the discussion above that it is the emitter workfunction, the intercept is actually that of the *collector*. The argument for why is not so simple, but can be explained as follows. When two metals are brought together, they have a contact potential between them that forms as electrons flow from one to another until the flow stops (if you have every screwed the wrong screw into a metal and seen it later corrode, this is the reason why). This capacitor-like set up creates an intrinsic potential difference, which is not measured on the voltmeter. Hence, for an electron to leave the emitter and be collected at the collector, it must go from outside the collector to inside. This requires passing through the workfunction of the collector. The kinetic energy must be large enough for the emitted electron to pass through this barrier. Hence, it is the workfunction of the collector that determines the stopping voltage. We discuss this further in the problems. One can measure the potential difference between the emitter and collector $W_e - W_c$ to correct the data to determine the work function of the emitter, but this requires additional experiments and a careful set-up to do so. Confused? So was Einstein, so you are in good company. Become a surface scientist and these ideas will become commonplace to you. We have a problem that discusses these ideas further too, so you can get some practice with them.

Of course, back at that time, with the light sources being broadband, such an experiment could not be done. In fact, most physicists preferred Lenard's trigger model and rejected Einstein's particle picture. One highly vocal critic was Robert Millikan, a US physicist who first worked at the University of Chicago and then moved to the California Institute of Technology. He already was famous for an ingenious measurement of the electric charge, which we will discuss in Sec. 1.2. He worked hard to get more monochromatic light sources to test Einstein's prediction. Millikan's image along with his results, are shown in Fig. 1.6. One can see excellent agreement with the prediction.

In the early 1900s, this was enough information to establish photons as having particle properties. This picture was further enhanced when Arthur Compton showed that photons scatter off of electrons as particles too (a problem often covered when studying collisions in special relativity). But, the situation is more complicated than originally thought. In the 1960's it was shown that one can obtain all of the experimental results seen for the photoelectric effect using a quantum description for the electrons and a classical electric field description for the photons [6]. Note that this does not exclude the particle picture for photons, it just shows that the photoelectric effect alone cannot establish it. So, why did we spend so much time discussing the photoelectric effect? Because it leads to a critically important technology—it can be used to detect individual photons.

This is the next story we will develop and it will show how photons are firmly established once and for all. We start with the detection of individual photons. The one critical aspect from the photoelectric effect that we learned is that if photons exist then the photoelectric effect has single-photon sensitivity, because one photon in will release one electron out. This can then be used to detect the single photon. What we need to do is to amplify the signal so it is large enough to be seen with

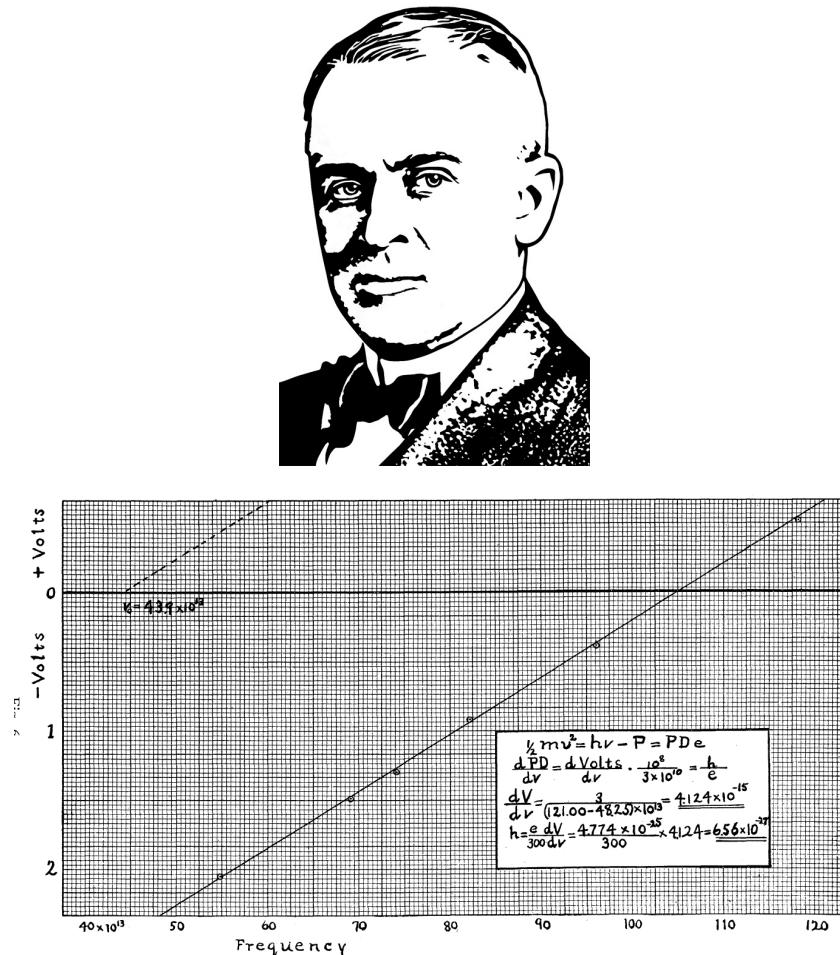


Fig. 1.6: Top: Robert Millikan in 1928. Bottom: Experimental data for six different colors of light showing the linear dependence of the kinetic energy of the electrons on the frequency with a slope of \hbar/e . Millikan won the Nobel prize in 1923 for this work and the oil-drop experiment. Note that Millikan is another scientist who had unpleasant world views. He strongly supported the eugenics movement in the United States and there were issues with his other famous experiment discussed in the next section. [Top image: Photograph by Harris and Ewing, courtesy of AIP Emilio Segrè Visual Archives, Gift of Hugh Logan and converted to a stencil by the authors. Bottom: © American Physical Society, used with permission and extracted from [5].]

classical measuring devices. This combination of detect and amplify is called a photomultiplier tube and it is a clever way to detect individual photons.

The strategy is simple. Take the single electron and use a process to multiply it by a factor of two to three at each stage. Use 30 or so stages and you have a huge pulse of electrons that can be easily measured. The amplification is done by accelerating the electrons to hit metal electrodes, which then emit multiple electrons and then repeating it. The schematic setup is shown in Fig. 1.7. There you can see the thin metal plate to the left where photons are converted to electrons and then the series of dynodes where the electron signal is amplified. Note that the photon is destroyed in this process, so this is called a destructive measurement of the photon.

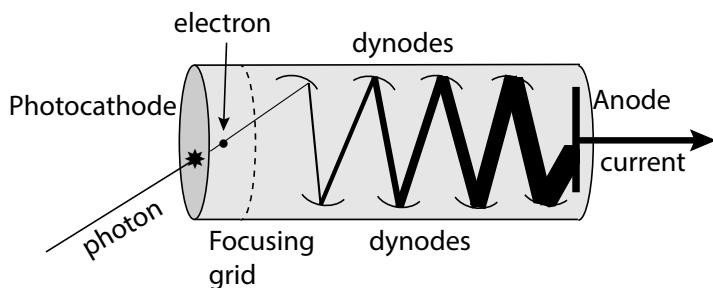


Fig. 1.7: Schematic of a photomultiplier tube. The photocathode metal, which is struck by a photon and emits an electron is on the left. The curved electrodes, called dynodes have a large voltage difference between them. Electrons are focused onto the dynode with enough energy that they will emit two to three electrons for each electron that hits it. The cascade ends with a sizable current pulse that can be measured using classical equipment.

This procedure of finding a way to have single quantum sensitivity and amplifying the signal so it can be measured is the most common way of setting up a measurement that can detect individual quanta. We will see it repeated many times.

So, we now know how to detect photons. How do we make a single photon light source? Is it just extremely dim light? It was initially thought to be just that. The argument goes as follows: create an attenuated light signal so that on average less than one photon is in the device at any given moment. This experiment was done in 1909 by Geoffrey Taylor [7]. Taylor shined light onto a needle and collected the light on a photograph. He attenuated the light so much that one of his images took three months to generate enough photons hitting the film that the diffraction pattern could be seen. This experiment must correspond to single photons, doesn't it? It turns out it does not, and we will explain why shortly. Physicists did not know this in the early 1900's, so Taylor's experiment was influential in showing that photons could create diffraction images one photon at a time.

Let us first think about alternative ways to make single photon light sources. We have to give you one more fact—atoms emit light only at very specific frequencies,

which vary from one atom to another. The prevailing explanation was that the atom has discrete energy levels and electrons transitioning from one level to another would emit a single photon, with an energy given by the energy difference between the two atomic energy levels. This idea was a critical part of the Bohr model for the atom, if you have heard of that before. This seems to indicate that we can easily make single photons sources by just creating atomic transitions. The problem is that we do not know when an atom is excited and when it emits the photon. If we have many atoms in a gas and we excite them, then the photons would be emitted randomly from the different atoms, and we would not be able to just observe individual photons as more than one could be emitted during our measurement window. If we want a single-photon light source, we have to be sure we have one and only one photon emitted during the experiment.

Finding a way to do this took until the late 1970's. A popular choice was the calcium cascade heralded single-photon light source. This is an atom that is excited by intense laser light to a state, where it emits two electrons—one about 10 nanoseconds before the other. The two photons have different energies so by selectively measuring the first photon, we know the second will emerge within about 10 nanoseconds later. The set-up is shown schematically in Fig. 1.8.

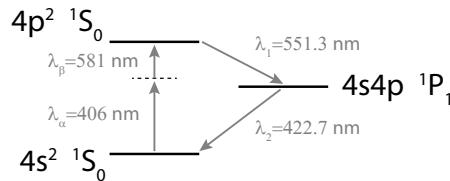


Fig. 1.8: Calcium cascade heralded single-photon light source. Two photons are needed to excite from the $4s^2 \ ^1S_0$ state to the $4p^2 \ ^1S_0$ state. This is accomplished by using two intense lasers with wavelengths of 406 nm and 581 nm shining onto a gas of calcium atoms. Then a photon of 551.3 nm wavelength will be emitted first and within 10 nanoseconds or so, a second photon of wavelength 422.7 nm is emitted.

One can immediately see the benefits of this approach. While the emission of the first photon is random, if we can detect it, we know a precise time window where the second photon will be emitted. This allows us to activate our system to be ready to perform a measurement only for a short time window after the photon is heralded. There are a few things to note about this approach. First, we do not have the photons emerge “on demand.” Instead, they emerge randomly. Second, there may be many reasons why we do not detect the first photon, or we detect the first, but not the second, so the process is never even close to 100% efficient; we just need it to be efficient enough. Third, we need to have the rate of heralded photon emission to be low enough that it is not likely to have two heralded events within our measurement

window. Fortunately, these issues are all able to be dealt with, so this source can be used for single photon experiments.

Our explanation makes it seem clear that this can create a single-photon source that we can work with. But, we are physicists, so we want to verify that it really is a single-photon source. How can we do this? Recall that a photon can be measured once and only once in a destructive counting experiment (such as with a photomultiplier tube). So, if we can create a situation where the photon can be measured with the same probability in two different detectors, then we should only see single measurements and never coincident measurements. Such an experiment is called a G2 experiment and it is shown in Fig. 1.9. We describe it in detail next.

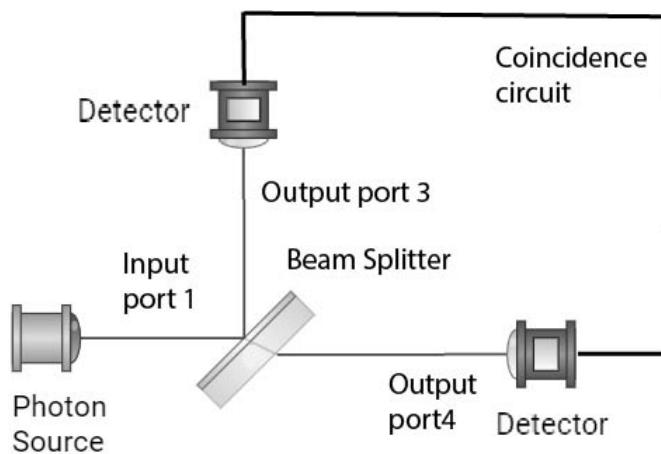


Fig. 1.9: In the G2 experiment, we send a photon toward a 50–50 beam splitter from a photon source on input port 1. Then, we set up detectors on the reflected and the transmitted paths, corresponding to the two output ports. We count the photons measured in each detector as well as the number of times we see coincidences. Analyzing this data confirms whether we have a classical light source of dim light, or a quantum light source of single photons.

The idea is simple. Let the source photon pass through a beam splitter where there is an equal probability for the photon to be reflected or transmitted. Then we measure whether it was detected on the reflected path or the transmitted path. We will never see it detected on both. The full quantum analysis is more complex and nuanced than what we described above, but we will get into that later in the book. This is our expectation for a single-photon light source, where we can restrict our measurements only to the times when the single photon will be emitted.

What about a classical light source? If there is a probability of p for a single photon to be in the experiment at a given time, then the probability for two photons to be there goes like p^2 . Similarly, if I measure the probability for the photon to go on

the reflected path as p_r and along the transmitted path to be p_t , then the probability for us to see both should be the product $p_c = p_r \times p_t$, where c refers to coincidences. This is, of course, a quite different prediction. Can we test the two? Yes, we can. What we want to do is compare the probability to be measured in either with the probability for coincidences. If the experiment is run for a time T , and we have N_H heralded responses where we should have a photon enter the apparatus, then we will find N_r and N_t reflected path detector and transmitted path detector counts. The probabilities are the ratios of the numbers counted with the numbers heralded. We also measure the number coincidences within the measurement window N_c . We then need to compare the probabilities as follows. Define

$$\alpha = \frac{P_c}{P_r P_t} = \frac{N_c N_H}{N_r N_t}. \quad (1.8)$$

For a single-photon source, we expect this ratio to be close to zero. For a classical light source, we expect the ratio to be close to one.

This experiment was performed by Nobel laureate Alain Aspect and Phillippe Grangier in the 1980's. Their picture is in Fig. 1.10 along with the results for the measurement on the heralded calcium cascade light source [8]. One can see that they are able to obtain results with $\alpha \approx 0.05$. You might ask why isn't it zero? The answer is that the photomultiplier tubes have dark counts, where they go off without a photon triggering them. Also, we could have two atoms emit photons within the same measurement window. And so on. Now, with more modern equipment, and better single-photon sources, one can do even better. We use data from an experiment run by Kiko Galvez with a collection time of 42 s, which found $N_H = 2,914,321$, $N_r = 108,972$, $N_t = 120,788$, and $N_c = 128$. You can see that we easily find $\alpha = 0.028$. So, this experiment verifies that we really have a single-photon light source. Note that the Galvez results use a slightly different set-up than the classic G2 experiment, but the final interpretation is the same.

What about the classical light source of dim light (even from lasers)? Here, we dim the light and call the heralded counts as the counts in a time interval that goes straight to a detector from the source, with no beam splitter. Then we have data from Aspect and Grangier [9], which gives us $N_H = 9.55 \times 10^8$, $N_r = 8.32 \times 10^5$, $N_t = 9.14 \times 10^5$, and $N_c = 840$, yielding $\alpha = 1.05$. This clearly acts like a classical light source.

Why the difference? Because light likes to bunch together. This is called photon bunching. It means that when we have a classical light source, we will have multiple photons emitted from time to time and we cannot have a proper single-photon light source. The single-photon sources behave like they are antibunched, so they really are isolated photons.

This then establishes, without a shadow of a doubt, that single photons really exist and that we can perform experiments with them. This result, stemming from the 1980's ushered in the second quantum revolution, where we can measure, manipulate, and control individual quanta. We are currently deep into the second quantum revolution and anticipate many new technologies to come with it. You may be fa-

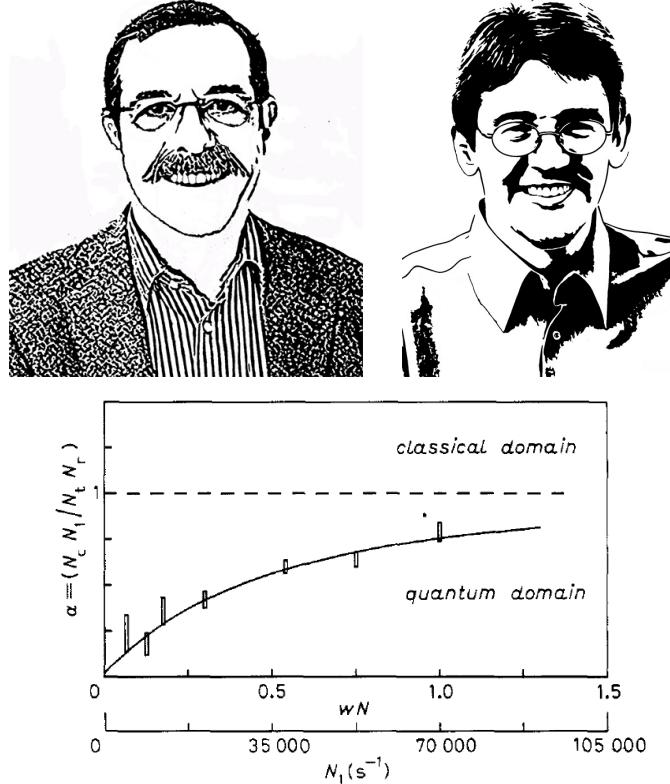


Fig. 1.10: Alain Aspect (top left) and Phillippe Grangier (top right) along with their measurement of α for a calcium cascade heralded single-photon source (bottom); Aspect won the Nobel prize in 2022. Note how their best result has $\alpha \approx 0.05$. Any value of $\alpha < 1$ (taking experimental error into account) means one has a quantum source of light. [Aspect image used with permission, Grangier image used with permission from optica, and experimental image used with permission from Institute of Physics and appearing in [8]. Images of Aspect and Grangier were converted to stencils by the authors.]

miliar with some already that have already appeared such as the global positioning system (GPS) or quantum computers.

1.2 What are electrons?

Now that we have started to explore what photons are, we have another fundamental particle to discover—the electron. We mentioned earlier about the photoelectric effect, where shining light onto a metallic surface would emit electrons if the frequency of the light was high enough. One could also get a beam of electrons to be emitted by just creating a high enough voltage between a cathode and an anode. These beams were called cathode rays and the vacuum tubes within which they lived were called cathode ray tubes (not too surprisingly, given his skill with this type of apparatus, they were originally called Lenard rays). Once the vacuum in these tubes was good enough, one could see that the cathode rays bent in response to a magnetic field. This means they are charged particles moving with some velocity. Here, we can see the beam with our eye and perform experiments on it, even if we cannot see the individual electrons.

How do we determine the charge, mass, and velocity of these individual particles if we cannot directly see them? This requires working with some clever ideas from electromagnetism. First, recall that the Lorentz force law on a particle of mass m , charge q and moving at a velocity \vec{v} is

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}. \quad (1.9)$$

The clever idea is to note that if the electric and magnetic fields are perpendicular, then we can set them up so that their forces are opposite and cancel. Consider the set-up in Fig. 1.11, with the cathode ray running from left to right, the electric field vertical, and the magnetic field out of the page. Then $\vec{v} \times \vec{B}$ points down, opposite to the electric field, and the two forces can be canceled by adjusting one of the field strengths. In this case, the cathode ray will go straight from left to right. The velocity is given by $|\vec{E}|/|\vec{B}|$. Note that we are assuming that the particles in the beam travel at a nearly uniform speed. Such a device is called a Wien filter. It is a way to measure velocity (or equivalently momentum if the mass is known) by measuring the position (of the beam).

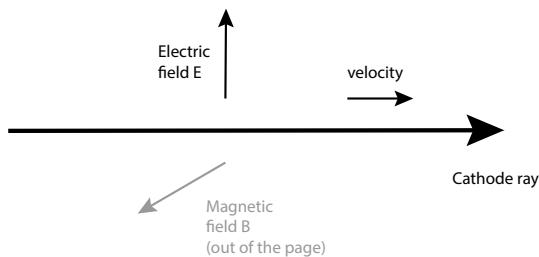


Fig. 1.11: Schematic of the forces on the cathode ray. The electric field is vertical, the magnetic field is out of the page, and the velocity is left to right. One can see that $\vec{v} \times \vec{B}$ is directed opposite to \vec{E} .

John. J. Thomson was the first to discover the electron (see Fig. 1.12), although he was not completely convinced that he had done so [10]. After seeing the cathode ray being deflected by fields, he used a thermodynamic measurement to infer the approximate mass of the particle and found them to be about 1000 times smaller than atoms. He is most famous for his measurement of the m/e ratio though, which was done by separately examining a beam with a vertical electric field, which bends the beam up or down, and a magnetic field out of the page that does the same. If we assume the electric field is uniform and of strength E , and acts over a length l along the beam (as we would have for a field created by parallel metal plates on top and bottom of the beam), then the acceleration of magnitude qE/m acts for a time l/v and imparts a velocity in the vertical direction of size $v_{\perp} = qEl/mv$. If we measure the deflection after some additional time of flight (Thomson did this by letting the beam hit a spherical glass chamber coated with fluorescent material as shown in Fig. 1.12), the angle will be $\theta \approx v_{\perp}/v$ or

$$\theta = \frac{qEl}{mv^2}. \quad (1.10)$$

Similarly, if we use Helmholtz coils to create a uniform magnetic field over the same region, then the magnetic field gives a force of magnitude qvB and a similar analysis yields the angle of deflection ϕ to satisfy

$$\phi = \frac{qBl}{mv}. \quad (1.11)$$

The idea is to set the magnetic field so its deflection is the same as the electric field deflection. Note that the v appears to the inverse square power in the θ equation but only inversely in the ϕ equation. We want to eliminate it to make things simpler. This can be done by writing $\phi = \theta^2/\theta$ or

$$\frac{qEl}{mv^2} = \left(\frac{qBl}{mv} \right)^2 \frac{1}{\phi}, \quad (1.12)$$

or

$$\frac{m}{q} = \frac{B^2 l}{E \phi}. \quad (1.13)$$

Note the clever way to modify the expressions to determine the ratio of mass to charge.

Even though Thomson has estimates of the mass of the electron, one needed a more accurate value. This was achieved when Robert Millikan (see Fig. 1.6) successfully measured the charge of the electron [11]. He did this using the strategies we identified before—first find a mechanism that has single quantum sensitivity and then amplify the signal until it can be observed with classical equipment. Here, what Millikan did was to measure the charge of the electron by attaching electrons onto small drops of oil and placing them in a vertical electric field, designed to counter the downward force due to gravity. By using a telescope to image the size of the drop, and knowing

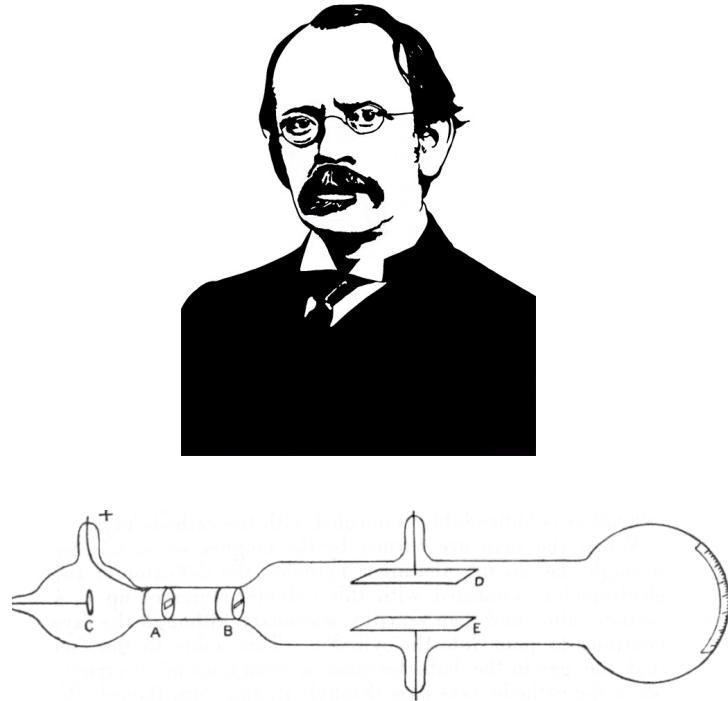


Fig. 1.12: Top: John J., Thomson around 1900. Bottom: Crooke's tube used for the m/e experiment. Note how the cathode ray is created and focused from the left, bent in the center, and read out on the right. [Thomson image is public domain as is the experimental apparatus, which comes from [10]. Thomson image converted to a stencil by the authors.]

its density, the mass of the drop could be estimated. Then one can measure the net force on the drop by determining its acceleration and subtracting the gravitational force to find the electric force. Dividing by the known electric field yields the charge. The experiment works even better if the drops can be made to float, so that the forces are equal. Millikan repeated the experiment many times and discovered the charge on the oil drop was always a multiple of a single fundamental charge, which he identified with the charge of the electron.

There are a few issues associated with this experiment. First, how do I make small sized oil drops? Second, how do I attach a charge to them? And third, how do I illuminate them so they can be seen? It turns out that many of these technical details were worked out by Millikan's student Harvey Fletcher, who was not an author on the original work. Millikan and Fletcher were both at the University of Chicago at the time, and the University policy is that Ph.D. students need to publish a solo

authored article to graduate. The two agreed that Millikan would publish the first as a solo author and Fletcher the second article as a solo author. In this case, the first article won a Nobel prize! You can read more of the history of their relationship in a wonderful article that discusses all of the details [12]. The original experiments were using water, but the droplets evaporated too fast. Fletcher suggested using watch oil, which is designed not to evaporate. When he sprayed the oil between two brass plates attached to a high voltage, he could see some move up and some down—the friction of the spraying attached charges automatically to the oil, allowing the experiment to just work. At this point, Millikan dropped working with another student to work solely with Fletcher. What is clear is that the original idea was Millikan's and Fletcher provided critical technical expertise to help carry it out. Millikan also faced some controversy related to bias in how he selected some of his data. None of this detracts anything from the experiment itself, which is beautiful example of an interaction-free experiment that has single quantum sensitivity and clearly establishes that charges are discrete multiples of the fundamental charge of the electron.

1.3 Discussion

In our introduction to quantum mechanics, we have discussed the particle nature of quanta—the photon and the electron. They are manifest in two different ways as well. For the photon, the key is that it can be measured once and only once. For the electron, that its charge is quantized and discrete. The experiments that lead to these particle pictures for both are based heavily on classical physics and classical ideas. Along the way, we also saw that some of the classical ideas no longer held. This is a continuing theme in quantum mechanics. Classical ideas can help describe much of the material, but eventually give way to quantum phenomena, which are somewhat different. By basing our discussions on the classical physics that we already know, we are helping us build what is unique and new about the quantum world, while also recognizing that much of our classical intuition still holds.

Even though the particle nature of the photon and the electron are emphasized here, both objects also display wave properties. This is the wave-particle duality of quantum mechanics, which will be discussed more thoroughly later in the book. We hope you have found this introduction to the quantum world intriguing. We have a lot more cool ideas and cool results to show you, so be ready for a wild ride!

Each chapter will end with problems that support the materials discussed in the chapter. Some of these problems will seem similar to the types of problems you have worked on before in an introductory course, but they may require you to work more independently than in the past where you will need to put together ideas you have seen before to be able to solve them. This process helps you become more sophisticated in your ability to solve problems. Others will ask you to explain or write short essays about different phenomena. This may seem to be very different from what you did before. The reality is that in Physics, the ideas are just as important as the equations,

and you need practice to develop the skill to work with these powerful ideas. The problems should help you with this as well.

Chapter 1 problems

Problem 1.1 Photoelectric effect by absorbing energy directly from light. Assume a light beam is turned on that emits energy at the rate of 10^6 eV/s and extends over a beam area of 0.0001 m^2 . Assume that the energy is uniformly given to each unit cell of the metal, whose dimension on the surface is a square of length 3 \AA . Once the energy deposited to the electron in the unit cell reaches the magnitude of the work function of the emitter (take it to be 4 eV), the electron will be emitted. How long does it take for the electron to be emitted if it absorbs all of the energy directly from the light? Note that this model predicts that the electrons are also emitted once they exceed a threshold, so they all will be emitted with about the same kinetic energy.

Problem 1.2 Resonance or near resonance. Using the same numbers as in the text, plot the maximum amplitude of the steady state harmonic oscillator near resonance as a function of the absolute value of the detuning (a log-log plot is appropriate for this). Determine how close the detuning needs to be to reach a displacement of 1 nm and 10 nm. Compute the time it takes the resonant case to reach the same displacement. Approximately how many oscillations are needed for the resonant case to reach this displacement?

Problem 1.3 Damping. We chose to ignore damping effects. Most metals at room temperature have a damping rate on the order of $10^{14}/\text{s}$. How many oscillations of the electric field occur during a time equal to the inverse of the damping rate? How does this compare to the number of oscillations needed to achieve the displacement required for emission from the metal?

Problem 1.4 Coherent versus incoherent light. We assumed coherent light (periodic drive), but the light sources of that time were not coherent. This means that the light acts like a smooth oscillation only for a time on the order of the coherence time of the source. Then the phase of the electric field is randomized and it starts over. How does this affect the classical model?

Problem 1.5 Einstein and photons. Explain how Einstein's theory supports photons being particles.

Problem 1.6 Which workfunction? Because the emitter and collector plates are connected through the circuit, charges move to equalize the chemical potential between the plates. This places a charge difference on the two plates and a potential

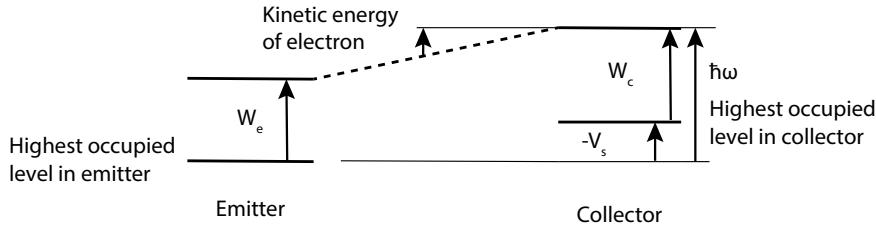


Fig. 1.13: Schematic of the energy levels in the emitter and collector. The photon energy is $\hbar\omega$. The emitter workfunction, and the kinetic energy of the electron are shown starting on the left. The dashed line approximates how the kinetic energy of the electron is reduced as it approaches the collector due to the difference in chemical potentials. One can clearly see that $|V_s| + W_c = \hbar\omega$, implying it is the work function of the collector that is directly measured. This can be corrected to give the work function of the emitter.

is present, even if the voltage is set at zero. Using the diagram in Fig. 1.13, which shows the energies involved, argue why the measurement in the Millikan experiment yields the workfunction of the collector plate, not the emitter plate. One can measure this potential difference, which Millikan did, to correct this issue. See Ref. [13] for more details on this subtle point.

Problem 1.7 Heralded photon sources. Write an essay that describes how you can make the heralded photon source work. For simplicity, assume all heralded photons emitted from the atom that travel to the left will be detected, while all secondary electrons emitted to the right will go to the experiment. Do not forget that light travels at a finite speed and electrical signals in wires do not travel that fast.

Problem 1.8 Data for the G2 experiment. The two tables summarize data for different experiments on the calcium cascade light source and on the light-emitting-diode light source. Calculate α for each of these cases and explain what your results mean.

Problem 1.9 Dim classical light. The argument that if your intensity is so low that a single photon is emitted every microsecond, and your apparatus is one m long, so we should never have two photons in the apparatus at the same time is a compelling argument for dim classical light acting like a single-photon source. First use a quantitative analysis to explain why we should only see single photons. Then, explain why an examination of the statistics of the emitted photons will show that it is not a single-photon light source.

Problem 1.10 Estimating the deflection for the m/e experiment. In the text, we showed a calculation for how to estimate the vertical velocity imparted to the electron. The discussion might not have been familiar to you. Let us go over just how it works. To start, we will make an approximation that the horizontal velocity is large and that the vertical force acts uniformly and only in the region of length l . Then, to determine the acceleration, and thereby the final vertical velocity, we need only determine the time the particle is in the region of length l . Explain how to do this. Explain further why this is a good approximation, in general. Verify that the equations worked out in the text are correct.

Problem 1.11 Alternative m/e experiment. Assume we used the Wien filter idea and determined the approximate horizontal velocity in the cathode ray. Could another experiment be used to find the m/e ratio instead of the way Thomson did it?

Problem 1.12 Oil-drop experiment. If the density of the oil used in the experiment is the same as that of water, and the radius of the oil drop ranges from 10 microns to 100 microns, determine how large the force from a parallel plate capacitor with a separation of 1 cm and a voltage of up to 1000 V is applied to it. Is this sufficient to get the drop to float if it has just a single excess electron on it?

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