

QUANTUM SUPERMAPS ARE CHARACTERIZED BY LOCALITY

Matt Wilson^{1,2} Giulio Chiribella^{2,3,4,5}, Aleks Kissinger²

July 21, 2023

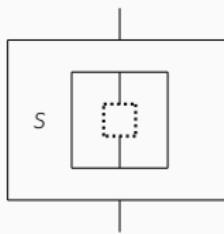
1. Department of Computer Science, University College London
2. Department of Computer Science, University of Oxford
3. HKU-Oxford Joint Laboratory for Quantum Information and Computation
4. QICI Quantum Information and Computation Initiative, Department of Computer Science,
Department of Computer Science, The University of Hong Kong
5. Perimeter Institute for Theoretical Physics

QUANTUM CHANNELS AND SUPERCHANNELS

A quantum channel:



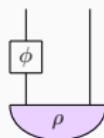
a quantum superchannel:



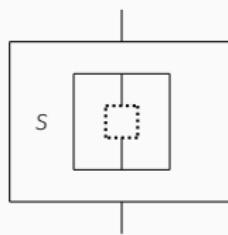
G. Chiribella, P. Perinotti, and G. M. D'Ariano, *Europhysics Letters* 83, 30004 2008

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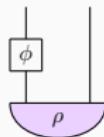
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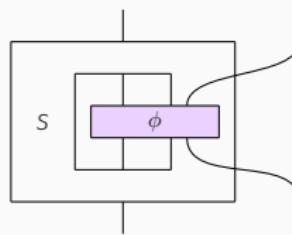
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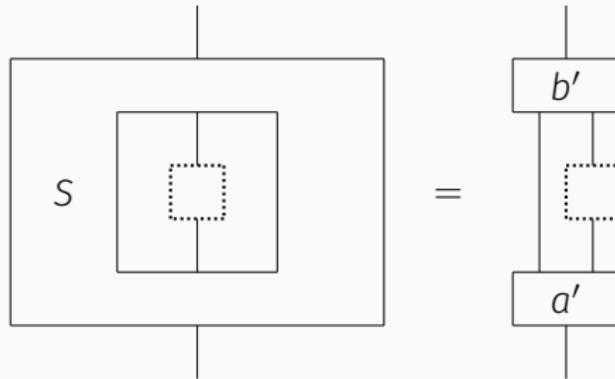
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CIRCUIT DECOMPOSITION

Circuit decomposition theorem:

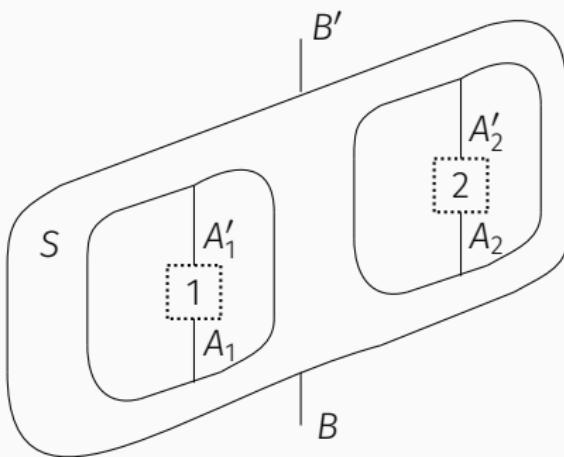


Also true for:

- Classical superchannels
- Pure superchannels

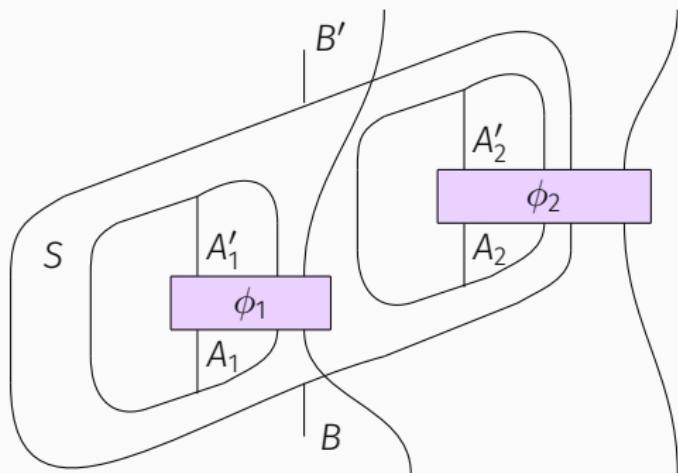
MULTI-INPUT SUPERMAPS

Multi-inputs:



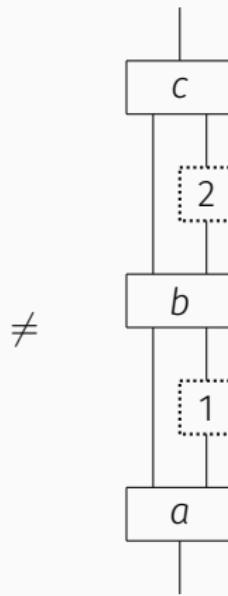
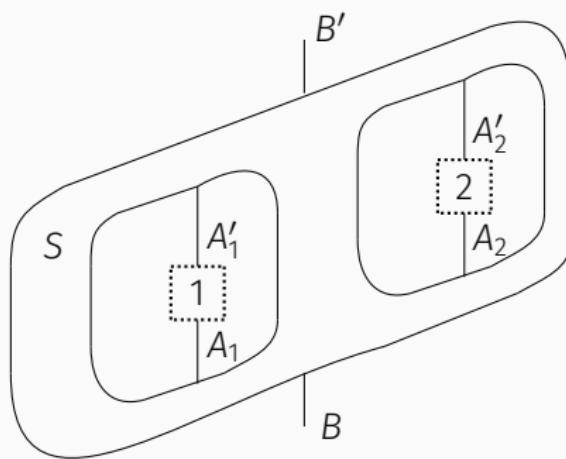
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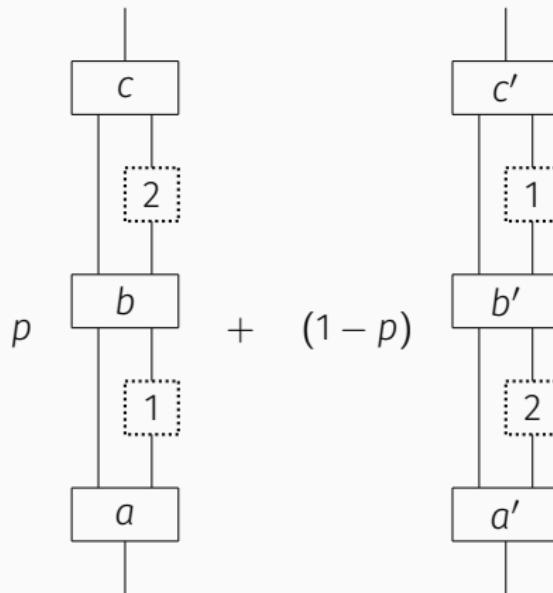
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Circuit decomposition?



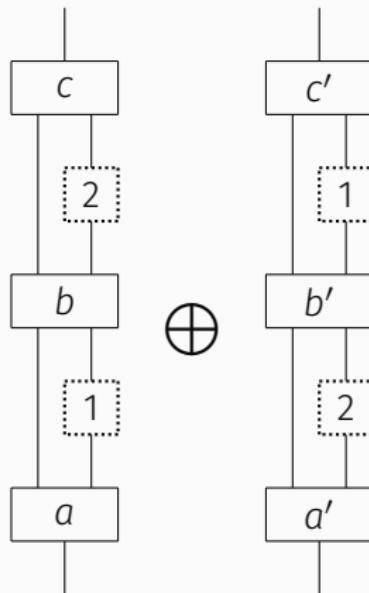
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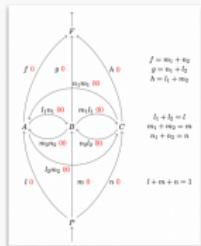
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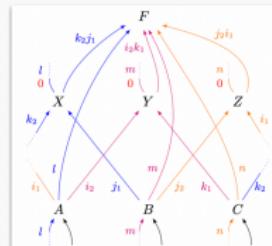
G. Chiribella, G. M. D'Ariano, P. Perinotti, and B. Valiron, arXiv 2009, PRA 2013

MULTI-INPUT SUPERMAPS

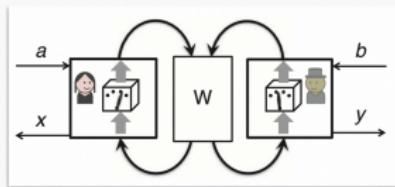
Grenoble [a]



Lugano [b]



OCB [c]



[a] J Wechs, H. Dourdent, A. Abbott, C. Branciard PRX Quantum 2, 030335 2021

[b] Ämin Baumeler, Stefan Wolf, IEEE International Symposium on Information Theory 2014

[c] O. Oreshkov, F. Costa, and Ć. Brukner, Nat Comms 2012

MULTI-INPUT SUPERMAPS: CONSEQUENCES

Computational advantage from quantum-controlled ordering of gates

Mateus Araújo,^{1,2} Fabio Costa,^{1,2} and Časlav Brukner,^{1,2}

¹Faculty of Physics, University of Vienna, Boltzmanngasse 5, 1090 Vienna, Austria
²Institute for Quantum Optics and Quantum Information (IQOQI),
Austrian Academy of Sciences, Boltzmanngasse 3, 1090 Vienna, Austria

(Dated: June 11, 2020)

Quantum computation is performed by applying gates in a specific order. In general, a control quantum system is used to switch the order of operations in a circuit with fixed order. Here we propose an interferometric method for perfect discrimination of no-signalling channels via quantum superposition of causal structures.

Center for Quantum Information and

Giulio Chiribella,
Information Sciences, Tsinghua Univ., Institute

A no-signalling channel t_{no} is compatible with two input and output channels. A strict hierarchy between parallel, sequential, and indefinite-causal-order strategies for channel discrimination

Jessica Barreiro,^{1,*} Mie Murao,^{2,3} and Mario Túlio Quintino,^{4,5}

¹Institute for Quantum Optics and Quantum Information (IQOQI),
Austrian Academy of Sciences, Boltzmanngasse 5, 1090 Vienna, Austria

²Graduate School of Science, The University of Tokyo, Hongo 7-3-1, Bunkyo-ku, Tokyo 113-0033, Japan

³Yonsei Center for Quantum Science and Technology (VCQT), Faculty of Physics, University of Vienna, Boltzmanngasse 5, A-1090 Vienna, Austria

(Dated: November 18, 2021)

We present an instance of a task of minimum-error discrimination of two qubit-qubit quantum channels for which a sequential strategy outperforms any parallel strategy. We then obtain two new classes of strategies for channel discrimination that involve indefinite causal orders and show that these even outstrip methods based on the performance of all four strategies. Our proof technique employs a general method of arbitrary-seeded proofs. We also provide a systematic method for finding pairs of channels that showcase this phenomenon, demonstrating that the hierarchy between the strategies is not exclusive to our main example.

Enhanced Communication With the Assistance of Indefinite Causal Order

Daniel Ebler,^{1,2} Sina Sabik,¹ and Giulio Chiribella^{3,4,5}

¹Department of Computer Science The University of Hong Kong, Pokfulam Road, Hong Kong
²HKU Shenzhen Institute of Research and Innovation, Kejiaodong 1000, Shenzhen, China

³Department of Computer Science, University of Oxford, 2一阵路, Oxford OX1 3QU, United Kingdom
⁴Wolfson Building, Parks Road, Oxford OX1 3QU, United Kingdom

⁵Canadian Institute for Advanced Research, CIFAR Program in Quantum Information Sciences, Toronto, ON M5G 1Z8

In quantum Shannon theory, the way information is encoded and decoded takes advantage of the lack of causality, while the way communication channels are intertwined is assumed to be classical. In this Letter we relax the assumption that quantum channels are combined causally, showing that a quantum communication network where quantum channels are combined in a superposition of different orders can achieve tasks that are impossible in conventional quantum Shannon theory. In particular, we show that two copies of a recently developed channel become able to transmit information when they are combined in a quantum superposition of two alternative orders. This striking result comes from the intuition that two communication channels become using them in different orders should not make any difference. The failure of this intuition stems from the fact that a single noisy channel can be a random mixture of elementary, non-commuting processes, whose order (or lack thereof) can affect the ability to transmit information.

Exponential Communication Complexity Advantage from Quantum Superposition of the Direction of Communication

Philippe Allard Guérin,^{1,2} Adrien Feix,^{1,2} Mateus Araújo,^{1,2} and Časlav Brukner^{1,2}

¹Faculty of Physics, University of Vienna, Boltzmanngasse 5, 1090 Vienna, Austria
²Institute for Quantum Optics and Quantum Information (IQOQI),
Austrian Academy of Sciences, Boltzmanngasse 3, 1090 Vienna, Austria

(Dated: October 8, 2018)

In communication complexity, a number of distant parties have the task of calculating a distributed function of their inputs, while minimizing the amount of communication between them. It is known that with quantum resources, such as entanglement and quantum channels, one can obtain significant improvements in the complexity of some tasks. In this work we study the role of the quantum superposition of the direction of communication as a resource for communication complexity. We present a tripartite communication task for which such a superposition allows for an exponential saving in communication, compared to one-way quantum (or classical) communication; the advantage also holds when we allow for protocols with bounded error probability.

A HOLE IN THE FRAMEWORK

Current definitions use combinations of ...

G. Chiribella, G. M. D'Ariano, P. Perinotti, and B. Valiron, arXiv 2009, PRA 2013

O. Oreshkov, F. Costa, and Č. Brukner, Nat Comms 2012

A. Kissinger, S. Uijlen 441 2021

A HOLE IN THE FRAMEWORK

Current definitions use combinations of ...

- Linearity

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A HOLE IN THE FRAMEWORK

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- Linearity
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- Choi-Jamiolkowski Isomorphisms
- Compact closure ($\text{cups} \cup$ and $\text{caps} \cap$)

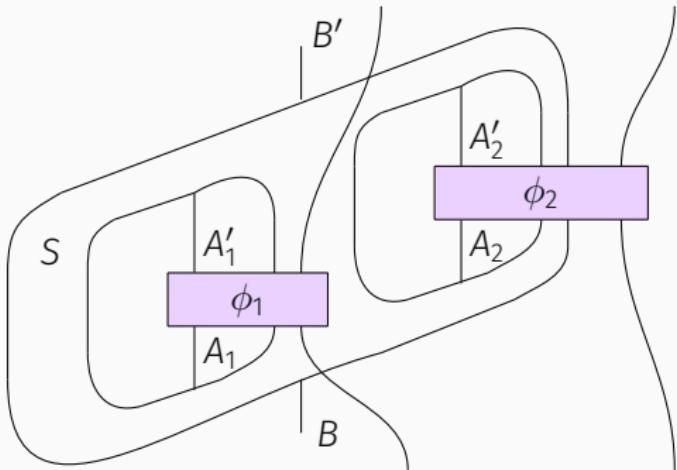
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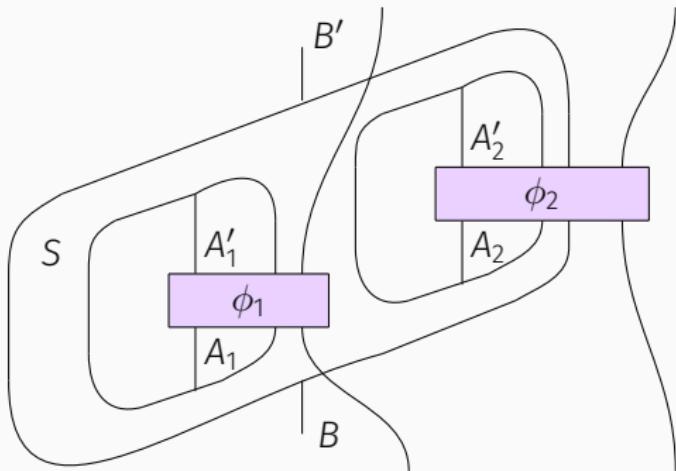
A HOLE IN THE FRAMEWORK

What kind of structure?



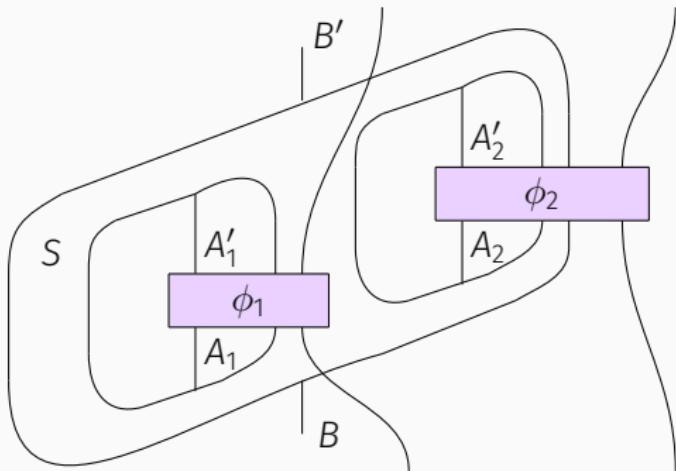
A HOLE IN THE FRAMEWORK

What kind of structure? Process-theoretic structure!



A HOLE IN THE FRAMEWORK

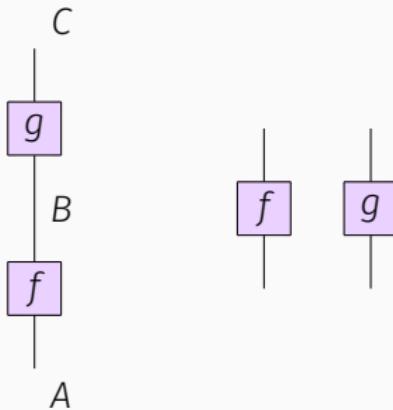
What kind of structure? Process theoretic structure!



General Hilbert spaces and operational probabilistic theories?

A HOLE IN THE FRAMEWORK

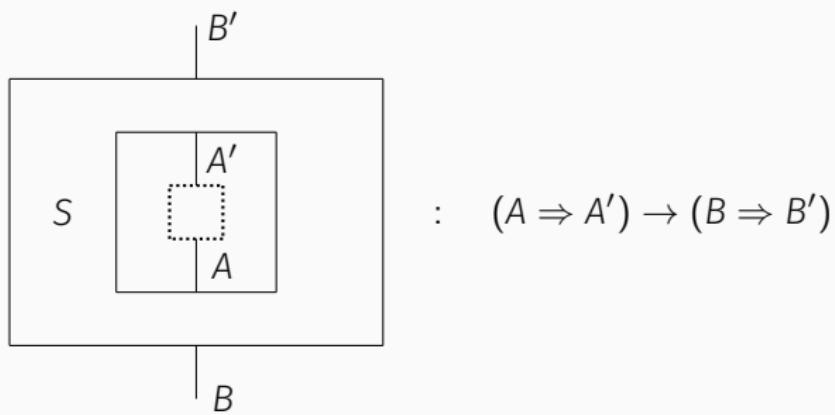
A process theory (symmetric monoidal category):



Meaning a theory \mathbf{C} with sets of processes $\mathbf{C}(A, B)$:

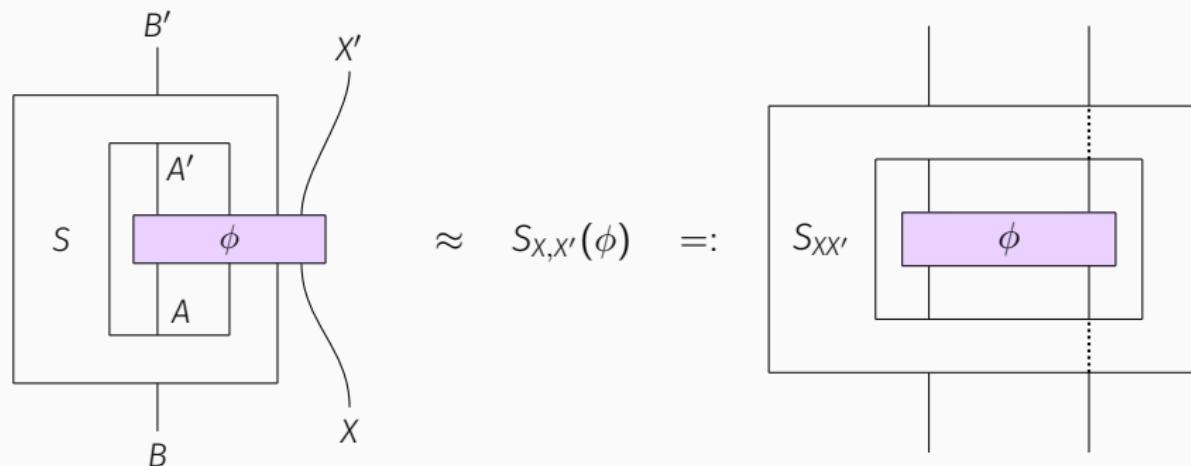
- Sequential composition $g \circ f$ and parallel composition $f \otimes g$
- Swaps

PROPOSED SOLUTION



PROPOSED SOLUTION

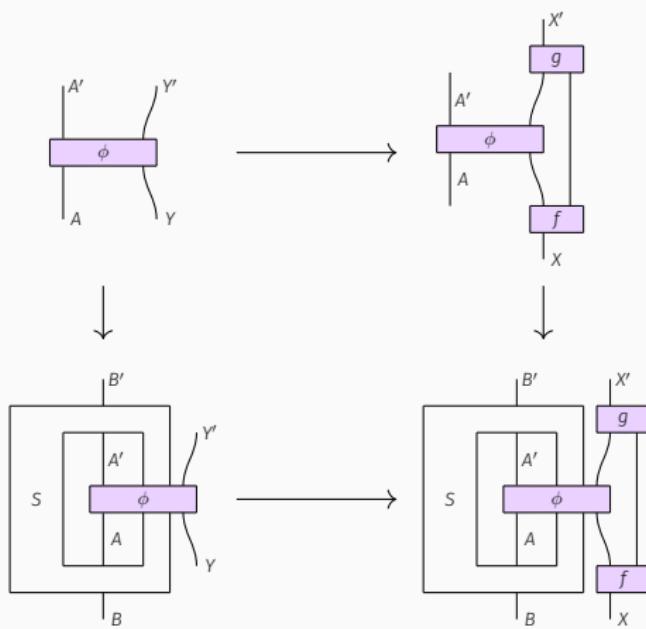
The picture suggests a family of functions



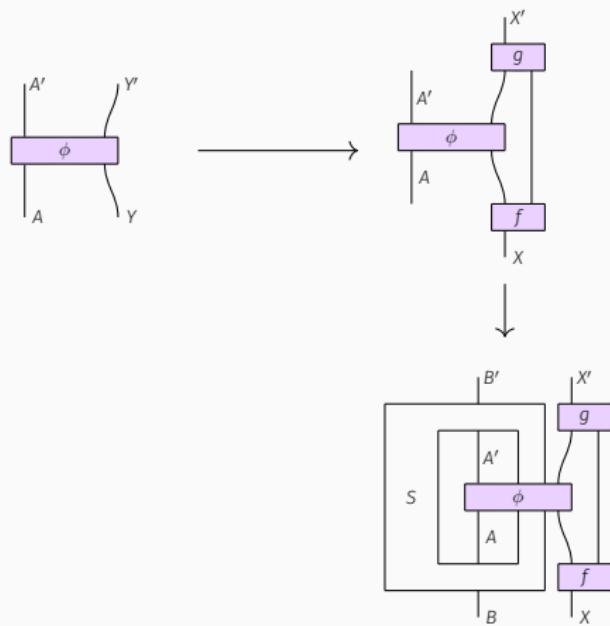
of type

$$S_{X,X'} : \mathbf{C}(A \otimes X, A' \otimes X') \rightarrow \mathbf{C}(B \otimes X, B' \otimes X')$$

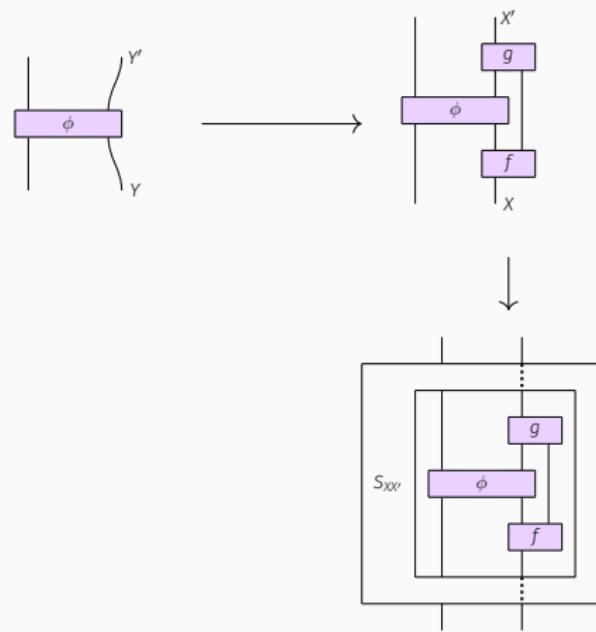
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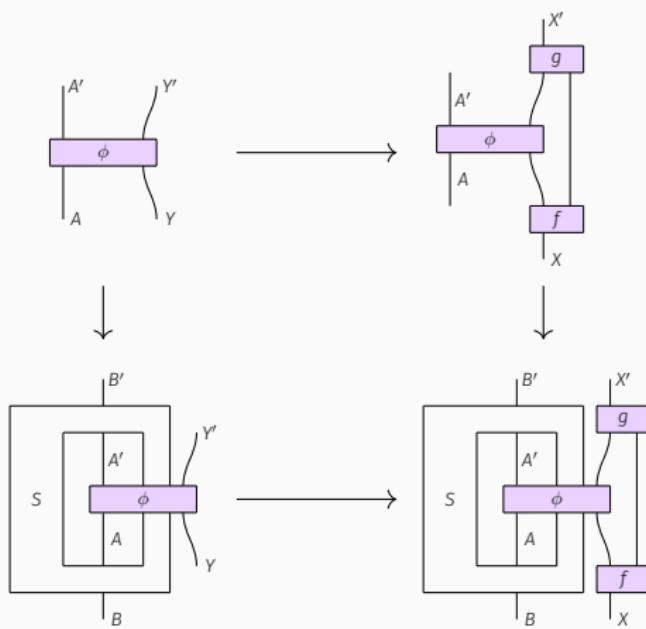
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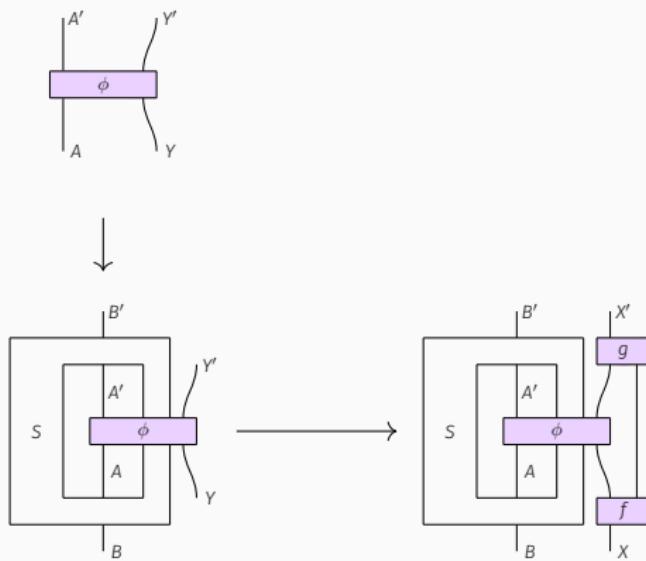
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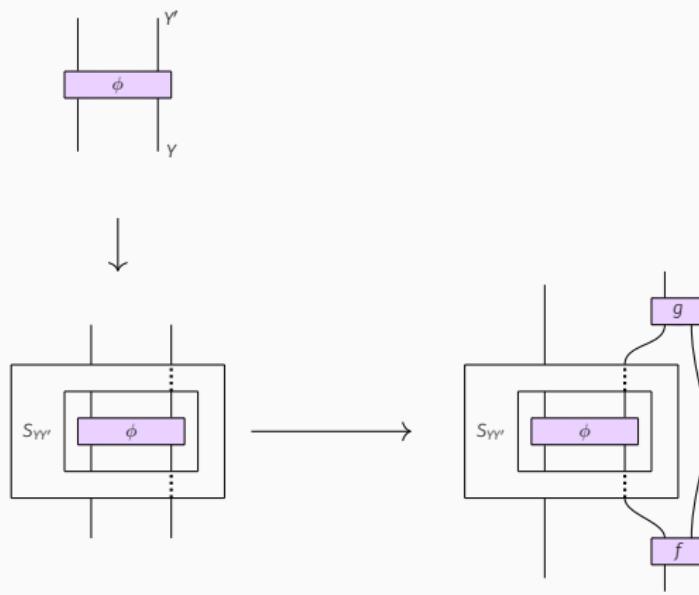
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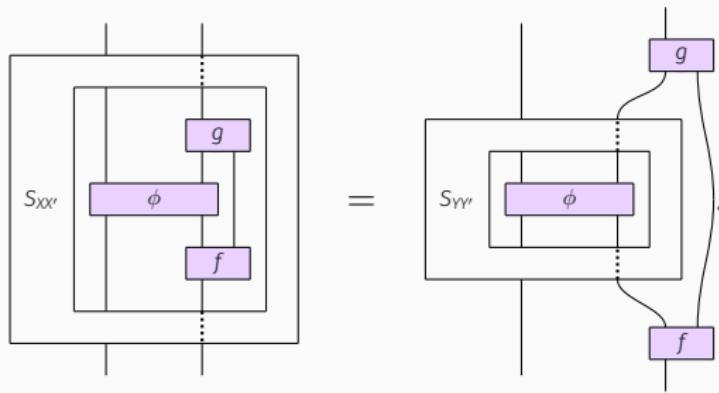


LOCALLY APPLICABLE TRANSFORMATIONS

Locality: Algebraically

$$S_{X,X'}((i \otimes g) \circ (\phi \otimes i) \circ (i \otimes f)) = (i \otimes g) \circ (S_{Y,Y'}(\phi) \otimes i) \circ (i \otimes f)$$

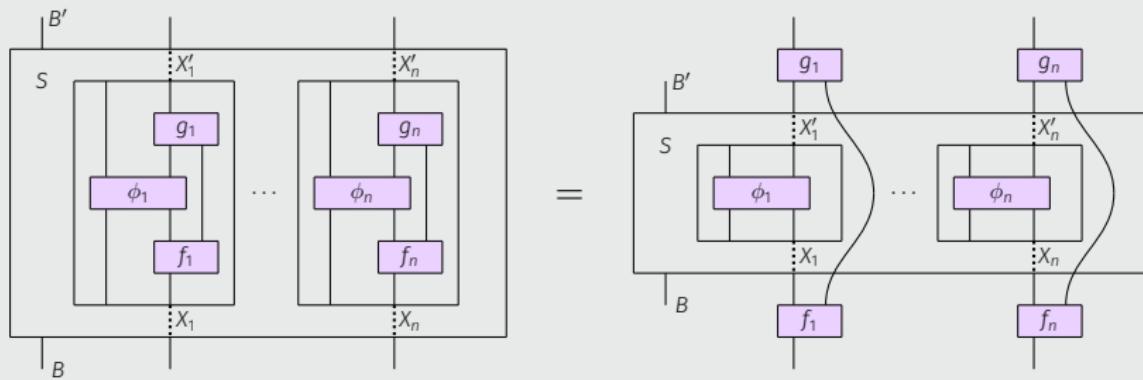
Locality: Diagrammatically



MULTI-INPUT LOCALLY APPLICABLE TRANSFORMATIONS

Definition

A *lot* of type $S : (A_1 \Rightarrow A'_1) \dots (A_n \Rightarrow A'_n) \longrightarrow (B \Rightarrow B')$ is a family of functions $S_{X_1 \dots X_n}^{X'_1 \dots X'_n}$ satisfying:



RECONSTRUCTION THEOREM

Theorem

There is a one-to-one correspondence between locally applicable transformations on quantum channels and deterministic quantum superchannels of type

$$(A_1 \Rightarrow A'_1) \dots (A_n \Rightarrow A'_n) \longrightarrow (B \Rightarrow B')$$

In categorical language, there is an equivalence of multicategories

$$\mathsf{lot[QC]} \cong \mathsf{QS}$$

CATEGORICAL SUPERMAPS

Saunders Mac Lane (Allegedly (Allegedly)):

- *I didn't invent categories to study functors; I invented them to study natural transformations.*

$$\mathbf{C}(A-, A' =) : \mathbf{C}^{op} \times \mathbf{C} \rightarrow \mathbf{SET}$$

Theorem

The quantum superchannels of type

$$(A_1 \Rightarrow A'_1) \dots (A_n \Rightarrow A'_n) \longrightarrow (B \Rightarrow B')$$

are the natural transformations of type

$$\times_{i=1}^n \mathbf{QC}(A_i - i, A'_i = i) \longrightarrow \mathbf{QC}(B -_1 \dots -_n, B'_1 =_1 \dots =_n)$$

OUTLOOK

We have a bare-minimum axiom for supermaps on any theory of processes:

- Arbitrary Hilbert spaces?
- Post-quantum causal structures?
- Reconstruction for all of Higher-Order Quantum Theory?

this axiom can be explained to:

- Category theorists (natural transformations)
- Your friends (circuit diagrams)

the only concept needed is the purely compositional one of **local applicability**.

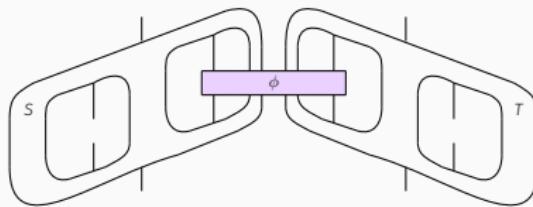
THANK-YOU FOR LISTENING!

<https://arxiv.org/abs/2205.09844>

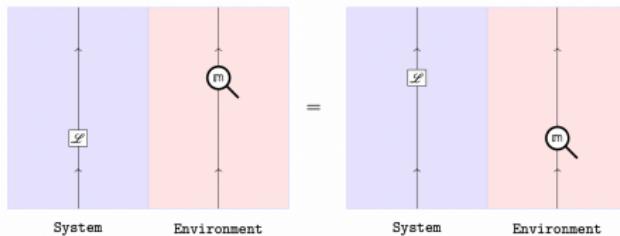


FOLLOW-ON PROJECTS

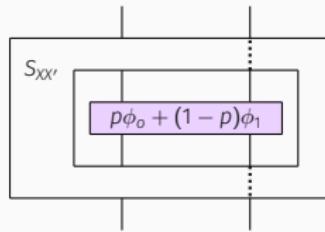
- Free Polycategories for Unitary Supermaps of Arbitrary Dimension



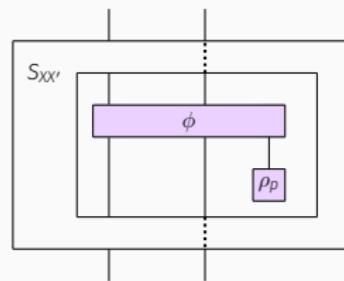
- On the Origin of Linearity and Unitarity in Quantum Theory



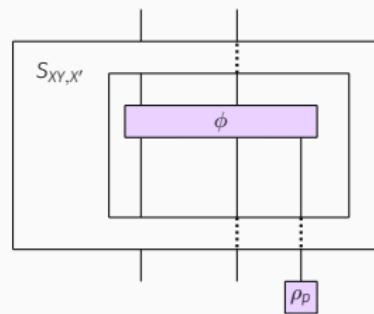
CONVEX LINEARITY



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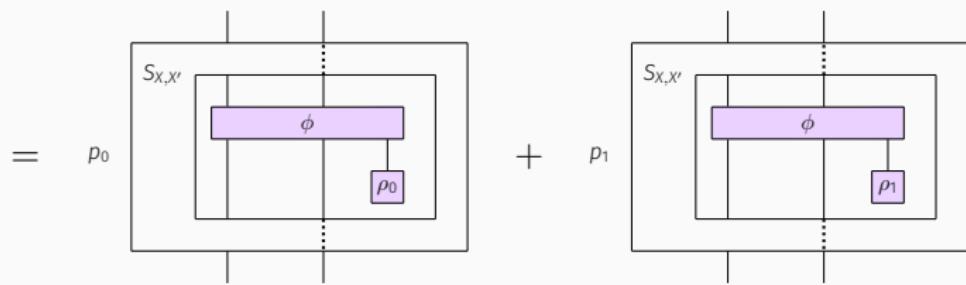


CONVEX LINEARITY

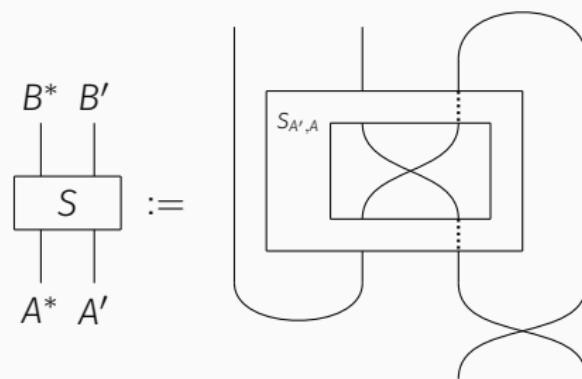
$$= p_0 \begin{array}{c} S_{XY,X'} \\ \boxed{\phi} \\ \rho_0 \end{array} + p_1 \begin{array}{c} S_{XY,X'} \\ \boxed{\phi} \\ \rho_1 \end{array},$$

The diagram illustrates the concept of convex linearity. It shows two terms being added together. Each term consists of a large rectangle labeled $S_{XY,X'}$ containing a central purple box labeled ϕ . Below each ϕ box is a small purple square labeled ρ_0 or ρ_1 . The addition is indicated by a plus sign between the two terms.

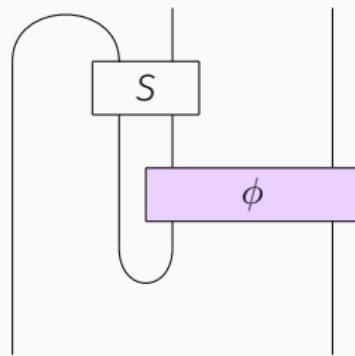
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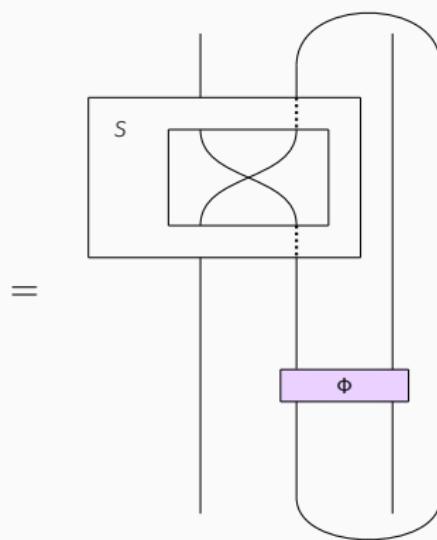
USING CUPS AND CAPS



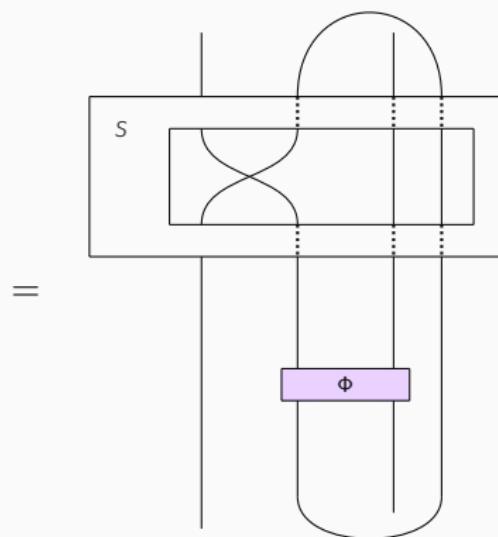
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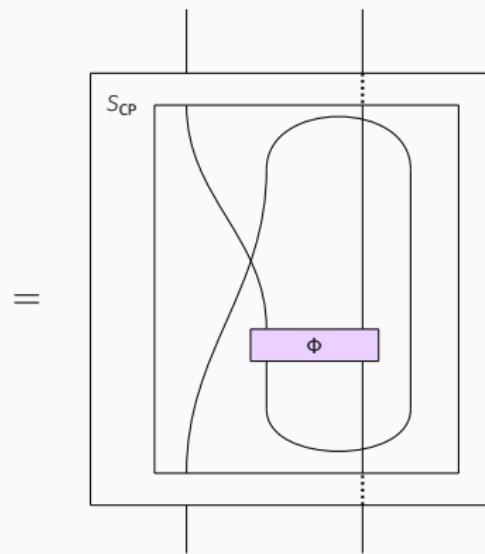
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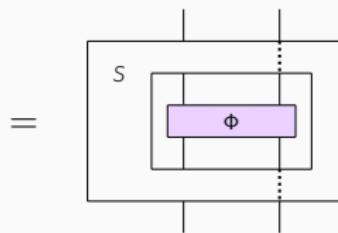
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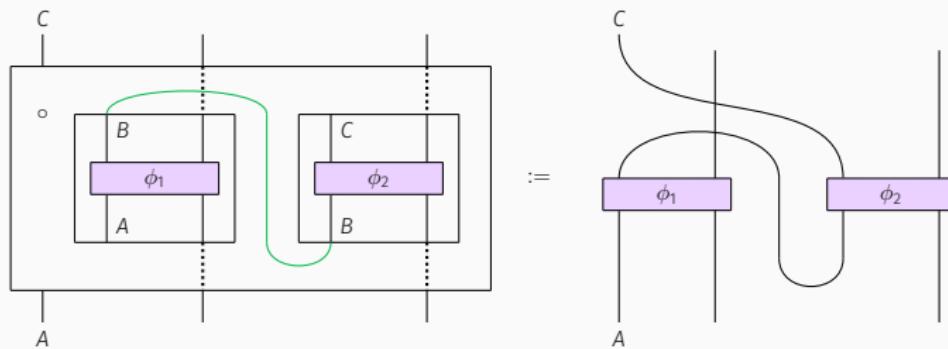


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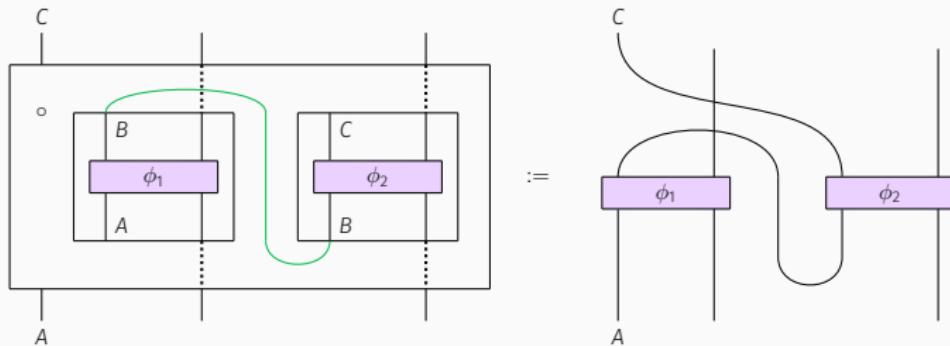
ENRICHED STRUCTURE FOR SUPERMAPS

Locally-applicable transformations enrich the category on which they act



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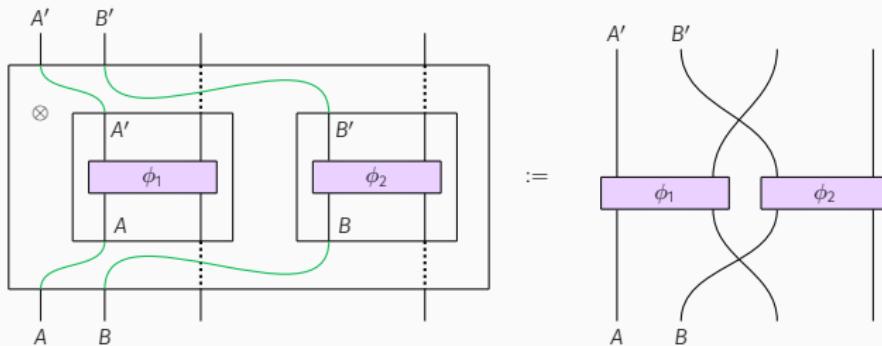
Locally-applicable transformations enrich the category on which they act



Formally have constructed a $\text{lot}[C]$ -category C

ENRICHED STRUCTURE FOR SUPERMAPS

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Formally have constructed a $\text{lot}[C]$ -monoidal category C