Non canonical polarizations of Gravitational Waves

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We hereby propose an alternative method for the detection of gravitational waves (GWs), focused on the variations of the time coordinate carried by the GW itself, postulating the theoretical possibility of tackling the detection challenge from this perspective, i.e. using clocks rather than rulers. By explicitly working outside of the transverse-traceless gauge, we propose how events with well-defined time duration, when hit by a GW, would consequently be expected to show a difference in their characteristic time, as measured from the rest frame of an outside observer, whose clock is to remain unaffected by the GW. This constitutes a theoretically viable way in the sense of detecting the passing of the wave itself and may prove relevant as a standalone method for GWs detection other than laser interferometers, or as well be implemented as a complementary but independent system of signal triggering, improving the statistical significance of existing methods. A simple but physically realistic scenario in which the appropriate conditions for the generation and detection of GWs with time dilation are met is presented, along with the conceptual design of an experimental detector.

I. INTRODUCTION

The detection of GWs with laser interferometers [1, 2] has proven immensely successful for several years now [3–6]. Interferometers measure the phase shifts of laser beams, and it is important to stress that the phase shift originates from a variation of the time flow induced by the GW. Because in any locally inertial reference frame the speed of light c is constant, the phase shift is then translated in the contraction and expansion of the distance coordinate, or, in other words, the length of the interferometer's arm. For instance, in the transverse-traceless (TT) gauge, spatial coordinates are left invariant at first-order, while what it changes is the laser's proper travel time, i.e. its phase as measured by an observer for whom the speed of light is equal to c. The problem of associating coordinates' measurements to physical variables is in general not a trivial one, and especially arduous in the context of GW detection. Spacetime is, in a sense, *undivided*, and yet we usually expect to be able to isolate a particular observable, a spatial or time coordinate, as if it were an independent quantity.

When hit by a GW, in the language of special relativity, an event in a 3+1 D spacetime is subject to a strain, usually parameterized in linearized general relativity formalism by $h_{\mu\nu}$, which, assuming the idealized situation of adopting a weak field approximation and of being in an inertial frame, is assumed as a perturbation of a flat Minkowski metric $\eta_{\mu\nu}$ as

$$g_{\mu\nu}(\mathbf{x}) = \eta_{\mu\nu} + h_{\mu\nu}(\mathbf{x}). \tag{1}$$

Keeping in the weak field approximation the strain $h_{\mu\nu}(\mathbf{x})$ propagates in vacuum ($T_{\mu\nu} = 0$) as a wave,

$$\Box h_{\mu\nu}(\mathbf{x}) = 0 \tag{2}$$

at speed $c = 1$. In order to make explicit the existence of a GW in the most straightforward way, it is fairly common practice to move onto the TT gauge

$$h^{0,\mu} = 0, \quad h^i{}_i = 0, \quad \partial^j h_{ij} = 0, \tag{3}$$

which identifies the metric h^{TT}_{ij}. The "transverse" condition, in particular, means that the maximal strain effects reside in the planes orthogonal to the direction of propagation of the GW; in other words, in this specific gauge, the GW oscillations are orthogonal to the 4-vector U^μ of an observer moving towards the propagation direction of the GW, i.e.

$$A_{\mu\nu} U^\nu = 0 \tag{4}$$

(with $A_{\mu\nu}$ the generic GW amplitude); this alone does not say anything about possible lesser effects at different angles with respect to the propagation direction. Moreover, one should also try to be more explicit about the meaning assigned to the TT gauge itself: as was indeed pointed out in [7, 8] the notion of transverse-traceless might refer either to a decomposition of the metric perturbation h_{ij} which is local in momentum space and nonlocal in physical space or to that part of h_{ij} obtained with a projection operator P_i^j local in physical space. The two notions are only interchangeable under specific conditions and are in general not equivalent. Once the appropriate notion of TT gauge is adopted, the two independent constant amplitudes, h_+ and $h_\times$, can be derived, representing the possible polarizations of the GW, which are at an angle of $\pi/4$ to each other and −it's worth pointing out− emerge from and after choosing the gauge. On this note, it was shown [9, 10] that the canonical double polarization of GWs is not the sole possible solution as, for instance, spin-1 GWs might be obtained as exact solutions of Einstein's equations (with cosmic strings or GRBs as sources). Although tentative, this result suggests a possibility in the sense of non-canonical GW polarizations and observables when working outside the constraints of the TT gauge.

More recently, some attempt in the direction of a search of nontensorial GWs has been carried out by the LIGO-Virgo collaboration [11], where without relying on any specific theory of gravity, a search for possible GWs with up to six different polarizations was performed assuming a generic metric theory. Although inconclusive, this work showed that the possibility of finding new solutions outside the canonical framework of the GR+TT gauge is not completely out of the equation, and could still be worth pursuing.

Let's start by considering that during the passage of a GW what gets displaced are not just test masses over a spatial distance but the entire spacetime interval

$$ds^2 = g_{\mu\nu}(\mathbf{x}) dx^\mu dx^\nu \tag{5}$$

between events in the perturbed spacetime. Indeed, as GWs traverse spacetime they are locally modifying the gravitational field through perturbations of the Riemann tensor $R_{\mu\alpha\nu\beta}$: not only distances are affected from the point of view of an external observer but also times are expected to do the same. Although the existence of a GW is usually derived from the $\partial_\mu \bar{h}^\mu{}_\nu = 0$ condition, an equally valid criterion is to work with the Riemann tensor itself. In fact, during the passing of a GW, Eq. (5) can be recast in a more appropriate form −using gaussian coordinates− as

$$ds^2 = \Big(\eta_{\mu\nu} + \frac{1}{3} R_{\mu\alpha\nu\beta}\, x^\alpha x^\beta\Big) dx^\mu dx^\nu. \tag{6}$$

More explicitly, it might prove useful to relax the usual TT gauge approach and formulate a hypothesis on what could be a possible alternative physical observable resulting from this choice; it's conceivable that this could lead to the emergence of one or more different polarizations of the GW. In this sense, our claim is that both approaches are correct, but explanations of the different emerging observables could vary based on the gauge choice. It is equally important to point out that whatever the perturbed metric will be, it will be essential to express it in terms of the Riemann invariant (at a linearized level) tensor, rather than the Christoffel symbols.

For example, a minimal modification of the gauge choice could be the following one. After fixing the Landau-Lorenz gauge, we can use one of the four residual degrees of freedom to impose the vanishing of the trace. Then, there remain 3 further degrees of freedom usually used to impose the vanishing of the h_{0j} terms. Before imposing such constraints, for a wave $h_{\mu\nu}(z - ct)$ moving along the direction z, we have

$$h'_{00} + h'_{30} = 0, \tag{7}$$
$$h'_{03} + h'_{33} = 0, \tag{8}$$
$$h'_{01} + h'_{31} = 0, \tag{9}$$
$$h'_{02} + h'_{32} = 0, \tag{10}$$

where a prime indicates derivative w.r.t. the argument, and 3 is associated to z. The first two equations, together with the traceless condition, show that we must have $h_{00} = h_{33}$ and $h_{11} = -h_{22}$. From the last two equations, we see that if we use two of the residual freedoms to fix $h_{01} = h_{02} = 0$, then also $h_{31} = h_{32} = 0$. At this point, we are left with the waveform

$$h_{\mu\nu} = \begin{pmatrix} h_o(w) & 0 & 0 & h_o(w) \\ 0 & h_+(w) & h_\times(w) & 0 \\ 0 & h_\times(w) & -h_+(w) & 0 \\ h_o(w) & 0 & 0 & h_o(w) \end{pmatrix}, \tag{11}$$

$w = z - ct$, and still one gauge freedom. Usually, this last gauge freedom is used to put $h_o = 0$, but we can instead use it to impose $a h_\times = b h_+$ for fixed $a, b \in \{0, 1, -1\}$ not both vanishing. For example, we could impose the vanishing of the $+$ mode thus remaining with the $\times$ and the o modes. In particular, the latter corresponds to the impossibility of keeping clocks along the z-axis synchronized during the passage of the gravitational wave.

The intuition behind them being different representations of the same wave comes from the equivalence principle. It's clear though, that in this gauge, which is manifestly a-synchronous, the *time-time* component of the amplitude, rather than being reabsorbed inside the $h_{+,\times}$ components of the strain, as it happens with the standard TT gauge, survives and can therefore be treated as an observable. We can call this gauge choice the A-synchronous Traceless (AT) gauge (having only one transverse mode and a "desynchronization" mode).

II. TIME DILATION AS A TECHNIQUE FOR THE DETECTION OF GRAVITATIONAL WAVES

The order of magnitude of $h_{\mu\nu}$ for astrophysical sources commonly falling in the LIGO band (stellar-mass black hole or neutron star mergers from some $10^{2\div3}$ Mpc luminosity distance) is generically assumed as a target for the sensitivity of laser interferometers, in that

$$\frac{\Delta l}{l} \simeq h_A \tag{12}$$

with A the $+$ and $\times$ polarizations (e.g., in the TT gauge), and l referring to one of the spatial coordinates (e.g. the length of one of the two arms of the interferometer).

For a typical LIGO-like event −say, a binary stellar black hole merger in the kHz− with a total burst duration (measured from Earth) of a few 10^2 ms, this would be the time interval over which the spacetime dilation is taking place and during which any measurement of distances and/or times should show some difference from the unperturbed conditions. Distance changes are what is currently observed with laser interferometers, while time changes are what is being proposed here.

Of course, a more persistent GW signal at lower frequencies, e.g. in the LISA band [12–14], or even in the μAres [15] μHz frequency band, would yield similar effects over longer timescales.

During the first three LIGO campaigns, the strain h_A of

confirmed GW events have been observed to show values usually between the order of magnitude of the instrumental sensitivity $S_n \simeq 10^{-21}$ and in the excess of 1.0×10^{-18}, [4], while the design Sensitivity of aLIGO currently exceeds 10^{-23}.
Our proposal is that if $h_{\mu\nu}$ is assumed to be acting over an unperturbed spacetime, one may then expect it to have a similar, if not the same, effect over a time interval, when measured by an external observer, i.e.

$$h_A \simeq \frac{\Delta t}{t} \simeq \frac{\Delta l}{l} \simeq 10^{-21\div-18} \tag{13}$$

where t is the coordinate time measured by an observer at infinity. The crucial point is that the proper time τ measured by a physical observer in the same rest frame of an event crossed by the trajectory of the GW will of course notice no alteration in its own rate. We point out that this is no different from two observers with identical clocks, one of them close to the event horizon of a black hole, the other at infinity, at rest in a flat Minkowski: while the two observers in their reference frames will observe their clock to progress at a rate of 1 second per second, the observer at infinity would, in principle, be able to observe the other clock in the proximity of the black hole advancing at a slower rate. Similarly, the observer near the black hole would observe the clock at infinity proceeding at a faster rate. Naturally, the assumption behind Eq. (13) is yet to be rigorously derived and proved. If this is indeed true, then what happens is that over timescales much shorter than the duration of the event itself ($\simeq$ 0.1 s, for an event like GW150914), the proper time contracts and expands in phase with the frequency of the GW with an amplitude given by h_A, but only an external observer unaffected by the GW would be able to detect such contractions. It is only intuitive to expect these contractions and expansions to cancel out over each period of the GW sinusoid.[1]
To restate the idea from the reference frame comoving with the GW, inside a stronger gravitational field, given by a positive GW strain, time will be slowed down, and likewise, it will be during both the ascending phase of single crests of the GW sinusoid – until the local maximum is reached where time will flow at its slowest rate– and the descending phase, where it starts accelerating back, until crossing the zero of the curve, where no time dilation happens. Similarly, when the strain is negative, during the descending phase, time flow will accelerate and at the local minimum, e.g. at the trough, time will flow at the fastest rate; during the ascending phase it will slow down again until the next zero; the process will then repeat for the subsequent period. Fig. 1 is adapted from [3] and makes an attempt at picturing the idea: where (or better, *when*) the strain is positive (colored in blue), the metric tensor is positively perturbed, meaning a stronger gravitational field, and a slower flowing of time when measured from an external observer. The vice-versa holds for negative strains (colored in orange).

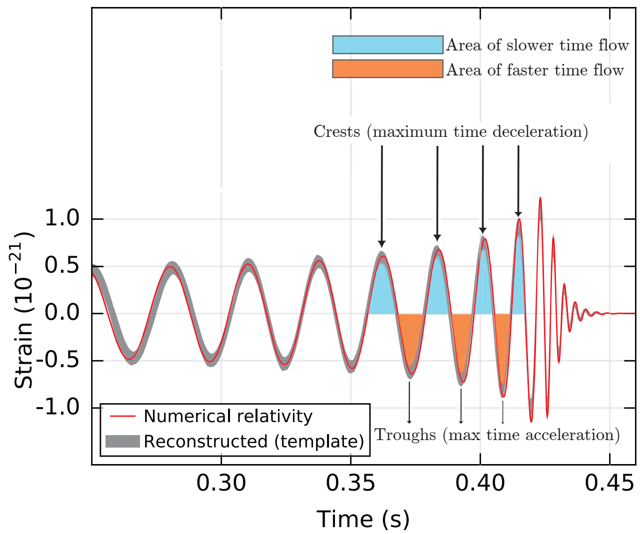

FIG. 1. Areas in blue (orange) show the time intervals over which the strain is positive (negative) and over which time measurements should show a deceleration (acceleration), meaning in other words that a millisecond should take a millisecond plus (minus) one part in the order of magnitude of h_A, say 10^{-21}, when measured by an observer at infinity. (Figure is an adaptation from [3].)

A. Conceptual difference with Pulsar Timing Arrays and Doppler shift tracking

It would be tempting at this point to see a strong similarity between this concept and what is generally referred to as Pulsar Timing Array (PTA) [21–26]. PTAs observe the incoming signal from distant pulsars, which are treated as highly regular clocks, and measure the time interval between the incoming pulses. Under unperturbed conditions the Δt between the pulses is constant; deviations from this regularity are interpreted as the passing of a GW between the source and the observer on Earth. The GW is therefore acting on the geodesic between them, resulting in a different travel time for the photons emitted by the pulsar, hence a shift in their time of arrival.
By contrast, what we are assuming here is a GW acting on the source of the Electromagnetic signal itself, and somehow not on the subsequent null geodesic covered by any EM signal coming out of it: it's the time dilation happening at the source to cause any subsequent measure of Δt at the observer rest frame, rather than an intervening GW on the path of a signal which originated regularly in

[1] The amplitude of the signal is not exactly constant though, more so during the final merger phase of a binary source; therefore one should not have exact cancellation of the signal between periods, but at least a residual effect in the measured time, analogous to what happens with the Christodoulou memory [16–20].

unperturbed conditions at the source. In other words, it is a differential, rather than integral, discrepancy.
A further possible misconception may arise with the notion of spacecraft Doppler tracking [27], where GWs act over the on-Earth observed Doppler shift of sinusoidal Electromagnetic signals to and from a spacecraft. Again GWs are acting on the photons' propagation, varying the metric along the signal path. The similarity with our proposal resides in the requirement for clocks to be as precise as the GW strain, i.e. with precision better than 1 part in 10^{18} for most sources. Yet again the difference is centered on the influence of the GW over the spatial, rather than the temporal component of the metric.
In a sense, a higher conceptual similarity can be found with the gravitational Aharonov-Bohm effect [28], namely the phase shifts induced by the gravitational influence of a close-by mass over proper time differences between freely falling, nonlocal trajectories in atom interferometers [29, 30]. The difference, in this case, is in the nature of the source, a mass, rather than a GW.

III. TOWARDS AN EXPERIMENTAL OBSERVATORY

Conceiving an experimental device capable of measuring time dilations in a way that can be useful for GWs detection is by itself quite a challenging task. The main difficulty is to have the GW influencing the timing of a physical process but not the clock measuring it. Furthermore, for any such detector to be conceptually different from an interferometer, one must make sure that the influence over any time measurement exerted by the GW doesn't stem from the spatial component being perturbed, altering the travel time of the signal.
Let us assume to possess a sample where some particular physical process is known to happen with characteristic time intervals τ^*, e.g. a perfect clock emitting particles at a constant rate; for instance, Wilczek's time crystals [31–35] could –at least ideally– serve this purpose. In theory, if such features are then continuously observed and therefore timed, being their τ^* known *a priori*, one should observe a different behavior, namely of the order of h_A, in terms of elapsed times when a GW is passing by the samples and altering their characteristic time when measured by an observer at infinity. The main difficulty is once again to have the external observer somehow unaffected by the passage of the GW wave itself, at least not during the time measurement of the process influenced by the GW, or in other words to be non-local in physical space with respect to the reference frame of the samples; if this were not so, the external observer's clock would be in turn affected by the GW, and although present, the phenomenon could not be detected.
A sufficiently precise clock to serve as detector is therefore the first prerequisite towards this program, and is the subject of the next subsection, where we will refer indistinctly to precision or resolution of the clock.

A. Timing resolutions

When dealing with a LIGO-like GW event in the kHz occurring altogether over a few 10^{-1} s (as is roughly the case for GW150914) and $h_A \simeq 10^{-21\div-18}$ it follows that the term Δt in Eq. (13) has to fall in the range of the $10^{-22\div-19}$ s; if dealing with a single period between crests of the wave (or more accurately between zeros of the amplitude), which can be assumed to happen over a few $\simeq 10^{-2}$ s [3], one finds a requirement for $\Delta t \simeq 10^{-23\div-20}$ s. This is the precision required from the measuring clock for an event of this kind. Table I lists some of the shortest characteristic timescales for processes known or predicted by particle and nuclear physics. The fact that such short timescales are predicted or inferred by fundamental physics does not straightforwardly nor automatically imply if and how they could be put to use in this context.

Duration [s]	Process
0.3×10^{-24}	Mean lifetime of the $\mathbf{W}\pm$ and $\mathbf{Z}$ bosons
1×10^{-24}	Top quark decay
1×10^{-24}	Gluon emission from a Top quark
9.1×10^{-23}	Half-life of ^{4}Li
7×10^{-21}	Neutron half-life in ^{9}He nuclear halo

TABLE I. List of some of the shortest known or predicted characteristic timescales of physical processes.

More attainable time measuring systems include processes of interactions of quick laser bursts with matter, with current technology reaching periodic times between pulses in the region of 10^{-18} s [36]. Similar timing resolutions in the attosecond have recently been claimed in high-harmonic spectroscopy with Gouy phase interferometers [37, 38]. Recent claims of timing precision measurements with uncertainty down to 9.7×10^{-19} obtained with optical atomic clocks based on quantum-logic spectroscopy [39], or even down to 2.5×10^{-19} obtained with imaging spectroscopy techniques [40], place the aforementioned required range inside current technological capabilities. A timing resolution around this order of magnitude would be more than adequate when trying to detect GWs at lower frequencies than those in the LIGO band. For a GW event falling in the LISA band, with frequency, say, of $\simeq 10^{-2}$ Hz, the actual required Δt in Eq. (13) would be $\simeq 10^{-19\div-16}$ s.
It is worth mentioning that in 2021 the quantum phase of a collectively excited nuclear state was tuned via transient magnons with a precision of 1×10^{-21} s, monitored interferometrically via quantum beats between different hyperfine-split levels [41]; moreover, in 2022 the gravitational redshift within a single millimeter scale sample of ultracold strontium was measured with a fractional frequency measurement uncertainty of 7.6×10^{-21} [42]. It is clear that a timing resolution in the zeptosecond or even higher would enable studying spatial gradients of the time clock, thus opening the door for a detailed waveform reconstruction and GW astrophysics of the source.

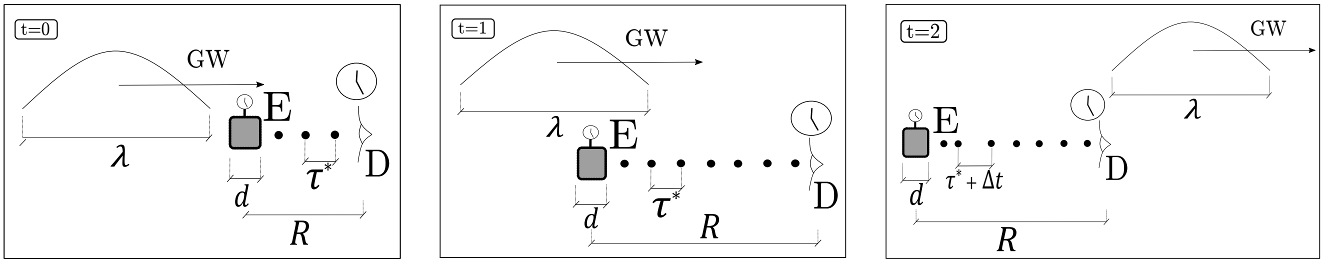

FIG. 2. Schematic view (not to scale) of an ideal detector, labeled "D", and an emitting source of particles, labeled "E", at times $t = 0, 1, 2$, i.e. before, during, and after the passing of a single GW crest, respectively. Note that the geometry of the system (R, d, λ) does not change over time.

All this being said, the actual scheming of an ideal device capable of arranging the samples and the observer so that they could firstly be sensible to a GW in terms of time dilation, and secondly be detectable by the external observer is not a trivial task and will be tentatively addressed in the next subsection.

B. Concept design

Let us assume to have at an initial time $t = 0$ (Fig. 2, left panel, not to scale) an incoming GW with wavelength λ, an ideal localized source (labeled 'E') of size d, emitting particles −say, electrons or photons− at a fixed rate, towards a detector (labeled 'D') placed at fixed distance R, consisting of a clock measuring their Δt of arrival. Let the system be in free fall, i.e. fully inertial, in an empty flat Minkowski background spacetime, where the curvature radius of the background can be written as:

$$\mathcal{R}_{\rm B} \equiv \left| \frac{1}{\sqrt{R^{\rm B}_{\mu\alpha\nu\beta}}} \right| \tag{14}$$

Let's further assume by now to consider a single crest (or trough) of the GW, rather than the entire waveform of several periods, and to evaluate its effect on the system as a standalone GW wavelet, or rather, a solitonic GW [43, 44]. The first requirement for such a system would be for it to observe the condition:

$$R \gg \lambda \gg d \tag{15}$$

Now the idea would be to have at $t = 1$ (Fig. 2, middle panel) the GW sweeping past the entire system, including the detecting clock, in a finite and short enough time, and the signal emitter to possess its own internal clock, related to a highly predictable physical process, determining the emission rate of the particles/photons. The crucial requirement is for the signal to be *generated* while under the influence of the GW, but to be *expelled* at a retarded time with respect to the transit of the entire GW, so as to exclude any contraction effect over the path length between the emitter and the detector. The delay is required to be long enough so that when the particles are finally expelled ($t = 2$, right panel of Fig. 2), they do so in a flat Minkowski background once again, as the GW has already left the system, but crucially encoding a difference in the Δt between their emission times, previously registered inside the device while under the influence of the GW. If this were not so, the internal clock would not be able to encode the time dilation brought about by the GW, since 1 second during the GW passage would take again 1 second in the Minkowski conditions afterward. Equally essential is the requirement that the signal is actually "emitted", although still *inside* the device itself, only to later be expelled outside of it, with the strong caveat that the interdistance between each individual signal is to be accurately preserved. Because of the delay between production and expulsion, the particles can have in principle any speed, and massive particles, as well as photons, can be employed.
If the signal emission is continuous, it will not encode straight away the time dilation carried by the GW on the emitting device, but it will be affected nonetheless by the GW crossing its path, much like inside an interferometer or with PTAs. The information on time dilation alone will come later on when the GW has already left the system. The amount of delay between production and expulsion of the signal is the designer's choice and determines the GW frequency range available for detection.

C. More complex configurations

A natural next step toward a feasible detector would be to have several of the aforementioned systems, possibly pointing at the same detector. Let's imagine having them in some sort of distribution equidistant from the detector, let's say an atomic clock of arbitrary precision. Recent results in elementary quantum networks of entangled optical clocks [45] suggest a possibility in this sense. A simple distribution of particle emitters, all oriented towards a common detector, is depicted in Fig. 3. An incoming GW wavelet hitting the system of

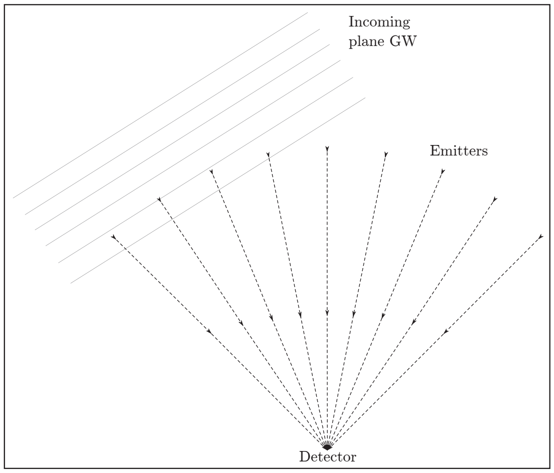

FIG. 3. Overhead view of a basic distribution of emitters with a common ideal detector hit by an incoming gravitational wave.

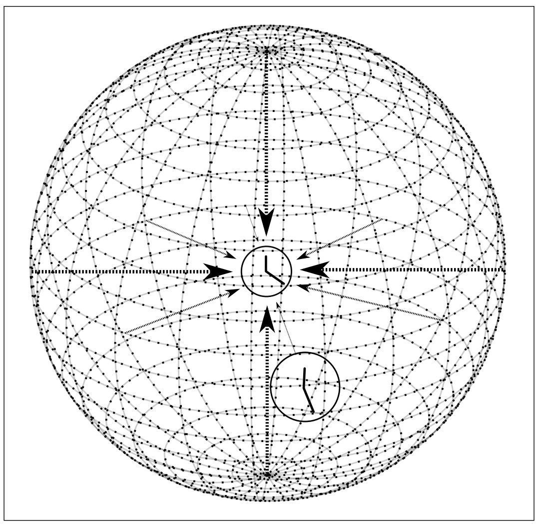

FIG. 4. Schematic view of a complete spherical detector.

emitters at an angle, would, at first, influence the timing of particle emission of some, and only some, of them; only at a later time would all the emitters be influenced by the GW, entailing a rough estimate of the direction of propagation of the GW.
Further development of this concept would be to have a distribution of samples in a 3D spatial volume, emitting signals towards a central detector, timing the difference of arrival time from the different directions. Having them all at the same distance from the center would result in a spherical configuration like the one in Fig. 4. This would produce a gradient of proper times, varying in phase with the transit of the GW, which could then be used to better reconstruct the direction of propagation of the GW, and therefore infer the sky localization of the source.

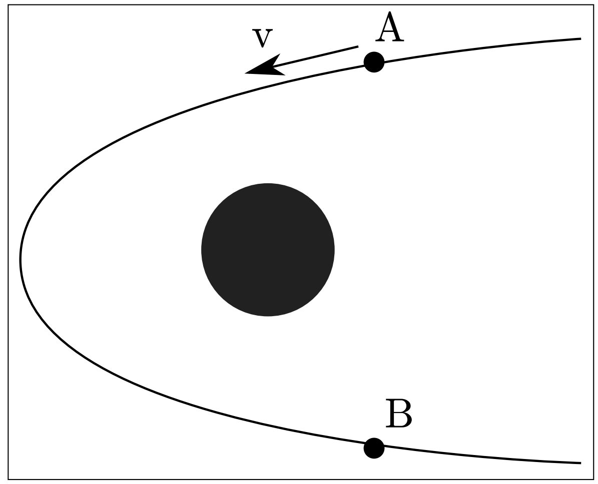

FIG. 5. Collision between a massive body and a particle.

D. Toy model results

In order to construct a physically realistic scenario in which a single GW crest (or trough) can be generated, we will assume the simple astrophysical case of an elastic collision, depicted in Fig. 5, between a massive body and a test particle, or an Extreme Mass Ratio Collision (EMRC), as our GW source. In particular, let's assume the massive body to have mass m_1 equal to that of Sagittarius A* (Sgr A*, $m_{\mathrm{Sgr\,A}^\star} = 4.3 \times 10^6\,\mathrm{M}_\odot$ [46]), the supermassive black hole at the Galactic Center, and the particle to be a compact object, for instance, a primordial black hole (PBH) with $m_2 = 1\,M_\odot$ [47–53], coming from past infinity, and directed to future infinity after the collision, which we define to be restricted between point A and point B in Fig. 5. Because of the high mass ratio, we can safely consider both the fixed scattering center and the center of mass of the two-body collision to be located inside the supermassive black hole. The initial and final 4-momenta of the PBH are P^μ and P'^μ, respectively. Following [54, 55] on the formalism of GW generation from two-body collisions, and from now on indicating with ω the GW frequency, the 4D Fourier transform of the corresponding energy-momentum tensor $T^{\mu\nu}(\mathbf{x}, t)$ in the non-relativistic limit is:

$$\tilde{T}_{ij}(\omega) \simeq -\frac{ic}{m\omega}(P_i P_j - P'_i P'_j) \tag{16}$$

The non-relativistic limit is valid for relative velocity between the two bodies $v \lesssim 0.3\,c$, a condition which is respected when assuming an impact parameter as small as $b = 2.5 \times 10^{-6}\,\mathrm{pc} = 6\,R_S$. We impose as a safe initial condition $v = 0.15\,c$, resulting in a collision total duration of approximately $t_c \approx b/v$. The collision can be approximated as instantaneous so that the spectrum needs to be cut at frequency

$$\omega_{\max} \sim \frac{2\pi}{t_c} \sim \frac{2\pi v}{b} = 3.7 \times 10^{-3}\,\mathrm{Hz} \tag{17}$$

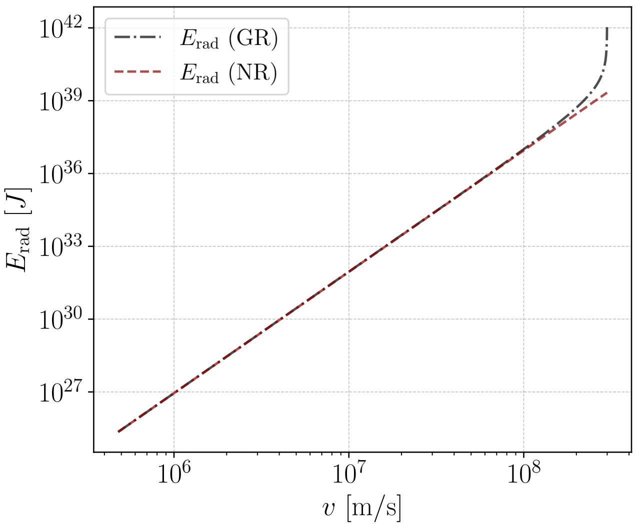

FIG. 6. Total radiated energy after the collision. The discrepancy between the full general relativistic and non-relativistic limit becomes evident only at $v > 0.3\,c$.

If the collision is happening in the (x, y) plane, most of the gravitational radiation will be emitted along the z direction, and will be described by an angular distribution with polar coordinates (θ, ϕ). If the test particle is scattered by the massive body by an angle θ_s, the non-relativistic energy spectrum integrated over the solid angle $d\Omega = d\cos\theta d\phi$ is given by:

$$\int \frac{dE}{d\omega d\Omega} d\Omega = \frac{dE}{d\omega} = \frac{8G}{5\pi c^5} \mu^2 v^4 \sin^2\theta_s \tag{18}$$

where $\mu = (m_1 m_2)/(m_1 + m_2)$ is the reduced mass. Integrating Eq. (18) up to $\omega_{\rm max}$ gives the total radiated energy in GWs after the collision:

$$E_{\rm rad} \sim \frac{16Gm^2}{5b} \left(\frac{v}{c}\right)^5 \sin^2\theta_s \tag{19}$$

In the general relativistic case, Eq. (16) is replaced by a full expression for $\tilde{T}^{\mu\nu}(\mathbf{k}, \omega)$, with the 4-momentum containing a Lorentz factor, and the expression for the angular distribution of the energy spectrum takes a more complicated form with additional factors expressing the bending of the radiation pattern in the direction of motion. Fig. 6 shows the total upper limit in total radiated energy as a function of relative velocity for the GR and non-relativistic cases, by assuming the scattering angle to be $\theta_s = \pi/2$; the result is expressed in Joules. The discrepancy between the two curves starts being significant only for values of $v > 0.3\,c$, which justifies the choice for adopting the non-relativistic limit. Expressing the (average) luminosity $\mathcal{L}$ as simply $E_{\rm rad}/t_c$, the relation between luminosity and energy flux is given by:

$$\mathcal{F} = \frac{\mathcal{L}}{4\pi D_L^2} \tag{20}$$

where for the luminosity distance D_L from Sgr A* we adopted 8.26×10^3 pc [46].

To obtain a first-order raw estimate of the expected GW strain, we plug this into the expression for the energy flux from [54], which strictly would only apply after averaging over several periods, which of course don't take place during a one-time collision, but can nevertheless be instructive about the expected magnitude of the result:

$$\mathcal{F} = \frac{1}{32\pi} \frac{c^3}{G} h_A^2 \omega_{\rm GW}^2 \tag{21}$$

Finally, substituting $\omega_{\rm max}$ for $\omega_{\rm GW}$, gives an estimate for the generic polarization strain:

$$h_A \simeq 6 \times 10^{-18} \tag{22}$$

Similarly the expression for the time derivative of h is given by [54]:

$$\mathcal{F} = \frac{1}{32\pi} |\dot{h}|^2 \frac{c^3}{G} \tag{23}$$

giving an estimate for $\dot{h} \simeq 2 \times 10^{-20}$. Keep in mind that these results are obtained for a scattering angle $\theta_s = \pi/2$ that maximizes the total radiated energy in Eq. (19). Different values of θ_s will ultimately entail a somewhat smaller value of the GW strain, but the order of magnitude of the results is largely preserved for a wide range of scattering angles. With the aforementioned initial conditions and time duration for the collision $t_c \approx 1$ ks, one can try and give a zeroth-order estimate for the waveform, once again borrowing improperly from the quadrupole formalism, which simply reads:

$$h(t) = h_A \cos(\omega_{\rm GW} t) \tag{24}$$

where substituting again $\omega_{\rm max}$ for $\omega_{\rm GW}$ and letting t vary from 0 to t_c, we find the resulting truncated waveform of Fig. 7. Since we are interested in the generation of a single GW crest (or trough), the approximation holds true only as long as the argument of the cosine in Eq. (24) doesn't exceed a certain threshold, or in other words, a full sinusoid is not able to form. Because we defined $\omega_{\rm GW}$ in terms of the relative velocity v, this results in a constraint on the degeneracy of the two parameters v and t_c appearing inside the cosine. The shaded area in Fig. 8 shows the points in such parameter space where this condition is observed, i.e. the resulting sinusoid doesn't exceed a half period; the edge, in particular, indicates the points where the waveform −when centered on $t = 0$− is truncated exactly at the change of sign of its second derivative, which maximizes the integrated GW strain, or in our language, the time dilation. Of course the closer the points are to the edge of the shaded area, the more developed the GW crest (or trough, with a simple phase shift of $\pm\pi$) will be; the viceversa holds true. Conversely, points just outside of it are still permitted, but will result in a sinusoid with components of the opposite sign, inducing an increasing degree of cancellation of the time dilation produced by the GW; total cancellation will happen when a full period of the sinusoid is formed (see footnote 1).

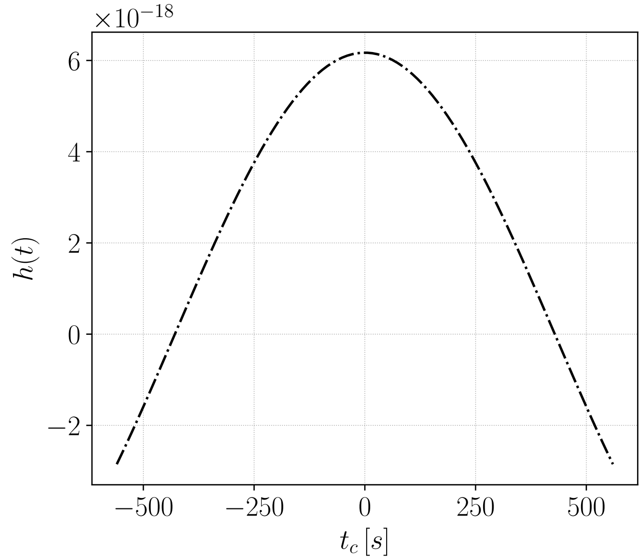

FIG. 7. Waveform resulting from a collision with our chosen initial conditions. The approximation holds true only when the argument of the cosine does not exceed a certain threshold, which implies conditions on both v and t_c.

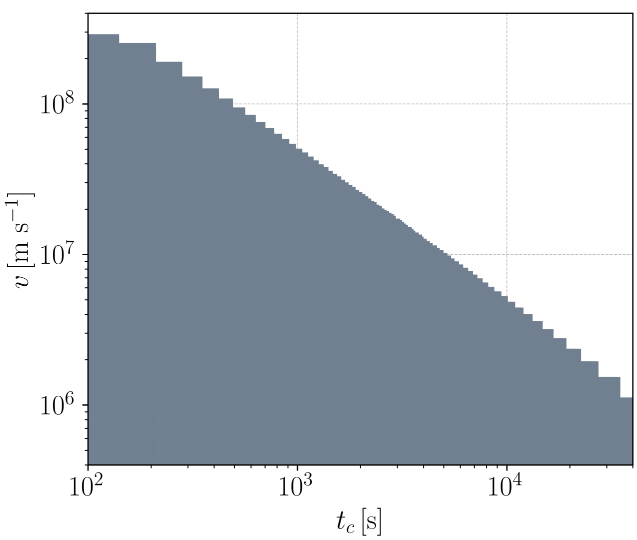

FIG. 8. Parameter space showing the possible values of v and t_c for which only a single crest or trough of the GW waveform can develop.

IV. CONCLUSIONS

Detection of GWs with laser interferometers is nowadays an affirmed technique. Nevertheless, a second path towards reaching the same result taking advantage of unexplored aspects of the same underlying physics may be permitted. In this work we have proposed a new approach to GWs detection based on the observation of time dilations, as the spacetime perturbation carried by the GW acts in principle both over distances as well as time intervals, the only invariant ultimately being simply the Riemann, and the observables being themselves a choice emerging as a result of the adopted gauge. By means of defining an AT gauge, in lieu of the canonic TT, we have obtained an explicit expression for the GW strain in terms of time dilation; the rank-2 tensorial nature of the GW is preserved, with one independent mixed term appearing in addition to the *time-time* component as the remaining degree of freedom inside the polarization matrix. We have presented the concept design for an ideal detector, which in reality would have to deal with the technological limitations in terms of a sufficiently constant rate of signal emission, the capability of preserving the inter-signal separation, as well as a sufficiently precise detecting clock, with the non-obvious question regarding the versatility of such a system to become part of a feasible GW detection system in the vicinity of Earth, where the background metric is not Minkowski. Finally, we described a possible realistic astrophysical scenario, an extreme mass ratio collision, in which the conditions for the generation of GWs appropriate for their detection with time dilation are immediately met. The natural extension of this concept to the detection of periodic GW signals requires further development, mostly on technical grounds, and is left for future research.

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