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Noise and Decoherence of Primordial Graviton From Minimum Uncertainty States

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We have investigated quantum noise and decoherence due to primordial gravitons with minimum uncertainty initial states. This condition allows the initial state to be in the form of an entanglement or a superposition state. We got that the increasing number of gravitons in the initial state of entanglement can increase the effective strain corresponding to the quantum noise and can reduce the dimensions of the experimental setup system. The existence of non-diagonal elements in the initial superposition state allows the distribution of the quantum noise to be non-Gaussian. In addition, these non-diagonal terms also cause the quantum noise to occur for a very long time compared to the initial state in the form of a Bunch-Davies vacuum.

Keyword: *Minimum Uncertainty, Primordial Graviton, Noise and Decoherence*

I. INTRODUCTION

One of the predictions from Einstein's general theory of relativity is the phenomenon of gravitational waves [1, 2]. When there is a compact concentration of energy, spacetime will bend and if the concentration of energy changes, it will produce dynamics in the curvature of spacetime, that will propagate in all directions at the speed of light. The propagation of the dynamic curvature of space-time in all directions is what is known as gravitational waves. In the last decades, gravitational waves have been attempted to be detected. Until 2015, the LIGO detector has succeeded to detect gravitational waves for the first time [3]. This success has become a new hope for scientists to detect other phenomena related to gravitational waves in the future. One of those phenomena is looking for the quantum characteristics of gravitational waves as graviton particles. This phenomenon is interesting because graviton particles are one of the consequences that arise from the quantization of gravity. As we know, there has been no satisfactory theory for the quantum theory of gravity up until now. Some scientists even believe that gravity does have not to be quantized at all [4]. Even so, a lot of scientists are still trying to develop a mechanism to detect gravitons.

Several studies related to graviton detection are still being developed today [5–14]. The interesting one is the research proposed by Parikh *et. al.* [5]. They submitted a study proposal to detect graviton through the quantum noise effect produced by these particles on classical masses with the same vein as quantum Brownian motion [15, 16]. The idea of this detection is based on the assumption that detecting individual gravitons is considered to be impossible [17]. In his research, Parikh *et. al.* [5] studied the behavior of a gravitational wave detector in response to a quantized gravitational field. The detector will be modeled as two objects that are geodesically separated. Then, using the Feynman-Vernon influence functional method [18, 19] the quantum noise generated from the quantized gravitational field can be studied. Furthermore, this study was expanded by S. Kanno *et. al.* [8], where they review decoherence events that occur between the detector and the environment with a quantized gravitational field to determine how long quantum noise could be detected. Apart from that, there was also another research conducted by H. T. Cho and B. L. Hu [14], where the study was expanded by considering all possible modes of graviton including polarization. From these studies, there are similar results where quantum noise can be detected if the initial quantum state is squeezed.

Gravitational waves could originate from several places, for example, black holes [20–22], neutron stars [23, 24], or from the early universe when the inflation mechanism occurred [25–27]. The gravitational wave that originates from the early universe is called a primordial gravitational wave. Some of the best cosmological inflation models that can explain the structure of our universe with great precision predict that the universe started from a quantum state [28–33]. This means that the primordial gravitational waves also originate from the quantum state. So, there would be gravitons generated in the early universe. Interestingly, due to inflation at the beginning of the universe, it is believed that the initial quantum state was squeezed as the universe expanded [34]. This means that if gravitons came from the beginning of the universe, using the quantum noise method it should be possible to detect gravitons. The proposal to detect primordial gravitons was put forward by S. Kanno *et. al* [9]. In their proposal, they proposed an experimental setup that could be used to detect noise generated

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from primordial gravitons. The result shows that the quantum noise of gravitons from the early universe can be detected during an interval of 20 seconds. However, the quantum initial state of the graviton used is only the Bunch-Davies vacuum. This quantum state is the state that is generally chosen to explain the initial state of the universe, but it does not rule out the possibility that the initial quantum state could be another state that could increase the possibility of detecting gravitons. In this study, we want to try to expand our understanding of this research by assuming the initial state in minimal uncertainty condition, where the unique properties of quantum physics may be included.

The Heisenberg uncertainty principle is a concept in quantum mechanics regarding measurement. Where is usually formulated in the context of the probability interpretation for two ideal quantum measurements of the observables. In the case of gravitons, several observables that can be analyzed are their polarization properties. Like the electromagnetic field, the measurement operators of the graviton polarization can be represented as Stokes operators [35]. There are four Stokes operators, each of which will define the linear, circular, and total polarization of the gravitons. If it is assumed that the quantum state in the early universe that generates gravitons is in a minimum uncertainty condition, then one of the possible states is the entanglement, as in the research [36–39]. It is known well that the entanglement state is a unique condition in quantum physics that does not exist in classical physics. This means that if the initial state is assumed to be in a condition of minimal uncertainty, the quantum state can be in the form of an entanglement state, which should increase the quantumness of the primordial graviton so that it will be able to increase the possibility of its detection. Not only entanglement but the minimum uncertainty state of the stoke operator is also supposed to have other unique quantum characteristics such as superposition and interference.

The organization of this paper is as follows. First of all in section II we will briefly review the quantum noise operator due to gravitons as in research [8]. Followed by determining the minimum uncertainty states of graviton based on their polarization in section III. In section IV we will calculate the quantum noise correlation using the quantum initial states obtained in the previous section. Followed by the calculation of the decoherence time in section V using an experimental setup such as [9]. Then in section VI is the conclusion. In Appendix A, we will write down some details regarding the calculation of quantum noise correlation, and Appendix B will describe the influence functional method used to determine decoherence functional due to gravitons.

II. QUANTUM NOISE FROM GRAVITON

In this section, we will briefly review quantum noise caused by gravitons based on research from S. Kanno *et al.* [8]. Quantum noise is a consequence that arises due to an interaction between the system (detector) and the environment within the framework of quantum mechanics. In his research, it is assumed that the system is an interferometer with two mirrors in the gravitational wave environment. If the gravitational waves are quantized (in the form of graviton particles), it is expected that there will be interactions between the system and the environment. So that the quantum noise would appear when the measurements are made by the interferometer. This quantum noise can verify the quantum characteristic of gravitational waves or the existence of the graviton particle.

Mathematically, the quantum noise can be obtained by assuming the interferometer mirror is a particle in Fermi normal coordinates system. In this coordinates system, there are two particles, where one of those (a particle with a geodesic γ_τ) will be used as a frame of reference. While the other particles (particles with geodesic $\gamma_{\tau'}$) are test particles that have a position $x^i(t) = \xi^i(t)$ to the reference frame at point $P(0,t)$. The use of these Fermi normal coordinates is based on the Einstein equivalence principle, where if only one particle is considered, the effect of gravitational waves cannot be affected.

Next, consider the geodesic action of the test particles. By using the metric in Fermi coordinates for the second order of x^i as

$$\begin{aligned} ds^2 \;\simeq\; & (-1 - R_{0i0j}x^ix^j)dt^2 - \frac{4}{3}R_{0jik}x^jx^kdtdx^i + \\ & \Big(\delta_{ij} - \frac{1}{3}R_{ikjl}x^kx^l\Big)dx^idx^j. \end{aligned} \tag{1}$$

The test particle action will be obtained as

$$S_p \simeq \int_{\gamma_{\tau'}} dt\Big[\frac{m}{2}\dot{\xi^i}^2 - \frac{m}{2}R_{0i0j}(0,t)\xi^i\xi^j\Big]. \tag{2}$$

Because the mirror particle is in an environment with gravitational waves, the Riemann tensor R_{0i0j} can be obtained by using a flat metric space and time with small perturbations h_{ij} that satisfies the transverse traceless gauge ($\partial_i h^{ij} = 0$, and $h^i_i = 0$). So if the perturbation h_{ij} is represented in Fourier space, we get action

$$S_p \simeq \int_{\gamma_{\tau'}} dt\Big[\frac{m}{2}\dot{\xi^i}^2 - \frac{m}{2}\frac{\kappa}{\sqrt{V}}\sum_A\sum_{\mathbf{k}\leq\Omega_m}\ddot{h}^A_{\mathbf{k}}(t)\; e^A_{\mathbf{k},ij}\;\xi^i\xi^j\Big], \tag{3}$$

where $\sum_{\mathbf{k}\leq\Omega_m}$ represents the sum of $\mathbf{k}$ modes, with UV cutoff $\Omega_m \sim \xi^{-1}$. In general, the total action can be written as the sum of the actions of the test particles plus the gravitational action ($S = S_p + S_g$) which can be written as follows

$$\begin{aligned} S \;\simeq\; & \int dt\sum_{\mathbf{k},A}\Big[\frac{1}{2}\dot{h}^A_{\mathbf{k}}(t)\dot{h}^{*A}_{\mathbf{k}}(t) - \frac{1}{2}k^2h^A_{\mathbf{k}}(t)h^{*A}_{\mathbf{k}}(t)\Big] \\ & + \int dt\Big[\frac{m}{2}\dot{\xi^i}^2 - \frac{m}{2}\frac{\kappa}{\sqrt{V}}\sum_A\sum_{\mathbf{k}\leq\Omega_m}\ddot{h}^A_{\mathbf{k}}(t)\; e^A_{\mathbf{k},ij}\;\xi^i\xi^j\Big]. \end{aligned} \tag{4}$$

The gravitational action in the first term is just an action with the equation of motion in the form of a harmonic oscillator. By using the interaction picture, the field $h^A_{I,\mathbf{k}}(t)$ can be quantized by defining it in terms of the annihilation and creation operators

$$\hat{h}^A_{\mathbf{k},I}(t) = \hat{a}^A_{\mathbf{k}} u_k(t) + \hat{a}^{A\dagger}_{-\mathbf{k}} u^*_k(t) \tag{5}$$

Where $u_k(t)$ is the solution to the equation of motion which has the form $u_k(t) = (1/\sqrt{2k})e^{-ikt}$ and the annihilation and creation operators would satisfy $[\hat{a}^A_{\mathbf{k}} \hat{a}^{A'\dagger}_{\mathbf{k}'}] = \delta_{A,A'}\delta_{\mathbf{k}\mathbf{k}'}$. Next, the field $\hat{h}^A_{I,\mathbf{k}}(t)$ will be divided into two parts, namely the "classic" part $\left(h_{cl,\mathbf{k}}(t) \equiv \langle \hat{h}^A_{\mathbf{k},I}(t)\rangle\right)$ and the "quantum" part which can be written as

$$\delta\hat{h}^A_{\mathbf{k},I}(t) = \hat{h}^A_{\mathbf{k},I}(t) - h_{cl,\mathbf{k}}(t). \tag{6}$$

From this equation, one could say that $\delta\hat{h}^A_{I,\mathbf{k}}(t)$ will define the quantum fluctuations of gravitational waves due to graviton particles. To review the expression of the quantum noise, we will look for the equation of motion of the test particle from the action of equation (4). Then we will look for terms from the equation of motion where the quantum fluctuations $\delta\hat{h}^A_{I,\mathbf{k}}(t)$ will have an effect. From this term, we can define quantum noise caused by graviton particles. The equation of motion of the test particle where ξ^i is promoted as operator $\hat{\xi}^i$ can be written

$$\begin{aligned}\ddot{\hat{\xi}}^i(t) - \frac{1}{2}\ddot{h}^{cl}_{ij}(0,t)\hat{\xi}^j(t) + \frac{\kappa^2 m}{40\pi}\left[\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl}\right]\hat{\xi}^j(t)\frac{d^5}{dt^5}\left\{\hat{\xi}^k(t)\hat{\xi}^l(t)\right\} \\ = -\frac{\kappa}{\sqrt{V}}\sum_A \sum_{\mathbf{k}\le\Omega_m} k^2\ e^A_{\mathbf{k},ij}\delta\hat{h}^A_{\mathbf{k},I}(t)\ \xi^j + \frac{\kappa^2 m}{20\pi^2}\Omega_m\left[\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl}\right]\hat{\xi}^j(t)\frac{d^4}{dt^4}\left\{\hat{\xi}^k(t)\hat{\xi}^l(t)\right\}\end{aligned} \tag{7}$$

This result is similar to the calculation obtained by Parikh *et. al* [5]. The term where quantum fluctuations will affect is in the first term on the right side of the equation (7), so quantum noise can be defined as follows

$$\hat{N}_{ij}(t) = \frac{\kappa}{\sqrt{V}}\sum_A \sum_{\mathbf{k}\le\Omega_m} k^2\ e^A_{\mathbf{k},ij}\delta\hat{h}^A_{\mathbf{k},I}(t). \tag{8}$$

Where the quantum noise caused by graviton $\hat{N}_{ij}(t)$ will always be there as long as gravitational waves are quantized.

III. MINIMUM UNCERTAINTY OF GRAVITON

In quantum mechanics, if there are two hermitian operators $\hat{X}_1$ and $\hat{X}_2$, the Heisenberg uncertainty relations can be expressed as $\left(\Delta\hat{X}_1\right)^2\left(\Delta\hat{X}_2\right)^2 \ge \frac{1}{4}|\left\langle[\hat{X}_1,\hat{X}_2]\right\rangle|^2$. A quantum state can be said to fulfill its minimal conditions or minimal uncertainty if its quantum state is a solution to the eigenequation [44]

$$\left(\hat{X}_1 - i\lambda\hat{X}_2\right)|\psi\rangle = \beta\,|\psi\rangle\,, \tag{9}$$

where β is the eigenvalue and λ is the squeezing parameter of the solution of the eigenstate.The squeezed parameter λ will determine the initial quantum state of the system in its minimal uncertainty condition. To see why λ is defined as the squeezing parameter, it can be explained by looking for the variance of the $\hat{X}_1$ and $\hat{X}_2$ operators based on the minimal uncertainty relation (9). The variance can be written as follows

$$\left(\Delta\hat{X}_1\right)^2 = \frac{1}{2}\lambda\ \left|\langle\psi|\,[\hat{X}_1,\hat{X}_2]\,|\psi\rangle\right| \tag{10}$$

and

$$(\Delta\hat{X}_2)^2 = \frac{1}{2\lambda}\ \left|\langle\psi|\,[\hat{X}_1,\hat{X}_2]\,|\psi\rangle\right|. \tag{11}$$

It can be seen that when $\lambda < 1$ the variance of equation (11) will be greater than equation (10). Based on the definition of a squeezed state [45, 47], a quantum state can be said to be squeezed if the variance of one operator that satisfies the minimum uncertainty is smaller than the variance of the other operators, this means that when $\lambda < 1$ the quantum state will be squeezed. It also applies when $\lambda > 1$ except that the variance of equation (10) will be larger than equation (11). When $\lambda = 0$ the stated will not be squeezed because the two variances are the same. Furthermore, for simplicity of calculation, the value of λ will be limited to $0 \le \lambda \le 1$.

To find out the minimum uncertainty relation of primordial graviton, the arbitrary operators $\hat{X}_1$ and $\hat{X}_2$ in equation (9) are chosen to be operators that define the polarization of gravitational waves. Like the photon, the polarization of gravitational waves can be defined by the Stokes operators. Using the representation of annihilation and creation operators, the Stokes operator can be expressed as

$$\begin{aligned}
\hat{\mathcal{P}}_k^{(0)} &= \hat{a}_{\mathbf{k}}^{\oplus\dagger}\hat{a}_{\mathbf{k}}^{\oplus} + \hat{a}_{\mathbf{k}}^{\otimes\dagger}\hat{a}_{\mathbf{k}}^{\otimes}, \quad \hat{\mathcal{P}}_k^{(1)} = \hat{a}_{\mathbf{k}}^{\oplus\dagger}\hat{a}_{\mathbf{k}}^{\oplus} - \hat{a}_{\mathbf{k}}^{\otimes\dagger}\hat{a}_{\mathbf{k}}^{\otimes} \\
\hat{\mathcal{P}}_k^{(2)} &= \hat{a}_{\mathbf{k}}^{\oplus\dagger}\hat{a}_{\mathbf{k}}^{\otimes} + \hat{a}_{\mathbf{k}}^{\oplus}\hat{a}_{\mathbf{k}}^{\otimes\dagger}, \quad \hat{\mathcal{P}}_k^{(3)} = -i\big(\hat{a}_{\mathbf{k}}^{\oplus}\hat{a}_{\mathbf{k}}^{\otimes\dagger} + \hat{a}_{\mathbf{k}}^{\oplus\dagger}\hat{a}_{\mathbf{k}}^{\otimes}\big).
\end{aligned} \tag{12}$$

The operator $\mathcal{P}_k^{(0)}$ defines the total intensity of all modes of the gravitons. $\mathcal{P}_k^{(1)}$ and $\mathcal{P}_k^{(2)}$ are intensity operators of linear polarization. Meanwhile, $\mathcal{P}_k^{(3)}$ is an operator for measuring the intensity of circular polarization. The last three Stokes operators will have a commutation relation $[\hat{\mathcal{P}}_k^{(i)}, \hat{\mathcal{P}}_k^{(j)}] = 2i\epsilon^{ijk}\hat{\mathcal{P}}_k^{(k)}$ which satisfies the SU(2) algebra with $(i, j, k) \equiv 1, 2, 3$. However, for operator $\mathcal{P}_k^{(0)}$ the commutation relations with other operators will always be zero, which means the operator $\mathcal{P}_k^{(0)}$ cannot be used to study minimal uncertainty.

Next, we will look for the minimum uncertainty quantum initial state by calculating the eigentstates of the equation (9). Chose, the operators $\hat{X}_1 = \hat{P}_k^{(2)}$ and $\hat{X}_2 = \hat{P}_k^{(3)}$. The minimum uncertainty relations will be obtained as

$$\left[(1+\lambda)\hat{a}_{\mathbf{k}}^{\oplus\dagger}\hat{a}_{\mathbf{k}}^{\otimes} + (1-\lambda)\hat{a}_{\mathbf{k}}^{\otimes\dagger}\hat{a}_{\mathbf{k}}^{\oplus}\right]|\psi\rangle = \beta\,|\psi\rangle\,. \tag{13}$$

There are three possible values of λ namely $\lambda = 0$, $\lambda = 1$ and $0 < \lambda < 1$. For $\lambda = 1$ it will apply

$$\left[\hat{a}_{\mathbf{k}}^{\oplus\dagger}\hat{a}_{\mathbf{k}}^{\otimes} + \hat{a}_{\mathbf{k}}^{\oplus}\hat{a}_{\mathbf{k}}^{\otimes\dagger}\right]|\psi\rangle = \beta\,|\psi\rangle\,. \tag{14}$$

In order to obtain a solution, the general form of the quantum state is taken $|\psi\rangle = \sum_{n,m}\mathcal{C}_{nm}\,|n_{\mathbf{k}}^{\oplus}, m_{\mathbf{k}}^{\otimes}\rangle$. Where $\mathcal{C}_{nm}$ is a constant. Substitute back into equation (14), then multiply by ${}_I\langle i_{\mathbf{k}}^{\oplus}, j_{\mathbf{k}}^{\otimes}|$ from the left, become

$$\sqrt{i(j+1)}\;\mathcal{C}_{(i-1)\,(j+1)} + \sqrt{(i+1)(j-1)}\;\mathcal{C}_{(i+1)\,(j-1)} = \beta\;\mathcal{C}_{ij}. \tag{15}$$

The constant β is chosen to be the total number of gravitons ($N = n + m$), then the magnitude of the constant $\mathcal{C}_{nm} = \binom{N}{n}^{1/2}$. The solution to the eigenstate become

$$\begin{aligned}
|\psi\rangle &= \sum_{n=0}^{N}\left[\frac{1}{\sum_{n'=0}^{N}\binom{N}{n'}}\right]^{1/2}\binom{N}{n}^{1/2}|(N-n)_{\mathbf{k}}^{\oplus},\; n_{\mathbf{k}}^{\otimes}\rangle\,. \\
&= \sum_{n=0}^{N}\mathcal{E}_N\binom{N}{n}^{1/2}|(N-n)_{\mathbf{k}}^{\oplus},\; n_{\mathbf{k}}^{\otimes}\rangle\,.
\end{aligned} \tag{16}$$

where $\mathcal{E}_N$ is a normalization constant. This state is an entanglement between the polarization of modes $\oplus$ and $\otimes$. As explained in the introduction, the state of minimal uncertainty from the polarization of gravitational waves can be a state of entanglement. For the constant $\lambda = 1$, then equation (13) becomes $2\hat{c}_{\mathbf{k}}^{\oplus\dagger}\hat{c}_{\mathbf{k}}^{\otimes}\,|\psi\rangle = \beta\,|\psi\rangle$. There will be a solution if $\beta = 0$ with the quantum state being a vacuum state for the polarization of mode $\otimes$ and for the mode polarization $\oplus$, the quantum state can be any state ($|\psi\rangle = |\phi^{\oplus}, 0^{\otimes}\rangle$). Meanwhile, if the λ constant is $0 < \lambda < 1$, the solution will only be a vacuum for both $\oplus$ and $\beta = 0$ polarization modes with $\otimes$.

Uniquely, because equation (13) depends on the value of λ, the solution can be expressed in a more general form as a superposition of possible states. Say for the constant $\lambda = 1$ the solution is a vacuum state, so there are only two possible quantum states, namely the entanglement state (16) and the vacuum state itself. So the solution of the equation if expressed in superposition form is

$$\begin{aligned}
|\psi\rangle &= \tanh u\;\;|0^{\oplus}, 0^{\otimes}\rangle + (1-\tanh^2 u)^{1/2} \\
&\quad\times\sum_{n=0}^{N}\mathcal{E}_N\binom{N}{n}^{1/2}|(N-n)_{\mathbf{k}}^{\oplus},\; n_{\mathbf{k}}^{\otimes}\rangle\,.
\end{aligned} \tag{17}$$

Where $u \equiv c\,\lambda$ and c is an arbitrary positive real number with value $c >> 1$, so when $0 < \lambda \leq 1$ the vacuum state is the solution and when $\lambda = 0$ the solution is the entanglement state (16). According to this definition, the constant u has a range of values of $0 \leq u < \infty$. If it is assumed that the gravitational waves in the environment of the interferometer have only one of the polarization modes, then the initial quantum state of the primordial graviton will be traced out for one of the modes. The density matrix from equation (17) will be (the mode of polarization $\otimes$ is selected)

$$\begin{aligned}
\rho^{\otimes} &= Tr_{\oplus}\Big(\,|\psi\rangle\,\langle\psi|\,\Big) \\
&= \tanh^2 u\,|0\rangle\,\langle 0| + \tanh u\;(1-\tanh^2 u)^{1/2} \\
&\quad\times\mathcal{E}_N\Big[\,|0\rangle\,\langle N| + |N\rangle\,\langle 0|\Big] + (1-\tanh^2 u) \\
&\quad\times\sum_{n=0}^{N}\mathcal{E}_N^2\binom{N}{n}|n\rangle\,\langle n|\,.
\end{aligned} \tag{18}$$

The index $\mathbf{k}$ and polarization index are omitted to avoid future confusion. In superposition form, the density matrix for one of the polarizations will produce non-diagonal terms that define the interference from the vacuum and the entanglement. These interference terms are a unique condition in quantum mechanics that does not exist in classical physics. The existence of these interference terms should affect the noise that produces by gravitons. The entanglement state (16) will also be satisfied if $\hat{X}_1 = \hat{\mathcal{P}}_k^{(2)}$ and $\hat{X}_2 = \hat{\mathcal{P}}_k^{(1)}$.

Not only the entanglement (16) but there is another entanglement state that also satisfies the minimum uncertainty relation. Selected $\hat{X}_1 = \hat{\mathcal{P}}_k^{(3)}$ and $\hat{X}_2 = \hat{\mathcal{P}}_k^{(2)}$, using the same method then taking $\beta = N$ and $\lambda = 1$, then we get the entanglement state

$$|\psi\rangle = \sum_{n=0}^{N}(-1)^N\mathcal{E}_N\binom{N}{n}^{1/2}|(N-n)_{\mathbf{k}}^{\oplus},\; n_{\mathbf{k}}^{\otimes}\rangle\,. \tag{19}$$

This solution is similar to the entanglement (16), but it can be negative if the total gravitons are odd. For $0 \le \lambda < 1$ the solution is a vacuum. So that when expressed in the form of a superposition becomes

$$|\psi\rangle = \tanh u \; |0^{\oplus}, 0^{\otimes}\rangle + (1-\tanh^2 u)^{1/2} \times \sum_{n=0}^{N} (-1)^N \mathcal{E}_N \binom{N}{n}^{1/2} |(N-n)^{\oplus}_{\mathbf{k}},\; n^{\otimes}_{\mathbf{k}}\rangle, \quad (20)$$

where the constant u will be defined differently , namely $u \equiv c(1-\lambda)$ which has the range $0 \le u < \infty$. The density matrix would be similar to equation (18) with the interference terms would be negative if the total gravitons are odd. The density matrix for one mode of polarization in this case can be written as follows

$$\rho^{\otimes} = \tanh^2 u \, |0\rangle\langle 0| + (-1)^N \tanh u \, (1-\tanh^2 u)^{1/2} \times \mathcal{E}_N \Big[|0\rangle\langle N| + |N\rangle\langle 0| \Big] + (1-\tanh^2 u) \times \sum_{n=0}^{N} \mathcal{E}_N^2 \binom{N}{n} |n\rangle\langle n|. \quad (21)$$

This entanglement solution would also be satisfied when $\hat{X}_1 = \hat{\mathcal{P}}_k^{(1)}$ and $\hat{X}_2 = \hat{\mathcal{P}}_k^{(3)}$.

In the next section, by defining the minimum uncertainty state of equations (16), (17), (19), or (20) as an initial state the quantum noise correlation will be calculated. We also calculated the decoherence time to find out how long the quantum noise from the primordial graviton could be able to detect.

IV. QUANTUM NOISE CORRELATION FROM MINIMUM UNCERTAINTY PRIMORDIAL GRAVITON

To calculate the quantum noise correlation due to graviton particles from the early universe (primordial gravitons), squeezed formalism will be used, as in research [12, 34]. This formalism is based on the cosmological inflation model, in which the initial quantum state will experience squeezing as the universe expands rapidly. Although this squeezed representation still faces a lot of criticism [40–43], it is still often used in the study of quantum states at the beginning of the universe. Mathematically the squeezing process will be represented as a two-mode squeezed operator as follows

$$\hat{S}^A(\zeta) = \sum_{\mathbf{k},A} \frac{1}{V} \exp\left[\zeta^* \; \hat{a}^A_{\mathbf{k}} \hat{a}^A_{-\mathbf{k}} - \zeta \; \hat{a}^{A\dagger}_{\mathbf{k}} \hat{a}^{A\dagger}_{-\mathbf{k}}\right], \quad (22)$$

with $\zeta \equiv r_k e^{i\varphi_k}$, where r_k and φ_k is a squeezing and phase angle parameter. In general, the squeezing parameter r_k and the phase φ_k depend on k. However, for simplicity, we regard these variables as constant so that the squeezing parameter can have a value of $0 \le r_k$ and the phase angle parameter of $0 \le \varphi_k \le 2\pi$. It should be noted that the squeeze parameter r_k here is different from the squeeze parameter in the initial state (λ) due to the minimum uncertainty.

In this session, the calculation of quantum noise correlation will be divided into two parts. Where the first part will calculate the quantum noise correlation for two points, followed by three points in the second part.

A. Two Point Correlation

Two point anticommutation correlation function ($\{\hat{X}, \hat{Y}\} = (\hat{X}\hat{Y} + \hat{Y}\hat{X})/2$) at limit $L_x, L_y, L_z \to \infty$, can be written as follows

$$\Big\langle \big\{\hat{N}_{ij}(t), \hat{N}_{kl}(t')\big\} \Big\rangle = Tr\Big(\hat{S}^{\otimes\dagger} \big\{\hat{N}_{ij}(t), \hat{N}_{kl}(t')\big\} \hat{S}^{\otimes} \rho^{\otimes}\Big) = \frac{\kappa^2}{10\pi^2}\Big(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk} - \frac{2}{3}\delta_{ij}\delta_{kl}\Big) \times \int_0^{\Omega_m} dk \; k^6 P(t,t',k), \quad (23)$$

where $P(t,t',k)$ in one of the polarization modes is

$$Tr\Big(\hat{S}^{\otimes\dagger} \big\{\hat{h}^{\otimes}_{\mathbf{k},I}(t), \hat{h}^{\otimes}_{\mathbf{k}',I}(t')\big\} \hat{S}^{\otimes} \rho^{\otimes}\Big) = \delta_{\mathbf{k}+\mathbf{k}',0} P(t,t',k). \quad (24)$$

If it is assumed that the initial state of the primordial graviton is a state of minimum uncertainty, then the initial state can be chosen from one of the solutions. If the selected state is the entanglement (16), the density matrix for one of the polarization modes becomes

$$\rho^{\otimes} = \sum_{n=0}^{N} \mathcal{E}_N^2 \binom{N}{n}^{1/2} |n\rangle\langle n| \,. \quad (25)$$

By using this density matrix will be obtained

$$P(t,t',k) = \frac{(N+1)}{2k}\Big[\cos(k(t-t'))\cosh 2r_k - \cos(k(t-t') - \varphi_k)\sinh 2r_k\Big] \quad (26)$$

This result is nothing more than a generalized form of the correlation function for the Bunch-Davies vacuum [8]. Where is the two-point correlation function of this entanglement state is $(N+1)$ times greater than the vacuum state. The consequence of this multiplier factor can be seen by observing the effective strain (h_{eff}) of the correlation as

$$h_{eff} \equiv \frac{N(f)}{(2\pi f)^2} \quad (27)$$

with

$$N(f) = \Big(\int dt \Big\langle \big\{\hat{N}^{ij}(t), \hat{N}_{ij}(0)\big\} \Big\rangle e^{2\pi i f t}\Big)^{1/2}, \quad (28)$$

so that

$$h_{eff} = (N+1)\pi^2 \frac{(2f)^{5/2}}{M_{pl}} e^{r_k} \approx (2N+2)\times 10^{-42}\left(\frac{f}{1\text{Hz}}\right)^{-1/2} e^{r_k}\ \text{Hz} \quad (29)$$

For primordial gravitational waves with a frequency of 100 Hz (which is the characteristic frequency of the LIGO detector), the magnitude of the effective strain is $h_{eff}(f)|_{f\sim 100\ Hz} \sim (N+1)\ 10^{-23}\ Hz^{-1/2}$, with the size of the squeezing parameter is chosen as $e^{r_K} \sim 10^{18}$ based on the inflation mechanism. This strain sensitivity is already in the range of the LIGO detector, Which is known to be $10^{-23}\ Hz^{-1/2}$. But with the presence of the multiplier factor $(N+1)$, the size of the effective strain can be bigger, so it should make it easier to detect the quantum noise by the LIGO detector if the total particle $N > 0$. Similar results will be obtained if the initial state used is equation (19).

If the chosen initial state is the density matrix of equation (18) or (21) where the initial state has interference terms (as an example it will be calculated for total particles $N = 2$) then we will get

$$\begin{aligned} P_{N=2}(t,t',k) = {} & \frac{(3-2\tanh^2 u)}{2k}\Big[\cos(k(t-t'))\cosh 2r_k - \cos(k(t-t')-\varphi_k)\sinh 2r_k\Big] + \\ & \frac{\sqrt{2}\tanh u(1-\tanh^2 u)^{1/2}}{2k}\Big[2\cos(k(t+t'))\cosh^2 r_k - 2\cos(k(t+t')-2\varphi_k) \\ & \sinh^2 r_k - \Big\{\cos(k(t-t')+\varphi_k) + \cos(k(t-t')-\varphi_k)\Big\}\sinh 2r_k\Big] \end{aligned} \quad (30)$$

In general, this result is not much different from equation (26) if the total particle is $N = 2$. But in this result, there is a correction term in the second term of the equation. Where this term appears due to the interference terms in the initial density matrix. The magnitude of the correction term will be much smaller than the first term because the correction term contains a multiplier factor $\sqrt{2}\tanh u(1-\tanh^2 u)^{1/2}$. If the value of the u constant is chosen to be 10, then the correction factor will be in the order of 10^{-5}.

B. Three Point Correlation

The interference terms in the initial state with superposition (equation (18) or (21)) will not significantly affect the two-point correlation function. However, if the correlation function reviewed is more than two, then the interference terms can have a significant influence. For example, when studying about three-point correlation function. If the total number of the particles is $N = 1$ or $N = 3$, the three-point correlation function will not be zero. In this subsection, we will calculate the bispectrum quantity as an example. Where the quantum noise operator used is expressed in Fourier space $(\hat{N}^A_{\mathbf{k}}(t) = k^2\delta\hat{h}^A_{\mathbf{k}}(t)/2)$. So the bispectrum for one of the polarization modes can be written as

$$S(\omega_1,\omega_2) = \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2\ \ Tr\Big(\hat{S}^{\otimes\dagger}\Big\{\delta\hat{N}^{\otimes}_{\mathbf{k}}(0), \{\hat{N}^{\otimes}_{\mathbf{k}}(t_1), \hat{N}^{\otimes}_{\mathbf{k}}(t_2)\}\Big\}\hat{S}^{\otimes}\rho^{\otimes}\Big)\ e^{-i(\omega_1 t_1+\omega_2 t_2)}. \quad (31)$$

By choosing the initial state density matrix as equation (18) or (21) and the phase parameter $\varphi_k = 0$ as a simplification, then for the total particles $N = 3$ we get

$$\begin{aligned} S_{N=3}(\omega_1,\omega_2) = {} & \pm\frac{k^5\tanh u(1-\tanh^2 u)^{1/2}}{12\sqrt{2k}}\Big(\cosh r_k - \sinh r_k\Big)\Big[\cosh 2r_k\Big(\delta(\omega_1+k)\delta(\omega_2+k) + \\ & \delta(\omega_1-k)\delta(\omega_2-k)\Big) + \sinh 2r_k\Big(\delta(\omega_1-k)\delta(\omega_2+k) + \delta(\omega_1+k)\delta(\omega_2-k)\Big)\Big], \end{aligned} \quad (32)$$

the $\pm$ sign in this result refers to the choice of the initial state. If the initial state is the equation (18), then the bispectrum will have a positive value and vice versa for the initial state (21). The bispectrum quantity will be localized at four frequency points depending on the size of the variable k, as an example is shown in Figure (1)[46]. From each figure (1(a) and 1(b))), it can be seen those four points will have the same bispectrum quantity.

Next, we will calculate the bispectrum quantity with the same density matrix but with a total particle is $N = 1$, obtain

$$\begin{aligned} S_{N=1}(\omega_1,\omega_2) \;=\; & \pm\frac{k^5 \tanh u(1-\tanh^2 u)^{1/2}}{12\sqrt{2k}}\Big(\cosh r_k - \sinh r_k\Big)\Big[A1\ \delta(\omega_1+k)\delta(\omega_2+k) + \\ & A2\ \delta(\omega_1+k)\delta(\omega_2-k) + A3\ \delta(\omega_1-k)\delta(\omega_2+k) + A4\ \delta(\omega_1-k)\delta(\omega_2-k)\Big], \end{aligned} \tag{33}$$

where $A1, A2, A3$, and $A4$ are shown in Appendix A. Just as for the total particles $N = 3$, the bispectrum quantity for $N = 1$ will be localized at four frequency points. However, each point will have a different size as shown in Figure (2).

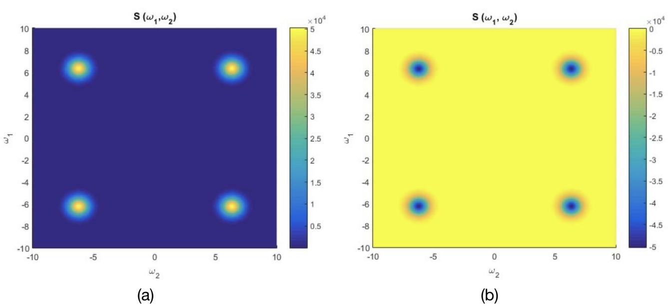

FIG. 1: The plot of bispectrum equation (32) with $e^{r_k} = 10^5$, $k = 2\pi$ and $u = 10$. Figure (a) is the bispectrum plot with the initial state (18) and Figure (b) is the bispectrum plot with the initial state (21)

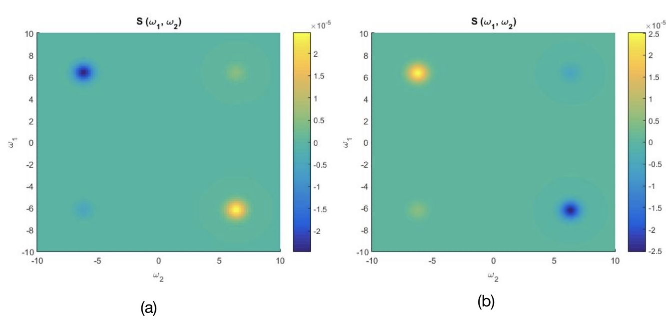

FIG. 2: The plot of bispectrum equation (33) with $e^{r_k} = 10^5$, $k = 2\pi$ and $u = 10$. Figure (a) is the bispectrum plot with the initial state (18) and Figure (b) is the bispectrum plot with the initial state (21)

From these two examples, it is shown that the interference term of the density matrix initials allows the correlation function that is larger than the two points to have a value not equal to zero. This means that the quantum noise due to the minimal uncertainty of the primordial graviton could have non-Gaussian distribution.

V. DECOHERENCE FROM MINIMUM UNCERTAINTY PRIMORDIAL GRAVITON

Quantum noise from primordial gravitons can be detected as long as the quantum interference effect of the system is still present or decoherence has not occurred yet due to interactions with the environment. For this reason, we will look for the expression of decoherence functional to find out how long it takes for the quantum noise to be detected. In order to make the discussion more simple, an experimental setup of the system (test particle or mirror) was chosen to have a form similar to research [9]. In that experimental setup, the Michelson equal arm interferometer is used, which has two macroscopic suspended mirrors at the end of each interferometer arm. When the laser interferometer is fired there will be two possible paths, namely towards mirror 1 or mirror 2. Furthermore, it is assumed that when a photon from the laser interferometer hits one of the mirrors an oscillation will occur which is described as the following semiclassical state

$$\begin{aligned} \vec{\xi_1}(t) &= (\xi_1, 0, 0), \qquad \xi_1 = A\cos\omega t \\ \vec{\xi_2}(t) &= (0, \xi_1, 0), \qquad \xi_2 = A\cos\omega t. \end{aligned} \tag{34}$$

Where ω and A are the frequency and amplitude that can be generated from the oscillations of each mirror. In the Hilbert space representation, there will be two possible bases namely $|0\rangle$ which is the basis vector when the photon does not hit the mirror and $|\vec{\xi_i}\rangle$ is the basis vector when the photon hits one of the mirrors ($i = 1, 2$). Overall the quantum state of the system can be expressed in the superposition form as follows

$$|\Psi(t_i)\rangle = \frac{1}{\sqrt{2}}\Big(|\vec{\xi_1}\rangle \otimes |0\rangle + |0\rangle \otimes |\vec{\xi_2}\rangle\Big). \tag{35}$$

In the density matrix form we can write $\rho_m(t_i) = |\Psi(t_i)\rangle\langle\Psi(t_i)|$. As shown in Figure (3), the system will be

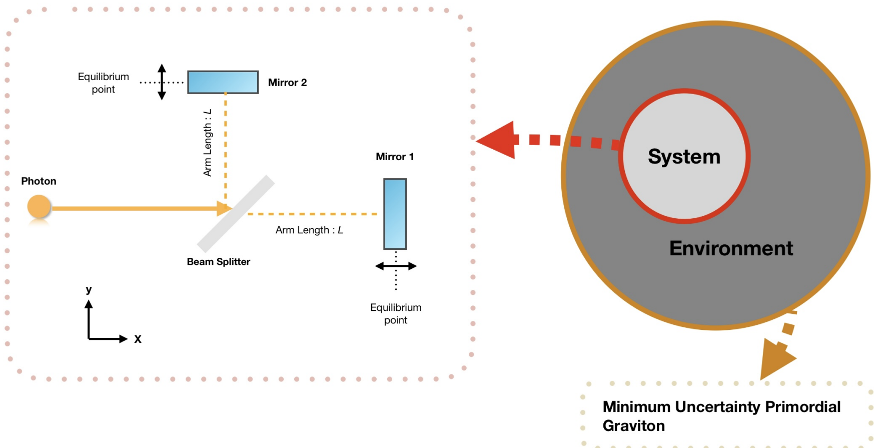

FIG. 3: The Michelson equal arm interferometer (the experimental setup (ρ_m)) in the environment filled with graviton originated from the early universe in the minimum uncertainty initial state ($\rho^{\otimes}$).

in an environment filled with gravitons originating from the early of the universe with an initial state of minimum uncertainty. This means that the total density matrix of the system and environment can be described as follows

$$\begin{aligned}\rho(t_i) &= \rho_m(t_i)\otimes\rho^{\otimes}\\ &= \Big(\rho_{11}(t_i)+\rho_{22}(t_i)+\rho_{12}(t_i)+\rho_{21}(t_i)\Big)\\ &\quad\otimes\Big(\rho^{\otimes}_{Diag}+\rho^{\otimes}_{Non-Diag}\Big), \end{aligned}\tag{36}$$

where $\rho^{\otimes}$ can be chosen from the solutions of minimum uncertainty states. In general, the density matrix $\rho^{\otimes}$ can be separated based on diagonal and non-diagonal elements.

The density matrix of the system and environment evolves with time based on the Langevin equation of geodesic deviation of the mirror in the presence of gravitons. Due to the interaction between the mirror and gravitons, the quantum state of the system and the environment will be entangled. This means to obtain the quantum state of the mirror at a certain time, we will trace out the degrees of freedom of the graviton environment from the total density matrix as in Appendix (B). We will obtain the density matrix of the mirror at a certain time affected by gravitons (for example, we chose the total of the graviton $N=3$ and ρ_f to be equation (18) or (21)) as

$$\begin{aligned}\rho_m(t_f) \approx\ & \rho_{11}(t_i)+\rho_{22}(t_i)+\Big(\exp\Big\{i\Phi_{Diag\ N=3}\Big\}+\exp\Big\{i\Phi_{Non-Diag\ N=3}\Big\}\Big)\ \rho_{12}(t_i)\\ &+\Big(\exp\Big\{-i\Phi^*_{Diag\ N=3}\Big\}+\exp\Big\{-i\Phi^*_{Non-Diag\ N=3}\Big\}\Big)\ \rho_{12}(t_i)\end{aligned}\tag{37}$$

The influence of the environmental gravitons will only affect the interference terms of the mirrors. However, unlike research [9], the influence phase functional in this case will be separated into two parts based on the diagonal $\big(i\Phi_{Diag}[\xi_1,\xi_2]\big)$ and non-diagonal $\big(i\Phi_{Non-Diag}[\xi_1,\xi_2]\big)$ terms of the environmental density matrix. The real part of the influence phase functional will be a factor that influences the existence of the interference terms of the matter density matrix. Because the real part is time-dependent, at a certain time the interference terms will disappear. The process of losing the interference terms is known as decoherence. Which is the real part of the influence phase functional is called the decoherence functional and the time when the interference term is zero is called the decoherence time. In this study there will be two influence phase functional as shown in equation (37), meaning that there will also be two decoherences functional, each of which will have the form

$$\Gamma_{Diag}(t_f)=\frac{m^2}{8}\int_0^{t_f}dt\int_0^{t_f}dt'\ Tr\Big(\hat{S}^{\otimes\dagger}\{\hat{N}_{ij}(t),\hat{N}_{kl}(t')\}\hat{S}^{\otimes}\ \rho^{\otimes}_{Diag}\Big)\ \Delta(\xi^i\xi^j)(t)\ \Delta(\xi^k\xi^l)(t')\tag{38}$$

and

$$\Gamma_{Non-Diag\ N=3}(t_f) = \ln\Bigg[\Bigg|\frac{m^3}{24}\int_0^{t_f} dt \int_0^{t_f} dt' \int_0^{t_f} dt''\ Tr\Big(\hat{S}^{\otimes\dagger}\Big\{\hat{N}_{ij}(t),\big\{\hat{N}_{kl}(t'),\hat{N}_{mn}(t'')\big\}\Big\}\hat{S}^{\otimes}\ \rho^{\otimes}_{Non-Diag\ N=3}\Big) \times\Delta(\xi^i\xi^j)(t)\ \Delta(\xi^k\xi^l)(t')\ \Delta(\xi^m\xi^n)(t'')\Bigg|^{-1}\Bigg] \equiv \ln\Big[\ \big|\mathcal{N}(t_f)\big|^{-1}\Big] \quad (39)$$

Here $\Delta(\xi^i\xi^j)(t) = \xi_1^i(t)\xi_1^j(t) - \xi_2^i(t)\xi_2^j(t)$, that will be determined based on the experimental setup in Figure (3). Equation (38) is the decoherence functional that arises due to the influence of the diagonal terms of the environmental density matrix. This expression will apply not only to the diagonal density matrices of equations (18) or (21) for $N = 3$, but to all environmental density matrices of primordial gravitons which have only diagonal elements. It is different from equation (39) which this expression specifically appears for the non-diagonal density matrices (18) and (21) with the total graviton particles $N = 3$. The existence of the squeezing operator on both decoherence functionals is due to gravitons in the environment originating from the beginning of the universe.

A. Entanglement Primordial Graviton

Furthermore, the decoherence functional will be calculated if the environmental state is the entanglement of equations (16) or (19). The density matrix for one of the polarization modes will take the form of an equation (25). Which density matrix only has diagonal elements. It means the influence functional $\exp\big\{i\Phi_{Non-Diag\ N=3}\big\} \approx 0$. So there will only be one decoherence functional, namely equation (38). Based on the experimental setup in Figure (3), there will be two possible forms of $\Delta(\xi^i\xi^j)(t)$ that are

$$\Delta\xi_1^2(t) = (L + A\cos\omega t)^2 - L^2, \quad (40)$$
$$\Delta\xi_2^2(t) = L^2 - (L + A\cos\omega t)^2. \quad (41)$$

If the oscillation amplitude is much smaller than the length of the interferometer arm, it can be approximated that $\Delta\xi_i^2 \sim 2LA$. To calculate the decoherence functional, the two-point correlation function of equation (23) will be used. Then based on the conventional inflation scenario, $\sinh 2r_k \simeq \cosh 2r_k \simeq (k_c/k)^4$, where $k_c = 2\pi f_c$ and f_c is the cutoff frequency of primordial gravitational waves. The bound on the cutoff frequency from CMB is $f_c \lesssim 10^9$ Hz. Obtained

$$\int_0^{\Omega_m} dk\ k^6 P(t,t',k) \approx \Big(N+1\Big)\ k_c^4\ \Omega_m^2\ \frac{x\sin(x)+\cos(x)-1}{x^2}, \quad (42)$$

with $x = \Omega_m(t-t')$. Substituting into equation (38) and using the same method as in the research [9], then the decoherence functional becomes

$$\Gamma_{Diag}(t_f) = \Big(N+1\Big)\ \frac{4\pi^3}{5}\Big(\frac{m}{M_p}\Big)^2 (L\ f_c)^4 \Big(\frac{A}{L}\Big)^2 (\omega\ t_f) \quad (43)$$

Similar to the two-point correlation function in section (4), this decoherence functional is a general form of decoherence functional for the environment in the form of a Bunch-Davies vacuum. Where there will be a multiplier factor $(N+1)$. The role of this multiplier can be seen when reviewing the decoherence time t_f. For $N = 0$, the decoherence time (the condition when $\Gamma_{Diag}(t_f) \simeq 1$) will be 20 seconds when the parameters $\omega = 10^3$ Hz, $L = 40$ km , $m = 40$ kg, $f_c = 10^9$ Hz, and $A = 10/\sqrt{2m\omega}$ are selected. This means that when $N \neq 0$, the bigger the number of N, the decoherence time will be reduced to $20/(N+1)$ seconds. However, if you want to maintain the decoherence time for 20 seconds, you can reduce the dimension of the interferometer arm (L) or the mass of the mirror (m) by $(N+1)^{-1/2}$ times.

B. Superposition Primordial Graviton

In this section, the decoherence functional will be calculated for the environmental density matrix in the form of equation (18) or (21). These two density matrices will have two decoherence functionals. Where the interference terms of the mirror density matrix will not be zero if one of its decoherence functionals value is small $(\Gamma(t_f) < 1)$. For the diagonal decoherence functional $(\Gamma_{Diag}(t_f))$, the expression is not much different from equation 43 when $N = 3$ (just replacing the multiplier factor $(N+1)$ with $(4 - 3\tanh^2 u)$), meaning that the decoherence time of this function will only last for a maximum of 20 seconds. Meanwhile, for non-diagonal decoherence functional $(\Gamma_{Non-Diag\ N=3}(t_f))$, different results will be obtained.

To start the discussion, we will calculate the three-point correlation from the non-diagonal decoherence functional of equation (39), which will take the following form

$$Tr\Big(\hat{S}^{\otimes\dagger}\Big\{\hat{N}_{ij}(t),\{\hat{N}_{kl}(t'),\hat{N}_{mn}(t'')\}\Big\}\hat{S}^{\otimes}\ \rho^{\otimes}_{Non-Diag\ N=3}\Big)$$
$$=\frac{\Big(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}-\frac{2}{3}\delta_{ij}\delta_{kl}\Big)}{10\pi^2M_p^3}\int_0^{\Omega_m}dk\ k^8\ e^{\otimes}_{\mathbf{k},mn}\ F(t,t',t'',k), \tag{44}$$

with $F(t,t',t'',k)$ is

$$Tr\Big(\hat{S}^{\otimes\dagger}(\zeta)\Big\{\delta\hat{h}^{\otimes}_{\mathbf{k},I}(t),\{\delta\hat{h}^{\otimes}_{\mathbf{k}',I}(t'),\delta\hat{h}^{\otimes}_{\mathbf{k}'',I}(t'')\}\Big\}\hat{S}^{\otimes}(\zeta)\ \rho^{\otimes}_{Non-Diag\ N=3}\Big)=\delta_{\mathbf{k}-\mathbf{k}',0}\ \delta_{\mathbf{k}'+\mathbf{k}'',0}\ F\ (t,t',t'',k) \tag{45}$$

and the polarisation tensor $e^{\otimes}_{\mathbf{k},mn}$ will look like [48]

$$e^{A}_{\mathbf{k},mn}=\frac{1}{\sqrt{2}}\begin{pmatrix}1 & \mp i & 0\\ \mp i & 1 & 0\\ 0 & 0 & 0\end{pmatrix}. \tag{46}$$

The $\pm$ sign in the matrix refers to the choice of the polarization mode of the primordial gravitational wave. Furthermore, because $\Delta(\xi^i\xi^j)(t)$ is only non-zero when the index $i=j$ as shown in equations (40) and (41), then the part of the polarization tensor that affects the calculation of the three-point correlation function is only the diagonal elements. Taking that $\cosh 3r_k\sim(k_c/k)^6$ we get

$$F(t,t',t'',k)\ \approx\ \frac{\sqrt{6}}{12\ k^{3/2}}\tanh u\big(1-\tanh^2 u\big)^{1/2}\times\Bigg[\Big(\frac{k_c}{k}\Big)^6\ \cos\Big(k\big(t+t'+t''\big)\Big)\Bigg] \tag{47}$$

Substituting into equation (44) then equation (39), using some integration and choosing the UV cutoff $\Omega_m\sim\xi^{-1}\equiv L^{-1}$ we find

$$\mathcal{N}(t_f)^{-1}\ \approx\ \frac{3\sqrt{3}m^2\pi^4}{5M_p^3}\big(A\ f_c^2\big)^3\ \tanh u\big(1-\tanh^2 u\big)^{1/2}\times\Bigg(\frac{(t_f\ L^{-1/2})-2L^{3/2}\ (4\omega^2\ t_f^2+2)}{8\omega^2L^2-1}\Bigg). \tag{48}$$

The non-diagonal decoherence functional can be obtained by calculating the natural logarithm of the absolute value of $\mathcal{N}(t_f)^{-1}$. If the same parameters are taken as in section (5.1) then the decoherence time is in the order $\sim\big(\tanh u\sqrt{1-\tanh^2 u}\ \big)^{-1/2}10^9$ second. This result is much longer than the decoherence time due to the diagonal decoherence functional. Unless the constant $u\sim 0$ or $\tanh u\sim 1$. (this condition is when the non-diagonal density matrix ($\rho^{\otimes}_{Non-Diag}$) vanish). This means that the interference terms of the mirror density matrix will last much longer in the presence of non-diagonal terms of the environment density matrix. So the quantum noise will be able to be detected for a longer period of time. Although this result looks quite large for a decoherence process but based on research by V. A. De Lorenci and L. H. Ford [13] this result is still within the range of possible decoherence time induced by graviton.

VI. CONCLUSION

In this work, we have investigated the quantum noise and decoherence due to gravitons from the early universe with minimum uncertainty in the initial state to expand the research previously conducted by S. Kanno et al [8]. In their research, the initial quantum state was limited to the Bunch-Davies vacuum. By taking the minimum uncertainty conditions, the initial quantum states can be expanded, so that it can be either an entanglement or a superposition state between vacuum and entanglement. To find these states, we use quantum operators that can define the polarization of gravitational waves, namely Stoke operators. In this research, we assumed that the graviton in the detector mirror environment is only one of the polarization modes so that the other mode in the initial state will be traced out, which makes the superposition state have non-diagonal or interference terms in the density matrix. In addition, because the gravitons originate from the early universe (primordial graviton), squeezed formalism will be used as the consequence of cosmological inflation. The initial quantum state will evolve based on the squeezed operator.

To investigate the quantum noise we calculated the correlation of the quantum noise operator due to the graviton. We got, for the initial entanglement state, the increasing number of gravitons will increase the effective strain corresponding to the quantum noise. This means the quantum noise would be more likely to be detected by the LIGO detector when the initial state has more gravitons. For the initial state of superposition, the existence of non-diagonal terms causes a quantum noise correlation of more than two to be possible to have a non-zero value. An example is the three-point correlation on the bispectrum quantity in section (3). So the quantum noise distribution produced by a state of minimal uncertainty can be non-Gaussian.

Meanwhile, for decoherence, we use the influence functional method to calculate the decoherence time to find out how long the quantum noise can be detected. Using the same experimental setup as [9], we found if the initial state of the environmental density matrix is in the form of entanglement, an increase in the number of gravitons can reduce the dimensions of the interferometer arm or the mass of the mirror of the experimental setup that can detect the quantum noise for 20 seconds as in the Bunch-Davies vacuum initial state. If the environmental conditions are in the form of a superposition, then the presence of the non-diagonal term will provide an additional term for the influence functional. So there will be two decoherence functions, one of which allows the detection of quantum noise over a very long time.

For future work, this research can still be developed by considering the possibility of a larger quantum mode that occurred as a result of nonlinear time evolution in the early universe. As we know, squeezed formalism is a description of linear time evolution obtained from Bogoulibov transformation with two modes only.

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Appendix A: Details of $A1, A2, A3$ and $A4$

$$\begin{aligned} A1 &= 3\cosh^2 r_k - 5\cosh r_k \sinh r_k + 2\sinh^2 r_k + \tanh^2 u\big(1-\tanh^2 u\big)\big(5\cosh^2 r_k - 7\cosh r_k \sinh r_k \\ &\quad +3\sinh^2 r_k\big) + \frac{1}{\sqrt{2}}\tanh^2 u\big(1-\tanh^2 u\big)^{3/2}\big(3\cosh^2 r_k - 8\cosh r_k \sinh r_k + 2\sinh^2 r_k\big) \end{aligned} \tag{A1}$$

$$\begin{aligned} A2 &= 5\sinh^2 r_k - 5\cosh r_k \sinh r_k + \tanh^2 u\big(1-\tanh^2 u\big)\big(3\cosh r_k^2 - 8\cosh r_k \sinh r_k \\ &\quad +3\sinh^2 r_k\big) + \frac{1}{\sqrt{2}}\tanh^2 u\big(1-\tanh^2 u\big)^{3/2}\big(4\cosh r_k^2 - 7\cosh r_k \sinh r_k + 3\sinh^2 r_k\big) \end{aligned} \tag{A2}$$

$$\begin{aligned} A3 &= 5\cosh^2 r_k - 5\cosh r_k \sinh r_k + \tanh^2 u\big(1-\tanh^2 u\big)\big(3\cosh r_k^2 - 8\cosh r_k \sinh r_k \\ &\quad +3\sinh^2 r_k\big) + \frac{1}{\sqrt{2}}\tanh^2 u\big(1-\tanh^2 u\big)^{3/2}\big(4\cosh r_k^2 - 7\cosh r_k \sinh r_k + 3\sinh^2 r_k\big) \end{aligned} \tag{A3}$$

$$\begin{aligned} A4 &= 2\cosh^2 r_k - 5\cosh r_k \sinh r_k + 3\sinh^2 r_k + \tanh^2 u\big(1-\tanh^2 u\big)\big(5\cosh^2 r_k - 7\cosh r_k \sinh r_k \\ &\quad +3\sinh^2 r_k\big) + \frac{1}{\sqrt{2}}\tanh^2 u\big(1-\tanh^2 u\big)^{3/2}\big(5\cosh^2 r_k - 7\cosh r_k \sinh r_k + 4\sinh^2 r_k\big) \end{aligned} \tag{A4}$$

Appendix B: Influence Functional Method

This section of the Appendix will explain the influence functional method as in the research [49], for determining the expression of the decoherence functional. However, in this explanation, the environmental density matrix will be assumed to be in the form of equation (18) or (21) which has non-diagonal or interference terms. This Appendix will be divided into two-part. The B.1 part, will be explained functional decoherence in the QED case. The B.2 part is for the gravity case.

1. Influence Functional in QED

Before discussing the decoherence functional of the gravitational case, a case with the same analogy will be reviewed. Consider a matter (ρ_m) that is in an environment with electromagnetic field radiation. In the initial state, the density matrix can be written as $\rho(t_i) = \rho_m(t_i) \otimes \rho_f(t_i)$, where $\rho_f(t)$ is the environmental density matrix. If we want to calculate the matter density matrix at a final time $\rho_m(t_f)$, then

$$\rho_m(t_f) = Tr_f\bigg(T_\leftarrow \exp\Big(\int_{t_i}^{t_f} d^4x \mathcal{L}(x)\Big)\rho(t_i)\bigg). \tag{B1}$$

Where $T_\leftarrow$ is the time-ordering operator and $\mathcal{L}(x)$ is the Liouville super operator which has the relation

$$\mathcal{L}(x)\rho = -i[\mathcal{H}(x), \rho] \tag{B2}$$

$\mathcal{H}(x)$ denotes the Hamiltonian density at spacetime coordinates $x = x^\mu = (x^0, \vec{x})$. We choose the coulomb gauge in the following which means that the Hamiltonian density takes the form $\mathcal{H}(x) = \mathcal{H}_C(x) + \mathcal{H}_{tr}(x)$, with

$$\mathcal{H}_C(x) = \frac{1}{2}\int \ d^3y \frac{j^0(x^0,\vec{x})j^0(x^0,\vec{y})}{4\pi|\vec{x}-\vec{y}|}, \tag{B3}$$

and

$$\mathcal{H}_{tr}(x) = j^\mu(x)A_\mu(x). \tag{B4}$$

$\mathcal{H}_{tr}(x)$ represent the Hamiltonian density of the interaction of the matter current density $j^\mu(x)$ with the transverse radiation field $A^\mu(x)$. Next, the time-ordering operator $T_\leftarrow$ will be decomposed into a time-ordering operator $T^j_\leftarrow$ for matter current and a time-ordering operator $T^A_\leftarrow$ for electromagnetic fields. With some derivation and defined current super operator $J^\mu_+(x)\rho \equiv j^\mu(x)\rho$ and $J^\mu_-(x)\rho \equiv \rho j^\mu(x)$, then equation (B1) becomes

$$\begin{aligned}\rho_m(t_f) \ &= \ T^j_\leftarrow\Bigg(\exp\Bigg[\int_{t_i}^{t_f} d^4x\mathcal{L}_C(x) - \frac{1}{2}\int_{t_i}^{t_f} d^4x \\ &\times \int_{t_i}^{t_f} d^4x'\theta(t-t') \ [A_\mu(x), A_\nu(x')] \\ &\times J^\mu_+(x)J^\nu_+(x') + \frac{1}{2}\int_{t_i}^{t_f} d^4x \int_{t_i}^{t_f} d^4x'\theta(t-t') \\ &\times [A_\mu(x), A_\nu(x')]J^\mu_-(x)J^\nu_-(x')\Bigg]W[J_+,J_-]\Bigg).\end{aligned} \tag{B5}$$

Where

$$W[J_+,J_-] = Tr_f\Bigg\{\exp\Bigg[\int_{t_i}^{t_f} d^4x\mathcal{L}_{tr}(x)\rho(t_i)\Bigg]\Bigg\}. \tag{B6}$$

This functional is part of the density matrix of matter at any time ($\rho_m(t_f)$) that contains the correlation between the matter field and electromagnetic radiation. In general, the initial radiation density matrix $\rho_f(t_i)$ will be chosen as a thermal equilibrium state. Where the density matrix of that state has only diagonal elements. In this study, the environmental state of the graviton is in conditions of minimum uncertainty that allow for non-diagonal elements in the density matrix. So $\rho_f(t_i)$ will be expanded and assumed to be the same as equation (18)or(21). Furthermore, using an exponential expansion in equation (B6) is obtained

$$\begin{aligned}W[J_+,J_-] \ &= \ Tr_f\Bigg\{\Bigg[1 + \int d^4x\mathcal{L}_{tr}(x) + \frac{1}{2!}\int d^4x \int d^4x' \\ &\times\mathcal{L}_{tr}(x)\mathcal{L}_{tr}(x') + \frac{1}{3!}\int d^4x \int d^4x' \int d^4x'' \\ &\times\mathcal{L}_{tr}(x)\mathcal{L}_{tr}(x')\mathcal{L}_{tr}(x'') + ...\ \Bigg] \\ &\rho_m(t_i)\otimes\rho_f(t_i)\Bigg\}\end{aligned} \tag{B7}$$

The density matrix of matter $\rho_f(t_i)$ in general can be split into a diagonal part and a non-diagonal part $\Big(\rho_f(t) \equiv \rho_{f,Diag}(t) + \rho_{f,Non-Diag}(t)\Big)$. If it is assumed that the total particle N is odd, then the diagonal density matrix $\rho_{f,Diag}(t)$ will only work on even order Liouville super operator and the non-diagonal density matrix $\rho_{f,Non-Diag}(t)$ will only work on odd order Liouville super operators. This means that for $N = 3$, the functional $W[J_+,J_-]$ can be written as

$$\begin{aligned}W[J_+,J_-] \ &= \ \Bigg(\exp\Bigg[\frac{1}{2}\int d^4x \int d^4x' \ \ Tr_f\Bigg\{\mathcal{L}_{tr}(x)\mathcal{L}_{tr}(x') \\ &\times\rho_{f,Diag\ N=3}(t_i)\Bigg\}\Bigg] + \frac{1}{3!}\int d^4x \int d^4x' \\ &\times \int d^4x''Tr_f\Bigg\{\mathcal{L}_{tr}(x)\mathcal{L}_{tr}(x')\mathcal{L}_{tr}(x'') \\ &\times\rho_{f,Non-Diag\ N=3}(t_i)\Bigg\}\Bigg)\rho_m(t_i).\end{aligned} \tag{B8}$$

Where the Liouville super operators with odd order are limited only to the third. The first term of the equation can be obtained by using the Wick theorem for the Gaussian correlation function. Where the even-order correlation function can always be expressed in the form of a two-point correlation. From this result, if it is substituted back into equation (B5), then

$$\begin{aligned}\rho_m(t_f) \ &= \ T^j_\leftarrow\Bigg(\exp\Big\{i\Phi_{Diag\ N=3}[J_c,J_a]\Big\} \\ &+\exp\Big\{i\Phi_{Non-Diag\ N=3}[J_c,J_a]\Big\}\Bigg)\rho_m(t_i).\end{aligned} \tag{B9}$$

$i\Phi_{Diag\ N=3}[J_c,J_a]$ is the influence phase functional resulting from the diagonal density matrix $\Big(\rho_{f,Diag}(t)\Big)$ and $i\Phi_{Non-Diag\ N=3}[J_c,J_a]$ is influence phase functional caused by the no-diagonal density matrix $\Big(\rho_{f,Non-Diag}(t)\Big)$. Each of those influence phase functional are

$$\begin{aligned}i\Phi_{Diag\ N=3} \ &= \ \int_{t_i}^{t_f} d^4x\mathcal{L}_C(x) + \frac{1}{2}\int_{t_i}^{t_f} d^4x \int_{t_i}^{t_f} d^4x' \\ &\Bigg[\ iD(x-x')_{\mu\nu}\ J^\mu_c(x)J^\nu_a(x') \\ &-Tr_f\Big(\big\{A_\mu(x), A_\nu(x')\big\}\ \rho_{f,Diag\ N=3}(t_i)\Big) \\ &\times J^\mu_c(x)J^\nu_c(x')\Bigg]\end{aligned} \tag{B10}$$

and

$$
\begin{aligned}
i\Phi_{Non-Diag\ N=3} &= \int_{t_i}^{t_f} d^4x \mathcal{L}_C(x) + \frac{1}{2}\int_{t_i}^{t_f} d^4x \int_{t_i}^{t_f} d^4x' \Big[\ iD(x-x')_{\mu\nu}\ \Big(J_c^\mu(x)J_a^\nu(x') + J_a^\mu(x)J_c^\nu(x')\Big)\Big] \\
&- \ln\Bigg[\ \Bigg|\frac{1}{6}\int_{t_i}^{t_f} d^4x \int_{t_i}^{t_f} d^4x' \int_{t_i}^{t_f} d^4x''\ Tr_f\Big(\Big\{A_\mu(x), \{A_\nu(x'), A_\gamma(x'')\}\Big\}\rho_{f,Non-Diag\ N=3}(t_i)\Big) \\
&\times J_c^\mu(x)J_c^\nu(x')J_c^\gamma(x'')\Bigg|^{-1}\Bigg] \pm i\frac{\pi}{2}, \qquad \text{(B11)}
\end{aligned}
$$

with $D(x-x')_{\mu\nu} \equiv i[A_{\mu(x)}, A_\nu(x')]$. The influence phase functional $i\Phi_{Diag\ N=3}[J_c, J_a]$ is none other than the influence phase functional that is the same as obtained from the research of H. Breuer and F. Petruccione [49]. When the initial density matrix only has diagonal terms, so $i\Phi_{Non-Diag\ N=3}[J_c, J_a]$ will be zero. The last term in $i\Phi_{Non-Diag\ N=3}[J_c, J_a]$ appears as a result of the complex logarithm when expressing the second term of equation (B8) in the form of exponential. To find the degree of measurement of decoherence, the matter density matrix has to be defined. It is assumed, that the time-dependent quantum state of matter can be described in Figure (4). Where a charged particle in the initial state can move to the final state through two trajectories which will represent two amplitude probabilities that can be described by two wave packets $|\Psi_1(t)\rangle$ and $|\Psi_2(t)\rangle$. Based on the superposition principle, the wave function at the initial time can be written as follow

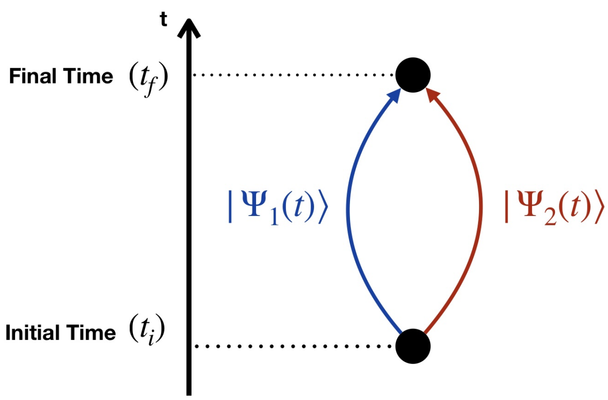

FIG. 4: The trajectory of charge particle from initial time (t_i) into some final time (t_f).

$$
|\Psi(t_i)\rangle_m = |\Psi_1(t_i)\rangle + |\Psi_2(t_i)\rangle\,. \qquad \text{(B12)}
$$

So the density matrix becomes

$$
\rho_m(t_i) = \rho_{11}(t_i) + \rho_{22}(t_i) + \rho_{12}(t_i) + \rho_{21}(t_i). \qquad \text{(B13)}
$$

If $j^\mu|\Psi(t)\rangle \approx s^\mu(x)|\Psi(t)\rangle$, with $s^\mu(x)$ is the classical current density then

$$
\begin{aligned}
J_c^\mu(x)\ \rho_{11} &\approx J_c^\mu(x)\ \rho_{22} \approx 0,\ \ J_c^\mu(x)\rho_{12} \approx \big[s_1^\mu(x) - s_2^\mu(x)\big] \\
J_a^\mu(x)\rho_{12} &\approx \big[s_1^\mu(x) + s_2^\mu(x)\big]
\end{aligned}
$$

and

$$
\mathcal{L}_C(x)\ \rho_{11} \approx \mathcal{L}_C(x)\ \rho_{22} \approx 0 \qquad \text{(B14)}
$$

Substituted the matrix density equation (B13) into equation (B9) is obtained

$$
\begin{aligned}
\rho_m(t_f) &\approx \rho_{11}(t_i) + \rho_{22}(t_i) + \Big(\exp\Big\{i\Phi_{Diag\ N=3}[s_1, s_2]\Big\} + \exp\Big\{i\Phi_{Non-Diag\ N=3}[s_1, s_2]\Big\}\Big)\ \rho_{12}(t_i) \\
&+ \Big(\exp\Big\{-i\Phi^*_{Diag\ N=3}[s_1, s_2]\Big\} + \exp\Big\{-i\Phi^*_{Non-Diag\ N=3}[s_1, s_2]\Big\}\Big)\ \rho_{12}(t_i) \qquad \text{(B15)}
\end{aligned}
$$

Where each influence phase functional now becomes

$$
\begin{aligned}
i\Phi_{Diag\ N=3} &= \frac{1}{2}\int_{t_i}^{t_f} d^4x \int_{t_i}^{t_f} d^4x' \Big[\ iD(x-x')_{\mu\nu}\ \big[s_1^\mu(x) - s_2^\mu(x)\big]\big[s_1^\nu(x') + s_2^\nu(x')\big] + \\
&- Tr_f\Big(\big\{A_\mu(x), A_\nu(x')\big\}\ \rho_{f,Diag\ N=3}(t_i)\Big)\ \big[s_1^\mu(x) - s_2^\mu(x)\big]\big[s_1^\nu(x') - s_2^\nu(x')\big]\Big] \qquad \text{(B16)}
\end{aligned}
$$

and

$$
\begin{aligned}
i\Phi_{Non-Diag\ N=3} \ = \ & \frac{1}{2}\int_{t_i}^{t_f} d^4x \int_{t_i}^{t_f} d^4x' \Bigg[\ iD(x-x')_{\mu\nu}\ \bigg(\big[s_1^{\mu}(x) - s_2^{\mu}(x)\big]\big[s_1^{\nu}(x') + s_2^{\nu}(x')\big] \\
& + \big[s_1^{\mu}(x) + s_2^{\mu}(x)\big]\big[s_1^{\nu}(x') - s_2^{\nu}(x')\big]\bigg)\Bigg] - \ln\Bigg[\ \bigg|\frac{1}{6}\int_{t_i}^{t_f} d^4x \int_{t_i}^{t_f} d^4x' \int_{t_i}^{t_f} d^4x'' \\
& Tr_f\Big(\big\{A_{\mu}(x), \big\{A_{\nu}(x'), A_{\gamma}(x'')\big\}\big\}\ \rho_{f,Non-Diag\ N=3}(t_i)\Big)\big[s_1^{\mu}(x) - s_2^{\mu}(x)\big] \\
& \times\big[s_1^{\nu}(x') - s_2^{\nu}(x')\big]\big[s_1^{\gamma}(x'') - s_2^{\gamma}(x'')\big]\bigg|^{-1}\ \Bigg] \pm\ i\frac{\pi}{2}. \qquad \text{(B17)}
\end{aligned}
$$

These two influence phase functionals will only affect the interference terms of the density matrix of the matter. Where the real part of each influence phase functional could suppress the interference terms to be zero, which means there is information that will be lost due to environmental effects (decoherence). Therefore the real part of those functionals can be defined as decoherence functional. In this case, there will be two decoherence functional that arises as a consequence of the presence of diagonal and non-diagonal terms of the environmental density matrix. Those two decoherences functional can be expressed as follows

$$
\Gamma_{Diag}[\ \Delta s\ ] = \frac{1}{2}\int_{t_i}^{t_f} d^4x \int_{t_i}^{t_f} d^4x'\ Tr_f\Big(\big\{A_{\mu}(x), A_{\nu}(x')\big\}\ \rho_{f,Diag}(t_i)\Big)\ \Delta s^{\mu}(x)\Delta s^{\nu}(x') \qquad \text{(B18)}
$$

and

$$
\begin{aligned}
\Gamma_{Non-Diag\ N=3}[\ \Delta s\ ] \ = \ \ln\Bigg[\ \bigg|\frac{1}{6}\int_{t_i}^{t_f} d^4x \int_{t_i}^{t_f} d^4x' \int_{t_i}^{t_f} d^4x''\ Tr_f\Big(\Big\{A_{\mu}(x), \big\{A_{\nu}(x'), A_{\gamma}(x'')\big\}\Big\}\rho_{f,Non-Diag\ N=3}(t_i)\Big) \\
\times\Delta s^{\mu}(x)\Delta s^{\nu}(x')\Delta s^{\gamma}(x'')\bigg|^{-1}\ \Bigg], \qquad \text{(B19)}
\end{aligned}
$$

where $\Delta s^{\mu}(x) \equiv s_1^{\mu}(x) - s_2^{\mu}(x)$. For the decoherence functional equation (B18), the notation $N = 3$ is omitted, because this equation would have the same form for all environment density matrices that only have diagonal elements.

2. Influence Functional in Gravity

Next, we will look for the expression of decoherence functional for the gravity case as in section (2). Where there are two test particles represented in the Fermi coordinates with a quantized gravitational waves environment (graviton). Formally, the expression of decoherence functional will be similar to the QED case. The time evolution that affects the correlation of the system with the environment is caused by a similar interaction Hamiltonian. In the case of gravity, the interaction Hamiltonian can be obtained from the last action term of equation (4). In the representation of the quantum noise, the interaction Hamiltonian can be expressed as

$$
\hat{H}_{int} = -\frac{m}{2}\ \hat{N}_{ij}(t)\ \hat{\xi}^i(t)\hat{\xi}^j(t). \qquad \text{(B20)}
$$

By using the same method as the QED case for $N = 3$ with the same enviromental densiti matrix, the two functional decoherences will be obtained as follows

$$
\Gamma_{Diag}(t_f) = \frac{m^2}{8}\int_0^{t_f} dt \int_0^{t_f} dt'\ Tr\Big(\big\{\hat{N}_{ij}(t), \hat{N}_{kl}(t)\big\}\ \rho^{\otimes}_{Diag}\Big)\ \Delta(\xi^i\xi^j)(t)\ \Delta(\xi^k\xi^l)(t') \qquad \text{(B21)}
$$

and

$$\Gamma_{Non-Diag\ N=3}(t_f) \ = \ \ln\Bigg[\Bigg|\frac{m^3}{24}\int_0^{t_f}\ dt\int_0^{t_f}\ dt'\int_0^{t_f}\ dt''\ Tr\Bigg(\Big\{\hat{N}_{ij}(t),\big\{\hat{N}_{kl}(t'),\hat{N}_{mn}(t'')\big\}\Big\}\rho^{\otimes}_{Non-Diag\ N=3}\Bigg)$$

$$\Delta(\xi^i\xi^j)(t)\ \Delta(\xi^k\xi^l)(t')\ \Delta(\xi^m\xi^n)(t'')\Bigg|^{-1}\Bigg], \tag{B22}$$

with $\Delta(\xi^i\xi^j)(t) = \xi_1^i(t)\xi_1^j(t) - \xi_2^i(t)\xi_2^j(t)$, that will be determined based on the experimental setup.

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