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RESEARCH ARTICLE

Fundamental decoherence from quantum gravity: a pedagogical review

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Abstract We present a discussion of the fundamental loss of unitarity that appears in quantum mechanics due to the use of a physical apparatus to measure time. This induces a decoherence effect that is independent of any interaction with the environment and appears in addition to any usual environmental decoherence. The discussion is framed self consistently and aimed to general physicists. We derive the modified Schrödinger equation that arises in quantum mechanics with real clocks and discuss the theoretical and potential experimental implications of this process of decoherence.

1 Introduction

As ordinarily formulated, quantum mechanics involves and idealization. The idealization is the use of a perfect classical clock to measure times. Such a device clearly does not exist in nature, since all measuring devices are subject to some level of quantum fluctuations. Therefore the equations of quantum mechanics, when cast in terms of the variable that is really measured by a clock in the laboratory, will differ from the traditional Schrödinger description. Although this is an idea that arises naturally in ordinary quantum mechanics, it is of paramount importance when one is discussing

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quantum gravity. This is due to the fact that general relativity is a generally covariant theory where one needs to describe the evolution in a relational way. One ends up describing how certain objects change when other objects, taken as clocks, change. At the quantum level this relational description will compare the outcomes of measurements of quantum objects. Quantum gravity is expected to be of importance in regimes (e.g. near the big bang or a black hole singularity) in which the assumption of the presence of a classical clock is clearly unrealistic. The question therefore arises: is the difference between the idealized version of quantum mechanics and the real one just of interest in situations when quantum gravity is predominant, or does it have implications in other settings? We will argue that indeed it does have wider implications. Some of them are relevant to conceptual questions (e.g. the problem of measurement in quantum mechanics or the black hole information paradox) and there might even be experimental implications.

Although we have discussed several of these issues in previous papers [1–3], the latter were written with an audience of relativists and quantum gravity experts in mind, and involving a particular approach to quantum gravity we have been pioneering [4,5]. Since many of the results are really robust and independent of details of quantum gravity, we are giving a presentation in this paper with minimal references to issues that may be unfamiliar to physicists from outside the research area of gravitational physics.

The plan of this paper is as follows: in the next section we will derive the form of the evolution equation of quantum mechanics when the time variable, used to describe it, is measured by a real clock. In Sect. 3 we will consider a fundamental bound on how accurate can a real clock be and the implications it has for quantum mechanics in terms of real clocks and its consequences. Section 4 discusses the implications of the formalism.

2 Quantum mechanics with real clocks

Given a physical situation described by a (multi-dimensional) phase space q, p, we start by choosing a "clock". By this we mean a physical quantity (more precisely a set of quantities, like when one chooses a clock and a calendar to monitor periods of more than a day) that we will use to keep track of the passage of *time*. An example of such a variable could be the angular position of the hand of an analog watch. Let us denote it by $T(q, p)$. We then identify some physical variables that we wish to study as a function of time. We shall call them generically $O(q, p)$ ("observables"). We then proceed to quantize the system by promoting all the observables and the clock variable to self-adjoint quantum operators acting on a Hilbert space. The latter is defined once a well defined inner product is chosen in the set of all physically allowed states. Usually it consists of squared integrable functions $\psi(q)$.

Notice that we are not in any way modifying quantum mechanics. We assume that the system has an evolution in terms of an external parameter t, which is a classical variable, given by a Hamiltonian and with operators evolving with Heisenberg's equations (it is easier to present things in the Heisenberg picture, though it is not mandatory to use it for our construction). Then the standard rules of quantum mechanics and its probabilistic nature apply.

We will call the eigenvalues of the "clock" operator T and the eigenvalues of the "observables" O. We define the projector associated to the measurement of the time variable within the interval $[T_0 - \Delta T, T_0 + \Delta T]$,

$$P_{T_0}(t) = \int_{T_0-\Delta T}^{T_0+\Delta T} dT \sum_k |T, k, t >< T, k, t| \tag{1}$$

where k denotes the eigenvalues of the operators that form a complete set with $\hat{T}$ (the eigenvalues can have continuous or discrete spectrum, in the former case the sum should be replaced by an integral). We have assumed a continuous spectrum for T therefore the need for the integral over an interval on the right hand side. The interval ΔT is assumed to be very small compared to any of the times intervals of interest in the problem, in particular the time separating two successive measurements. Similarly we introduce a projector associated with the measurement of the observable O,

$$P_{O_0}(t) = \int_{O_0-\Delta O}^{O_0+\Delta O} dO \sum_j |O, j, t >< O, j, t| \tag{2}$$

with j the eigenvalues of a set of operators that form a complete set with $\hat{O}$. These projectors have the usual properties, i.e., $P_a(t)^2 = P_a(t)$, $\sum_a P_a(t) = 1, \forall t$ and $P_a(t)P_{a'}(t) = 0$ if the intervals surrounding a and a' do not have overlap.

We would like now to ask the question "what is the probability that the observable O take a given value O_0 given that the clock indicates a certain time T_0 ". Such question is embodied in the conditional probability,

$$\begin{aligned} &\mathcal{P}\left(O \in [O_0 - \Delta O, O_0 + \Delta O] | T \in [T_0 - \Delta T, T_0 + \Delta T]\right) \\ &\quad = \lim_{\tau\to\infty} \frac{\int_{-\tau}^{\tau} dt \,\mathrm{Tr}\,(P_O(t) P_T(t) \rho P_T(t))}{\int_{-\tau}^{\tau} dt \,\mathrm{Tr}\,(P_T(t)\rho)} \end{aligned} \tag{3}$$

where we have used the properties of both projectors and the integrals over t in the right hand side are taken over all its possible values. The reason for the integrals is that we do not know for what value of the external ideal time t the clock will take the value T_0. In this expression ρ is the density matrix of the system. One has to take some obvious cares, like for instance to choose a clock variable (or set of variables) that do not take twice the same value during the relevant lifetime of the experiment one is considering.

The above expression is general, it will apply to any choice of "clock" and "system" variables we make. The relational evolution of the conditional probabilities will be complicated and will bear little resemblance to the usual evolution of probabilities in ordinary quantum mechanics unless we make a "wise" selection of the clock and system variables. What we mean by this is that we would like to choose as clock variables a subsystem that interacts little with the system we want to study and that behaves semiclassically with small quantum fluctuations. Namely, the physical clock will be correlated with the ideal time in such a way to produce the usual notion of time. In such a regime one expects to recover ordinary Schrödinger evolution (plus small corrections) even if one is using a "real" clock. Let us consider such a limit in detail. We will assume that we divide the density matrix of the whole system into a product form between clock and system, $\rho = \rho_{\rm cl} \otimes \rho_{\rm sys}$ and the evolution will be given by a unitary operator also of product type $U = U_{\rm cl} \otimes U_{\rm sys}$.

Up to now we have considered the quantum states as described by a density matrix at a time t. Since the latter is unobservable, we would like to shift to a description where we have density matrices as functions of the observable time T. To do this, we recall the expression for the usual probability in the Schrödinger representation of measuring the value O at a time t,

$$\mathcal{P}\left(O|t\right) \equiv \frac{\mathrm{Tr}\left(P_O(0)\rho(t)\right)}{\mathrm{Tr}\left(\rho(t)\right)} \tag{4}$$

where the projector is evaluated at $t = 0$ since in the Schrödinger picture operators do not evolve. We would like to get a similar expression in terms of the real clock. To do this we consider the conditional probability (3), and make explicit the separation between clock and system,

$$\begin{aligned}
&\mathcal{P}\left(O \in [O_0 \pm \Delta O] | T \in [T_0 \pm \Delta T]\right) \\
&= \lim_{\tau\to\infty} \frac{\int_{-\tau}^{\tau} dt\, \mathrm{Tr}\left(U_{\rm sys}(t)^\dagger P_O(0) U_{\rm sys}(t) U_{\rm cl}(t)^\dagger P_T(0) U_{\rm cl}(t) \rho_{\rm sys} \otimes \rho_{\rm cl}\right)}{\int_{-\tau}^{\tau} dt\, \mathrm{Tr}\left(P_T(t)\rho_{\rm cl}\right) \mathrm{Tr}\left(\rho_{\rm sys}\right)} &(5)\\
&= \lim_{\tau\to\infty} \frac{\int_{-\tau}^{\tau} dt\, \mathrm{Tr}\left(U_{\rm sys}(t)^\dagger P_O(0) U_{\rm sys}(t) \rho_{\rm sys}\right) \mathrm{Tr}\left(U_{\rm cl}(t)^\dagger P_T(0) U_{\rm cl}(t) \rho_{\rm cl}\right)}{\int_{-\tau}^{\tau} dt\, \mathrm{Tr}\left(P_T(t)\rho_{\rm cl}\right) \mathrm{Tr}\left(\rho_{\rm sys}\right)}. &(6)
\end{aligned}$$

We define the probability that the resulting measurement of the clock variable T correspond to the value t,

$$\mathcal{P}_t(T) \equiv \frac{\mathrm{Tr}\left(P_T(0) U_{\rm cl}(t) \rho_{\rm cl} U_{\rm cl}(t)^\dagger\right)}{\int_{-\infty}^{\infty} dt\, \mathrm{Tr}\left(P_T(t)\rho_{\rm cl}\right)}, \tag{7}$$

and notice that $\int_{-\infty}^{\infty} dt \mathcal{P}_t(T) = 1$. We now define the evolution of the density matrix,

$$\rho(T) \equiv \int_{-\infty}^{\infty} U_{\rm sys}(t) \rho_{\rm sys} U_{\rm sys}(t)^\dagger \mathcal{P}_t(T) \tag{8}$$

where we dropped the "sys" subscript in the left hand side since it is obvious we are ultimately interested in the density matrix of the system under study, not that of the clock. Noting that

$$\mathrm{Tr}\left(\rho(T)\right) = \int_{-\infty}^{\infty} dt\, \mathcal{P}_t(T) \mathrm{Tr}\left(\rho_{\mathrm{sys}}\right) = \mathrm{Tr}\left(\rho_{\mathrm{sys}}\right), \tag{9}$$

one can equate the conditional probability (5) to the ordinary probability of quantum mechanics (4). We have therefore ended with the standard probability expression with an "effective" density matrix in the Schrödinger picture given by $\rho(T)$. By its very definition, it is immediate to see that in the resulting evolution unitarity is lost, since one ends up with a density matrix that is a superposition of density matrices associated with different t's and that each evolve unitarily according to ordinary quantum mechanics.

Now that we have identified what will play the role of a density matrix in terms of a "real clock" evolution, we would like to see what happens if we assume the "real clock" is behaving semiclassically. To do this we assume that $\mathcal{P}_t(T) = f(T - T_{\mathrm{max}}(t))$, where f is a function that decays very rapidly for values of T far from the maximum of the probability distribution T_{max}. To make the expressions as simple as possible, let us assume that $T_{\mathrm{max}}(t) = t$, i.e. the peak of the probability distribution is simply at t. More general dependences can of course be considered, altering the formulas minimally (for a more complete treatment see [2]). We will also assume that we can approximate f reasonably well by a Dirac delta, namely,

$$f(T - t) = \delta(T - t) + a(T)\delta'(T - t) + b(T)\delta''(T - t) + \ldots, \tag{10}$$

where the first term has a unit coefficient so the integral of the probability is unit and we assume $b(T) > 0$ so it represents extra width with respect to the Dirac delta.

We now consider the evolution of the density matrix,

$$\rho(T) = \int_{-\infty}^{\infty} dt\, \rho_{\mathrm{sys}}(t)\mathcal{P}_t(T) = \int_{-\infty}^{\infty} dt\, \rho_{\mathrm{sys}}(t) f(T - t) \tag{11}$$

and associating a Hamiltonian with the evolution operator $U(t) = \exp(iHt)$, we get,

$$\rho(T) = \rho_{\mathrm{sys}}(T) + a(T)[H, \rho_{\mathrm{sys}}(T)] - b(T)[H, [H, \rho_{\mathrm{sys}}(t)]], \tag{12}$$

and we notice that there would be terms involving further commutators if we had kept further terms in the expansion of $f(T - t)$ in terms of the Dirac deltas.

We can now consider the time derivative of this expression, and get,

$$\frac{\partial \rho(T)}{\partial T} = i\left(-1 + \frac{\partial a(T)}{\partial T}\right)[H, \rho(T)] + \left(a(T) - \frac{\partial b(T)}{\partial T}\right)[H, [H, \rho(T)]]. \tag{13}$$

If we had considered a symmetric distribution (it is natural to consider such distributions since on average one does not expect an effect that would lead systematically to values grater or smaller than the mean value), we see that one would have obtained the traditional evolution to leading order plus a corrective term,

$$\frac{\partial \rho(T)}{\partial T} = i[\rho(T), H] + \sigma(T)[H, [H, \rho(T)]]. \tag{14}$$

and the extra term is dominated by the rate of change of the width of the distribution $\sigma(T) = \partial b(T)/\partial T$.

An equation of this form has been considered in the context of decoherence due to environmental effects, it is called the Lindblad equation [6],

$$\frac{d}{dt}\rho = -i[H, \rho] - \mathcal{D}(\rho), \tag{15}$$

with

$$\mathcal{D}(\rho) = \sum_n [D_n, [D_n, \rho]], \quad D_n = D_n^\dagger, \quad [D_n, H] = 0, \tag{16}$$

and in our case there is only one D_n that is non-vanishing and it coincides with H. This is a desirable thing, since it implies that conserved quantities are automatically preserved by the modified evolution. Other mechanisms of decoherence coming from a different set of effects of quantum gravity have been criticized in the past because they fail to conserve energy [7,8]. It should be noted that Milburn arrived at a similar equation as ours from different assumptions [9]. Egusquiza, Garay and Raya derived a similar expression from considering imperfections in the clock due to thermal fluctuations [10]. It is to be noted that such effects will occur in addition to the ones we discuss here. Corrections to the Schrödinger equation from quantum gravity have also been considered in the context of WKB analyses [11].

What is the effect of the extra term? To study this, let us pretend for a moment that $\sigma(T)$ is constant. That is, the distribution in the clock variable has a width that grows linearly with time. In that case, the evolution equation is exactly solvable. If we consider a system with energy levels, the elements of the density matrix in the energy eigenbasis is given by,

$$\rho(T)_{nm} = \rho_{nm}(0)e^{-i\omega_{nm}T}e^{-\sigma\omega_{nm}^2 T} \tag{17}$$

where $\omega_{nm} = \omega_n - \omega_m$ is the Bohr frequency corresponding to the levels n, m. We therefore see that the off-diagonal elements of the density matrix go to zero exponentially at a rate governed by σ, i.e. by how badly the clock's wavefunction spreads. It is clear that a pure state is eventually transformed into a completely mixed state by this process.

The origin of the lack of unitarity is the fact that definite statistical predictions are only possible by repeating an experiment. If one uses a real clock, which has thermal and quantum fluctuations, each experimental run will correspond to a different value of the evolution parameter. The statistical prediction will therefore correspond to an average over several intervals, and therefore its evolution cannot be unitary.

In a real experiment, there will be decoherence in the system under study due to interactions with the environment, that will be superposed on the effect we discuss. Such interactions might be reduced by cleverly setting up the experiment. The decoherence we are discussing here however, is completely determined by the quality of the clock used. It is clear that if one does experiments in quantum mechanics with poor clocks, pure states will evolve into mixed states very rapidly. The effect we are discussing can therefore be magnified arbitrarily simply by choosing a lousy clock. This effect has actually been observed experimentally in the Rabi oscillations describing the exchange of excitations between atoms and field [12–14].

3 Fundamental limits to realistic clocks

We have established that when we study quantum mechanics with a physical clock (a clock that includes quantum fluctuations), unitarity is lost, conserved quantities are still preserved, and pure states evolve into mixed states. The effects are more pronounced the worse the clock is. Which raises the question: is there a fundamental limitation to how good a clock can be? This question was first addressed by Salecker and Wigner [15]. Their reasoning went as follows: suppose we want to build the best clock we can. We start by insulating it from any interactions with the environment. An elementary clock can be built by considering a photon bouncing between two mirrors. The clock "ticks" every time the photon strikes one of the mirrors. Such a clock, even completely isolated from any environmental effects, develops errors. The reason for them is that by the time the photon travels between the mirrors, the wavefunctions of the mirrors spread. Therefore the time of arrival of the photon develops an uncertainty. Salecker and Wigner calculated the uncertainty to be $\delta t \sim \sqrt{t/M}$ where M is the mass of the mirrors and t is the time to be measured (we are using units where $\hbar = c = 1$ and therefore mass is measured in 1 s^{-1}). The longer the time measured the larger the error. The larger the mass of the clock, the smaller the error.

So this tells us that one can build an arbitrarily accurate clock just by increasing its mass. However, Ng and Van Damme [16,17] pointed out that there is a limit to this. Basically, if one piles up enough mass in a concentrated region of space one ends up with a black hole. Some readers may ponder why do we need to consider a concentrated region of space. The reason is that if we allow the clock to be more massive by making it bigger, it also deteriorates its performance. For instance, in the case of two mirrors and a photon, if one makes the mirror big, there will be uncertainty in its position due to elastic effects like sound waves traveling across it, which will negate the effect of the additional mass (see the discussion in [18] in response to [19]).

A black hole can be thought of as a clock (as we will see it turns out to be the most accurate clock one can have). It has normal modes of vibration that have frequencies that are of the order of the light travel time across the Schwarzschild radius of the black hole. (It is amusing to note that for a solar sized black hole the frequency is in the kilohertz range, roughly similar to that of an ordinary bell). The more mass in the black hole, the lower the frequency, and therefore the worse its performance as a clock. This therefore creates a tension with the argument of Salecker and Wigner, which required more mass to increase the accuracy. This indicates that there actually is a "sweet spot" in terms of the mass that minimizes the error. Given a time to be measured, light traveling at that speed determines a distance, and therefore a maximum mass one could fit into a volume determined by that distance before one forms a black hole. That is the optimal mass. Taking this into account one finds that the best accuracy one can get in a clock is given by $\delta T \sim T_{\text{Planck}}^{2/3} T^{1/3}$ where $t_{\text{Planck}} = 10^{-44} s$ is Planck's time and T is the time interval to be measured. This is an interesting result. On the one hand it is small enough for ordinary times that it will not interfere with most known physics. On the other hand is barely big enough that one might contemplate experimentally testing it, perhaps in future years.

With this absolute limit on the accuracy of a clock we can quickly work out an expression for the $\sigma(T)$ that we discussed in the previous section [20,3]. It turns out to be $\sigma(T) = \left(\frac{T_{\text{Planck}}}{T_{\max} - T}\right)^{1/3} T_{\text{Planck}}$. With this estimate of the absolute best accuracy of a clock, we can work out again the evolution of the density matrix for a physical system in the energy eigenbasis. One gets

$$\rho(T)_{nm} = \rho_{nm}(0) e^{-i\omega_{nm} T} e^{-\omega_{nm}^2 T_{\text{Planck}}^{4/3} T^{2/3}}. \tag{18}$$

So we conclude that *any* physical system that we study in the lab will suffer loss of quantum coherence at least at the rate given by the formula above. This is a fundamental inescapable limit. A pure state inevitably will become a mixed state due to the impossibility of having a perfect classical clock in nature.

4 Possible experimental implications

Given the conclusions of the previous section, one can ask what are the prospects for detecting the fundamental decoherence we propose. At first one would expect them to be dim. It is, like all quantum gravitational effects, an "order Planck" effect. But it should be noted that the factor accompanying the Planck time can be rather large. For instance, if one would like to observe the effect in the lab one would require that the decoherence manifest itself in times of the order of magnitude of hours, perhaps days at best. That requires energy differences of the order of $10^{10} eV$ in the Bohr frequencies of the system. Such energy differences can only be achieved in "Schrödinger cat" type experiments, but are not outrageously beyong our present capabilities. Among the best candidates today are Bose–Einstein condensates, which can have 10^6 atoms in coherent states. However, it is clear that the technology is still not there to actually detect these effects, although it could be possible in forthcoming years.

A point that could be raised is that atomic clocks currently have an accuracy that is less than a decade of orders of magnitude worse than the absolute limit we derived in the previous section. Couldn't improvements in atomic clock technology actually get better than our supposed absolute limit? This seems unlikely. When one studies in detail the most recent proposals to improve atomic clocks, they require the use of entangled states [21] that have to remain coherent. Our effect would actually prevent the improvement of atomic clocks beyond the absolute limit!

Another point to be emphasized is that our approach has been quite naive in the sense that we have kept the discussion entirely in terms of non-relativistic quantum mechanics with a unique time across space. It is clear that in addition to the decoherence effect we discuss here, there will also be decoherence spatially due to the fact that one cannot have clocks perfectly synchronized across space and also that there will be fundamental uncertainties in the determination of spatial positions. We have not studied this in detail yet, but it appears that this type of decoherence could be even more promising from the point of view of experimental detection [22].

5 Conceptual implications

The fact that pure states evolve naturally into mixed states has conceptual implications in at least three interesting areas of physics. We will discuss them separately.

5.1 The black hole information paradox

The black hole information paradox appeared when Hawking [23] noted that when quantum effects are taken into account, black holes emit radiation like a black body with a temperature $T_{\rm BH} = \hbar/(8\pi kGM)$ where M is the black hole mass, k is Boltzmann's constant and G is Newton's constant. As the black hole radiates, it loses mass, and therefore its temperature increases. This process continues until the black hole eventually evaporates completely and the only thing left is outgoing purely thermal radiation. Now, suppose one had started with a pure quantum state of enough mass that it collapses into a black hole. After the evaporation process, one is left with a mixed state (the outgoing purely thermal radiation). In ordinary quantum mechanics this presents a problem, since pure states cannot evolve into mixed states. (For further discussion and references on the paradox see [24–26]).

On the other hand, we have argued that due to the lack of perfectly classical clocks, quantum mechanics really implies that pure states do evolve into mixed states. The question is: could the effect be fast enough to render the black hole information paradox effectively unobservable? On one hand we have argued that our effect is small. But it is also true that black holes usually take a very long time to evaporate. Of course, a full calculation of the evaporation of a black hole would require a detailed modelling including quantum effects of gravity that no one is in a position of carrying out yet. We have done a naive estimate [3,20] of how our effect would take place in the case of an evaporating black hole. To this aim we have assumed the black hole is a system with energy levels (this is a common assumption in many quantum gravity scenarios), and that most of the Hawking radiation is coming from a transition between two dominant energy levels separated by a characteristic frequency dependent on the temperature as

$$\omega_{12}(t) = \frac{1}{(8\pi)^2 \, t_P} \left(\frac{t_P}{T_{\rm max} - t} \right)^{1/3} \tag{19}$$

with $T_{\rm max}$ the lifetime of the black hole (how long it takes to evaporate) and the subscript 12 denotes that it is the transition frequency between the two states of the system.

Although this model sounds simple-minded it just underlies the robustness of the calculation: it just needs that the black hole have discrete energy levels characterized by a separation determined by the temperature of the black hole. It is general enough to be implemented either assuming the Bekenstein spectrum of area or the spectrum stemming from loop quantum gravity [27] (see for a discussion of both spectra). We assume that we start with the black hole in a pure state which is a superposition of different energy eigenstates (there is no reason to assume that the black hole is exactly in an energy eigenstate, which would imply a stationary state with no radiation being emitted; as soon as one takes into account the broadening of lines due to interaction one has to consider a superposition of states within the same broadened level with a time dependent separation with a similar behavior). Therefore the density matrix has off-diagonal elements.

A detailed calculation [3] for the evolution of the density matrix shows that,

$$|\rho_{12}(T_{\max})| \sim |\rho_{12}(0)| \left(\frac{M_P}{M_{\rm BH}}\right)^{\frac{2}{3}}. \tag{20}$$

For astrophysical sized black holes, where $M_{\rm BH}$ is of the order of the mass of the Sun, this indicates that the off diagonal elements are suppressed by the time of evaporation by 10^{-28}, rendering the information puzzle effectively unobservable. What happens for smaller black holes? The effect is smaller. So can one claim that there still is an information puzzle for smaller black holes? This is debatable. After all, we do expect decoherence from other environmental effects to be considerably larger than the one we are considering here. If one makes the holes too small, then none of these calculations apply, and in fact the traditional Hawking evaporation is not an adequate description, since one has to take into account full quantum gravity effects. So we can say that the paradox is effectively eliminated for large black holes and we cannot say for sure for smaller ones using this simplified analysis.

5.2 The measurement problem in quantum mechanics

A potential conceptual application of the fundamental decoherence that we discussed that has not been exploited up to now is in connection with the measurement problem in quantum mechanics. The latter is related to the fact that in ordinary quantum mechanics the measurement apparatus is assumed to be always in an eigenstate after a measurement has been performed. The usual explanation [28] for this is that there exists interaction with the environment. This selects a preferred basis, i.e., a particular set of quasi-classical states that commute, at least approximately, with the Hamiltonian governing the system–environment interaction. Since the form of the interaction Hamiltonians usually depends on familiar "classical" quantities, the preferred states will typically also correspond to the small set of "classical" properties. Decoherence then quickly damps superpositions between the localized preferred states when only the system is considered. This is taken as an explanation of the appearance to a local observer of a "classical" world of determinate, "objective" (robust) properties.

The main problem with such a point of view is how is one to interpret the local suppression of interference in spite of the fact that the total state describing the system–environment combination retains full coherence. One may raise the question whether retention of the full coherence could ever lead to empirical conflicts with the ascription of definite values to macroscopic systems. The usual point of view is that it would be very difficult to reconstruct the off diagonal elements of the density matrix in practical circumstances. However, at least as a matter of principle, one could indeed reconstruct such terms (the evolution of the whole system remains unitary [29]).

Our mechanism of fundamental decoherence could contribute to the understanding of this issue. In the usual system-environment interaction the off-diagonal terms of the density matrix oscillate as a function of time. Since the environment is usually considered to contain a very large number of degrees of freedom, the common period of oscillation for the off-diagonal terms to recover non-vanishing values is very large, in many cases larger than the life of the universe. This allows to consider the problem solved in practical terms. When one adds in the effect we discussed, since it suppresses exponentially the off-diagonal terms, one never has the possibility that the off-diagonal terms will see their initial values restored, no matter how long one waits.

More generally, the environment-induced decoherence leads naturally to a reduced density matrix for the quantum system plus the measurement apparatus that is approximately diagonal and therefore very similar to a statistical mixture of pure states corresponding to different outcomes of the measurement. That implies that a measurement of an observable that only pertains to the system plus the measurement device cannot discriminate between the total pure state and a mixed state. However, as it was extensively discussed by d'Espagnat the formal identity between the reduced density matrix and a mixed-state density matrix is frequently misinterpreted as implying that the system is in a mixed state. As the system is entangled with the environment the total system is still described by a pure state and no individual definite state or set of possible states may be attributed to a portion of the total system. As it is well known, measurements on the environment will always allow us, in principle, to distinguish between the reduced and mixed state density matrices.

The combined effect of these two forms of decoherence could allow to understand the physical transition from a reduced density matrix to a mixed state. In fact, the precise unitary evolution of the total system is broken by the clock-induced decoherence destroying the correlations. What remains to be studied is whether this effect is sufficiently fast to avoid any possibility of distinguishing, not only for all practical purposes but also on theoretical basis, between these two kinds of density matrices.

In a nutshell, this is how our effect might help understand the measurement problem. In a future paper we will expand more on this and on other issues related to the measurement problem.

5.3 Quantum computing

In quantum computing, when one performs operations one is evolving quantum states. If one wishes the computers to perform faster, one needs to expend extra energy to evolve the quantum states. Based on this premise, Lloyd [30] presented a fundamental limitation to how fast quantum computers can be. Using the Margolus–Levitin [31] theorem he notes that in order to perform a computation in a time δT one needs to expend at least an energy $E \geq \pi\hbar/(2\Delta T)$. As a consequence, a system with an average energy E can perform a maximum of $n = 2E/(\pi\hbar)$ operations per second. For an "ultimate laptop" (a computer of a volume of one liter and one kilogram of weight) the limit turns out to be 10^{51} operations per second.

Such results assume the evolution is unitary. When it is not, as we have argued in this paper, erroneous computations are carried out. Since the rate of decoherence we discussed increases with increased energy differences, the rate of erroneous computations increases the faster one wishes to make the computer.

Can't one error correct? After all, one expects quantum computers to have errors due to decoherence from environmental factors. One can indeed error-correct. But there are limitations to how fast this can be done. At its most basic level error correction is achieved by duplicating calculations and comparing results. This requires spatial communication, which is limited by the speed of light. Our point is that one cannot simply error correct one's way out of the fundamental decoherence effects.

For instance, for a NOT gate our effect implies that after a time $T \sim \pi/(2E)$ we will have,

$$|\psi_0><\psi_0| \rightarrow (1-\epsilon)|\psi_1><\psi_1| + \epsilon|\psi_0><\psi_0|, \tag{21}$$

with $\epsilon = 4T_{\text{Planck}}^{4/3} T^{2/3} E^2$. How does this effect influence, for instance, the "ultimate laptop"?

We have to distinguish a bit between serial and parallel computing. In serial computing one achieves speed by increasing the energy in each qubit. This enhances our decoherence effect and significantly affects the performance. In a parallel machine one increases the speed by operating simultaneously on many qubits with lower energies per qubit therefore lowering the importance of the effect we introduced. For a machine with L qubits and a number of simultaneous operations d_P one gets,

$$n \leq \left(\frac{1}{t_{\text{P}}}\right)^{4/7} \left(\frac{cL}{R}\right)^{3/7} d_p^{4/7} \sim 10^{47}\text{op/s}, \tag{22}$$

where the last estimate was obtained by taking the values of parameters for the "ultimate laptop" (for more details see [32]).

This is actually four orders of magnitude stronger than the bound that Lloyd found. If one had chosen a serial machine, the bound would have been tighter, 10^{42} operations per second.

We therefore see that although the effect we introduced is far from being achievable in quantum computers built in the next few years, it can limit the ultimate computing power of quantum computer. This is quite remarkable, given that it is a limit obtained involving gravity. Few people could have foreseen that gravity would play any role in quantum computation.

6 Discussion

We have argued that the use of realistic clocks in quantum mechanics implies that pure states evolve into mixed states. Another way of putting this is that we are allowing quantum fluctuations in our clock. Similar ideas have been considered by Bonifacio, with a different formulation [33]. In quantum gravity and quantum cosmology it is natural to consider the clock to be part of the system under study. This is what motivated our interest in these issues, but it is clear that the core of the phenomenon can be described without references to quantum gravity, and that is what we have attempted to do in this presentation.

In the immediate future the most attractive possibility is to generalize these results to consider spatial decoherence due to the lack of a universal clock across spatial points. This may open further possibilities for experimentally observing these effects.

Even in the absence of a direct possibility of detecting these effects, they can have important conceptual implications, as we have illustrated with the black hole information puzzle, quantum computing and the problem of measurement in quantum mechanics.

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