About Shu Chen *et. al.* paper: arXiv:2107.02709v3

1 Overview

- It is well known that when the Fisher zeros touches the temperature real axis, a phenomenon called quantum phase transition occur, which happens only in the thermodynamic limit.
- In the case of DQPT, the same thing happens. The exact zeros of Loschmidt echo(LE) in a finite system only appear in the complex time plane. When the system size tends to infinity, the zeros approach the real time axis for quenches crossing the QPT point.
- One may ask whether exact zeros of LE can occur in the real time axis for a finite size quantum system? Their results **show that exact zeros of LE exist even in finite size quantum systems when the post-quench parameter takes some discrete values**.
- Once we know when the exact zeros of LE occur, it is natural to explore the minimum time of an initial state evolving to its orthogonal state which corresponds to the time for the emergence of the first exact zero of LE. This is directly connected to QSL, since it measures the time a system take to evolve from a state to the corresponding orthogonal state.
- QSL was already discussed in the context of DQPT by Markus Heyl [Phys. Rev. B 95, 060504(R) (2017)], but only concerning the dynamics of the quantum critical state. As we shall clarify, if the initial state is the quantum critical state, no exact zeros of LE can be found.
- We also pay particular attention to the maximum and minimum values of QSL of τ_{QSL}, as the maximum of τ_{QSL} is related to the quench dynamics close to the critical point and **the minimum of τ_{QSL} gives the important message of how fast DQPT could happen**. When the size of the system tends to infinity, we find that the maximum value of τ_{QSL} approaches infinity when the quench parameter approaches to the critical point, which is independent of the initial state. However, the behavior of the minimum value of τ_{QSL} is distinct if the initial state is chosen in different phase.

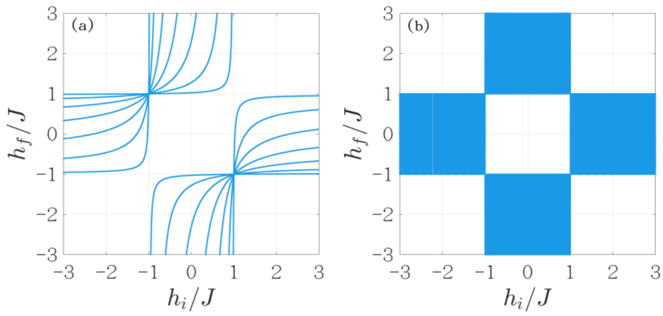

Figure 1. The combination of h_i/J and h_f/J which fulfill Eq. (7). (a) $L = 14$ and (b) $L = 400$.

2 Results

After diagonalizing the transverse field Ising model, we calculate the Loschmidt echo given by

$$\mathcal{L}(t) = |\langle\psi_i|e^{-iH_f t}|\psi_i\rangle|^2 = \prod_{k\in\mathcal{K}^+_{aPBC}} \left[1 - \sin^2(2\delta\theta_k)\sin^2(2E_{kf}t)\right] \tag{1}$$

where $\delta\theta_k = \theta_{kf} - \theta_{ki}$,

$$\theta_{ki} = \frac{1}{2}\arctan\frac{J\sin(k)}{J\cos(k) + h_i} \tag{2}$$

is the Bogoliubov angle of the pre-quench hamiltonian and

$$\theta_{kf} = \frac{1}{2}\arctan\frac{J\sin(k)}{J\cos(k) + h_f} \tag{3}$$

the Bogoliubov angle of the post-quench hamiltonian. Also $\mathcal{K}^+_{aPBC} = \{\frac{+\pi(2m-1)}{L},\ m = 1, \cdots, \frac{L}{2}\}$. The equation (1) is zero when

$$\frac{h_f}{J} = -\frac{J + h_i\cos(k)}{h_i + J\cos(k)}, \tag{4}$$

then, given the pre-quench parameter h_i, we can get a series of h_f determined by equation (4) for various k. When the post-quench parameter take these discrete values, we have $\mathcal{L}(t) = 0$ at

$$t = t^*_n = \frac{\pi}{2E_{kf}}\left(n + \frac{1}{2}\right) \tag{5}$$

with

$$E_{kf}/J = \sqrt{(\cos(k) + h_f/J)^2 + \sin^2(k)}. \tag{6}$$

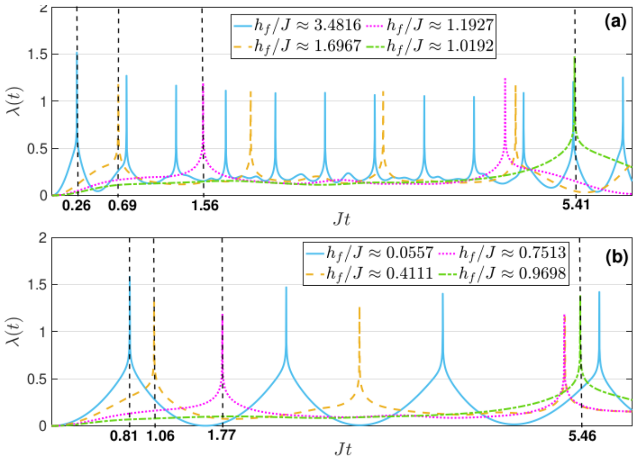

Figure 3. Rate function $\lambda(t)$ for $L = 22$. Black dashed lines guide the first time for the appearance of the exact zero of LE for every h_f/J. The prequench parameter is (a) $h_i/J = 0.3$ and (b) $h_i/J = 2$.

Then, the QSL time will correspond to $n = 0$ in equation (5), so

$$\tau_{\mathrm{QSL}} = \frac{\pi}{4E_{kf}}. \tag{7}$$

We can see from figure 3-(a) that the QSL time decreases when h_f/J increases, when we quench from the region $0 < h_i/J < 1$ to the region $h_f/J > 1$. On the other hand, when we quench from the region $h_i/J > 1$ to the region $0 < h_f/J < 1$, the QSL time decreases with the decrease of h_f/J as shown in figure 3-(b) and it doesn't rely on the system size. It follows that the QSL time increases as h_f/J approaches the critical point $h_f/J = 1$

About the analysis of $\tau_{\max} = \max(\tau_{\mathrm{QSL}})$, they conclude that $\tau_{\max} \to \infty$ when $L \to \infty$ with $|h_f/J| \to 1$. This means that we can't observe DQPT in a finite time if we quench the system from a non-critical phase to the critical phase with $|h_f/J| = 1$, i.e., no DQPT occurs in a finite time. They also conclude that $\tau_{\max}$ increases linearly with system size according to

$$\tau_{\max} = \frac{L}{4J} \tag{8}$$

which can be regarded as an upper bound to the QSL time.

For any ground state of 1D TFIM, it is also interesting to ask how fast could the ground state achieve its orthogonal state as we quench the parameter of the system? The answer of the question is given by the minimal value of QSL time denoted as $\tau_{\min} = \min(\tau_{\mathrm{QSL}})$. In figure (5), we demonstrate that if the initial state lies in the paramagnetic phase [Fig 5-(a)] ($h_i/J > 1$), $\tau_{\min}$ approaches some finite values as the system size increases. However, $\tau_{\min}$ would approach to zero if the initial state lies in the ferromagnetic phase ($h_i/J < 1$) [Fig 5-(b)].

To see how $\tau_{\min}(L)$ changes with h_i/J, we can calculate the mean value of $\tau_{\min}(L)$ numerically

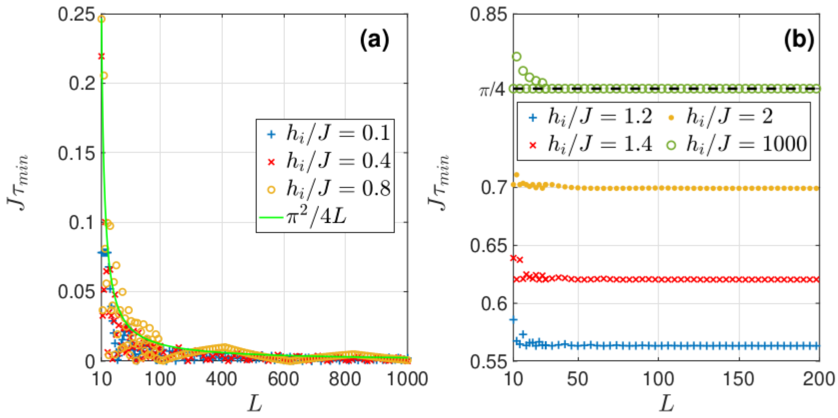

Figure 5. $J\tau_{min}$ versus system size L. The prequench parameter is (a) $|h_i/J| > 1$ and (b) $|h_i/J| < 1$.

from L_{min} and L_{max} and denote it as

$$\bar{\tau}_{\min} = \frac{1}{L_{max} - L_{min}} \sum_{L=L_{min}}^{L_{max}} \tau_{\min}(L) \tag{9}$$

and the variance of $\tau_{\min}(L)$ as

$$\sigma^2_{\tau_{min}} = \frac{1}{L_{max} - L_{min}} \sum_{L=L_{min}}^{L_{max}} \tau^2_{\min}(L) - \bar{\tau}^2_{\min}. \tag{10}$$

We count from $L_{min} = 10$ to $L_{max} = 10000$ and show the numerical results of $J\bar{\tau}_{min}$ and $J^2\sigma^2_{\tau_{min}}$ with respect to pre-quench parameter in figures 6-(a) and 6-(b). It can be observed that both $\bar{\tau}_{min}$ and $\sigma^2_{\tau_{min}}$ have an abrupt change in the critical point $h_i/J = 1$.

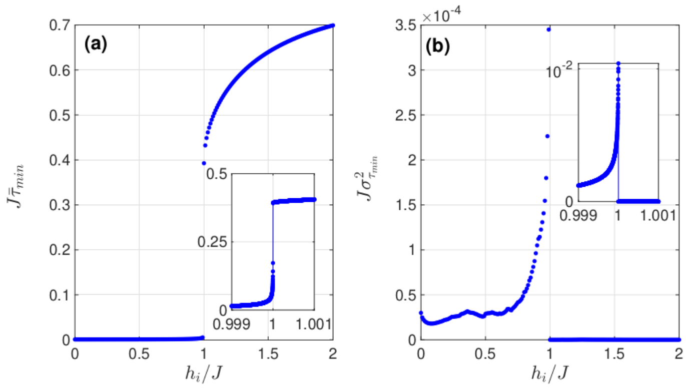

Figure 6. (a) $J\bar{\tau}_{min}$ with respect to h_i/J; (b) $J^2\sigma^2_{\tau_{min}}$ with respect to h_i/J. Here we count the size of system from $L_{min} = 10$ to $L_{max} = 10000$.