Fluctuation-Dissipation Relation and Hawking Effect

Rabin Banerjee[a] and Bibhas Ranjan Majhi[b]
[a]S. N. Bose National centre for Basic Sciences, JD Block, Sector III, Salt Lake, Kolkata-700098, India
[b]Department of Physics, Indian Institute of Technology Guwahati, Guwahati 781039, Assam, India
e-mails: rabin@bose.res.in, bibhas.majhi@iitg.ac.in

Abstract

We show a direct connection between Kubo's fluctuation-dissipation relation and Hawking effect that is valid in any dimensions. The relevant correlators, computed from the known expressions for the stress tensor, are shown to satisfy the Kubo relation, from which the temperature of a black hole as seen by an observer at an arbitrary distance is abstracted. This reproduces the Tolman temperature and hence the Hawking temperature as that measured by an observer at infinity.

I. Introduction

Quantisation of fields on a classical background, containing a horizon, leads to the fact that particles can radiate from the horizon [1, 2, 3]. Moreover, it is now well known that such phenomenon depends on a particular observer and is connected to the non-unique definition of vacuum in non-inertial frame or in curved spacetime. One of the classic examples in this context is the Hawking effect [1] – the thermal radiation observed by a static observer at infinity in a black hole spacetime in the Kruskal or Unruh vacuum. While there are several approaches to analyse this phenomenon, each has its own merits and/or demerits, but none is truly clinching. This is the primary reason that new avenues are still being explored.

The fluctuation–dissipation theorem is a general result of statistical thermodynamics that yields a concrete relation between the fluctuations in a system that obeys detailed balance and the response of the system to applied perturbations (see [4] for a review). It has been effectively used to study various processes like Brownian motion in fluids, Johnson noise in electrical conductors and, as shall become clear very soon, relevantly, Kirchoff's law of thermal radiation. Since black holes satisfy the condition of detailed balance and the emitted radiation is thermal, it is likely that new insights into the phenomenon of Hawking radiation could be obtained by using the fluctuation-dissipation relation.

Our motivation in this paper is to exploit the fluctuation-dissipation relation to obtain the Hawking temperature. In fact we are able to obtain the more general Tolman temperature [5, 6], that is valid for an observer at an arbitrary distance. There are different versions of the theorem but the one suited for our analysis was given by Kubo [4, 7, 8]. In simple terms this relation is able to provide the temperature of the heat bath from a study of the correlators of the fluctuations of the force of the emitted particles, as measured by the detector. We shall apply this relation to black holes. Treating the black holes as a heat bath, it is possible to compute the fluctuations of the force of the emitted particles as seen by an observer at an arbitrary distance. Using Kubo's relation we derive the temperature, which turns out to be the Tolman temperature. Putting the detector at infinity immediately yields the Hawking temperature [1].

An essential ingredient in our calculation is the structure of the energy momentum tensor. The force is computed by taking the time variation of the space component of the four momentum which in turn is defined from the energy momentum tensor. Since our analysis is very near to the horizon, where the spacetime is effectively $(1+1)$ dimensions [9, 10, 11], we shall concentrate on the two dimensional stress tensor in a curved background. Classically this is not well defined and recourse has to taken to some regularisation to include quantum effects. Incidentally the method discussed here was applied earlier by one of the authors, in a collaborative work [12, 13], in a classical treatment. It is useful to recall that the fluctuation dissipation relation remains valid both for classical and quantum systems. Other relevant applications were based on the path integral fluctuation-dissipation formalism developed in [14]. The Minkowski vacuum was modelled as a thermal bath with respect to an accelerated observer, so that any particle in it is executing a Brownian like motion [15, 16, 17, 18, 19]. For the quantum case which is relevant here, there are two possibilities. Either the theory is non-chiral or it is chiral. In the first case the stress tensor satisfies diffeomorphism invariance but lacks conformal invariance leading to a nonvanishing trace of the stress tensor [20, 21, 22, 23]. For the chiral theory both conformal and diffeomorphism symmetries are broken [24, 25, 26, 27]. We have done the analysis for either situation and find the correct Tolman/Hawking temperatures.

The organisation of the paper is as follows. In the next section, we shall briefly discuss our general formalism of computing fluctuations of force and their use in Kubo's relation. In sections III and IV, respectively, we derive the Hawking effect from the non-chiral and chiral theories. The last section contains our conclusions and a look into future possibilities.

II. Fluctuations of Force and Kubo's relation

Here we outline our general strategy for obtaining the Hawking temperature from Kubo's relation. The first thing is to define the force that will lead to the force correlators. The space components of the four momentum as measured by the observer in its own frame which perceives the particles is defined as

$$p^{\alpha} = \int d^3\mathbf{x} T^{0\alpha}(\tau, \mathbf{x})\delta(\mathbf{x} - \mathbf{x}_D) = T^{0\alpha}(\tau, \mathbf{x}_D) \ , \tag{1}$$

where $\mathbf{x}_D$ is the position of the detector in its own frame and this is not changing. Therefore, the above quantity will be only function of its proper time τ. Then the force, as measured by this detector turns out to be

$$F^{\alpha}(\tau) = \frac{dp^{\alpha}}{d\tau} = \frac{dT^{0\alpha}}{d\tau} \ . \tag{2}$$

The fluctuating part of this force is

$$F^{\alpha}_{fluc}(\tau) = F^{\alpha} - < F^{\alpha} > = \frac{dT^{0\alpha}}{d\tau} - \frac{d}{d\tau} < T^{0\alpha} > \ . \tag{3}$$

Next we define the fluctuating force-force two point correlation function as

$$R^{\alpha\beta}(\tau_2; \tau_1) = < F^{\alpha}_{fluc}(\tau_2) F^{\beta}_{fluc}(\tau_1) > \ . \tag{4}$$

In the above $< \cdots >$ refers to the expectation value of the quantity of interest with respect to the relevant state. It is now possible to proceed with the calculations in a general way. More precisely, for the accelerated frame case, the stress-tensor components are defined in Rindler frame while the states are vacuum states corresponding to Minkowski observer.

Specialising for black holes, the operators are defined in static Schwarzschild coordinates and the states will be considered as Kruskal and Unruh vacua. Moreover, near the horizon, the arbitrary dimensional stationary black holes are effectively two dimensional and hence conformally flat [9, 10, 11]. The effective metric in Schwarzschild coordinates, Kruskal null-null coordinates and in Eddington null-null coordinates is given by

$$\begin{aligned} ds^2 &= -f(r)dt^2 + \frac{dr^2}{f(r)}; \quad \text{Schwarzschild} \\ &= \frac{f(U,V)}{\kappa^2 UV} dU dV; \quad \text{Kruskal null-null} \\ &= -f(u,v) du dv; \quad \text{Eddington null-null} , \end{aligned} \tag{5}$$

where f is the metric coefficient and $\kappa = f'(r_H)/2$ is the surface gravity and $r = r_H$ is the horizon. The relations among these sets of coordinates are as follows:

$$dr^* = \frac{dr}{f(r)}; \quad u = t - r^*; \quad v = t + r^*; \tag{6}$$

$$U = -\frac{1}{\kappa} e^{-\kappa u}; \quad V = \frac{1}{\kappa} e^{\kappa v} . \tag{7}$$

Thus in Eq. (4) we have to take only the R^{11} component. Also, since the correlators are translationally invariant, it is a function of only the difference of the proper times, $(\tau_2 - \tau_1)$. Let us now represent the Fourier transform of the correlator $R^{11}(\tau_2 - \tau_1)$ by $K(\omega)$.

Now as usual [4, 7, 8], the symmetric and antisymmetric combinations are taken as

$$K^{\pm}(\omega) = K(\omega) \pm K(-\omega) \tag{8}$$

Then Kubo's fluctuation dissipation relation states that these two are related by,

$$K^{+}(\omega) = \coth\left(\frac{\omega}{2T}\right) K^{-}(\omega) , \tag{9}$$

where T is the temperature of the heat bath which, in our case, is the black hole. In the next sections we will explicitly compute the correlators, both for nonchiral and chiral cases, and obtain the temperature from the relation (9).

III. Non-chiral theory:

At the quantum level both the trace and the covariant divergence of the stress tensor cannot be made vanishing. Since diffeomorphism symmetry is more fundamental in gravitational theories, a regularisation is done such that the trace of energy-momentum tensor is non-vanishing and given by [20, 21]: $T^a_a = c_w R$. However, it is covariantly conserved, i.e. $\nabla_a T^{ab} = 0$. The value of the proportionality constant is $c_w = 1/(24\pi)$. In the (trace) anomaly based approach of discussing the Hawking effect [23], which is valid only for two dimensions, use is made of this result. However we are interested in the explicit form of the stress tensor. This is derived from the anomalous effective action [22],

$$S_P = -\frac{c_w}{4} \int d^2x \sqrt{-g} (-\phi \Box \phi + 2R\phi) , \tag{10}$$

where,

$$\Box \phi = R . \tag{11}$$

By taking appropriate functional derivatives [22],

$$
\begin{aligned}
T_{ab} &= \frac{2}{\sqrt{-g}}\frac{\delta S_P}{\delta g^{ab}} = \frac{c_w}{2}\Big[\nabla_a\phi\nabla_b\phi - 2\nabla_a\nabla_b\phi \\
&+ g_{ab}(2R - \frac{1}{2}\nabla_c\phi\nabla^c\phi)\Big] . \qquad (12)
\end{aligned}
$$

Now the equation for the scalar field (11) under the background (5) yields

$$\frac{\partial^2\phi}{\partial t^2} - \frac{\partial^2\phi}{\partial r^{*2}} = -fR . \qquad (13)$$

Since the metric is static, we choose the ansatz for the solution as $\phi(t, r^*) = e^{-i\omega t}F(r^*)$, where ω is the energy of the scalar mode and $F(r^*)$ is an unknown function to be determined. Substituting this in (13) we obtain

$$\frac{d^2F}{dr^{*2}} + \omega^2 F = fRe^{i\omega t} . \qquad (14)$$

Since our analysis is very near to the horizon where R is finite, the right hand side of the above can be neglected compared to the terms on the left hand side and then the solutions for F are $F = e^{\pm i\omega r^*}$. Under this limit, the modes are identical to the usual ones and hence the mode expansion of ϕ is same as for free massless scalar field. Therefore the positive frequency Wightman functions, corresponding to respective vacuum states, will be same as those for the free massless scalar field. So the expressions for the positive frequency Wightman functions corresponding to Kruskal and Unruh vacuums are as follows [28]:

$$G_K^+(x_2; x_1) = -\frac{1}{4\pi}\ln[(\Delta U - i\epsilon)(\Delta V - i\epsilon)] ; \qquad (15)$$

$$G_U^+(x_2; x_1) = -\frac{1}{4\pi}\ln[(\Delta U - i\epsilon)(\Delta v - i\epsilon)] . \qquad (16)$$

Kruskal vacuum: For Kruskal vacuum the expectation value of T^{tr}, as measured by the Schwarzschild static observer, vanishes (see Appendix C of [13] for details). So the fluctuating part of the force is given by the first term of (3). Therefore, the fluctuating force-force correlator, as measured by the Schwarzschild static observer, is given by

$$
\begin{aligned}
& R_K^{11}(\tau_2;\tau_1) = \frac{d}{d\tau_2}\frac{d}{d\tau_1} < T^{tr}(\tau_2)T^{tr}(\tau_1) > \\
& = \frac{f^2(r_s)}{16}\frac{d}{d\tau_2}\frac{d}{d\tau_1}\Big[\Big(g^{uv}(\tau_2)\Big)^2\Big(g^{uv}(\tau_1)\Big)^2 e^{-2\kappa(u_2+u_1)} \\
& \times < T_{UU}(\tau_2)T_{UU}(\tau_1) > \Big] \\
= \;& [f(r_s)]^{-2}\frac{d}{d\tau_2}\frac{d}{d\tau_1}\Big[e^{-2\kappa(u_2+u_1)} < T_{UU}(\tau_2)T_{UU}(\tau_1) > \Big] , \qquad (17)
\end{aligned}
$$

where $g^{uv} = -(2/f(r))$ has been used while the observer is static at $r = r_s$. In going from first to second equality, only T_{uu} (i.e. T_{UU}) component was considered. This is because this component, which is corresponding to outgoing modes, leads to the flux of emitted particles from the horizon; while T_{vv}, related to ingoing modes, does not contribute to this flux. Now using (12) one finds

$$T_{UU}(x) = \frac{c_w}{2}\Big[(\partial_U\phi)(\partial_U\phi) - 2\partial_U^2\phi + \frac{2}{A}(\partial_U A)(\partial_U\phi)\Big] , \qquad (18)$$

where $A = f(U,V)/(UV)$. With this, one finds by using the Wick's theorem

$$
\begin{aligned}
&< T_{UU}(x_2)T_{UU}(x_1) >= \Big(\frac{c_w}{2}\Big)^2\Big[4\partial_2^2\partial_1^2 G(x_2;x_1) + 2\Big(\partial_2\partial_1 G(x_2;x_1)\Big)^2 \\
- &\frac{4\partial_1 A_1}{A_1}\partial_2^2\partial_1 G(x_2;x_1) - \frac{4\partial_2 A_2}{A_2}\partial_2\partial_1^2 G(x_2;x_1) \\
+ &\frac{4\partial_2 A_2 \partial_1 A_1}{A_2 A_1}\partial_2\partial_1 G(x_2;x_1)\Big] \ . \qquad (19)
\end{aligned}
$$

In the above we used the following notations: $\partial_i \equiv \partial_{U_i}$ and $\phi_i \equiv \phi(x_i)$ with $i = 1,2$. The expectation value is taken here with respect to the Kruskal vacuum. Other terms vanish as $< \phi >= 0$. In the above we denoted $< \phi_2\phi_1 >= G(x_2;x_1)$ which is the Green's function corresponding to the differential equation (11) for field ϕ. Here for Kruskal vacuum, this is given by (15). Substituting this we find the terms of the expression (19) as

$$
\begin{aligned}
\Big(\partial_2\partial_1 G(x_2;x_1)\Big)^2 &= \frac{1}{16\pi^2}\frac{1}{(U_2-U_1)^4} \ ; \\
\partial_2^2\partial_1^2 G(x_2;x_1) &= \frac{3}{2\pi}\frac{1}{(U_2-U_1)^4} \ ; \\
\partial_2^2\partial_1 G(x_2;x_1) &= \frac{1}{2\pi}\frac{1}{(U_2-U_1)^3} \ ; \\
\partial_2\partial_1^2 G(x_2;x_1) &= -\frac{1}{2\pi}\frac{1}{(U_2-U_1)^3} \ ; \\
\partial_2\partial_1 G(x_2;x_1) &= -\frac{1}{4\pi}\frac{1}{(U_2-U_1)^2} \ ; \qquad (20)
\end{aligned}
$$

Hence the correlator (17) turns out to be

$$
\begin{aligned}
R_K^{11}(\tau_2;\tau_1) &= -16\kappa^6[f(r_s)]^{-3}\Big(\frac{c_w}{2}\Big)^2\Big(\frac{1}{8\pi^2}+\frac{6}{\pi}\Big)\frac{5+4\sinh^2(\frac{\kappa}{2\sqrt{f(r_s)}}\Delta\tau)}{\sinh^6(\frac{\kappa}{2\sqrt{f(r_s)}}\Delta\tau)} \\
&+ \frac{2\kappa^6[f(r_s)]^{-3}\Big[(\frac{f'(r_s)}{2\kappa})^2-1\Big]}{\pi}\Big(\frac{c_w}{2}\Big)^2\frac{3+2\sinh^2(\frac{\kappa}{2\sqrt{f(r_s)}}\Delta\tau)}{\sinh^4(\frac{\kappa}{2\sqrt{f(r_s)}}\Delta\tau)} \ ; \qquad (21)
\end{aligned}
$$

where we have used the transformation $U = -(1/\kappa)e^{-\kappa u}$, along with $\partial_U A/A = (1/U)((f'(r)/2\kappa)-1)$ while a prime denotes differentiation with respect to r coordinate and $\Delta u = u_2 - u_1 = (t_2 - r_s^*) - (t_1 - r_s^*) = \Delta t = \Delta\tau/\sqrt{f(r_s)}$.

<u>*Unruh vacuum*</u>: For Unruh vacuum, the expression for $R_U^{11}(\tau_2;\tau_1)$ is again given by (17) where the vacuum expectation has to be calculated with respect to Unruh vacuum. This is because $< T^{tr} >$ for a Schwarzschild static observer is constant (see Appendix C of [13] for a detailed analysis) and hence the second part of (3) vanishes. The two point correlation function for stress-tensor component in this expression is again expressed in terms of positive frequency Wightman function, which is given by (16). Since the derivatives will be with respect to U, only the ΔU part of G^+ contributes and hence the final expression for $R_U^{11}(\tau_2;\tau_1)$ comes out to be same as that of Kruskal vacuum; i.e. Eq. (21).

IV. Chiral theory:

Contrary to the previous nonchiral case, here both trace and diffeomorphism anomalies exist: $T^a_a = \frac{c_w}{2}R$; $\nabla_b T^{ab} = \frac{c_w}{4}\bar{\epsilon}^{ac}\nabla_c R$. This is the covariant form of the anomaly which, as was shown by [29, 30, 31, 32], is more effective than the consistent form of the anomaly, in analysing Hawking effect. Here the effective action and corresponding energy-momentum tensor are evaluated in [27]. The form of the stress-tensor is given by,

$$T_{ab} = \frac{c_w}{2}\Big[D_a G D_b G - 2D_a D_b G + \frac{1}{2}g_{ab}R\Big]\ , \tag{22}$$

where G satisfies $\Box G = R$. The chiral derivative is defined as $D_a = \nabla_a \pm \bar{\epsilon}_{ab}\nabla^b$. Here $+(-)$ corresponds to ingoing (outgoing) mode and $\bar{\epsilon}_{ab} = \sqrt{-g}\epsilon_{ab}$ (or $\bar{\epsilon}^{ab} = -\frac{\epsilon^{ab}}{\sqrt{-g}}$) is an anti-symmetric tensor while ϵ_{ab} is the usual Levi-Civita symbol in $(1+1)$ dimensions.

Since we are interested in outgoing modes, the negative sign of D_a operator will be considered. In this case then we have $D_U = 2\nabla_U$ and $D_V = 0$. This can be checked using the expression for $\bar{\epsilon}_{ab} = \sqrt{-g}\epsilon_{ab}$ with $\epsilon_{UV} = 1$. Then T_{UU}, as obtained from (22), becomes identical to the non-chiral case. Moreover, G satisfies the equation which is identical to ϕ. So the correlator for the fluctuation of the force, in both vacua, will be identical to the form (21). Only the over all multiplicative constant factor is different.

Note that in all cases, the form of the correlators for the fluctuation of the force are identical:

$$R^{11}(\tau_2;\tau_1) = C\frac{5+4\sinh^2(\frac{B}{2}\Delta\tau)}{\sinh^6(\frac{B}{2}\Delta\tau)} + C_0\frac{3+2\sinh^2(\frac{\kappa}{2\sqrt{f(r_s)}}\Delta\tau)}{\sinh^4(\frac{\kappa}{2\sqrt{f(r_s)}}\Delta\tau)}\ ; \tag{23}$$

where C and C_0 are unimportant over all constants (different for different cases) and,

$$B = \kappa/\sqrt{f(r_s)}\ . \tag{24}$$

Moreover, the correlator is time translational invariant as it depends only on the difference of detector's proper time. This is a signature of the thermal equilibrium between the detector and the thermal bath seen by this detector. It also helps us to express the quantity in it's Fourier space which we shall do later to find the equilibrium fluctuation-dissipation relation.

The Fourier transformation of (23) is given by

$$\begin{aligned} K(\omega) &= C\int_{-\infty}^{+\infty} d(\Delta\tau)\frac{5e^{i\omega\Delta\tau}}{\sinh^6(\frac{B\Delta\tau}{2} - i\epsilon)} \\ &+ C\int_{-\infty}^{+\infty} d(\Delta\tau)\frac{4e^{i\omega\Delta\tau}}{\sinh^4(\frac{B\Delta\tau}{2} - i\epsilon)} \\ &+ C_0\int_{-\infty}^{+\infty} d(\Delta\tau)\frac{3e^{i\omega\Delta\tau}}{\sinh^4(\frac{B\Delta\tau}{2} - i\epsilon)} \\ &+ C_0\int_{-\infty}^{+\infty} d(\Delta\tau)\frac{2e^{i\omega\Delta\tau}}{\sinh^2(\frac{B\Delta\tau}{2} - i\epsilon)}\ . \end{aligned} \tag{25}$$

The above one consists of four integrations. All of them can be evaluated by the standard formula [33]

$$\begin{aligned} &\int_{-\infty}^{+\infty} dx\frac{e^{-i\rho x}}{\sinh^{2n}(x - i\epsilon)} \\ &= \frac{(-1)^n}{(2n-1)!}\Big(\frac{2\pi}{\rho}\Big)\frac{1}{e^{\pi\rho}-1}\prod_{k=1}^{n}\Big[\rho^2 + 4(n-k)^2\Big]. \end{aligned} \tag{26}$$

Then one finds

$$K(\omega) = \left(\frac{2\omega}{B}\right)^3 \left[\left\{\left(\frac{2\omega}{B}\right)^2 + 4\right\}\frac{\pi C}{6B} - \frac{2\pi C_0}{B}\right]\frac{1}{e^{-\frac{2\pi\omega}{B}} - 1} \ . \tag{27}$$

Now use of (8) yields

$$K^+(\omega) = \coth\left(\frac{\pi\omega}{B}\right)K^-(\omega) \ , \tag{28}$$

which is identical to Kubo's fluctuation-dissipation relation (9) with the temperature identified as

$$T = \frac{B}{2\pi} \ . \tag{29}$$

Using (24) one can check that this corresponds to,

$$T = \kappa/(2\pi\sqrt{f(r_s)}) \tag{30}$$

which is the correct value of the Tolman expression [6, 5]. For the detector located at infinity, $r_s \to \infty$, $f(r_s) \to 1$, the above result simplifies to,

$$T = \kappa/2\pi \tag{31}$$

which is the familiar Hawking expression [1],

V. Conclusions

A new approach for analysing the Hawking effect has been given in this paper which is based on the fluctuation dissipation relation as formulated by Kubo. It is general enough to include any stationary metric, any dimensions and also to yield the Tolman temperature, which is the result of measurement by an observer at an arbitrary distance from the black hole horizon. Expectedly, the result for an observer at infinity is easily derived, thereby giving the Hawking temperature. It is universal in the sense that it does not depend on how the effective action yielding the stress tensor is regularised. Thus it was applicable both for nonchiral and chiral couplings. In the literature [23, 9, 10, 29, 30, 31, 32], stress tensor based approaches have used either one or the other but a holistic treatment was lacking.

Contrary to several other approaches, this is a physically motivated derivation of the Hawking effect by directly computing the force of the emitted spectrum on the detector and brings it in line with other phenomena of statistical thermodynamics. For instance, the present approach shows that the thermal heat bath characterising a black hole is Brownian in nature. Naturally such an approach is expected to yield further insights into the interpretation of black holes as thermodynamic objects.

It is possible to extend this analysis in other ways. As an example, the back reaction effect might be taken into account. This would change the force (and its fluctuations) as perceived by the detector. The application of the Kubo relation would then yield a correction to the Hawking temperature that determines the greyness of the otherwise blackbody radiation.

References

[1] S. W. Hawking, Nature **248**, 30 (1974).

[2] W. G. Unruh, Phys. Rev. D **14**, 870 (1976).

[3] S. Takagi, Prog. Theor. Phys. Suppl. **88**, 1 (1986).

[4] R. Kubo, Rep. Prog. Phys. **29**, 255 (1966).

[5] R. C. Tolman, Phys. Rev. **35**, 904 (1930).

[6] R. Tolman and P. Ehrenfest, Phys. Rev. **36**, no. 12, 1791 (1930).

[7] R. Kubo, M. Toda and N. Hashitsume, "Statistical Physics II: Nonequilibrium Statistical Mechanics," Springer-Verlag, Berlin Heidelberg, (1985)

[8] F. Reif, "Fundamentals of Statistical and Thermal Physics," McGraw-Hill, New York, (1965).

[9] S. P. Robinson and F. Wilczek, Phys. Rev. Lett. **95**, 011303 (2005) [gr-qc/0502074].

[10] S. Iso, H. Umetsu and F. Wilczek, Phys. Rev. Lett. **96**, 151302 (2006) [hep-th/0602146].

[11] B. R. Majhi, "Quantum Tunneling in Black Holes," PhD Thesis, University of Calcutta, India, arXiv:1110.6008 [gr-qc].

[12] A. Adhikari, K. Bhattacharya, C. Chowdhury and B. R. Majhi, Phys. Rev. D **97**, no. 4, 045003 (2018) [arXiv:1707.01333 [gr-qc]].

[13] A. Das, S. Dalui, C. Chowdhury and B. R. Majhi, arXiv:1902.03735 [gr-qc].

[14] A. O. Caldeira and A. J. Leggett, "Path integral approach to quantum Brownian motion," Physica **121A**, 587 (1983)

[15] W. G. Unruh and W. H. Zurek, Phys. Rev. D **40** (1989) 1071.

[16] D. J. Raine, D. W. Sciama and P. G. Grove, Proc. R. Soc. Land. **A 435** (1991) 205-215.

[17] F. Hinterleitner, Fortsch. Phys. **43** (1995) 207.

[18] H. C. Kim and J. K. Kim, Phys. Rev. D **56** (1997) 3537 [gr-qc/9704030].

[19] H. C. Kim, Phys. Rev. D **59** (1999) 064024 [gr-qc/9808056].

[20] S. Deser, M. J. Duff and C. J. Isham, Nucl. Phys. B **111**, 45 (1976).

[21] M. J. Duff, Nucl. Phys. B **125**, 334 (1977).

[22] A. M. Polyakov, Phys. Lett. B **103**, 207 (1981).

[23] S. M. Christensen and S. A. Fulling, Phys. Rev. D **15**, 2088 (1977).

[24] W. A. Bardeen and B. Zumino, Nucl. Phys. B **244**, 421 (1984).

[25] L. Alvarez-Gaume and P. H. Ginsparg, Annals Phys. **161** (1985) 423 Erratum: [Annals Phys. **171** (1986) 233].

[26] L. Alvarez-Gaume and E. Witten, Nucl. Phys. B **234**, 269 (1984).

[27] H. Leutwyler, Phys. Lett. **153B**, 65 (1985) Erratum: [Phys. Lett. **155B**, 469 (1985)].

[28] N. D. Birrell and P. C. W. Davies, "Quantum Fields in Curved Space," Cambridge University Press, Cambridge, England, (1982).

[29] R. Banerjee and S. Kulkarni, Phys. Rev. D **77**, 024018 (2008) [arXiv:0707.2449 [hep-th]].

[30] R. Banerjee and S. Kulkarni, Phys. Lett. B **659**, 827 (2008) [arXiv:0709.3916 [hep-th]].

[31] R. Banerjee and S. Kulkarni, Phys. Rev. D **79**, 084035 (2009) [arXiv:0810.5683 [hep-th]].

[32] R. Banerjee and B. R. Majhi, Phys. Rev. D **79**, 064024 (2009) [arXiv:0812.0497 [hep-th]].

[33] T. Padmanabhan and T. P. Singh, Class. Quant. Grav. **4**, 1397 (1987).