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Non-thermal Unruh radiation for flavour neutrinos

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Abstract. In the quantum field theory framework, both flavour mixing and Unruh effect have been shown to arise from Bogoliubov transformations connecting inequivalent Hilbert spaces. In the present work, we study how these transformations combine when field mixing for an accelerated observer (Rindler observer) is considered. In particular, a simplified two–flavour model involving Dirac neutrino fields is analyzed. In spite of such basic setting, we find that the spectrum of Unruh radiation gets significantly modified, losing its characteristic thermal behaviour. Exploiting this result, the possibility of fixing new constraints on the neutrino squared mass differences is investigated.

1. Introduction

The topics of neutrino flavour mixing and oscillations have been increasingly investigated since Pontecorvo's pioneering idea [1]. Although the theoretical basis of neutrino mixing have so far been well-established [2] and experimental developments have successfully confirmed the occurrence of flavour oscillations [3], the full picture has not yet been revealed. Puzzling questions, indeed, are still open, such as the origin of neutrino masses in the Standard Model or the non-trivial condensate structure of the vacuum for mixed fields. The latter issue, in particular, has been widely investigated after the unitary inequivalence between mass and flavour vacua in quantum field theory (QFT) was pointed out [4], thereby showing the limits of the quantum mechanical (QM) approach in the analysis of neutrino mixing.

Up to now, the QFT formalism for mixed neutrinos has been developed only within Minkowski framework. Such an approach would be strictly workable only for solar, atmospheric and reactor neutrinos. In order to get valuable information on the structure of the early universe and the properties of super-large celestial objects, however, cosmological neutrinos need to be considered. Because of their passage in the vicinity of supermassive black holes and star clusters, for these particles a treatment in the context of QFT on curved background is required. A first step along this direction has been recently taken in Ref. [5], where the effects of gravity on mixing transformations have been simulated with a uniform acceleration for the case of bosonic fields.

In the present work, starting from these preliminary results, the QFT of two mixed neutrino fields is analyzed in a uniformly accelerated frame. Despite such a minimal setting, the Bogoliubov transformation arising from the Rindler spacetime structure is found to non–trivially combine with the one hiding in field mixing[1]. As a result, Unruh radiation loses its original

[1] Bogoliubov transformations are isomorphisms of the canonical (anti-)commutation relation algebra. In other

thermal nature, potentially providing new constraints on the neutrino squared mass differences.

The paper is organized as follows: in Sec. 2 we briefly review the second quantization of the free Dirac spinor field in Minkowski spacetime (MS). Sec. 3 is devoted to the discussion of the field quantization in Rindler spacetime (RS) and the related Unruh effect. The QFT of two mixed neutrinos is analyzed in Sec. 4, both for inertial and uniformly accelerated observers. Conclusions are shortly summarized in the last section.

Throughout all the work, the metric $\eta_{\mu\nu} = \mathrm{diag}(+1,-1,-1,-1)$ and natural units $\hbar = c = 1$ are used. Moreover, the following notation is adopted: $x = \{t, \mathbf{x}\}$, $\mathbf{x} = \{x^1, \vec{x}\}$, $\vec{x} = \{x^2, x^3\}$.

2. Field quantization in MS: plane–wave and boost–mode representations

In MS the standard plane–wave expansion of the free Dirac neutrino field reads

$$\psi(x) = \sum_{r=1,2} \int d^3k \left[a^r_{\mathbf{k}}\, \psi^{r+}_{\mathbf{k}}(x) \,+\, b^{r\dagger}_{\mathbf{k}}\, \psi^{r-}_{\mathbf{k}}(x) \right], \tag{1}$$

where $\psi^{r+}_{\mathbf{k}} = N_k\, u^r_{\mathbf{k}}\, e^{-ik\cdot x}$ and $\psi^{r-}_{\mathbf{k}} = N_k\, v^r_{\mathbf{k}}\, e^{+ik\cdot x}$ are the positive and negative frequency plane waves, respectively, with $\omega_k = \sqrt{m^2 + |\mathbf{k}|^2}$ and $N_k = (2\pi)^{-3/2}\sqrt{\frac{m}{\omega_k}}$ (see Ref. [6] for the convention used for the spinors $u^r_{\mathbf{k}}$ and $v^r_{\mathbf{k}}$). The $a^r_{\mathbf{k}}$ and $b^r_{\mathbf{k}}$ are the annihilation operators for the ordinary Minkowski vacuum: $a^r_{\mathbf{k}}|0\rangle_M = b^r_{\mathbf{k}}|0\rangle_M = 0$. They are assumed to be canonical: $\left\{a^r_{\mathbf{k}}, a^{s\dagger}_{\mathbf{k}'}\right\} = \left\{b^r_{\mathbf{k}}, b^{s\dagger}_{\mathbf{k}'}\right\} = \delta_{rs}\, \delta^3(k-k')$, with all other anticommutators vanishing.

In order to analyze the connection between the Minkowski and Rindler quantum constructions, an alternative quantization in MS comes in handy. Since the time evolution of the Rindler observer corresponds to an infinite succession of Minkowski boost transformations [7], we expand the spinor field as follows

$$\psi(x) = \sum_{j=1,2} \int d^3\kappa \left[c^j_\kappa\, \Psi^{j-}_\kappa(x) \,+\, d^{j\dagger}_\kappa\, \Psi^{j+}_\kappa(x) \right], \tag{2}$$

where

$$\Psi^{j\mp}_\kappa(x) = \frac{1}{2}\, N^\mp_\kappa \left[X^j_k\, e^{\pm\frac{i\pi}{2}\left(i\Omega - \frac{1}{2}\right)} \int_{-\infty}^{+\infty} d\theta\, e^{i\,\mu_k [x^1 \sinh\theta \,\mp\, t\cosh\theta]}\, e^{\mp\left(i\Omega - \frac{1}{2}\right)\theta} \right.$$
$$\left. +\, Y^j_k\, e^{\pm\frac{i\pi}{2}\left(i\Omega + \frac{1}{2}\right)} \int_{-\infty}^{+\infty} d\theta\, e^{i\,\mu_k [x^1 \sinh\theta \,\mp\, t\cosh\theta]}\, e^{\mp\left(i\Omega + \frac{1}{2}\right)\theta} \right] e^{i\vec{k}\cdot\vec{x}}, \tag{3}$$

(the spinors X^j_k and Y^j_k are defined as in Ref. [7]). These modes are eigenfunctions of the Lorentz momentum operator M_{01}, with eigenvalue Ω. The normalization is defined by $N^\mp_\kappa = e^{\pm\frac{1}{2}\pi\Omega}/(2\pi\sqrt{\mu_k})$, where the subscript κ stands for $\kappa = \{\Omega, \vec{k}\}$.

It is not difficult to prove that the unfamiliar quantization in Eq. (2) is equivalent to the plane–wave construction above introduced. From Eq. (3), indeed, it arises that the boost–modes Ψ^{j-}_κ (Ψ^{j+}_κ) are linear combination of positive (negative) frequency plane waves alone, with $\omega_k = \mu_k\cosh\theta$ ($\omega_k = -\mu_k\cosh\theta$) and $k_1 = \mu_k\sinh\theta$. This means that the transformation connecting the two quantum schemes does not mix the positive and negative frequency modes. Projecting the spinor field in Eq. (1) on Ψ^{j-}_κ and exploiting the orthonormality of plane–waves with respect to the scalar product in MS, we thus obtain

$$c^j_\kappa = (\Psi^{j-}_\kappa, \psi)_M = \sum_{r=1,2} \int_0^{+\infty} dk_1\, F_{j,r}(k_1, \Omega)\, a^r_{\mathbf{k}}, \tag{4}$$

words, they are linear transformations of ladder operators that preserve the algebraic relations among them.

where

$$F_{j,r} = \frac{2\,\pi^2}{\omega_k}\, N_\kappa^-\, N_k \left[\left(\frac{\omega_k + k_1}{\omega_k - k_1}\right)^{i\frac{\Omega}{2}+\frac{1}{4}} e^{i\frac{\pi}{2}\left(i\Omega+\frac{1}{2}\right)}\, X_k^{j\dagger} \;+\; \left(\frac{\omega_k + k_1}{\omega_k - k_1}\right)^{i\frac{\Omega}{2}-\frac{1}{4}} e^{i\frac{\pi}{2}\left(i\Omega-\frac{1}{2}\right)}\, Y_k^{j\dagger}\,\right] u_{\mathbf{k}}^r$$

(a similar connection between the b and d operators can be derived by projecting the field on Ψ_κ^{j+}). From the previous relations, it arises that the vacuum annihilated by c_κ^j and $a_{\mathbf{k}}^r$ is the same. Moreover, by a straightforward calculation, it is possible to show that the transformation in Eq. (4) is canonical. These two statements enable us to definitively conclude that the quantum formalisms above introduced are equivalent. Therefore, we are allowed to use the latter in our derivation of Unruh effect.

3. Field quantization in RS: Unruh effect

Let us now turn to the problem of quantizing the Dirac spinor field in non–inertial frames. As known, the natural framework for describing the motion of an accelerated observer in flat spacetime is the Rindler background. The analysis of flavour mixing for such an observer thus requires a preliminary discussion about the causal structure induced by this metric.

In Minkowski coordinates, the line element has the well known form $ds^2 = (dt)^2 - (dx^1)^2 - \sum_{j=2,3}(dx^j)^2$. Under the coordinate transformation $t = \xi\sinh\eta,\ x^1 = \xi\cosh\eta$, it becomes[2]

$$ds^2 = (dt)^2 - (dx^1)^2 - \sum_{j=2,3}(dx^j)^2 \underset{Rindler\ coord.}{\longrightarrow} ds^2 = \xi^2 d\eta^2 - d\xi^2 - \sum_{j=2,3}(dx^j)^2 . \qquad (5)$$

The trajectory of an observer moving with constant proper acceleration a along the x^1–axis is given by $\xi(\tau) = \text{const} \equiv a^{-1}$, $\vec{x}(\tau) = \text{const}$, where τ is the proper time measured along the line. From Eq. (5), it thus follows that a Rindler observer in the right wedge $R = \{x|x^1 > |t|\}$ is causally disconnected from one in the left sector $L = \{x|x^1 < -|t|\}$. The null hyperplane $t = |x^1|$ $(t = -|x^1|)$, indeed, acts for him as future (past) event horizon.

Bearing this in mind, let us now solve the Dirac equation in the wedge R; similar calculations can be performed in the other three sectors of MS (see, for example, Refs. [7, 8]). Using the tetrad formalism, the Dirac equation in Rindler coordinates takes the form

$$i\partial_\eta\psi = \left(-i\xi\,\alpha^j\partial_j - \frac{1}{2}\,i\,\alpha^1 + m\,\xi\,\beta\right)\psi, \qquad (6)$$

where $\alpha^j = \gamma^{\bar{0}}\,\gamma^{\bar{j}}$ and $\beta = \gamma^{\bar{0}}$. Here $\gamma^{\bar{\mu}}$ are the analogous in Rindler spacetime of the Dirac matrices γ^μ, satisfying the generalized condition $\gamma^{\bar{\mu}}\,\gamma^{\bar{\nu}} + \gamma^{\bar{\nu}}\,\gamma^{\bar{\mu}} = 2g^{\bar{\mu}\bar{\nu}}$ with $g^{\bar{\mu}\bar{\nu}}$ given in Eq. (5). We look for solutions of positive frequency with respect to the Rindler time η, in the form

$$\Psi_\kappa^{j\,R}(\eta,\xi,\vec{x}) = N_\Omega\left(X_k^j\,K_{i\Omega-\frac{1}{2}}(\mu_k\xi) \;+\; Y_k^j\,K_{i\Omega+\frac{1}{2}}(\mu_k\xi)\right)e^{-i\Omega\eta}e^{i\vec{k}\cdot\vec{x}}, \qquad j = 1,2, \qquad (7)$$

where $N_\Omega = \frac{1}{4\pi^2\sqrt{\mu_k}}\sqrt{\cosh(\pi\Omega)}$, $K_{i\Omega\pm\frac{1}{2}}$ is the modified Bessel function and $\kappa \equiv \{\Omega, \vec{k}\}$.

In order to analyze the relationship between the Minkowski and Rindler quantum constructions, it is useful to express the solutions of Eq. (7) in Minkowski coordinates. In this connection, let us observe that, after the coordinate transformation $(t, x^1) \to (\eta, \xi)$, the tetrads - and therefore the spinors - undergo an orthogonal rotation of the following form: $\Psi(t, x^1) = \exp\left(\frac{1}{2}\gamma^{\bar{0}}\gamma^{\bar{1}}\eta\right)\Psi(\eta,\xi)$. Eq. (7) thus becomes

$$\Psi_\kappa^{j\,R}(t,x^1,\vec{x}) = N_\Omega\left(X_k^j\,K_{i\Omega-\frac{1}{2}}(\mu_k\xi)\;e^{-\left(i\Omega-\frac{1}{2}\right)\eta} \;+\; Y_k^j\,K_{i\Omega+\frac{1}{2}}(\mu_k\xi)\,e^{-\left(i\Omega+\frac{1}{2}\right)\eta}\right)e^{i\vec{k}\cdot\vec{x}}. \qquad (8)$$

[2] Notice that $\vec{x} \equiv \{x^2, x^3\}$ is common to both sets of coordinates.

It must be clear that this equation is still defined only in the right wedge R. However, as shown in Ref. [7], it could be linked to the solutions of the Dirac equation in the other sectors of MS by analytically continuing these functions across the horizons. The two possible paths of analytical continuation, one rotating clockwise from the region R to L, the other counterclockwise, correspond to the two different global representations of the boost eigenfunctions $\Psi_\kappa^{j\mp}$ in Eq. (3).

Holding all the necessary tools, the Unruh effect for the free spinor field can now be promptly derived. Following the standard procedure first used by Unruh [9], let us quantize the field in terms of solutions of the Dirac equation which are eigenfunctions of the Rindler Hamiltonian (that is to say, of the Lorentz boost generator in Rindler coordinates), orthonormal and analytic in the whole MS. The functions we are looking for are the following

$$\mathcal{R}_\kappa^j = \frac{1}{\sqrt{2\cosh(\pi\Omega)}}\left(e^{\frac{\pi\Omega}{2}}\,\Psi_\kappa^{j-} + e^{-\frac{\pi\Omega}{2}}\,\Psi_\kappa^{j+}\right), \quad \mathcal{L}_\kappa^j = \frac{1}{\sqrt{2\cosh(\pi\Omega)}}\left(e^{-\frac{\pi\Omega}{2}}\,\Psi_\kappa^{j-} - e^{\frac{\pi\Omega}{2}}\,\Psi_\kappa^{j+}\right), \tag{9}$$

where $\Psi_\kappa^{j\pm}$ are the globally defined modes in Eq. (3). One can easily verify that the functions $\mathcal{R}_\kappa^j$ vanish in the left wedge, whereas they consistently reduce to the Rindler solutions of Eq. (8) in the right sector, thereby providing the correct global representation of the modes for a uniformly accelerated observer in RS (the inverse behaviour, of course, is manifested by the $\mathcal{L}_\kappa^j$).

Now, inverting Eq. (9) with respect to $\Psi_\kappa^{j\pm}$ and inserting into Eq. (2), we have

$$\psi(x) = \sum_{j=1,2}\int d^3\kappa\left[r_\kappa^j\,\mathcal{R}_\kappa^j + r_\kappa^{j\dagger}\,\mathcal{R}_{\tilde\kappa}^j + l_\kappa^j\,\mathcal{L}_{\tilde\kappa}^j + l_\kappa^{j\dagger}\,\mathcal{L}_\kappa^j\right], \tag{10}$$

where $\tilde\kappa \equiv \{-\Omega, \vec{k}\}$ and

$$r_\kappa^j = \frac{c_\kappa^j\,e^{\frac{\pi\Omega}{2}} + d_\kappa^{j\dagger}\,e^{-\frac{\pi\Omega}{2}}}{\sqrt{2\cosh(\pi\Omega)}}, \qquad l_\kappa^{j\dagger} = \frac{c_\kappa^j\,e^{-\frac{\pi\Omega}{2}} - d_\kappa^{j\dagger}\,e^{\frac{\pi\Omega}{2}}}{\sqrt{2\cosh(\pi\Omega)}}. \tag{11}$$

The so obtained *thermal Bogoliubov transformations* allow us to determine the spectrum of Rindler quanta in the inertial vacuum. A straightforward calculation leads to

$$_M\langle 0|r_\kappa^{i\dagger}\,r_{\kappa'}^j|0\rangle_M = {}_M\langle 0|l_\kappa^{i\dagger}\,l_{\kappa'}^j|0\rangle_M = \frac{1}{e^{2\pi\Omega}+1}\,\delta_{ij}\,\delta^3(\kappa-\kappa') \equiv \frac{1}{e^{\frac{a\,\Omega}{T}}+1}\,\delta_{ij}\,\delta^3(\kappa-\kappa'), \tag{12}$$

where in the last step we have taken into account the fact that the proper energy of the quanta detected by a Rindler observer with acceleration a is $a\Omega$, rather than Ω. From Eq. (12) it thus follows that the vacuum radiation perceived by the Rindler observer has a thermal spectrum, according to Fermi–Dirac statistics with temperature $T = \frac{a}{2\pi}$ [9].

4. QFT of mixed fields: inertial and accelerated frames

In a simplified two–flavour model, mixing transformations for Dirac neutrino fields read

$$\psi_e(x) = \psi_1(x)\,\cos\theta + \psi_2(x)\,\sin\theta, \qquad \psi_\mu(x) = -\psi_1(x)\,\sin\theta + \psi_2(x)\,\cos\theta. \tag{13}$$

Here ψ_χ, $\chi = e, \mu$ are the neutrino fields with definite flavours; ψ_i, $i = 1, 2$ are the free neutrino fields with definite masses m_i; θ is the mixing angle.

Exploiting the completeness of the plane-waves $\psi_{\mathbf{k},}^{r\pm}$, the following free field–like expansions for flavour fields in MS can be adopted [4]

$$\psi_\ell(x) = \sum_{r=1,2}\int d^3k\,e^{i\mathbf{k}\cdot\mathbf{x}}\left[a_{\mathbf{k},\ell}^r(t)\,u_{\mathbf{k},\sigma}^r(t) + b_{-\mathbf{k},\ell}^{r\dagger}(t)\,v_{-\mathbf{k},\sigma}^r(t)\right], \quad (\ell,\sigma) = (e,1), (\mu,2), \tag{14}$$

where the notation: $u^r_{\mathbf{k},\sigma}(t) \equiv u^r_{\mathbf{k},\sigma}\, e^{-i\omega_k t}$, $v^r_{-\mathbf{k},\sigma}(t) \equiv v^r_{-\mathbf{k},\sigma}\, e^{+i\omega_k t}$ has been introduced.

For field quanta moving along the same direction as the Rindler observer (i.e., in the reference frame such that $\mathbf{k} = (k, 0, 0)$), the *flavour operator* $a^r_{\mathbf{k},e}$ assumes the form

$$a^r_{\mathbf{k},e}(t) = \cos\theta\, a^r_{\mathbf{k},1} \,+\, \sin\theta\, \left(U^*_{\mathbf{k}}(t)\; a^r_{\mathbf{k},2} \,-\, V_{\mathbf{k}}(t)\; b^{-r\dagger}_{-\mathbf{k},2}\right) \tag{15}$$

(similar expressions hold for the other ladder operators in Eq. (14)), where the mixing Bogoliubov coefficients $U_{\mathbf{k}}$ and $V_{\mathbf{k}}$ are given by

$$U_{\mathbf{k}}(t) \,=\, \left(\frac{\omega_{k,1}+m_1}{2\omega_{k,1}}\right)^{\frac{1}{2}} \left(\frac{\omega_{k,2}+m_2}{2\omega_{k,2}}\right)^{\frac{1}{2}} \left(1 + \frac{k^2}{(\omega_{k,1}+m_1)(\omega_{k,2}+m_2)}\right) e^{i(\omega_{k,2}-\omega_{k,1})\,t}, \tag{16}$$

$$V_{\mathbf{k}}(t) \,=\, \left(\frac{\omega_{k,1}+m_1}{2\omega_{k,1}}\right)^{\frac{1}{2}} \left(\frac{\omega_{k,2}+m_2}{2\omega_{k,2}}\right)^{\frac{1}{2}} \left(\frac{k}{\omega_{k,2}+m_2} - \frac{k}{\omega_{k,1}+m_1}\right) e^{i(\omega_{k,2}+\omega_{k,1})\,t}. \tag{17}$$

By use of these equations, one can easily prove that flavour operators, just like the corresponding ones for definite mass fields, satisfy the canonical anticommutation relations at equal times.

The foregoing discussion is completely within the usual Minkowski framework. Thus it arises the question about how the above formalism gets modified for an accelerated observer. For this purpose, using the same approach as in Sec. 2, let us quantize the mixed fields in Eq. (13) as follows

$$\psi_\ell(x) = \sum_{j=1,2} \int d^3\kappa \left[c^j_{\kappa,\ell}\, \Psi^{j-}_{\kappa,\sigma}(x) \,+\, d^{j\dagger}_{\kappa,\ell}\, \Psi^{j+}_{\kappa,\sigma}(x)\right], \quad (\ell,\sigma) = (e,1), (\mu,2), \tag{18}$$

where $\Psi^{j\mp}_{\kappa,\sigma}$ are the boost–modes in Eq. (3) and

$$c^j_{\kappa,\ell} = \sum_{r=1,2} \int_0^{+\infty} dk_1\, F_{j,r,\sigma}(k_1,\Omega)\, a^r_{\mathbf{k},\sigma}, \qquad (\ell,\sigma) = (e,1), (\mu,2) \tag{19}$$

as for free fields (see the corresponding transformation in Eq. (4)). Notice that for quanta such that $\mathbf{k} = (k, 0, 0)$, the subscript κ reduces to $\kappa = (\Omega, 0, 0)$.

Equation (18) provides a springboard for analyzing flavour mixing in accelerated frames. By extending the Unruh quantum construction in Eq. (10) to the mixed fields, indeed, we are immediately allowed to write down for them the following expansions

$$\psi_\ell(x) = \sum_{j=1,2} \int d^3\kappa \left[r^j_{\kappa,\ell}\, \mathcal{R}^j_{\kappa,\sigma} \,+\, r^{j\dagger}_{\kappa,\ell}\, \mathcal{R}^j_{\tilde\kappa,\sigma} \,+\, l^j_{\kappa,\ell}\, \mathcal{L}^j_{\tilde\kappa,\sigma} \,+\, l^{j\dagger}_{\kappa,\ell}\, \mathcal{L}^j_{\kappa,\sigma}\right], \quad (\ell,\sigma) = (e,1), (\mu,2), \tag{20}$$

where the flavour operators for the Rindler observer arise from the interplay between the mixing and thermal Bogoliubov transformations in Eqs. (11) and (19), respectively, i.e.

$$r^j_{\kappa,\ell} = \frac{1}{\sqrt{2\cosh(\pi\Omega)}} \sum_{r=1,2} \int_0^{+\infty} dk_1 \left(e^{\frac{\pi\Omega}{2}}\, F_{j,r,\sigma}(\Omega,k_1)\, a^r_{\mathbf{k},\sigma} + e^{-\frac{\pi\Omega}{2}}\, G_{j,r,\sigma}(\Omega,k_1)\; b^{r\dagger}_{\mathbf{k},\sigma}\right), \tag{21}$$

with

$$G_{j,r} \,=\, \frac{2\,\pi^2}{\omega_k}\, N^+_\kappa\, N_k \Bigg[\left(\frac{\omega_k+k_1}{\omega_k-k_1}\right)^{i\frac{\Omega}{2}+\frac{1}{4}} e^{i\frac{\pi}{2}\left(i\Omega+\frac{1}{2}\right)}\; X^{j\dagger}_k \,+\, \left(\frac{\omega'+k'_1}{\omega'-k'_1}\right)^{i\frac{\Omega}{2}-\frac{1}{4}} e^{i\frac{\pi}{2}\left(i\Omega-\frac{1}{2}\right)}\; Y^{j\dagger}_k \Bigg] v^r_{\mathbf{k}}$$

(similarly for $l^j_{\kappa,\ell}$). In the simplest case of $t = \eta = 0$, the spectrum of mixed neutrinos in the inertial vacuum thus assumes the form

$$
{}_M\langle 0|r^{i\dagger}_{\kappa,\chi}\, r^{j}_{\kappa',\chi}|0\rangle_M = \frac{1}{e^{\frac{a\,\Omega}{T}}+1}\,\delta_{ij}\,\delta(\kappa-\kappa') \,+\, \sin^2\theta\left[\frac{e^{\frac{\pi\,(\Omega+\Omega')}{2}}}{2\sqrt{\cosh(\pi\Omega)\,\cosh(\pi\Omega')}}\,N^{ij}_{F,F}(\Omega,\Omega') - \frac{e^{-\frac{\pi\,(\Omega+\Omega')}{2}}}{2\sqrt{\cosh(\pi\Omega)\,\cosh(\pi\Omega')}}\,N^{ij}_{G,G}(\Omega,\Omega')\right]\delta_{ij}, \qquad (22)
$$

where

$$
N^{ij}_{F,F} = \sum_{r=1,2}\int F^{*}_{i,r}(k_1,\Omega)\,F_{j,r}(k_1,\Omega')\,|V_{\mathbf{k}}|^2, \qquad N^{ij}_{G,G} = \sum_{r=1,2}\int G^{*}_{i,r}(k_1,\Omega)\,G_{j,r}(k_1,\Omega')\,|V_{\mathbf{k}}|^2.
$$

One can immediately verify that Eq. (22) consistently reduces to the standard Unruh result in Eq. (12) for $\theta \to 0$ and/or $m_1 = m_2$ (see Eq. (17)), as it must be in absence of mixing. In the realistic limit $\frac{\Delta m^2}{m^2} \equiv \frac{|m_2^2-m_1^2|}{m^2} \ll 1$, however, it can be further manipulated, thus giving [10]

$$
{}_M\langle 0|r^{i\dagger}_{\kappa,\chi}\, r^{j}_{\kappa',\chi}|0\rangle_M = \frac{1}{e^{\frac{a\,\Omega}{T}}+1}\,\delta_{ij}\,\delta(\kappa-\kappa') \,+\, \mathcal{O}\left(\frac{\Delta m^2}{m^2}\right). \qquad (23)
$$

Therefore, for mass differences that are comparable with the acceleration, the vacuum radiation experienced by the Rindler observer gets non-trivially modified, resulting in the sum of the usual Unruh contribution plus non–thermal corrections arising from flavour mixing and depending only on the parameter $\Delta m^2/m^2$. Beyond the conceptual relevance of such a result, Eq. (23) can thus be exploited to fix new potential constraints on the neutrino squared mass differences.

An heuristic interpretation for the origin of the modified spectrum in Eq. (23) can be found in Ref. [5], where the same scenario has been analyzed for the case of bosonic mixed fields.

5. Conclusions

The QFT of two mixed neutrinos has been analyzed in an accelerated frame as a basis for investigating gravitational effects on mixing transformations. Despite this minimal setting, the Bogoliubov transformation arising from the Rindler spacetime structure has been found to suggestively combine with the one built in field mixing. As a result, Unruh radiation loses its thermality, thus providing an alternative way to constrain the neutrino squared mass differences.

We stress that the formalism developed in this work provides a useful tool for analyzing neutrino flavour oscillations in the context of QFT on curved background. Following what has been done in Ref. [4] within Minkowski framework, indeed, the oscillation probability formulas for an accelerated observer can be derived by defining the one flavour neutrino state and then performing similar calculations to those in Eq. (22). Work is in progress along this direction.

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