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Neutrino oscillations in Unruh radiation

M. Blasone [a,b], G. Lambiase [a,b], G.G. Luciano [a,b,*], L. Petruzziello [a,b,*]

[a] Dipartimento di Fisica, Università di Salerno, Via Giovanni Paolo II, 132 84084 Fisciano, Italy
[b] INFN, Gruppo collegato di Salerno, Italy

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A B S T R A C T

The study of the decay of an accelerated proton recently provided a "theoretical proof" of the Unruh effect. On the basis of general covariance of Quantum Field Theory, indeed, it was found that the decay rates in the inertial and comoving frames do coincide only when the thermal nature of the accelerated vacuum is taken into account. Such an analysis was then extended to the case with mixed neutrinos. In this paper, we show that, by further embedding neutrino oscillations in the above framework, the requirement of general covariance necessarily entails the use of flavor neutrinos as asymptotic states, as well as the occurrence of neutrino oscillations in the Unruh thermal bath.

1. Introduction

In the history of Physics, the adoption of principles has revealed to be a formidable investigation tool. Although intimately related to the phenomenological realm from which they stem, once elevated to the status of postulates, physical principles act as lighthouses for the development of a consistent theoretical apparatus. Paradigmatic examples are the principle of conservation of energy, which led for instance to the discovery of the neutrino, and the principle of constancy of speed of light, at the basis of Special Relativity.

Recently, general covariance was advocated to exhibit that the internal consistency of quantum field theory (QFT) unavoidably requires the existence of the Unruh effect, also known as Fulling-Davies-Unruh effect [1–3]. Indeed, starting from the statement that acceleration can influence even the proper lifetime of stable particles [4], in a series of remarkable papers [5] it was shown that the tree-level decay rate of an accelerated proton via the inverse β-decay is frame-independent only when the thermal nature of the vacuum for a non-inertial observer is considered.

The fact that a theoretical requirement leads to the specific form of the ground state for an accelerated observer should be regarded as a considerable result, especially in view of the perplexities which have been sometimes raised about the physical significance of the Unruh effect [6]. Such a skepticism is enhanced by the lack of direct evidences of this phenomenon, as it also happens for the case of the Hawking radiation [7]. In fact, at present, the most likely arena for (indirect) experimental tests of these effects is given by analogue gravity [8].

In the aforemention studies on the proton decay, the emitted neutrino was treated as massless and only in Ref. [9] as a massive particle. In these works, however, neutrino mixing was not taken into account. This was done for the first time in Ref. [10], where a discrepancy between the proton decay rate in the inertial and comoving frames was claimed to arise. Subsequently, it was proved that general covariance does indeed hold in the above analysis: this was shown by employing either flavor [11] or mass [12] eigenstates for neutrinos, thus leaving an essential ambiguity on the very nature of asymptotic neutrino states.

In this paper we show that, due to the occurrence of neutrino oscillations in the problem at hand, general covariance leads to the conclusion that the correct states to describe the asymptotic behavior of neutrinos must be the flavor ones, and that the Unruh thermal bath is made up of oscillating neutrinos. Throughout the work, we shall use natural units $\hbar = c = 1$ and the Minkowski metric with the mostly negative signature.

2. General formalism

Let us start by setting the framework. Following the approach and the notation of Refs. [5], the proton $|p\rangle$ and the neutron $|n\rangle$ can be viewed as unexcited and excited states of the nucleon.

* Corresponding authors.
E-mail addresses: blasone@sa.infn.it (M. Blasone), lambiase@sa.infn.it (G. Lambiase), gluciano@sa.infn.it (G.G. Luciano), lpetruzziello@na.infn.it (L. Petruzziello).

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In addition, we assume to deal with particles that are energetic enough to possess a well-defined trajectory. In these conditions, it is possible to employ the Fermi theory of current-current interaction, where we consider a quantum leptonic and a classical hadronic current $\hat{J}^{\mu}_{\ell}\hat{J}_{h,\mu} \to \hat{J}^{\mu}_{\ell}\hat{J}^{(cl)}_{h,\mu}$, with

$$\hat{J}^{(cl)}_{h,\mu} = \hat{q}(\tau)u_{\mu}\delta(x)\delta(y)\delta(u-a^{-1})\,, \tag{1}$$

(for the definition of the lepton current $\hat{J}^{\mu}_{\ell}$, see below). Here,[1] $\tau = v/a$ is the nucleon's proper time (with v being the Rindler time coordinate), a its proper acceleration and $u = a^{-1} = \text{const}$ represents the spatial Rindler coordinate which denotes the world line of the uniformly accelerated particle. The four-velocity of the nucleon u^{μ} is given by $u^{\mu} = (a, 0, 0, 0)$, $u^{\mu} = (\sqrt{a^2t^2+1}, 0, 0, at)$, in Rindler and Minkowski coordinates, respectively. In accordance with Refs. [5,13], the Hermitian monopole $\hat{q}(\tau)$ is given by $\hat{q}(\tau) \equiv e^{i\hat{H}\tau}\hat{q}_0 e^{-i\hat{H}\tau}$, where $\hat{H}$ is the nucleon Hamiltonian and $\hat{q}_0$ is used to reconstruct the effective Fermi constant $G_F \equiv \langle p|\hat{q}_0|n\rangle$.

Assuming to deal with a simplified two-flavor model, the interaction of the charged leptons $\hat{\Psi}_{\alpha}$ and neutrinos $\hat{\Psi}_{\nu_\alpha}$ $(\alpha = e, \mu)$ with the nucleon current $\hat{J}^{(cl)}_{h,\mu}$ is described by the Fermi action

$$\hat{S}_I \equiv \sum_{\alpha=e,\mu}\int d^4x\sqrt{-g}\,\hat{J}^{(cl)}_{h,\lambda}\left(\hat{\bar{\Psi}}_{\nu_\alpha}\gamma^{\lambda}\hat{\Psi}_{\alpha} + \hat{\bar{\Psi}}_{\alpha}\gamma^{\lambda}\hat{\Psi}_{\nu_\alpha}\right), \tag{2}$$

where $g \equiv \det(g_{\mu\nu})$ and γ^{λ} are the gamma matrices in Dirac representation (e.g., see Ref. [14]). In Eq. (2), neutrino fields with definite flavors are related to the ones with definite masses by the standard transformations

$$\hat{\Psi}_{\nu_e} = \cos\theta\,\hat{\Psi}_{\nu_1} + \sin\theta\,\hat{\Psi}_{\nu_2}, \qquad \hat{\Psi}_{\nu_\mu} = -\sin\theta\,\hat{\Psi}_{\nu_1} + \cos\theta\,\hat{\Psi}_{\nu_2}. \tag{3}$$

In what follows, we shall focus only on the process involving the production of a positron. The case in which the proton decays into a neutron, an anti-muon and a muon neutrino can be treated analogously and in an independent way.

3. Inertial frame

In the inertial frame, the process to be considered is (inverse β-decay)

$$\text{(i)} \quad p \to n + e^{+} + \nu_e\,, \tag{4}$$

which is pictorially represented in Fig. 1. In this frame, the fermion field is expanded as

$$\hat{\Psi} = \sum_{\sigma=\pm}\int \frac{d^3k}{4\pi^{\frac{3}{2}}}\left[e^{-ik^{\mu}x_{\mu}}u^{(\omega)}_{\sigma}\hat{b}_{\mathbf{k}\sigma} + e^{ik^{\mu}x_{\mu}}u^{(-\omega)}_{-\sigma}\hat{d}^{\dagger}_{\mathbf{k}\sigma}\right], \tag{5}$$

where σ is the polarization, $\omega = \sqrt{k^2+m^2}$ is the Minkowski frequency and $u^{(\omega)}_{\sigma}$ is the spinor defined as [11]

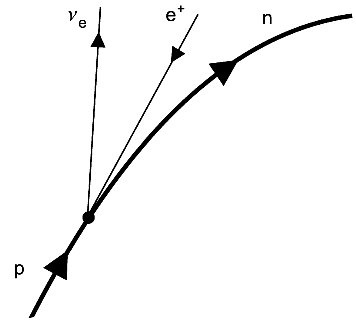

Fig. 1. Decay process (i) in the inertial frame. Time flows in the vertical direction.

$$u^{(\pm\omega)}_{+}(\mathbf{k}) = \frac{1}{\sqrt{\omega(\omega\pm m)}}\begin{pmatrix} m\pm\omega \\ 0 \\ k^z \\ k^x + ik^y\end{pmatrix},$$
$$u^{(\pm\omega)}_{-}(\mathbf{k}) = \frac{1}{\sqrt{\omega(\omega\pm m)}}\begin{pmatrix} 0 \\ m\pm\omega \\ k^x - ik^y \\ -k^z\end{pmatrix}. \tag{6}$$

The tree-level transition amplitude for the process (i) reads [11]

$$\begin{aligned}\mathcal{A}^{(\nu_e)}_{(\text{i})} &\equiv \langle n|\otimes\langle e^{+},\nu_e|\hat{S}_I|0\rangle\otimes|p\rangle \\ &= \frac{G_F}{2^4\pi^3}\Big[\cos^2\theta\,\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_1},\omega_e) + \sin^2\theta\,\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_2},\omega_e)\Big],\end{aligned} \tag{7}$$

where for simplicity we have omitted the $\mathbf{k}$- and σ-dependence of the lepton states, we have assumed equal momenta and polarizations for neutrinos with definite masses and

$$\begin{aligned}&\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_j},\omega_e) \\ &\quad = \int_{-\infty}^{+\infty} d\tau\, u_{\mu}\left[\bar{u}^{(+\omega_{\nu_j})}_{\sigma_\nu}\gamma^{\mu}u^{(-\omega_e)}_{-\sigma_e}\right] \\ &\qquad \times e^{i\left[\Delta m\tau + a^{-1}\left(\omega_{\nu_j}+\omega_e\right)\sinh a\tau - a^{-1}\left(k^z_\nu + k^z_e\right)\cosh a\tau\right]}, \quad j = 1, 2.\end{aligned} \tag{8}$$

Note that the asymptotic flavor state $|\nu_e\rangle$ in Eq. (7) has been expressed in terms of the corresponding mass states $|\nu_i\rangle$ $(i = 1, 2)$ by means of Pontecorvo mixing transformations [15]

$$\begin{aligned}|\nu_e\rangle &= \cos\theta\,|\nu_1\rangle + \sin\theta\,|\nu_2\rangle, \\ |\nu_\mu\rangle &= -\sin\theta\,|\nu_1\rangle + \cos\theta\,|\nu_2\rangle\,.\end{aligned} \tag{9}$$

Now, the differential transition probability takes the form $d^6\mathcal{P}^{(\nu_e)}_{(\text{i})}/d^3k_\nu\,d^3k_e \equiv \sum_{\sigma_e,\sigma_\nu}\left|\mathcal{A}^{(\nu_e)}_{(\text{i})}\right|^2$ and the transition rate is given by $\Gamma \equiv \mathcal{P}/T$, with $T \equiv \int_{-\infty}^{+\infty}d\tau$ being the nucleon total proper time.

In Ref. [11], it has been shown that the decay rate for the process (i) is

$$\Gamma^{(\nu_e)}_{in} = \cos^4\theta\,\Gamma_1 + \sin^4\theta\,\Gamma_2 + \cos^2\theta\sin^2\theta\,\Gamma_{12}\,, \tag{10}$$

where we have introduced the shorthand notation

$$\Gamma_j \equiv \frac{1}{T}\sum_{\sigma_\nu,\sigma_e}\frac{G_F^2}{2^8\pi^6}\int d^3k_\nu\int d^3k_e\left|\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_j},\omega_e)\right|^2, \quad j = 1, 2, \tag{11}$$

[1] We assume the acceleration to occur along the z-direction. With this choice, the Rindler coordinates (v, x, y, u) are related with the Minkowski coordinates (t, x, y, z) as follows: $t = u\sinh v$, $z = u\cosh v$, with x and y untouched.

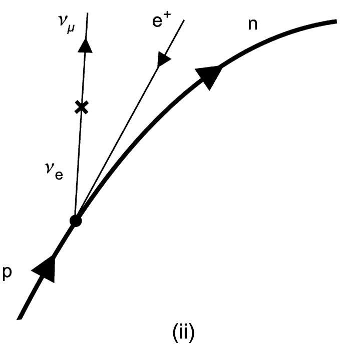

Fig. 2. Decay process (ii) in the inertial frame.

$$\Gamma_{12} \equiv \frac{1}{T} \sum_{\sigma_\nu,\sigma_e} \frac{G_F^2}{2^8\pi^6} \int d^3k_\nu \times \int d^3k_e \Big[\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_1},\omega_e)\,\mathcal{I}^*_{\sigma_\nu\sigma_e}(\omega_{\nu_2},\omega_e) + \text{c.c.}\Big]. \tag{12}$$

The aim of the calculation contained in Ref. [11] is to exhibit the equality between Eq. (10) and its counterpart in the accelerated frame (see below), which guarantees the validity of the principle of general covariance.

At this point, it must be emphasized that in the above calculation an infinite proper time interval is considered, which allows for the emitted electron neutrino to oscillate.[2] Thus, we should tale into account not only the process in Eq. (4), but also the following one:

$$\text{(ii)} \quad p \to n + e^+ + \nu_\mu\,. \tag{13}$$

The above relation must be intended in the sense of Fig. 2: although it is true that the lepton charge must necessarily be conserved in the vertex (at tree-level), as soon as the outgoing neutrino is produced, there is a non-vanishing probability that it undergoes oscillations. We remark that this process has not been included in the analysis of Ref. [11], without affecting, however, the validity of the results there contained.

The transition amplitude for the process (ii) is now given by

$$\mathcal{A}^{(\nu_\mu)}_{\text{(ii)}} = \langle n| \otimes \langle e^+, \nu_\mu|\hat{S}_I|0\rangle \otimes |p\rangle \tag{14}$$
$$= -\frac{G_F}{2^4\pi^3}\cos\theta\sin\theta\Big[\mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_1},\omega_e) - \mathcal{I}_{\sigma_\nu\sigma_e}(\omega_{\nu_2},\omega_e)\Big].$$

In terms of Γ, the quantity $\mathcal{A}^{(\nu_\mu)}_{\text{(ii)}}$ of Eq. (14) associated to the process (ii) leads to the following transition rate:

$$\Gamma^{(\nu_\mu)}_{in} = \cos^2\theta\sin^2\theta\,(\Gamma_1 + \Gamma_2 - \Gamma_{12})\,, \tag{15}$$

where the three terms in the r.h.s. have already been defined in Eqs. (11) and (12). We notice that the above transition rate is proportional to $\sin^2 2\theta$, thus showing that it is originated by interference.

Finally, observe that the total decay rate in the inertial frame reads

$$\Gamma_{in} \equiv \Gamma^{(\nu_e)}_{in} + \Gamma^{(\nu_\mu)}_{in} = \cos^2\theta\,\Gamma_1 + \sin^2\theta\,\Gamma_2\,. \tag{16}$$

[2] We remark that we are describing the spacetime evolution of neutrinos by means of plane waves rather than wave packets, so that no decoherence scale appears in our analysis.

4. Accelerated frame

From the point of view of an observer comoving with the proton, the only way to make the particle's decay possible is to suppose the existence of a thermal bath of electrons, neutrinos and the corresponding antiparticles [11]. In such conditions, three channels have to be considered to match the inertial process (4), i.e.

$$\text{(iii)}\ p^+ + e^- \to n + \nu_e\,, \quad \text{(iv)}\ p^+ + \overline{\nu}_e \to n + e^+\,, \quad \text{(v)}\ p^+ + e^- + \overline{\nu}_e \to n\,. \tag{17}$$

In order to calculate the proton's decay rate in the accelerated frame, we need the expansion for the fermion fields in Rindler coordinates, that is

$$\hat{\Psi} = \sum_{\sigma=\pm} \int_0^{+\infty} d\omega \int \frac{d^2k}{(2\pi)^{\frac{3}{2}}} \Big[e^{i(-\omega v/a + k_\alpha x^\alpha)} u^{(\omega)}_\sigma \hat{b}_{\mathbf{w}\sigma} + e^{i(\omega v/a + k_\alpha x^\alpha)} u^{(-\omega)}_{-\sigma} \hat{d}^\dagger_{\mathbf{w}\sigma}\Big], \tag{18}$$

where $k_\alpha x^\alpha = k_x x + k_y y$, ω is the Rindler frequency which can assume arbitrary positive values and does not satisfy any dispersion relation, $\mathbf{w} = (\omega, k^x, k^y)$ and $u^{(\omega)}_\sigma$ is defined as [11]

$$u^{(\omega)}_+(u,\mathbf{w}) = \sqrt{\frac{a\cosh(\pi\omega/a)}{\pi l}} \times \begin{pmatrix} ilK_{i\omega/a-1/2}(ul) + mK_{i\omega/a+1/2}(ul) \\ -(k^x + ik^y)K_{i\omega/a+1/2}(ul) \\ ilK_{i\omega/a-1/2}(ul) - mK_{i\omega/a+1/2}(ul) \\ -(k^x + ik^y)K_{i\omega/a+1/2}(ul) \end{pmatrix}, \tag{19}$$

$$u^{(\omega)}_-(u,\mathbf{w}) = \sqrt{\frac{a\cosh(\pi\omega/a)}{\pi l}} \times \begin{pmatrix} (k^x - ik^y)K_{i\omega+1/2}(ul) \\ ilK_{i\omega/a-1/2}(ul) + mK_{i\omega/a+1/2}(ul) \\ -(k^x - ik^y)K_{i\omega+1/2}(ul) \\ -ilK_{i\omega/a-1/2}(ul) + mK_{i\omega/a+1/2}(ul) \end{pmatrix}. \tag{20}$$

Here $K_{i\omega/a\pm1/2}(ul)$ represents the modified Bessel function of second kind and complex order, and $l \equiv \sqrt{m^2 + (k^x)^2 + (k^y)^2}$.

Let us then consider the scattering process (iii) in Eq. (17) (see also Fig. 3); similar calculations can be carried out for the processes (iv) and (v). By straightforward calculations, we get [11]

$$\mathcal{A}^{(\nu_e)}_{\text{(iii)}} \equiv \langle n| \otimes \langle \nu_e|\hat{S}_I|e^-\rangle \otimes |p\rangle \tag{21}$$
$$= \frac{G_F}{(2\pi)^2}\Big[\cos^2\theta\,\mathcal{J}^{(1)}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e) + \sin^2\theta\,\mathcal{J}^{(2)}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e)\Big],$$

where we have assumed equal frequencies and polarizations for neutrino states with definite masses, and

$$\mathcal{J}^{(i)}_{\sigma_\nu\sigma_e}(\omega_\nu,\omega_e) \equiv \delta\big(\omega_e - \omega_\nu - \Delta m\big)\,\bar{u}^{(i,\omega_\nu)}_{\sigma_\nu}\gamma^0 u^{(\omega_e)}_{\sigma_e}\,, \tag{22}$$

with $i = 1, 2$. The spinor component related to the neutrino field contains the information on the mass of ν_i and, by means of the current hypothesis, this is the only difference between the functions $\mathcal{J}^{(i)}$ with different indexes.

Now, the sum of the transition rates for the three processes in Eqs. (17) yields [11]

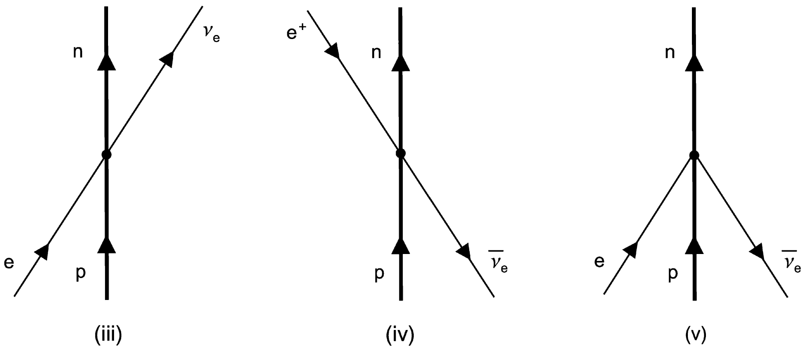

Fig. 3. Decay processes (iii), (iv) and (v) in the accelerated frame.

$$\Gamma_{acc}^{(\nu_e)} = \cos^4\theta\,\widetilde{\Gamma}_1 + \sin^4\theta\,\widetilde{\Gamma}_2 + \cos^2\theta\sin^2\theta\,\widetilde{\Gamma}_{12}\,, \tag{23}$$

where

$$\widetilde{\Gamma}_j \equiv \mathcal{N}\int_{-\infty}^{+\infty} d\omega\,\mathcal{R}_j(\omega), \qquad j=1,2\,, \tag{24}$$

and $\mathcal{N} \equiv 2\pi^{-7}a^{-2}G_F^2 e^{-\pi\Delta m/a}$. The functions $\widetilde{\Gamma}_{12}$ and $\mathcal{R}_j$ in Eqs. (23) and (24) are defined by

$$\mathcal{R}_j(\omega) = \int d^2k_\nu d^2k_e\, l_{\nu_j}\, l_e \left|K_{\frac{i}{a}\tilde{\omega}+\frac{1}{2}}\left(\frac{l_{\nu_j}}{a}\right)K_{\frac{i}{a}\omega+\frac{1}{2}}\left(\frac{l_e}{a}\right)\right|^2 + m_{\nu_j}m_e\mathrm{Re}\left[\int d^2k_\nu d^2k_e K^2_{\frac{i}{a}\tilde{\omega}-\frac{1}{2}}\left(\frac{l_{\nu_j}}{a}\right)K^2_{\frac{i}{a}\omega+\frac{1}{2}}\left(\frac{l_e}{a}\right)\right], \tag{25}$$

and

$$\begin{aligned}\widetilde{\Gamma}_{12} = \frac{\mathcal{N}}{\sqrt{l_{\nu_1}l_{\nu_2}}}\int d\omega d^2k_e d^2k_\nu \Bigg\{ & l_e\left|K_{\frac{i}{a}\omega+\frac{1}{2}}\left(\frac{l_e}{a}\right)\right|^2 \\ & \times\left(\kappa_\nu^2 + m_{\nu_1}m_{\nu_2} + l_{\nu_1}l_{\nu_2}\right) \\ & \times \mathrm{Re}\left[K_{\frac{i}{a}\tilde{\omega}+\frac{1}{2}}\left(\frac{l_{\nu_1}}{a}\right)\times K_{\frac{i}{a}\tilde{\omega}-\frac{1}{2}}\left(\frac{l_{\nu_2}}{a}\right)\right] \\ & + m_e\left(l_{\nu_1}m_{\nu_2} + l_{\nu_2}m_{\nu_1}\right) \\ & \times \mathrm{Re}\left[K^2_{\frac{i}{a}\omega+\frac{1}{2}}\left(\frac{l_e}{a}\right)K_{\frac{i}{a}\tilde{\omega}-\frac{1}{2}}\left(\frac{l_{\nu_1}}{a}\right)K_{\frac{i}{a}\tilde{\omega}-\frac{1}{2}}\left(\frac{l_{\nu_2}}{a}\right)\right]\Bigg\},\end{aligned} \tag{26}$$

respectively, where we have used the shorthand notation $\tilde{\omega} \equiv \omega - \Delta m$.

In Ref. [11] it was shown that the decay rate $\Gamma_{acc}^{(\nu_e)}$ in Eq. (23) is in agreement with the corresponding expression $\Gamma_{in}^{(\nu_e)}$ in the inertial frame (see Eq. (10)). In particular, one can prove that $\Gamma_i = \widetilde{\Gamma}_i$ for $i=1,2$, whereas Γ_{12} and $\widetilde{\Gamma}_{12}$ are equal to each other only up to a first-order expansion in the parameter $\delta m \equiv m_{\nu_2} - m_{\nu_1}$, where m_{ν_i} represents the mass of the i-th neutrino state. In such an approximation, Pontecorvo states (9) can be identified with the exact flavor neutrino states, defined as eigenstates of flavor charges [16,17].

On the other hand, in Sec. 3 we have seen that an additional contribution to the proton decay rate has to be considered to take account of flavor oscillations. In the inertial frame, such a contribution is provided by the process (ii) in Fig. 2. Guided by the principle of general covariance, we now look for the corresponding interactions in the comoving frame which should lead to the same expression of the decay rate.

To this aim, we consider the following three channels as potential candidates for the non-inertial counterpart of the decay (13) (see Fig. 4)

$$\begin{aligned}&\text{(vi) } p^+ + e^- \to n + \nu_\mu, \quad \text{(vii) } p^+ + \overline{\nu}_\mu \to n + e^+, \\ &\text{(viii) } p^+ + e^- + \overline{\nu}_\mu \to n\,.\end{aligned} \tag{27}$$

Note that, while the process (vi) is of the same type of (ii), since it simply involves the oscillation of the emitted electron neutrino, the processes (vii) and (viii) are essentially due to the oscillation of a muon antineutrino that is already present in the Unruh thermal bath.

In order to legitimate the validity of our assumption, we need to perform the same calculations leading to the decay rate in Eq. (23). The outcome of this procedure turns out to be

$$\Gamma_{acc}^{(\nu_\mu)} = \cos^2\theta\,\sin^2\theta\left(\widetilde{\Gamma}_1 + \widetilde{\Gamma}_2 - \widetilde{\Gamma}_{12}\right). \tag{28}$$

Therefore, in light of the discussion after Eq. (26), which allow us to state that $\Gamma_{in}^{(\nu_e)} = \Gamma_{acc}^{(\nu_e)}$, it is possible to infer that such an equivalence holds also for the decay rates in Eqs. (15) and (28). Moreover, if we compute the total (comoving) decay rate which includes neutrino oscillations, we deduce that

$$\Gamma_{acc} = \Gamma_{acc}^{(\nu_e)} + \Gamma_{acc}^{(\nu_\mu)} = \cos^2\theta\,\widetilde{\Gamma}_1 + \sin^2\theta\,\widetilde{\Gamma}_2\,. \tag{29}$$

By comparing the above result with the total (inertial) decay rate in Eq. (16), we find that

$$\Gamma_{in} = \Gamma_{acc}\,, \tag{30}$$

which means that such a result does not depend on the quantities Γ_{12} and $\widetilde{\Gamma}_{12}$, whose treatment would require additional computational effort [11].

Remarkably, Eq. (30) not only involves a generalization of the analysis of the inverse β-decay to the case in which the emitted neutrino undergoes oscillations, but also corroborates our guess of selecting the processes in Eqs. (27) as the counterpart for the decay (ii) in the inertial frame. Hence, the maintenance of the principle of general covariance results in the necessity of having a thermal bath containing oscillating neutrinos.

5. Conclusions

In this paper, we have extended the study of the inverse β-decay firstly introduced in Refs. [5] and then developed in Refs. [10–12] to the case in which neutrino oscillations are taken into account. On the basis of the requirement of general covariance of QFT, we have shown that the Unruh radiation "seen" by the accelerated proton must necessarily be made up of oscillating neutrinos. This is a novel feature which had been surprisingly

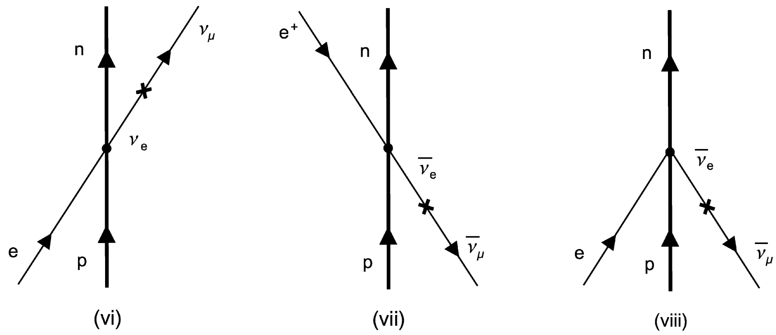

Fig. 4. Decay processes (vi), (vii) and (viii) in the accelerated frame. Oscillations of neutrinos in the Unruh thermal bath are considered in the last two diagrams.

neglected in the previous literature on this topic and that here emerges in a very natural way.

A further interesting observation that can be deduced from our analysis is related to the identities Eqs. (16) and (29) that are true for the inertial and the comoving frame, respectively. For this purpose, we recall that the decay rates appearing in the aforementioned equations have been computed by employing neutrino flavor states as asymptotic states. However, we note that similar relations also hold for the quantities $\Gamma^{(\nu_1)}$ and $\Gamma^{(\nu_2)}$ calculated in Ref. [12] using neutrino mass eigenstates as fundamental objects. We then have

$$\Gamma^{(\nu_1)} + \Gamma^{(\nu_2)} = \Gamma^{(\nu_e)} + \Gamma^{(\nu_\mu)}, \tag{31}$$

where $\Gamma^{(\nu_1)}$ and $\Gamma^{(\nu_2)}$ are inclusive of the elements of Pontecorvo matrix. The above equality has to be regarded both in the inertial and the comoving frames. Such an equation constitutes a consistency check for the correctness of the calculations in Refs. [11] and [12]. The physical meaning of Eq. (31) can be understood by considering the charges for mixed neutrino fields as derived from Noether's theorem [17]. Indeed, by denoting with

$$Q_i = \int d^3x\, \Psi^\dagger_{\nu_i}(x)\Psi_{\nu_i}(x), \quad i = 1, 2, \tag{32}$$

the conserved charges for the neutrino fields with definite masses and with

$$Q_\alpha(t) = \int d^3x\, \Psi^\dagger_{\nu_\alpha}(x)\Psi_{\nu_\alpha}(x), \quad \alpha = e, \mu, \tag{33}$$

the (time-dependent) flavor charges, one can see that $Q = \sum_i Q_i = \sum_\alpha Q_\alpha(t)$, where Q represents the total charge [17]. The above relation can be interpreted as the conservation of the total lepton number. On the one hand, this can be viewed as the sum of two separately conserved family lepton numbers, when no mixing is present; on the other hand, the same conserved number is obtained by the sum of non-conserved flavor charges, which are associated to oscillations.

Apart from its relevance in the context of neutrino mixing and oscillations, we stress that the Unruh effect provides an excellent benchmark for both testing well-established predictions and pointing out novel effects in fundamental physics, as it combines such wide domains as general relativity, quantum field theory and thermodynamics. For instance, in Refs. [18] it has been shown that the Unruh spectrum may exhibit exotic non-thermal corrections even within the standard QFT, thus emphasizing how such a framework represents an active forge of still unexplored scenarios. A similar non-thermal behavior has been obtained in Refs. [19], where Planck scale effects on the Hawking/Unruh bath have been derived in the context of the generalized uncertainty principle [20]. Further features of the Unruh effect may be addressed in connection with possible modification of the oscillation probability formula in accelerated frames [21] and with entanglement properties for accelerated observers, whose implications have been investigated also in the context of black hole physics [22]. In particular, in Ref. [23], it has been proved that entanglement turns out to be an observer-dependent quantity in non-inertial frames due to the Unruh radiation. The question thus arises as to how this setting is modified in the presence of mixed neutrinos, particularly in view of the discussion of Refs. [24]. The entanglement among neutrinos and other products of the decays in which they arise is also relevant in connection with the issue concerning the "ontological" nature of neutrino states – mass or flavor [25]. For more details, see Refs. [26,27].

As a final remark, we stress that in the current work we have made use of the simplest framework of neutrino mixing among two generations. The extension to three flavors is in principle straightforward and represents one of the future directions of our investigation. We envisage that the presence of CP violation may introduce interesting additional features which would enrich the non-trivial structure of the Unruh radiation.

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