Fluctuations and dissipation for a mirror in vacuum

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Abstract. A mirror in vacuum is submitted to a radiation pressure exerted by scattered fields. It is known that the resulting mean force is zero for a motionless mirror, but not for a mirror moving with a non-uniform acceleration. We show here that this force results from a motional modification of the field scattering while being associated with the fluctuations of the radiation pressure on a motionless mirror. We consider the case of a scalar field in a two-dimensional spacetime and characterize the scattering upon the mirror by frequency dependent transmissivity and reflectivity functions obeying unitarity, causality and high frequency transparency conditions. We derive causal expressions for dissipation and fluctuations and exhibit their relation for any stationary input. We recover the known damping force at the limit of a perfect mirror in vacuum. Finally, we interpret the force as a mechanical signature of the squeezing effect associated with the mirror's motion.

1. Introduction

Even in the vacuum state, the electromagnetic field exhibits quantum fluctuations [1] which manifest themselves through the macroscopic Casimir forces [2, 3].

These forces can be understood as resulting from the radiation pressure exerted by the scattered fluctuations and they depend upon the reflection coefficients which characterize the boundaries. Assuming that the boundaries are transparent at high frequencies, which is certainly the case for any real mirrors, one obtains expressions free from the divergences usually associated with the infiniteness of the vacuum energy [4].

In this formulation of the Casimir effect, the force is related to the vacuum stress tensor evaluated on the boundaries and is itself a fluctuating quantity. As illustrated by the Langevin theory of Brownian motion [5], any fluctuating force has a long term cumulative effect. Here a motional force for a mirror in vacuum can be deduced from linear response theory [6] and it is connected to the fluctuations through some 'fluctuation–dissipation relations'. A force has been derived for a perfectly reflecting mirror moving in a two-dimensional (2D) spacetime [7, 8]; it is dissipative and

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proportional to the third time derivative of the mirror's position q (in a linear approximation with respect to q):

$$F = \hbar q''' / (6\pi c^2) \tag{1}$$

(from now on, we use natural units where $c = 1$; however, we keep $\hbar$ as a scale for vacuum fluctuations). This force results from a motional modification of the vacuum stress tensor and is connected to the Casimir forces. Actually, both effects are present when the motion of two mirrors is studied [8–10]. However, expression (1) of the force does not exhibit the causal properties which are expected from the linear response theory.

A related effect has been studied in great detail since it limits the sensitivity of the interferometers designed for gravitational wave detection [11–14]. When irradiated by a laser wave, a mirror undergoes a fluctuating radiation pressure [15] as well as a damping force proportional to its velocity and to the laser intensity [16]. However, the discussion of these effects has not taken into account the fact that they remain at the limit of a null laser intensity; the radiation pressure fluctuates also in the vacuum and this causes an extra mirror's damping.

In the present paper, we study the simplest case where a point-like mirror is placed in the stationary state of a scalar field in a 2D spacetime (with vacuum as a particular case). The field scattering upon the mirror is characterized by frequency dependent transmissivity and reflectivity functions obeying unitarity, causality and high frequency transparency conditions.

First, we derive the radiation pressure exerted upon a motionless mirror. Then, we study the motional modification of the field scattering (at first order in the mirror's displacement) and obtain a causal expression for the motional force. We exhibit the relation connecting this force with the fluctuations of the radiation pressure computed for a motionless mirror. These results are demonstrated for any stationary state of the input fields. At the end of the paper, we give the particular expressions for the vacuum state. Equation (1) is reproduced at frequencies well below the reflection cutoff. Finally, the motional force is connected with the squeezing of vacuum field, as put into evidence by the expression of the effective Hamiltonian describing the mirror's motion in the linear approximation.

2. Notations

In a 2D spacetime (time coordinate t, space coordinate x), a free scalar field is the sum of two counterpropagating components $(\hat{\varphi}(t - x) + \hat{\psi}(t + x))$; we will write these two components in a column matrix:

$$\hat{\Phi}_x(t) = \begin{pmatrix} \hat{\varphi}(t - x) \\ \hat{\psi}(t + x) \end{pmatrix}.$$

We will consider that any function $\hat{f}$ defined in the time domain and its Fourier transform f are related through:

$$\hat{f}(t) = \int \frac{\mathrm{d}\omega}{2\pi} f(\omega) \mathrm{e}^{-\mathrm{i}\omega t}.$$

For example, the Fourier development of the column Φ_x is related to the standard annihilation and creation operators corresponding to the two propagation directions:

$$\Phi_x(\omega) = \begin{pmatrix} \varphi(\omega)e^{i\omega x} \\ \psi(\omega)e^{-i\omega x} \end{pmatrix} = e^{i\eta\omega x}\Phi(\omega)$$

$$\varphi(\omega) = [\hbar/2|\omega|]^{1/2}[\theta(\omega)a_\omega + \theta(-\omega)a_{-\omega}^\dagger]$$

$$\psi(\omega) = [\hbar/2|\omega|]^{1/2}[\theta(\omega)b_\omega + \theta(-\omega)b_{-\omega}^\dagger]$$

$$\eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2}$$

The abbreviated notations $\hat{\Phi}$, Φ are used for the values of $\hat{\Phi}_x$, Φ_x evaluated at $x = 0$. The commutations relations of the Fourier components of the fields are:

$$[\varphi(\omega), \varphi(\omega')] = [\psi(\omega), \psi(\omega')] = 2\pi\delta(\omega + \omega')\hbar/(2\omega)$$

$$[\varphi(\omega), \psi(\omega')] = 0. \tag{3}$$

We will use specific notations for the fields $\bar{\Phi}$ evaluated at the time-dependent mirror position q_t (shortened notation for $\hat{q}(t)$), written as a function of the mirror proper time τ, as well as for its Fourier transform $\bar{\Phi}$:

$$\bar{\Phi}(\tau) = \hat{\Phi}_{q_t}(t) = \{e^{-x\eta\partial_t}\hat{\Phi}(t)\}_{x=q_t} \qquad d\tau = (1 - q_t'^2)^{1/2}\,dt \qquad \bar{\Phi}(\tau) = \int \frac{d\omega}{2\pi}\,\bar{\Phi}(\omega)e^{-i\omega\tau}.$$

As $\bar{\Phi}$ and $\hat{\Phi}$ are related through a phase modulation, there is no simple relation between $\dot{\bar{\Phi}}$ and Φ, except in the particular case of a motionless or uniformly moving mirror. In order to deal with this transformation, we will perform a first-order expansion in a modification δq_t of the mirror's trajectory around $q = 0$:

$$\bar{\Phi}(t) = \hat{\Phi}(t) - \delta q_t \eta \partial_t \hat{\Phi}(t) \tag{4a}$$

(as second-order terms are neglected, the proper time τ and the laboratory time t coincide). Equivalently, in the frequency domain:

$$\bar{\Phi}(\omega) = \int \frac{d\omega'}{2\pi} [2\pi\delta(\omega - \omega') + \delta q(\omega - \omega')\eta i\omega']\Phi(\omega'). \tag{4b}$$

The energy and impulsion densities correspond to two counterpropagating energy fluxes:

$$\hat{e}_x(t) = \hat{\varphi}'(t-x)^2 + \hat{\psi}'(t+x)^2 \qquad \hat{p}_x(t) = \hat{\varphi}'(t-x)^2 - \hat{\psi}'(t+x)^2.$$

Their mean values may be written in terms of the covariance matrix (the elements of this matrix are the two point correlation functions of the fields):

$$\hat{C}_{x,x'}(t, t') = \langle\hat{\Phi}_x(t)\hat{\Phi}_{x'}(t')^T\rangle \qquad \langle\hat{e}_x(t)\rangle = \{\mathrm{Tr}[\partial_t\partial_{t'}\hat{C}_{x,x}(t, t')]\}_{t'=t}$$

$$\langle\hat{p}_x(t)\rangle = \{\mathrm{Tr}[\eta\partial_t\partial_{t'}\hat{C}_{x,x}(t, t')]\}_{t'=t}$$

(Tr stands for the trace operation on square matrices and X^T for the transpose of X). The same expressions written in the frequency domain will be useful, particularly:

$$C_{x,x'}(\omega, \omega') = \langle\Phi_x(\omega)\Phi_{x'}(\omega')^T\rangle = e^{i\eta\omega x}C(\omega, \omega')e^{i\eta\omega'x'}$$

$$\langle\hat{e}_x(t)\rangle = \int \frac{d\omega}{2\pi} \int \frac{d\omega'}{2\pi} e^{-i\omega t - i\omega't}i\omega i\omega'\mathrm{Tr}[C_{x,x}(\omega, \omega')] \tag{5a}$$

$$\langle \hat{p}_x(t) \rangle = \int \frac{d\omega}{2\pi} \int \frac{d\omega'}{2\pi}\, e^{-i\omega t - i\omega' t} i\omega i\omega'\, \mathrm{Tr}[\eta C_{x,x}(\omega, \omega')]. \tag{5b}$$

For a stationary state, the covariance matrices depend only upon one parameter:

$$\hat{C}(t, t') = \hat{c}(t - t') \qquad C(\omega, \omega') = 2\pi\delta(\omega + \omega')c(\omega). \tag{6}$$

We will often write the covariances in terms of the anticommutators (which characterize the various states of the fields) and of the commutators (which do not depend upon the state; see equation (3)):

$$c(\omega) = c_+(\omega) + c_-(\omega) \tag{7a}$$

$$c_+(\omega) = \tfrac{1}{2}[c(\omega) + c(-\omega)^{\mathrm{T}}] = c_+(-\omega)^{\mathrm{T}} \tag{7b}$$

$$c_-(\omega) = \tfrac{1}{2}[c(\omega) - c(-\omega)^{\mathrm{T}}] = I\hbar/(4\omega) \tag{7c}$$

(I is the unit matrix).

3. Scattering upon a motionless mirror

In the limiting case of perfect reflection, the field is constrained to be zero at the mirror's position q so that the input and output fields (see figure 1) are related through:

$$\hat{\psi}_{\mathrm{out}}(t + q) = -\hat{\varphi}_{\mathrm{in}}(t - q) \qquad \hat{\varphi}_{\mathrm{out}}(t - q) = -\hat{\psi}_{\mathrm{in}}(t + q)$$

$$\Phi_{\mathrm{out}}(\omega) = e^{-i\eta\omega q} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} e^{i\eta\omega q} \Phi_{\mathrm{in}}(\omega)$$

For a partly transmitting mirror, the scattering of the field is described by a frequency dependent S-matrix:

$$\Phi_{\mathrm{out}}(\omega) = e^{-i\eta\omega q}\tilde{S}(\omega)e^{i\eta\omega q}\Phi_{\mathrm{in}}(\omega) \tag{8}$$

$$\tilde{S}(\omega) = \begin{pmatrix} \tilde{s}(\omega) & \tilde{r}(\omega) \\ \tilde{r}(\omega) & \tilde{s}(\omega) \end{pmatrix}.$$

For clarity, we denote $\tilde{S}$ the S-matrix in the proper frame (same convention as for the fields). As a consequence of the translational invariance of a stationary state, all the results will be independent of q and we shall suppose from now on that $q = 0$.

The matrix $\tilde{S}$ is supposed to obey the following conditions [4]: it is real in the temporal domain, causal and unitary ($\tilde{S} = \tilde{S}^{\mathrm{T}}$):

$$\tilde{S}(-\omega) = \tilde{S}(\omega)^*$$

$\tilde{S}(\omega)$ is analytic (and regular) for $\mathrm{Im}(\omega) > 0$
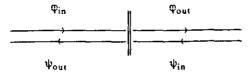

Figure 1. The mirror scatters the two counterpropagating fields.

$$\tilde{S}(\omega)\tilde{S}(\omega)^\dagger = 1.$$

.Finally, the mirror is supposed transparent at high frequencies:

$$\tilde{S}(\omega) \to I \qquad \text{for } \omega \to \infty. \tag{9}$$

This assumption will allow regularization of the ultraviolet divergences associated with the infiniteness of the vacuum energy. It must be noted that the perfect mirror ($s = 0$ and $r = -1$ at all frequencies) does not obey this condition. So, it will be preferable to consider the perfect mirror as the limit of a model obeying the transparency condition (for example a mirror perfectly reflecting at frequencies below a reflection cutoff).

4. Mean radiation pressure upon a motionless mirror

The force $\hat{F}(t)$ may be evaluated as the difference between the radiation pressures exerted upon the left and right sides of the mirror (at rest at $q = 0$). In a 2D spacetime, the component T_{xx} of the stress tensor is equal to the energy density and one gets:

$$\hat{F}(t) = \hat{\varphi}_{\text{in}}'^2(t) + \hat{\psi}_{\text{out}}'^2(t) - \hat{\varphi}_{\text{out}}'^2(t) - \hat{\psi}_{\text{in}}'^2(t).$$

This force can also be considered as the difference between the impulsion densities of the input and output fields evaluated at the mirror's position:

$$\hat{F}(t) = \hat{p}_{\text{in}}(t) - \hat{p}_{\text{out}}(t). \tag{10}$$

For a perfect mirror, the force is twice the impulsion density which would exist at the location of the mirror in its absence [9]. The mean value of this force is zero in the vacuum state. However, we shall see later on that the instantaneous radiation pressure has irreducible quantum fluctuations.

For a partly transmitting mirror, the force $\hat{F}(t)$ is still given by the difference (10) between the input and output impulsion densities but we have now to evaluate the output fields by using the input–output relation (8):

$$C_{\text{out}}(\omega, \omega') = \tilde{S}(\omega)C_{\text{in}}(\omega, \omega')\tilde{S}(\omega').$$

One gets an expression of the force having the same form as equation (5):

$$\langle \hat{F}(t) \rangle = \int \frac{\mathrm{d}\omega}{2\pi} \int \frac{\mathrm{d}\omega'}{2\pi} e^{-i\omega t - i\omega' t} i\omega i\omega' \text{Tr}[\mathscr{F}(\omega, \omega')C_{\text{in}}(\omega, \omega')]$$

where $\mathscr{F}$ is a square matrix:

$$\mathscr{F}(\omega, \omega') = \eta - \tilde{S}(\omega')\eta\tilde{S}(\omega) = \begin{pmatrix} \alpha(\omega, \omega') & \beta(\omega, \omega') \\ -\beta(\omega, \omega') & -\alpha(\omega, \omega') \end{pmatrix}$$

$$\alpha(\omega, \omega') = 1 - \tilde{s}(\omega)\tilde{s}(\omega') + \tilde{r}(\omega)\tilde{r}(\omega')$$

$$\beta(\omega, \omega') = \tilde{s}(\omega)\tilde{r}(\omega') - \tilde{r}(\omega)\tilde{s}(\omega'). \tag{11}$$

The matrix $\mathscr{F}$ obeys the following properties which will be used thereafter:

$$\mathscr{F}(\omega, \omega')^{\text{T}} = \mathscr{F}(\omega', \omega) = \eta\mathscr{F}(\omega, \omega')\eta \tag{12a}$$

$$\mathscr{F}(\omega, \omega')^* = \mathscr{F}(-\omega', -\omega) \tag{12b}$$

$$\mathcal{F}(\omega, \omega')\mathcal{F}(\omega, \omega')^\dagger = \mathcal{F}(\omega, \omega')\eta + \eta\mathcal{F}(\omega, \omega')^\dagger \qquad (13a)$$

$$\mathcal{F}(\omega, \omega')^\dagger\mathcal{F}(\omega, \omega') = \eta\mathcal{F}(\omega, \omega') + \mathcal{F}(\omega, \omega')^\dagger\eta. \qquad (13b)$$

Using equations (7), the force is written in terms of the field anticommutators:

$$\langle\hat{F}(t)\rangle = \int \frac{d\omega}{2\pi} \int \frac{d\omega'}{2\pi} e^{-i\omega t - i\omega' t'} i\omega i\omega' \mathrm{Tr}[\mathcal{F}(\omega, \omega')C_{+,\mathrm{in}}(\omega, \omega')] \qquad (14)$$

The force may be written in the temporal domain:

$$\langle\hat{F}(t)\rangle = \{\partial_t\partial_{t'} \mathrm{Tr}[\eta\hat{C}_{\mathrm{in}}(t, t') - \eta\hat{C}_{\mathrm{out}}(t, t')]\}_{t'=t}$$

with:

$$\mathrm{Tr}[\eta\hat{C}_{\mathrm{in}}(t, t') - \eta\hat{C}_{\mathrm{out}}(t, t')] = \int dt'' \int dt''' \mathrm{Tr}[\hat{\mathcal{F}}(t'', t''')\hat{C}_{\mathrm{in}}(t - t'', t' - t''')]$$

$$\hat{\mathcal{F}}(t, t') = \delta(t')\eta\delta(t) - \bar{S}(t')\eta\bar{S}(t). \qquad (15)$$

It clearly appears in these expressions that the force exerted upon the mirror is a retarded function of the input stress tensor ($\bar{S}$ is a causal function and $\hat{\mathcal{F}}(t, t')$ is zero as soon as $t < 0$ or $t' < 0$).

The expression (14) provides the mean force for any input state. For a stationary input (see equation (6)), one gets a simpler expression:

$$\langle\hat{F}\rangle = \int \frac{d\omega}{2\pi} \omega^2 \mathrm{Tr}[\mathcal{F}(\omega, -\omega)c_{+,\mathrm{in}}(\omega)].$$

One can evaluate also the energy exchange between the field and the mirror; it is the difference between the energy densities of the input and output fields:

$$\hat{G}(t) = \hat{e}_{\mathrm{in}}(t) - \hat{e}_{\mathrm{out}}(t).$$

One finds that $\hat{G}$ is given by equation (14) with $\mathcal{F}$ replaced by:

$$\mathcal{G}(\omega, \omega') = I - \bar{S}(\omega')\bar{S}(\omega).$$

As $\bar{S}$ is unitary:

$$\mathcal{G}(\omega, -\omega) = 0.$$

Therefore, the energy exchange is zero in any stationary state:

$$\langle\hat{G}\rangle = \int \frac{d\omega}{2\pi} \omega^2 \mathrm{Tr}[\mathcal{G}(\omega, -\omega)c_{\mathrm{in}}(\omega)] = 0. \qquad (16)$$

5. Scattering upon a moving mirror

At perfect reflection, the field is still zero on a moving mirror [9] and the input–output relations have a simple form for the fields evaluated along the mirror's trajectory:

$$\hat{\psi}_{\mathrm{out}}(t + q_t) = -\hat{\varphi}_{\mathrm{in}}(t - q_t) \qquad \hat{\varphi}_{\mathrm{out}}(t - q_t) = -\hat{\psi}_{\mathrm{in}}(t + q_t)$$

$$\hat{\psi}_{\mathrm{out}}(\tau) = -\hat{\varphi}_{\mathrm{in}}(\tau) \qquad \hat{\varphi}_{\mathrm{out}}(\tau) = -\hat{\psi}_{\mathrm{in}}(\tau)$$

$$\bar{\Phi}_{\mathrm{out}}(\omega) = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \bar{\Phi}_{\mathrm{in}}(\omega).$$

These relations describe the Doppler shift associated with the mirror's motion (Lorenz transformation for the frequencies) and the dilatation of the derived fields $\hat{\varphi}'$ and $\hat{\psi}'$ (Lorentz transformation for the fields) [9].

For a partly transmitting mirror, the S-matrix (defined previously for a motionless mirror) describes the scattering of the field evaluated along the trajectory†:

$$\tilde{\Phi}_{\text{out}}(\omega) = \tilde{S}(\omega)\tilde{\Phi}_{\text{in}}(\omega).$$

The matrix S is deduced in a first order expansion in a mirror's displacement δq_t by using the transformation (4):

$$\Phi_{\text{out}}(\omega) = \int \frac{d\omega'}{2\pi} [2\pi\delta(\omega - \omega')\tilde{S}(\omega) + \delta S(\omega, \omega')]\Phi_{\text{in}}(\omega')$$

$$\delta S(\omega, \omega') = i\omega' \delta q(\omega - \omega')[\tilde{S}(\omega)\eta - \eta\tilde{S}(\omega')]. \tag{17}$$

6. Force exerted upon a moving mirror

Taking the mirror's motion into account, the force can be written as:

$$\langle \hat{F}(t) \rangle = \langle \hat{p}_{q_t, \text{in}}(t) - \hat{p}_{q_t, \text{out}}(t) \rangle - q_t' \langle \hat{e}_{q_t, \text{in}}(t) - \hat{e}_{q_t, \text{out}}(t) \rangle$$

where the densities are evaluated at the mirror's position. In a first-order expansion in δq_t, it becomes:

$$\langle \hat{F}(t) \rangle = \langle \hat{p}_{\text{in}}(t) - \hat{p}_{\text{out}}(t) \rangle - \delta q_t \partial_t \langle \hat{G}(t) \rangle - \delta q_t' \langle \hat{G}(t) \rangle$$

$$\langle \hat{G}(t) \rangle = \langle \hat{e}_{\text{in}}(t) - \hat{e}_{\text{out}}(t) \rangle$$

where the densities are evaluated at $x = 0$, the second term represents the variation of the impulsion densities in a translation (for a free field $\partial_x p + \partial_t e = 0$) and the third term is the correction proportional to energy densities and to the mirror's velocity. The energy modification $\langle \hat{G} \rangle$ has to be evaluated at the zeroth order. From now on, we will restrict ourselves to stationary inputs, in which case $\langle \hat{G} \rangle$ is zero (see equation (16)). The two corrections associated with it will be forgotten in the expression of $\langle \hat{F} \rangle$.

We will eventually compute the mean force as:

$$\langle \delta\hat{F}(t) \rangle = -\langle \delta\hat{p}_{\text{out}}(t) \rangle = -\{\partial_t \partial_{t'} \text{Tr}[\eta \delta \hat{C}_{\text{out}}(t, t')]\}_{t=t'}$$

where the variations of the output fields are due to the modifications δS of the S-matrix in the laboratory:

$$\langle \delta\hat{F}(t) \rangle = \int \frac{d\omega}{2\pi} \int \frac{d\omega'}{2\pi} e^{-i\omega t - i\omega' t'} \omega\omega' \text{Tr}[\eta \delta C_{\text{out}}(\omega, \omega')]$$

† This assumption could be justified by considering a conformal transformation from the laboratory to a 'proper frame' [8] which preserves the two counterpropagating components and is chosen so that the mirror is at rest and the time is the mirror's proper time in the proper frame.

$$\delta C_{\text{out}}(\omega, \omega') = \int \frac{d\omega''}{2\pi} [\delta S(\omega, \omega'') C_{\text{in}}(\omega'', \omega') \tilde{S}(\omega')^{\text{T}} + \tilde{S}(\omega) C_{\text{in}}(\omega, \omega'') \delta S(\omega', \omega'')^{\text{T}}]$$

that is, for a stationary input (see equations (6) and (17)):

$$\delta C_{\text{out}}(\omega, \omega') = -i\omega' \delta q(\omega + \omega')[\tilde{S}(\omega)\eta - \eta\tilde{S}(-\omega')] c_{\text{in}}(-\omega') \tilde{S}(\omega')$$
$$-i\omega \delta q(\omega + \omega') \tilde{S}(\omega) c_{\text{in}}(\omega) [\eta\tilde{S}(\omega') - \tilde{S}(-\omega)\eta]. \tag{18}$$

Using the unitarity of $\tilde{S}$, one obtains:

$$\text{Tr}[\eta \delta C_{\text{out}}(\omega, \omega')] = \delta q(\omega + \omega')\text{Tr}[\mathcal{F}(\omega, \omega')[i\omega c_{\text{in}}(\omega)\eta + i\omega' \eta c_{\text{in}}(-\omega')]].$$

We will write the motional force as a symmetric integral over the two frequencies:

$$\langle \delta \hat{F}(t) \rangle = \int \frac{d\omega}{2\pi} \int \frac{d\omega'}{2\pi} e^{-i\omega t - i\omega' t} \chi(\omega, \omega') \delta q(\omega + \omega')$$

$$\chi(\omega, \omega') = \tfrac{1}{2}\omega\omega' \text{Tr}[\mathcal{F}(\omega, \omega')[i\omega c_{\text{in}}(\omega)\eta + i\omega' \eta c_{\text{in}}(-\omega')]]$$
$$+ \tfrac{1}{2}\omega\omega' \text{Tr}[\mathcal{F}(\omega', \omega)[i\omega' c_{\text{in}}(\omega')\eta + i\omega \eta c_{\text{in}}(-\omega)]].$$

Transposing the matrices inside the second line and using the properties (12), one transforms $\chi(\omega, \omega')$ into a function of the anticommutators (see equation (7)):

$$\chi(\omega, \omega') = \omega\omega' \text{Tr}[\mathcal{F}(\omega, \omega')[i\omega c_{+,\text{in}}(\omega)\eta + i\omega' \eta c_{+,\text{in}}(-\omega')]]$$
$$= \omega\omega' \text{Tr}[\mathcal{F}(\omega, \omega') i\omega c_{+,\text{in}}(\omega)\eta + \mathcal{F}(\omega', \omega) i\omega' c_{+,\text{in}}(\omega')\eta]. \tag{19}$$

Finally, the force appears as a linear response to the mirror's motion:

$$\langle \delta F(\omega) \rangle = \chi(\omega) \delta q(\omega) \tag{20}$$

with a susceptibility given by:

$$\chi(\omega) = \int \frac{d\omega'}{2\pi} \chi(\omega', \omega - \omega'). \tag{21}$$

These expressions generalize the motional force (1) known for a perfect mirror in the vacuum to the case of a partly transmitting mirror in an arbitrary stationary input field. Later on, we shall see that the force (1) is recovered as an approximate result.

We have shown that the motional force is a consequence of the transformation of the fields by the moving mirror. Actually, this transformation is a squeezing effect (the particular case where the input state is the vacuum is discussed later on). In order to squeeze the field, the mirror has to exchange energy with it. A motional force thus appears as the signature of the squeezing effect.

6. Interpretation of the motional force in the comoving frame

It is instructive to compute the force in the comoving frame:

$$\langle \bar{F}(\tau) \rangle = \langle \bar{p}_{\text{in}}(\tau) - \bar{p}_{\text{out}}(\tau) \rangle = \{\partial_\tau \partial_{\tau'} \text{Tr}[\eta \bar{C}_{\text{in}}(\tau, \tau') - \eta \bar{C}_{\text{out}}(\tau, \tau')]\}_{\tau = \tau'}.$$

This expression differs from the force computed in the laboratory but the corrections are seen to depend upon the energy exchange $\langle \hat{G} \rangle$ and they can be forgotten (see the

previous discussion). The mean force is the same in the laboratory or in the comoving frame in a first-order expansion in δq_t.

In the comoving frame, one can use the expression of the force computed for a motionless mirror but the input stress tensor has to be modified because of the mirror's motion:

$$\langle \delta \bar{F}(t) \rangle = \int \frac{\mathrm{d}\omega}{2\pi} \int \frac{\mathrm{d}\omega'}{2\pi} \, \mathrm{e}^{-\mathrm{i}\omega t - \mathrm{i}\omega' t} \mathrm{i}\omega \mathrm{i}\omega' \mathrm{Tr}[\mathscr{F}(\omega, \omega') \delta \hat{C}_{\mathrm{in}}(\omega, \omega')].$$

The apparent stress tensor is obtained from the transformation (4) of the fields:

$$\delta \hat{C}_{\mathrm{in}}(t, t') = -\delta q_t \eta \partial_t \hat{C}_{\mathrm{in}}(t, t') - \partial_{t'} \hat{C}_{\mathrm{in}}(t, t') \eta \delta q_{t'}$$

that is, for a stationary input:

$$\delta \hat{C}_{\mathrm{in}}(\omega, \omega') = \delta q(\omega + \omega')[-\mathrm{i}\omega c_{\mathrm{in}}(\omega)\eta - \mathrm{i}\omega' \eta c_{\mathrm{in}}(-\omega')]. \tag{22}$$

This gives exactly the same force as previously.

So the force exerted upon a moving mirror can be computed in the laboratory by considering that the S-matrix is modified (see equation (17)) or in the comoving frame by considering that the input stress tensor is modified (see equation (22)). It clearly appears in the comoving frame that the force is a causal function of the mirror's trajectory (see equation (15)). We will discuss this point more precisely in the particular case of a vacuum input.

7. Fluctuations of the radiation pressure upon a motionless mirror

The appearance of a force for a moving mirror can actually be guessed by inspecting the situation where the mirror is at rest. Indeed, we shall now exhibit the quantitative relation between the motional force and the noise spectrum of the force computed for a motionless mirror. This connection can be considered as a 'fluctuation–dissipation' theorem for a mirror which scatters field fluctuations. We will thus check in this context that the motional force can be deduced from linear response theory.

As a qualitative introduction to the problem of force fluctuations, we consider the expression (10) which relates the force and the impulsion densities $\hat{p}_{\mathrm{in}}$ and $\hat{p}_{\mathrm{out}}$ of the input and output fields. We see that the instantaneous impulsion density has irreducible quantum fluctuations. For example the two counterpropagating energy fluxes are statistically independent random variables in the vacuum state and the force fluctuations do not vanish.

We come now to a quantitative evaluation of the correlation function $\hat{C}_{\mathrm{FF}}$ of the force exerted upon a partly transmitting mirror:

$$\hat{C}_{\mathrm{FF}}(t) = \langle \hat{F}(t)\hat{F}(0) \rangle - \langle \hat{F} \rangle^2.$$

Using the input–output relation (8), one obtains an operatorial expression of the force analogous to equations (14):

$$\hat{F}(t) = \int \frac{\mathrm{d}\omega}{2\pi} \int \frac{\mathrm{d}\omega'}{2\pi} \, \mathrm{e}^{-\mathrm{i}\omega t - \mathrm{i}\omega' t} \mathrm{i}\omega \mathrm{i}\omega' \mathrm{Tr}[\mathscr{F}(\omega, \omega') \Phi_{\mathrm{in}}(\omega) \Phi_{\mathrm{in}}(\omega')^{\mathrm{T}}]. \tag{23}$$

It follows that $\hat{C}_{\mathrm{FF}}$ depends upon four-points correlation functions of the fields. Inserting the expressions (2) of the input fields in terms of the annihilation and

creation operators, we compute the four-point functions (Φ_α stands for a component φ or ψ of the input field; we suppose that the input field is in a stationary Gaussian state; this is the case for the vacuum (this property is equivalent to the Wick rules [17])):

$$\langle\Phi_\alpha(\omega)\Phi_\beta(\omega')\Phi_{\alpha'}(\omega'')\Phi_{\beta'}(\omega''')\rangle - \langle\Phi_\alpha(\omega)\Phi_\beta(\omega')\rangle\langle\Phi_{\alpha'}(\omega'')\Phi_{\beta'}(\omega''')\rangle$$

$$= 2\pi\delta(\omega+\omega'')c_{\alpha\alpha'}(\omega)2\pi\delta(\omega'+\omega''')c_{\beta\beta'}(\omega')$$

$$+ 2\pi\delta(\omega+\omega''')c_{\alpha\beta'}(\omega)2\pi\delta(\omega'+\omega'')c_{\beta\alpha'}(\omega').$$

The autocorrelation function of the force thus comes out as (using the properties (12)):

$$\hat{C}_{FF}(t) = \int\frac{d\omega}{2\pi}\int\frac{d\omega'}{2\pi}e^{-i\omega t-i\omega' t}C_{FF}(\omega,\omega') \tag{24}$$

$$C_{FF}(\omega,\omega') = 2\omega^2\omega'^2\operatorname{Tr}[\mathcal{F}(\omega,\omega')c_{in}(\omega)\mathcal{F}(\omega,\omega')^\dagger c_{in}(\omega')^\mathrm{T}]. \tag{25}$$

It appears that C_{FF} is a symmetric function of the two frequencies:

$$C_{FF}(\omega,\omega') = C_{FF}(\omega',\omega).$$

One obtains the noise spectrum of the force as the Fourier transform of the autocorrelation function (24):

$$C_{FF}(\omega) = \int\frac{d\omega'}{2\pi}C_{FF}(\omega',\omega-\omega'). \tag{26}$$

The explicit evaluation of this noise spectrum for a mirror in the vacuum will be given later on.

8. Commutator of the force operator

In order to exhibit the fluctuation–dissipation relations, we have to compute the mean value of the commutator of the force operator:

$$\hat{\xi}_{FF}(t) = (2\hbar)^{-1}[\hat{C}_{FF}(t) - \hat{C}_{FF}(-t)].$$

We write it:

$$\hat{\xi}_{FF}(t) = \int\frac{d\omega}{2\pi}\int\frac{d\omega'}{2\pi}e^{-i\omega t-i\omega' t}\xi_{FF}(\omega,\omega') \tag{27}$$

$$\xi_{FF}(\omega,\omega') = (2\hbar)^{-1}[C_{FF}(\omega,\omega') - C_{FF}(-\omega',-\omega)] \tag{28}$$

Using the properties (12), one transforms this expression into:

$$\xi_{FF}(\omega,\omega') = \hbar^{-1}\omega^2\omega'^2\operatorname{Tr}[\mathcal{F}(\omega,\omega')c_{in}(\omega)\mathcal{F}(\omega,\omega')^\dagger c_{in}(\omega')^\mathrm{T}]$$

$$- \hbar^{-1}\omega^2\omega'^2\operatorname{Tr}[\mathcal{F}(\omega,\omega')c_{in}(-\omega)^\mathrm{T}\mathcal{F}(\omega,\omega')^\dagger c_{in}(-\omega')].$$

One then writes the covariances in terms of anticommutators and commutators (see equation (7)):

$$\xi_{FF}(\omega,\omega') = \tfrac{1}{2}\omega\omega'\operatorname{Tr}[\mathcal{F}(\omega,\omega')\omega c_{+,in}(\omega)\mathcal{F}(\omega,\omega')^\dagger$$

$$+ \mathcal{F}(\omega,\omega')\mathcal{F}(\omega,\omega')^\dagger\omega' c_{+,in}(\omega')^\mathrm{T}].$$

We could have derived this expression directly from the field commutators. Therefore, it is correct even when the input fields are not Gaussian variables.

Finally, one uses the property (13) to obtain (compare with the expression 19):

$$\xi_{FF}(\omega, \omega') = (2i)^{-1}[\chi(\omega, \omega') - \chi(-\omega, -\omega')]. \tag{29}$$

A comparison with equation (28) shows that there is a connection between the force fluctuations computed for a motionless mirror and the mean force computed for a moving mirror.

9. Fluctuation–dissipation relation

Defining the Fourier transform of the force commutator (27):

$$\xi_{FF}(\omega) = \int \frac{d\omega'}{2\pi} \xi_{FF}(\omega', \omega - \omega') \tag{30}$$

one deduces from equations (28) and (29):

$$\xi_{FF}(\omega) = (2\hbar)^{-1}[C_{FF}(\omega) - C_{FF}(-\omega)] = (2i)^{-1}[\chi(\omega) - \chi(-\omega)]. \tag{31}$$

This constitutes the fluctuation–dissipation relation for a mirror submitted to the radiation pressure of scattered field fluctuations.

This establishes that the motional force can be derived from the correlation functions using the linear response theory. Considering that a classical modification δq_t of the mirror's trajectory corresponds to an effective perturbation of the Hamiltonian:

$$\delta \hat{H}(t) = -\hat{F}(t)\delta q_t \tag{32}$$

where $\hat{F}(t)$ is the force operator, we get the motional force (20) by using the formulae of linear response theory given in the appendix. The susceptibility $\chi(\omega)$ is actually the retarded response function while the function $\xi_{FF}(\omega)$ is the spectral density associated with the effective perturbation (32).

The fluctuation–dissipation relation (31) provides the spectral density ξ_{FF} as soon as the noise spectrum or the susceptibility is known. Using the analytic properties of the response functions, the latter can therefore be deduced from the noise spectrum. The force for a moving mirror can always be guessed by inspecting the fluctuations of the force upon a motionless mirror.

The converse is not true in general. However, if the input state corresponds to a thermal equilibrium, the anticommutator of the force (the fluctuations) can be obtained from the commutator (the susceptibility). The vacuum state is the equilibrium state at zero temperature and the noise spectrum C_{FF} can effectively be deduced in this case from the spectral density ξ_{FF}, as we see now.

10. The case of vacuum fluctuations

The covariance matrix corresponding to the vacuum state is easily derived from the expressions (2) of the fields in terms of the annihilation and creation operators:

$$c_{vac}(\omega) = I\theta(\omega)\hbar/(2\omega) \qquad c_{+, vac}(\omega) = I\hbar/(4|\omega|).$$

This covariance matrix is scalar so that the mean radiation pressure (14) is zero (as expected).

The function χ (see equation (19)) corresponding to the vacuum is:

$$\chi_{\text{vac}}(\omega, \omega') = \tfrac{1}{2}i\hbar\omega\omega'[\varepsilon(\omega) + \varepsilon(\omega')]a(\omega, \omega')$$

where ε is the sign function:

$$\varepsilon(\omega) = \theta(\omega) - \theta(-\omega)$$

and a the diagonal term of the matrix $\mathcal{F}$ (see equation (11)). It follows that the susceptibility for a mirror which scatters the vacuum fluctuations may be written (see equation (21)):

$$\chi_{\text{vac}}(\omega) = i\hbar \int_0^\omega \frac{d\omega'}{2\pi}\, \omega'(\omega-\omega')a(\omega', \omega-\omega'). \tag{33}$$

If the frequency ω is such that $s = 0$ and $r = -1$ between 0 and ω, a may be replaced by 2 in equation (33) and the response function reduces to:

$$\chi_{\text{vac}}(\omega) = i\hbar\omega^3/(6\pi). \tag{34}$$

This means that the damping force in the vacuum can be approximated by equation (1), in the limiting case of a perfect mirror, at frequencies below the reflection cutoff. In other words, the coarse grained force (force averaged over a time longer than the reflection delay) is proportional to q'''. However, it has to be emphasized that the exact expression (33) is causal which is not the case for the approximated one (34). Regularized and causal expressions are obtained only when considering partly transmitting mirrors [4].

The force (1) is identical to the linear approximation (first-order expansion in the mirror displacement δq) of the non-linear expression obtained by Fulling and Davies for a perfectly reflecting mirror [8]. It is worth noting that it is also the non-relativistic limit ($q' \ll c = 1$) of this expression. This suggests that the domain of validity of the first-order expansion corresponds to a non-relativistic mirror velocity.

The vacuum fields may be considered as Gaussian random variables [17] so that we can effectively compute the force correlations from the field covariance matrix. One gets from equations (25) and (13):

$$C_{\text{FFvac}}(\omega, \omega') = \hbar^2\theta(\omega)\theta(\omega')\omega\omega'[a(\omega, \omega') + a(\omega, \omega')^*].$$

One then deduces the noise spectrum of the force:

$$C_{\text{FF}}(\omega) = 2\hbar\theta(\omega)\xi_{\text{FF}}(\omega) \tag{35a}$$

$$\xi_{\text{FF}}(\omega) = (2i)^{-1}[\chi_{\text{vac}}(\omega) - \chi_{\text{vac}}(-\omega)]. \tag{35b}$$

Clearly, the fluctuation–dissipation relation is obeyed in the vacuum state. Simple results are obtained when the reflection is perfect at frequencies between 0 and ω:

$$C_{\text{FF}}(\omega) = \hbar^2/(3\pi)\theta(\omega)\omega^3 \tag{36}$$

$$\xi_{\text{FF}}(\omega) = \hbar/(6\pi)\omega^3. \tag{37}$$

The connection between the variation of C_{FF} as ω^3 and the variation of the damping force as q''' is a manifestation of the fluctuation–dissipation relation. As already noted, the noise spectrum (35) is more regular at high frequencies than the approximation (36) as a consequence of the transparency condition (9).

Two properties of the noise spectrum (35) have to be emphasized. First, C_{FF} is zero at the limit of a null frequency, which means that the force fluctuations are averaged to zero when integrated over a long time. Note that the input impulsion density $\hat{p}_{in}$ also vanishes when integrated over a long time.

Second, the relation (35) between the noise spectrum C_{FF} and the spectral density ξ_{FF} implies that the noise spectrum contains only positive frequency components, because the vacuum is the zero temperature state. It follows that the vacuum can damp the mirror's motion but cannot excite it.

11. Connection with squeezing

We have developed a formalism where the scattering is characterized by frequency dependent coefficients. The effect of the mirror's motion is described by a modification of the S-matrix in the laboratory or by a transformation of the input stress tensor in the comoving frame. It can be noted that the scattering formalism has already been used for dealing with vacuum fluctuations in accelerated frames or in curved space [18–23].

In this scattering approach, the damping force for a mirror in the vacuum appears as connected to squeezing [24, 25]. In order to put this point into evidence, we write the secular part (component at zero frequency) of the effective Hamiltonian (32) as follows (see the operatorial expression (23) of the force):

$$\delta H(0) = \int dt \delta \hat{H}(t)$$

$$= \int \frac{d\omega}{2\pi} \int \frac{d\omega'}{2\pi} \delta q(-\omega - \omega') \omega \omega' \, \mathrm{Tr}[\mathscr{F}(\omega, \omega') \Phi_{in}(\omega) \Phi_{in}(\omega')^{T}].$$

Considering as an example the case where the mirror oscillates at a fixed frequency $2\omega_0$:

$$\delta q_t = \delta q_0 \cos(2\omega_0 t)$$

one recognizes an effective Hamiltonian giving rise to a squeezing effect [1].

Actually, the motional force constitutes a mechanical consequence of the squeezing. As the input field is the vacuum state, the mirror has to give energy to the field in order to squeeze it and this damps its motion.

In the laboratory frame, the modification of the output covariance matrix which results from the effective Hamiltonian is given by equation (18):

$$\delta C_{out}(\omega, \omega') = \tfrac{1}{2} i\hbar \delta q(\omega + \omega')[\theta(\omega) - \theta(-\omega')]\mathscr{F}(\omega', \omega).$$

For the oscillating mirror, this is non-zero only if the two frequencies ω and ω' have the same sign and if their sum is $\pm 2\omega_0$. It follows that the squeezing effect vanishes when the oscillation frequency goes to zero. A mirror moving slowly in the vacuum does not appreciably squeeze it and the motional force (1) is very small at low frequencies.

The connection of the motional force with squeezing can also be analysed in the comoving frame. Now, the apparent input state (22) is itself squeezed when the input is the vacuum in the laboratory frame. The motional force thus appears as a

mechanical manifestation of the fact that the mirror scatters (with the unmodified
S-matrix) squeezed field fluctuations.

12. Conclusion

We have given explicit expressions of the correlation functions and of the motional
force experienced by a mirror in the vacuum state. When the mirror is irradiated by a
coherent wave, the same method leads to a mean radiation pressure and to extra
fluctuations [15]. It also provides an extra damping force, proportional to the mirror's
velocity and to the coherent field intensity [16]. These two results are related through
the fluctuation–dissipation theorem which would hold also for a mirror in a thermal
field.

   We have only considered in this paper the one mirror problem. The situation
where two mirrors scatter the same field fluctuations seems attractive. The mean
Casimir force is well known for motionless mirrors. It is expected to be modified when
the mirrors are moving [7, 8]. It must also exhibit fluctuations [26]. These fluctuations
have to be connected with the motional effect in the same manner as in the one mirror
problem. Finally, the motional dependence of the Casimir effect is associated to the
squeezing effect due to the mirrors' motion. The formalism developed in the present
paper will be applied to the two mirrors problem in a forthcoming paper.

Acknowledgments

Appendix. The relations of linear response theory

A classical modification δq_t of the mirror's trajectory corresponds to an effective
perturbation (32) of the Hamiltonian. The linear response theory [6] provides the
variation of the mean force:

$$\langle \delta \hat{F}(t) \rangle = \int dt' \hat{\chi}_{FF}^{R}(t - t') \delta q_{t'}$$

The response function $\hat{\chi}_{FF}^{R}$ (the abbreviated notation $\hat{\chi}$ was used previously) is the
retarded susceptibility; it is related to the force commutator $\hat{\xi}_{FF}$ and to the correlation
function $\hat{C}_{FF}$ (correlation functions are supposed stationary):

$$\hat{\chi}_{FF}^{R}(t) = 2i\theta(t)\hat{\xi}_{FF}(t) \qquad \hat{\xi}_{FF}(t) = (2\hbar)^{-1}[\hat{C}_{FF}(t) - \hat{C}_{FF}(-t)]$$

$$\hat{C}_{FF}(t) = \langle \hat{F}(t)\hat{F}(0) \rangle - \langle \hat{F} \rangle^2.$$

It is also possible to define an advanced response:

$$\hat{\chi}_{FF}^{A}(t) = -2i\theta(-t)\hat{\xi}_{FF}(t) = \hat{\chi}_{FF}^{R}(-t).$$

   These relations have a simple form in the spectral domain:

$$\langle \delta F(\omega) \rangle = \chi_{FF}^{R}(\omega)\delta q(\omega)$$

$$\chi_{FF}^{R}(\omega) = \xi_{FF}(\omega) + i\xi_{FF}(\omega)$$

$$\chi_{FF}^{A}(\omega) = \xi_{FF}(\omega) - i\xi_{FF}(\omega) = \chi_{FF}^{R}(\omega)^* = \chi_{FF}^{R}(-\omega).$$

The dispersive part ξ_{FF} of the susceptibility functions is obtained from the spectral density ξ_{FF} through a dispersion relation:

$$\xi_{FF}(\omega) = \int \frac{d\omega'}{\pi} \mathscr{P} \frac{\xi_{FF}(\omega')}{\omega' - \omega}.$$

The response function χ_{FF}^{R} (respectively χ_{FF}^{A}) is analytic (and regular) in the upper half plane $\text{Im}(\omega) > 0$ (respectively in the lower half plane $\text{Im}(\omega) < 0$). As $\hat{F}$ is a Hermitean operator, ξ_{FF} is a real and odd function of ω while ξ_{FF} is a real and even function of ω.

The general form of the relation between the noise spectrum C_{FF} and the retarded susceptibility χ_{FF}^{R} is:

$$\xi_{FF}(t) = (2\hbar)^{-1}[\hat{C}_{FF}(t) - \hat{C}_{FF}(-t)] = (2i)^{-1}[\hat{\chi}_{FF}^{R}(t) - \hat{\chi}_{FF}^{R}(-t)]$$

which leads to equation (31) in the frequency domain:

$$\xi_{FF}(\omega) = (2\hbar)^{-1}[C_{FF}(\omega) - C_{FF}(-\omega)] = (2i)^{-1}[\chi_{FF}^{R}(\omega) - \chi_{FF}^{R}(-\omega)].$$

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