Remarks on: 'Theory of Neutrino Oscillations' (hep-ph/0311241) by C.Giunti, the comments by L.B.Okun et al in hep-ph/0312151, and Giunti's reply in hep-ph/0312180

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Abstract

As pointed out previously, some of the hypotheses used in the standard discussion of the quantum mechanics neutrino oscillations, recently summarised in the paper 'Theory of Neutrino Oscillations' by C.Giunti, lack any physical foundation and lead to neglect of both a factor of two correction in the contribution of neutrino propagation to the oscillation phase and an important contribution to the latter from the propagator of the source particle.

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C.Giunti has performed a valuable service in making a clear and concise statement in Ref. [1] (TNO) of the basic physical assumptions underlying the standard quantum mechanical treatment of neutrino oscillations. Of particular importance are the assumptions A2 and A3 of TNO which are the essential ones in order to derive the standard result for the oscillation phase. However, A2, as written, is either factually incorrect, or, on the most generous interpretation highly likely to be misinterpreted. Indeed, it is the incorrect interpretation that is used in the standard derivation. In fact the initial and final states in the amplitudes of any standard model process are, uniquely, the mass eigenstates $|\nu_k\rangle$ not the 'neutrino flavour eigenstate' $|\nu_\alpha\rangle$ defined in Eqn(1) of TNO. Since the MNS matrix element, $U_{\alpha k}$, necessarily appears in the amplitude when the state $|\nu_k\rangle$ is created or destroyed, in association with a charged lepton with label $\alpha = e, \mu, \tau$, one can introduce, formally, the state $|\nu_\alpha\rangle$ in order to write, in a compact way, a general expression for the charged weak current or the corresponding Lagrangian, always bearing in mind, however, that it has no actual physical significance, as the state does not appear in the amplitude of any standard model process. Now what is done in the standard derivation is to assume that this formal, physically non-existent state <u>is actually created by the neutrino production process at some fixed time</u>. Since the detection process evidently occurs at another fixed time, the implicit assumption is made that all the mass eigenstates have the same velocity, which is just what is explicitly stated in A3, so that two assumptions are certainly consistent with each other. Two remarks follow. Firstly the state $|\nu_\alpha\rangle$ is <u>not</u> produced, because there is no physical reason why it should be. As already stated above only the mass eigenstates $|\nu_k\rangle$ appear in physical amplitudes as initial or final states.. As I have shown in a recent paper [2], assuming that such states are created in pion decay gives a prediction for the ratio $\Gamma(\pi \to e\nu)/\Gamma(\pi \to \mu\nu)$ completely excluded by experiment. Interestingly enough, Giunti has himself recently given arguments based on quantum field theory why such states are *not* produced in pion decay [3], wthout, apparently, realising the contradiction with the assumption A2 of TNO! Secondly, which is essentially what is pointed out in the comment of Okun et al [4], the assumptions A2 and A3 of TNO are in clear contradiction with A4. On the one hand, it is assumed (A2 and A3) that the space-time velocities, L/T, are the same for all mass eigenstates, on the other hand (A4) they must have different kinematical velocities, p/E_k. Aristotle made a wise remark that seems quite appropriate to this situation[a]

`In inventing a model we may assume what we wish, but should avoid impossiblities.`

Since, however, the different mass eigenstates do not have to be produced at the same time in the different path amplitudes whose interference results in the 'neutrino oscillation' phenomenon, the hypotheses A2 and A3 are generally incorrect so that there in no contradiction with A4. I am, of course, using here Feynman's space-time formulation of QM which is the one best adapted to flavour oscillation problems, for the theoretical description of the problem.

Thus Assumptions A2 and A3 are usually false. A coherent state $|\nu_\alpha\rangle$ is <u>not</u> produced [2, 3], and although the source-detector distance, L, is the same for all states $|\nu_k\rangle$, the times-of-flight, T, (and hence the times of decay of the source) are different in the different amplitudes: $T_k/L = p_k/E_k$. Taking into account the different decay times of the source increases the contribution of the neutrino propagators to the interference phase by a factor of two and results in a numerically important contibution to the interference phase from the propagator of the source particle. There is therefore in fact no incompatiblity between the space-time and kinematical velocities, and no need, as asserted by Giunti in TNO and Ref. [5], to introduce an *ad hoc* gaussian 'spatial wave-packet' to circumvent the problem.

There should be nothing shocking in the idea that a source can decay at different (but unobserved) times in quantum interference phenomena. Consider a Michelson interferometer with a 1m path difference between the two arms. The times of decay of the source atom that produces the photon that 'interferes with itself' must obviously differ by 2m/c = 6.7 nsec when interference fringes are observed.

Unfortunately, practitioners of the theory of neutrino oscillations appear to have little familiarity with Feynman's space-time approach to QM, being instead usually experts in text-book second-quantised field theory, which has limited relevance to space-time flavour oscillations. This is not the case, however, for atomic physics theorists. In concluding below I will briefly describe an atomic physics experiment that is an almost perfect analogue of the two-flavour neutrino oscillation problem. Feynman's path amplitude method has been used to predict quantum interference effects in the experiment in very good agreement with observation.

One has the impression that neutrino physics theorists are stuck, for their conceptial

[a] On Giunti's website 'Neutrino Unbound': http://www.nu.to.infn.it/ a fair number of wise quotations are to be found, even one in connection with a paper of mine. I therefore consider that I have the right to include at least one here

understanding of QM, with Heisenberg in 1930. It is as though Dirac's [6] and Feynman's [7] discoveries of the true role of space-time in QM were never made. At this very early period in the history of QM, statements like the following can often be found [8]:

`There exists a body of exact mathematical laws but these cannot be interpreted as expressing simple relationships between objects in space and time.`

Or, [9]:

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EITHER:

  Causal relationship expressed by mathematical laws

  BUT:  physical description of space-time impossible

OR

  Phenomena described in terms of space and time

  BUT:   Uncertainty Principle
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The Feynman path amplitude formulation actually gives a mathematical law expressing a causal relationship (i.e. ordered in time) between elementary processes *in* space-time! The above quotations a very reminiscent of the woolly arguments invoked in blanket fashion by Giunti to avoid thinking clearly about the space time aspects of the QM of neutrino oscillations.

For example, misuse of the Heisenberg Uncertanity relations [1]:

`The wave packet treatment of neutrino oscillations is also necessary for a correct description of the momentum and energy uncertainties necessary for the coherent production and detection of different massive neutrinos [12,13,9] whose interference generates the oscillations`

It is not the wave functions of the neutrinos that are 'coherent in the oscillation phenomenon, but rather the path amplitudes for the whole experiment. The neutrinos are only unobserved intermediate states in these amplitudes.

or [5]:

One is free to define 'so-called space velocities $\overline{v} = x/t$ of massive neutrinos which are trivially identical. However I think that such a definition is useless. Neutrinos are not classical objects for which $v = x/t$. It is pretty clear that the uncertainty principle forbids any relation of this type

In fact, according to quantum field theory, on-shell particles *do* propagate classically in space-time; only highly virtual ones show different behaviour. Notice the blanket invocation of 'Heisenberg uncertainty' to block all further discussion. Actually the relation $v = x/t$ is absolutely essential for the correcct quantum mechanical description of neutrino oscillations.

and [5]

`Of course energy and momentum are not exactly defined in the production process, in order to allow the coherent production of different massive neutrinos, but it is pretty unlikely that the uncertainty in energy and momentum could generate the equal energy constraint.`

In relativistic quantum field theory energy and momentum are both exactly conserved at all vertices. 'Heisenberg uncertainty' (see below) only appears as a smearing of the masses of virtual particles about pole values. Another, desperate, last resort invocation of the 'Uncertainty Principle'!

As discussed in detail in my paper Ref. [10], the misuse of spatial wavepackets by Giunti and many other authors is related to a confusion of QM with classical wave theory, for which the concepts of phase velocity, group velocity and wave packets are useful and meaningful concepts. In the Dirac-Feynman formulation of QM they become almost irrelevant. Still, the Heisenberg Uncertainty Principle is an important part, when correctly used, of QM. I describe below its application to neutrino production in pion decay. In fact the neutrinos in $\pi \to \mu\nu$ (not in $\pi \to e\ \nu$) are described by a momentum wave packet. It has however nothing to do with the Fourier transform of the arbitary gaussian spatial function that is parachuted into the theory by Giunti and many other authors.

It is a consequence of quantum field theory that on-shell particles, or virtual particles propagating over macroscopic spatial separations, do so in a classical manner [11]. The dominant trajectories in Feynman path integrals are the classical ones. Consider the space-time description of pion decay, $\pi \to \mu\nu$, in QM. The Heisenberg Uncertainty Principle (so dear to the hearts of neutrino physics theorists!) should be correctly invoked in three different ways in the description of this process: (i) the decay width Γ and the decay lifetime τ certainly respect the energy-time Uncertainty Relation $\Gamma\tau = 1$; (ii) as a result of the finite pion lifetime the physical mass of the pion differs from the pole mass. In accordance with the energy-time Uncertainty Relation the mass smearing, given by a Breit-Wigner amplitude, is inversely proportional to the decay lifetime; (iii) due to the finite muon decay lifetime, the physical mass of the muon is also smeared around its pole value. Since the muon is unobserved, overall energy-momentum conservation leads to a neutrino momentum wave-packet in the case of $\pi \to \mu\nu$ but not for $\pi \to e\ \nu$, since the electron has an infinite lifetime. The effects of this wave packet are calculated in my paper Ref. [12]. They are tiny. The upshot is, that once the neutrinos are produced (respecting, of course, the law $\Gamma\tau = 1$) they propagate in space-time as essentially classical particles. This wave packet is the only one that is relevant to pion decay. In particular there is no *ad hoc* gaussian spatial wave packet describing the source. The uncertainty in the position of the source particle, a property of the initial state, and therefore the same in all path amplitudes, gives only a small, incoherent, correction to the oscillation phase. Again, this effect is calculated in Ref.[2] and found to be very small.

I would like to remark that I do agree with Giunti that the 'equal energy' hypothesis proposed in Ref. [4] is not relevant to resolving the evident contradiction between assumptions A2 and A3 of TNO as compared to A4. As shown in Ref. [10] sufficient assumptions to obtain the standard oscillation phase are A2 or A3 (since A2 implies A3). The result at, $O(m_\nu^2)$, is independent of the kinematical assumptions such as A4 (equal momenta, equal energies or exact energy-momentum conservation). In fact, in the papers by Stodolsky [13] and Lipkin [14] cited ini Ref. [4], not onlt is the equal energy assumption made, but the temporal part of the neutrino propagator phase (necessarily non-zero in the laboratory system by Lorentz invariance) is, arbitarily and incorrectly, set to zero.

A misnomer which occurs widely in Refs. [1, 4, 5] is 'plane wave' for 'space-time propagator'. This function has meaning only as a factor in a path amplitude. Interpreted as a wavefunction it is non square-integrable and therefore devoid of any physical significance.

The atomic physics experiment that provides a close analogy with a two flavour neutrino oscillation problem is called the 'Photodetachment Microscope [15].A coherent source of electrons of fixed energy is provided by a negative ion beam irradiated by a laser. The detached electron moves in a constant electric field before detection. Just two classical trajectories link the point of emission to any point on a plane detector oriented perpendicularly to the electric field direction. Quantum interference effects are observed between the path amplitudes corresponding to the two trajectories. A good pedagogical description can be found in Ref. [16] where the appropriate path integral formula[b]r :

$$\psi(\vec{r}, t_f) = \int_{-\infty}^{t_f} \exp[i\frac{\epsilon t_i}{\hbar}] \exp[i\frac{S_{cl}(\vec{r}, t_i, t_f)}{\hbar}] dt_i$$

is given.

In this formula ϵ is the energy of the detached electron and S_{cl} the classical action corresponding to an electron trajectory. Note particularly the time integral on the RHS of the equation. The first exponential function is the propagator of the coherent source (analagous to that of a coherent neutrino source) the second represents the propagator of the electron in the electric field. In practice it is well approximated by the contributions of the two classical trajectories mentioned above, corresponding to values of t_i with a fixed separation. These are the analogues of the propagators of different neutrino mass eigenstates. A typical value of the difference in t_i between the two trajectories, quoted in Ref. [16] is 160 psec for a time-of-flight of 117 nsec. The laws of physics are the same in this and in any neutrino oscillation experiment. Contrary to the claims in TNO and Ref. [5]], no *ad hoc* 'wave packets' are required for a correct quantum mechanical description of such systems. The standard formula for the oscillation phase which, as discussed in Ref. [10] may well be correct for heavy quark oscillations, is then in most cases, not so [2] for neutrino oscillations.

Finally, it is remarked that as described in Ref. [2] a clear experimental discrimination between the standard and path amplitude formulae of the oscillation phase is provided by long-linebase 'ν_μ disappearence' experiments with pion and kaon source beams.

[b] A similar formula has recently been proposed for the neutrino oscillation problem by Pažma and Vanko [17]. The corresponding oscillation phase was not, however, derived.

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