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Quantum quenches as classical critical films

A. GAMBASSI^{1(a)} and P. CALABRESE²
¹ *SISSA, International School for Advanced Studies and INFN - via Bonomea 265, 34136 Trieste, Italy, EU*
² *Dipartimento di Fisica dell'Università di Pisa and INFN - Pisa, Italy, EU*

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Abstract – We study the unitary time evolution of the order parameter of a quantum system after a sudden quench in the parameter which drives the transition. By mapping the dynamics onto the imaginary time path integral in a film geometry we derive the full mean-field *non-equilibrium phase diagram* for a one-component order parameter. The recently discovered non-equilibrium transition is identified with the shifted critical point in films and therefore it is generally expected to occur in more than one spatial dimension. We also find that anharmonic oscillations of the order parameter are a general feature of the mean-field dynamics after the quench.

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Introduction. – The experimental realization [1] of many-body cold atomic systems evolving in the absence of dissipation triggered a significant theoretical activity aimed at understanding the fate of a quantum interacting system initially prepared in a state which is not a Hamiltonian eigenstate. A global *quantum quench* is the prototypical instance: the system is prepared at time $t = 0$ in the ground state of a Hamiltonian H_0 and then it is let evolve with a Hamiltonian H which differs from H_0 *everywhere* in space (*e.g.*, due to a sudden quench of an external field). The absence of a solid and general theoretical framework for studying the ensuing non-equilibrium evolution motivated the investigation of simple one-dimensional models with exactly solvable dynamics (see, *e.g.*, refs. [2–6], but this list is far from being exhaustive). Several aspects of their evolution have been explored both analytically and numerically [7], as reviewed in ref. [8]. Only few studies of models in spatial dimension $d > 1$ are presently available [3,9,10].

One of the central issues concerning the time evolution after a quench is whether a stationary state exists at long times. It is commonly believed [2,3,5,7,11,12] that, in case it does, it is characterized by correlation functions which resemble those of either a thermal state or a so-called generalized Gibbs ensemble. However, a stationary asymptotic state does not always exist since the order parameter of the system might display persistent anharmonic oscillations, as it occurs in the case of the mean-field

(MF) approximation for superfluid bosons [13–15], for the Hubbard model [16] and for some exactly solvable pairing models [17–19].

A general approach to the study of quench problems, proposed in ref. [3], consists in mapping the real-time evolution of a d -dimensional quantum system onto a boundary problem in $d + 1$ dimensions in imaginary time. With this framework the quantum evolution governed by critical Hamiltonians [3,20] or by massive integrable field theories in $d = 1$ [21], has been successfully investigated.

In this paper we consider such a mapping for a quench within a generic effective Landau-Ginzburg (LG) Hamiltonian, which is expected to capture the universal properties of a rather large class of quantum critical points, encompassing some of the cases mentioned above. For a one-component order parameter, *i.e.*, within the Ising universality class, we present a remarkably simple complete MF solution for the order parameter evolution which, as we shall see, accounts comprehensively for all the qualitative features observed in real-time MF analyses¹. The main result of our investigation is that the novel “dynamical transition” (which has no equilibrium counterpart) reported in refs. [14,16] is the manifestation, in real time, of a rather well-known effect in confined critical systems, *i.e.*, the critical point shift in films [22].

¹In the case of systems with n -component vector order parameter and $O(n)$ symmetry —such as those discussed in refs. [13–17]— the analysis reported here applies to the only component of the order parameter which does not vanish when the $O(n)$ symmetry is broken by the boundaries.

(a) E-mail: gambassi@sissa.it

Remarkably, the universality of this phenomenon allows us to conclude that the dynamical transition is not a MF artifact but it occurs even in the presence of fluctuations, an important issue which was left open by all the analytical approaches considered so far in the literature. In addition, we find that anharmonic oscillations are general features of the MF evolution and we characterize quantitatively their features (form, amplitude, and period) in terms of the quench parameters. However, the existence of the transition beyond MF does not guarantee that these oscillations do actually withstand the effects of quantum fluctuations. Experiments and additional theoretical studies should be able to provide a conclusive answer to this subtle and difficult question.

The mapping to the film geometry. – Following ref. [3], consider a d -dimensional system prepared in a state $|\Psi_0\rangle$ which is not an eigenstate of the Hamiltonian H controlling the time evolution. The expectation value of a local operator $\mathcal{O}(x)$ at time t is given by the path integral

$$\langle \mathcal{O}(t, x) \rangle = Z^{-1} \langle \Psi_0 | e^{iHt - \epsilon H} \mathcal{O}(x) e^{-iHt - \epsilon H} | \Psi_0 \rangle, \quad (1)$$

where $Z = \langle \Psi_0 | e^{-2\epsilon H} | \Psi_0 \rangle$ and the damping factors $e^{-\epsilon H}$ have been introduced in order to make the path integral representation of $\langle \mathcal{O}(t, x) \rangle$ absolutely convergent. Equation (1) can be represented by an analytically continued path integral in imaginary time τ in which the field Ψ takes the boundary values Ψ_0 for $\tau = \tau_{1,2} = \pm\epsilon - it$. The operator \mathcal{O} is inserted at $\tau = 0$. The width of the film is $\tau_2 - \tau_1 = 2\epsilon$. In the course of the calculation $\tau_{1,2}$ must be considered as real numbers and only at the very end they are continued to their effective complex values. In this way, the real-time non-equilibrium evolution of a d -dimensional system is mapped onto the thermodynamics of a $(d+1)$ -dimensional field theory in a film geometry in which the initial state $|\Psi_0\rangle$ plays the role of a boundary condition at both boundaries. This approach can be used to study the time-evolution of a generic operator \mathcal{O} . However, in what follows we will concentrate on the order parameter $\phi(t, x)$.

When the system is close to a (quantum) phase transition, we can take advantage of the renormalization group (RG) theory of boundary critical phenomena [23] in order to describe the system in terms of the LG Hamiltonian for the order parameter $\phi(\tau, x)$ in a film [23]:

$$\mathcal{H}[\phi] = \int d^d x \int_{\tau_1}^{\tau_2} d\tau \left[\frac{1}{2} (\vec{\partial}\phi)^2 + \frac{r}{2} \phi^2 + \frac{g}{4!} \phi^4 \right] + \int d^d x [\mathcal{B}(\phi(\tau_1, x)) + \mathcal{B}(\phi(\tau_2, x))], \quad (2)$$

$[\vec{\partial} \equiv (\vec{\nabla}, \partial_\tau)]$ where $\mathcal{B}(\phi) \equiv \frac{1}{2} c \phi^2 - h_s \phi$ represents the boundary density, r controls the distance from the critical point, and $g > 0$ ensures the stability for $r < 0$. c is related to the surface enhancement, *i.e.*, to the difference between the interaction strengths within the boundaries and within the bulk, whereas h_s is a symmetry-breaking

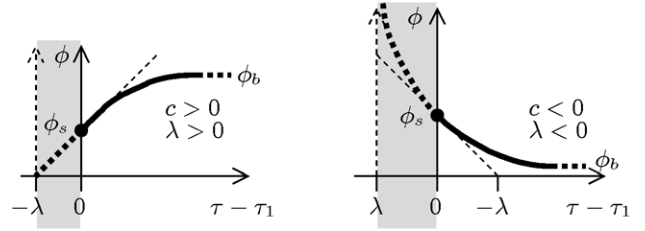


Fig. 1: Order parameter profiles close to the boundary, for $h_s = 0$ and $c > 0$ (left), $c < 0$ (right). The RG invariant boundary conditions, *i.e.*, $\phi_s^* = 0$ and $\phi_s^* = \pm\infty$, are effectively imposed at $\tau = \tau_1 - |\lambda|$. For h_s large enough the profile looks like the right one.

field acting at both confining surfaces. The values of c and h_s determine the actual boundary conditions for the field ϕ . This is easily understood in the semi-infinite geometry $\tau_2 \rightarrow \infty$, taking into account the MF boundary condition $-\partial_\tau \phi_s + c \phi_s - h_s = 0$ implied by eq. (2) for $\phi_s \equiv \phi(\tau_1, x)$, whereas $\phi_b \equiv \phi(\tau \rightarrow \infty, x)$ attains its bulk value $\phi_b = \sqrt{-6r/g}$ for $r < 0$, and $\phi_b = 0$ for $r \geq 0$. For $h_s = 0$ and $\phi_s \neq 0$, $|\phi|$ increases upon leaving the boundary if $c > 0$, as shown by the left panel of fig. 1. The order parameter profile extrapolated outside the film vanishes for $\tau = \tau_1 - \lambda$ (see fig. 1), where λ is the so-called *extrapolation length* $\lambda \simeq 1/c$ for c large enough [23]. If $c < 0$, instead, $|\phi|$ decreases upon leaving the surface, as shown by the right panel of fig. 1, and the (negative) extrapolation length λ determines the position $\tau = \tau_1 + \lambda$ at which the MF order parameter profile *diverges*. In order to have $\phi_s \neq 0$ with $h_s = 0$, one needs either $r > 0$ and $-c > r^{1/2}$ or $r < 0$, which correspond to $\phi_b = 0$ or $\phi_b \neq 0$, respectively. A large enough $h_s \neq 0$ causes an enhancement of $|\phi|$ close to the surface, so that ϕ_s does not vanish independently of the value of ϕ_b and the profile looks like the one depicted on the right panel of fig. 1.

Accounting for fluctuations requires a RG analysis [23] which yields the following conclusions for translational invariant boundary conditions for $h_s = 0$, the possible fixed-point values of c are $c^* = \infty, 0, -\infty$, with $c^* = \pm\infty$ stable and $c^* = 0$ unstable. In particular $c^* = +\infty$ leads to Dirichlet (D) boundary condition for ϕ at the boundaries of the film, *i.e.*, ϕ_s flows under RG towards $\phi_s^* = 0$. The fixed point with $c^* = -\infty$ describes both the cases $(-c > r^\Phi, h_s = 0)$ [23] and $(c < \infty, h_s \neq 0)$ and leads to a divergence of the order parameter profile upon approaching the surface: $\phi(\tau \rightarrow 0, x) \simeq \tau^{-\beta/\nu}$ (with β and ν the standard critical exponents), *i.e.*, ϕ_s flows to $\phi_s^* = \pm\infty$, referred to as \pm boundary conditions. The kind of asymptotic boundary conditions can be also expressed in terms of ϕ_s and ϕ_b . If $\phi_s = 0$, the corresponding boundary condition is Dirichlet. On the other hand, if $\phi_s \neq 0$, the order parameter profile looks like on the left or right of fig. 1, depending on whether $\phi_s < \phi_b$ or $\phi_s > \phi_b$, which correspond respectively to D and + boundary condition.

Going back to the two boundaries problem, the most important feature of the phase diagram is the *critical point shift* [22]. This is easily understood in the case of classical critical points for which $r = T - T_c$, being T_c the critical temperature in the *bulk*. For DD boundary conditions, the suppression of order at the surfaces prevents the film from ordering and therefore $\phi = 0$ everywhere even slightly below T_c . However, upon further decreasing T , the increasingly strong tendency towards ordering overcomes the effects of the boundaries and the whole film orders. This occurs at the shifted critical point $T = T_c(L) < T_c$, which depends on the thickness L of the film, and is such that $T_c(L) - T_c \sim L^{-1/\nu}$. For $++$ boundary conditions, such a point is additionally shifted off-coexistence and it can be reached by applying a non-zero bulk magnetic field which overcomes the order at the surface.

Mean-field order parameter profiles – Let us consider the LG Hamiltonian (2) in a film geometry of thickness $L \equiv \tau_2 - \tau_1$ and introduce the rescaled MF order parameter profile $\phi(\tau) = \frac{1}{L} \sqrt{\frac{3!}{g}} \psi((\tau - \tau_1)/L)$, which satisfies the equation $-\psi'' + \bar{r}\psi + \psi^3 = 0$, where $\bar{r} \equiv rL^2$. For DD boundary conditions, $\psi(0) = \psi(1) = 0$, whereas for $++$ boundary conditions, $\psi(x \rightarrow 0) \simeq x^{-1}$ and $\psi(x \rightarrow 1^-) \simeq (1-x)^{-1}$. The differential equation can be solved analytically. For $++$ boundary conditions and $\bar{r} > \bar{r}_c = -\pi^2$ (high-temperature phase H) one finds [24]

$$\psi_{++}(x) = 2\sqrt{2}K \frac{\text{dn}(2Kx; k)}{\text{sn}(2Kx; k)}, \quad (3)$$

where $K \equiv K(k)$ is the complete elliptic integral of the first kind and the elliptic modulus $k = k(\bar{r})$ solves $\bar{r} = (2K)^2(2k^2 - 1)$, dn and sn are the standard Jacobi elliptic functions. For $\bar{r} \leq \bar{r}_c$ (low-temperature phase L), instead, one finds [24]

$$\psi_{++}(x) = 2\sqrt{2}K [\text{sn}(2Kx; k)]^{-1}, \quad (4)$$

where k satisfies $\bar{r} = -(2K)^2(k^2 + 1)$.

For DD boundary conditions, ψ vanishes for $\bar{r} \geq \bar{r}_c$. For $\bar{r} < \bar{r}_c$, instead, the film orders and the order parameter is [25]

$$\psi_{DD}(x) = 2\sqrt{2}K k \text{sn}(2Kx; k), \quad (5)$$

where now k solves $\bar{r} = -(2K)^2(k^2 + 1)$. In fig. 2(a) we plot ψ for the cases of $++$ and DD boundary conditions.

The analytic continuation. – For the purpose of extracting the asymptotic behavior of $\phi(t)$ for $t \gg \tau_0$ we replace $|\Psi_0\rangle$ by the appropriate RG-invariant boundary condition $|\Psi_0^*\rangle$ to which it flows. The difference is taken into account, to leading order, by assuming that the RG-invariant boundary conditions are not imposed at $\tau = \tau_{1,2}$ but at $\tau = \tau_{1,2} \mp \tau_0$, where τ_0 is the absolute value of the extrapolation length. In the quantum non-equilibrium problem, τ_0 is of the order of the correlation length in the initial state [3]. The effect of introducing τ_0 is simply to replace ϵ by $\epsilon + \tau_0$ and the limit $\epsilon \rightarrow 0^+$ can

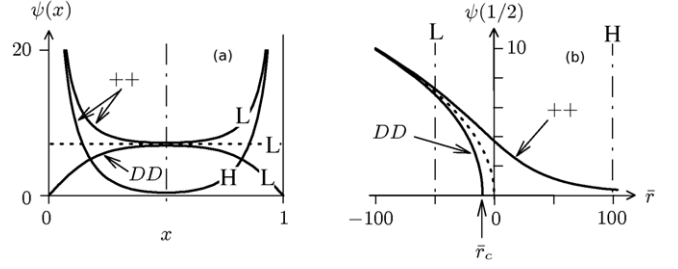


Fig. 2: (a) Order parameter profiles $\psi(x)$ across a film with DD and $++$ boundary conditions. The profiles denoted by L and H correspond, respectively, to the low- and high-temperature phases. (b) Value of the order parameter in the middle of the film as functions of \bar{r} . L and H indicate the values of \bar{r} corresponding to $\psi(x)$ in panel (a). In both panels, the dotted lines refer to the bulk order parameter.

now be taken, resulting in a film of effective thickness $L = 2\tau_0$. In order to determine the time evolution of the order parameter ϕ one has simply to continue analytically the expressions provided above for ψ to the complex position $\tau = \tau_0 + it$ within the film. Such a continuation is straightforwardly done by using standard formulas for the Jacobi functions [26]. For a quench with $++$ boundary conditions to a point below $r_c = -\pi^2/(4\tau_0^2)$, one finds

$$[L] \quad \begin{cases} \phi_+(t) = a K \text{dn}(Kt/\tau_0; k'), \\ r/|r_c| = -(2K/\pi)^2(k^2 + 1) < -1, \end{cases} \quad (6)$$

where $k' \equiv \sqrt{1 - k^2}$ and $a \equiv 2\sqrt{3}/(\tau_0\sqrt{g})$. $\phi_+(t)$ oscillates between aK and akK with period $T_k \equiv 2\tau_0 K'/K$, where $K' \equiv K(k')$. For $++$ boundary conditions, but with $r > r_c$:

$$[H] \quad \begin{cases} \phi_+(t) = a k' K \text{cn}(Kt/\tau_0; k'), \\ r/|r_c| = (2K/\pi)^2(2k^2 - 1) > -1. \end{cases} \quad (7)$$

$\phi_+(t)$ oscillates with period $2T_k$ between $-ak'K$ and $ak'K$. The qualitative change in the evolution below and above the boundary point occurs at $r = r_c$ via an exponential relaxation

$$\phi_+(t) \propto [\cosh(\pi t/2\tau_0)]^{-1} \quad (8)$$

which is similar to the result at criticality in one dimension² and is characterized by a decay time $\tau_{\text{exp}} \equiv 2\tau_0/\pi$. Upon approaching r_c ,

$$T_k(r \rightarrow r_c) \simeq \tau_{\text{exp}} \ln(24/|1 - r/r_c|) \quad (9)$$

²We mention that from the mathematical standpoint, anharmonic oscillations in one spatial dimension are absent because for critical phenomena in a two-dimensional film geometry the Jacobi functions which characterize the mean-field expressions encountered so far for the order parameter profile are replaced by trigonometric functions (see, e.g., [27]). The analytic continuation of the latter involve hyperbolic functions, which result in an exponential relaxation towards the corresponding asymptotic values. Physically, this is a consequence of the absence of ordered phases in classical one-dimensional systems and therefore the two low-temperature regions disappear.

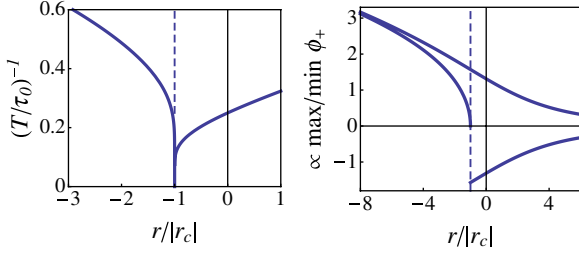


Fig. 3: (Colour on-line) Left: inverse period of the oscillations of $\phi_+(t)$ in units of τ_0^{-1} as a function of r/r_c . For $r \rightarrow r_c$ the period diverges logarithmically. Right: maximum and minimum value of $\phi_+(t)$ (up to the multiplicative constant a) as a function of r/r_c .

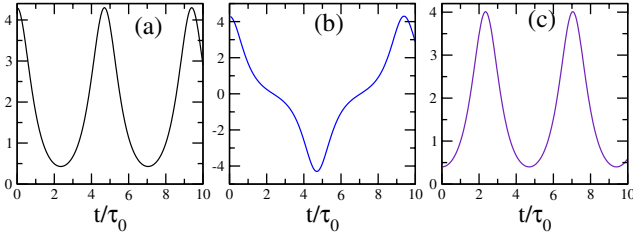


Fig. 4: (Colour on-line) Order parameter ϕ (in arbitrary units) as a function of t/τ_0 : (a) $\phi_+(t)$ for $r < r_c$ is always positive. (b) $\phi_+(t)$ for $r > r_c$ oscillates symmetrically around 0. (c) $\phi_D(t)$ for $r < r_c$. Note that $\phi_D(t) = 0$ for $r > r_c$.

and, therefore, the periods T_k and $2T_k$ of the oscillations (6) and (7) —respectively, above and below r_c — diverge logarithmically with a prefactor set by the exponential decay time τ_{exp} . For DD boundary conditions we have instead

$$[\text{H}] \quad \phi_D(t) = 0 \quad \text{for} \quad r > r_c, \quad (10)$$

and

$$[\text{L}] \quad \begin{cases} \phi_D(t) = a K \text{dn}(Kt/\tau_0 - K'; k'), \\ r/r_c = -(2K/\pi)^2(k^2 + 1) < -1, \end{cases} \quad (11)$$

so that $\phi_D(t)$ oscillates with period T_k between aK and akK . For $r < r_c$, $\phi_D(t)$ differs from $\phi_+(t)$ only by a half-period shift. Figure 3 shows the period and the amplitude of the oscillations of ϕ_+ whereas in fig. 4 we present the three typical non-vanishing evolutions. In the case of $++$ boundary conditions a mean-field analysis of the critical evolution ($r=0$) was presented in ref. [3], where anharmonic oscillations were reported. Here we have extended the analysis to the entire phase diagram with generic r and boundary conditions, with a careful characterization of the period of the emerging anharmonic oscillations as a function of the quench parameters. This careful study allowed us to highlight the appearance of the dynamical mean-field transition and reveal its very nature.

Discussion. — Our results depend explicitly on the parameter τ_0 , which determines the value of $r_c \propto \tau_0^{-2}$. In ref. [3] it has been argued that τ_0 is proportional to

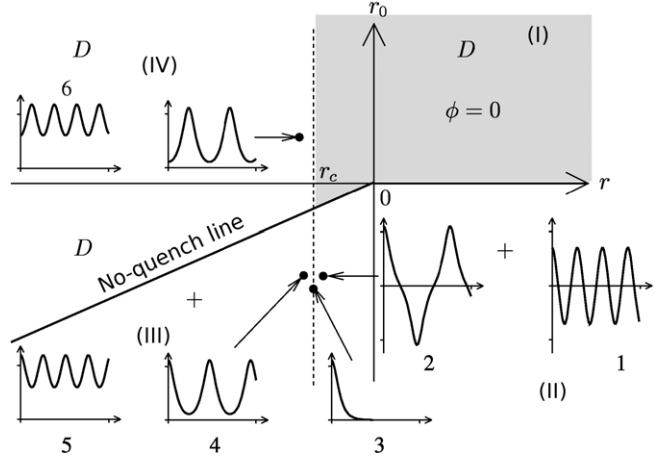


Fig. 5: Mean-field non-equilibrium phase diagram. We identify four “phases”: (I) $r > r_c$ and $r_0 > 0$, where $\phi(t)$ vanishes identically; (II) $r > r_c$ and $r_0 < 0$, where $\phi(t)$ shows anharmonic oscillations around zero; (III) and (IV) where $\phi(t)$ shows anharmonic oscillations with a non-zero average. These latter two regions are separated by the no-quench line $r = r_0$ and, qualitatively, the corresponding $\phi(t)$ differ only by a half-period shift. Upon approaching the line $r = r_c$, the period of oscillation diverges according to eq. (9) and $\phi(t)$ decays exponentially to zero with decay time τ_{exp} .

the spatial correlation length ξ_0 in the initial state, the inverse of the initial mass gap. Within the mean-field LG formalism ξ_0 is proportional to $|r_0|^{-1/2}$, where r_0 is the initial value of r in eq. (2). Accordingly, $r\tau_0^2$ is proportional to $r/|r_0|$ and therefore $r_c \propto r_0$, but with unknown proportionality factors. Although we cannot fix the exact value of τ_0 , the *non-equilibrium phase diagram*, which gives the appropriate asymptotic boundary conditions corresponding to a certain choice of the quench parameters r_0 and r , can be understood on physical grounds. In the case of disordered initial state $r_0 > 0$, one always expects Dirichlet boundary conditions in the film, independently of the final value r . If such a value exceeds the boundary transition point r_c , the order parameter ϕ vanishes at all times. Conversely, for $r < r_c$, one has anharmonic oscillations the features of which depend on r_0 through $\tau_0 \propto |r_0|^{-1/2}$. When the quantum system is in an initial state with $r_0 < 0$, the asymptotic boundary conditions depend on the value of r . From the general features of the profiles in the film (see fig. 2), we expect that if $\phi_0 > \phi_{\text{eq}}$, the corresponding asymptotic boundary conditions is $++$, whereas it is DD for $\phi_0 < \phi_{\text{eq}}$, where ϕ_{eq} and ϕ_0 correspond, respectively, to ϕ_b and ϕ_s . The crossover between these two regimes occurs on the *no-quench line* $r = r_0$. All these considerations lead to the non-equilibrium phase diagram sketched in fig. 5 (where, in the spirit of our analysis, r , r_0 , and τ_0 are considered as independent parameters). Remarkably, this phase diagram accounts for all the qualitative features found via real-time MF approaches [14,16]. In particular, in the Bose-Hubbard model [14], for quenches within the ordered

(superfluid) phase, $\langle \phi^2(t) \rangle$ displays two types of oscillations separated by the point of “dynamical transition” at which an exponential relaxation occurs, with decay time τ_{exp} . These two different kind of oscillations actually correspond to “phases” (II) and (III) in fig. 5, as can be realized by comparing the time evolution of the square modulus of the order parameter presented in fig. 2(A) and (C) of ref. [14] (dashed lines) with the one which can be readily inferred for ϕ^2 from fig. 4(a) and (b), respectively. Upon approaching the dynamical transition, the period T_k of these oscillation —expressed in terms of the corresponding τ_{exp} — was found [14] to diverge according to the leading term of eq. (9). In addition, it turned out that for a fixed initial value U_0 of the coupling constant U of the Bose-Hubbard model the dynamical transition occurs at a value $U^d(U_0) = (U_0 + U_c)/2$ of the coupling U after the quench [14], where U_c is the value of U at which the model undergoes a Mott-superfluid transition in equilibrium. (The notation here is slightly different from the one used in ref. [14].) In terms of the distance $r \propto U - U_c$ from the critical point, the dynamical transition therefore occurs at $r_c \propto U^d(U_0) - U_c = (U_0 - U_c)/2 \propto r_0$, where $r_0 \propto U_0 - U_c$ is the value of r right before the quench. The fact that $r_c \propto r_0$ provides an additional evidence of the correctness of the picture presented here, which allows us to conclude that the point of dynamical transition highlighted in ref. [14] is nothing but $r = r_c$, i.e., the shifted critical temperature in the corresponding classical film. This implies that the dynamical transition is not a MF artifact and therefore it has to be expected generically for $d > 1$. This could also be the case for the dynamical transition points with anharmonic oscillations found in the MF evolution of the Hubbard [15,16] and pairing models [17,18]. The MF analysis discussed here highlights two distinctive features of this dynamical transition: first, the period of the anharmonic oscillations of the order parameter diverges upon approaching it and, second, it separates a region ((I)+ (II)) within which the asymptotic time average of the order parameter vanishes from a region ((III) + (IV)) within which it does not. Fluctuations beyond MF are likely to affect and completely wipe out these oscillations, resulting either in a relaxation of the order parameter towards some asymptotic value or even to some chaotic dynamics in which oscillations (damped or not) with various frequencies are involved, as suggested by the results of ref. [14]. Within the present approach, however, it is difficult to establish whether these fluctuations are relevant only below an upper critical dimensionality (as it is the case of equilibrium critical phenomena) or, instead, they destroy the oscillations in generic dimensionality. Nonetheless, the RG and scaling analysis of the classical system in the film geometry —which accounts for critical fluctuations— suggests that the presence of the dynamical transition is robust in any dimensionality ($\neq 1$) and that this transition, as its classical counterpart, is characterized by the second of the two features mentioned above. However,

the critical-point shift r_c is affected by fluctuations and indeed it scales as $r_c \sim \tau_0^{-1/\nu}$, where ν is the equilibrium critical exponent governing the divergence of the order parameter correlation length ξ of the $(d+1)$ -dimensional classical system without boundaries. This fact provides an additional example [9,28] of how the dynamical behavior after a quantum quench may display some of the universal features of the corresponding classical equilibrium model.

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