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Unruh effect and particle decay

Giuseppe Gaetano Luciano^{1,2}

¹Università di Salerno, Via Giovanni Paolo II, 132 I-84084 Fisciano (SA), Italy.

²INFN, Sezione di Napoli, Gruppo collegato di Salerno, I-84084 Fisciano (SA), Italy.

E-mail: gluciano@sa.infn.it

Abstract. We review the study of the inverse β -decay of uniformly accelerated protons in the context of neutrino flavor mixing. Letting ourselves be guided by some core theoretical principles, such as the general covariance of Quantum Field Theory and the conservation law of the family lepton numbers built into the Standard Model, we infer non-trivial results on the asymptotic nature of neutrinos.

1. Introduction

The *lifetime* of a particle is usually listed among its inherent properties. For some particles like pions or muons, it can be estimated by knowing the interactions they experience. By contrast, other particles such as electrons or protons are stable, at least based on the predictions of the Standard Model (SM). Despite such a rooted belief, decay properties are not fundamental at all, since they can be significantly altered by external influences. For instance, one can affect the lifetime of muons or, even more strikingly, spoil the overall stability of protons, by exposing them to a sufficiently large *acceleration*. This was first highlighted by Ginzburg and Syrovatskii [1] and later settled by Müller in a toy model approach with spinless fields [2].

Few years later these quite speculative works, the decay of *uniformly accelerated protons* (inverse β -decay)

$$i) \quad p \rightarrow n + e^+ + \nu_e, \quad (1)$$

served as a test bench for Unruh effect [3], which predicts that a uniformly accelerated observer associates a thermal spectrum to the vacuum of inertial observers. Indeed, it is a well-established fact that in Quantum Field Theory (QFT) the mean proper lifetime of a particle must be frame-independent, since it is a scalar. On this basis, in Ref. [4] the decay time for the process Equation (1) was evaluated both in the laboratory frame (where the proton is accelerated) and in the comoving frame (where the proton is at rest), concluding that the only way to preserve the covariance is by assuming that the hadron interacts with electrons and (anti-)neutrinos popping out from Unruh thermal bath in the latter case.

The above analysis was carried out by considering massless neutrinos. Nevertheless, neutrinos are massive and interact in weak (i.e. flavor) eigenstates that are superpositions of mass eigenstates through Pontecorvo transformation [5, 6]. This feature was successfully included in a series of recent works [7, 8, 9, 10, 11, 12, 13], although the issue of the asymptotic nature of neutrinos – either flavor or mass – was left open. Further progress was later made in Refs. [14, 15], where the analysis of the inverse β -decay was refined by taking into account neutrino oscillations and the related CP symmetry violation.



In this work, we review the study of the inverse β -decay in the context of neutrino mixing. By invoking consistency with very first principles and laws built into the extended SM, we show that the correct way to describe asymptotic neutrinos is by means of flavor states. Contrary to some claims in the literature, these states are actually well-defined, at least in the relativistic limit, as we have been taught since Pontecorvo's pioneering works on mixing [5, 6].

The remainder of the work is organized as follows: in Sect. 2 we set the stage for the computation of the proton decay rate. Explicit calculation are carried out in Sect. 3. Sect. 4 is devoted to conclusions and outlook. Throughout the whole analysis, we use Minkowski metric with the time-like signature and set $k_B = \hbar = c = 1$.

2. The general framework

Let us introduce the basic ingredients for the study of the inverse β -decay. More details can be found in Refs. [8, 14, 15]. By following Ref. [4], we consider the neutron n and proton p as excited and unexcited states of a unique quantum system, the nucleon. Moreover, we suppose that these two particles move along a well-defined trajectory, the Rindler hyperbola, which features the relativistic uniformly accelerated motion in Minkowski spacetime. Such an assumption holds true as far as the momenta of both the emitted positron e^+ and neutrino ν_e obey the *no-recoil condition* $|\mathbf{k}_{e,\nu}| \ll m_p, m_n$, where m_p and m_n are the masses of the proton and neutron, respectively.

For accelerations a small enough with respect to the masses of the intermediate vector bosons W^\pm and Z^0 , the interaction action can be described by a semiclassical Fermi-like effective theory, i.e.

$$\hat{S}_I = \int d^4x \sqrt{-g} \hat{J}_{h,\lambda} \hat{J}_l^\lambda, \quad (2)$$

where g is the determinant of the metric tensor. Here the hadron current $\hat{J}_{h,\lambda}$ is defined as

$$\hat{J}_{h,\lambda} = \hat{q}(\tau) u_\lambda \delta(x) \delta(y) \delta(u - 1/a), \quad (3)$$

where $\hat{q}(\tau) = e^{i\hat{H}\tau} \hat{q}(0) e^{-i\hat{H}\tau}$ is the monopole operator, \hat{H} is the nucleon hamiltonian such that $\hat{H}|p\rangle = m_p|p\rangle$ and $\hat{H}|n\rangle = m_n|n\rangle$ and $G_F = |\langle n|\hat{q}(0)|p\rangle|$ is the Fermi constant. u^λ represents the nucleon four-velocity along the Rindler trajectory, which is parameterized by the condition $u = 1/a$, where u is the Rindler spatial coordinate. The nucleon proper time τ is related to the coordinate time v by $\tau = v/a$.¹

On the other hand, the lepton current \hat{J}_l^λ is given by

$$\hat{J}_l^\lambda = \sum_{\ell=e,\mu,\tau} \left(\hat{\bar{\Psi}}_{\nu_\ell} \gamma^\lambda \hat{\Psi}_\ell + \hat{\bar{\Psi}}_\ell \gamma^\lambda \hat{\Psi}_{\nu_\ell} \right), \quad (4)$$

where $\hat{\Psi}_\ell$ and $\hat{\Psi}_{\nu_\ell}$ are the charged lepton and neutrino Dirac quantum fields of flavor ℓ . The γ 's are the gamma matrices in the Dirac representation [8].

According to Pontecorvo's original analysis of two-flavor mixing [5, 6] and its later extension to three generations [16], in the SM neutrinos produced in weak interactions are weak eigenstates $|\nu_\ell\rangle$. In general, these states do not have a well-defined mass and can be written as linear

¹ Here we are assuming that the acceleration is along the z -axis. In this setting, the Rindler coordinates (v, x, y, u) can be expressed in terms of Minkowski ones (t, x, y, z) by $t = u \sinh v$, $z = u \cosh v$, with x and y left unchanged. Notice also that the nucleon four-velocity takes the form $u^\lambda = (a, 0, 0, 0)$ and $u^\lambda = (\sqrt{a^2 t^2 + 1}, 0, 0, at)$ in Rindler and Minkowski coordinates, respectively.

combinations of the three massive states $|\nu_j\rangle$ ($j = 1, 2, 3$) as

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \end{pmatrix} = U \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \end{pmatrix}, \quad (5)$$

where U is the unitary Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix parameterized by the three mixing angles θ_{12} , θ_{23} , θ_{13} and the CP violating phase δ_{CP} [15].

A comment is now in order: since the transformation Equation (5) is unitary, one might think that the flavor and mass representations are physically equivalent. Actually, this remains true as long as flavor mixing is analyzed in quantum mechanics. On the other hand, in QFT it was shown that mixing transformations at level of ladder operators are by far more complicated, as they exhibit the structure of a rotation nested into a Bogoliubov transformation [17]. As a result, the Fock space for the fields with definite flavors becomes unitarily inequivalent to the Fock space for the fields with definite masses, giving rise to physically different representations of canonical commutation relations and genuinely field theoretical phenomena² [19, 20, 21, 22, 23]. In turn, this poses the problem of choosing the “right” representation for mixed neutrinos.

By using the above tools, in the next Section we compute the tree-level decay rate for the process Equation (1). In compliance with the conservation of the lepton number for each family in the interaction vertices, we employ neutrino flavor states for the evaluation of the scattering matrix. Notice that this approach is simply dictated by a law embedded into the extended SM and does not require any empirical model involving measurement issues.

3. Laboratory and comoving decay rates

In this Section we aim at computing the proton decay rate both in the laboratory (inertial) and in the comoving frame. In the first case the process is kinematically allowed by the accelerating source, which supplies the proton with the missing energy to convert into a neutron, a positron and an electron neutrino (see Eq. Equation (1)). By contrast, in the comoving frame the proton is at rest: in this context, what actually makes the decay possible is the interaction with a thermal bath of electrons and (anti-)neutrinos due to Unruh effect [3]. Therefore, new decay channels open up, which allow to preserve the otherwise broken general covariance of QFT (we shall introduce these new channels below).

3.1. Laboratory frame

By using the scattering-matrix formalism, one can show that the tree-level transition amplitude for the process Equation (1) takes the form [15]

$$\mathcal{A}^{(i)} \equiv \langle n | \otimes \langle e^+, \nu_e | \hat{S}_I | 0 \rangle \otimes | p \rangle = \frac{G_F}{2^4 \pi^3} \sum_{j=1}^3 |U_{ej}|^2 \mathcal{I}_{\sigma_\nu \sigma_e}(\omega_{\nu_j}, \omega_e), \quad (6)$$

where U_{ej} is the generic element of the PMNS matrix and we have implemented the mixing transformation Equation (5) on both the neutrino state and field. Henceforth, the superscript referred to the transition amplitudes and rates will label the decay process under consideration. The function $\mathcal{I}_{\sigma_\nu \sigma_e}$ is given by [15]

$$\mathcal{I}_{\sigma_\nu \sigma_e}(\omega_{\nu_j}, \omega_e) = \int_{-\infty}^{+\infty} d\tau u_\lambda \left[\bar{u}_{\sigma_\nu}^{(+\omega_{\nu_j})} \gamma^\lambda u_{-\sigma_e}^{(-\omega_e)} \right] e^{i[\Delta m \tau + a^{-1}(\omega_{\nu_j} + \omega_e) \sinh a\tau - a^{-1}(k_\nu^z + k_e^z) \cosh a\tau]}. \quad (7)$$

² This is a direct consequence of Haag’s theorem in QFT [18].

Here $u_\sigma^{(\pm\omega)}$ is the Dirac spinor of polarization $\sigma = \pm$ and frequency $\omega = \sqrt{\mathbf{k}^2 + m^2}$ [8], while $\Delta m = m_n - m_p$ denotes the difference between the nucleon masses. Notice that the integration is performed over the (infinite) proper time of the nucleon.

The scalar decay rate is defined by

$$\Gamma^{(i)} \equiv \frac{1}{T} \sum_{\sigma_e, \sigma_\nu} \int d^3 k_\nu \int d^3 k_e |\mathcal{A}^{(i)}|^2, \quad (8)$$

where T is the total nucleon proper time. By inserting Eq. Equation (6) into Equation (8), we are led to [15]

$$\begin{aligned} \Gamma^{(i)} &= |U_{e1}|^4 \Gamma_1 + |U_{e2}|^4 \Gamma_2 + |U_{e3}|^4 \Gamma_3 \\ &+ \left(|U_{e1}|^2 |U_{e2}|^2 \Gamma_{12} + |U_{e1}|^2 |U_{e3}|^2 \Gamma_{13} + |U_{e2}|^2 |U_{e3}|^2 \Gamma_{23} + \text{c.c.} \right), \end{aligned} \quad (9)$$

where we have used the shorthand notation

$$\begin{aligned} \Gamma_j &\equiv \frac{1}{T} \frac{G_F^2}{2^8 \pi^6} \sum_{\sigma_\nu, \sigma_e} \int d^3 k_\nu \int d^3 k_e |\mathcal{I}_{\sigma_\nu \sigma_e}(\omega_{\nu_j}, \omega_e)|^2 \\ &= \frac{G_F^2}{a \pi^6 e^{\pi \Delta m/a}} \int d^3 k_\nu \int d^3 k_e \left\{ K_{2i\Delta m/a} \left(\frac{2(\omega_{\nu_j} + \omega_e)}{a} \right) + \frac{m_{\nu_j} m_e}{\omega_{\nu_j} \omega_e} \text{Re} \left[K_{2i\Delta m/a+2} \left(\frac{2(\omega_{\nu_j} + \omega_e)}{a} \right) \right] \right\}, \end{aligned} \quad (10)$$

$$\Gamma_{jk} \equiv \frac{1}{T} \frac{G_F^2}{2^8 \pi^6} \sum_{\sigma_\nu, \sigma_e} \int d^3 k_\nu \int d^3 k_e \mathcal{I}_{\sigma_\nu \sigma_e}(\omega_{\nu_j}, \omega_e) \mathcal{I}_{\sigma_\nu \sigma_e}^*(\omega_{\nu_k}, \omega_e), \quad (11)$$

with $K_{i\nu}(x)$ being the modified Bessel function of second kind.

The above considerations only represent the first step in the history of the study of the inverse β -decay with neutrino mixing. Indeed, in parallel to this approach, a similar analysis was carried out in Ref. [10] based on the use of neutrino mass representation. Notice that, in doing so, the decay rate Equation (9) would simply be $\Gamma = |U_{ei}|^2 \Gamma_i$, with the subscript i depending on which of the three mass eigenstates is employed. In the absence of experimental evidence which could definitively solve the flavor-mass controversy³, in Ref. [14] the question was asked about which of the two representations could be ruled out on the basis of a full theoretical consistency. A potential test bench was initially recognized in the phenomenon of flavor oscillations.

It has been proved experimentally that, due to their non-vanishing masses, neutrino can oscillate during the spacetime propagation [24, 25]. We then expect that also the processes

$$ii) \quad p \rightarrow n + e^+ + \nu_\mu, \quad iii) \quad p \rightarrow n + e^+ + \nu_\tau, \quad (12)$$

do contribute to the total decay rate. In the flavor-basis setting, these processes can be naturally described by replacing the state $|\nu_e\rangle$ by $|\nu_\mu\rangle$ or $|\nu_\tau\rangle$ in Eq. Equation (6). Calculations follow as above, yielding [15]

$$\begin{aligned} \Gamma^{(ii)} &= |U_{\mu 1}|^2 |U_{e1}|^2 \Gamma_1 + |U_{\mu 2}|^2 |U_{e2}|^2 \Gamma_2 + |U_{\mu 3}|^2 |U_{e3}|^2 \Gamma_3 \\ &+ \left(U_{\mu 1}^* U_{e1} U_{\mu 2} U_{e2}^* \Gamma_{12} + U_{\mu 1}^* U_{e1} U_{\mu 3} U_{e3}^* \Gamma_{13} + U_{\mu 2}^* U_{e2} U_{\mu 3} U_{e3}^* \Gamma_{23} + \text{c.c.} \right), \end{aligned} \quad (13)$$

³ It must be noted that protons are not likely to decay in (current) laboratory conditions. For instance, for typical LHC/CERN accelerations, the proton mean lifetime is $\tau_p \simeq 10^{3 \cdot 10^8} \text{ yr}$ [4], a time out of reach even for the longest-lived physicist.

$$\begin{aligned} \Gamma^{(iii)} = & |U_{\tau 1}|^2 |U_{e1}|^2 \Gamma_1 + |U_{\tau 2}|^2 |U_{e2}|^2 \Gamma_2 + |U_{\tau 3}|^2 |U_{e3}|^2 \Gamma_3 \\ & + \left(U_{\tau 1}^* U_{e1} U_{\tau 2} U_{e2}^* \Gamma_{12} + U_{\tau 1}^* U_{e1} U_{\tau 3} U_{e3}^* \Gamma_{13} + U_{\tau 2}^* U_{e2} U_{\tau 3} U_{e3}^* \Gamma_{23} + \text{c.c.} \right). \end{aligned} \quad (14)$$

Clearly, since the decays Equation (1), Equation (12) are mutually exclusive, the total rate in the laboratory frame is simply given by the incoherent sum of the three probabilities Equation (8), Equation (13) and Equation (14), i.e.

$$\Gamma^{\text{lab}} = \Gamma^{(i)} + \Gamma^{(ii)} + \Gamma^{(iii)} = |U_{e1}|^2 \Gamma_1 + |U_{e2}|^2 \Gamma_2 + |U_{e3}|^2 \Gamma_3, \quad (15)$$

where we have exploited the unitarity of the PMNS matrix. From this equation, it arises that the sum of the decay rates over the three flavors is the same as the sum over the three masses, up to the mixing coefficients. The physical explanation for such a result is clear: when integrating over an infinite time interval, we are effectively averaging over the oscillation probabilities, throwing any time-dependence of the decay rate away. In order to get some meaningful clue on the nature of neutrinos, one should perform the integration over a finite time. However, due to technicalities, work along this direction is still under active investigation.

Despite such a seemingly fruitless attempt, neutrino oscillations offered a further way out. Indeed, in the three-flavor model it is well-known that oscillations can violate CP symmetry, due to the fact that the PMNS matrix has complex elements [26, 27]. These effects have been recently measured with great accuracy using long-baseline neutrino and anti-neutrino oscillations observed by the T2K experiment (see [28] and references therein) and can be quantified by introducing the so-called Jarlskog invariant J

$$\text{Im} [U_{\delta i} U_{\gamma i}^* U_{\delta j}^* U_{\gamma j}] \equiv J \sum_{\lambda, k} \varepsilon_{\delta \gamma \lambda} \varepsilon_{ijk}, \quad (16)$$

where $\delta, \gamma, \lambda = \{e, \mu, \tau\}$ and $i, j, k = \{1, 2, 3\}$. Since this is the unique phase-independent measure of CP violation that can be built from the PMNS matrix, all observables quantifying CP violation cannot but be proportional to J . One may then wonder whether this feature can be exploited to tackle the flavor-mass dichotomy.

CP violation effects in the above framework were studied in Ref. [15]. Here we outline the main contents. In order to keep our considerations as general as possible, we consider the scattering matrix \hat{S}_{weak} for a given charged-current weak interaction with a neutrino in the final state (the specific case of the inverse β -decay is obtained for \hat{S}_{weak} equal to \hat{S}_I in Eq. Equation (2)). To quantify CP violation, we assume that the outgoing neutrino, emitted for instance with flavor e , is detected after a certain distance with flavor μ . For this process, the transition amplitude in the flavor representation is

$$\mathcal{A}_{\nu_e, \nu_\mu} = \text{out} \langle \nu_\mu, \dots | S_{\text{weak}} (\bar{\psi}_{\nu_e} \dots) | \dots \rangle_{\text{in}}, \quad (17)$$

where the dots stand for the other fields and states involved in the interaction. By using the mixing transformation Equation (5), this can be cast as

$$\mathcal{A}_{\nu_e, \nu_\mu} = \sum_{j=1}^3 U_{\mu j}^* U_{e j} \mathcal{A}_j, \quad \mathcal{A}_j \equiv \text{out} \langle \nu_j, \dots | S_{\text{weak}} (\bar{\psi}_{\nu_j} \dots) | \dots \rangle_{\text{in}}. \quad (18)$$

The decay rate is then

$$\begin{aligned} \Gamma_{\nu_e, \nu_\mu} \sim & |\mathcal{A}_{\nu_e, \nu_\mu}|^2 = |U_{\mu 1}|^2 |U_{e1}|^2 |\mathcal{A}_1|^2 + |U_{\mu 2}|^2 |U_{e2}|^2 |\mathcal{A}_2|^2 + |U_{\mu 3}|^2 |U_{e3}|^2 |\mathcal{A}_3|^2 \\ & + \left(U_{\mu 1}^* U_{e1} U_{\mu 2} U_{e2}^* \mathcal{A}_1 \mathcal{A}_2^* + U_{\mu 1}^* U_{e1} U_{\mu 3} U_{e3}^* \mathcal{A}_1 \mathcal{A}_3^* + U_{\mu 2}^* U_{e2} U_{\mu 3} U_{e3}^* \mathcal{A}_2 \mathcal{A}_3^* + \text{c.c.} \right). \end{aligned} \quad (19)$$

where we have omitted the sum over polarizations and the integration over momenta to simplify the notation.

Let us now consider the mirror-symmetric process (Parity transformation) and swap particles with antiparticles (Charge conjugation). By evaluating the decay rate $\Gamma_{\bar{\nu}_e, \bar{\nu}_\mu}$ as above, we can finally compute the CP quantifier $A_{CP}^{(e, \mu)} = \Gamma_{\nu_e, \nu_\mu} - \Gamma_{\bar{\nu}_e, \bar{\nu}_\mu}$ as

$$A_{CP}^{(e, \mu)} = 4J \left\{ -\text{Im} [\mathcal{A}_1 \mathcal{A}_2^*] + \text{Im} [\mathcal{A}_1 \mathcal{A}_3^*] - \text{Im} [\mathcal{A}_2 \mathcal{A}_3^*] \right\}. \quad (20)$$

As expected, it is non-vanishing (which means that our formalism is capable of describing CP violation in neutrino oscillations) and proportional to the Jarlskog invariant J (which implies that $A_{CP}^{(e, \mu)}$ does not depend on the particular parameterization of the PMNS matrix).

On the other hand, one may try to repeat the same considerations by resorting to the asymptotic mass states. However, as shown in Ref. [15], it is unclear how to take into account CP violation effects in that context, as one would trivially get $A_{CP} = 0$.

3.2. Comoving frame

Let us now describe the proton decay from the point of view of a coaccelerated observer (Rindler observer). As commented above, in the comoving rest-frame the process Equation (1) is forbidden by the energy conservation. Hence, at first glance it would seem that the static proton has no available phase space to decay. However, because of Unruh effect, the new following channels

$$iv) \quad p + e^- \rightarrow n + \nu_e, \quad v) \quad p + \bar{\nu}_e \rightarrow n + e^+, \quad vi) \quad p + e^- + \bar{\nu}_e \rightarrow n, \quad (21)$$

are allowed. We stress that in the above processes a proton lying at rest with the Rindler observer decays into a neutron by the absorption (and possible emission) of leptons from (to) the Unruh thermal bath. In particular, in the process (iv) the proton picks up an electron and decays into a neutron and an electron neutrino (for more details, see Ref. [4]). In what follows, we focus on this process, but similar considerations can be performed for the interactions (v) and (vi).

By following the same scheme as in the laboratory frame, the transition tree-level amplitude for the decay (iv) reads [15]

$$\mathcal{A}^{(iv)} \equiv \langle n | \otimes \langle \nu_e | \hat{S}_I | e^- \rangle \otimes | p \rangle = \frac{G_F}{(2\pi)^2} \sum_{j=1}^3 |U_{ej}|^2 \mathcal{J}_{\sigma_\nu \sigma_e}^{(j)}(\omega_\nu, \omega_e), \quad (22)$$

where

$$\mathcal{J}_{\sigma_\nu \sigma_e}^{(j)}(\omega_\nu, \omega_e) = \delta(\omega_e - \omega_\nu - \Delta m) \bar{u}_{\sigma_\nu}^{(\omega_\nu)} \gamma^0 u_{\sigma_e}^{(\omega_e)}, \quad (23)$$

and we have assumed equal frequencies, transverse momenta and polarizations for neutrino states with definite mass. Note that the Dirac lepton fields must now be quantized according to the Rindler-Fulling procedure [8] and indeed $\omega_{\nu(e)}$ denotes the frequency relative to Rindler time. Furthermore, we have to consider that the probability to absorb (emit) a particle of frequency ω from (to) Unruh thermal bath is provided by the statistical weight $n_F(\omega) = (e^{\omega/T_U} + 1)^{-1}$ ($\tilde{n}_F(\omega) = 1 - n_F(\omega)$), where $T_U = a/2\pi$ is the Unruh temperature [3] (see Refs. [29, 30, 31, 32, 33] for some recent applications). For instance, the transition rate for the process (iv) is given by $n_F(\omega)$ times the square modulus of the amplitude Equation (22) [8].

By performing similar computations for the processes (v) and (vi) and adding up the resulting decay rates, we get

$$\Gamma^{(iv)} + \Gamma^{(v)} + \Gamma^{(vi)} = |U_{e1}|^4 \tilde{\Gamma}_1 + |U_{e2}|^4 \tilde{\Gamma}_2 + |U_{e3}|^4 \tilde{\Gamma}_3 + \left(|U_{e1}|^2 |U_{e2}|^2 \tilde{\Gamma}_{12} + |U_{e1}|^2 |U_{e3}|^2 \tilde{\Gamma}_{13} + |U_{e2}|^2 |U_{e3}|^2 \tilde{\Gamma}_{23} + \text{c.c.} \right), \quad (24)$$

where the diagonal terms are

$$\tilde{\Gamma}_j \equiv \frac{2G_F^2}{a^2 \pi^7 e^{\pi \Delta m/a}} \int d\omega \left\{ \int d^2 k_\nu l_{\nu_j} \left| K_{i(\omega-\Delta m)/a+1/2} \left(\frac{l_{\nu_j}}{a} \right) \right|^2 \int d^2 k_e l_e \left| K_{i\omega/a+1/2} \left(\frac{l_e}{a} \right) \right|^2 \right. \\ \left. + m_{\nu_j} m_e \text{Re} \left[\int d^2 k_\nu K_{i(\omega-\Delta m)/a-1/2}^2 \left(\frac{l_{\nu_j}}{a} \right) \int d^2 k_e K_{i\omega/a+1/2}^2 \left(\frac{l_e}{a} \right) \right] \right\}, \quad (25)$$

while the off-diagonal contribution read

$$\tilde{\Gamma}_{jk} = \frac{G_F^2}{a^2 \pi^7 e^{\pi \Delta m/a}} \int d\omega \left\{ \int d^2 k_e l_e \left| K_{i\omega/a+1/2} \left(\frac{l_e}{a} \right) \right|^2 \int \frac{d^2 k_\nu}{\sqrt{l_{\nu_j} l_{\nu_k}}} \left((k_\nu^x)^2 + (k_\nu^y)^2 + m_{\nu_j} m_{\nu_k} + l_{\nu_j} l_{\nu_k} \right) \right. \\ \times \left[K_{i(\omega-\Delta m)/a+1/2} \left(\frac{l_{\nu_j}}{a} \right) K_{i(\omega-\Delta m)/a-1/2} \left(\frac{l_{\nu_k}}{a} \right) \right] + \frac{m_e}{\sqrt{l_{\nu_j} l_{\nu_k}}} \int d^2 k_e \int d^2 k_\nu (l_{\nu_j} m_{\nu_k} + l_{\nu_k} m_{\nu_j}) \\ \left. \times \left[K_{i\omega/a+1/2}^2 \left(\frac{l_e}{a} \right) K_{i(\omega-\Delta m)/a-1/2} \left(\frac{l_{\nu_j}}{a} \right) K_{i(\omega-\Delta m)/a-1/2} \left(\frac{l_{\nu_k}}{a} \right) \right] \right\}. \quad (26)$$

Here we have defined the reduced frequency $l_{e(\nu)} = \sqrt{(k^x)^2 + (k^y)^2 + m_{e(\nu)}^2}$.

As in the laboratory frame, the total decay rate for the comoving observer also gets non-vanishing contributions from the flavor-violating processes

$$vii) \quad p + e^- \rightarrow n + \nu_\mu, \quad viii) \quad p + \bar{\nu}_\mu \rightarrow n + e^+, \quad ix) \quad p + e^- + \bar{\nu}_\mu \rightarrow n, \quad (27)$$

due to the $\nu_e \rightarrow \nu_\mu$, $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ oscillations, and from the processes

$$x) \quad p + e^- \rightarrow n + \nu_\tau, \quad xi) \quad p + \bar{\nu}_\tau \rightarrow n + e^+, \quad xii) \quad p + e^- + \bar{\nu}_\tau \rightarrow n, \quad (28)$$

due to the $\nu_e \rightarrow \nu_\tau$, $\bar{\nu}_e \rightarrow \bar{\nu}_\tau$ oscillations. We emphasize that, whilst the processes (vii) and (x) are of the same type as (ii) and (iii) since they simply describe the oscillation of the emitted neutrino, the channels $(viii)$, (ix) , (xi) and (xii) are characterized by new physics with respect to (v) and (vi) , since they show that also the (anti-)neutrinos in the Unruh bath undergo oscillations. This non-trivial result was first pointed out in Ref. [14].

Now, the decay rate for the processes Equation (27), Equation (28) can be evaluated by following the same steps as in Eqs. Equation (22)-Equation (26). We have [15]

$$\Gamma^{(vii)} + \Gamma^{(viii)} + \Gamma^{(ix)} = |U_{\mu 1}|^2 |U_{e1}|^2 \tilde{\Gamma}_1 + |U_{\mu 2}|^2 |U_{e2}|^2 \tilde{\Gamma}_2 + |U_{\mu 3}|^2 |U_{e3}|^2 \tilde{\Gamma}_3 + \left(U_{\mu 1}^* U_{e1} U_{\mu 2} U_{e2}^* \tilde{\Gamma}_{12} + U_{\mu 1}^* U_{e1} U_{\mu 3} U_{e3}^* \tilde{\Gamma}_{13} + U_{\mu 2}^* U_{e2} U_{\mu 3} U_{e3}^* \tilde{\Gamma}_{23} + \text{c.c.} \right), \quad (29)$$

$$\Gamma^{(x)} + \Gamma^{(xi)} + \Gamma^{(xii)} = |U_{\tau 1}|^2 |U_{e1}|^2 \tilde{\Gamma}_1 + |U_{\tau 2}|^2 |U_{e2}|^2 \tilde{\Gamma}_2 + |U_{\tau 3}|^2 |U_{e3}|^2 \tilde{\Gamma}_3 + \left(U_{\tau 1}^* U_{e1} U_{\tau 2} U_{e2}^* \tilde{\Gamma}_{12} + U_{\tau 1}^* U_{e1} U_{\tau 3} U_{e3}^* \tilde{\Gamma}_{13} + U_{\tau 2}^* U_{e2} U_{\tau 3} U_{e3}^* \tilde{\Gamma}_{23} + \text{c.c.} \right), \quad (30)$$

Finally, the total decay rate in the comoving frame reads

$$\Gamma^{\text{com}} \equiv \sum_{s=iv}^{xii} \Gamma^{(s)} = |U_{e1}|^2 \tilde{\Gamma}_1 + |U_{e2}|^2 \tilde{\Gamma}_2 + |U_{e3}|^2 \tilde{\Gamma}_3, \quad (31)$$

where we have used once again the unitarity of the mixing matrix.

In order for our formalism to be generally covariant, we must have the equality between the laboratory and comoving scalar decay rates for each neutrino flavor. In this regard, we refer to Ref. [8], where it was shown that

$$\Gamma_j = \tilde{\Gamma}_j, \quad (32)$$

$$\Gamma_{jk} = \tilde{\Gamma}_{jk}, \quad (33)$$

with the last equality being valid at least in the (realistic) approximation of small neutrino mass differences. By means of the above relations, it follows that

$$\Gamma^{(i)} = \Gamma^{(iv)} + \Gamma^{(v)} + \Gamma^{(vi)}, \quad (34)$$

$$\Gamma^{(ii)} = \Gamma^{(vii)} + \Gamma^{(viii)} + \Gamma^{(ix)}, \quad (35)$$

$$\Gamma^{(iii)} = \Gamma^{(x)} + \Gamma^{(xi)} + \Gamma^{(xii)}, \quad (36)$$

which finally imply

$$\Gamma^{\text{lab}} = \Gamma^{\text{com}}. \quad (37)$$

Therefore, we find that the use of asymptotic flavor neutrinos complies with the general covariance of QFT as well⁴.

4. Conclusion and outlook

We have gone through the study of the inverse β -decay in the context of neutrino mixing. By computing the scalar decay rate both in the laboratory and in the comoving frames, we have shown that the description of asymptotic neutrinos by means of flavor states allows to get consistency with *i*) the general covariance of QFT, *ii*) the phenomenologically observed flavor oscillations and *iii*) the related CP violations effects. We stress that our approach relies solely on principles built into the Theory. On the other hand, one might think of resorting to the mass representation on the basis of considerations involving measurement issues. Clearly, due to the absence of experimental evidences, neither of the two scenarios can be definitively ruled out at present. However, if one accepts the mass description as the correct one, then a major revision of the SM and of its extension including CP violation is required, since this description would imply a non-conservation of the family lepton numbers in the interaction vertices, as well as possible inconsistencies with the description of CP violation in neutrino oscillations. Of course, we would be willing to contemplate this possibility, given that the (extended) SM is still in progress. Nevertheless, before moving along such a drastic direction, all the more conventional paths must be pursued. The flavor representation above all.

Finally, we emphasize that a conceptually similar situation happens in the Stueckelberg-Horwitz-Piron framework [34, 35], where the choice of mass eigenstates turns out to be likewise

⁴ Actually, we notice that the same conclusion would be reached by working in the mass representation. Indeed, by considering the state $|\nu_i\rangle$ as asymptotic, the comoving decay rate for the three channels involving, e.g. the electron neutrino, would be $\Gamma = |U_{ei}|^2 \tilde{\Gamma}_i$, which is still consistent with the corresponding result in the laboratory (see the discussion below Eq. Equation (11)). This explains why the only requirement of general covariance is not enough to discriminate between the two representations.

less compelling. In that case, indeed, the mass spectrum may be continuous on the whole real line and does not actually characterize anything about different particles. This aspect and other issues related to the flavor-mass controversy will be deeply investigated elsewhere.

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