Graviton Noise Correlation in Nearby Detectors

Maulik Parikh, a,b Francesco Settia

^aDepartment of Physics, Arizona State University, AZ 85287, United States

E-mail: maulik.parikh@asu.edu, fsetti@asu.edu

ABSTRACT: We consider quantum gravity fluctuations in a pair of nearby gravitational wave detectors. Quantum fluctuations of long-wavelength modes of the gravitational field induce coherent fluctuations in the detectors, leading to correlated noise. We determine the variance and covariance in the lengths of the arms of the detectors, and thereby obtain the graviton noise correlation. We find that the correlation depends on the angle between the detector arms as well as their separation distance.

^bBeyond: Center for Fundamental Concepts in Science, Arizona State University, Tempe, Arizona 85287, USA

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1 Introduction

It is widely expected on theoretical grounds that gravity must ultimately obey the laws of quantum mechanics [1-4]. By contrast there is, as yet, no compelling experimental evidence for the quantization of gravity. Smoking-gun proofs of the quantization of gravity are remarkably challenging to find [5]. Indeed, it is challenging enough to find any distinctive observational signature of quantum gravity, smoking-gun or otherwise. In particular, it has been argued by Dyson that single-graviton detection is virtually impossible [6–8]. However, it is not necessary to detect individual gravitons to detect observational signatures of quantum gravity: just as Brownian noise can be observed even when collisions with individual molecules cannot, it can be that graviton noise is observable even when individual gravitons are not. In a series of papers, Parikh, Wilczek, and Zahariade (PWZ) calculated the effect of a quantized spacetime metric on a pair of free-falling particles [9-11]; they found that quantum-gravitational fluctuations induce stochastic fluctuations (i.e. noise) in the separation of the particles. The particle separation, which in classical general relativity would be governed by the geodesic deviation equation, now obeys a Langevin-like stochastic equation, a kind of quantum geodesic deviation equation. These results, which were extended by Cho and Hu [12] and obtained by a different approach by Kanno et al [13, 14], are exciting because they indicate the presence of quantum-gravitational fluctuations ("the noise of gravitons") in the separation of the mirrors of a gravitational wave interferometer. The statistical properties of the noise depend on the quantum state of the gravitational field and can be greatly enhanced for certain classes of states, notably squeezed states. These results have been corroborated by several other authors [15–22]. In addition, PWZ's work has inspired much theoretical effort to better understand the behavior of the noise due to gravitons. For example, researchers have calculated the quantum corrections to the trajectory of a free-falling particle [23], and the quantum corrections in the Raychaudhuri

equation [24–26]. Furthermore, PWZ's formalism has been used to study the quantization of cylindrical gravitational waves [27], and the interest in squeezed states has led to a study on whether Virgo-LIGO data could be compatible with a gravitational field in such a quantum state [28].

These calculations are the beginning of attempts to extract a precise observational signature of the quantization of gravity that experimentalists could search for. In order for the noise to be observable, however, two requirements need to be met. First, the amplitude of the noise must be large enough to be observable. As mentioned, this requires the gravitational field to be in an especially noisy state, such as a squeezed state; an open theoretical problem is determining whether there are realistic astrophysical scenarios that might give rise to such states. Second, the noise needs to be distinguishable from the numerous other sources of noise that gravitational wave interferometers are subject to.

Although graviton noise has a characteristic spectrum (depending on the state), it can nevertheless be challenging to distinguish it from the many other types of noise present in gravitational wave detectors, such as thermal noise, seismic noise, electronic noise, and photon shot noise, among others [29]. To address this problem PWZ suggested that the quantum noise due to gravitons might be correlated in different detectors. This would help in the isolation of graviton noise because other types of noise are generically uncorrelated between different detectors. Intuitively, quantum fluctuations in long-wavelength modes of the gravitational field have the same coupling to nearby detectors and therefore lead to correlated stochastics. In this paper, we analyze the correlations in graviton noise, deriving the noise covariance, the standard deviation, and the correlation. We predict the existence of correlated quantum-gravitational noise between nearby detector arms, with the correlation dependent on the angle between the arms and their separation distance. Based on the angular dependence of the graviton noise correlation, we suggest a configuration of gravitational wave interferometers to distinguish quantum-gravitational noise from environmental noise.

2 Derivation of the Correlation

2.1 Obtaining the Equations of Motion

Consider a system with four masses m_0 , m_1 , m_2 , and m_3 . Let m_0 and m_1 be the mirrors of the first detector, and let m_2 and m_3 be the mirrors of the second detector. Let the metric be $g_{\mu\nu} = \eta_{\mu\nu} + \kappa h_{\mu\nu}$ where $\kappa^2 = 16\pi G$ and $h_{\mu\nu}$ is the metric perturbation due to gravitons.

We set up Fermi normal coordinates at the origin, so that m_0 is stationary. In addition, let ξ^i be the vector from m_0 to m_1 , η^i be the vector from m_0 to m_2 , and ζ^i be the vector from m_0 to m_3 . Since we are in Fermi normal coordinates, the components of these vectors are proper lengths. In addition, $\xi = \sqrt{\delta_{ij}\xi^i\xi^j} = \xi^3$, is the arm length of the first detector, while we define the arm length of the second detector to be $\rho = \sqrt{\delta_{ij}\rho^i\rho^j}$, where $\rho^i = \zeta^i - \eta^i$.

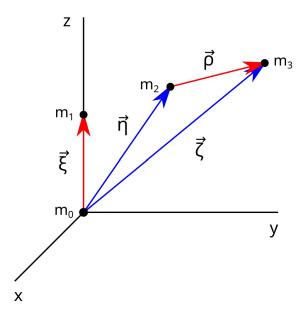


Figure 1. In the above image, m_0 and m_1 are the mirrors of the first detector, while m_2 and m_3 are the mirrors of the second detector. $\vec{\xi}$ and $\vec{\rho}$, shown in red, are the arms of the two detectors while $\vec{\eta}$ and $\vec{\zeta}$, shown in blue, are the positions of the mirrors of the second detector. Since we are working in Fermi normal coordinates, all lengths are proper lengths.

Working in linearized gravity, it is straightforward to show that the classical action for the system is

$$S_{\text{total}} = S_{\text{grav}} + S_{m_0} + S_{m_1} + S_{m_2} + S_{m_3}$$

$$= -\frac{1}{4} \int d^4 x \, \partial_\alpha h_{\mu\nu}(x) \partial^\alpha h^{\mu\nu}(x) + \int dt \, \frac{\delta_{ij}}{2} [m_1 \dot{\xi}^i \dot{\xi}^j + m_2 \dot{\eta}^i \dot{\eta}^j + m_3 \dot{\zeta}^i \dot{\zeta}^j]$$

$$+ \frac{\kappa}{4} \ddot{h}_{ij}(x) [m_1 \xi^i \xi^j + m_2 \eta^i \eta^j + m_3 \zeta^i \zeta^j],$$
(2.1)

where the metric perturbation can be decomposed into modes using the following equation

$$h_{\mu\nu}(x) = \int dk^3 \sum_{s=+,\times} \epsilon_{\mu\nu}^{(s)}(\vec{k}) h^{(s)}(\vec{k},t) e^{i\vec{k}\cdot\vec{x}}$$
 (2.2)

Notice that in equation (2.1) we implicitly applied a dipole approximation in order to factor out the second time derivative of the metric perturbation in the last line. This dipole approximation is needed because we are only interested in the modes that are correlated in the two detectors. Because of this approximation, the integral in equation (2.2) is only over the modes that have a wavelength much larger than the dimensions of the system.

Now, suppose that the masses are initially in a quantum state $|A\rangle$ and that the gravitational field is initially in a state $|\Psi\rangle$. Then, the probability that the system is in final state $|B\rangle$ is given by the following path integral

$$P_{\Psi}(A \to B) \sim \int \mathcal{D}\xi \, \mathcal{D}\xi' \, \mathcal{D}\eta \, \mathcal{D}\eta' \mathcal{D}\zeta \, \mathcal{D}\zeta' \left\{ e^{\frac{i}{\hbar} \int dt \frac{1}{2} [m_1(\dot{\xi}^2 - \dot{\xi}'^2) + m_2(\dot{\eta}^2 - \dot{\eta}'^2) + m_3(\dot{\zeta}^2 - \dot{\zeta}'^2)]} \right.$$

$$\left. F_{\Psi}[\xi, \xi', \eta, \eta', \zeta, \zeta'] \right\}$$
(2.3)

where F_{Ψ} is the Feynman-Vernon influence functional [30]. Generalizing Cho and Hu's calculations [12] to the case of 4 stationary masses, we find that the influence functional can be written in terms of the noise kernel

$$K_{ijkl}^{\Psi}(t,t') = \frac{1}{2} \frac{d^2}{dt^2} \frac{d^2}{dt'^2} \int d^3k \, d^3k' \int d^3x \, d^3x' \, e^{-i\vec{k}\cdot\vec{x}} e^{-i\vec{k}'\cdot\vec{x}'} \sum_{s} \epsilon_{ij}^{(s)}(\vec{k}) \epsilon_{kl}^{(s)}(\vec{k}') G_{\Psi}^{(1)}(x,x')$$
(2.4)

with

$$G_{\Psi}^{(1)}(x,x') = \langle \Psi | \{ h(x), h(x') \} | \Psi \rangle \tag{2.5}$$

Next, it is possible to rewrite the part of the influence functional that involves the noise kernel using a trick due to Feynman and Vernon. As discussed in [9,12], this introduces in the action a stochastic tensor \mathcal{N}_{ij} that has the following properties

$$P[\mathcal{N}] = \mathcal{C}e^{-\frac{1}{2}\int \mathcal{N}_{ij}[(K^{\Psi})^{-1}]^{ijkl}\mathcal{N}_{kl}}$$
(2.6)

$$\langle \mathcal{N}_{ij}(t) \rangle_s = \int \mathcal{D}\mathcal{N} P[\mathcal{N}] \mathcal{N}_{ij}(t) = 0$$
 (2.7)

$$\langle \mathcal{N}_{ij}(t)\mathcal{N}_{kl}(t')\rangle_s = \int \mathcal{D}\mathcal{N} P\left[\mathcal{N}\right]\mathcal{N}_{ij}(t)\mathcal{N}_{kl}(t') = K_{ijkl}^{\Psi}(t,t'),$$
 (2.8)

where (2.6) is a Gaussian probability density, and the subscript s in (2.7) and (2.8) denotes a statistical average. Using the new form of the action that contains the stochastic tensor, we can obtain the equations of motion by evaluating the saddle point of the path integral, which is obtained by taking a derivative of the above action with respect to ξ , η , or ζ , and setting it equal to zero. By taking another derivative with respect to time, and dropping the radiation reaction terms, we obtain

$$\ddot{\xi}^{i}(t) - 2\alpha \delta^{ip} \mathcal{N}_{pj}(t) \xi^{j}(t) = 0$$

$$\ddot{\eta}^{i}(t) - 2\alpha \delta^{ip} \mathcal{N}_{pj}(t) \eta^{j}(t) = 0$$

$$\ddot{\zeta}^{i}(t) - 2\alpha \delta^{ip} \mathcal{N}_{pj}(t) \zeta^{j}(t) = 0$$
(2.9)

where $\alpha = \kappa/2\sqrt{2}(2\pi)^3$. These are Langevin-like equations in which the arm length of the detectors is affected by the stochastic tensor. Notice that the equations of motion are uncoupled. This means that the motion of one mass does not influence the motion of the other mass. This is what we expect since, by ignoring the radiation reaction terms, we made the approximation that the masses emit no gravitational waves when they move, and so they have no way of interacting with one another. Also, this implies that any correlation in the motion of the masses is indeed due to the background gravitons.

2.2 Solving the Equations of Motion to Obtain the 2-point Functions

Hi by Following [24], we will solve the equations of motion perturbatively. Let us begin by solving the equation of motion for m_1 . The equations of motion for the other masses are solved in an analogous way.

Let $\xi^i(t) = \xi^i_0(t) + \xi^i_1(t) + \xi^i_2(t) + \dots$, where the superscript indicates the spatial vector component and the subscripts indicate the order, which corresponds to the number of

stochastic tensor factors. Then, the equation for ξ becomes

$$\ddot{\xi}_0^i(t) = 0\,, (2.10)$$

$$\ddot{\xi}_1^i(t) = 2\alpha \,\delta^{ij} \mathcal{N}_{jk}(t) \,\xi_0^k(t) \,, \tag{2.11}$$

$$\ddot{\xi}_2^i(t) = 2\alpha \,\delta^{ij} \mathcal{N}_{jk}(t) \,\xi_1^k(t) \,, \tag{2.12}$$

Integrating with respect to time, we find

$$\xi_1^i(t) = 2\alpha \,\delta^{ij} \int_0^t dt' \int_0^{t'} dt'' \,\mathcal{N}_{jk}(t'') \,\xi_0^k \tag{2.13}$$

$$\xi_2^i(t) = (2\alpha)^2 \, \delta^{ij} \delta^{kl} \int_0^t dt' \int_0^{t'} dt'' \int_0^{t''} dt''' \int_0^{t'''} dt'''' \, \mathcal{N}_{jk}(t'') \mathcal{N}_{lm}(t'''') \, \xi_0^m, \tag{2.14}$$

where we assumed that the unperturbed arm length ξ_0^i is a constant. The equations for $\eta_1^i(t)$ and $\eta_2^i(t)$, and $\zeta_1^i(t)$ and $\zeta_2^i(t)$ are the same as (2.13) and (2.14), but with η_0^i and ζ_0^i replacing ξ_0^i .

We can now calculate the 2-point functions. It turns out that all we need to obtain the noise correlation are the 2-point functions of $\xi_1^i(t)$, $\eta_1^i(t)$, and $\zeta_1^i(t)$. In the next section, we will show why this is the case. For now, we will just calculate the various 2-point functions that we will use later.

Since we will not use terms like $\xi_2^i(t)$ and other higher order expansion terms, we will relabel $\xi_1^i(t)$, $\eta_1^i(t)$, and $\zeta_1^i(t)$ as $\delta \xi^i(t)$, $\delta \eta^i(t)$, and $\delta \zeta^i(t)$, respectively, so that

$$\xi^{i}(t) = \xi_{0}^{i}(t) + \delta \xi^{i}(t) + \mathcal{O}(\mathcal{N}^{2}) + \dots,
\eta^{i}(t) = \eta_{0}^{i}(t) + \delta \eta^{i}(t) + \mathcal{O}(\mathcal{N}^{2}) + \dots,
\zeta(t)^{i} = \zeta_{0}^{i}(t) + \delta \zeta^{i}(t) + \mathcal{O}(\mathcal{N}^{2}) + \dots.$$
(2.15)

Let's start by finding $\langle \delta \xi^i(t) \delta \eta^j(t) \rangle_s$. Using equation (2.13) and (2.8), we find

$$\left\langle \delta \xi^{i}(t) \delta \eta^{j}(t') \right\rangle_{s} = (2\alpha)^{2} \, \delta^{ik} \delta^{jl} \xi_{0}^{m} \eta_{0}^{n} \int_{0}^{t} dt' \int_{0}^{t'} dt'' \int_{0}^{t'^{3}} dt'^{4} \int_{0}^{t'^{4}} dt'' \int_{0}^{t'^{$$

where the noise kernel in the Minkowski vacuum is given by

$$K_{ijkl}^{0}(t,t') = -\frac{32\pi^{4}}{15} [2\delta_{ij}\delta_{kl} - 3(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})] \int_{0}^{\Lambda} d\omega \,\omega^{5} \cos[\omega(t-t')], \tag{2.17}$$

with Λ being the cut-off frequency. We impose a cut-off frequency because if not regularized, this integral would be divergent. Additionally, since we are only interested in the correlated modes, the cut-off is naturally $\Lambda \sim L^{-1}$, where L is the largest separation between any two masses in our apparatus. For example, in Figure 1, $L = |\vec{\zeta}|$. We also point out that the noise kernel for different quantum states that are rotationally invariant have the same factors of Kronecker delta's present in equation (2.17). This means that although the variance of the quantum noise is different, its angular dependency is the same. For instance, this is true for thermal states and squeezed states [12,13]. Additionally, the noise

kernel for a squeezed state with squeeze parameter r would also be enhanced by a factor of e^r . Evaluating the integrals and taking the limit $t' \to t$, we find

$$\left\langle \delta \xi^{i}(t) \delta \eta^{j}(t) \right\rangle_{s} = A(\Lambda, t) \delta^{ik} \delta^{jl} \xi_{0}^{m} \eta_{0}^{n} [-2\delta_{km} \delta_{ln} + 3(\delta_{kl} \delta_{mn} + \delta_{kn} \delta_{ml})], \tag{2.18}$$

where we defined the unitless time-dependent amplitude

$$A(\Lambda, t) \simeq \frac{\kappa^2}{240\pi^2} \Lambda^4 t^2 \tag{2.19}$$

More information about this calculation can be found in [12] and [24]. Since there are different ways of regularizing the integral in equation (2.17), we only keep the first term in $A(\Lambda, t)$. More information about this can be found in our very recent paper [26].

Using equation (2.18) we can now find the 2-point functions for all components. Since $\xi^i(t)$ is along the z-axis, we know that its only nonzero component is $\xi^3(t)$, which is also equal to its magnitude. In addition, we only need to consider $\delta \xi^3(t)$, since we cannot measure $\delta \xi^1(t)$ and $\delta \xi^2(t)$ due to the fact that these fluctuations are perpendicular to the path of the laser in the detector. Then, the only 2-point functions of interest involving $\delta \xi^i(t)$ and $\delta \eta^i(t)$ are

$$\begin{split} \left\langle \delta \xi^3(t) \delta \eta^1(t) \right\rangle_s &= -2 \, \xi_0^3 \eta_0^1 \, A(\Lambda, t) \\ \left\langle \delta \xi^3(t) \delta \eta^2(t) \right\rangle_s &= -2 \, \xi_0^3 \eta_0^2 \, A(\Lambda, t) \\ \left\langle \delta \xi^3(t) \delta \eta^3(t) \right\rangle_s &= +4 \, \xi_0^3 \eta_0^3 \, A(\Lambda, t). \end{split} \tag{2.20}$$

Notice that the 2-point functions for perpendicular components are proportional to -2, while the 2-point functions for parallel components are proportional to +4. In addition, all 2-point functions of $\delta \xi^i(t)$ and $\delta \eta^j(t)$ are directly proportional to corresponding factors of ξ^i_0 and η^j_0 . Finally, all 2-point functions are proportional to the amplitude $A(\Lambda, t)$. The same is true for all other 2-point functions involving $\delta \xi^3(t)$ and $\delta \zeta^i(t)$. Specifically,

$$\begin{split} \left\langle \delta \xi^3(t) \delta \zeta^1(t) \right\rangle_s &= -2 \, \xi_0^3 \zeta_0^1 \, A(\Lambda, t) \\ \left\langle \delta \xi^3(t) \delta \zeta^2(t) \right\rangle_s &= -2 \, \xi_0^3 \zeta_0^2 \, A(\Lambda, t) \\ \left\langle \delta \xi^3(t) \delta \zeta^3(t) \right\rangle_s &= +4 \, \xi_0^3 \zeta_0^3 \, A(\Lambda, t). \end{split} \tag{2.21}$$

2.3 Obtaining the Noise Correlation

Our goal in this section is to obtain the graviton noise correlation in the two detectors, which is

$$\operatorname{corr}(\delta \xi, \delta \rho) = \frac{\operatorname{cov}(\delta \xi, \delta \rho)}{\sqrt{\operatorname{Var}(\delta \xi)} \sqrt{\operatorname{Var}(\delta \rho)}}$$
 (2.22)

Notice that in this equation $\delta \xi$ and $\delta \rho$ are not vectors, they are magnitudes. This is because we are interested in the fluctuations in the detectors' arms, not in the fluctuations in their Cartesian components.

Let's start by determining $cov(\delta \xi, \delta \rho)$. To do this, let's momentarily go back to our original notation where $\xi(t) = \xi_0(t) + \xi_1(t) + \xi_2(t) + \dots$ and $\rho(t) = \rho_0(t) + \rho_1(t) + \rho_2(t) + \dots$

By definition, we have

$$cov(\xi(t), \rho(t)) = \langle [\xi(t) - \langle \xi(t) \rangle_s] [\rho(t) - \langle \rho(t) \rangle_s] \rangle_s
= \langle [\xi_0(t) + \xi_1(t) + \xi_2(t) + \dots - \langle \xi_0(t) + \xi_1(t) + \xi_2(t) + \dots \rangle_s]
[\rho_0(t) + \rho_1(t) + \rho_2(t) + \dots - \langle \rho_0(t) + \rho_1(t) + \rho_2(t) + \dots \rangle_s] \rangle_s
= \langle \xi_1(t) \rho_1(t) \rangle_s + \mathcal{O}(\mathcal{N}^3) + \mathcal{O}(\mathcal{N}^4) + \dots
= \langle \xi_1(t) \rho_1(t) \rangle_s = cov(\delta \xi(t), \delta \rho(t)),$$
(2.23)

where we used the fact that $\langle \delta \xi(t) \rangle_s = \langle \delta \rho(t) \rangle_s = 0$. This means that, to leading order in the stochastic tensor \mathcal{N} , the covariance of the detectors arm length is equal to the covariance of the noise. The same is also true for the correlation. In addition, the above equation also demonstrated why we can neglect $\xi_2(t)$, $\rho_2(t)$, and all higher order terms.

Next, using the fact that $\vec{\xi}(t)$ is along the z-axis, we find

$$cov(\delta\xi(t),\delta\rho(t)) = \langle \delta\xi(t)\delta\rho(t)\rangle_s = \langle \left[\delta\vec{\xi}(t)\cdot\vec{\xi_0}/\xi_0\right] \left[\delta\vec{\rho}(t)\cdot\vec{\rho_0}/\rho_0\right]\rangle_s
= (\langle \delta\xi^3(t)\delta\rho^1(t)\rangle_s \rho_0^1 + \langle \delta\xi^3(t)\delta\rho^2(t)\rangle_s \rho_0^2 + \langle \delta\xi^3(t)\delta\rho^3(t)\rangle_s \rho_0^3)/\rho_0$$
(2.24)

The 2-point functions in the above equation are evaluated exactly like the ones in the previous section. For example,

$$\begin{split} \left\langle \delta \xi^{3}(t) \delta \rho^{1}(t) \right\rangle_{s} &= \left\langle \delta \xi^{3}(t) [\delta \zeta^{1}(t) - \delta \eta^{1}(t)] \right\rangle_{s} \\ &= \left\langle \delta \xi^{3}(t) \delta \zeta^{1}(t) \right\rangle_{s} - \left\langle \delta \xi^{3}(t) \delta \eta^{1}(t) \right\rangle_{s} \\ &= -2 \, \xi_{0}^{3} (\zeta_{0}^{1} - \eta_{0}^{1}) \, A(\Lambda, t) \\ &= -2 \, \xi_{0}^{3} \rho_{0}^{1} \, A(\Lambda, t) \end{split} \tag{2.25}$$

With similar calculations, we determine

$$\langle \delta \xi^3(t) \delta \rho^2(t) \rangle_s = -2 \, \xi_0^3 \rho_0^2 \, A(\Lambda, t) \langle \delta \xi^3(t) \delta \rho^3(t) \rangle_s = +4 \, \xi_0^3 \rho_0^3 \, A(\Lambda, t)$$
(2.26)

Substituting these into equation (2.24), we determine

$$cov(\delta\xi(t), \delta\rho(t)) = \left[-2\left(\rho_0^1\right)^2 - 2\left(\rho_0^2\right)^2 + 4\left(\rho_0^3\right)^2 \right] A(\Lambda, t) \, \xi_0/\rho_0 \tag{2.27}$$

Next, we need to find $Var(\delta \xi(t))$ and $Var(\delta \rho(t))$. With a calculation extremely similar to the one of equation (2.23), we determine

$$\operatorname{Var}(\xi(t)) \approx \operatorname{Var}(\delta \xi(t)) = \langle \delta \xi(t) \delta \xi(t) \rangle_{s}$$

$$\operatorname{Var}(\rho(t)) \approx \operatorname{Var}(\delta \rho(t)) = \langle \delta \rho(t) \delta \rho(t) \rangle_{s}$$
(2.28)

Since $\vec{\xi}(t)$ is along the z-axis,

$$\operatorname{Var}(\delta \xi(t)) = \left\langle \delta \xi^{3}(t) \delta \xi^{3}(t) \right\rangle_{s} = +4 \left(\xi_{0} \right)^{2} A(\Lambda_{\xi}, t), \tag{2.29}$$

where $\Lambda_{\xi} \sim \xi^{-1}$ is cut-off frequency of the first detector. Similarly, it is straightforward to show that

$$\operatorname{Var}(\delta\rho(t)) = \left\langle \delta\rho^{3}(t)\delta\rho^{3}(t)\right\rangle_{s} = +4\left(\rho_{0}\right)^{2}A(\Lambda_{\rho}, t), \tag{2.30}$$

where $\Lambda_{\rho} \sim \rho_0^{-1}$ is cut-off frequency of the second detector. The details of this calculation are shown in the appendix.

Substituting equations (2.27), (2.29), and (2.30) into equation (2.22), we determine

$$corr(\delta\xi(t), \delta\rho(t)) = \frac{A(\Lambda, t)}{\sqrt{A(\Lambda_{\xi}, t)A(\Lambda_{\rho}, t)}} \frac{\left[-\left(\rho_0^1\right)^2 - \left(\rho_0^2\right)^2 + 2\left(\rho_0^3\right)^2\right]}{2\left(\rho_0\right)^2}$$
(2.31)

Finally, after converting the above equation from Cartesian coordinates to spherical coordinates and writing the amplitudes A in terms of the arm lengths of the detectors and the size of the system, we obtain

$$\operatorname{corr}(\delta \xi(t), \delta \rho(t)) \simeq \frac{\xi_0^2 \rho_0^2}{L^4} \left[\frac{1}{2} \left(3 \cos^2 \theta - 1 \right) \right], \tag{2.32}$$

where the polar angle θ is the angle between the two detectors, since $\vec{\xi}(t)$ is along the z-axis. Also, as previously mentioned, $\operatorname{corr}(\xi(t), \rho(t)) = \operatorname{corr}(\delta \xi(t), \delta \rho(t))$.

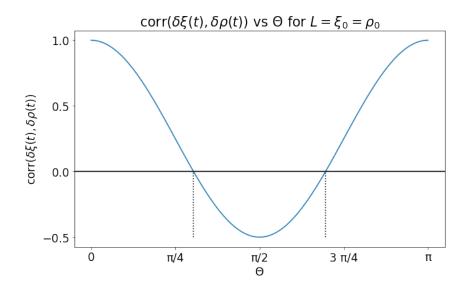


Figure 2. Plot of the correlation $\operatorname{corr}(\delta\xi(t),\delta\rho(t))$ vs the polar angle θ for $L=\xi_0=\rho_0$ as described by equation (2.32). When the detectors are aligned the noise correlation is 1, while when they are orthogonal the noise correlation is -1/2. The correlation is zero at $\theta=\cos^{-1}\left(\pm 1/\sqrt{3}\right)$, which correspond to roughly 54.7° and 125.3°.

2.4 Phenomenology

Equation (2.32) indicates that the correlation of the graviton noise in the two detectors is constant in time and it depends on the arm length of the detectors, the size of the system, and the angle between the orientation of detectors. Since the size of the system is always smaller than or equal to the arm length of the individual detectors, the factor that involves the ξ , ρ , and L is never larger than one, and it is equal to one only if $L = \xi_0 = \rho_0$. The closer the detectors are, the larger this factor is. On the contrary, this ratio becomes zero in the limit that the detectors are infinitely far away from each other.

Figure 2 contains the plot of the correlation $\operatorname{corr}(\delta\xi(t),\delta\rho(t))$ vs the polar angle θ for the special case $L=\xi_0=\rho_0$. As we can see, a maximum correlation of 1 occurs when theta is 0 or π , meaning that the correlation is maximized when the detectors are aligned. On the contrary, a correlation of -1/2 occurs when the detectors are orthogonal so that θ is $\pi/2$. The correlation is never below this value. Finally, the correlation is zero for values of θ of roughly 54.7° and 125.3°.

3 Discussion

We have calculated the correlation in the fundamental quantum-gravitational noise between a pair of gravitational wave detector arms. While the amplitude of the noise depends on the quantum state of the gravitational field, the correlation has a characteristic form, at least for states with rotational symmetry such as the vacuum state. We found, unsurprisingly, that the correlation falls off as the arms are separated in space. The correlation otherwise depends only on the angle between the orientations of the detector arms: it is maximal when they are aligned and zero for $\cos^2\theta = 1/3$ i.e. for an angle of 54.7° or 125.3°. This suggests that, in order to maximally distinguish the gravitational noise from other sources of noise, an ideal configuration of gravitational wave detectors could consist of three arms, two of which are aligned with the third at an angle of 54.7° or 125.3°. The detectors should be close enough to maximize the correlation while still being far enough apart to eliminate correlated noise from other environmental sources. Under these conditions, noise that was correlated between the two aligned detectors but uncorrelated with the third one would likely be graviton noise. Conversely, our result can be used to identify the correlated environmental noise by assessing whether there is any correlation in the noise of the detectors placed at an angle of 54.7° or 125.3°: if such correlations are observed, then the observed noise cannot be due to gravitons.

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A Appendix

Here are the calculations to derive the variance of $\delta \rho(t)$ given in equation (2.30). Since $\langle \delta \rho(t) \rangle_s = 0$, we have

$$\operatorname{Var}(\delta\rho(t)) = \langle \delta\rho(t)\delta\rho(t)\rangle_{s}$$

$$= \langle [\delta\vec{\rho}(t) \cdot \vec{\rho_{0}}/\rho_{0}] [\delta\vec{\rho}(t) \cdot \vec{\rho_{0}}/\rho_{0}]\rangle_{s}$$

$$= \left[\langle \delta\rho^{1}(t)\delta\rho^{1}(t)\rangle_{s} (\rho_{0}^{1})^{2} + \langle \delta\rho^{2}(t)\delta\rho^{2}(t)\rangle_{s} (\rho_{0}^{2})^{2} + \langle \delta\rho^{3}(t)\delta\rho^{3}(t)\rangle_{s} (\rho_{0}^{3})^{2} + 2\langle \delta\rho^{1}(t)\delta\rho^{2}(t)\rangle_{s} \rho_{0}^{1}\rho_{0}^{2} + 2\langle \delta\rho^{1}(t)\delta\rho^{3}(t)\rangle_{s} \rho_{0}^{1}\rho_{0}^{3} + 2\langle \delta\rho^{2}(t)\delta\rho^{3}(t)\rangle_{s} \rho_{0}^{2}\rho_{0}^{3} \right] / (\rho_{0})^{2}$$

$$(A.1)$$

Thus, we need to evaluate the 2-point functions involving all possible combinations of the components of $\delta \rho^i(t)$. For example,

$$\begin{split} \left\langle \delta \rho^{1}(t) \delta \rho^{1}(t) \right\rangle_{s} &= \left\langle \left[\delta \zeta^{1}(t) - \delta \eta^{1}(t) \right] \left[\delta \zeta^{1}(t) - \delta \eta^{1}(t) \right] \right\rangle_{s} \\ &= \left\langle \delta \zeta^{1}(t) \delta \zeta^{1}(t) \right\rangle_{s} + \left\langle \delta \eta^{1}(t) \delta \eta^{1}(t) \right\rangle_{s} - 2 \left\langle \delta \zeta^{1}(t) \delta \eta^{1}(t) \right\rangle_{s} \end{split} \tag{A.2}$$

or

$$\begin{split} \left\langle \delta \rho^{1}(t) \delta \rho^{2}(t) \right\rangle_{s} &= \left\langle \left[\delta \zeta^{1}(t) - \delta \eta^{1}(t) \right] \left[\delta \zeta^{2}(t) - \delta \eta^{2}(t) \right] \right\rangle_{s} \\ &= \left\langle \delta \zeta^{1}(t) \delta \zeta^{2}(t) \right\rangle_{s} + \left\langle \delta \eta^{1}(t) \delta \eta^{2}(t) \right\rangle_{s} \\ &- \left\langle \delta \zeta^{1}(t) \delta \eta^{2}(t) \right\rangle_{s} - \left\langle \delta \zeta^{2}(t) \delta \eta^{1}(t) \right\rangle_{s} \end{split} \tag{A.3}$$

From these two examples, we infer that in order to evaluate $\langle \delta \rho^i(t) \delta \rho^j(t) \rangle_s$, we need to evaluate $\langle \delta \zeta^i(t) \delta \eta^j(t) \rangle_s$ for all possible values of i and j.

Following the same procedure shown in the previous sections, it is easy to obtain equations for $\delta \eta^i(t)$ and $\delta \zeta^i(t)$ analogous to (2.13). We can then combine these and obtain

$$\left\langle \delta \zeta^{i}(t) \delta \eta^{j}(t') \right\rangle_{s} = (2\alpha)^{2} \, \delta^{ik} \delta^{jl} \zeta_{0}^{m} \eta_{0}^{n} \int_{0}^{t} dt' \int_{0}^{t'} dt'' \int_{0}^{t^{(3)}} dt^{(4)} \int_{0}^{t^{(4)}} dt^{(5)} K_{kmln}^{\Psi}(t, t'), \quad (A.4)$$

where the noise kernel is the same as the one from equation (2.17). Then, it follows that

$$\langle \delta \zeta^{i}(t) \delta \eta^{j}(t) \rangle_{c} = A(\Lambda, t) \delta^{ik} \delta^{jl} \zeta_{0}^{m} \eta_{0}^{n} [-2\delta_{km}\delta_{ln} + 3(\delta_{kl}\delta_{mn} + \delta_{kn}\delta_{ml})], \tag{A.5}$$

where $A(\Lambda, t)$ is given in equation (2.19). We can now evaluate the 2-point functions for all values of i and j.

Suppose that i=1. Then,

$$\langle \delta \zeta^{1}(t) \delta \eta^{j}(t) \rangle_{c} = A(\Lambda, t) \delta^{11} \delta^{jl} \zeta_{0}^{m} \eta_{0}^{n} [-2\delta_{1m}\delta_{ln} + 3(\delta_{1l}\delta_{mn} + \delta_{1n}\delta_{ml})]$$
 (A.6)

Using this, we obtain

$$j = 1 : \left\langle \delta \zeta^{1}(t) \delta \eta^{1}(t) \right\rangle_{s} / A(\Lambda, t) = \delta^{11} \delta^{11} \zeta_{0}^{m} \eta_{0}^{n} [-2\delta_{1m} \delta_{1n} + 3(\delta_{11} \delta_{mn} + \delta_{1n} \delta_{m1})]$$

$$= -2\zeta_{0}^{1} \eta_{0}^{1} + 3 \left(\vec{\zeta}_{0} \cdot \vec{\eta}_{0} + \zeta_{0}^{1} \eta_{0}^{1} \right)$$

$$= 3\vec{\zeta}_{0} \cdot \vec{\eta}_{0} + \zeta_{0}^{1} \eta_{0}^{1}$$

$$j = 2 : \left\langle \delta \zeta^{1}(t) \delta \eta^{2}(t) \right\rangle_{s} / A(\Lambda, t) = \delta^{11} \delta^{22} \zeta_{0}^{m} \eta_{0}^{n} [-2\delta_{1m} \delta_{2n} + 3(\delta_{12} \delta_{mn} + \delta_{1n} \delta_{m2})]$$

$$= -2\zeta_{0}^{1} \eta_{0}^{2} + 3\zeta_{0}^{2} \eta_{0}^{1}$$

$$j = 3 : \left\langle \delta \zeta^{1}(t) \delta \eta^{3}(t) \right\rangle_{s} / A(\Lambda, t) = \delta^{11} \delta^{33} \zeta_{0}^{m} \eta_{0}^{n} [-2\delta_{1m} \delta_{3n} + 3(\delta_{13} \delta_{mn} + \delta_{1n} \delta_{m3})]$$

$$= -2\zeta_{0}^{1} \eta_{0}^{3} + 3\zeta_{0}^{3} \eta_{0}^{1}$$

$$= -2\zeta_{0}^{1} \eta_{0}^{3} + 3\zeta_{0}^{3} \eta_{0}^{1}$$

From this example, we infer that

$$\left\langle \delta \zeta^{i}(t) \delta \eta^{j}(t) \right\rangle_{s} / A(\Lambda, t) = \begin{cases} 3\vec{\zeta}_{0} \cdot \vec{\eta}_{0} + \zeta_{0}^{i} \eta_{0}^{j} & \text{if } i = j \\ -2\zeta_{0}^{i} \eta_{0}^{j} + 3\zeta_{0}^{j} \eta_{0}^{i} & \text{if } i \neq j \end{cases}$$
(A.8)

Also, completely analogous rules apply to $\left\langle \delta\zeta^i(t)\delta\zeta^j(t)\right\rangle_s$ and $\left\langle \delta\eta^i(t)\delta\eta^j(t)\right\rangle_s$.

Using these we can now evaluate $\left\langle \delta \rho^i(t) \delta \rho^j(t) \right\rangle_s$ for all values of i and j. For example,

$$\begin{split} \left< \delta \rho^{1}(t) \delta \rho^{1}(t) \right>_{s} / A(\Lambda, t) &= \left[\left< \delta \zeta^{1}(t) \delta \zeta^{1}(t) \right>_{s} + \left< \delta \eta^{1}(t) \delta \eta^{1}(t) \right>_{s} - 2 \left< \delta \zeta^{1}(t) \delta \eta^{1}(t) \right>_{s} \right] / A(\Lambda, t) \\ &= 3 \left(\zeta_{0} \right)^{2} + \left(\zeta_{0}^{1} \right)^{2} + 3 \left(\eta_{0} \right)^{2} + \left(\eta_{0}^{1} \right)^{2} - 6 \vec{\zeta}_{0} \cdot \vec{\eta}_{0} - 2 \zeta_{0}^{1} \eta_{0}^{1} \\ &= 3 \left(\vec{\zeta}_{0} - \vec{\eta}_{0} \right)^{2} + \left(\zeta_{0}^{1} - \eta_{0}^{1} \right)^{2} \\ &= 3 \left(\rho_{0} \right)^{2} + \left(\rho_{0}^{1} \right)^{2} \end{split} \tag{A.9}$$

and

$$\begin{split} \left< \delta \rho^{1}(t) \delta \rho^{2}(t) \right>_{s} / A(\Lambda, t) &= \left[\left< \delta \zeta^{1}(t) \delta \zeta^{2}(t) \right>_{s} + \left< \delta \eta^{1}(t) \delta \eta^{2}(t) \right>_{s} \\ &- \left< \delta \zeta^{1}(t) \delta \eta^{2}(t) \right>_{s} - \left< \delta \zeta^{2}(t) \delta \eta^{1}(t) \right>_{s} \right] / A(\Lambda, t) \\ &= \left(-2 \zeta_{0}^{1} \zeta_{0}^{2} + 3 \zeta_{0}^{2} \zeta_{0}^{1} \right) + \left(-2 \eta_{0}^{1} \eta_{0}^{2} + 3 \eta_{0}^{2} \eta_{0}^{1} \right) \\ &- \left(-2 \zeta_{0}^{1} \eta_{0}^{2} + 3 \zeta_{0}^{2} \eta_{0}^{1} \right) - \left(-2 \zeta_{0}^{2} \eta_{0}^{1} + 3 \zeta_{0}^{1} \eta_{0}^{2} \right) \\ &= \zeta_{0}^{1} \zeta_{0}^{2} + \eta_{0}^{1} \eta_{0}^{2} - \zeta_{0}^{1} \eta_{0}^{2} - \zeta_{0}^{2} \eta_{0}^{1} \\ &= \left(\zeta_{0}^{1} - \eta_{0}^{1} \right) \left(\zeta_{0}^{2} - \eta_{0}^{2} \right) \\ &= \rho_{0}^{1} \rho_{0}^{2} \end{split} \tag{A.10}$$

From these calculations, we infer the following rules

$$\left\langle \delta \rho^{i}(t) \delta \rho^{j}(t) \right\rangle_{s} / A(\Lambda, t) = \begin{cases} 3 \left(\rho_{0} \right)^{2} + \left(\rho_{0}^{i} \right)^{2} & \text{if } i = j \\ \rho_{0}^{i} \rho_{0}^{j} & \text{if } i \neq j \end{cases}$$
(A.11)

Finally, we can use these rules to finish evaluating $Var(\delta \rho(t))$. We obtain

$$Var(\delta\rho(t)) = \left[\left\langle \delta\rho^{1}(t)\delta\rho^{1}(t) \right\rangle_{s} \left(\rho_{0}^{1}\right)^{2} + \left\langle \delta\rho^{2}(t)\delta\rho^{2}(t) \right\rangle_{s} \left(\rho_{0}^{2}\right)^{2} \right.$$

$$\left. + \left\langle \delta\rho^{3}(t)\delta\rho^{3}(t) \right\rangle_{s} \left(\rho_{0}^{3}\right)^{2} + 2 \left\langle \delta\rho^{1}(t)\delta\rho^{2}(t) \right\rangle_{s} \rho_{0}^{1}\rho_{0}^{2}$$

$$\left. + 2 \left\langle \delta\rho^{1}(t)\delta\rho^{3}(t) \right\rangle_{s} \rho_{0}^{1}\rho_{0}^{3} + 2 \left\langle \delta\rho^{2}(t)\delta\rho^{3}(t) \right\rangle_{s} \rho_{0}^{2}\rho_{0}^{3} \right] / \left(\rho_{0}\right)^{2}$$

$$= \left[\left(3 \left(\rho_{0}\right)^{2} + \left(\rho_{0}^{1}\right)^{2} \right) \left(\rho_{0}^{1}\right)^{2} + \left(3 \left(\rho_{0}\right)^{2} + \left(\rho_{0}^{2}\right)^{2} \right) \left(\rho_{0}^{2}\right)^{2} \right.$$

$$\left. + \left(3 \left(\rho_{0}\right)^{2} + \left(\rho_{0}^{3}\right)^{2} \right) \left(\rho_{0}^{3}\right)^{2} + 2 \rho_{0}^{1}\rho_{0}^{2}\rho_{0}^{1}\rho_{0}^{2}$$

$$\left. + 2 \rho_{0}^{1}\rho_{0}^{3}\rho_{0}^{1}\rho_{0}^{3} + 2 \rho_{0}^{2}\rho_{0}^{3}\rho_{0}^{2}\rho_{0}^{3} \right] A(\Lambda, t) / \left(\rho_{0}\right)^{2}$$

$$= \left[3 \left(\rho_{0}\right)^{2} \left[\left(\rho_{0}^{1}\right)^{2} + \left(\rho_{0}^{2}\right)^{2} + \left(\rho_{0}^{3}\right)^{2} \right] + \left[\left(\rho_{0}^{1}\right)^{2} + \left(\rho_{0}^{2}\right)^{2} + \left(\rho_{0}^{3}\right)^{2} \right]^{2} \right] A(\Lambda, t) / \left(\rho_{0}\right)^{2}$$

$$= 4 \left(\rho_{0}\right)^{2} A(\Lambda, t). \tag{A.12}$$

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