Unruh effect as a noisy quantum channel

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We studied the change of the nonlocal correlation of the entanglement in Rindler space-time by showing that the Unruh effect can be interpreted as a noisy quantum channel having a complete positive and trace preserving map with an "operator sum representation." It is shown that the entanglement fidelity is obtained in analytic form from the operator sum representation, which agrees well numerically with the entanglement monotone and the entanglement measure obtained previously. Nonzero entropy exchange between the system Q and region II of the Rindler wedge indicates the nonlocal correlation between causally disconnected regions. We have also shown the subadditivity of entropies numerically.

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I. INTRODUCTION

The Unruh effect [1–4] discovered over four decades ago predicts that a noninertial observer in an accelerated motion would see the Minkowski vacuum as a thermal bath of excited particles. The discovery of the Unruh effect is regarded as one of the monumental achievements of our understanding of quantum field theory in curved space-time despite the lack of direct experimental confirmation. Recently, there has been renewed interest in the Unruh effect, especially associated with the entanglement harvesting [5–8] and the detection of the possible signature of the Unruh effect in quantum radiation [9–11]. Iso *et al.* [11] pointed out that this quantum radiation is related to the nonlocal correlation nature of the Minkowski vacuum state, which has its origin in the entanglement of the state between the left and right Rindler wedges.

In this work, we study the change of the nonlocal correlation of the entanglement in Rindler space from a quantum information point of view by showing that the Unruh effect can be interpreted as a noisy quantum channel having a complete positive and trace preserving map with an "operator sum representation." The setting, in which Alice and Rob are two observers, one inertial and the other noninertial, describes the entanglement between two modes of a free scalar field from the point of their detectors [12–14]. When a noninertial observer, Rob, is under the influence of the acceleration, the measure of entanglement seen by the noninertial observer is affected by the presence of quantum thermal fields. The state observed by an inertial observer Alice and a noninertial observer Rob is a $2 \times \infty$-dimensional space in which case the necessary and sufficient criteria for the entanglement are not so well established [14]. When a quantum system is coupled to the Unruh radiation, it is inevitably treated in an infinite-dimensional space, in which case only a Gaussian state has an entanglement measure [15–17]. For this reason, Alsing and Milburn [12] used an indirect measure of entanglement as they calculated teleportation fidelity. Fuentes-Schuller and Mann [13] calculated the lower bound of entanglement. Ahn and Kim [14] studied an entanglement measure by calculating the symplectic eigenvalues of the matrix obtained through the partial transposition of the variance matrix.

II. THEORETICAL MODEL

Here, we obtain the entanglement fidelity directly from the operator sum representation [18] of the complete positive superoperator ε^Q, which acts on the initial density operator ρ^Q in analytical form. It is shown that our analytical result agrees very well with the entanglement monotone [13] and the entanglement measure [14] obtained numerically. We assume that the quantum state Q describes an entanglement between Alice and Rob in stationary states, i.e., the state in which Rob also stays stationary without acceleration. We will describe the evolution of the system Q by allowing Rob to experience uniform acceleration a through the acceleration parameter r defined by $\tanh r = \exp(-2\pi\Omega)$, $\Omega = |k|c/a = \omega_k/a$, where k is the wave vector, c the speed of light, and a the uniform acceleration. We consider the real, scalar field of the modes, s and k, in the Minkowski and Rindler space-time, respectively. Let Alice be an observer at event P with zero velocity in the Minkowski space-time and let noninertial observer Rob be moving with positive uniform acceleration in the z direction with respect to Alice (Fig. 1). We assume that Alice has a detector which only detects mode s and Rob has a detector sensitive only to mode k as in Ref. [13], so we are assuming that there is no global mode detected by Alice and Rob and the Hilbert spaces for Alice and Rob are independent. If Rob is under a uniform acceleration, the corresponding ground state should be specified in Rindler coordinates [19–22] in order to describe what Rob observes. Let us denote the ground states, which Alice and Rob detect in the Minkowski space-time as $|O_A^s\rangle_M$ and $|O_R^k\rangle_M$ (Fig. 1), respectively. Then the ground state from the noninertial point of view can be written

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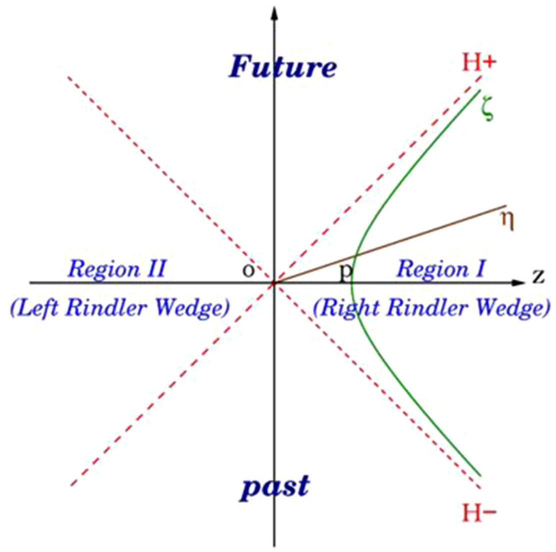

FIG. 1. Rindler space-time. In regions I and II, time coordinates $\eta =$ const. are straight lines through the origin. Space coordinates $\zeta =$ const. are hyperbolas with null asymptotes H_+ and H_-, which act as event horizons. The Minkowski coordinates t, z and Rindler coordinates η, ζ are given by $t = a^{-1}\exp(a\zeta)\sinh a\eta$ and $z = a^{-1}\exp(a\zeta)\cosh a\eta$, where a is a uniform acceleration. Alice and Rob initially share a two-mode squeezed state at the event P. We consider the case of Alice in stationary and Rob (green hyperbola) under uniform acceleration.

as $|O_R^k\rangle_M = \frac{1}{\cosh r}\sum_{n=0}^{\infty}\tanh^n r|n_R^k\rangle_{\rm I}\otimes|n_R^k\rangle_{\rm II}$, with $|n_R^k\rangle_{\rm I}$ and $|n_R^k\rangle_{\rm II}$ the mode decompositions in Rindler regions I and II, respectively [13,14]. The excited state for Rob in Minkowski space-time in mode k is obtained by applying the Minkowski creation operator $a_{kR}^{\dagger}$ to the vacuum state successively [14]. For example,

$$\begin{aligned}|1_R^k\rangle_M &= a_{kR}^{\dagger}|O_R^k\rangle_M,\\ |2_R^k\rangle_M &= \frac{1}{\sqrt{2}}(a_{kR}^{\dagger})^2|O_R^k\rangle,\ldots,\ |m_R^k\rangle_M = \frac{1}{\sqrt{m!}}(a_{kR}^{\dagger})^m|O_R^k\rangle.\end{aligned} \tag{1}$$

The particle creation and annihilation operators for the Rindler space-time are expressed as $b_{k\sigma}^{\dagger}$ and $b_{k\sigma}$, respectively. Here, the subscript $\sigma =$ I or II takes into account the fact that the space-time has an event horizon, so that it is divided into two causally disconnected Rindler wedges I and II (Fig. 1). The Minkowski operators $a_{kR}^{\dagger}$ and a_{kR} can be expressed in terms of the Rindler operators $b_{k\sigma}^{\dagger}$ and $b_{k\sigma}$ by Bogoliubov transformations [14]:

$$\begin{aligned}a_{kR}^{\dagger} &= b_{k\rm I}^{\dagger}\cosh r - b_{k\rm II}\sinh r = G_k b_{k\rm I}^{\dagger}G_k^{\dagger},\\ a_{kR} &= b_{k\rm I}\cosh r - b_{k\rm II}^{\dagger}\sinh r = G_k b_{k\rm I}G_k^{\dagger},\end{aligned} \tag{2}$$

with $G_k = \exp\{r(b_{k\rm I}^{\dagger}b_{k\rm II}^{\dagger} - b_{k\rm I}b_{k\rm II})\}$. Then, the Minkowski ground state $|O_R^k\rangle_M$ seen by the Rindler observer, i.e., Rob, is given by $|O_R^k\rangle_M = G_k(|O\rangle_{\rm I}\otimes|O\rangle_{\rm II})$. Here $|O\rangle_{\rm I}$ and $|O\rangle_{\rm II}$ are the Rindler vacuum states of regions I and II, regarding Rob. This is the basis of the Unruh effect, which says that a noninertial observer with uniform acceleration would see thermal quantum fields. In other words, Rob would see the quantum bath populated by thermally excited states of mode k. The quantum fields arising from the solution of the Klein-Gordon equation can be described either in the Minkowski space-time or the Rindler space-time and the equivalence of two solutions is obtained by matching them on H_- (Fig. 1) as described in Appendix C. An Unruh-DeWitt detector model with localized modes with compact support can be used to construct independent Hilbert space for Alice and Rob and is described in Appendix D. The resulting Bogoliubov transformation may be more complicated than the one described in (2) and will be the subject of future study. Modified Bogoliubov transformation within the first-order approximation is described by Landulfo and Matsas [23] and Kok and Yurtserver [24].

The excited states for Rob in Minkowski space-time are now given by [14]

$$\begin{aligned}a_{kR}^{\dagger}|O_R^k\rangle_M &= G_k b_{kI}^{\dagger}(|O\rangle_{\rm I}\otimes|O\rangle_{\rm II}),\ldots,(a_{kR}^{\dagger})^m|O_R^k\rangle_M\\ &= G_k(b_{k\rm I}^{\dagger})^m(|O\rangle_{\rm I}\otimes|O\rangle_{\rm II}).\end{aligned} \tag{3a}$$

For example $|1_R^k\rangle_M$ is given by

$$|1_R^k\rangle_M = \frac{1}{\cosh^2 r}\sum_{n=0}^{\infty}\tanh^n r|(n+1)_k\rangle_{\rm I}\otimes|n_k\rangle_{\rm II}. \tag{3b}$$

We now consider the system Q' described by

$$\rho^{Q'} = {\rm Tr}_{\rm II}(|\psi\rangle\langle\psi|), \tag{4}$$

where

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|O_A^s\rangle_M\otimes|1_R^k\rangle + |1_A^s\rangle_M\otimes|O_R^k\rangle\right), \tag{5}$$

and ${\rm Tr}_{\rm II}$ denotes the partial trace over all modes of Rindler wedge II except for modes 2 of mode k. We are considering detectors sensitive to a single Minkowski mode s for Alice and k for Rob. The initial quantum state ρ^Q is given by

$$\begin{aligned}\rho^Q &= \lim_{a\to 0}[{\rm Tr}_{\rm II}(|\psi\rangle\langle\psi|)]\\ &= \tfrac{1}{2}(|01\rangle + |10\rangle)(\langle 01| + \langle 10|),\end{aligned} \tag{6}$$

where $|nm\rangle = |n_A^s\rangle\otimes|m_R^k\rangle$. Here the limit $a\to 0$ means considering the initial stationary state for Rob before the acceleration. Equation (5) describes an entanglement between Alice and Rob in Minkowski space-time. The state of Rob under acceleration is also entangled between the Rindler wedge states I and II. Since we are tracing out for the Rindler wedge state II, we are mostly considering the entanglement between Alice and Rob.

Here, we would like to treat the Unruh effect as a noisy quantum channel [18] where the system Q prepared in an initial state ρ^Q is described by the dynamical process, after which the system is in $\rho^{Q'}$. The dynamical process is described by a map ε^Q, so that the evolution is [18]

$$\rho^Q \to \rho^{Q'} = \varepsilon^Q(\rho^Q). \tag{7}$$

The map ε^Q is a channel between the initial state of the field and the field seen by an accelerating Rob.

If the map ε^Q is given by

$$\varepsilon^Q(\rho^Q) = \sum_n A_n^Q\rho^Q A_n^{Q\dagger}, \tag{8}$$

where A_n^Q is an operator on the Hilbert space of Q only, then the map is a complete positive map [18,22]. From Eqs. (1)–(5), we obtain, after some mathematical manipulation (Appendix A) [13],

$$\rho^{Q'} = \frac{1}{\cosh^2 r}\sum_{n=0}^{\infty}(\tanh^2 r)^n \rho_n = \rho_{AR}, \tag{9}$$

$$\rho_n = \frac{1}{2}\bigg\{|1n\rangle\langle 1n| + \frac{\sqrt{n+1}}{\cosh r}(|1n\rangle\langle 0(n+1)| + |0(n+1)\rangle\langle 1n|) + \frac{(n+1)}{\cosh^2}|0(n+1)\rangle\langle 0(n+1)|\bigg\}. \tag{10}$$

By comparing Eqs. (6) and (10), we obtain

$$A_n^Q = \frac{1}{\sqrt{n!}}\frac{\tanh^n r}{\cosh^2 r}(\cosh r)^{\hat{n}_A} \otimes (b_I^\dagger)^n, \tag{11}$$

where $\hat{n}_A = a_A^\dagger a_A$ is a number operator acting on Alice's Hilbert space. From this one can see that the Unruh effect can be described by a completely positive map acting on the quantum state Q of Alice and Rob, when both parties are in the stationary state, i.e., zero acceleration for Rob. Let $|\phi^Q\rangle$ be a quantum state of ρ^Q; then after some manipulation we obtain

$$\begin{aligned}
&\sum_{n=0}^{\infty}\langle\phi^Q|A_n^{Q\dagger}A_n^Q|\phi^Q\rangle \\
&= \frac{1}{2}\frac{(\tanh^2 r)^n}{\cosh^2 r}\left[\langle 1n| + \frac{\sqrt{n+1}}{\cosh r}\langle 0(n+1)|\right] \\
&\quad\times\left[|1n\rangle + \frac{\sqrt{n+1}}{\cosh r}|0(n+1)\rangle\right] \\
&= \frac{1}{2\cosh^2 r}\sum_{n=0}^{\infty}(\tanh^2 r)^n\left(1 + \frac{n+1}{\cosh^2 r}\right) = 1 \\
&= \mathrm{Tr}\left(\sum_{n=0}^{\infty}A_n^Q|\phi^Q\rangle\langle\phi^Q|A_n^{Q\dagger}\right) = \mathrm{Tr}\rho^{Q'}.
\end{aligned} \tag{12}$$

This indicates that the map is trace preserving. The map is complete positive and trace preserving, and as a result can be represented by an operator sum representation [18]. Conversely, if the map can be represented by the operator sum representation, the map is (i) trace preserving, (ii) Hermiticity preserving, and (iii) complete positive. Moreover, the operator sum representation is independent of the specific density operator. The Unruh effect transforms the stationary entangled state into the mixed state in Rindler space by a complete positive trace preserving map. Here, we have used the following relations:

$$\begin{aligned}
\frac{1}{\cosh^2 r}\sum_{n=0}^{\infty}(\tanh^2 r)^n &= \frac{1}{\cosh^2 r}\frac{1}{(1-\tanh^2 r)} \\
&= \frac{1}{\cosh^2 r}\frac{\cosh^2 r}{(\sinh^2 r - \cosh^2 r)} = 1,
\end{aligned} \tag{13}$$

and

$$\begin{aligned}
&\frac{1}{\cosh^4 r}\sum_{n=0}^{\infty}(n+1)(\tanh^2 r)^n \\
&= \frac{1}{\cosh^4 r}\frac{d}{d(\tanh^2 r)}\sum_{n=0}^{\infty}(\tanh^2)^{n+1} \\
&= \frac{1}{\cosh^4 r}\frac{d}{d(\tanh^2 r)}\frac{\tanh^2 r}{1-\tanh^2 r} \\
&= \frac{1}{\cosh^4 r}\frac{1}{(1-\tanh^2 r)^2} = 1.
\end{aligned} \tag{14}$$

According to Schumacher [18], for a complete positive and trace preserving map, the entanglement fidelity F_e which measures how successfully the quantum channel preserves the entanglement of Q can be represented by

$$F_e = \sum_n \left(\mathrm{Tr}\rho^Q A_n^Q\right)\left(\mathrm{Tr}\rho^Q A_n^{Q\dagger}\right). \tag{15}$$

From Eqs. (6) and (11), we obtain

$$\begin{aligned}
\mathrm{Tr}\rho^Q A_n^Q &= \mathrm{Tr}\bigg[\frac{1}{2}(|10\rangle + |01\rangle)(\langle 10| + \langle 01|) \\
&\quad\times\frac{1}{\sqrt{n!}}\frac{\tanh^n r}{\cosh^2 r}(\cosh r)^{\hat{n}_A}\otimes(b_I^\dagger)^n\bigg] \\
&= \frac{1}{2}\frac{1}{\cosh r}\left(1 + \frac{1}{\cosh r}\right)\delta_{n,0},
\end{aligned} \tag{16}$$

and as a result

$$\begin{aligned}
F_e &= \sum_n \left(\mathrm{Tr}\rho^Q A_n^Q\right)\left(\mathrm{Tr}\rho^Q A_n^{Q\dagger}\right) \\
&= \frac{1}{4}\frac{1}{\cosh^2 r}\left(1 + \frac{1}{\cosh r}\right)^2.
\end{aligned} \tag{17}$$

When Rob is in a stationary state, $a \to 0$ and $\cosh r \to 1$. Then, from Eq. (17) the entanglement fidelity approaches unity; i.e., $F_e \to 1$. On the other hand, when the value of the acceleration is large, then $\cosh r$ is increasing monotonically and the entanglement fidelity also decreases monotonically approaching zero for a very large value of the acceleration (Fig. 2). Our analytical result for the entanglement fidelity agrees very well with the entanglement monotone obtained by Feuntes-Schuller and Mann [13] and Ahn and Kim [14].

Since the final state $|\psi\rangle$ is a pure state, the von Neumann entropy $S(|\psi\rangle\langle\psi|) = 0$ and as a result, we obtain

$$S(\rho^{Q'}) = S(\rho_{AR}) = S(\rho_{\mathrm{II}}), \tag{18}$$

where $\rho_{\mathrm{II}} = \mathrm{Tr}_{A\mathrm{I}}(|\psi\rangle\langle\psi|)$. The entropy defined by Eq. (18) is called an entropy exchange S_e [18], which is common entropy for two initially uncorrelated systems. Another measure of correlation is the mutual information $I(\rho_{AR})$, which is defined by [13]

$$I(\rho_{AR}) = S(\rho_A) + S(\rho_R) - S(\rho_{AR}), \tag{19}$$

where $\rho_A = \mathrm{Tr}_{\mathrm{I}}(\rho_{AR})$ and $\rho_R = \mathrm{Tr}_A(\rho_{AR})$. The detailed expressions for entropies are given by (Appendix A) [13]

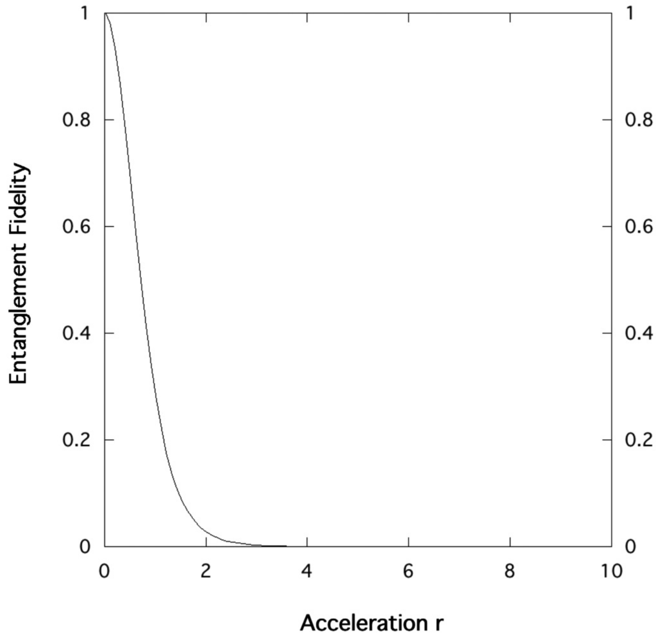

FIG. 2. Entanglement fidelity F_e versus acceleration r. This measure of entanglement is obtained in analytical form, i.e., $F_e = \frac{1}{4\cosh^2 r}(1+\frac{1}{\cosh^2 r})^2$, as a function of the acceleration r from the operator sum representation [18]. The results agree well with the entanglement monotone [13,14].

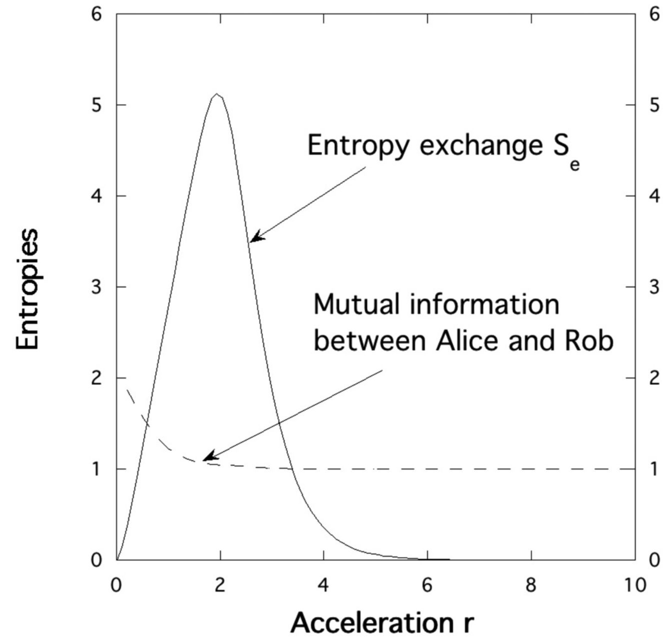

FIG. 3. Comparison of mutual information $I(\rho_{AR})$ and entropy exchange S_e. The mutual information is a measure of total correlation between Alice and Rob in their entangled state while the entropy exchange is a common entropy for two initially uncorrelated systems. A maximally mixed state of maximally entangled states has mutual information equal to 1 [13].

$$S(\rho_{AR}) = -\sum_n a_n\left(1+\frac{n+1}{\cosh^2 r}\right)\log_2\left[a_n\left(1+\frac{n+1}{\cosh^2 r}\right)\right], \tag{20}$$

$$S(\rho_R) = -\sum_n a_n\left(1+\frac{n}{\sinh^2 r}\right)\log_2\left[a_n\left(1+\frac{n}{\sinh^2 r}\right)\right], \tag{21}$$

$$S(\rho_A) = 1, \tag{22}$$

with

$$a_n = \frac{(\tanh^2 r)^n}{2\cosh^2 r}. \tag{23}$$

The mutual information $I(\rho_{AR})$ is a measure of total correlation between Alice and Rob in their entangled state. In Fig. 3 we plot the entropy exchange (solid line) and the mutual information (dashed line) as a function of the acceleration r. As acceleration increases the mutual information is approaching unity, which indicates that the states become more mixed by way of von Neumann entropy [13]. A maximally mixed state of maximally entangled states has mutual information equal to 1 [13]. From Eq. (6), the eigenvalues of the reduced density matrix ρ_{AR} for $r \to 0$ are 0, 0, 0, 1, and as a result we have $S(\rho_{AR}) = 0$. On the other hand, when the acceleration becomes infinite, we have $a_n(1+\frac{n+1}{\cosh^2 r}) \to 0$ and as a result

$$a_n\left(1+\frac{n+1}{\cosh^2 r}\right)\log_2\left[a_n\left(1+\frac{n+1}{\cosh^2 r}\right)\right] \to 0, \tag{24}$$

and we obtain $S(\rho_{AR}) \to 0$. The peak value of the entropy exchange exceeds 2 and this is the amount of correlation that Alice and Rob's entangled states have with the quantum bath due to the Unruh effect. In Fig. 4, we show the subadditivity [18] $S_e = S(\rho_{AR}) \leqslant S(\rho_A) + S(\rho_R)$, numerically. According

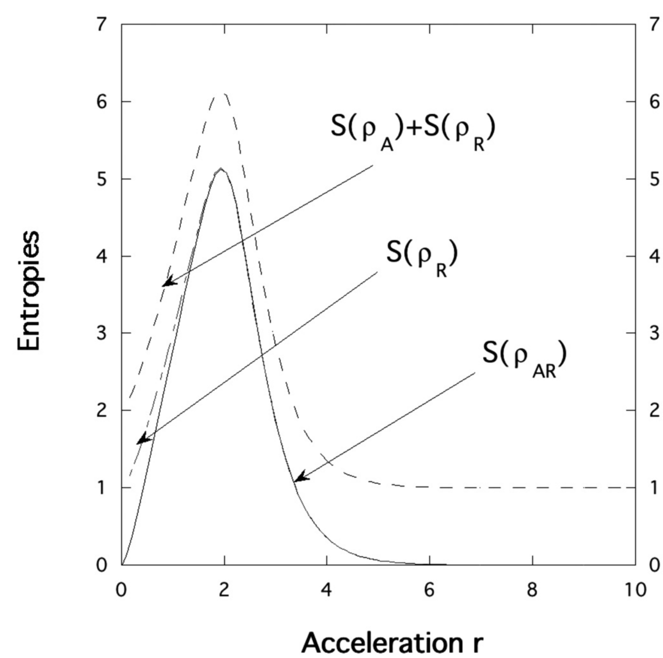

FIG. 4. Numerical proof of subadditivity for the entropy $S(\rho_{AR}) \leqslant S(\rho_A) + S(\rho_R)$.

to the interpretation of nonrelativistic quantum information theory [18], the entropy exchange characterizes the information exchange between the system Q and the external world during the evolution given by ε^Q. Since regions I and II of the Rindler wedges are causally disconnected and the entropy exchange is the information exchange between the system Q and the causally disconnected external world, i.e., region II of the Rindler wedge, they can be interpreted as a measure of nonlocal correlation.

III. SUMMARY

In summary, we studied the change of the nonlocal correlation of the entanglement in Rindler space-time by showing that the Unruh effect can be interpreted as a noisy quantum channel having a complete positive and trace preserving map with an "operator sum representation." It is shown that the entanglement fidelity is obtained in analytic form, which agrees well with entanglement monotone [13] and the entanglement measure [14], numerically. Nonzero entropy exchange between the system Q and region II of the Rindler wedge indicates the nonlocal correlation between causally disconnected regions. We have also shown subadditivity of entropies nu merically.

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APPENDIX A: DERIVATION OF EQ. (9)

From (1)–(5), we have

$$|\psi\rangle\langle\psi| = \frac{1}{2}\sum_{n,n'}\frac{\tanh^n r\tanh^{n'} r}{\cosh^2 r}\left\{\frac{\sqrt{n+1}}{\cosh r}|0(n+1)\rangle + |1n\rangle\right\}_{\mathrm{I}} \times\left\{\langle 1n'| + \frac{\sqrt{n'+1}}{\cosh r}\langle 0(n'+1)|\right\}_{\mathrm{I}} \otimes |n\rangle_{\mathrm{II}}\langle n'|. \tag{A1}$$

If we take the partial trace of (A1) with respect to the Rindler state of wedge II, we get

$$\begin{aligned}&\mathrm{Tr}_{\mathrm{II}}(|\psi\rangle\langle\psi|)\\ &= \frac{1}{\cosh^2 r}\sum_n \tanh^{2n} r\frac{1}{2}\left[\frac{\sqrt{n+1}}{\cosh r}|0(n+1)\rangle + |1n\rangle\right]\\ &\quad\times\left[\frac{\sqrt{n+1}}{\cosh r}\langle 0(n+1)| + \langle 1n|\right]\\ &= \frac{1}{\cosh^2}\sum_n \tanh^{2n}\rho_n.\end{aligned} \tag{A2}$$

APPENDIX B: DERIVATION OF EQ. (20)

We need to find the eigenvalues of ρ_n which are given by

$$\rho_n = \begin{pmatrix} 0 & 0 & 0 & 0\\ 0 & \frac{n+1}{2\cosh^2 r} & \frac{\sqrt{n+1}}{2\cosh r} & 0\\ 0 & \frac{\sqrt{n+1}}{2\cosh r} & \frac{1}{2} & 0\\ 0 & 0 & 0 & 0\end{pmatrix}, \tag{B1}$$

in the bases $\{|0n\rangle, |0(n+1)\rangle, |1n\rangle, |1(n+1)\rangle\}$.

From

$$\det|\rho_n - \lambda I| = \lambda^3\left(\lambda - \frac{1}{2} - \frac{n+1}{2\cosh^2 r}\right) = 0 \tag{B2}$$

we obtain the eigenvalues

$$\lambda = 0,\ 0,\ 0,\ \frac{1}{2} + \frac{n+1}{2\cosh^2 r}. \tag{B3}$$

The von Neumann entropy is then given by

$$S(\rho_{AR}) = -\sum_n a_n\left(1 + \frac{n+1}{\cosh^2 r}\right)\log_2\left[a_n\left(1 + \frac{n+1}{\cosh^2 r}\right)\right], \tag{B4}$$

with

$$a_n = \frac{\tanh^{2n} r}{2\cosh^2 r}. \tag{B5}$$

APPENDIX C: REVISIT TO BOGOLIUBOV TRANSFORMATION

Here, we follow Unruh and Wald's [25] and Birrell and Davies's [19] derivation closely. The solutions of the Klein-Gordon equation, which are positive frequency with respect to inertial time, are those with positive frequency with respect to η on H_- (Fig. 1), whereas the solutions which are positive frequency with respect to Rindler time in region I are those whose value on the portion of H_- with $t - z < 0$ is positive with respect to η. Let $\phi_{\mathrm{I}k}$ be a single-mode solution to the Klein-Gordon equation which on H_- is given by

$$\phi_{\mathrm{I}k} = \begin{cases}\Phi_k e^{-i\omega_k\eta}, & t - z < 0\\ 0, & t - z > 0\end{cases}, \tag{C1}$$

where Φ_k is the mode function. Then $\phi_{\mathrm{I}k}$ gives rise to a purely positive-frequency solution in region I. Similarly, the solution which is valid in region II is given by

$$\phi_{\mathrm{II}k} = \begin{cases}0, & t - z < 0\\ \Phi_k^* e^{i\omega_k\eta}, & t - z > 0\end{cases}. \tag{C2}$$

The corresponding positive-frequency solution in Minkowski space-time is given by

$$\psi_{Mk} = F_k e^{-i\omega_k t}. \tag{C3}$$

By matching solutions on H_- we obtain

$$F_k = \frac{\phi_{\mathrm{I}k} + e^{-\pi\omega_k/a}\phi^*_{\mathrm{II}k}}{(1 - e^{-2\pi\omega_k/a})^{1/2}}. \tag{C4}$$

From the above equations, the field operator for the Minkowski space-time is given by

$$a_{kR}(F_k) = \frac{b_{kI}(\phi_{Ik}) - e^{-\pi\omega_k/a} b^{\dagger}_{kII}(\phi_{IIk})}{(1 - e^{-2\pi\omega_k/a})^{1/2}} = b_{kI}\cosh r - b^{\dagger}_{kII}\sinh r. \tag{C5}$$

APPENDIX D: UNRUH-DeWITT DECTOR MODEL FOR THE LOCAL MODES

In this section, we describe the Unruh-DeWitt detector model [25] for local modes when one (or both) is in a local mode with compact support. We extend the approach of Landulfo and Matsas [23] and Kok and Yurtserver [24]. We model Alice's qubit in Minkowski space-timeby a two-level detector. As Rob is accelerated, his detector would see the thermally excited Rindler photons, as a result, the detector proper Hamiltonian for Alice and Rob is defined as [23]

$$H_A = \Omega A^{\dagger} A, \tag{D1}$$

and

$$H_R = \sum_k \omega_k R^{\dagger}_k R_k, \tag{D2}$$

where $A^{\dagger}$, A are the creation and annihilation operators for Alice, respectively, and $R^{\dagger}_k$, R_k are the creation and annihilation operators for Rob, respectively. The interaction Hamiltonian between the detector and a massless scalar field operator $\phi(x)$ is defined as [23]

$$H_{\text{int}}(t) = \varepsilon(t)\int_{\Sigma_t} d^3\vec{x}\sqrt{-g}\phi(x)[\psi(\vec{x})D + \bar{\psi}(\vec{x})D^{\dagger}], \tag{D3}$$

where $g = \det(g_{\mu\nu})$, $g_{\mu\nu}$ is the metric tensor, $D = A, R$, $\vec{x}$ the coordinates defined on the Cauchy hypersurface Σ_t, t the Minkowski time, $\varepsilon(t) \in C^{\infty}_0(R)$ is a smooth compact-support real-valued function which keeps the detector switched on for a finite amount of proper time, and $\psi \in C^{\infty}_0(\Sigma_t)$ is a smooth compact-support complex-valued function which models the fact that the detector interacts only with the field in a neighborhood of its world line. In the interaction picture, the state defined at the future null infinity is given by [23]

$$\begin{aligned}|\Psi^{D\phi}_{\infty}\rangle &= T\exp\left[-i\int_{-\infty}^{\infty} dt' H^{\mathrm{I}}_{\text{int}}(t')\right]|\Psi^{D\phi}_{-\infty}\rangle \\ &= T\exp\left[-i\int d^4x\sqrt{-g}\phi(x)(fD + \bar{f}D^{\dagger})\right]|\Psi^{D\phi}_{-\infty}\rangle,\end{aligned} \tag{D4}$$

where $f = \varepsilon(t)e^{-i\Omega t}\psi(\vec{x})$ is a compact-support complex function defined in Minkowski space-time.

We also have [23,26]

$$\phi(f) = \int d^4x\sqrt{-g}\phi(x)f = i[a(\bar{\lambda}) - a^{\dagger}(\lambda)], \tag{D5}$$

which is an operator valued distribution defined by smearing out the field operator by the testing function f. Here $a(\bar{\lambda})$ and $a^{\dagger}(\lambda)$ are annihilation and creation operators of λ modes, respectively. From (D4) and (D5), we obtain [23]

$$|\Psi^{D\phi}_{\infty}\rangle = \exp[a(\lambda)D - a^{\dagger}(\bar{\lambda})D^{\dagger} - a^{\dagger}(\lambda)D + a(\bar{\lambda})D^{\dagger}]|\Psi^{D\phi}_{-\infty}\rangle. \tag{D6}$$

The above describes the excitation and deexcitation of an Unruh-DeWitt detector associated with the absorption and emission, respectively, of a particle as "naturally" defined by the observers comoving with the detector, i.e., Minkowski and Rindler particles for inertial and uniformly accelerated observers, respectively [23]. The corresponding Hilbert space for Alice and Rob can be constructed independently following Landulfo and Matsas [23].

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