Relativistic Entropy Production for Quantum Field in Cavity

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A non-uniformly accelerated quantum field in a cavity undergoes the coordinate transformation of annihilation and creation operators, known as the Bogoliubov transformation. In this Letter, we consider the entropy production of a quantum field in a cavity induced by the Bogoliubov transformation. Dividing modes in the cavity into the system and environment, we obtain the lower bound of the entropy production which is defined by the sum of the von Neumann entropy in the system and the dissipated heat in the environment. The obtained lower bound is a signature of a refined second law of thermodynamics for a quantum field in a cavity and can be interpreted as the Landauer principle, which gives the thermodynamic cost of changing information contained in the system. Moreover, it gives an upper bound for the quantum mutual information, which can quantify the extent of information scrambling in the cavity due to the acceleration.

Introduction.—The validity of classical mechanics is challenged when processes occur in a huge spatial scale, where the relativistic effects become non-negligible. A prominent example is that clocks in satellites of the global positioning system (GPS) tick more quickly than those on the ground due to speed and gravity, and thus the accuracy of GPS is deteriorated without consideration of the relativistic effects. Conventionally, quantum mechanics has been concerned with processes that occur in an inertial and small system, where the relativistic effects do not come into play. Recently, entanglement distribution was performed between a satellite and receivers on the ground that are located 1200km apart [1]. Considering the fact that quantum mechanics already operates far beyond a laboratory spatial scale, relativistic quantum information recently attracts more attentions, where relativistic effects, e.g., Unruh [2] and dynamical Casimir effects [3], on quantum information are studied using quantum field theory [4–9].

Quantum thermodynamics is an extension of stochastic thermodynamics operating at a mesoscopic scale to a quantum microscopic scale. It generalizes the notion of stochastic work, heat, and entropy to the quantum domain, and several thermodynamic relations, e.g., Jarzynski equality [10], fluctuation theorem [11–13], and thermodynamic uncertainty relation [14, 15], have been shown to hold in the quantum domain as well [16– 18. Recently, the thermodynamic quantities and relations are further generalized to quantum field theory. References [19] and [20] derived the Jarzynski equality for quantum field theory in flat spacetime using twopoint and indirect measurements, respectively. Regarding quantum thermodynamics for quantum field theory in curved spacetime, Ref. [21] investigated the work exerted by the expanding universe. Moreover, Ref. [22] considered several quantum thermodynamic quantities for a quantum field in a cavity that undergoes acceleration.

In the present paper, we study the entropy produc-

tion [23] in a quantum field confined in a cavity which undergoes acceleration. In thermodynamics, the entropy production quantifies the extent of irreversibility of the system and plays a fundamental role in quantifying the thermodynamic cost of thermal machines. The nonnegativity of entropy production is a signature of the second law of thermodynamics and directly implies the Landauer principle [24], which evaluates the relation between information, quantified by entropy, and dissipated heat. Despite its importance in quantum thermodynamics, the entropy production due to acceleration has not been investigated so far. We consider a quantum field in a cavity that undergoes an arbitrary acceleration inducing the coordinate transformation referred to as the Bogoliubov transformation. Defining the entropy production by the sum of entropy and dissipated heat in a quantum field, we derive the lower bound of the entropy production, which is a refinement of the second law for the quantum field in a cavity and leads to the Landauer principle. Moreover, using the obtained inequality, we can obtain an upper bound for the quantum mutual information that can quantify the extent of information scrambling due to acceleration.

Methods.—We consider (1+1) dimensional Minkowski space [25]. Suppose that a cavity contains a massless scalar quantum field, where the cavity has length L>0. The confined quantum field model has been extensively employed in relativistic quantum information. The scalar quantum field satisfies Klein-Goldon equation in a curved spacetime [26]. The field Φ admits the mode expansion with respect to $\{\phi_n\}_{n=1}^\infty$ as $\hat{\Phi} = \sum_n (\hat{a}_n \phi_n + \hat{a}_n^\dagger \phi_n^*)$, where \hat{a}_n and \hat{a}_n^\dagger are annihilation and creation operators, respectively. They satisfy the canonical commutation relations $[\hat{a}_n, \hat{a}_m^\dagger] = \delta_{n,m}$ and $[\hat{a}_n, \hat{a}_m] = [\hat{a}_n^\dagger, \hat{a}_m^\dagger] = 0$. ϕ_n constitutes an orthonormal basis with respect to the inner product (ϕ_m, ϕ_n) . A coordinate transformation between different observers, induced by an acceleration, for instance, can be modeled by the Bogoliubov trans-

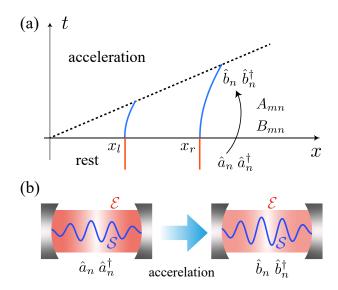


FIG. 1. Illustration of relativistic quantum field in a cavity. (a) Example of a cavity trajectory in the Minkovski space with a co-moving frame (t,x). x_l and x_r are boundaries of the cavity, where $x_r - x_l = L > 0$. The cavity is inertial for t < 0 and begins to accelerate for $t \ge 0$. Due to the acceleration, the annihilation and creation operators undergoes the Bogoliubov transformation specified by A_{mn} and B_{mn} . (b) Separation of the system and environment. The modes specified by \mathcal{S} corresponds to the system and the remaining modes \mathcal{E} to the environment.

formation, which transforms modes $\{\phi_n\}$ in the original coordinate to modes $\{\tilde{\phi}_n\}$ in a different coordinate. The field $\hat{\Phi}$ can also be expanded with $\{\tilde{\phi}_n\}_{n=1}^{\infty}$: $\hat{\Phi} = \sum_n (\hat{b}_n \tilde{\phi}_n + \hat{b}_n^{\dagger} \tilde{\phi}_n^*)$, where \hat{b}_n and \hat{b}_n^{\dagger} are different annihilation and creation operators, respectively, satisfying the canonical commutation relation $[\hat{b}_n, \hat{b}_m^{\dagger}] = \delta_{n,m}$ and $[\hat{b}_n, \hat{b}_m] = [\hat{b}_n^{\dagger}, \hat{b}_m^{\dagger}] = 0$. \hat{a}_n and \hat{b}_n are related via

$$\hat{b}_m = \sum_n (A_{mn}^* \hat{a}_n + B_{mn}^* \hat{a}_n^{\dagger}), \tag{1}$$

where A_{mn} and B_{mn} are Bogoliubov coefficients defined by $A_{mn} \equiv (\tilde{\phi}_m, \phi_n)$ and $B_{mn} \equiv -(\tilde{\phi}_n, \phi_m^*)$. The matrices $A = \{A_{mn}\}$ and $B = \{B_{mn}\}$ should satisfy the Bogoliubov identities $AA^{\dagger} - BB^{\dagger} = \mathbb{1}$ and $AB^{\top} - BA^{\top} = 0$, where $\mathbb{1}$ is the identity matrix. The annihilation operator \hat{a}_n defines the vacuum state $|0\rangle$ via $\hat{a}_n |0\rangle = 0$ for all $n \in \{1, 2, \cdots\}$. Therefore, the vacuum state $|0\rangle$ is an eigenstate with a vanishing eigenvalue of the annihilation operator. One of the most prominent properties of the Bogoliubov transformation is that the vacuum states of different coordinates do not agree in general. Indeed, the vacuum state $|\tilde{0}\rangle$ for \hat{b}_n is given by $\hat{b}_n |\tilde{0}\rangle = 0$ for all n, which does not agree with $|0\rangle$ in general. Therefore, the vacuum state of a coordinate may be populated with particles with respect to another coordinate.

Suppose that the cavity is in an inertial frame at the initial state. In relativistic quantum information, we are

usually only interested in one or two modes in the cavity, and the other remaining modes are regarded as the environment [6, 22]. Therefore, following Ref. [22], we divide the whole modes into the system \mathcal{S} and the environment \mathcal{E} . A set of modes in the system \mathcal{S} is defined by $\mathcal{S} = \{n_{s_1}, n_{s_2}, \ldots, n_{s_K}\}$ where $n_{s_i} \in \{1, 2, \cdots\}$ is an index of the system mode and K is the number of modes of the system, and a set of modes in the environment \mathcal{E} comprises the other remaining modes, i.e., $\mathcal{E} = \{n \in \{1, 2, \cdots\} | n \notin \mathcal{S}\}$. A set of the whole cavity modes is thus $\mathcal{C} = \mathcal{E} \cup \mathcal{S} = \{1, 2, \cdots\}$.

Since the quantum field is an infinite-dimensional system, the calculation of thermodynamic quantities is difficult. We employ the Gaussian state formalism [27–29], which is widely employed in relativistic quantum information, to quantify thermodynamic quantities. Let $\hat{\xi} \equiv [\hat{a}_1, \hat{a}_2, ..., \hat{a}_1^{\dagger}, \hat{a}_2^{\dagger}, ...]^{\top}$. Then the covariance matrix $\sigma = \{\sigma_{mn}\}$ is defined by

$$\sigma_{mn} \equiv \langle \hat{\xi}_m \hat{\xi}_n^{\dagger} + \hat{\xi}_n^{\dagger} \hat{\xi}_m \rangle - 2 \langle \hat{\xi}_m \rangle \langle \hat{\xi}_n^{\dagger} \rangle, \qquad (2)$$

where $\langle \bullet \rangle$ denotes the expectation value. Let σ^f and σ^i be the covariance matrices after and before the coordinate transformation, respectively (throughout this Letter, variables with superscripts f and i are defined in the same way). Under the Bogoliubov transformation, the covariance matrix after the transformation is

$$\sigma^f = S\sigma^i S^\dagger, \tag{3}$$

where S is a complex symplectic transformation, specified by the Bogoliubov matrices A and B:

$$S = \begin{bmatrix} A & B \\ B^* & A^* \end{bmatrix}. \tag{4}$$

Thermodynamic quantities of interest can be represented by the covariance matrix σ .

Results.—Let us consider a massless quantum field in a cavity that is in an inertial frame at the initial state. According to the Klein–Goldon equation, the field mode ϕ_n $(n \in \{1, 2, \dots\})$ is

$$\phi_n(x,t) = \frac{1}{\sqrt{n\pi}} \sin\left[\omega_n(x-x_l)\right] e^{-i\omega_n t}.$$
 (5)

Here, we choose a co-moving frame (t,x) where the boundaries of the cavity are x_l and x_r for any t where $x_r-x_l=L>0$ (see Fig. 1(a)), and the Dirichlet boundary condition is imposed at the boundaries. The Hamiltonian operators of the system and environment are defined by $\hat{H}_s \equiv \sum_{n \in \mathcal{S}} \omega_n \hat{a}_n^\dagger \hat{a}_n$ and $\hat{H}_e \equiv \sum_{n \in \mathcal{E}} \omega_n \hat{a}_n^\dagger \hat{a}_n$, respectively, where $\omega_n \equiv \pi n/L$ is the angular frequency of the field mode. We assume that the initial states of the cavity are prepared to be thermal states. Specifically, the initial density operator of the cavity is $\rho_c^i = \rho_s^i \otimes \rho_e^i$, where ρ_s^i and ρ_e^i are both thermal states of the system and environment, respectively, given by

$$\rho_s^i = \frac{1}{Z_s(\beta_s^i)} e^{-\beta_s^i \hat{H}_s}, \quad \rho_e^i = \frac{1}{Z_e(\beta_e^i)} e^{-\beta_e^i \hat{H}_e}, \tag{6}$$

with $Z_e(\beta_e^i) \equiv \operatorname{Tr}_e[e^{-\beta_e^i\hat{H}_e}]$ and $Z_s(\beta_s^i) \equiv \operatorname{Tr}_s[e^{-\beta_s^i\hat{H}_s}]$. Here, β_s^i and β_e^i denote the initial inverse temperature of the system and environment, respectively. Let $\eta_n \equiv \coth(\beta_s \omega_n/2)$ and $\nu_n \equiv \coth(\beta_e \omega_n/2)$ be symplectic eigenvalues of the system and environment, respectively. The initial Gaussian states of the system and environment are defined respectively by

$$\sigma_s^i = \operatorname{diag}([\eta_n]_{n \in \mathcal{S}}, [\eta_n]_{n \in \mathcal{S}}), \tag{7}$$

$$\sigma_e^i = \operatorname{diag}([\nu_n]_{n \in \mathcal{E}}, [\nu_n]_{n \in \mathcal{E}}). \tag{8}$$

Using the symplectic eigenvalues, the mean energy of the initial environment state is

$$\operatorname{Tr}_e\left[\rho_e^i \hat{H}_e\right] = \sum_{n \in \mathcal{E}} \omega_n \frac{\sigma_{nn}^i - 1}{2}.$$
 (9)

where the mean environmental energy after the transformation $\operatorname{Tr}_e[\rho_e^f\hat{H}_e]$ can be calculated in a similar manner, where $\rho_s^f \equiv \operatorname{Tr}_e[\rho_c^f] = \operatorname{Tr}_e[\hat{U}\rho_c^i\hat{U}^\dagger]$. The von Neumann entropy is defined by $\mathbb{S} \equiv -\operatorname{Tr}_s[\rho_s\ln\rho_s]$, where Tr_s is the trace with respect to the system (the environmental trace is defined analogously). In the covariance matrix formalism, the von Neumann entropy can be represented by a symplectic eigenvalue of the system [30]

$$S = \sum_{n \in S} \left\{ \mathfrak{s}_{+} \left(\eta_{n} \right) - \mathfrak{s}_{-} \left(\eta_{n} \right) \right\}, \tag{10}$$

where $\mathfrak{s}_{\pm}(x) \equiv \{(x \pm 1)/2\} \ln \{(x \pm 1)/2\}.$

After preparing the initial state, the cavity undergoes arbitrary acceleration. In Fig. 1(a), we show an example of the trajectory for t>0, where the cavity starts to accelerate satisfying the rigidity of the cavity. The coordinate transformation due to the acceleration is modeled by the Bogoliubov transformation. Let $\hat{\zeta} \equiv [\hat{b}_1, \hat{b}_2, ..., \hat{b}_1^{\dagger}, \hat{b}_2^{\dagger}, ...]^{\top}$. The Bogoliubov transformation on the operators $\hat{\xi}$ can be unitarily implemented as follows [27, 31, 32]:

$$\hat{\zeta} = S\hat{\xi} = \hat{U}^{\dagger}(S)\hat{\xi}\hat{U}(S),\tag{11}$$

where S is a symplectic transformation defined by Eq. (4) and U(S) is a unitary operator satisfying $\hat{U}(S_1 + S_2) = \hat{U}(S_1)\hat{U}(S_2)$. Equation (11) is the Heisenberg picture of the creation and annihilation operators. Therefore, in the Schrödinger picture, the density operator of the whole cavity evolves unitarily as $\rho_c^f = \hat{U}(S)\rho_c^i\hat{U}^{\dagger}(S)$. In quantum thermodynamics, the dissipated heat is often defined by the energy difference in the environment between the final and initial states:

$$\Delta \mathbb{Q} \equiv \text{Tr}_e[\rho_e^f \hat{H}_e] - \text{Tr}_e[\rho_e^i \hat{H}_e] = \sum_{n \in \mathcal{E}} \omega_n \frac{\sigma_{nn}^f - \sigma_{nn}^i}{2}.$$
(12)

where $\rho_e^f \equiv \text{Tr}_s[\rho_c^f] = \text{Tr}_s[\hat{U}\rho_c^i\hat{U}^{\dagger}]$ is the final density operator of the environment. Entropy difference between the initial and final states is

$$\Delta S \equiv S(\rho_s^f) - S(\rho_s^i), \tag{13}$$

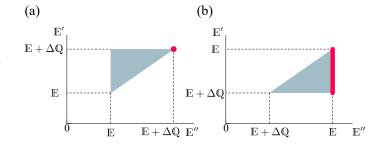


FIG. 2. Integral regions of Eq. (18) for (a) $\Delta \mathbb{Q} > 0$ and (b) $\Delta \mathbb{Q} < 0$. Pink regions denote values of \mathbb{E}'' and \mathbb{E}' at which $\operatorname{Var}_{\beta}[\hat{H}_e]$ gives the maximum.

where the covariance matrix representation of \$\\$ follows from Eq. (10).

We next define the entropy production for a quantum field in a cavity. The entropy production plays fundamental roles in stochastic and quantum thermodynamics. The entropy production can be defined in several ways [23, 33]; for instance, in stochastic thermodynamics, it can be quantified by the probability ratio between forward and backward processes, or it may be defined by a total entropy including both the system and environment. In quantum domain, due to the high degree of freedom in modeling, there are many definitions of entropy production [23]. Here we define the entropy production by $\beta_e^i \Delta \mathbb{Q} + \Delta \mathbb{S}$, which is the sum of the dissipated heat [Eq. (12)] and the von Neumann entropy of the system [Eq. (13)]. Because the modes in the cavity undergo Bogoliubov transformation, the density operator of the whole cavity ρ_c evolves via the corresponding unitary operator [Eq. (11)]. Therefore, from Refs. [34, 35], the following relation holds [36]:

$$\beta_e^i \Delta \mathbb{Q} + \Delta \mathbb{S} = I + D(\rho_e^f || \rho_e^i) \ge D(\rho_e^f || \rho_e^i) \ge 0, \quad (14)$$

where $D(\rho_e^f||\rho_e^i) \equiv \operatorname{Tr}\left[\rho_e^f \ln \rho_e^f\right] - \operatorname{Tr}\left[\rho_e^f \ln \rho_e^i\right]$ is the quantum relative entropy and $I \equiv S(\rho_s^f) + S(\rho_e^f) - S(\rho_c^f)$ is the quantum mutual information. The quantum relative entropy and the quantum mutual information are always non-negative [37] (non-negativity of the quantum mutual information was used in the second line of Eq. (14)). Equation (14) shows that the entropy production is nonnegative under a coordinate transformation induced by the acceleration, which is a second law for a quantum field in a cavity.

We can refine Eq. (14) by using the fact that the environment comprises a bosonic quantum field. To calculate the lower bound of $D(\rho_e^f||\rho_e^i)$, we follow Ref. [35]. Using the Pitágoras relation [35], which can be proved via straight-forward calculation, $D(\rho_e^f||\rho_e^i)$ is bounded from below by

$$D(\rho_e^f||\rho_e^i) = D(\rho_e^f||\rho_e^{\text{th}}) + D(\rho_e^{\text{th}}||\rho_e^i) \ge D(\rho_e^{\text{th}}||\rho_e^i).$$
(15)

Here ρ_e^{th} is a thermal state which gives the same energy as ρ_e^f . Let $\mathbb{E}(\beta)$ be the mean energy of the environment with

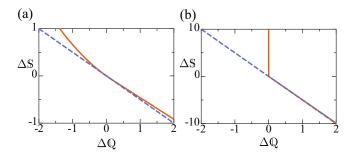


FIG. 3. Region plot of the Landauer bound for the cavity with (a) $\beta_e^i = 0.5$ and (b) $\beta_e^i = 5.0$. The solid line denotes Eq. (21) and the dashed line denotes $\Delta \$ \ge -\beta_e^i \Delta \mathbb{Q}$. Regions above the solid lines are physically feasible. The other parameter is L=1.

respect to a thermal state having the inverse temperature β :

$$\mathbb{E}(\beta) \equiv \frac{1}{Z_e(\beta)} \operatorname{Tr}_e \left[e^{-\beta \hat{H}_e} \hat{H}_e \right]. \tag{16}$$

Then, we have $\mathbb{E} + \Delta \mathbb{Q} = \operatorname{Tr}_e[\hat{H}_e \rho_e^f] = \operatorname{Tr}_e[\hat{H}_e \rho_e^{\text{th}}]$, where $\mathbb{E} = \mathbb{E}(\beta_e^i) = \operatorname{Tr}_e[\rho_e^i \hat{H}_e]$. Since $(d/d\beta)\mathbb{E}(\beta) < 0$, β_e^{th} satisfying $\mathbb{E}(\beta_e^{\text{th}}) = \mathbb{E} + \Delta \mathbb{Q}$ can be uniquely specified given $\Delta \mathbb{Q}$. Therefore, given $\Delta \mathbb{Q}$, $\rho_e^{\text{th}} = Z_e(\beta_e^{\text{th}})^{-1} \exp[-\beta_e^{\text{th}} \hat{H}_e]$ can be uniquely identified, indicating that $D(\rho_e^{\text{th}}||\rho_e^i)$ in Eq. (15) can be calculated given $\Delta \mathbb{Q}$. The relative entropy admits the following expression:

$$D(\rho_e^{\text{th}}||\rho_e^i) = \beta_e^i \Delta \mathbb{Q} - \left[S(\rho_e^{\text{th}}) - S(\rho_e^i) \right]. \tag{17}$$

From Ref. [35], $D(\rho_e^{\text{th}}||\rho_e^i)$ can be further calculated into

$$D(\rho_e^{\text{th}}||\rho_e^i) = \beta_e^i \Delta \mathbb{Q} - \int_{\mathbb{E}}^{\mathbb{E} + \Delta \mathbb{Q}} \frac{d\mathbb{S}(\mathbb{E}')}{d\mathbb{E}'} d\mathbb{E}'$$

$$= \beta_e^i \Delta \mathbb{Q} - \int_{\mathbb{E}}^{\mathbb{E} + \Delta \mathbb{Q}} \beta(\mathbb{E}') d\mathbb{E}'$$

$$= \int_{\mathbb{E}}^{\mathbb{E} + \Delta \mathbb{Q}} d\mathbb{E}' \int_{\mathbb{E}}^{\mathbb{E}'} \frac{d\mathbb{E}''}{\operatorname{Var}_{\beta(\mathbb{E}'')}[\hat{H}_e]}, \quad (18)$$

where $\operatorname{Var}_{\beta}[\hat{H}_e]$ is the variance of \hat{H}_e with respect to a thermal state of the environment with the inverse temperature β :

$$\operatorname{Var}_{\beta}[\hat{H}_e] = \frac{1}{4} \sum_{n \in \mathcal{E}} \omega_n^2 \operatorname{csch}\left(\frac{\beta \omega_n}{2}\right)^2. \tag{19}$$

Here $\beta(\mathbb{E})$ is the inverse temperature β which satisfies $\mathbb{E} = \operatorname{Tr}_e \left[Z_e(\beta)^{-1} e^{-\beta \hat{H}_e} \hat{H}_e \right]$, which can be uniquely identified since $\mathbb{E}(\beta)$ is a monotonically decreasing function. In Eq. (18), we used

$$\frac{d\mathbb{S}}{d\mathbb{E}} = \frac{d\mathbb{S}}{d\beta} \frac{d\beta}{d\mathbb{E}} = \left(-\beta \operatorname{Var}_{\beta}[\hat{H}_e]\right) \left(-\operatorname{Var}_{\beta}[\hat{H}_e]\right)^{-1} = \beta(\mathbb{E}).$$
(20)

We calculate the maximum value of $\operatorname{Var}_{\beta}[\hat{H}_e]$ within the integral domain $\int_{\mathbb{E}}^{\mathbb{E}+\Delta\mathbb{Q}} d\mathbb{E}' \int_{\mathbb{E}}^{\mathbb{E}'} d\mathbb{E}''$. We need to consider two cases, $\Delta\mathbb{Q}>0$ and $\Delta\mathbb{Q}<0$, separately. Since $\beta(\mathbb{E})$ is a monotonically decreasing function of \mathbb{E} , $\operatorname{Var}_{\beta}[\hat{H}_e]$ gives the maximum for $\Delta\mathbb{Q}>0$ and $\mathbb{Q}<0$ at points indicated by Fig. 2(a) and (b), respectively. Taking the maximum, we obtain a refined second law for accelerated cavities as follows:

$$\Delta \mathbb{S} + \beta \Delta \mathbb{Q} \ge \begin{cases} \frac{(\Delta \mathbb{Q})^2}{2 \operatorname{Var}_{\beta_e^i}[\hat{H}_e]} & \Delta \mathbb{Q} \le 0\\ \frac{(\Delta \mathbb{Q})^2}{2 \operatorname{Var}_{\beta(\mathbb{E} + \Delta \mathbb{Q})}[\hat{H}_e]} & \Delta \mathbb{Q} > 0 \end{cases}, \quad (21)$$

which is the main result of this Letter. In Eq. (21), the variance term for $\Delta \mathbb{Q} < 0$ does not depend on $\Delta \mathbb{Q}$ whereas that for $\Delta Q > 0$ does. Equation (21) holds for an arbitrary Bogoliubov transformation, indicating that Eq. (21) should hold for any acceleration undergone by the cavity. Equation (21) is a refinement of the second law for the quantum field in the cavity. Although the above calculation follows Ref. [35], the lower bound of Eq. (21) differs from Ref. [35] since the cavity is infinite-dimensional system whereas Ref. [35] concerns finite-dimensional systems. While Eq. (21) is the statement for the entropy production, it can be regarded as a statement between the information change in the system and the dissipated energy in the environment, i.e., Eq. (21) can be identified as the Landauer principle for a quantum field in the cavity which undergoes acceleration. The Landauer principle concerns the entropy decrease in the system, quantified by $-\Delta$ \$, and gives the lower bound of the heat dissipation for realizing the entropy decrease. Figure 3 plots Eq. (21) for two inverse temperature settings in (a) $\beta_e^i = 0.5$ and (b) $\beta_e^i = 5.0$ (explicit parameters are shown in the caption) with the solid lines. The regions above the solid lines are the feasible regions predicted by Eq. (21). In Fig. 3, the dashed lines represent the lower bound of $\Delta S + \beta_e^i \Delta Q \geq 0$, which is the naive second law. As can be seen, the area of the negative heat region decreases as the temperature decreases. Moreover, for $\Delta \mathbb{Q} > 0$, Eq. (21) is tighter than the naive second law for the higher temperature case.

Another consequence of Eq. (21) is a relation to information scrambling [38, 39]. The extent of scrambling is often quantified by out-of-order correlators. It has been proposed that the extent of scrambling can be alternatively quantified by the quantum mutual information [40]. The cavity undergoing a nonuniform acceleration can be identified as a process of information spreading. Suppose the cavity only has modes in S and the other remaining modes are empty. After a nonuniform acceleration, the other remaining modes are populated due to the Bogoliubov transformation, which is a reminiscent of the scrambling process. The mutual information I is bounded from above by $I \leq \Delta S + \beta_e^i \Delta Q - D(\rho_e^{\text{th}} || \rho_e^i)$.

From Eq. (21), we have

$$I \leq \begin{cases} \beta \Delta \mathbb{Q} + \Delta \mathbb{S} - \frac{(\Delta \mathbb{Q})^2}{2 \operatorname{Var}_{\beta_e^i} [\hat{H}_e]} & \Delta \mathbb{Q} \leq 0\\ \beta \Delta \mathbb{Q} + \Delta \mathbb{S} - \frac{(\Delta \mathbb{Q})^2}{2 \operatorname{Var}_{\beta(\mathbb{E} + \Delta \mathbb{Q})} [\hat{H}_e]} & \Delta \mathbb{Q} > 0 \end{cases}$$
(22)

Equation (22) shows that the extent of scrambling induced by the cavity acceleration can be bounded from above by the entropy Δ \$ and the dissipated heat $\mathbb Q$ in the cavity.

Conclusion.—In this Letter, we obtained a lower bound for the entropy production of a quantum field in a cavity that undergoes acceleration. We first identified the cavity mode of interest as the system and the other remaining modes as the environment, and defined the entropy production by the sum of the von Neumann entropy of the system and the dissipated heat. The nonnegativity of the entropy production is a signature of the second law, and, moreover, it gives the statement of the Landauer principle for the accelerated cavity.

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Supplementary Material for "Relativistic Landauer Principle for Quantum Field in Cavity"

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This supplementary material describes the calculations introduced in the main text. Equation and figure numbers are prefixed with S (e.g., Eq. (S1) or Fig. S1). Numbers without this prefix (e.g., Eq. (1) or Fig. 1) refer to items in the main text.

S1. QUANTITIES OF QUANTUM FIELD

Let us consider the thermal state of the Hamiltonian $\hat{H} = \sum_n \omega_n \hat{a}_n^{\dagger} \hat{a}_n$. The density operator is

$$\rho^{\text{th}} \equiv \frac{1}{Z(\beta)} e^{-\beta \hat{H}},\tag{S1}$$

where β is the inverse temperature and $Z(\beta) \equiv \text{Tr}[e^{-\beta \hat{H}}]$. The number operator $\hat{\mathfrak{n}}_n \equiv \hat{a}_n^{\dagger} \hat{a_n}$ admits the eigendecomposition

$$\hat{\mathfrak{n}}_n |\mathfrak{n}_n\rangle = \mathfrak{n}_n |\mathfrak{n}_n\rangle. \tag{S2}$$

which implies that $\hat{\mathfrak{n}}_n$ can be represented by $\hat{\mathfrak{n}}_n = \sum_{\mathfrak{n}_n} \mathfrak{n}_n |\mathfrak{n}_n\rangle \langle \mathfrak{n}_n|$. By using this representation, terms in ρ^{th} are given by

$$e^{-\beta \hat{H}} = \prod_{n=1}^{\infty} e^{-\beta \omega_n \hat{\mathfrak{n}}_n} = \prod_{n=1}^{\infty} \sum_{\mathfrak{n}_n} e^{-\beta \omega_n \mathfrak{n}_n} |\mathfrak{n}_n\rangle \langle \mathfrak{n}_n|,$$
 (S3)

and

$$\operatorname{Tr}[e^{-\beta \hat{H}}] = \prod_{n} \left(\sum_{\mathfrak{n}_{n}} e^{-\beta \omega_{n} \mathfrak{n}_{n}} \right) = \prod_{n} \frac{e^{\beta \omega_{n}}}{e^{\beta \omega_{n}} - 1}.$$
 (S4)

Let us consider the expectation of $\hat{a}_n^{\dagger}\hat{a}_n$ with respect to the thermal state ρ^{th} :

$$\langle \hat{a}_n^{\dagger} \hat{a}_n \rangle = \text{Tr} \left[\hat{\mathfrak{n}}_n \rho^{\text{th}} \right] = \frac{\sum_{\mathfrak{n}_n} \mathfrak{n}_n e^{-\beta \omega_n \mathfrak{n}_n}}{\sum_{\mathfrak{n}_n} e^{-\beta \omega_n \mathfrak{n}_n}} = \frac{1}{e^{\beta \omega_n} - 1}.$$
 (S5)

Since $\sigma_{nn} = \langle \hat{a}_n^{\dagger} \hat{a}_n + \hat{a}_n \hat{a}_n^{\dagger} \rangle = 2 \langle \hat{a}_n^{\dagger} \hat{a}_n \rangle + 1$, the covariance matrix becomes

$$\sigma_{nn} = 2 \langle \hat{a}_n^{\dagger} \hat{a}_n \rangle + 1 = \coth\left(\frac{\beta \omega_n}{2}\right).$$
 (S6)

By using Eq. (S5), the mean of \hat{H} is

$$Tr[\rho^{th}\hat{H}] = \sum_{n} \frac{\omega_n}{e^{\beta\omega_n} - 1}.$$
 (S7)

Similarly, the second moment of \hat{H} is

$$\operatorname{Tr}\left[\rho^{\operatorname{th}}\hat{H}^{2}\right] = \sum_{n} \frac{(e^{\beta\omega_{n}} + 1)\omega_{n}^{2}}{(e^{\beta\omega_{n}} - 1)^{2}} + \sum_{n \neq m} \left(\frac{\omega_{n}}{e^{\beta\omega_{n}} - 1}\right) \left(\frac{\omega_{m}}{e^{\beta\omega_{m}} - 1}\right). \tag{S8}$$

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By using Eqs. (S7) and (S8), the variance of \hat{H} is given by

$$\operatorname{Var}[\hat{H}] = \operatorname{Tr}\left[\rho^{\operatorname{th}}\hat{H}^{2}\right] - \operatorname{Tr}\left[\rho^{\operatorname{th}}\hat{H}\right]^{2} = \sum_{n} \frac{e^{\beta\omega_{n}}\omega_{n}^{2}}{(e^{\beta\omega_{n}} - 1)^{2}} = \frac{1}{4}\sum_{n} \omega_{n}^{2}\operatorname{csch}\left(\frac{\beta\omega_{n}}{2}\right)^{2}.$$
 (S9)

 $\mathbb{E}(\beta)$ in the main text is

$$\mathbb{E}(\beta) = \frac{1}{Z_e(\beta)} \operatorname{Tr}_e \left[e^{-\beta \hat{H}_e} \hat{H}_e \right] = \sum_{n \in \mathcal{E}} \frac{\omega_n}{e^{\beta \omega_n} - 1}.$$
 (S10)

Its derivative is

$$\frac{d}{d\beta}\mathbb{E}(\beta) = -\frac{e^{\beta\omega_n}\omega_n^2}{(e^{\beta\omega_n} - 1)^2} < 0,$$
(S11)

which implies that the inverse function $\beta(\mathbb{E})$ can be defined.

S2. EQUATION (14) IN THE MAIN TEXT

For readers convenience, we show the derivation of Eq. (14), which was proved in Refs. [1, 2]. We are going to show the following relation:

$$\beta_e^i \Delta \mathbb{Q} + \Delta \mathbb{S} = I + D(\rho_e^f || \rho_e^i), \tag{S12}$$

where $D(\rho_e^f||\rho_e^i)$ and I are the quantum relative entropy and the quantum mutual information, respectively, defined by

$$D(\rho_e^f || \rho_e^i) \equiv \text{Tr}\left[\rho_e^f \ln \rho_e^f\right] - \text{Tr}\left[\rho_e^f \ln \rho_e^i\right], \tag{S13}$$

$$I \equiv \mathbb{S}(\rho_s^f) + \mathbb{S}(\rho_e^f) - \mathbb{S}(\rho_c^f). \tag{S14}$$

As mentioned in the main text, the whole cavity undergoes a unitary transformation: $\rho_c^f = U \rho_c^i U^{\dagger}$. Therefore, the von Neumann entropy of the whole cavity is invariant under the transformation: $\mathbb{S}(\rho_c^f) = \mathbb{S}(\rho_c^i) = \mathbb{S}(\rho_s^i) + \mathbb{S}(\rho_e^i)$. Therefore, we have

$$I + D(\rho_e^f || \rho_e^i) = \Delta S + \text{Tr}_e \left[(\rho_e^i - \rho_e^f) \ln \rho_e^i \right], \tag{S15}$$

where $\Delta S \equiv S(\rho_s^f) - S(\rho_s^i)$. Since $\rho_e^i = Z_e(\beta_e^i)^{-1} e^{-\beta_e^i \hat{H}_e}$, the second term in Eq. (S15) is

$$\operatorname{Tr}_{e}\left[\left(\rho_{e}^{i}-\rho_{e}^{f}\right)\ln\rho_{e}^{i}\right] = \operatorname{Tr}_{e}\left[\left(\rho_{e}^{i}-\rho_{e}^{f}\right)\left(-\beta_{e}^{i}\hat{H}_{e}\right)\right] = \beta_{e}^{i}\Delta\mathbb{Q},\tag{S16}$$

where $\Delta \mathbb{Q} \equiv \text{Tr}_e[\rho_e^f \hat{H}_e] - \text{Tr}_e[\rho_e^i \hat{H}_e]$. Equation (S12) directly follows from Eqs. (S15) and (S16).

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