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Inertial effects on neutrino oscillations and decoherence: A wave packet approach

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ABSTRACT

We study the impact of inertial effects on neutrino oscillations and decoherence. Toward this end, we describe the propagation of neutrino wave packets by employing the density matrix formalism. We show that the acceleration and angular velocity of the observer's reference frame significantly influence neutrino coherence, the effects being in principle testable in strong regime.

1. Introduction

Neutrino physics is among the most active fields of research in particle physics [1–8]. Despite the intensive study, the intimate nature of these particles is yet to be fully understood and even more questions have been raised after the discovery of flavor mixing and oscillations [9–15]. Recently, the quantum field theoretical (QFT) analysis of neutrino mixing [16–18] has revealed the shortcomings of the original quantum mechanical (QM) treatment by highlighting the problem of the unitary inequivalence between the Fock spaces for flavor and mass fields, respectively. Implications of this issue have been largely explored in the literature [19–24], sometimes with conflicting results (see, for instance, [25,26] and [27,28]).

The above studies have been carried out in Minkowski spacetime. The effects of gravitationally induced quantum mechanical phases in neutrino oscillations have been discussed in [29–33], showing that the gravitational field of an external source adds a non-trivial contribution to the phase difference. This has fostered the development of a covariant formalism [34] based on Stodolsky's definition of the QM phase [35], which has inspired later investigation in extended theories of gravity [36–38] and, by virtue of the equivalence principle, in non-inertial frames [39–41]. Remarkably, inertial effects on neutrino oscillations have proved to be appreciable for solar neutrinos [39].

The analysis of neutrino oscillations typically relies on the plane wave approximation. However, such description does not account for neutrinos being localized particles. The first consistent approach based on wave packets (WP) of finite size has been elaborated in [42] and subsequently refined in [43–47] for both vacuum and matter oscillations [48–50]. In this framework it has been found that the use of WPs induces decoherence among neutrino mass eigenstates, giving rise to a maximal coherence length beyond which the interference requested for oscillations fades out. Spacetime effects on neutrino decoherence have been addressed in [51–53] using the density matrix formalism with Gaussian WPs. As a result, it has been shown that the characteristic coherence length of neutrino oscillations processes is significantly affected, the corrections being sizable in strong gravity regimes. Gravitational decoherence has also been investigated in connection with neutrino lensing (see, for instance, [54] and [55]).

The aim of this paper is to study the influence of inertial effects on neutrino oscillations and WP decoherence. The motivation for this analysis is plain to see: because of Earth's motion, any stationary laboratory on Earth accelerates and rotates relative to the local inertial frames. Therefore, any theoretical and/or experimental study precise enough should in principle take into account corrections arising from such effects. Following [51], we employ the density matrix approach to compute the effective neutrino coherence length in terms of the local energy and proper distance

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traveled by neutrinos. The possibility to test our predictions is finally discussed through some examples.

The layout of the paper is the following: in Sec. 2 we review the standard density matrix approach for WP decoherence in Minkowski spacetime. For this purpose we closely follow [50,51]. Section 3 is devoted to the study of inertial effects on neutrino oscillations and decoherence. Conclusions and perspectives are finally summarized in Sec. 4. Unless specified otherwise, the natural units $\hbar = c = 1$ are used throughout. Furthermore, the signature of the metric is chosen to be mostly negative $(+,-,-,-)$.

2. Neutrino decoherence in flat spacetime

Since Pontecorvo's theoretical predictions [9–11], it is well-known that neutrino flavor states (eigenstates of the interaction) can be written as coherent superpositions of mass states (eigenstates of the propagation) according to the mixing transformations

$$|\nu_\ell(t,\vec{p})\rangle = \sum_{i=1,2} U^*_{\ell i}\,|\nu_i(t,\vec{p})\rangle\,. \tag{1}$$

Here, we are considering for simplicity a model with only two neutrino generations of flavor (mass) $\ell = e, \mu$ $(i = 1, 2)$. Furthermore, we are assuming neutrinos to be relativistic Dirac particles. However, the same considerations and results can be quite straightforwardly extended to three generations, as well as to the case of neutrinos being Majorana particles. The rotation matrix U provides a unitary transformation parameterized by the mixing angle θ.

The time-evolution of the state $|\nu_i(t,\vec{p})\rangle$ with 3-momentum $\vec{p}$ is governed by the Schrödinger-like equation

$$i\frac{d}{dt}|\nu_i(t,\vec{p})\rangle = H|\nu_i(t,\vec{p})\rangle, \tag{2}$$

of Hamiltonian H. Following [51], here we neglect both matter- and self-interactions terms, so that H reduces to the free relativistic Hamiltonian of eigenvalue $E_i(\vec{p}) = \sqrt{m_i^2 + |\vec{p}|^2}$.

For our next purposes, it is convenient to express the i-th neutrino state in Eq. (1) in terms of its Fourier anti-transform in the coordinate space

$$|\nu_i(t,\vec{x})\rangle = \int\limits_{\vec{p}} e^{i\vec{p}\cdot\vec{x}}|\nu_i(t,\vec{p})\rangle\,, \tag{3}$$

where we have used the shorthand notation $\int_{\vec{p}} \equiv (2\pi)^{-3}\int d^3p$.

It is worth emphasizing that in the following analysis we neglect the spin structure of neutrinos, since it is in principle not relevant for our purposes of studying inertial effects on neutrino flavor oscillations and decoherence. In passing, we mention that effects of gravity-spin coupling on neutrinos have been largely addressed in the literature (see, e.g. [56–58]). By virtue of the equivalence principle, we expect that similar implications may also be induced by acceleration. A more detailed investigation of these effects in the wave-packet formalism will be developed elsewhere.

In the usual treatment of flavor oscillations, neutrino mass eigenstates are described by plane waves [59,60]. Although quite successful in explaining a wide range of experimental results, the plane wave approximation is not self-consistent, giving rise to a number of theoretical paradoxes [61]. The limits of applicability of this approximation have been discussed in detail in [61–63]. To overcome these problems, one should then consider an approach based on wave packets [42–50]. In this approach the flavor states in Eq. (1) are written as superpositions of mass eigenstate WPs, each picked at the momentum $\vec{p}_i$ with distribution amplitude $f_{\vec{p}_i}(\vec{p})$. For $t = 0$, the i-th WP component reads

$$|\nu_i(0,\vec{p})\rangle = f_{\vec{p}_i}(\vec{p})|\nu_i\rangle, \tag{4}$$

with normalization $\langle\nu_j|\nu_i\rangle = \delta_{ij}$ and $\int_{\vec{p}}|f_{\vec{p}_i}(\vec{p})|^2 = 1$.

In this setting, the states of neutrino with definite flavor in the coordinate space take the form

$$|\nu_\ell(t,\vec{x})\rangle = \sum_{i=1,2} U^*_{\ell i}\,\psi_i(t,\vec{x})|\nu_i\rangle\,, \tag{5}$$

where we have used Eqs. (2)-(4) and $\psi_i(t,\vec{x}) \equiv \int_{\vec{p}} e^{i\vec{p}\cdot\vec{x}} f_{\vec{p}_i}(\vec{p}) e^{-iE_i(\vec{p})t}$.

In our investigation of neutrino flavor evolution, we resort to the density matrix formalism to describe decoherence. With reference to the state in Eq. (5), the one-body density matrix is defined by [50,51]

$$\rho^{(\ell)}(t,\vec{x}) \equiv |\nu_\ell(t,\vec{x})\rangle\langle\nu_\ell(t,\vec{x})|\,, \tag{6}$$

whose jk-element is

$$\rho^{(\ell)}_{jk}(t,\vec{x}) \equiv \langle\nu_j|\rho^{(\ell)}(t,\vec{x})|\nu_k\rangle = U^*_{\ell j}U_{\ell k}\,\psi_j(t,\vec{x})\psi^*_k(t,\vec{x})\,. \tag{7}$$

A comment is now in order: in the standard picture of neutrino oscillations, the mechanism of flavor changing originates from the interference between different mass states in coherent superposition. Clearly, in the plane wave approximation the distance over which mass neutrinos interfere coherently is formally infinite due to the infinite spatial extension of plane waves. In other terms, oscillations can always occur, no matter the distance between the source and detection points. In the WP approach this is not true anymore. To account for neutrinos being localized particles, one defines a *coherence length* L_{coh} as the distance at which massive neutrinos cease to overlap. More rigorously, denoting by σ_x the WP space width, the coherence length is the distance at which the separation Δx between the mass eigenstates WPs centroids is at least σ_x.[1]

To derive the coherence length explicitly, we use the density matrix formalism with Gaussian WPs of amplitude

$$f_{\vec{p}_i}(\vec{p}) = \left(\frac{2\pi}{\sigma_p^2}\right)^{3/4} e^{-\frac{(\vec{p}-\vec{p}_i)^2}{4\sigma_p^2}}\,, \qquad \sigma_p = (2\sigma_x)^{-1}\,, \tag{8}$$

where σ_p denotes the momentum spread. Substitution of Eq. (8) into (7) yields [51]

$$\rho^{(\ell)}_{jk}(t,\vec{x}) = N^{(\ell)}_{jk}\int\limits_{\vec{p},\vec{q}} e^{-i[E_j(\vec{p})-E_k(\vec{q})]t}\,e^{i(\vec{p}-\vec{q})\cdot\vec{x}}\,e^{-\left[\frac{(\vec{p}-\vec{p}_j)^2}{4\sigma_p^2}+\frac{(\vec{q}-\vec{q}_k)^2}{4\sigma_p^2}\right]}, \tag{9}$$

where

$$N^{(\ell)}_{jk} \equiv \left(\frac{2\pi}{\sigma_p^2}\right)^{3/2} U^*_{\ell j}U_{\ell k}\,. \tag{10}$$

The above integrals can be solved by expanding $E_j(\vec{p})$ around the WP central momentum $\vec{p}_j$, i.e.

$$E_j(\vec{p}) \simeq E_j + (\vec{p}-\vec{p}_j)\cdot\vec{v}_j\,, \tag{11}$$

where $E_j \equiv E_j(\vec{p}_j)$ and $\vec{v}_j = \partial E_j/\partial p\big|_{\vec{p}=\vec{p}_j}$ is the group velocity of the j-th mass eigenstate WP (similarly for $E_k(\vec{q})$). We then obtain

$$\rho^{(\ell)}_{jk}(t,\vec{x}) = \frac{N^{(\ell)}_{jk}}{(2\sqrt{\pi}\sigma_x)^6}\,e^{-i(E_{jk}t-\vec{p}_{jk}\cdot\vec{x})}\,e^{-\left[\frac{(\vec{x}-\vec{v}_jt)^2}{4\sigma_x^2}+\frac{(\vec{x}-\vec{v}_kt)^2}{4\sigma_x^2}\right]}, \tag{12}$$

[1] Here we are assuming equal WP spatial separation for the mass eigenstates WPs.

where $E_{jk} \equiv E_j - E_k$ and $\vec{p}_{jk} \equiv \vec{p}_j - \vec{q}_k$.

Now, since in oscillation experiments we are typically interested in neutrino decoherence as a function of the traveled distance, Eq. (12) must be integrated over time. This gives[2]

$$\rho^{(\ell)}_{jk}(\vec{x}) \equiv \int dt\, \rho^{(\ell)}_{jk}(t,\vec{x}) = A^{(\ell)}_{jk}\, \rho^{\rm osc}_{jk}(\vec{x})\, \rho^{\rm damp}_{jk}(\vec{x})\,, \tag{13}$$

where

$$A^{(\ell)}_{jk} = \frac{U^*_{\ell j} U_{\ell k}}{\sqrt{2\pi}\, v\, \sigma_x^2}\, e^{-\frac{(E_{jk}\sigma_x)^2}{v^2}}\,, \tag{14}$$

$$\rho^{\rm osc}_{jk}(\vec{x}) = e^{i\left(\vec{p}_{jk} - \frac{2E_{jk}\vec{v}_g}{v^2}\right)\cdot\vec{x}}\,, \tag{15}$$

$$\rho^{\rm damp}_{jk}(\vec{x}) = e^{-\frac{(\vec{v}_j-\vec{v}_k)^2 x^2}{4v^2\sigma_x^2}}\,. \tag{16}$$

The amplitude $A^{(\ell)}_{jk}$ is a constant factor depending on the velocity $v^2 = v_j^2 + v_k^2$. It does not affect flavor oscillations. The contribution $\rho^{\rm osc}_{jk}(\vec{x})$ is the oscillation term with the extra factor $2E_{jk}\vec{v}_g/v^2$, where $\vec{v}_g = (\vec{v}_j + \vec{v}_k)/2$ is the average group velocity of the mass eigenstate WPs. The last term is responsible for damping. In particular, the coherence length can be computed as the distance at which the density matrix is suppressed by a factor e^{-1}, obtaining in the relativistic limit [51]

$$L_{\rm coh} \simeq \frac{4\sqrt{2}E^2}{|m_k^2 - m_j^2|}\sigma_x\,, \tag{17}$$

where $E \simeq |\vec{p}|$ is the average energy between the j-th and k-th mass eigenstates WPs. It is worth noticing that this expression agrees with the heuristic estimate given in [50], up to a numerical factor.

3. Inertial effects on neutrino decoherence

The formula (17) for the coherence length has been derived in the case of neutrino oscillations in flat geometry. Spacetime effects on neutrino decoherence have been recently analyzed in [51] by extending the density matrix approach to a generic curved background. The strategy consists in describing the spacetime evolution of neutrino WPs through the Stodolsky's covariant quantum mechanical phase [35] as

$$|\nu_i(P,D)\rangle = e^{-i\Phi_i(P,D)}|\nu_i(P)\rangle\,, \tag{18}$$

where $P(t,\vec{x}_P)$ and $D(t,\vec{x}_D)$ are the production and detection points, respectively and

$$\Phi_i(P,D) = \int_P^D p^{(i)\mu} g_{\mu\nu}\, dx^\nu\,, \tag{19}$$

is the phase factor for the i-th mass eigenstate WP in the metric $g_{\mu\nu}$. Here the canonical 4-momentum $p^{(i)}_\mu$ conjugated to the coordinate x^μ is defined as usual by $p^{(i)}_\mu = m_i\, g_{\mu\nu}\, dx^\nu/ds$, where ds is the line element along the trajectory of the i-th neutrino mass eigenstate. It satisfies the mass-shell condition

$$p^{(i)}_\mu\, p^{(i)\,\mu} = m_i^2\,. \tag{20}$$

[2] Alternatively, one can compute the averaged density matrix as a function of the time by integrating Eq. (12) over space. This has been done in [50], obtaining a similar result through the identification $t \simeq |\vec{x}|$.

In [51] calculations have been developed explicitly for neutrinos in a spherically symmetric geometry (Schwarzschild spacetime) and in the strong gravity regime. While interesting and worthy of further investigation, this analysis is mostly relevant in astrophysical situations, such as for neutrinos propagating close to neutron stars or in the presence of supernova explosions. Here we consider the more tangible situation of a neutrino experiment performed on the Earth. As discussed in Sec. 1, any laboratory system attached to the Earth is only approximately an inertial frame due to Earth's rotation and gravity. Thus, at least in principle, inertial effects cannot be entirely disregarded in any high-precision theoretical and/or experimental study of decoherence.

Following [64,65], let us describe the coordinate system in non-inertial reference frames by the line element

$$\begin{aligned} ds^2 = &\left[\left(1+\vec{a}\cdot\vec{x}\right)^2 + \left(\vec{\omega}\cdot\vec{x}\right)^2 - \left(\vec{\omega}\cdot\vec{\omega}\right)\left(\vec{x}\cdot\vec{x}\right)\right]dt^2 \\ &- 2\,dt\,d\vec{x}\cdot\left(\vec{\omega}\wedge\vec{x}\right) - d\vec{x}\cdot d\vec{x}\,, \end{aligned} \tag{21}$$

where $\vec{a}$, $\vec{\omega}$ are the linear acceleration and angular velocity of the frame and $x^\mu = (t,\vec{x})$ the local coordinates for the observer at the origin. As discussed in [65], this coordinate system makes physical sense only in the immediate neighborhood of the observer, i.e. for

$$|ax| \ll 1\,, \qquad |\omega x| \simeq |\omega y| \ll 1\,. \tag{22}$$

In what follows we shall develop exact calculations at first, taking account of this approximation at a later stage.

Without loss of generality, we assume $\vec{a} \equiv (a,0,0)$ and $\vec{\omega} \equiv (0,0,\omega)$, with both a and ω time-independent. The line element becomes

$$\begin{aligned} ds^2 = &\left[(1+a\,x)^2 - \omega^2\left(x^2+y^2\right)\right]dt^2 \\ &- 2\left(-\omega\, y dx + \omega\, x dy\right)dt - dx^2 - dy^2 - dz^2\,. \end{aligned} \tag{23}$$

Under these assumptions, the relevant components of the 4-momentum $p^{(i)}_\mu$ are

$$p^{(i)}_t = m_i\left\{\left[(1+ax)^2 - \omega^2\left(x^2+y^2\right)\right]\frac{dt}{ds} + \omega\, y\frac{dx}{ds} - \omega\, x\frac{dy}{ds}\right\}, \tag{24}$$

$$p^{(i)}_x = m_i\left(\omega\, y\frac{dt}{ds} - \frac{dx}{ds}\right), \tag{25}$$

$$p^{(i)}_y = m_i\left(-\omega\, x\frac{dt}{ds} - \frac{dy}{ds}\right), \tag{26}$$

$$p^{(i)}_z = -m_i\frac{dz}{ds}\,. \tag{27}$$

Since the metric tensor does not depend on t, the canonical momentum component

$$p^{(i)}_t \equiv E_i(\vec{p})\,, \tag{28}$$

is a constant of motion corresponding to the neutrino energy as measured by an inertial observer at rest at the origin [40,66]. It is related to the energy E_{loc} for an observer at the generic position $\vec{x}$ in the metric (23) through the transformation that connects the local Minkowski frame $\left\{x^{\hat{a}}\right\}$ to the general frame $\left\{x^\mu\right\}$ [65]

$$x^\mu = e^\mu_{\hat{a}}\, x^{\hat{a}}\,, \qquad g_{\mu\nu} e^\mu_{\hat{a}} e^\nu_{\hat{b}} = \eta_{\hat{a}\hat{b}}\,, \tag{29}$$

where the vierbein fields for the metric (23) are given by[3] [64]

[3] We use the convention of denoting general (local Lorentz frame) coordinates by Greek (hatted Latin) letters.

$$e_0^{\hat{0}} = 1 + ax\,, \quad e_m^{\hat{0}} = 0\,, \quad e_0^{\hat{k}} = \varepsilon^{\hat{k}\hat{l}\hat{m}} \omega^{\hat{l}} x^{\hat{m}}\,, \quad e_l^{\hat{k}} = \delta_l^k \qquad k, l, m = \{1, 2, 3\}\,, \tag{30}$$

(the vierbein components $e_{\hat{a}}^{\mu}$ are calculated by use of the orthonormality condition $e_{\mu}^{\hat{a}} e_{\hat{b}}^{\mu} = \delta_{\hat{b}}^{\hat{a}}$).

We focus on the case of neutrinos propagating parallel to the x-axis. By setting $dy/ds = dz/ds = 0$, the mass-shell relation (20) reads

$$\left[(1+ax)^2 - \omega^2\left(x^2+y^2\right)\right]\left(\frac{dt}{ds}\right)^2 - \left(\frac{dx}{ds}\right)^2 + 2\omega\, y \frac{dt}{ds}\frac{dx}{ds} = 1\,, \tag{31}$$

where now $y = const$. Combination of Eqs. (24) and (28) gives us

$$\frac{dt}{ds} = \frac{E_i(\vec{p})}{m_i\left[(1+ax)^2 - \omega^2\left(x^2+y^2\right)\right]} - \frac{\omega\, y}{\left[(1+ax)^2 - \omega^2\left(x^2+y^2\right)\right]}\frac{dx}{ds}\,, \tag{32}$$

which can be inserted into Eq. (31) to obtain

$$\frac{dx}{ds} = \pm\sqrt{\frac{E_i^2(\vec{p})}{m_i^2\left[(1+ax)^2 - \omega^2 x^2\right]} + \frac{y^2\omega^2}{(1+ax)^2 - \omega^2 x^2} - 1}\,. \tag{33}$$

Here the $+$ $(-)$ sign refers to neutrinos propagating toward increasing (decreasing) values of x as s increases. For the sake of calculation, we consider the positive solution, but analogous considerations hold true in the opposite case.

We are now ready to compute the quantum mechanical phase in Eq. (19). Toward this end, we use Eqs. (32) and (33) and retain terms up to the quadratic order in ax, ωx, ωy, consistently with the metric (23). The phase for the i-th WP component becomes

$$\Phi_i(P,D) \simeq E_i(\vec{p})\left[t_{PD} - f_{PD}(a,\omega)\right] + \frac{1}{2}\frac{m_i^2}{E_i(\vec{p})} g_{PD}(a,\omega)\,, \tag{34}$$

where we have exploited the approximation of relativistic neutrinos along the trajectory [51]. The following shorthand notation has been introduced

$$f_{PD}(a,\omega) \equiv x_{PD}\left[1 + \frac{a^2 X_{PD}^2}{3} - \omega\, y + \frac{\omega^2\left(X_{PD}^2 + 6y^2\right)}{6} + \frac{a\,(x_D + x_P)\,(-1+2\omega\, y)}{2}\right], \tag{35}$$

$$g_{PD}(a,\omega) \equiv x_{PD}\left[1 + \frac{a}{2}\,(x_D + x_P) - \frac{\omega^2 X_{PD}^2}{6}\right], \tag{36}$$

$$X_{PD}^2 \equiv x_D^2 + x_P\, x_D + x_P^2\,, \tag{37}$$

where $t_{PD} \equiv t_D - t_P$ and $x_{PD} \equiv x_D - x_P$. In passing, we notice that x_{PD} is exactly the proper distance $L_p(P,D)$ traveled by neutrinos between the source and detection points in the metric (23), since by definition

$$L_p(P,D) \equiv \int_P^D \sqrt{-g_{xx}}\,dx = x_{PD}\,. \tag{38}$$

Contrary to the standard Minkowski treatment, where $\Phi_i(P,D)$ only depends on the distance $x_D - x_P$, here we find a dependence of Eq. (34) on both $x_D \pm x_P$. This could be somehow expected, since the metric (23) is no longer translation-invariant. We remark that a similar behavior for the neutrino oscillation phase in non-inertial frames is exhibited in [40]. Clearly, for $a, \omega \to 0$, we have $f_{PD}(a,\omega) = g_{PD}(a,\omega) \to x_{PD}$, thus recovering the well-known Minkowski expression for the phase.

Now, the phase difference between the j-th and k-th neutrino WPs is

$$\Phi_{jk}(P,D) \simeq \left[E_j(\vec{p}) - E_k(\vec{q})\right]\left[t_{PD} - f_{PD}(a,\omega)\right] + \frac{1}{2}\left(\frac{m_j^2}{E_j(\vec{p})} - \frac{m_k^2}{E_k(\vec{q})}\right) g_{PD}(a,\omega)\,. \tag{39}$$

With the aid of Eq. (11), this relation can be further manipulated to give

$$\Phi_{jk}(P,D) \simeq E_{jk}\left[t_{PD} - f_{PD}(a,\omega)\right] + \frac{1}{2}\left(\frac{m_j^2}{E_j} - \frac{m_k^2}{E_k}\right) g_{PD}(a,\omega) + \vec{v}_j\cdot(\vec{p}-\vec{p}_j)(t_{PD}-\lambda_j) - \vec{v}_k\cdot(\vec{q}-\vec{q}_k)(t_{PD}-\lambda_k)\,, \tag{40}$$

where

$$\lambda_i(P,D) \equiv f_{PD}(a,\omega) + \frac{m_i^2}{2E_i} g_{PD}(a,\omega)\,. \tag{41}$$

Remarkably, Eq. (40) has the same structure as the phase shift found in [51], the only difference being the definition of the λ-function.

Let us now compute the one-body density matrix (9) with the QM phase expressed as in Eq. (40). A straightforward calculation leads to

$$\begin{aligned}\widetilde{\rho}_{jk}^{(\ell)}(P,D) &= N_{jk}^{(\ell)} \int_{\vec{p},\vec{q}} e^{-i\Phi_{jk}(P,D)}\, e^{-\left[\frac{(\vec{p}-\vec{p}_j)^2}{4\sigma_p^2} + \frac{(\vec{q}-\vec{q}_k)^2}{4\sigma_p^2}\right]} \\ &= \frac{N_{jk}^{(\ell)}}{(2\sqrt{\pi}\sigma_x)^6}\, e^{-iE_{jk}[t_{PD}-f_{PD}(a,\omega)]}\, e^{i\left(\frac{m_k^2}{2E_k} - \frac{m_j^2}{2E_j}\right)g_{PD}(a,\omega)} \\ &\quad\times e^{-\sigma_p^2\left[v_k^2(t_{PD}-\lambda_k)^2 + v_j^2(t_{PD}-\lambda_j)^2\right]},\end{aligned} \tag{42}$$

where we have introduced the tilde to distinguish the density matrix in the non-inertial frame from the corresponding Minkowski expression. Once again, the averaged density matrix is obtained by integrating $\widetilde{\rho}_{jk}^{(\ell)}$ over time. Similarly to Eq. (13), the ensuing expression can be factorized as

$$\widetilde{\rho}_{jk}^{(\ell)}(x_P, x_D) = A_{jk}^{(\ell)}\, \widetilde{\rho}_{jk}^{\,\mathrm{osc}}(x_P, x_D)\, \widetilde{\rho}_{jk}^{\,\mathrm{damp}}(x_P, x_D)\,. \tag{43}$$

The amplitude $A_{jk}^{(\ell)}$ exhibits the same expression as in Eq. (14). The second factor is now given by

$$\widetilde{\rho}_{jk}^{\,\mathrm{osc}}(x_P, x_D) = e^{i\left(\frac{m_k^2}{2E_k} - \frac{m_j^2}{2E_j}\right)g_{PD}(a,\omega)}\; e^{-i\frac{E_{jk}}{v^2}\left(v_j^2\frac{m_j^2}{2E_j^2} + v_k^2\frac{m_k^2}{2E_k^2}\right)g_{PD}(a,\omega)}\,. \tag{44}$$

As discussed below Eq. (16), $\widetilde{\rho}_{jk}^{\,\mathrm{osc}}$ is responsible for flavor oscillations. In the plane wave approximation and for $\omega \to 0$, this reproduces the result in Eq. (29) of [40], where inertial effects on neutrino oscillations have been studied for the case of pure linear acceleration. Finally, for relativistic neutrinos the term inducing damping can be approximated as

$$\widetilde{\rho}_{jk}^{\,\mathrm{damp}}(x_P, x_D) \simeq e^{-\frac{(m_k^2 - m_j^2)^2}{32E^4\sigma_x^2} g_{PD}^2(a,\omega)}\,, \tag{45}$$

where E is the average energy between the neutrino WPs as defined below Eq. (17).

Now, Eq. (45) allows us to derive the effective neutrino coherence length as measured by non-inertial observers. Demanding the density matrix to be suppressed by a factor e^{-1}, we get

$$g_{PD}(a,\omega)=\frac{4\sqrt{2}E^2\sigma_x}{|m_k^2-m_j^2|}\,. \tag{46}$$

By use of Eqs. (36) and (38), one can show that the effective coherence length takes the form

$$\widetilde{L}_{coh}^{PD}\simeq\frac{4\sqrt{2}E_{loc}^2(x_D)\,\sigma_x}{|m_k^2-m_j^2|}\left[1+\frac{a}{2}F_{PD}(\omega)+\frac{1}{4}a^2x_{PD}^2+G_{PD}(\omega)\right], \tag{47}$$

where we have recast the result in terms of the local energy $E_{loc}(x_D)$ at the detector as defined at the beginning of this section and

$$F_{PD}(\omega)\equiv(1+2\omega\,y)\,x_{PD}+2x_D\,, \tag{48}$$

$$G_{PD}(\omega)\equiv\frac{1}{6}\omega\left[-\omega\left(x_D^2-x_P^2\right)+\omega x_Dx_P+6y\,(2+\omega\,y)\right.$$
$$\left.-10\omega x_D^2\right]. \tag{49}$$

Therefore, we find that the acceleration and angular velocity of the observer's reference frame affect the neutrino coherence length in a non-trivial way. Once more, we point out that the result has been approximated to the second order in ax, $\omega\,x$, $\omega\,y$.

Some comments are needed here: first, we notice that the effective coherence length $\widetilde{L}_{coh}^{PD}$ correctly reduces to L_{coh} in Eq. (17) for $a,\omega\to 0$, since in this limit the additional contributions in Eq. (47) vanish and $E_{loc}\to E$. Furthermore, F_{PD} and G_{PD} contain terms depending solely on x_D. As discussed below Eq. (38), this follows from the fact that metric (23) is no longer invariant under space translations. The same spurious dependence on the detector position is found, for instance, in the expressions of the proper neutrino oscillation length [31] and coherence length [52] in Schwarzschild spacetime.

3.1. Applications

To quantify the size of the above corrections, we apply the formula (47) to some specific examples, taking care of all the adopted approximations. First, we consider the case of atmospheric neutrinos. In this framework it is known that flavor oscillations can be well-described by a two-flavor model ($\nu_\mu\leftrightarrow\nu_\tau$), since the mixing angle θ_{13} is much smaller than θ_{23} and θ_{12} and two of three mass states are very close in mass compared to the third ($\Delta m_{21}^2\ll\Delta m_{32}^2\approx\Delta m_{31}^2$ in the normal mass hierarchy [67]). Atmospheric neutrinos are typically produced in hadronic showers from the interaction of cosmic rays with nuclei in the atmosphere. We focus on *downward-going* neutrinos produced from interactions above the detector, for which matter effects have no relevance. These neutrinos fly over distances of order 10^2 km before being detected (we set the impact parameter $y\simeq 0$). Assuming that the detector co-moves with the Earth, it will rotate with an angular velocity $\omega\simeq 10^{-5}$ rad/s, experiencing an acceleration of order of Earth's gravity $a\simeq 10\,\mathrm{m/s^2}$.

We quantify inertial effects on neutrino decoherence by computing the relative difference between the effective coherence length (47) and the corresponding Minkowski expression, i.e.

$$\delta\equiv\frac{|\widetilde{L}_{\rm coh}^{PD}-\widetilde{L}_{\rm coh}^{PD}(a=\omega=0)|}{\widetilde{L}_{\rm coh}^{PD}(a=\omega=0)}\,. \tag{50}$$

By inserting the above numerical values, we find $\delta_{atm}\simeq 10^{-12}$, the dominant contribution coming from a. As expected, this is far below the sensitivity of the current experiments on neutrino oscillations and decoherence [1,2,6,7], thus validating the use of the plane wave approximation in theoretical computations.

On the other hand, let us consider solar neutrinos. Notice that the two-flavor description is appropriate also in this case for discussing the transition $\nu_e\leftrightarrow\nu_x$, where ν_x is a superposition of ν_μ and ν_τ. For these neutrinos typical flight distances are of order of the distance from Earth to Sun, i.e. 1.5×10^{11} m. By repeating the same computation as above, we find $\delta_{sol}\simeq 10^{-5}$, the rotational and acceleration contributions being now of the same order. Though still quite small, this value is not too far from the sensitivity of the next-generation solar neutrino experiments [68] (see also [69] for a broader overview on reactor neutrino experiments). This suggests that inertial effects might play a non-negligible rôle in future high-precision tests on solar neutrinos.

Clearly, one might even consider situations where inertial effects become much more prominent, such as astrophysical regimes (for instance, neutrinos accelerated by supernova explosions). From Eq. (47), we expect that in this case corrections to the coherence length become by far more relevant, consistently with the result of [51] in gravity scenarios, where it is shown that the more massive the gravity source, the higher the impact on the coherence length. In this sense, our comparison with [51] must be intended as qualitatively, more than quantitatively. However, a rigorous analysis of these regimes is better suited to be explored going beyond the present approximation. We are currently working on this aspect and results along this direction will appear.

4. Conclusions and perspectives

We have investigated the impact of inertial effects on neutrino oscillations and WP decoherence. To this aim we have used the formalism developed in [51], which extends the density matrix approach for Minkowski spacetime to a generic background geometry. Calculations have been performed explicitly for the metric (23) describing the coordinate system for an accelerating and rotating observer. By working in a suitable approximation, we have derived the effective neutrino coherence length and we have found that it is non-trivially modified with respect to the standard expression. Corrections have been quantified in some specific examples, showing that inertial effects might be relevant in the next-generation solar neutrino experiments.

Clearly, the above study only provides a first step in the study of inertial (and, more general, gravitational) effects on neutrino decoherence. Some issues require further consideration. For instance, we have assumed that both the linear acceleration and angular velocity of the frame are constant. However, a more realistic approach should allow for these quantities to vary over time. The difficulty in this case is that we can no longer consider the energy as a constant of motion (see the discussion below Eq. (27)). Furthermore, our analysis has been carried out for vacuum oscillations. Future studies should inevitably take into account the effects of neutrino interactions with matter and self-interactions as well. A more technical aspect to address is the development of a fully covariant formalism. Indeed, the one-body density matrix introduced above relies on the use of non-covariant Gaussian WPs to describe neutrino mass eigenstates [51]. Of course, a treatment based on a suitable covariant definition of WPs is necessary to explore spacetime effects on decoherence in a more consistent relativistic fashion. At this stage, we can envisage that such a description could affect the definition of neutrino WP space-width as well. This would demand the introduction of a proper space-width, similarly to what happens to the definition of proper distances as opposed to coordinate distances.

Finally, let us remark that neutrino decoherence can also be induced by phenomena distinct from WP propagation (see for instance [70]). Among these, a potential source of coherence degradation is the weak (and as yet undetected) coupling with the environment, which might lead neutrinos to experience stochastic perturbations resulting from the would-be fluctuating nature of spacetime at Planck scale [71]. This has been recently discussed in [72], where it has been argued that the search for neutrino decoherence may open a rare window on Planck scale physics. The study of these issues is very demanding and will be deepened elsewhere.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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