# Quantum generalisation of Einstein's Equivalence Principle can be verified with entangled clocks as quantum reference frames

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The Einstein Equivalence Principle (EEP) is of crucial importance to test the foundations of general relativity. When the particles involved in the test exhibit quantum properties, it is unknown whether this principle still holds. A possibility introduced in Ref. [1] is that the EEP holds in a generalised form for particles having an arbitrary quantum state. The core of this proposal is the ability to transform to a Quantum Reference Frame (QRF) associated to an arbitrary quantum state of a physical system, in which the metric is locally inertial. Here, we show that this extended EEP, initially formulated in terms of the local expression of the metric field in a QRF, can be verified in an interferometric setup via tests on the proper time of entangled clocks. We find that the violation of the generalised EEP corresponds to the impossibility of defining dynamical evolution in the frame of each clock. The violation results in a modification to the probabilities of measurements calculated in the laboratory frame, and hence can be verified in an interferometric setting.

#### I. INTRODUCTION

The Einstein Equivalence Principle (EEP) is one of the cornerstones of general relativity. Its status in conjunction with quantum theory is still debated: several proposals to extend it to the quantum regime have been advanced, while some authors have argued that the EEP is incompatible with situations when either matter (i.e. the particles used to test it) or gravity acquire quantum properties [1–15]. Recently, it was proposed that the EEP holds in a generalised form when the particles are in an arbitrary quantum state and the gravitational field is in a superposition of classical states [1]. Key to this generalised EEP is the possibility to transform to the reference frame associated to an arbitrary quantum system, namely a Quantum Reference Frame (QRF) in the formulation introduced in Ref. [16] (see also related work [1, 17–39]).

Thanks to this QRF transformation, the metric field can be made locally minkowskian in any QRF associated to an arbitrary quantum state, and in any superposition of classical gravitational fields. This definition identifies the notion of a Quantum Local Inertial Frame (QLIF), extending the usual notion of a Local Inertial Frame to reference frames in a quantum superposition or entangled relative to each other and to gravitational fields in a quantum superposition. This generalised EEP for QRFs states that a local measurement cannot distinguish [40] whether the spacetime is flat or curved, or in a superposition thereof [1], namely

In any and every Quantum Locally Inertial Frame (QLIF), anywhere and anytime in the universe, all the

(nongravitational) laws of physics must take on their familiar non-relativistic form.

Here, we show how such EEP for QRFs can be verified, in the classical Newtonian spacetime limit describing the Earth gravitational field, in an interferometric test involving entangled quantum clocks [41–49]. Specifically, we implement the transformation to the QLIF of one of the clocks with the formulation for Spacetime QRFs (SQRFs) introduced in Ref. [33]. In this formulation, we describe the dynamical evolution of quantum systems, each having a quantum state in position/momentum and an internal state acting as a clock, from the perspective of a QRF associated to any of these quantum systems. This allows us to cast the definition of a QLIF in terms of the time shown by a clock at the origin of a QRF. We further show that this formulation is equivalent to the standard quantum mechanical treatment in the laboratory frame.

We generalise the three aspects of the EEP to QRFs, namely

(Q-WEP) The local effects of (quantum) motion in a superposition of uniform gravitational fields are indistinguishable from those of an observer in flat spacetime that undergoes a quantum superposition of accelerations [16].

(Q-LLI) The outcome of any local nongravitational experiment is independent of the velocity of the freely falling quantum reference frame in which it is performed.

(Q-LPI) The outcome of any local nongravitational experiment is independent of the position of the quantum reference frame in which it is performed.

By building an explicit model for the violation of the principle, we show that it is sufficient to test two aspects and the classical version of the third in order to have a complete test of the generalised EEP for QRFs.

The rest of the paper is organized as follows. Section II is dedicated to a brief review of the Spacetime Quantum Reference Frames formalism. Section III shows how stan-

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dard tests of the EEP can be generalized to test also the quantum features of the EEP for QRFs, using entangled clocks in an atomic clock interferometer. In Section IV we provide a model for violations of the EEP for QRFs and in Section V we verify that atomic clock interferometers are sensitive to the violation parameters of our model, proving that such interferometers can be used to test the principle.

### II. SPACETIME QUANTUM REFERENCE FRAMES

In this section we review the Spacetime Quantum Reference Frames (SQRFs) formalism, introduced in Ref. [33]. This is a generalisation of the QRFs formalism [16] that additionally treats space and time on an equal footing and gives a timeless and fully relational description of a set of physical systems from the point of view of one of them. It adopts elements of Covariant Quantum Mechanics [50] and the Page-Wootters mechanism [51, 52]. Moreover, it accounts for both external and internal degrees of freedom of quantum systems, that are both employed in building the SQRF description.

The SQRFs formalism is a timeless formulation of a set of quantum systems, where the dynamical evolution of the systems is encoded in a set of constraints and emerges through a procedure which fixes the redundancies induced by the constraints. Such a procedure has the physical interpretation of a reduction to the QRF of one of the quantum systems considered. Each system has an external Hilbert space, corresponding to the position or momentum of the system, and an internal Hilbert space corresponding to a clock. Both external and internal degrees of freedom are used to identify the QRF: the external ones are used to fix the transformation to the QRF, and the internal ones identify the proper time in the QRF of each system. The formalism is completely relational, meaning that there is no external spacetime structure beside the relations between the particles, as an effect of imposing the constraints.

Concretely, we consider a system of N non-interacting quantum particles of mass  $m_I$ ,  $I=1,\ldots,N$  in the weak gravitational field generated by a particle L of mass  $m_L$ , as depicted in Figure 1. Each particle lives on a Hilbert space  $\mathcal{H}_J = \mathcal{H}_{\mathbf{J}}^{ext} \otimes \mathcal{H}_{C_J}$ ,  $J=1,\ldots,N,L$  where  $\mathcal{H}_{\mathbf{J}}^{ext} \simeq L^2(\mathbb{R}^2)$  corresponds to the position or momentum of the system in (1+1)D (one time coordinate and one space coordinate), and  $\mathcal{H}_{C_J} \simeq L^2(\mathbb{R})$  to the internal state of the clock, ticking according to its proper time. The physical state of the N-particle system and the laboratory L satisfies the constraint

$$\hat{C} |\Psi\rangle_{ph} = 0, \tag{1}$$

where  $\hat{C}$  is a linear combination of first-class constraints [53], namely constraints that commute with each

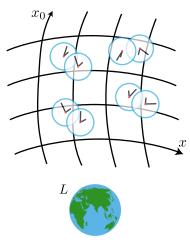


FIG. 1: A system of N non-interacting quantum particles in the gravitational field generated by a mass L. Each particle has both external and internal degrees of freedom, the latter acting as clocks. Therefore, the particles can be in a superposition of different positions, each associated to a different gravitational time dilation.

other. Specifically,

$$\hat{C} = \sum_{J=1}^{L} \mathcal{N}_J \hat{C}_J + z_\mu \hat{f}^\mu,$$
 (2)

where the spacetime labels  $\mu=0,1$  refer to a (1+1)-dimensional spacetime. In the following, the spatial component is written in boldface, while the 2-vector is in plain text, e.g.  $x^{\mu}=\{x^0,\mathbf{x}\}$ . The constraints describe both the dynamics of the particles and the global symmetries of the model, namely spacetime global translations: the symmetry constraints  $\hat{f}^{\mu}$  enforce that total momentum and total energy are null, i.e., that the model is globally translational-invariant in space and time. This condition corresponds to having a model where the dynamics is relational [54–56]. The constraints  $\hat{C}_J$  enforce the general-relativistic dispersion relation of each particle.

The gravitational field sourced by the mass L is described in the Newtonian limit in terms of the metric

$$g_{00} = 1 + 2 \frac{V(\mathbf{x} - \mathbf{x_L})}{c^2},$$
 (3)

$$g_{01} = g_{10} = 0, (4)$$

$$g_{11} = -1, (5)$$

and the constraints in this regime are

$$\hat{C}_I = \sqrt{g^{00}(\hat{\mathbf{x}}_I - \hat{\mathbf{x}}_L)}\hat{p}_0^I - \omega_{\mathbf{p}}^I, \tag{6}$$

$$\hat{C}_L = \hat{p}_0^L - m_L c - \frac{\hat{\mathbf{p}}_L^2}{2m_L c},\tag{7}$$

$$\hat{f}^{0} = \sum_{I=1}^{N} \left[ \hat{p}_{0}^{I} + \Delta(\hat{\mathbf{x}}_{I} - \hat{\mathbf{x}}_{L}, \hat{\mathbf{p}}_{I}) \frac{\hat{H}_{I}}{c} \right] + \hat{p}_{0}^{L} + \frac{\hat{H}_{L}}{c}, \quad (8)$$

$$\hat{f}^{1} = \sum_{I=1}^{N} \hat{\mathbf{p}}_{I} + \hat{\mathbf{p}}_{L}. \tag{9}$$

The operators  $\hat{x}_I^\mu$  and  $\hat{p}_{\nu}^I$  satisfy  $[\hat{x}_I^\mu,\hat{p}_{\nu}^I]=i\hbar\,\delta_{\nu}^\mu$ , for  $I=1,\ldots,N$ , where  $\hat{x}_I^0$  is the coordinate time operator.  $\hat{H}_I$  is the internal Hamiltonian for each particle, associated to the internal evolution of the clock. Moreover,  $\hat{\omega}_{\mathbf{p}}^I=m_Ic~\hat{\gamma}_I$  where  $\hat{\gamma}_I=\sqrt{1+\frac{\hat{\mathbf{p}}_I^2}{m_I^2c^2}}$ . Here  $\Delta(\hat{\mathbf{x}}_I-\hat{\mathbf{x}}_L,\hat{\mathbf{p}}_I)$ , with  $I=1,\cdots,N$  is the worldline operator of a quantum relativistic particle in a weak gravitational field, and is responsible for the time dilation of the quantum clock, as explained in detail in Ref. [33], and it reads  $\Delta(\hat{\mathbf{x}}_I-\hat{\mathbf{x}}_L,\hat{\mathbf{p}}_I)=\sqrt{g_{00}(\hat{\mathbf{x}}_I-\hat{\mathbf{x}}_L)}\,\hat{\gamma}_I^{-1}$ . All calculations are performed keeping only the leading terms in  $O\left(\frac{V}{c^2}\right),\,O\left(\frac{p}{mc}\right)$ , and discarding factors  $O\left(\frac{p^2V}{m^2c^4}\right)$ .

In addition to the original formulation given in Ref. [33], we here additionally consider the dynamics of a laboratory L on the surface of the Earth, through the dynamical constraint  $\hat{C}_L$ . The Earth is responsible for sourcing the gravitational field, and the position of the laboratory is constrained to the Earth: no relative motion exists of the laboratory relative to the Earth, hence we treat the two as a single quantum system. For all practical purposes, the mass  $m_L$  can be taken to be equal to the mass of the Earth, meaning that  $m_L \gg m_J$ , with  $J=1,\cdots,N$ . For this reasons, we treat the laboratory as a classical reference frame.

We choose the initial coordinates such that the metric is locally flat at the origin of the laboratory, and that the motion of the laboratory is described in the low-velocity limit. As a result, the dynamical constraint  $\hat{C}_L$  is Newtonian, as in Eq. (7).

The introduction of the dynamics for the mass L is useful to give a description of the system from the point of view of the laboratory, and to see what a specific state in the perspective of the laboratory looks like in the perspective of another particle. We expect the dynamics from the point of view of L to be the standard Newtonian dynamics of quantum clocks in a weak gravitational field, as described e.g. in Ref. [41].

The perspective of a particle is obtained by applying a unitary operator  $\hat{\mathcal{T}}_i$  to the physical state, to transform the coordinates to the relative coordinates with respect to particle i, and set the metric to be flat at the position of the particle. Finally, the resulting state is projected on a state to fix the origin of the reference frame to be in

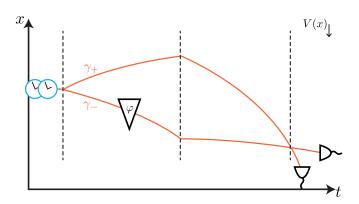


FIG. 2: Two atomic clocks in an interferometer vertically placed in a gravitational field V(x). The setup is made of two beam splitters, a mirror, a controllable phase shifter  $\varphi$ , and two detectors. The upper trajectory is called  $\gamma_+$  and the lower  $\gamma_-$ .

the position of the particle. For example, for particle 1:

$$|\psi\rangle^{(1)} = \langle q_1 = 0 | \hat{\mathcal{T}}_1 | \Psi \rangle_{ph}, \qquad (10)$$

where the explicit form of the operator  $\hat{T}_1$  is given in Appendix A 1. Through a lengthy calculation, one can show that the state in the perspective of particle 1 is equal to a "history state" [25, 33, 52]: an entangled state associating to each time state  $|\tau_1\rangle_{C_1}$  of the internal clock of the particle a state  $|\psi^{(1)}(\tau_1)\rangle$  describing all the other particles at time  $\tau_1$  as (see Appendix A 1 for the derivation)

$$|\psi\rangle^{(1)} \propto \int d\tau_1 |\psi^{(1)}(\tau_1)\rangle |\tau_1\rangle,$$
 (11)

where  $|\psi^{(1)}(\tau_1)\rangle = e^{-\frac{i}{\hbar}\hat{H}^{(1)}\tau_1} |\psi_0\rangle^{(1)}$ ,  $|\psi_0\rangle^{(1)}$  being the initial state of the system of N particles.

#### III. TEST FOR THE EEP FOR QRFS

An atomic clock in a weak gravitational field can be described [41, 57–60] via the mass-energy equivalence, by promoting the mass of the clock to an operator  $\hat{M}=m\,\hat{\mathbb{1}}+\frac{\hat{H}}{c^2}$ , where the rest mass m can be defined as the static part of the total mass-energy, while  $\hat{H}$  includes the dynamical contributions responsible for the dynamics of the internal state. Performing the substitution  $m\to\hat{M}$  in the standard Newtonian Hamiltonian and keeping terms up to  $O(c^{-2})$  we obtain

$$\hat{H}_{tot} = mc^2 + \frac{\hat{\mathbf{p}}^2}{2m} + mV(\hat{\mathbf{x}}) + \hat{H}\left(1 + \frac{V(\hat{\mathbf{x}})}{c^2} - \frac{\hat{\mathbf{p}}^2}{2m^2c^2}\right).$$
(12)

We consider the setup depicted in Fig. 2, where two atomic clocks, A and B, enter an interferometer in an entangled state from the perspective of the laboratory

L. The interferometer is placed vertically in the gravitational field  $V(\hat{\mathbf{x}})$  of the Earth. Each clock acquires a different proper time according to the path it takes in the interferometer [41, 58], resulting in a relative phase before the second beam splitter. Formally, each system X = A, B, L lives on a Hilbert space  $\mathcal{H}_X = \mathcal{H}_{\mathbf{X}}^{ext} \otimes \mathcal{H}_{C_X}$ , where  $\mathcal{H}_{\mathbf{X}}^{ext} \simeq L^2(\mathbb{R}^2)$  corresponds to the position or momentum of the system in (1 + 1)D (one time coordinate and one space coordinate), and  $\mathcal{H}_{C_X} \simeq L^2(\mathbb{R})$  to the internal state of the clock, ticking according to its proper time. The physical implications of the violation of the EEP for ORFs are best illustrated by the SORFs formalism [33], briefly reviewed in the previous section. This formalism recovers the standard quantum theory description from the perspective of the laboratory, and additionally allows us to describe the situation from the perspectives of the systems A and B.

The state from the point of view of the laboratory is

$$|\psi\rangle^{(L)} \propto \int d\tau_L \, |\psi^{(L)}(\tau_L)\rangle_{AB} \, |\tau_L\rangle_{C_L} \,,$$
 (13)

where  $|\psi^{(L)}(\tau_L)\rangle_{AB} = e^{-\frac{i}{\hbar}\hat{H}^{(L)}\tau_L} |\psi_0\rangle_{AB}^{(L)}, |\psi_0\rangle_{AB}^{(L)}$  is the initial state of the systems A and B in the full Hilbert space  $\mathcal{H}_A \otimes \mathcal{H}_B$ , and the Hamiltonian is found to be  $\hat{H}^{(L)} = \sum_{I=A,B} (m_I c^2 + \hat{H}_I^{(L)})$ , with

$$\hat{H}_{I}^{(L)} = \frac{\hat{\mathbf{p}}_{I}^{2}}{2m_{I}} + m_{I}V(\hat{\mathbf{x}}_{I}) + \hat{H}_{I}\left(1 + \frac{V(\hat{\mathbf{x}}_{I})}{c^{2}} - \frac{\hat{\mathbf{p}}_{I}^{2}}{2m_{I}^{2}c^{2}}\right),\tag{14}$$

coinciding with the standard Hamiltonian of Eq. (12). The state at an arbitrary time t in the perspective of the laboratory can be retrieved with a projection on the state of the clock, namely  $|\psi^{(L)}(t)\rangle_{AB} = c_L \langle t|\psi\rangle^{(L)} = e^{-\frac{i}{\hbar}\hat{H}^{(L)}t} |\psi_0\rangle^{(L)}_{AB}$ . Hence the predictions of the SQRF formalism coincide with standard quantum mechanics in the perspective of the laboratory.

We now illustrate how LPI is extended to QRFs (Q-LPI). For simplicity, we restrict to the regime in which the external degrees of freedom can be treated in the semiclassical limit, meaning that they appear only as fixed functions inside the Hamiltonian [41, 58]: their time evolution consists only in a phase. Under this approximation, we can simplify the evolution of the clocks in the interferometer by assigning them two states, e.g.  $\{|x_{+}\rangle_{\mathbf{A}}, |x_{-}\rangle_{\mathbf{A}}\}$ , corresponding to the upper and lower path  $\gamma_{\pm}$  respectively.

The initial state after the first beam splitter is (see Fig. 2)

$$|\psi_0\rangle_{AB}^{(L)} = \frac{|x_+\rangle_{\mathbf{A}} |x_+\rangle_{\mathbf{B}} + e^{2i\varphi} |x_-\rangle_{\mathbf{A}} |x_-\rangle_{\mathbf{B}}}{\sqrt{2}} |\tau_{in}\rangle_{C_A} |\tau_{in}\rangle_{C_B},$$
(15)

where  $\varphi$  is a phase that can be experimentally controlled, and from now on we choose  $\tau_{in}=0$  for simplicity. The evolution inside the interferometer is different for each trajectory  $\gamma_{\pm}$ , and is given by the time evolution operator

 $\hat{U}_{\pm}=e^{-\frac{i}{\hbar}\int_{\gamma_{\pm}}dt\hat{H}^{(L)}}$ . The state inside the interferometer, after the time evolution, is

$$|\psi^{(L)}\rangle_{AB} = \frac{1}{\sqrt{2}} (|x_{+}\rangle_{\mathbf{A}} |x_{+}\rangle_{\mathbf{B}} |\tau_{+}\rangle_{C_{A}} |\tau_{+}\rangle_{C_{B}} + e^{i(2\varphi + \sum_{j=A,B} \Delta\phi_{j})} |x_{-}\rangle_{\mathbf{A}} |x_{-}\rangle_{\mathbf{B}} |\tau_{-}\rangle_{C_{A}} |\tau_{-}\rangle_{C_{B}}),$$
(16)

where  $\Delta \phi_j = \frac{1}{\hbar} \int_{\Delta \gamma} dt \left( m_j c^2 + \frac{\mathbf{p}_j^2}{2m_j} + V(\mathbf{x}_j) \right)$  is the line integral along the closed path formed by the two arms of the interferometer, i.e.  $\int_{\Delta \gamma} (\cdots) = \int_{\gamma_+} (\cdots) - \int_{\gamma_-} (\cdots)$  and  $|\tau_{\pm}\rangle_{C_j} = e^{-\frac{i}{\hbar} \int_{\gamma_{\pm}} dt \left(1 + \frac{V(\mathbf{x}_j)}{c^2} - \frac{\mathbf{p}_j^2}{2m_j^2 c^2}\right) \hat{H}_{int}^j} |\tau_{in}\rangle_{C_j}, j = A, B$ . This means that, from the laboratory perspective, clocks A and B tick in a quantum superposition of times. The interferometric measurements are projections on the Hilbert space of the path alone (the clock is not measured)

$$|D_{\pm}\rangle_{\mathbf{A}\mathbf{B}} = \frac{|x_{+}\rangle_{\mathbf{A}}|x_{+}\rangle_{\mathbf{B}} \pm |x_{-}\rangle_{\mathbf{A}}|x_{-}\rangle_{\mathbf{B}}}{\sqrt{2}}.$$
 (17)

The measurement probabilities for two identical masses  $m_A = m_B = m$  are

$$P_{\pm} = \frac{1}{2} \left( 1 \pm |\langle \tau_{+} | \tau_{-} \rangle|^{2} \cos \left( 2\Delta \phi + 2\varphi + 2\varphi' \right) \right), \quad (18)$$

where  $\Delta \phi = \Delta \phi_A = \Delta \phi_B$  and  $\varphi'$  is defined by  $\langle \tau_+ | \tau_- \rangle_{C_A} = \langle \tau_+ | \tau_- \rangle_{C_B} = |\langle \tau_+ | \tau_- \rangle| \, e^{i \varphi'}$ .

An analogous "history state" to Eq. (13) describes the dynamical evolution in the perspective of the systems A or B. For example, the "history state" of B and L as seen from A is

$$|\psi\rangle^{(A)} \sim \int d\tau_A |\psi^{(A)}(\tau_A)\rangle_{BL} |\tau_A\rangle_{C_A},$$
 (19)

where  $|\psi^{(A)}(\tau_A)\rangle_{BL}=e^{-\frac{i}{\hbar}\hat{H}^{(A)}\tau_A}\,|\psi_0^{(A)}\rangle_{BL}$  and the notation is completely analogous to the one in Eq. (13). The Hamiltonian (see Appendix A 1 for details) is

$$\hat{H}^{(A)} = \sum_{j=B,L} \hat{T}_j + \sum_{J=A,B,L} \left( m_J c^2 + \hat{T}_J' \right) + m_B \hat{V}'(\hat{\mathbf{q}}_B, \hat{\mathbf{q}}_L) + - m_L V(\hat{\mathbf{q}}_L) + \hat{\Delta}_B^{(A)} \hat{H}_B + \hat{\Delta}_L^{(A)} \hat{H}_L,$$
(20)

where we have defined 
$$\hat{T}_j = \frac{\hat{\mathbf{k}}_j^2}{2m_j}$$
 for  $j = B, L$  and  $\hat{T}_J' = \frac{(\sum_{j=B,L} \hat{\mathbf{k}}_j)^2}{2m_A^2} m_J$  for  $J = A, B, L$ , and in addition  $\hat{V}'(\hat{\mathbf{q}}_B, \hat{\mathbf{q}}_L) = V(\hat{\mathbf{q}}_B - \hat{\mathbf{q}}_L) - V(\hat{\mathbf{q}}_L)$ ,  $\hat{\Delta}_B^{(A)} = \left(1 + \frac{\hat{V}'(\hat{\mathbf{q}}_B, \hat{\mathbf{q}}_L)}{c^2} + \frac{\hat{T}_A'}{m_A c^2} - \frac{\hat{T}_B}{m_B c^2}\right)$ ,  $\hat{\Delta}_L^{(A)} = \left(1 - \frac{V(\hat{\mathbf{q}}_L)}{c^2} + \frac{T_A'}{m_A c^2} - \frac{\hat{T}_L}{m_L c^2}\right)$ . For simplicity, in the following we neglect special-relativistic time dilation, because it is not related to the generalisation of LPI.

The quantum state in the laboratory frame in Eq. (16)

is transformed to the perspective of A as  $|\psi^{(A)}(\tau_A)\rangle_{BL} = \frac{1}{\sqrt{2}} (|\Psi(\tau_A, x_+, x_+)\rangle_{\mathbf{B}L} + |\Psi(\tau_A, x_-, x_-)\rangle_{\mathbf{B}L}) |\tau_A\rangle_{C_B}$ , where the explicit expression of  $|\Psi(\tau_A, x_\pm, x_\pm)\rangle_{\mathbf{B}L}$  is given in Appendix A 2. This shows that, even if the reading of time of  $C_A$  and  $C_B$  was unsharp in the laboratory frame, in the perspective of one of the clocks the time reading of the other clock is well-defined and there is no time dilation. This is an extension of the universality of gravitational redshift to quantum superpositions. We give another example of this extension in Appendix A 2.

We conclude that if two clocks are in spatial superposition, the time of the second clock is dilated according to the first clock, if there is a branch of the quantum superposition where the clocks are not at the same height. Moreover, the dilation factor coincides with the classical dilation factor  $\frac{\Delta\nu}{\nu} = \frac{\Delta V}{c^2}$ . Q-LPI reduces to standard LPI when quantum superpositions are not considered, as exemplified in Appendix A 2. A similar procedure leads to the generalization of LLI to Q-LLI, namely the superposition of special relativistic time dilations for particles in a superposition of momenta.

## IV. MODEL FOR VIOLATIONS OF THE EEP FOR QRFS

A violation of the EEP for QRFs can be quantified by generalising the model introduced in Ref. [11] to study violations of a different quantum version of the EEP. We show in Appendix B that this model, so generalised, can be used to test the EEP for QRFs in the framework of SQRFs.

The model for violations introduced in Ref. [11] uses three different types of mass-operators  $\hat{M}^{(s)} = m^{(s)} \mathbb{1} +$  $\frac{\hat{H}^{(s)}}{c^2}$ , with s=g,i,r, acting on the internal degrees of freedom of the clocks. In particular, the gravitational mass  $\hat{M}^{(g)}$  couples to the gravitational field, the inertial mass  $\hat{M}^{(i)}$  couples to the momentum, and the rest mass  $\hat{M}^{(r)}$  is the mass in the rest frame of the atomic clock. The EEP, in its generalised form, is valid if all matrix elements of these operators are the same, namely if and only if  $\hat{M}^{(g)} = \hat{M}^{(i)}$  (WEP),  $\hat{M}^{(r)} = \hat{M}^{(i)}$  (LLI), and  $\hat{M}^{(g)} = \hat{M}^{(r)}$  (LPI). To test the EEP to ORFs we generalise this model and relax the condition that the mass operator only acts on the internal clock Hilbert space. For example, the most general violation of Q-LPI should also include an explicit position dependence on the violation operator. An analogous conclusion can be drawn for Q-LLI for momentum-dependence. Therefore, we let  $\hat{H}^{(g)} = \hat{H}^{(g)}(\hat{\mathbf{x}}) = f(\hat{A}, \hat{\mathbf{x}})$ , and  $\hat{H}^{(i)} = \hat{H}^{(i)}(\hat{\mathbf{p}}) = \hat{H}^{(i)}(\hat{\mathbf{p}})$  $q(\hat{B}, \hat{\mathbf{p}})$ , namely  $\hat{H}^{(g)}(\hat{\mathbf{x}})$  is a generic function of the position operator  $\hat{\mathbf{x}}$  and of an operator  $\hat{A}$  that acts on the internal Hilbert space only, and analogously for  $\hat{H}^{(i)}(\hat{\mathbf{p}})$ .

The violations of the three conditions composing the

definition of the EEP are

$$m^{(i)}\hat{\mathbb{1}} + \hat{H}^{(i)}(\hat{\mathbf{p}}) \neq m^{(g)}\hat{\mathbb{1}} + \hat{H}^{(g)}(\hat{\mathbf{x}})$$
 (Q-WEP), (21)

$$\hat{H}^{(r)} \neq \hat{H}^{(i)}(\hat{\mathbf{p}})$$
 (Q-LLI), (22)

$$\hat{H}^{(r)} \neq \hat{H}^{(g)}(\hat{\mathbf{x}})$$
 (Q-LPI), (23)

where the rest mass does not appear in the second and third condition because it is not observable [11]. The resulting Hamiltonian is (up to the additive rest mass term)

$$\hat{H}_{tot} = \frac{\hat{\mathbf{p}}^2}{2m^{(i)}} + m^{(g)}V(\hat{\mathbf{x}}) + \hat{H}^{(r)} + + \hat{H}^{(g)}(\hat{\mathbf{x}}) \frac{V(\hat{\mathbf{x}})}{c^2} - \hat{H}^{(i)}(\hat{\mathbf{p}}) \frac{\hat{\mathbf{p}}^2}{2m^{(i)2}c^2}.$$
(24)

Note that the dependence on the phase-space operators does not introduce any ordering problem.

For convenience, we can parametrise the violation by defining a different but equivalent set of operators

$$\hat{\eta}(\hat{\mathbf{x}}, \hat{\mathbf{p}}) = \hat{\mathbb{1}} - \hat{M}^{(g)}(\hat{\mathbf{x}}) \, \hat{M}^{(i)-1}(\hat{\mathbf{p}}) \qquad (\text{WEP}), \qquad (25)$$

$$\hat{\beta}(\hat{\mathbf{p}}) = \hat{\mathbb{1}} - \hat{H}^{(i)}(\hat{\mathbf{p}}) \, \hat{H}^{(r)-1}$$
 (LLI), (26)

$$\hat{\alpha}(\hat{\mathbf{x}}) = \hat{\mathbb{1}} - \hat{H}^{(g)}(\hat{\mathbf{x}}) \, \hat{H}^{(r)-1} \tag{LPI}, \tag{27}$$

with  $\hat{\eta}$ ,  $\hat{\beta}$ , and  $\hat{\alpha}$  being invertible to our order of approximation. The different internal Hamiltonians in general do not commute, namely  $[\hat{H}^{(g)}(\hat{\mathbf{x}}), \hat{H}^{(r)}] \neq 0$ ,  $[\hat{H}^{(i)}(\hat{\mathbf{p}}), \hat{H}^{(r)}] \neq 0$ ,  $[\hat{H}^{(g)}(\hat{\mathbf{x}}), \hat{H}^{(i)}(\hat{\mathbf{p}})] \neq 0$ , but we assume the commutators to be sufficiently small such that the higher order commutators can be neglected, e.g.  $[\hat{H}^{(g)}(\hat{\mathbf{x}}), [\hat{H}^{(g)}(\hat{\mathbf{x}}), \hat{H}^{(r)}]] \frac{t}{\hbar} \ll [\hat{H}^{(g)}(\hat{\mathbf{x}}), \hat{H}^{(r)}]$ , and analogously for the others.

#### V. OBSERVATION OF THE VIOLATION OF THE EEP FOR QRFS

We now show that the notion of QLIF as defined in Ref. [1] is no longer valid when the EEP for QRFs is violated in the sense of the model that we have introduced. The notion of QLIF, namely the existence of a QRF associated to a quantum system in which, locally, the metric reduces to Minkowski, is the core of the generalisation of the EEP to QRFs. Detecting such a violation experimentally would imply that the EEP cannot be extended to QRFs, at least in the proposed form, and hence challenge the validity of a formulation of physics from the perspective of a QRF.

If the EEP for QRFs is violated, the constraint  $\hat{f}^0$  of

Eq. (8) is modified to

$$\hat{f}^{0} = \sum_{I=A,B} \hat{p}_{I}^{0} + \left(\sqrt{g_{00}(\hat{\mathbf{x}}_{I} - \hat{\mathbf{x}}_{L})} - 1\right) \frac{\hat{H}_{I}^{(g)}}{c} + \left(\sqrt{1 + \frac{\hat{\mathbf{p}}_{I}^{2}}{m_{I}^{2}c^{2}}} - 1\right) \frac{\hat{H}_{I}^{(i)}}{c} + \frac{\hat{H}_{I}^{(r)}}{c} + \hat{p}_{L}^{0}.$$
(28)

Notice that, even in the presence of a violation, it is always possible to take the perspective of the laboratory, in which the dynamics follows a standard Hamiltonian evolution. This is essential to be able to perform a test of the generalised EEP, and naturally follows from imposing the Galilean dynamical constraint of Eq. (7).

If any aspect of the EEP for QRFs (Q-LPI, Q-LLI, Q-WEP) is violated as in Eqs. (21-23) it is impossible to find a "history state" in the perspective of A or B. We illustrate this fact for Q-LPI, but the reasoning is the same for Q-LLI and Q-WEP and the explicit calculations for all cases are contained in Appendix B. Specifically, let us assume that  $\hat{H}_j^{(r)} \neq \hat{H}_j^{(g)}(\hat{\mathbf{x}}_j)$  but  $\hat{H}_j^{(r)} = \hat{H}_j^{(i)}(\hat{\mathbf{p}}_j)$  for j = A, B. When we reduce to the perspective of one of the systems, say A, we find a quantum state of the form

$$|\psi\rangle^{(A)} \sim \int d\tau_A dq_L dt_A |\chi(\tau_A, \mathbf{q}_L, t_A)\rangle_{BL} |\varphi(\tau_A, \mathbf{q}_L, t_A)\rangle_{C_A},$$
(29)

where the relevant aspect of the expression is that  $\varphi$  explicitly depends on the coordinate  $\mathbf{q}_L$ . The explicit form of  $\chi(\tau_A, \mathbf{q}_L, t_A)$  and  $\varphi(\tau_A, \mathbf{q}_L, t_A)$  is not relevant to our argument, and is reported in Appendix B. This result means that, in the QLIF of system A, the clock behaves differently according to where it is placed in the gravitational field. This is not compatible with the result that the metric is locally inertial in the QLIF associated to any quantum system, and hence shows that the EEP for QRFs in the formulation of Ref. [1] is violated. For simplicity of illustration, we study each aspect of the principle (Q-LPI, Q-LLI, Q-WEP) separately. These three aspects are not independent: a test of EEP for QRFs can be performed, e.g., by testing Q-LPI, Q-LLI, and the classical version of WEP, as explained in the following. We notice that to detect the violation in L, without comparing the clocks A and B in different QRFs, it is sufficient to use a single clock. In this case, the measurement probabilities are very similar to Eq. (18), and the derivation presented below still holds.

#### A. Violation of Q-LPI

For a violation of Q-LPI, we impose the condition  $\hat{H}^{(r)} \neq \hat{H}^{(g)}(\hat{\mathbf{x}})$  in Eq. (23). In the laboratory frame L, we obtain the Hamiltonian  $\hat{H}_{LPI}^{(L)} =$ 

$$\begin{split} \sum_{j=A,B} \left( m_j c^2 + \hat{H}_j^{LPI} \right), & \text{ with } \\ \hat{H}_j^{LPI} &= \frac{\mathbf{p}_j^2}{2m_j} + m_j V(\mathbf{x}_j) + \hat{H}_j^{(r)} + \\ &+ \hat{H}_j^{(g)}(\mathbf{x}_j) \frac{V(\mathbf{x}_j)}{c^2} - \hat{H}_j^{(r)} \frac{\mathbf{p}_j^2}{2m_s^2 c^2}. \end{split}$$

In the interferometric setup we have considered, the probabilities are formally the same as in Eq. (18), but the factor  $\langle \tau_+ | \tau_- \rangle_j'$ , j = A, B, is different. To the lowest order in the violation operator  $\hat{\alpha}(\mathbf{x})$ , discarding the commutators between  $\hat{\alpha}(\mathbf{x})$  and all the other operators, we find  $\Delta_{LPI} \langle \tau \rangle_j = \langle \tau_+ | \tau_- \rangle_j' - \langle \tau_+ | \tau_- \rangle_j$ , with

$$\Delta_{LPI} \langle \tau \rangle_{j} = -\frac{i}{\hbar} \int_{\Delta \gamma} dt \; \frac{V(\mathbf{x}_{\gamma})}{c^{2}}_{j} \langle \tau_{+} | \alpha(\mathbf{x}_{\gamma}) \hat{H}^{(r)} | \tau_{-} \rangle_{j} ,$$
(30)

where  $\Delta \gamma$  is the closed path formed by the two arms of the interferometer and  $\mathbf{x}_{\gamma}$  is the position evaluated along the (classical) trajectory of the system. Consequently, the probability explicitly depends on the matrix elements of the violation operator, which means that the experiment is sensitive to violations of Q-LPI.

#### B. Violation of Q-LLI

For a violation of Q-LLI, we impose the condition  $\hat{H}^{(r)} \neq \hat{H}^{(i)}(\hat{\mathbf{p}})$  in Eq. (22). With analogous notation to the case of Q-LPI, we obtain

$$\Delta_{LLI} \langle \tau \rangle_j = \frac{i}{\hbar} \int_{\Delta_{\gamma}} dt \; \frac{\mathbf{p}_{\gamma}^2}{m^2 c^2} \langle \tau_+ | \, \hat{\beta}(\mathbf{p}_{\gamma}) \hat{H}^{(r)} | \tau_- \rangle \,, \quad (31)$$

Thus, the experiment is sensitive to violations of LLI.

#### C. Violation of Q-WEP

The violation of Q-WEP is encoded in the operator  $\hat{\eta}(\mathbf{x}, \mathbf{p}) = \hat{\mathbb{1}} - \hat{M}^{(g)}(\mathbf{x}) \, \hat{M}^{(i)-1}(\mathbf{p})$ , where  $\hat{M}^{(g)}(\mathbf{x}) = m^{(g)} + (1 - \hat{\alpha}(\mathbf{x})) \hat{H}^{(r)}$  and  $\hat{M}^{(i)}(\mathbf{p}) = m^{(i)} + (1 - \hat{\beta}(\mathbf{p})) \hat{H}^{(r)}$ . Moreover, the standard WEP violation is  $\eta = 1 - \frac{m^{(g)}}{m^{(i)}}$ . Hence  $\hat{\eta}(x, p)$  is a function  $\hat{\eta} = \hat{\eta}\left(\hat{\alpha}(\mathbf{x}), \hat{\beta}(\mathbf{p}), \eta, m^{(i)}, \hat{H}^{(r)}\right)$ . Provided that the inertial mass  $m^{(i)}$  and the rest Hamiltonian  $\hat{H}^{(r)}$  are known,  $\hat{\eta}$  is fully determined by  $\hat{\alpha}(\mathbf{x}), \hat{\beta}(\mathbf{p})$  and the classical parameter  $\eta$ . Since we already showed that the Mach-Zehnder interferometer is sensitive to the coefficients of  $\hat{\alpha}(\mathbf{x})$  and  $\hat{\beta}(\mathbf{p})$ , it only remains to prove its sensitivity to  $\eta$ , namely that it can provide a classical standard WEP test.

Therefore, by imposing the condition for the mass of particle  $j=A, B \ m_j^{(g)} \neq m_j^{(i)}$  and by neglecting the rest-mass term, we find that the Hamiltonian in the

laboratory frame in Eq. (14) is modified to  $\hat{H}_{WEP}^{(L)} = \sum_{j=A,B} \hat{H}_{j}^{WEP}$ , where

$$\hat{H}_{j}^{WEP} = \frac{\mathbf{p}_{j}^{2}}{2m_{j}^{(i)}} + m_{j}^{(g)}V(\mathbf{x}_{j}) + \hat{H}_{j}\left(1 + \frac{V(\mathbf{x}_{j})}{c^{2}} - \frac{\mathbf{p}_{j}^{2}}{2m_{j}^{(i)2}c^{2}}\right)$$
(32)

The measurement probabilities are the same of Eq. (18), where

$$\Delta \phi_j = \frac{1}{\hbar} \int_{\Delta \gamma} dt \left[ (1 - \eta) m_j^{(i)} V(\mathbf{x}_j) + \frac{\mathbf{p}_j^2}{2m_j^{(i)}} \right]. \quad (33)$$

Thus, the setup can provide a test for WEP.

#### VI. CONCLUSION

In conclusion, we proposed a model for violations of the EEP for QRFs, and showed that an interferometric setup can be used to test this principle. We also established a connection among three in principle different extensions of the equivalence principle to the quantum regime, namely those in Refs. [1, 11, 16]. We showed that violations of the EEP for QRFs entails the impossibility of describing the dynamical evolution of the physical systems external to the QRF from the point of view of one of the quantum particles in the interferometer, and that this corresponds to breaking the notion of a Quantum Local Inertial Frame (QLIF). A verification of this generalised principle will allow us to gain a deeper insight into the interplay of the fundamental principles of quantum theory and general relativity.

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#### Appendix A: Spacetime Quantum Reference Frames

#### 1. The formalism

In this appendix we provide some more details on the Spacetime Quantum Reference Frames (SQRFs) formalism, introduced in Ref. [33]. As explained in the main text, the state from the perspective of the laboratory can be obtained via the definition of the original formalism, namely

$$|\psi\rangle^{(L)} = \langle q_L = 0 | \hat{\mathcal{T}}_L | \Psi \rangle_{nh} , \qquad (A1)$$

where we have introduced a transformation operator to the relative positions to L, analogously to the ones of the original framework, but with a flat metric, namely

$$\hat{\mathcal{T}}_L = e^{\frac{i}{\hbar}\hat{\mathbf{x}}_L(\hat{f}^1 - \hat{\mathbf{p}}_L)} e^{\frac{i}{\hbar}\hat{x}_L^0(\hat{f}^0 - \hat{p}_L^0)}. \tag{A2}$$

The calculations to obtain the state in the perspective of L are analogous to the original framework, namely we can write the physical state as

$$|\Psi\rangle_{ph} \propto \int d^{N+1} \mathcal{N} d^2 z \ e^{\frac{i}{\hbar} \mathcal{N}_J \hat{C}_J} e^{\frac{i}{\hbar} z_\mu \hat{f}^\mu} |\phi\rangle \,,$$
 (A3)

where  $|\phi\rangle$  can be written as

$$|\phi\rangle = \int \Pi_I \left[ d\mu(x_I) dE_I \right] d^2 x_L dE_L \phi(x_1, \dots, x_L, E_1, \dots, E_L) |x_1, \dots, x_L\rangle |E_1, \dots, E_L\rangle, \tag{A4}$$

and  $d\mu(x_I) = \sqrt{g_{00}(\mathbf{x}_I - \mathbf{x}_L)}d^2x_I$  is the covariant integration measure. We can then recover the quantum state from the laboratory perspective,  $|\psi\rangle^{(L)}$  of Eq. (A1), via a tedious but straightforward procedure detailed in Ref. [33]. The result is the history state

$$|\psi\rangle^{(L)} = \int d\tau_L e^{-\frac{i}{\hbar}\hat{H}^{(L)}\tau_L} |\psi_0^{(L)}\rangle |\tau_L\rangle, \qquad (A5)$$

where the Hamiltonian from the perspective of L is

$$\hat{H}^{(L)} = m_L c^2 + \frac{\left(\sum_I \hat{\mathbf{k}}_I\right)^2}{2m_L} + \sum_{I=1}^N \Delta(\hat{\mathbf{q}}_I, \hat{\mathbf{k}}_I)(\hat{\omega}_{\mathbf{k}}^I + \hat{H}_I) =$$

$$\sim \sum_{J=1}^L m_J c^2 + \sum_{I=1}^N \frac{\hat{\mathbf{k}}_I^2}{2m_I} + \sum_{I=1}^N m_I V(\hat{\mathbf{q}}_I) + \left(1 + \frac{V(\hat{\mathbf{q}}_I)}{c^2} - \frac{\hat{\mathbf{k}}_I^2}{2m_I^2 c^2}\right) \hat{H}_I.$$
(A6)

In going from the first to the second line of Eq. (A6) we have discarded the first kinetic term, since L is very massive, and approximated the world line operator  $\hat{\Delta}$  and  $\hat{\omega}_k$  to the first order in  $c^2$ . Moreover

$$|\psi_{0}^{(L)}\rangle = \int \Pi_{I}[d\mu(q_{I})dq_{I}^{\prime 0}\sqrt{g_{00}(\mathbf{q}_{I})}dE_{I}]e^{\frac{i}{\hbar}\sum_{I=1}^{N}(q_{I}^{\prime 0}-q_{I}^{0})\sqrt{g_{00}(\mathbf{q}_{I})}\omega_{\hat{\mathbf{k}}_{I}}} \times \times \phi_{0}^{(L)}(q_{1},\ldots,q_{N},E_{1},\ldots,E_{N})|q_{1}^{\prime 0},\mathbf{q}_{1},\ldots,q_{N}^{\prime 0},\mathbf{q}_{N}\rangle|E_{1},\ldots,E_{N}\rangle$$
(A7)

and the state  $\phi_0^{(L)}$  can be written as a complicated, but not informative expression in terms of the initial kinematical state  $|\phi\rangle$ . The interested reader can refer to Ref. [33] for details. The state in Eq. (A5) is a *history* state, where  $|\psi_0\rangle^{(L)}$  plays the role of the initial state of the system, and the state described by the laboratory clock at time t can be retrieved via the Page-Wootters mechanism, namely by projecting on the state of the internal clock  $|t\rangle_{C_L}$ .

The Hamiltonian of Eq. (A6) is the standard Newtonian Hamiltonian for free particles with the gravitational time dilation of the clocks due to their relative distance to the Earth and the special relativistic time dilation, coinciding with Eq. (12) and the Hamiltonian used in Refs. [41, 57–59]. Therefore, the introduction of the constraint of Eq. (7) provides a consistent way to describe the reference frame of the laboratory.

The novelty of the SQRFs framework is the possibility to describe the perspective of all the other particles as well in a completely analogous way to what we presented here for the laboratory. The transformation operator to the QRF associated to the relative positions to one of the particles, say particle 1, is

$$\hat{\mathcal{T}}_{1} = e^{-\frac{i}{\hbar} \frac{\log \sqrt{g_{00}(\hat{\mathbf{x}}_{L})}}{2} \sum_{J=1}^{L} (\hat{x}_{J}^{0} \hat{p}_{J}^{0} + \hat{p}_{J}^{0} \hat{x}_{J}^{0}) e^{\frac{i}{\hbar} \hat{\mathbf{x}}_{1} (\hat{f}^{1} - \hat{\mathbf{p}}_{1})} e^{\frac{i}{\hbar} \hat{x}_{1}^{0} (\hat{f}^{0} - \hat{p}_{1}^{0})}, \tag{A8}$$

and analogously to what we showed for the case of the transformation to the laboratory frame, the quantum state in the QRF of particle 1 is

$$|\psi\rangle^{(1)} = \langle q_1 = 0 | \hat{\mathcal{T}}_1 | \Psi \rangle_{ph} =$$

$$= \int d\tau_1 e^{-\frac{i}{\hbar} \hat{H}^{(1)} \tau_1} |\psi_0^{(1)}\rangle |\tau_1\rangle_{C_1}, \qquad (A9)$$

where the Hamiltonian is

$$\hat{H}^{(1)} = \hat{\gamma}_{\sum \mathbf{k},1} \left( \sum_{i} \sqrt{g'_{00}(\mathbf{q}_{i}, \mathbf{q}_{L})} (c\hat{\omega}_{\mathbf{k}}^{i} + \hat{\gamma}_{i}^{-1} \hat{H}_{i}) + c\hat{\omega}_{\sum \mathbf{k},1} + \sqrt{g^{00}(\mathbf{q}_{L})} \left( m_{L}c^{2} + \frac{\hat{\mathbf{k}}_{L}^{2}}{2m_{L}} + \hat{H}_{L} \right) \right)$$

$$\sim \sum_{J=1}^{L} m_{J}c^{2} + \sum_{j=2}^{L} \frac{\hat{\mathbf{k}}_{j}^{2}}{2m_{j}} + \sum_{J=1}^{L} \frac{(\sum_{j=2}^{L} \hat{\mathbf{k}}_{j})^{2}}{2m_{1}^{2}} m_{J} + \sum_{i=2}^{N} m^{(i)} (V(\hat{\mathbf{q}}_{i} - \hat{\mathbf{q}}_{L}) - V(\hat{\mathbf{q}}_{L})) - m_{L}V(\hat{\mathbf{q}}_{L}) +$$

$$+ \sum_{i=2}^{N} \left( 1 + \frac{V(\hat{\mathbf{q}}_{i} - \hat{\mathbf{q}}_{L}) - V(\hat{\mathbf{q}}_{L})}{c^{2}} + \frac{(\sum_{j=2}^{L} \hat{\mathbf{k}}_{j})^{2}}{2m_{1}^{2}c^{2}} - \frac{\hat{\mathbf{k}}_{i}^{2}}{2m_{i}^{2}c^{2}} \right) \hat{H}_{i} + \left( 1 - \frac{V(\hat{\mathbf{q}}_{L})}{c^{2}} + \frac{(\sum_{j=2}^{L} \hat{\mathbf{k}}_{j})^{2}}{2m_{L}^{2}c^{2}} - \frac{\hat{\mathbf{k}}_{L}^{2}}{2m_{L}^{2}c^{2}} \right) \hat{H}_{L}.$$

$$(A10)$$

The framework also allows us to describe a state in the QRF of a particle, and then reversibly transform it to the perspective of another particle. For example, if we wish to change from the QRF of the laboratory to the QRF of particle 1, this can be achieved via the invertible transformation

$$|\psi\rangle^{(1)} = \langle q_1 = 0 | \hat{\mathcal{T}}_1 \hat{\mathcal{T}}_L^{\dagger} | \psi \rangle^{(L)} | p_L = 0 \rangle. \tag{A11}$$

A pictorial representation of this change of perspective is given in Figure 3. Note that the SQRF transformation to

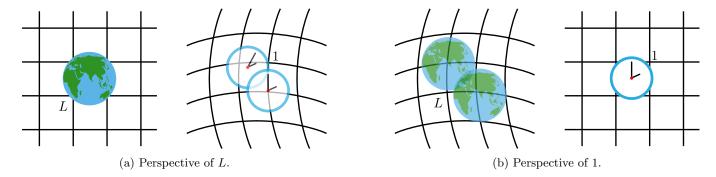


FIG. 3: Representation of an example of change of SQRF. In Figure 3a, particle 1 is seen in a superposition of states and in a curved spacetime from the point of view of particle L, namely the laboratory. In Figure 3b it is L to be in a superposition and in a curved spacetime, when it is seen in the SQRF of particle 1.

the perspective of a particle serve as an operational definition of its QLIF, since in that frame the metric is locally flat at the origin of the coordinates.

#### 2. Quantum extension of LPI

In this section we report in further detail the calculations regarding the Quantum extension of LPI of Section III. We employ the SQRFs framework described in Appendix A 1, using the extended constraints to include the laboratory as well. We focus on the Newtonian limit, namely we discard the special relativistic effects, including special relativistic time dilation, since we are only interested in gravitational time dilation.

We illustrate the simple case of a state in the perspective of the laboratory that is localised in position basis, namely

$$|\psi_0^{(L)}\rangle_{AB} = |x_1\rangle_{\mathbf{A}} |x_2\rangle_{\mathbf{B}} |\tau_{in} = 0\rangle_{C_A} |\tau_{in} = 0\rangle_{C_B}.$$
(A12)

This situation corresponds to having two quantum systems at two different heights  $\mathbf{x}_{1,2}$ , and to initialising the internal time of both of them is set to  $\tau_{in} = 0$ . The particles are not in a superposition, so we do not expect to see any quantum effects. An arbitrary quantum state, however, can be obtained by linear combination of this simple state.

The state in the perspective of particle A can be obtained with the reference frame transformation defined in Eq. (A11), namely

$$|\psi\rangle^{(A)} = {}_{\mathbf{A}} \langle q_A = 0 | \hat{\mathcal{T}}_A \hat{\mathcal{T}}_L^{\dagger} \int d\tau_L e^{-\frac{i}{\hbar} \hat{H}^{(L)} \tau_L} |\psi_0^{(L)}\rangle_{AB} |\tau_L\rangle_{C_L} |p_L = 0\rangle_L, \qquad (A13)$$

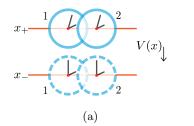
where  $\hat{T}_A$  and  $\hat{T}_L$  are defined in Eq. (A8) and Eq. (A2) respectively, and the Hamiltonian  $\hat{H}^{(L)}$  is defined in Eq. (A6).

The calculations can be performed by letting the operator  $\hat{\mathcal{T}}_A\hat{\mathcal{T}}_L^{\dagger}$  act on the Hamiltonian, and expanding both  $\langle q_A=0|$  and  $|p_L=0\rangle$  with a Fourier series. Then, we calculate the action of  $\hat{\mathcal{T}}_A\hat{\mathcal{T}}_L^{\dagger}$  on the states explicitly. The result is that the state in the perspective of particle A at time  $t_A$  is

$$|\psi(t_A)\rangle_{BL}^{(A)} = |\Psi(t_A, x_1, x_2)\rangle_{\mathbf{B}L} \left| \left( 1 + \frac{V(\mathbf{x}_2) - V(\mathbf{x}_1)}{c^2} \right) t_A \right\rangle_{C_B}, \tag{A14}$$

where  $\left|\left(1+\frac{V(\mathbf{x}_2)-V(\mathbf{x}_1)}{c^2}\right)t_A\right\rangle_{C_B}$  is the clock of the second particle B, while its external state is contained in  $|\Psi(t_A,x_1,x_2)\rangle_{\mathbf{R}_L}$ , along with the complete state of the laboratory:

$$|\Psi(t_A, x_1, x_2)\rangle_{\mathbf{B}L} = \left(1 - \frac{V(\mathbf{x}_1)}{c^2}\right)^2 e^{-\frac{i}{\hbar}\hat{H}'_{ext}\left(\left(1 - \frac{V(\mathbf{x}_1)}{c^2}\right)t_A + \frac{x_1^0}{c}\right)} \times \\ \times |\sqrt{g_{00}(\mathbf{x}_1)}(x_2^0 - x_1^0), \mathbf{x}_2 - \mathbf{x}_1\rangle_{\mathbf{B}} |-\sqrt{g_{00}(\mathbf{x}_1)}x_1^0, -\mathbf{x}_1\rangle_{\mathbf{L}} \left|\left(1 - \frac{V(\mathbf{x}_1)}{c^2}\right)t_A\right\rangle_{C_L}, \tag{A15}$$



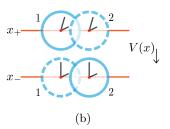


FIG. 4: On the left, representation of two atomic clocks in a superposition in a gravitational field, where in each branch of the superposition they are at the same height. On the right, representation of two atomic clocks in a superposition in a gravitational field, where in each branch of the superposition they are at different heights. The first branch of the superposition is represented with solid lines, the second branch with dashed lines.

where

$$\hat{H}'_{ext} = \sum_{J=A,B,L} m_J c^2 + m_A V(\hat{\mathbf{x}}_L) + m_B V(\hat{\mathbf{x}}_B - \hat{\mathbf{x}}_L) + \sum_{j=B,L} \frac{\hat{\mathbf{p}}_j^2}{2m_j} + \frac{\left(\sum_{j=2,M} \hat{\mathbf{p}}_j\right)^2}{2m_A}.$$
 (A16)

The result of Eq. (A14) is the usual expression of the gravitational redshift: given two clocks localised at two different heights, the proper time measured by a clock is dilated according to the other clock by a factor  $\nu' = \left(1 + \frac{\Delta V}{c^2}\right)\nu$ , where  $\nu$ ,  $\nu'$  are the frequencies of the first and second clock respectively. This result is employed in the main text to study Q-LPI when the clocks are entangled and follow the same path (Figure 4a). Here we give another example, namely the case in which in the laboratory frame the atoms are initially in the entangled state, represented in Figure 4b,

$$|\psi_0\rangle_{AB}^{(L)} = \frac{|x_+\rangle_{\mathbf{A}} |x_-\rangle_{\mathbf{B}} + |x_-\rangle_{\mathbf{A}} |x_+\rangle_{\mathbf{B}}}{\sqrt{2}} |\tau_{in}\rangle_{C_A} |\tau_{in}\rangle_{C_B},$$
(A17)

where in each branch they are at different heights. The state in the perspective of particle A can be calculated analogously to the previous case. We obtain a "history state" where now the clock on particle B is time-dilated from the perspective of A, i.e.,

$$|\psi^{(A)}(\tau_A)\rangle_{BL} = \frac{1}{\sqrt{2}} \left( |\Psi(\tau_A, x_+, x_-)\rangle_{\mathbf{B}L} |\tau_A + \Delta\tau\rangle_{C_B} + |\Psi(\tau_A, x_-, x_+)\rangle_{\mathbf{B}L} |\tau_A - \Delta\tau\rangle_{C_B} \right)$$
(A18)

where 
$$\Delta \tau = \frac{V(\mathbf{x}_+) - V(\mathbf{x}_-)}{c^2} \tau_A$$
.

These results show that in the QRF of the clocks there is a superposition of gravitational time-dilations, which is due to the relative delocalisation of the clocks. Moreover, when the wavefunctions of the particles A and B have non-negligible uncertainties in position basis in each path, we find that, from the perspective of A, an additional effect of the spread of the wave function in each branch of the interferometer results in a redshift factor for each possible position of particle B as seen from A. Therefore, the time of particle B is dilated according to particle A, however the effect due to such a delocalisation is much smaller than that across different branches.

A similar procedure can be performed to generalise LLI to Q-LLI. Note that the transformation to the QLIF of A does not map the momenta in the laboratory frame to the relative momenta with respect to A, due to the canonicity of the transformation. Hence, the functional relation between the relative velocities, defined as the time derivative of the position of B in the QRF of A, and the momentum operator that is canonically conjugated to the position operator does not have a straightforward physical interpretation. However, the time of B is not time-dilated as seen from A, and this holds true also for quantum superpositions of such states. Hence, we call this a genelarilation of LLI, namely Q-LLI.

#### Appendix B: Calculations for the model for violations

In this appendix we show that the model for violations we introduced in the main text can be employed to test the EEP for QRFs.

Specifically, we show that a model which generalises the one introduced in Ref. [11] can serve as a model for violation

of the EEP for QRFs. We formulate this model in terms of the conditions of Eq. (23)

$$m^{(i)}\hat{1} + \hat{H}^{(i)}(\hat{\mathbf{p}}) \neq m^{(g)}\hat{1} + \hat{H}^{(g)}(\hat{\mathbf{x}})$$
 (Q-WEP), (B1)

$$\hat{H}^{(r)} \neq \hat{H}^{(i)}(\hat{\mathbf{p}}) \tag{Q-LLI},$$

$$\hat{H}^{(r)} \neq \hat{H}^{(g)}(\hat{\mathbf{x}})$$
 (Q-LPI), (B3)

where i stands for "inertial", g for "gravitational", and r for "rest". It is easy to understand why this model can test a violation of the EEP for QRF by remembering that its formulation relies on a QRF transformation to a QLIF. In such a QLIF, the metric is locally minkowskian and, in turn, the behaviour of the clocks is independent of its position or velocity; hence, a violation of the EEP for QRFs implies a negation of this condition. We show in the following that this is achieved by imposing the previous equations, leading to a dependence of the time shown by the clock on the position or momentum operators. Clearly, different ways of violating the principle are possible, but because we are testing the violations employing the behaviour of quantum clocks, this is the most natural condition to impose.

Here, we investigate for the first time the effects of the violation not only in the perspective of the laboratory, but also in the QLIF of each particle. The result is that the violation of any condition is sufficient to break the SQRFs formalism, namely it prevents us to transform to the QLIF of a particle. Hence, the violations break the conditions for the validity of a description from a QLIF. Notice that it is necessary, in order to perform the test, that the perspective of the laboratory can be always described. In our model this is always true, as a consequence of imposing a Newtonian dynamical constraint for the laboratory, so we do not need to impose this condition separately. Intuitively, this happens because imposing a Newtonian constraint corresponds, as commented in Appendix A 1, to choosing a set of coordinates in which the metric is locally Minkowskian at the location of the laboratory.

To keep the technical calculations simple, we investigate the violation of each condition (Eqs. 21 - 23) separately, choosing the appropriate set of constraints according to what aspect we are studying. However, this has no influence on the conceptual bearing of our results.

#### 1. Violation of Q-LPI

In this section we study the violation of LPI, formalised in Eq. (B3). Since the condition involves only the gravitational Hamiltonian  $\hat{H}^{(g)}(\hat{\mathbf{x}})$  and the rest Hamiltonian  $\hat{H}^{(r)}$ , without any dependence on the inertial Hamiltonian, Q-LPI does not depend on special relativistic effects. Thus, we study the violation of Q-LPI in the Newtonian limit of the SQRFs formalism, where the gravitational field is weak, and special relativistic effects are discarded. Hence we employ the slow-speed limit of the constraints of Eqs. (6 - 9).

To introduce the violation of Q-LPI, we modify the constraints distinguishing the gravitational Hamiltonian from the rest Hamiltonian for each particle, namely

$$\hat{H}_{I}^{(g)}(\hat{\mathbf{x}}_{I} - \hat{\mathbf{x}}_{L}) \neq \hat{H}_{I}^{(r)}, \qquad I = A, B.$$
 (B4)

The separation between  $\hat{H}_{I}^{(g)}(\hat{\mathbf{x}}_{I} - \hat{\mathbf{x}}_{L})$  and  $\hat{H}_{I}^{(r)}$  is such that the gravitational Hamiltonian is coupled to the gravitational field, while the rest Hamiltonian is not. This results in a modification of the  $\hat{f}^{0}$  constraint, namely

$$\hat{f}^{0} = \sum_{I=A,B} \left[ \hat{p}_{I}^{0} + \frac{\hat{H}_{I}^{(r)}}{c} + \left( \sqrt{g_{00}(\hat{\mathbf{x}}_{I} - \hat{\mathbf{x}}_{L})} - 1 \right) \frac{\hat{H}_{I}^{(g)}(\hat{\mathbf{x}}_{I} - \hat{\mathbf{x}}_{L})}{c} \right] + \frac{\hat{H}_{L}^{(r)}}{c} + \hat{p}_{L}^{0}, \tag{B5}$$

indeed an expansion of the constraints for a weak gravitational field shows that  $\hat{H}_{I}^{(g)}(\hat{\mathbf{x}}_{I} - \hat{\mathbf{x}}_{L})$  is always coupled to the gravitational potential.

Note that it is not necessary to separate the Hamiltonians for the mass L, because the only Hamiltonian for L that appears in the constraints is the rest Hamiltonian, since the metric at the location of L is flat.

The state in the perspective of particle A can be obtained via the usual prescription in Eq. (A9), where the  $\hat{\mathcal{T}}_A$  operator is defined as in Eq. (A8), but  $\hat{f}^0$  is replaced by the new constraint defined in Eq. (B5).

This results in a quantum state from the perspective of particle A corresponding to

$$|\psi\rangle^{(A)} = \int d\tau_{A} d\mathbf{q}_{L} dt_{A} e^{-\frac{i}{\hbar} \hat{H}'^{(A)} \tau_{A}} |\tilde{\psi}_{0}^{(A)}(\mathbf{q}_{L}, t_{A})\rangle_{BL} \times e^{-\frac{i}{\hbar} \frac{V(\mathbf{q}_{L})}{c^{2}} (\tau_{A} - t_{A}) \hat{H}_{A}^{(g)}(\mathbf{q}_{L}) + \frac{1}{2\hbar^{2}} (\tau_{A} - t_{A}) [\hat{H}_{A}^{(g)}(\mathbf{q}_{L}), \hat{H}_{A}^{(r)}]} |\tau_{A} - (\tau_{A} - t_{A}) \frac{V(\mathbf{q}_{L})}{c^{2}} \rangle_{C_{A}},$$
(B6)

where the relationship between  $|\tilde{\psi}_0^{(A)}(\mathbf{q}_L,t_A)\rangle_{BL}$  and the standard initial state  $|\psi_0^{(A)}\rangle_{BL}$  is

$$|\psi_0^{(A)}\rangle_{BL} = \int dt_A d\mathbf{q}_L \, |\tilde{\psi}_0^{(A)}(\mathbf{q}_L, t_A)\rangle_{BL}, \tag{B7}$$

and the Hamiltonian is

$$\hat{H}^{\prime(A)} = \sum_{J=A,B,L} m_J c^2 + \sum_{j=B,L} \frac{\hat{\mathbf{k}}_j^2}{2m_j} + \frac{(\sum_{j=B,L} \hat{\mathbf{k}}_j)^2}{2m_A} + + m_B (V(\hat{\mathbf{q}}_B - \hat{\mathbf{q}}_L) - V(\hat{\mathbf{q}}_L)) - m_L V(\hat{\mathbf{q}}_L) + + \frac{V(\hat{\mathbf{q}}_B - \hat{\mathbf{q}}_L)}{c^2} \hat{H}_B^{(g)} (\hat{\mathbf{q}}_B - \hat{\mathbf{q}}_L) + \left(1 - \frac{V(\hat{\mathbf{q}}_L)}{c^2}\right) \hat{H}_B^{(r)} + \left(1 - \frac{V(\hat{\mathbf{q}}_L)}{c^2}\right) \hat{H}_L^{(r)}.$$
(B8)

Differently to the standard state in the perspective of particle A of Eq. (A9), the state in Eq. (B6) is not a history state, since it cannot be written in the form of Eq. (A9): this is the consequence of introducing a violation of Q-LPI. Therefore, it is not possible to identify the time measured by the first particle as its internal time, and consequently the SQRFs formalism does not hold anymore. In other words, the violation of Q-LPI implies that it is not possible to describe the system from the QLIF of particle A, and that Q-LPI is a necessary condition for the existence of a QLIF in the first place.

We conclude that the model that we introduced in the main text and in this Appendix is adequate to test Q-LPI when QLIFs are considered, namely to test Q-LPI within the context of the EEP for QRFs.

Note that it is still possible to obtain the history state in the perspective of the laboratory, because the Hamiltonian of the mass L is not modified by the violation of Q-LPI, since in the original constraints the metric for L is flat, and thus in the modified constraint of Eq. (B5) the gravitational Hamiltonian  $\hat{H}_L^{(g)}$  for the mass L does not appear.

The state in the perspective of the laboratory is

$$|\psi\rangle^{(L)} = \int d\tau_L e^{-\frac{i}{\hbar}\hat{H}'^{(L)}\tau_L} |\psi_0\rangle_{AB}^{(L)} |\tau_L\rangle_{C_L},$$
(B9)

which is still a history state. The modified Hamiltonian is

$$\hat{H}^{\prime(L)} = \sum_{I=A,B,L} m_J c^2 + \sum_{I=A,B} \left[ \frac{\hat{\mathbf{k}}_I^2}{2m_I} + m_I V(\hat{\mathbf{q}}_I) + \hat{H}_I^{(r)} + \frac{V(\hat{\mathbf{q}}_I)}{c^2} \hat{H}_I^{(g)}(\hat{\mathbf{q}}_I) \right].$$
(B10)

The fact that we are still able to give a description of the system in the perspective of the laboratory is not surprising, since the laboratory is a classical inertial frame in the Newtonian sense. In the next sections we show that this holds true not only for violations of Q-LPI, but also for the entire EEP for QRFs. This allows us to predict the dynamics in the perspective of the laboratory, and in particular in the main text we calculate the probabilities of an interferometric experiment, using the most general Hamiltonian where also special relativistic effects are considered, namely without performing the slow-speed limit.

#### 2. Violation of Q-LLI

A test for Q-LLI does not involve gravitation, but only special relativity, since the Q-LLI violation condition in Eq. (B2) regards the inertial and rest mass, but not the gravitational mass. Therefore, we study the violation of Q-LLI in the special relativistic limit of the SQRFs formalism, obtained from the constraints of Eqs. (6 - 9) when gravity is neglected.

The constraints should be modified to account for the violation of Q-LLI, by distinguishing the inertial Hamiltonian

from the rest Hamiltonian for each particle, namely

$$\hat{H}_{I}^{(i)}(\hat{\mathbf{p}}_{I}) \neq \hat{H}_{I}^{(r)}, \qquad I = A, B.$$
 (B11)

The separation is made in such a way that the inertial Hamiltonian is coupled to the momentum, while the rest Hamiltonian is not. The modified constraint is then

$$\hat{f}^{0} = \sum_{I=A,B} \left[ \hat{p}_{I}^{0} + \left( \hat{\gamma}_{I}^{-1} - 1 \right) \frac{\hat{H}_{I}^{(i)}(\hat{\mathbf{p}}_{I})}{c} + \frac{\hat{H}_{I}^{(r)}}{c} \right] + \hat{p}_{L}^{0} + \frac{\hat{H}_{L}^{(r)}}{c}, \tag{B12}$$

where 
$$\hat{\gamma}_I = \sqrt{1 + \frac{\hat{\mathbf{p}}_I^2}{m_I^2 c^2}}$$
 and  $\hat{\omega}_{\mathbf{p}}^I = m_I c \; \hat{\gamma}_I$ .

In order to obtain the state in the perspective of particle A, we employ the definition in Eq. (A9), where the  $\hat{\mathcal{T}}_A$  operator is

$$\hat{\mathcal{T}}_{A} = e^{\frac{i}{\hbar}\hat{\mathbf{x}}_{A}(\hat{\mathbf{p}}_{B} + \hat{\mathbf{p}}_{L})} e^{\frac{i}{\hbar}\hat{x}_{A}^{0} \left(\hat{p}_{B}^{0} + \hat{p}_{L}^{0} + \frac{\hat{H}_{L}^{(r)}}{c} + \sum_{I=A,B} \left(\hat{\gamma}_{I}^{-1} - 1\right) \frac{\hat{H}_{I}^{(i)}(\hat{\mathbf{p}}_{I})}{c} + \frac{\hat{H}_{I}^{(r)}}{c}\right)}.$$
(B13)

Thus we find

$$|\psi\rangle^{(A)} = \int d\tau_A d\mathbf{k}_B d\mathbf{k}_L dt_A e^{-\frac{i}{\hbar} \hat{H}^{\prime(A)} \tau_A} |\tilde{\psi}_0(\mathbf{k}_B, \mathbf{k}_L, t_A)\rangle_{BL}^{(A)} \times e^{-\frac{i}{\hbar} (1 - \gamma_{\sum \mathbf{k}}^A)(\tau_A - t_A) \hat{H}_A^{(i)}(\sum \mathbf{k}) + \frac{1}{2\hbar^2} (\tau_A - t_A)^2 (1 - \gamma_{\sum \mathbf{k}}^A)[\hat{H}_A^{(i)}(\sum \mathbf{k}), H_A^{(r)}]} \times |t_A + \gamma_{\sum \mathbf{k}}^A (\tau_A - t_A)\rangle_{C_A},$$
(B14)

where  $\hat{\gamma}_{\sum \mathbf{k}}^{A} = \sqrt{1 + \frac{\left(\sum_{i=B,L} \hat{\mathbf{k}}_{i}\right)^{2}}{m_{A}^{2}c^{2}}}$ , and we define for future convenience  $\omega_{\sum \mathbf{k}}^{\hat{A}} = m_{A}c \ \gamma_{\sum \mathbf{k}}^{\hat{A}}$ .

The relationship between the state  $|\tilde{\psi}_0(\mathbf{k}_B, \mathbf{k}_L, t_A)\rangle_{BL}^{(A)}$  and the standard initial state  $|\psi_0\rangle_{BL}^{(A)}$  is

$$|\psi_0\rangle_{BL}^{(A)} = \int d\mathbf{k}_B d\mathbf{k}_L dt_A |\psi_0(\mathbf{k}_B, \mathbf{k}_L, t_A)\rangle_{BL}^{(A)}, \qquad (B15)$$

and the Hamiltonian is

$$\hat{H}^{\prime(A)} = \hat{\gamma}_{\sum \mathbf{k}}^{A} \left( c\hat{\omega}_{\mathbf{k}}^{B} + m_{L}c^{2} + \frac{\hat{k}_{L}^{2}}{2m_{L}} + c\hat{\omega}_{\sum \mathbf{k}}^{A} + (\hat{\gamma}_{B}^{-1} - 1) \frac{\hat{H}_{B}^{(i)}(\hat{\mathbf{k}}_{B})}{c} + \frac{\hat{H}_{B}^{(r)}}{c} + \frac{\hat{H}_{L}^{(r)}}{c} \right),$$
(B16)

The state in Eq. (B14) is not a history state, as a consequence of the violation of Q-LLI. Thus, if Q-LLI is violated, it is not possible to describe the system from the point of view of particle A, and the SQRFs formalism does not work anymore: a violation of Q-LLI prevents the existence of a QLIF.

Notice that, in the presence of a violation, it is still possible to describe the laboratory, namely we still find a history state in the laboratory perspective, even if the QLIFs of the other particles do not exist anymore. The resulting Hamiltonian is

$$\hat{H}'^{(L)} = \sum_{I=A,B,L} m_I c^2 + \sum_{i=A,B} \left( c \hat{\omega}_{\mathbf{k}}^i + \left( \hat{\gamma}_i^{-1} - 1 \right) \frac{\hat{H}_i^{(i)}(\hat{\mathbf{k}}_i)}{c} + \frac{\hat{H}_i^{(r)}}{c} \right). \tag{B17}$$

#### 3. Violation of Q-WEP

The violation condition of Q-WEP in Eq. (B1) regards the inertial and gravitational mass, namely the parameters  $m^{(g)}$  and  $m^{(i)}$ , as well as the operators  $\hat{H}^{(g)}(\hat{\mathbf{x}})$ ,  $\hat{H}^{(i)}(\hat{\mathbf{p}})$ . Therefore, Q-WEP should be tested in the most general regime of the SQRFs formalism, where both general relativistic and special relativistic effects are significant.

Nevertheless, the three aspects of the model for violations of the EEP are not independent. In fact, it is easy to show that the conditions in Eqs. (B1 - B3) imply e.g. that if Q-LPI is valid and  $m^{(g)} = m^{(i)}$ , then validity of Q-WEP

coincides with validity of Q-LLI. Vice versa, if Q-LLI is valid and  $m^{(g)} = m^{(i)}$ , then Q-WEP coincides with Q-LLI. Therefore, we can reduce the test of Q-WEP to a test of Q-LLI and Q-LPI, that we already studied, and a test for the standard classical WEP, namely  $m^{(g)} = m^{(i)}$ .

Let us assume that Q-LLI is valid. Then, we can introduce a violation of Q-WEP in the SQRFs formalism with the conditions

$$m_I^{(g)} \neq m_I^{(i)}, \qquad I = A, B,$$
 (B18)

$$m_I^{(g)} \neq m_I^{(i)}, \qquad I = A, B,$$
 (B18)  
 $\hat{H}_I^{(g)}(\hat{\mathbf{x}}_I - \hat{\mathbf{x}}_L) \neq \hat{H}_I^{(r)}.$  (B19)

We already showed in Appendix B1 that the second condition prevents the SQRFs formalism to work. Therefore, a violation of WEP with this model implies that it is not possible to perform a transformation to the QLIF of a particle, and thus the model is adequate to analyze violations of WEP in the context of the EEP for QRFs. Note that the standard classical WEP, namely  $m_I^{(g)} = m_I^{(i)}$ , I = A, B, does not play a role in the SQRFs mechanism.

This is not the most general violation of WEP possible, since we assumed Q-LLI to be valid. Nevertheless, even this weaker violation is sufficient to break the formalism, so the same argument holds for the general case of a stronger violation where Q-LLI is not assumed. Indeed, we can repeat the argument by assuming Q-LPI but not Q-LLI, obtaining the same result.

In conclusion, we showed that the model for violations that we have considered in the main text and in this Appendix can be employed to test the EEP for QRFs in any QLIF.

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