Supplemental Material for "Loschmidt amplitude spectrum in Dynamical Quantum Phase Transitions"

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In this Supplemental Material, we present the analytical analysis of the Loschmidt amplitude spectrum (LAS) for dynamical quantum phase transitions (DQPTs) without crossing the underlying equilibrium phase transition in the one-dimensional (1D) XY model [S1] in Sec. S1. In Sec. S2, we present the general Loschmidt rate spectrum (LRS) with randomly excited k modes in the 1D transverse-field Ising model (TFIM).

S1. LOSCHMIDT AMPLITUDE SPECTRUM IN DQPT WITHOUT CROSSING THE EQUILIBRIUM CRITICAL POINT

Dynamical phase transitions was also shown to occur in some models where the quench is without crossing the model's equilibrium phase boundary. An example is the XY model, where the authors showed nonanalyitcal behaviors in the LR with ground state when one quenches the system within a single phase [S1]. The Hamiltonian of XY model is given by

$$H = -\sum_{j=1}^{N}\left[\left(\frac{1+\delta}{2}\right)\sigma_j^x\sigma_{j+1}^x + \left(\frac{1-\delta}{2}\right)\sigma_j^y\sigma_{j+1}^y\right] - g\sum_{j=1}^{N}\sigma_j^z. \tag{S1}$$

The domains at which the system is quenched to for an arbitrary prequench Hamiltonian (g_i, δ_i) to achieve the single-phase DQPT is defined by the inequality [S1]

$$\mathscr{D}(g_i,\delta_i) = \left\{(g_f,\delta_f)|2\delta_i\delta_f < 1 - g_i g_f - \sqrt{(g_i^2-1)(g_f^2-1)}\right\}. \tag{S2}$$

The XY model is diagonalizable using the same transformations in the TFIM as presented in the main text. We apply our scheme LAS in quasiparticle picture to one initial parameter pair $(g_i, \delta_i) = (0, 0.3)$, where the "transition point" reckoned using Eq. (S2) is $g = \sqrt{0.3276} \approx 0.573$. We choose the final parameter pair as $(g_f, \delta_f) = (0.8, 0.3)$ and observe four different contiguous excitations of k modes:

$$\begin{aligned}
&\text{(a)} \qquad \psi_n(g_i) = \prod_{k'=k_1}^{k_{m_a}} \eta_{k'}^\dagger\eta_{-k'}^\dagger\left|0(g_i)\right\rangle \qquad m_a = 1,2,\ldots,\frac{N}{2}\\
&\text{(b)} \qquad \psi_n(g_i) = \prod_{k'=k_{N/4-m_b}}^{k_{N/4+m_b+1}} \eta_{k'}^\dagger\eta_{-k'}^\dagger\left|0(g_i)\right\rangle \qquad m_b = 0,1,2,\ldots,\frac{N}{4}-1\\
&\text{(c)} \qquad \psi_n(g_i) = \prod_{k'=k_{N/2-m_c}}^{k_{N/2}} \eta_{k'}^\dagger\eta_{-k'}^\dagger\left|0(g_i)\right\rangle \qquad m_c = 0,1,2,\ldots,\frac{N}{2}-1\\
&\text{(d)} \qquad \psi_n(g_i) = \prod_{k'=k_{N/6}}^{k_{N/6+m_d}} \eta_{k'}^\dagger\eta_{-k'}^\dagger\left|0(g_i)\right\rangle \qquad m_d = 1,2,\ldots,\frac{N}{3},
\end{aligned} \tag{S3}$$

where the last excitation starts from around the middle of the lower-half k spectrum. Their respective rate functions are plotted in figure S1. The dynamics of the system around DQPT are very similar to that being observed in the TFIM in the main text: As in figure S1(c), there are barely excitations to very-high-energy excited states. The transient excitation of the quasiparticle states are mainly in the middle of the spectrum, as shown in figure S1(b),

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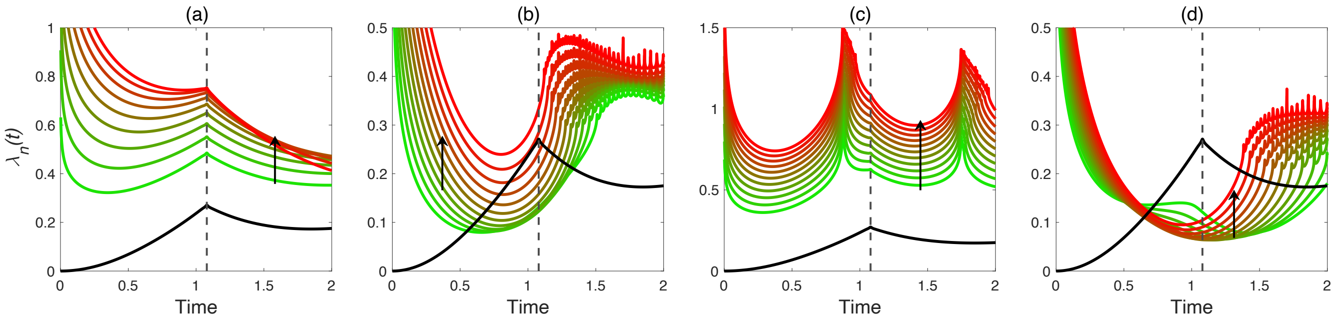

FIG. S1: LRS for four distinct excited state patterns stated in Eq. (S3) for an $N = 300$ system quenched from $(0, 0.3) \to (0.8, 0.3)$. Uparrows show the direction of increasing k occupation. Black lines are the rate function for ground state with dashed lines indicating the critical time.

followed by a rapid excitation shift to around the middle range of the lower-half k states as indicated in figure S1(d). The only qualitative difference of the dynamics happened during DQPT between TFIM and the XY model is that the dominating excitation in the vicinity of dynamical phase transition in the former model happens in lower-half modes, whereas the latter has a narrower range of excitation and lowest k mode is slightly higher at around $k_{N/6}$. The significance of the study is that it seems to be a general case that a downshift of momentum excitation should occur when an integrable system is quenched and DQPT is triggered. There may be other minor difference for the region of excitations the system has during quench, like the negligible contribution from the lower-range k mode excited states in XY model (figure S1(a)) compared to that in TFIM, but the crucial dynamics studied, namely the population re-distribution in the quasiparticle picture, persists in both cases.

S2. LOSCHMIDT RATE FOR RANDOMLY SELECTED EXCITED STATE

In the main text, the Loschmit rate function for an arbitrary excited state in the TFIM is derived in the momentum space as

$$\lambda_n(t) \sim -\frac{1}{N}\left\{ \sum_{k'} \ln\left[2T_{k'}^2(1-\cos(2\varepsilon_{k'}(g_f)t))\right] + \sum_{k\neq k'>0} \ln\left[1+T_k^4+2T_k^2\cos(2\varepsilon_k(g_f)t)\right]\right\} \tag{S4}$$

with k' representing the excited k modes. The expression contains two sums, with the former concerning all occupied momentum states and the latter concerning the empty momentum states. Since the k-modes are independent, one

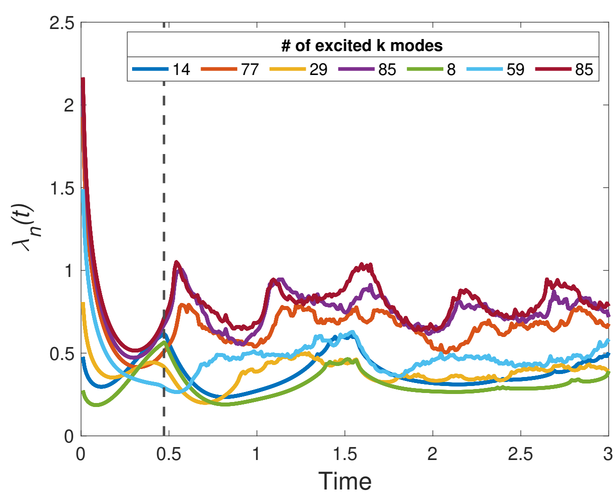

FIG. S2: LRS for seven randomly excited states in an $N = 300$ system. The quench is from $g = 0.1 \to 2$. Dashed line indicates the first critical time. Legend shows the number of excited k states.

can always observe the dynamics of an arbitrarily excited state using the above equation. We show the evolution of LRs for seven different randomly excited states in figure S2. Note that the excited k modes are chosen randomly from a discrete uniform distribution. Also one can hardly specify exactly which k states are occupied due to the tremendous number of ways for a system given size N to be excited (2^{N-1} possible excited states in a parity subspace for TFIM). To that, we label those excited states by only the number of occupied k modes, but by further inspection one can already draw some conclusions about the features of rate functions for different excited states. In particular, we divide them into several categories according to their transient growth in the LRs around the first critical time:

1. Ground-state-like LRs (green and blue lines in figure S2): A main big peak similar to the nonanalytical peak of the ground-state rate function with some additional spikes of nonanalyticities along the evolution originated from the nonanalyticites in the $\Lambda_{k'}(t)$ derived as

$$\Lambda_{k'}(t) = \frac{1}{N} \ln \left[2T_{k'}^2 (1 - \cos(2\varepsilon_{k'}(g_f)t)) \right] \tag{S5}$$

   with the associated critical time $t_m(k', g_f) = m\pi/\varepsilon_{k'}(g_f), m = 1, 2, \ldots$ (for details see the original manuscript);

2. low-range k-excited-like LRs (cyan line in figure S2): Opposite to ground-state-like LRs, they have the lowest valley instead of big peak around critical time implying the most probable groups of eigenstates the system will be in during dynamical phase transitions;

3. high-range k-excited-like LRs (purple and brown lines in figure S2): Magnitudes are high in general that they hardly contribute to the dynamics of DQPTs, and they have apparent nonanalytical behavior around DQPTs.

Notice however that the LRs for some excited states may have a mixture of these categories. To name a few, the middle-range k-mode excited state mentioned in the original manuscript and red line in figure S2 lie in between low-range k-excited-like and high-range k-excited-like LRs, namely the valley moves slightly away from the critical time but still these excited states contribute to the dynamics of the quench before DQPT, and the nonanalyticity starts becoming pronounced. The yellow line in figure S2, on the other hand, combines the features of ground-state-like and low-range k-excited-like LRs, where the big peak shrinks while the valley is moving closer to the critical time. To conclude, for a system that can be described by quasiparticles in momentum space, we observe several major features regarding the dynamical growth of Loschmidt rate for a general excited state. In particular, for such models, the Loschmidt rate for an arbitrarily excited state seems to either possess solely one of the categorized characteristics or a combination of those characteristics.

[S1] S. Vajna, and B. Dóra, Phys. Rev. B **89**, 161105(R) (2014).