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Testing gravity with neutrinos: From classical to quantum regime*

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In this paper, we survey the main characteristics that provide neutrinos with the capability of being the perfect candidate to test gravity. A number of potentially resourceful scenarios are analyzed, with particular emphasis on how the versatility of neutrinos lends itself to understand the multifaceted nature of the gravitational interaction, both at classical and quantum scales. As a common thread running through the two different regimes, we consider the fundamental principles underpinning General Relativity and its possible quantum extensions. Finally, we discuss some open problems and future perspectives.

Keywords: Neutrino mixing and oscillations; quantum gravity; relativity and gravitation.

1. Introduction

Neutrinos are the most elusive elementary particles in the Standard Model. Due to their extremely small mass and zero electric charge, they are capable of passing through ordinary matter with minimal interaction, representing a unique probe for investigating physics at length scales ranging from nuclei, to molecules and galaxies. Besides, the challenging search for direct evidences of the Cosmic Neutrino Background (CNB) may provide us with fundamental knowledge on the earliest stages of universe's existence. Due to these peculiar features, neutrinos can thus be regarded as unparalleled information messengers in many branches of physics.

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Among these, gravity theories and the related host of unsolved problems certainly represent one of the most demanding fields of research.

The aim of this work is twofold:

(i) on the one hand, neutrino physics is used as a test bench for predictions of General Relativity (GR) and its cornerstones at the classical and semi-classical level;
(ii) on the other hand, we discuss how the above framework may potentially unravel the unsettled riddles arising in the regime where GR and Quantum Mechanics should coexist.

2. Neutrino Physics and Classical Gravity

In the extended Standard Model, it is well known that neutrinos weakly interact in flavor states $|\nu_\alpha\rangle$ that are superpositions of mass states $|\nu_k\rangle$ according to[a] Ref. 1

$$|\nu_\alpha\rangle = \sum_{k=1,2} U_{\alpha k}(\theta)|\nu_k\rangle, \quad \alpha = e, \mu, \tag{1}$$

where $U_{\alpha k}$ is the generic element of Pontecorvo matrix.

Mass states propagate freely. In Minkowski spacetime, their evolution from a point $A(t_A, \mathbf{x}_A)$ to $B(t_B, \mathbf{x}_B)$ is governed by the phase factor $\varphi_k = E_k(t_B - t_A) - \mathbf{p}_k \cdot (\mathbf{x}_B - \mathbf{x}_A)$, where $E_k = \sqrt{m_k^2 + |\mathbf{p}_k|^2}$ and $\mathbf{p}_k$ denote the energy and three-momentum of the kth state of mass m_k, respectively. Accordingly, the phase shift φ_0 acquired by the mass eigenstates during the propagation leads to a nonvanishing flavor transition probability

$$\mathcal{P}_{\alpha\to\beta} = \sin^2(2\theta)\sin^2\left(\frac{\varphi_0}{2}\right), \tag{2}$$

where

$$\varphi_0 \simeq \frac{\Delta m^2}{2E_\ell} L_p, \quad \Delta m^2 = m_2^2 - m_1^2, \tag{3}$$

with E_ℓ being the (common) local energy of neutrinos and L_p the proper distance they travel.

Inspired by the detection of a Newtonian gravitational phase in a neutron-based interferometry experiment,[2] Stodolski[3] first investigated GR effects on the wave functions of particles propagating in curved background. Should the analysis of flavor oscillations be performed for neutrinos in the gravitational field of a source mass M, GR would then predict for the phase shift[4]

$$\varphi = \varphi_0 + \varphi_{\mathrm{GR}}(M). \tag{4}$$

In the simplest case of Schwarzschild spacetime and in the weak-field limit, one has

$$\varphi_{\mathrm{GR}}(M) \simeq \frac{\Delta m^2 L_p}{2E_\ell}\left[\frac{\mathrm{GM}}{r_B} - \frac{\mathrm{GM}}{L_p}\ln\left(\frac{r_B}{r_A}\right)\right]. \tag{5}$$

[a] We shall work within a simplified two-flavor scenario and in the approximation of relativistic neutrinos.

Although very small in size, the correction (4) may be, in principle, attainable in neutrino interferometry experiments,[5] thus allowing for a direct test of GR predictions via neutrino oscillations in regimes of weak gravity. However, describing the background gravitational field by simply using the spherically-symmetric Schwarzschild solution is not satisfactory in most situations. Astrophysical sources are expected to be rotating as well as endowed with shape deformations leading to effects which, in general, cannot be neglected. As a matter of fact, in order to render the above picture as realistic as possible, one should perform the analysis of neutrino oscillations in Kerr spacetime. A first step along this direction has been taken in Ref. 6, where gravity corrections to the phase shift have been computed in the slowly rotating, weak-field approximation and for the case of ultra-relativistic, spin-1/2 particles described by left-handed Weyl spinors. Further clues may come from the generalization of the above formalism to more exotic (quasi-spherical) geometries, where the assumption of a Kerr-like metric may lead to erroneous conclusions about the actual astrophysical processes that take place. For instance, neutrino oscillations in the field of rotating deformed neutron stars, white dwarfs and supermassive stars can be reasonably described by the Hartle–Thorne[7] or Zipoy–Voorhees metrics.[8]

The aforementioned analysis refers to vacuum flavor transitions and holds in the weak-field approximation. With proper refinements which embed matter enhancing effects (MSW effect) and higher-order gravity corrections, it can be safely extended to a variety of astrophysical environments. For instance, it is a well-established fact that neutrinos play a crucial rôle in stellar collapses and formation of black holes and neutron stars. Specifically, the theoretical models describing these phenomena[9] were proved to be accurate on the basis of the first-ever neutrino burst detection coming from the supernova SN1987A.[10] In this regard, it must be pointed out that, according to Ref. 11, the matter effects attributable to the high density of the collapsing stellar cores were believed to inhibit neutrino oscillations. Under a similar circumstance, the inevitable conclusion would be a permanent trapping of neutrinos due to the absence of flavor transition, which instead could permit a leakage from the surrounding astrophysical environment. However, the matter-induced suppression of oscillations was subsequently reconsidered in Ref. 12, showing that the relevance of neutrino oscillations in regimes of strong-gravity and, in particular, in supernova explosions, could strongly depend on the distribution of space regions where matter effects are factually prominent. Remarkable results supporting the key rôle of neutrinos in driving the collapse and explosion of massive stars have been recently summarized in Ref. 13 (and therein), where it has been argued that, due to their weakly interacting nature, these particles represent the only direct probe of the dynamics and thermodynamics at the center of a supernova. In particular, hydrodynamical simulations with most sophisticated neutrino transport have proved to be necessary to calculate detailed signal properties, which are required for the analysis of neutrino oscillations and neutrino-induced nucleosynthesis in supernovae, and for the potential detection of the Diffuse Supernova Neutrino Background (DSNB) and of neutrinos from a future Galactic supernova.

Furthermore, (heavy sterile) neutrinos are regarded as candidates for Dark Matter[14] and as probes to reveal the mysterious nature of dark energy,[15] which potentially offers precious hints toward the resolution of GR main puzzles. Among these, one of the most intriguing issues is represented by the intrinsic importance of the torsion T, which always equals zero in the context of GR. In spite of this, a number of works have been developed with the assumption $T \neq 0$, which gives rise to the so-called "torsion gravity" models.[16] The interest in such models can be ascribed to their capability of avoiding singularities, both at the quantum[17] and the cosmological[18] level. Even in this framework, the study of neutrino oscillations may have nontrivial implications. As shown in Ref. 19, the shape of the flavor transition probability can provide clues on the existence of a nonvanishing torsion, thus allowing to discriminate between the standard GR scenario and torsion gravity. In a two-flavor configuration, these effects (which are of the order of Planck scale) become manifest only when the superposed mass eigenstates have opposite spin, otherwise no discrepancy with standard GR results arises at all.

On the other hand, neutrino physics can provide valuable pieces of information about the principles underlying GR and other gravitational models. Indeed, from the first available data on astrophysical neutrinos, such particles have been constantly associated to the violation of the weak equivalence principle.[20,21] Additionally, from a more theoretical perspective, a similar scenario is encountered within the framework of exotic geometries and extended models of gravity.[22] All of these evidences lead to the awareness that the equivalence principle should be somehow modified when passing from classical to quantum regimes, as preliminarily pointed out in Ref. 23.

3. Neutrino Physics: From Semiclassical to Quantum Gravity

Along with the equivalence principle, general covariance represents another fundamental pillar of GR. Contrary to the former, however, such a principle still underpins most of the attempts of extending GR made so far. For instance, the generalization of Quantum Field Theory (QFT) to curved background is by construction generally covariant. Although this model only provides a semiclassical description of gravitational interaction, a plethora of its predictions are subjects of active investigation. Among these, the Hawking–Unruh radiation is certainly the most eloquent footprint of a possible nonclassical nature of gravity. In this regard, many studies predict that neutrinos expelled during black hole evaporation may nontrivially affect the emitted power and the lifetime of the source,[24] with phenomenological consequences which may be relevant for ruling out primordial black holes as Dark Matter candidates.[25] Furthermore, processes involving the production and/or absorption of neutrinos can be used as a theoretical tool for testing the existence of the Unruh effect as a consequence of the general covariance of

QFT,[26–29] as well as deviations of the Hawking–Unruh spectrum from a purely thermal behavior.[30]

In connection with the issue of fundamental principles, let us observe that another stimulating link between the "neutrino" and "gravity" worlds is provided by string theory's prediction of the existence of a minimum length at Planck scale $\lambda_P \simeq 10^{-35}\,$m, in compliance with the possible emergence of a discrete structure of spacetime. Implications of this requirement are extremely nontrivial, as they would affect most of the basic principles of modern physics, such as (local) Lorentz invariance and Heisenberg Uncertainty Principle (HUP). In all the cases, theoretical and experimental investigations involving neutrinos may shed light on such peculiar features. Specifically, detailed studies on Lorentz violation in neutrino oscillations have been proposed in Ref. 31. On the other hand, signatures of Planck-scale corrections to Pontecorvo oscillation formula have been addressed in Ref. 32 using a generalized commutator (GUP) of the form

$$[\hat{x}, \hat{p}] = i(1 + f(|p|^2)), \tag{6}$$

where p is the characteristic momentum of the physical system and $f(|p|^2) \to 0$ at energies far from Planck scale, so as to recover the standard quantum mechanical framework. In this perspective, a suggestive prediction has been conjectured in Ref. 33 on the basis of a GUP-modified de Broglie formula describing the wave-particle duality in the Planck regime, that is,

$$\lambda_{\mathrm{dB}} \sim \frac{1}{p} \overset{\mathrm{GUP}}{\rightarrow} \lambda \sim \frac{\lambda_P}{\tan^{-1}\left(\dfrac{\lambda_P}{\lambda_{\mathrm{dB}}}\right)} \begin{cases} \to \lambda_{\mathrm{dB}} & \text{for low-energy,} \\ \to \lambda_P & \text{at Planck scale.} \end{cases} \tag{7}$$

In fact, by attributing the origin of low-energy neutrino oscillations to the different de Broglie oscillation lengths associated with each mass eigenstate, it has been argued that the phenomenon of flavor changing may be *freezed* at Planck scale, owing to the saturation of Eq. (7) for all mass eigenstates. Nevertheless, due to the number of still open theoretical questions and the lack of experimental guidance at Planck energy, a definitive conclusion about the actual occurrence of the freezing of oscillations in all neutrino frameworks (equal energy, equal velocity or wave packet approaches) has not yet been reached.

Beyond theoretical conjectures, a more phenomenological investigation of non-standard features of gravity is related to the challenging detection of the CNB,[34,35] whose existence is supported only by strong indirect evidences to date.[36] Since relic neutrinos decoupled from matter few seconds after the Big Bang, it is possible to extract a great amount of data on the primordial features of the universe from them. In that stage, quantum and gravitational effects are expected to be comparably important, thus promoting such particles as unique witnesses of "exotic" gravity regimes that can no longer be reproduced in laboratory.

Finally, even though difficulties in testing the quantum nature of gravity seemed to relegate models such as QFT on curved background, string theory and loop quantum gravity to merely speculative formalisms until a few years ago, a promising way out has been recently offered by a series of experiments aiming at characterizing gravity as a *quantum coherent mediator*. The idea (which traces back to Feynman) is to consider two test masses prepared so as to exclude all types of perturbations from the environment and among each other, except for the mutual gravitational interaction. Then, if at a certain time a nonvanishing entanglement is measured between them, the only reason for this would be the exchange of a graviton, which would certify a sort of gravity quantumness.[37] In principle, a similar reasoning can be carried out also for superpositions of mass states[38] and, thus, for neutrinos. The advantage of using these particles is that they are only affected by the weak interaction and gravity, which significantly simplifies the realization of the experimental setup.

4. Future Perspectives

In this work, we have sorted through some of the scenarios where neutrinos act as a probe for testing gravity, both at classical and quantum scales. Even though most of them are genuine smoking guns since long time, several others are being proposed only in recent years. Let us mention some of the most promising ones:

(i) Gravitational waves (GW) have been shown to nontrivially affect neutrino spin and flavor oscillations. In particular, the case of neutrino interaction with stochastic GWs emitted by coalescing supermassive black holes has been discussed.[39] The question thus arises as to whether this mechanism can be exploited to gain information about physics of the GW emitting source through the detection of neutrinos undergoing oscillations in such a gravitational background;

(ii) In the last section, we have discussed implications of some proposed theories of quantum gravity for neutrino oscillations within the GUP framework. Note that, although the induced corrections are strongly suppressed by Planck energy, they may be experimentally detectable for ultra-high-energy cosmogenic neutrinos.[40] Therefore, we expect that the next-generation neutrino detectors may provide significant contributions in this direction.

Clearly, finding definite solutions to the above problems and framing them within a unified picture is a demanding, but at the same time intriguing, task. More investigation is inevitably required along this line.

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