Classical and Quantum Gravity

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To cite this article: L C Céleri *et al* 2009 *Class. Quantum Grav.* **26** 105014

IOP Publishing — Classical and Quantum Gravity

Class. Quantum Grav. **26** (2009) 105014 (13pp) doi:10.1088/0264-9381/26/10/105014

Action of the gravitational field on the dynamical Casimir effect

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Received 5 November 2008, in final form 3 March 2009

Published 29 April 2009

Online at stacks.iop.org/CQG/26/105014

Abstract

In this paper, we analyze the action of the gravitational field on the dynamical Casimir effect. We consider a massless scalar field confined in a cuboid cavity placed in a gravitational field described by a static and diagonal metric. With one of the plane mirrors of the cavity allowed to move, we compute the average number of particles created inside the cavity by means of the Bogoliubov coefficients computed through perturbative expansions. We apply our result to the case of an oscillatory motion of the mirror, assuming a weak gravitational field described by the Schwarzschild metric. The regime of parametric amplification is analyzed in detail, demonstrating that our computed result for the mean number of particles created agrees with specific associated cases in the literature. Our results, obtained in the framework of the perturbation theory, are restricted, under resonant conditions, to a short-time limit.

PACS numbers: 03.70.+k, 03.65.Sq, 04.20.Cv, 04.25.Nx

1. Introduction

Since Casimir's work [1], we know that vacuum zero-point fluctuations of a quantum field confined within a finite volume of space exert radiation pressure on the boundaries that confine the field [2]. In fact, the Casimir effect does not require material boundaries; it can be induced by any classical potential, such as a gravitational field, which is able to disturb the vacuum, changing the mode structure of this quantum field. In the case of an electromagnetic field inside a perfectly conducting Fabri–Perot cavity, this effect generates an attractive force between the plates which has been measured with high precision by Lamoureaux [3] and Mohideen and Roy [4]. Although Casimir's original analysis concerned the electromagnetic field, many authors have considered other fields, for example, a fermionic [5] or the Dirac field [6], the latter in the study of the quark confinement problem.

A great deal of effort has been devoted to the study of the Casimir effect in a curved background. The problem of a massless scalar field confined between two parallel plates placed in a weak and static gravitational field was considered in [7]. The author found that the gravitational interaction causes a small reduction in the Casimir energy, which leads to a weakening of the force between the plates. In [8], the force acting on a rigid Casimir cavity placed in a weak gravitational field was computed; it was found that the net force is in the opposite direction from the gravitational acceleration. An experiment to test the effects of the gravitational curvature on the vacuum energy has also been proposed in [8]. Recently, studying the case of an electromagnetic field inside a plane cavity placed in a weak and static gravitational field, Fulling *et al* [9] found that the Casimir energy gravitates as predicted by the equivalence principle, implying that the virtual field quanta follow geodesics [10].

When the boundaries or, equivalently, the classical potential confining the field, is time dependent, the dynamical counterpart of the Casimir effect takes place, revealing the striking feature of particle creation from the quantum vacuum. The dynamical Casimir effect (DCE) has been extensively studied [11], since the quantization of the radiation field in a cavity with moving, perfectly reflecting boundaries, performed by Moore [12] in the early 1970s. However, this phenomenon has not yet been observed experimentally, despite remarkable efforts [13].

The problem of the expanding universe exhibits strong similarities to the DCE and interesting achievements have been made in that subject [14, 15]. Parker [14] showed in 1969 that, in an expanding universe, particles are created from the vacuum [14]. In the same work, it was noted that the initial presence of bosons tends to increase the number of created bosons, while the opposite is true of fermions. Working with a brane model for the universe, Durrer [16] showed that gravitons are formed from vacuum, and Davies [17], studying the Rindler coordinate system in a flat spacetime, found that a uniformly accelerated observer would see a fixed boundary radiating energy. Actually, the idea of particle creation due to a nonstatic gravitational field was first discussed by Schrödinger [18], followed by DeWitt [19] and Imamura [20]; however, the first who gave a complete treatment of the problem was Parker [14]. It is interesting to mention that an analogy between the phenomenon of the production of particles in cosmological models and ion traps has recently been presented [21]. While gravitons, π mesons, protons and electrons are formed from the vacuum by the action of the gravitational field, a chain of ions confined by a time-dependent potential leads to the formation of phonons.

Here, we study the action of the gravitational field on the DCE. We consider a massless scalar field confined in a cuboid cavity placed in a gravitational field described by a static and diagonal metric. This restriction implies that the source of the gravitational field does not rotate. One of the plane mirrors of the cavity is allowed to move in accordance with the law $a(t) = a_0(1 + \epsilon f(t))$, while the remaining boundaries are fixed; $f(t)$ is an arbitrary function and ϵ is an exceedingly small quantity such that $\epsilon|f(t)| \ll 1$. The restriction $\epsilon|f(t)| \ll 1$ ensures that the variation of the cavity length in the direction of the gravitational field is much smaller than its proper length in this direction. Although the correction to the number of particle creation due to the gravitational field is expected to be exceedingly small on the earth's surface, we stress that our goal in the present work is focused on the relation between the Casimir effect and gravity. In fact, after the result in [9], that the Casimir energy gravitates, we find interesting to analyze its dynamical counterpart. Although we expect to find the same correction factor to the number of particle creation as that in [7, 9] for the Casimir energy—from the hypothesis of universality of the equivalence principle—it is worth confirming our expectation.

The paper is organized as follows. In section 2, we perform the quantization of the scalar field confined inside the cuboid cavity. In section 3, we compute the average number of particles created within the cavity, through the well-known Bogoliubov coefficients [22]. In section 4, we apply our general results to the case of a harmonic motion of the mirror, assuming a weak gravitational field described by the Schwarzschild metric. We emphasize that our resonant results for the average number of particle creation is restricted to a short-time limit. We present our concluding remarks in section 5. Throughout this paper we use natural units $c = G = \hbar = 1$, adopting the metric signature $(-, +, +, +)$. We also stress that we refer to the confining boundary of the scalar field as a *mirror*, because of the assumed Dirichlet boundary conditions.

2. Quantization of the massless scalar field

Let us consider a massless scalar field confined in a closed cuboid cavity, placed in a gravitational field described by a static and diagonal metric $g^{\mu\nu}$, with determinant g. Dirichlet boundary conditions are imposed on the scalar field at the mirrors. However, one of the plane mirrors of the cavity is allowed to move. The equation of motion for the scalar field ϕ in the vacuum (where the scalar curvature is null) is given by [23]

$$\partial_\nu(\sqrt{-g}g^{\mu\nu}\partial_\mu\phi) = 0. \tag{1}$$

The Lagrangian density that generates this equation of motion reads

$$\mathcal{L} = \tfrac{1}{2}\sqrt{-g}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi,$$

with the canonical conjugate momentum defined as

$$\pi = -\sqrt{-g}g^{00}\partial_0\phi. \tag{2}$$

Now, we introduce a reference system (x, y, z) with origin on a static mirror at $z = 0$, parallel to the moving one at $z = a(t)$, with $-\widehat{z}$ denoting the direction of the acceleration of gravity. To perform the field quantization, it is convenient to expand the scalar field in a complete and orthonormal set of instantaneous mode $\{u_{\mathbf{k}}(\mathbf{x}; t)\}$ of eigenfrequencies $\omega_{\mathbf{k}}$, $\mathbf{k} = (k_x, k_y, k_z)$ being the associated wave vector. We thus start with the classical scalar field $\phi(\mathbf{x}; t)$ and its canonical momentum given by

$$\phi(\mathbf{x}; t) = \sum_{\mathbf{k}}\sqrt{\frac{1}{2\omega_{\mathbf{k}}(t)}}[c_{\mathbf{k}}(t) + c^*_{\mathbf{k}}(t)]u_{\mathbf{k}}(\mathbf{x}; t), \tag{3}$$

$$\pi(\mathbf{x}; t) = \mathrm{i}\sqrt{-g}g^{00}\sum_{\mathbf{k}}\sqrt{\frac{\omega_{\mathbf{k}}(t)}{2}}[c_{\mathbf{k}}(t) - c^*_{\mathbf{k}}(t)]u_{\mathbf{k}}(\mathbf{x}; t), \tag{4}$$

where $c_{\mathbf{k}}(t)$ and $c^*_{\mathbf{k}}(t)$ are time-dependent complex coefficients and $\mathbf{x} = (x, y, z)$. The time dependence of the instantaneous modes and the corresponding eigenfrequencies is induced only by the moving mirror [24, 25]. As we deal with a cuboid cavity, we assume that $u_{\mathbf{k}}(\mathbf{x}; t)$ is a real function that obeys the following differential equations:

$$\left[-\sqrt{-g}g^{00}\omega^2_{\mathbf{k}} + \partial_i\sqrt{-g}g^{ij}\partial_j\right]u_{\mathbf{k}}(\mathbf{x}; t) = 0, \tag{5}$$

normalized by the inner product

$$-\int_{\mathcal{V}(t)}\mathrm{d}^3\mathbf{x}\sqrt{-g}g^{00}u_{\mathbf{k}'}(\mathbf{x}; t)u_{\mathbf{k}}(\mathbf{x}; t) = \delta_{\mathbf{k}\mathbf{k}'}, \tag{6}$$

where $i, j = x, y, z$ (from here on) and the integration is performed over the whole instantaneous cavity volume $\mathcal{V}(t)$. We assume that the mode functions satisfy the Dirichlet boundary conditions at the mirrors.

The quantization of the scalar field is performed in the canonical form, constructing a field operator Φ, associated with ϕ, by promoting the complex coefficients $c_{\mathbf{k}}$ and $c_{\mathbf{k}}^*$ to operators $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^\dagger$, respectively, and imposing the equal time commutation relations

$$[\Phi(\mathbf{x}; t), \Pi(\mathbf{x}'; t)] = \mathrm{i}\delta(\mathbf{x} - \mathbf{x}'), \tag{7a}$$

$$[\Pi(\mathbf{x}; t), \Pi(\mathbf{x}'; t)] = [\Phi(\mathbf{x}; t), \Phi(\mathbf{x}'; t)] = 0, \tag{7b}$$

where Π is the field operator associated with π. As a consequence of equations (6) and (7), the operators $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^\dagger$ satisfy the following commutation relations:

$$\left[a_{\mathbf{k}}(t), a_{\mathbf{k}'}^\dagger(t)\right] = \delta_{\mathbf{k}\mathbf{k}'}, \tag{8a}$$

$$[a_{\mathbf{k}}(t), a_{\mathbf{k}'}(t)] = \left[a_{\mathbf{k}}^\dagger(t), a_{\mathbf{k}'}^\dagger(t)\right] = 0, \tag{8b}$$

which are the usual boson commutation relations for the annihilation and creation operators. After the definition of these operators, we next compute the number of particles created inside the cavity by the DCE.

3. Average number of created particles

In this section, we will compute the mean number of particles created inside the cavity by the DCE, through the Bogoliubov coefficients $\alpha_{\mathbf{k}\mathbf{k}'}(t)$ and $\beta_{\mathbf{k}\mathbf{k}'}(t)$, defined by the relations

$$a_{\mathbf{k}}(t) = \sum_{\mathbf{k}'} \left[\alpha_{\mathbf{k}\mathbf{k}'}(t) a_{\mathbf{k}'}(t_0) + \beta_{\mathbf{k}\mathbf{k}'}(t) a_{\mathbf{k}'}^\dagger(t_0)\right], \tag{9a}$$

$$a_{\mathbf{k}}^\dagger(t) = \sum_{\mathbf{k}'} \left[\alpha_{\mathbf{k}\mathbf{k}'}^*(t) a_{\mathbf{k}'}^\dagger(t_0) + \beta_{\mathbf{k}\mathbf{k}'}^*(t) a_{\mathbf{k}'}(t_0)\right], \tag{9b}$$

which relate the annihilation and creation operators at time t, when the mirror ceases to move, to those at $t = t_0$, when the mirror starts to move. Our strategy is to find, with the help of equations (3) and (4), a set of differential equations for the operators $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^\dagger$. By comparing these equations with the equivalent set obtained directly from the time derivative of equation (9), we end up with the desired set of differential equations for the Bogoliubov coefficients. Expanding these equations in power series of the small parameter ϵ, we are able to find recurrence relations for both coefficients $\alpha_{\mathbf{k}\mathbf{k}'}(t)$ and $\beta_{\mathbf{k}\mathbf{k}'}(t)$, prompting their solutions in any desired order.

With the help of equation (6) it is straightforward to obtain, from the quantum version of equations (3) and (4), the relations

$$\begin{aligned} a_{\mathbf{k}}(t) = &-\sqrt{\frac{\omega_{\mathbf{k}}(t)}{2}} \int_{\mathcal{V}(t)} \mathrm{d}^3\mathbf{x} \sqrt{-g}\, g^{00} u_{\mathbf{k}}(\mathbf{x}; t) \Phi(\mathbf{x}; t) \\ &+ \mathrm{i}\sqrt{\frac{1}{2\omega_{\mathbf{k}}(t)}} \int_{\mathcal{V}(t)} \mathrm{d}^3\mathbf{x}\, u_{\mathbf{k}}(\mathbf{x}; t) \Pi(\mathbf{x}; t), \end{aligned} \tag{10a}$$

$$\begin{aligned} a_{\mathbf{k}}^\dagger(t) = &-\sqrt{\frac{\omega_{\mathbf{k}}(t)}{2}} \int_{\mathcal{V}(t)} \mathrm{d}^3\mathbf{x} \sqrt{-g}\, g^{00} u_{\mathbf{k}}(\mathbf{x}; t) \Phi(\mathbf{x}; t) \\ &- \mathrm{i}\sqrt{\frac{1}{2\omega_{\mathbf{k}}(t)}} \int_{\mathcal{V}(t)} \mathrm{d}^3\mathbf{x}\, u_{\mathbf{k}}(\mathbf{x}; t) \Pi(\mathbf{x}; t). \end{aligned} \tag{10b}$$

Taking the time derivative of these equations we obtain the above-mentioned set of differential equations for $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^{\dagger}$ as functions of $u_{\mathbf{k}}$, Φ and Π, as well as their time derivatives. Using relations (1), (2) and (5), we thus obtain

$$\begin{aligned}\dot{a}_{\mathbf{k}}(t) &= -\,\mathrm{i}\omega_{\mathbf{k}}a_{\mathbf{k}}(t) + \sum_{\mathbf{k}'}\left\{G_{[\mathbf{k}\mathbf{k}']}(t)a_{\mathbf{k}'}(t) + G_{(\mathbf{k}\mathbf{k}')}(t)a_{\mathbf{k}'}^{\dagger}(t)\right\},\\ \dot{a}_{\mathbf{k}}^{\dagger}(t) &= \ \mathrm{i}\omega_{\mathbf{k}}a_{\mathbf{k}}^{\dagger}(t) + \sum_{\mathbf{k}'}\left\{G_{[\mathbf{k}\mathbf{k}']}(t)a_{\mathbf{k}'}^{\dagger}(t) + G_{(\mathbf{k}\mathbf{k}')}(t)a_{\mathbf{k}'}(t)\right\},\end{aligned} \tag{11}$$

where we have defined the antisymmetric $G_{[\mathbf{k}\mathbf{k}']} = -G_{[\mathbf{k}'\mathbf{k}]}$ and symmetric $G_{(\mathbf{k}\mathbf{k}')} = G_{(\mathbf{k}'\mathbf{k})}$ parts of the coefficients

$$G_{\mathbf{k}\mathbf{k}'}(t) = \frac{1}{2}\frac{\dot{\omega}_{\mathbf{k}}}{\omega_{\mathbf{k}}}\delta_{\mathbf{k}\mathbf{k}'} + \sqrt{\frac{\omega_{\mathbf{k}}}{\omega_{\mathbf{k}'}}}\int_{\mathcal{V}(t)}\mathrm{d}^3\mathbf{x}\sqrt{-g}g^{00}\dot{u}_{\mathbf{k}'}(\mathbf{x};t)u_{\mathbf{k}}(\mathbf{x};t). \tag{12}$$

The equivalent set of equations for $a_{\mathbf{k}}$ and $a_{\mathbf{k}}^{\dagger}$, generated by differentiating the transformations (9), is simply given by

$$\dot{a}_{\mathbf{k}}(t) = \sum_{\mathbf{k}'}\left[\dot{\alpha}_{\mathbf{k}\mathbf{k}'}(t)a_{\mathbf{k}'}(t_0) + \dot{\beta}_{\mathbf{k}\mathbf{k}'}(t)a_{\mathbf{k}'}^{\dagger}(t_0)\right], \tag{13a}$$

$$\dot{a}_{\mathbf{k}}^{\dagger}(t) = \sum_{\mathbf{k}'}\left[\dot{\alpha}_{\mathbf{k}\mathbf{k}'}^{*}(t)a_{\mathbf{k}'}^{\dagger}(t_0) + \dot{\beta}_{\mathbf{k}\mathbf{k}'}^{*}(t)a_{\mathbf{k}'}(t_0)\right]. \tag{13b}$$

By substituting definitions (9) into equations (11), and comparing the result with equations (13), we obtain the set of differential equations for the Bogoliubov coefficients:

$$\begin{aligned}\dot{\alpha}_{\mathbf{k}\mathbf{k}'}(t) &= -\,\mathrm{i}\omega_{\mathbf{k}}\alpha_{\mathbf{k}\mathbf{k}'}(t) + \sum_{\mathbf{k}''}\{G_{[\mathbf{k}\mathbf{k}'']}(t)\alpha_{\mathbf{k}''\mathbf{k}'}(t) + G_{(\mathbf{k}\mathbf{k}'')}(t)\beta_{\mathbf{k}''\mathbf{k}'}^{*}(t)\},\\ \dot{\beta}_{\mathbf{k}\mathbf{k}'}(t) &= -\,\mathrm{i}\omega_{\mathbf{k}}\beta_{\mathbf{k}\mathbf{k}'}(t) + \sum_{\mathbf{k}''}\{G_{[\mathbf{k}\mathbf{k}'']}(t)\beta_{\mathbf{k}''\mathbf{k}'}(t) + G_{(\mathbf{k}\mathbf{k}'')}(t)\alpha_{\mathbf{k}''\mathbf{k}'}^{*}(t)\}.\end{aligned}$$

For reasons of mathematical convenience, we define $\Theta_{\mathbf{k}}(t) = \int_{t_0}^{t}\omega_{\mathbf{k}}(\tau)\mathrm{d}\tau$ and introduce the new coefficients

$$\begin{aligned}\tilde{\alpha}_{\mathbf{k}\mathbf{k}'} &= \mathrm{e}^{\mathrm{i}\Theta_{\mathbf{k}}}\alpha_{\mathbf{k}\mathbf{k}'},\\ \tilde{\beta}_{\mathbf{k}\mathbf{k}'} &= \mathrm{e}^{\mathrm{i}\Theta_{\mathbf{k}}}\beta_{\mathbf{k}\mathbf{k}'},\end{aligned}$$

leading to the simplified equations

$$\frac{\mathrm{d}\tilde{\alpha}_{\mathbf{k}\mathbf{k}'}}{\mathrm{d}t} = \sum_{\mathbf{k}''}[G_{[\mathbf{k}\mathbf{k}'']}\mathrm{e}^{\mathrm{i}[\Theta_{\mathbf{k}}-\Theta_{\mathbf{k}''}]}\tilde{\alpha}_{\mathbf{k}''\mathbf{k}'} + G_{(\mathbf{k}\mathbf{k}'')}\mathrm{e}^{\mathrm{i}[\Theta_{\mathbf{k}}+\Theta_{\mathbf{k}''}]}\tilde{\beta}_{\mathbf{k}''\mathbf{k}'}^{*}], \tag{14a}$$

$$\frac{\mathrm{d}\tilde{\beta}_{\mathbf{k}\mathbf{k}'}}{\mathrm{d}t} = \sum_{\mathbf{k}''}[G_{[\mathbf{k}\mathbf{k}'']}\mathrm{e}^{\mathrm{i}[\Theta_{\mathbf{k}}-\Theta_{\mathbf{k}''}]}\tilde{\beta}_{\mathbf{k}''\mathbf{k}'} + G_{(\mathbf{k}\mathbf{k}'')}\mathrm{e}^{\mathrm{i}[\Theta_{\mathbf{k}}+\Theta_{\mathbf{k}''}]}\tilde{\alpha}_{\mathbf{k}''\mathbf{k}'}^{*}]. \tag{14b}$$

We have omitted, for notational simplicity, the explicit time dependence of all parameters. To solve equations (14), we expand the Bogoliubov coefficients in a power series in ϵ, written as follows:

$$\tilde{\alpha}_{\mathbf{k}\mathbf{k}'} = \sum_{\lambda=0}^{\infty}\epsilon^{\lambda}\frac{1}{\lambda!}\lim_{\epsilon\to 0}\frac{\partial^{\lambda}\tilde{\alpha}_{\mathbf{k}\mathbf{k}'}}{\partial\epsilon^{\lambda}} = \sum_{\lambda=0}^{\infty}\epsilon^{\lambda}\tilde{\alpha}_{\mathbf{k}\mathbf{k}'}^{(\lambda)}, \tag{15a}$$

$$\tilde{\beta}_{\mathbf{k}\mathbf{k}'} = \sum_{\lambda=0}^{\infty}\epsilon^{\lambda}\tilde{\beta}_{\mathbf{k}\mathbf{k}'}^{(\lambda)}. \tag{15b}$$

As the coefficients $G_{\mathbf{kk}'}$ and $\Theta_{\mathbf{k}}$ also depend on ϵ (through $a(t)$), they must be expanded to compare orders in ϵ in equations (14), prompting the relations

$$G_{(\mathbf{kk}')}\mathrm{e}^{\mathrm{i}[\Theta_{\mathbf{k}}+\Theta_{\mathbf{k}'}]} = \sum_{\lambda=1}^{\infty}\epsilon^{\lambda}\Lambda_{\mathbf{kk}'}^{(\lambda)}, \tag{16a}$$

$$G_{[\mathbf{kk}']}\mathrm{e}^{\mathrm{i}[\Theta_{\mathbf{k}}-\Theta_{\mathbf{k}'}]} = \sum_{\lambda=1}^{\infty}\epsilon^{\lambda}\Xi_{\mathbf{kk}'}^{(\lambda)}. \tag{16b}$$

The sums in the above expansions start with $\lambda = 1$ because the coefficients $G_{\mathbf{kk}'}$ are proportional to $\dot{a}$ and, consequently, their lowest contribution is of first order, as expected.

By substituting equations (15) and (16) into equation (14) and comparing the terms of the same order in ϵ we find, with the help of the initial conditions $\tilde{\alpha}_{\mathbf{kk}'}^{(0)}(t) = \delta_{\mathbf{kk}'}$ and $\tilde{\beta}_{\mathbf{kk}'}^{(0)}(t) = 0$, the recurrence relations

$$\tilde{\alpha}_{\mathbf{kk}'}^{(\lambda)}(t) = \sum_{\mathbf{k}''}\sum_{\lambda'=0}^{\lambda-1}\int_{t_0}^{t}\mathrm{d}\tau\left\{\Xi_{\mathbf{kk}''}^{(\lambda-\lambda')}(\tau)\tilde{\alpha}_{\mathbf{k}''\mathbf{k}'}^{(\lambda')}(\tau) + \Lambda_{\mathbf{kk}''}^{(\lambda-\lambda')}(\tau)\tilde{\beta}_{\mathbf{k}''\mathbf{k}'}^{(\lambda')*}(\tau)\right\}, \tag{17a}$$

$$\tilde{\beta}_{\mathbf{kk}'}^{(\lambda)}(t) = \sum_{\mathbf{k}''}\sum_{\lambda'=0}^{\lambda-1}\int_{t_0}^{t}\mathrm{d}\tau\left\{\Xi_{\mathbf{kk}''}^{(\lambda-\lambda')}(\tau)\tilde{\beta}_{\mathbf{k}''\mathbf{k}'}^{(\lambda')}(\tau) + \Lambda_{\mathbf{kk}''}^{(\lambda-\lambda')}(\tau)\tilde{\alpha}_{\mathbf{k}''\mathbf{k}'}^{(\lambda')*}(\tau)\right\}, \tag{17b}$$

which provide the Bogoliubov coefficients to any desired order of parameter ϵ and, consequently, the average number of particles created inside the cavity. Assuming that the scalar field is initially in the vacuum state $|\{0_{\mathbf{k}}\}\rangle = \prod_{\mathbf{k}}|0_{\mathbf{k}}\rangle$, where $a_{\mathbf{k}}(t_0)|\{0_{\mathbf{k}}\}\rangle = 0$, the average number of particles created in the **k**th mode, computed from equations (9), is given by

$$\begin{aligned}\mathcal{N}_{\mathbf{k}}(t) &= \langle\{0_{\mathbf{k}'}\}|a_{\mathbf{k}}^{\dagger}(t)a_{\mathbf{k}}(t)|\{0_{\mathbf{k}'}\}\rangle \\ &= \sum_{\mathbf{k}'}|\beta_{\mathbf{k}'\mathbf{k}}(t)|^2 = \sum_{\mathbf{k}'}|\tilde{\beta}_{\mathbf{k}'\mathbf{k}}(t)|^2.\end{aligned} \tag{18}$$

In the following section, we apply this result to the particular case of an oscillatory motion of the mirror in a Schwarzschild background.

4. Oscillatory motion of the mirror

Let us consider a cavity of dimensions $a_0 \times a_0 \times a(t)$, $a(t)$ specifying the sinoidal law of motion

$$a(t) = a_0\left[1 + \epsilon\sin(\varpi t)\right],$$

where ϖ is the oscillation frequency of the mirror and $\epsilon \ll 1$. We also assume that the cavity is placed at a distance R from the gravitational source (outside the mass distribution), represented by a nonrotating spherical mass M. The gravitational field is thus described by the Schwarzschild metric which, in the isotropic coordinates and weak-field limit $M/r \ll 1$—r being the radial coordinate—is given by [23]

$$\mathrm{d}s^2 = -\left(1 - 2\frac{M}{r}\right)\mathrm{d}t^2 + \left(1 + 2\frac{M}{r}\right)\mathrm{d}\mathbf{r}^2. \tag{19}$$

Under the realistic restriction that the dimensions of the cavity are negligible compared to the size of the gravitational source, i.e., $a_0 \ll R$, we expand the line element (19) over a short distance z around R in the radial direction [7, 9] $r^2 = x^2 + y^2 + (z + R)^2$ up to order $(M/R)^2$, to obtain

$$\frac{M}{r} \simeq \chi - \gamma z, \tag{20}$$

where $\chi = M/R$ and $\gamma = M/R^2$ is the acceleration of gravity. With this expansion the line element (19) can be rewritten as

$$\mathrm{d}s^2 = -(1 - 2\chi + 2\gamma z)\,\mathrm{d}t^2 + (1 + 2\chi - 2\gamma z)\,\mathrm{d}\mathbf{r}^2. \tag{21}$$

Next, we consider only the first-order approximation, where $\gamma = 0$.

4.1. First-order approximation: a constant gravitational field

Considering the first-order approximation in the expansion (20), i.e. $\gamma = 0$, the instantaneous mode functions—that satisfy equations (5) and (6), as well as the Dirichlet boundary conditions at the mirrors—are given by

$$u_{\mathbf{k}}(\mathbf{x}; t) = \frac{2(1 - 2\chi)}{a_0} \sin[k_x(0)x] \sin[k_y(0)y] \sqrt{\frac{2}{a(t)}} \sin[k_z(t)z], \tag{22}$$

where $k_i(t) = (n_i\pi)/a(t)$ and n_i stands for the positive integers. The corresponding eigenfrequencies read

$$\omega_{\mathbf{k}}(t) = (1 - 2\chi)\sqrt{k_x^2(0) + k_y^2(0) + k_z^2(t)}. \tag{23}$$

As we assume that ϵ is a small number, we will compute the mean number of particles created up to second order in this parameter, which is the first non-zero contribution. This implies, as we can see from equation (18), that we have to compute the coefficient $\tilde{\beta}_{\mathbf{kk}'}$ up to first order in ϵ, given the result

$$\mathcal{N}_{\mathbf{k}}(t) \simeq \epsilon^2 \sum_{\mathbf{k}'} \left|\tilde{\beta}^{(1)}_{\mathbf{kk}'}(t)\right|^2 = \epsilon^2 \sum_{\mathbf{k}'} \left|\int_{t_0}^{t} \mathrm{d}\tau\, \Lambda^{(1)}_{\mathbf{kk}'}(\tau)\right|^2, \tag{24}$$

where we have used equation (17). With the definition $\mathbf{n}^2 = n_x^2 + n_y^2 + n_z^2$, the first-order approximation of the coupling coefficients $G_{\mathbf{kk}'}(t)$ in equation (16) read

$$\Lambda^{(1)}_{\mathbf{kk}'}(t) = \delta_{n_x,n'_x}\delta_{n_y,n'_y}\varpi\cos(\varpi t)$$
$$\times\, \mathrm{e}^{\mathrm{i}\omega_{n_z,n'_z}t}\left\{-\delta_{n_z,n'_z}\frac{n_z^2}{2\mathbf{n}^2} + (1 - \delta_{n_z,n'_z})(-1)^{n_z+n'_z}\frac{n_z n'_z}{n'^2_z - n_z^2}\frac{\mathbf{n} - \mathbf{n}'}{\sqrt{\mathbf{nn}'}}\right\},$$

where we have defined the frequencies

$$\omega_{\mathbf{n},\mathbf{n}'} = \delta_{n_x,n'_x}\delta_{n_y,n'_y}(1 - 2\chi)\frac{\pi}{a_0}(\mathbf{n} + \mathbf{n}').$$

By substituting the expressions for $\Lambda^{(1)}_{\mathbf{kk}'}(t)$ and $\omega_{\mathbf{n},\mathbf{n}'}$ into equation (24), we obtain the following expression for the mean number of particles created in a selected mode $\mathbf{k}$:

$$\mathcal{N}_{\mathbf{k}} = \frac{1}{4}\sum_{\mathbf{n}'}\epsilon^2\varpi^2 t^2 \mathcal{C}_{\mathbf{n},\mathbf{n}'}|f_{\mathbf{n},\mathbf{n}'}(\varpi, t)|^2,$$

where the constant coupling coefficients $\mathcal{C}_{\mathbf{n},\mathbf{n}'}$ are

$$\mathcal{C}_{\mathbf{n},\mathbf{n}'} = \delta_{n_x,n'_x}\delta_{n_y,n'_y}\left\{\frac{1}{4}\delta_{n_z,n'_z}\left(\frac{n_z^2}{\mathbf{n}^2}\right)^2 + \left(1 - \delta_{n_z,n'_z}\right)\frac{n_z^2 n_z^{2\prime}}{\left(n'^2_z - n_z^2\right)^2}\frac{(\mathbf{n} - \mathbf{n}')^2}{\mathbf{nn}'}\right\},$$

and the time-dependent function $f_{\mathbf{n},\mathbf{n}'}$ is given by

$$f_{\mathbf{n},\mathbf{n}'}(\varpi,t) = \delta_{n_x,n'_x}\delta_{n_y,n'_y}\left\{\frac{\exp[\mathrm{i}(\omega_{\mathbf{n},\mathbf{n}'}-\varpi)t]-1}{(\omega_{\mathbf{n},\mathbf{n}'}-\varpi)t}+\frac{\exp[\mathrm{i}(\omega_{\mathbf{n},\mathbf{n}'}+\varpi)t]-1}{(\omega_{\mathbf{n},\mathbf{n}'}+\varpi)t}\right\}. \quad (25)$$

As we can see from equation (25), $\mathcal{N}_{\mathbf{k}}$ is an oscillatory function in time, except when at least one of the resonance conditions $\varpi = \omega_{\mathbf{n},\mathbf{n}'}$ is satisfied. (Note that the second high-oscillatory term on the rhs of equation (25) can be disregarded within the rotating wave approximation.) Therefore, under the resonance condition, the mean number of particles created is a function that increases quadratically in time, given by

$$\lim_{\varpi\to\omega_{\mathbf{n},\mathbf{n}'}}\mathcal{N}_{\mathbf{k}} \simeq \tfrac{1}{4}\mathcal{C}_{\mathbf{n},\mathbf{n}'}(\epsilon\omega_{\mathbf{n},\mathbf{n}'}t)^2.$$

We remember that r and t are only coordinates, without any direct physical significance (see the discussions in [7, 9]). To obtain a measurable quantity, we must rewrite this result in terms of the proper time and length of the cuboid cavity, defined, in this static case, as $t_p = \int \mathrm{d}t\sqrt{-g_{00}}$ and $a_p = \int \mathrm{d}z\sqrt{g_{zz}}$, respectively. With this consideration we obtain the quantity

$$\omega_{\mathbf{n},\mathbf{n}'}t = \delta_{n_x,n'_x}\delta_{n_y,n'_y}\frac{\pi}{a_p}(\mathbf{n}+\mathbf{n}')t_p, \quad (26)$$

given, under the resonance condition, the expected mean number of particles created due to the DCE

$$\lim_{\varpi\to\omega_{\mathbf{n},\mathbf{n}'}}\mathcal{N}_{\mathbf{k}} \simeq \delta_{n_x,n'_x}\delta_{n_y,n'_y}\mathcal{C}_{\mathbf{n},\mathbf{n}'}\left(\epsilon\frac{\pi}{2a_p}(\mathbf{n}+\mathbf{n}')t_p\right)^2.$$

The above first-order result shows that a constant gravitational field does not modify the number of particles created due to the DCE inside the cavity (note that the coefficients $\mathcal{C}_{\mathbf{n},\mathbf{n}'}$ do not depend on t or $\omega_{\mathbf{n},\mathbf{n}'}$). The fact that a constant field does not change the static Casimir energy is a consequence of the lack of dependence of the physical results on the origin of the coordinate system [7, 9]. Therefore, any contribution of the gravitational field to the number of particles created will appear at least in a second-order approximation, where the spatial dependence of the metric arises. Next, we compute such a second-order correction for the number of particles.

4.2. Second-order approximation

In this section, considering the full metric shown in equation (21), we seek mode solutions in the form

$$u_{\mathbf{k}}(\mathbf{x};t) = \frac{2}{a_0}\sin[k_x(0)x]\sin[k_y(0)y]\xi_{\mathbf{k}}(z;t). \quad (27)$$

By substituting this ansatz solution into equation (5), the following differential equation for $\xi_{\mathbf{k}}$ results:

$$\partial_z^2\xi_{\mathbf{k}} - 4\gamma\omega_{\mathbf{k}}^2 z\xi_{\mathbf{k}} = -\Omega_{\mathbf{k}}^2\xi_{\mathbf{k}}, \quad (28)$$

where we have defined $\Omega_{\mathbf{k}}^2 = \omega_{\mathbf{k}}^2(1+4\chi) - (n_x\pi/a_0)^2 - (n_y\pi/a_0)^2$.

Now, following the reasoning in [7], it is convenient to perform the coordinate transformation

$$v_{\mathbf{k}}(z) = \left(\frac{\Omega_{\mathbf{k}}^2}{4\gamma\omega_{\mathbf{k}}^2} - z\right)\left(4\gamma\omega_{\mathbf{k}}^2\right)^{1/3}, \quad (29)$$

which leads to the following simplified form of the Airy differential equation:

$$\partial_v^2\xi_{\mathbf{k}}(v_{\mathbf{k}}) + v\xi_{\mathbf{k}}(v_{\mathbf{k}}) = 0,$$

whose solutions can be written in terms of a linear combination of Bessel functions of the first kind:

$$\xi_{\mathbf{k}}(v_{\mathbf{k}}) = \sqrt{v_{\mathbf{k}}}\left[A_{\mathbf{k}} J_{1/3}\left(\frac{2}{3}v_{\mathbf{k}}^{3/2}\right) + B_{\mathbf{k}} J_{-1/3}\left(\frac{2}{3}v_{\mathbf{k}}^{3/2}\right)\right].$$

By applying the boundary conditions and noting, from equation (29), that $v_{\mathbf{k}}(z,t) \gg 1$ for all values of t, we obtain the approximate solution [7]

$$\xi_{\mathbf{k}}(v_{\mathbf{k}}) \simeq N_{\mathbf{k}} v_{\mathbf{k}}^{-1/4}(z) \sin\left(\frac{2}{3}v_{\mathbf{k}}^{3/2}(z) - \frac{2}{3}v_{\mathbf{k}}^{3/2}(0)\right), \tag{30}$$

with the normalization factor $N_{\mathbf{k}}$ being fixed by equation (6), and the expression

$$\omega_{\mathbf{k}}(t) \simeq [1 - 2\chi + \gamma a(t)]\sqrt{k_x^2(0) + k_y^2(0) + k_z^2(t)}, \tag{31}$$

corroborating the above first-order approximation for $\omega_{\mathbf{k}}(t)$.

Therefore, by computing the coefficients $\Lambda^{(1)}_{\mathbf{k}\mathbf{k}'}$ through the mode functions written in equations (27) and (30), we finally obtain the second-order result for the mean number of particles created in mode $\mathbf{k}$, given by

$$\mathcal{N}_{\mathbf{k}} = \frac{1}{4}\sum_{\mathbf{n}'} \epsilon^2 \varpi^2 t^2 \mathcal{C}^{(2)}_{\mathbf{n},\mathbf{n}'} |f_{\mathbf{n},\mathbf{n}'}(\varpi, t)|^2, \tag{32}$$

where the function $f_{\mathbf{n},\mathbf{n}'}$ is defined in equation (25), but with the frequencies $\omega_{\mathbf{n},\mathbf{n}'}$ derived from equation (31) as

$$\omega_{\mathbf{n},\mathbf{n}'} = \delta_{n_x,n'_x}\delta_{n_y,n'_y}(1 - 2\chi + \gamma a_0)\frac{\pi}{a_0}(\mathbf{n} + \mathbf{n}').$$

A general expression for the coefficient $\mathcal{C}^{(2)}_{\mathbf{n},\mathbf{n}'}$ (valid for any value of the mirror frequency ϖ) can be computed with the help of equations (27) and (30). Its solution, though too cumbersome to be shown here, is straightforward. Therefore, we consider a special case of interest, the one that maximizes the number of particles created in a selected mode $\mathbf{k}$: the parametric amplification process, in which the frequency of the mirror oscillation is twice the stationary eigenfrequency of a given mode of the cavity, i.e., $\varpi = 2\omega_{\mathbf{k}}(0)$. As seen from equations (25) and (26), this condition implies that $\mathbf{k} = \mathbf{k}'$. Under these assumptions, the coefficients $\mathcal{C}^{(2)}_{\mathbf{n},\mathbf{n}}$ have the simple expression

$$\mathcal{C}^{(2)}_{\mathbf{n},\mathbf{n}} \simeq \frac{1}{4}\left(\frac{n_z^2}{\mathbf{n}^2} - \gamma a_0\right)^2, \tag{33}$$

so that the number of particles created in mode $\mathbf{k}$ is given by

$$\mathcal{N}_{\mathbf{k}} \simeq \left(\frac{\epsilon\omega_{\mathbf{k}}(0)t}{2}\right)^2\left(\gamma a_0 - \frac{n_z^2}{\mathbf{n}^2}\right)^2.$$

Rewriting this equation in terms of the proper length and time

$$a_0 \simeq a_p\left(1 + \chi + \gamma\frac{a_p}{2}\right),$$
$$t \simeq (1 + \chi - \gamma a_p)t_p,$$

we obtain our final result

$$\mathcal{N}_{\mathbf{k}} = \left[\frac{n_z^2}{\mathbf{n}^2}(1 - 4\chi) - \gamma a_p\left(1 + \frac{n_z^2}{\mathbf{n}^2}\right)\right]^2(\mathbf{n}\tau_p)^2, \tag{34}$$

where we have defined the dimensionless proper time variable $\tau_p = \epsilon\pi t_p/2a_0$. In the absence of gravity, equation (34) simplifies to $\mathcal{N}_{\mathbf{k}} = (n_z/\mathbf{n})^4(\mathbf{n}\tau_p)^2$, recovering the result in the literature for the DCE under the parametric amplification condition. For the fundamental mode, i.e. $n_x = n_y = n_z = 1$, we obtain

$$\mathcal{N}_{\mathbf{1}} = [1 - 4\chi - 2\gamma a_p]\tau_p^2, \tag{35}$$

instead of the simpler result $\mathcal{N}_{\mathbf{1}} = \tau_p^2$ given by the DCE in a flat spacetime or, as demonstrated above, within the first-order approximation $\gamma = 0$. This expression shows that the effect of the gravitational field on the DCE is simply to diminish the number of created particles. As expected, this fact is in agreement with the results obtained in [7, 9], where the authors demonstrate that the coupling with a gravitational field causes a weakening of the Casimir energy by the same factor $2\gamma a_p$. For a cuboid cavity on the earth's surface with a_0 around 1 cm, we verify that the effect of gravity is to change the number of photons produced by a small factor around 10^{-18}. As discussed in section 1, we were compelled to compute this small correction—expecting it to be the same as that in the Casimir energy [9]—only from a theoretical point of view: to investigate the universality of the equivalence principle. In this regard, an immediate extension of the present result is to consider an arbitrary direction of the surface of the oscillating mirror with respect to the gravitational acceleration.

Although we have considered only the parametric resonance regime—creating degenerate pairs of particles—other resonances can evidently be satisfied by the moving mirror: apart from the nondegenerate creation of pairs of particles in distinct modes $\mathbf{k}$ and $\mathbf{k}'$, under the resonance condition $\varpi = \omega_{\mathbf{k}} + \omega_{\mathbf{k}'}$, the scattering of particles between these modes also takes place under the condition $\varpi = |\omega_{\mathbf{k}} - \omega_{\mathbf{k}'}|$. However, in a three-dimensional cavity, a suitable choice of the cavity dimensions forbids both degenerate and nondegenerate resonances from occurring simultaneously. Therefore, it is reasonable to consider only the degenerate amplification process when analyzing the particle creation mechanism by the DCE under the action of a gravitational field [26]. Evidently, the mean number of particles created can be computed under any resonance condition from equation (32), by performing a numerical calculation of the second-order coefficients $\mathcal{C}^{(2)}_{\mathbf{n},\mathbf{n}'}$.

In figure 1, we plot $\mathcal{N}_{\mathbf{1}}$ against the dimensionless parameter γa_p for a fixed value $\tau_p = 1$ and $\chi = 0$. As expected from equation (35), the number of particles created in the fundamental mode decreases as γ increases. In figure 2, we plot $\mathcal{N}_{\mathbf{k}}$ against γa_p for a fixed value $\tau_p = 1/\mathbf{n}$ (during which the cavity performs $1/2\pi\epsilon$ oscillations), and for a few values of $\mathbf{n}$. Again as expected, since the energy of a given mode increases with $\mathbf{n}$, the number of particles created decreases as $\mathbf{n}$ increases.

In [7], Sorge computed the gravitational corrections to the Casimir vacuum energy density for a massless scalar field confined between two nearby parallel plates also placed in a weak and static gravitational field. It was verified that, to the first-order approximation, the Casimir energy is unaffected by a constant gravitational field. To the second-order approximation, however, setting $\chi = 0$, it was found that the gravitational interaction causes a small reduction in the value of the Casimir energy. Having the author fixed $\chi \neq 0$, he would obtain the same result to the second-order approximation, since the Casimir energy is given by the difference between the vacuum energies obtained in the presence and absence of the parallel plates. The contribution of χ to the vacuum energies, which is expected to be the same in the presence or absence of the plates, would cancelled out, differently from what happens with the computation of the number of particles carried out here.

5. Concluding remarks

In this paper, we analyzed the action of the gravitational field on the number of particles created in a massless scalar field by the dynamical Casimir effect. We considered a cuboid

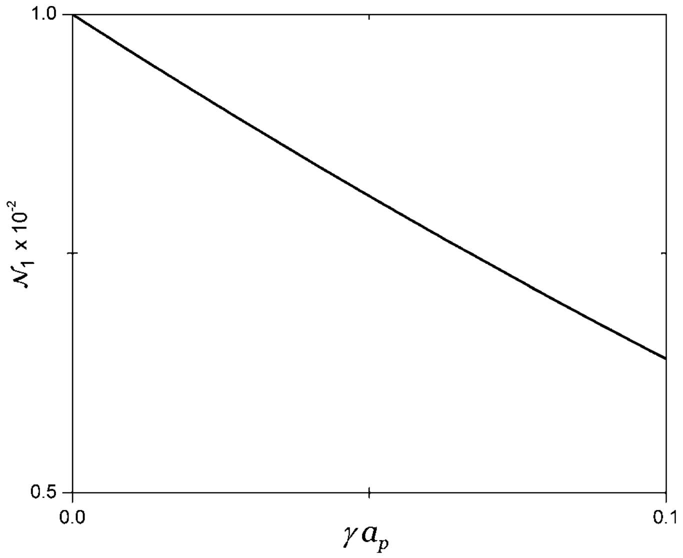

Figure 1. The mean number of particles created in the fundamental mode, $\mathcal{N}_{\mathbf{1}}/\tau_p^2$, plotted against γa_p, setting $\chi = 0$.

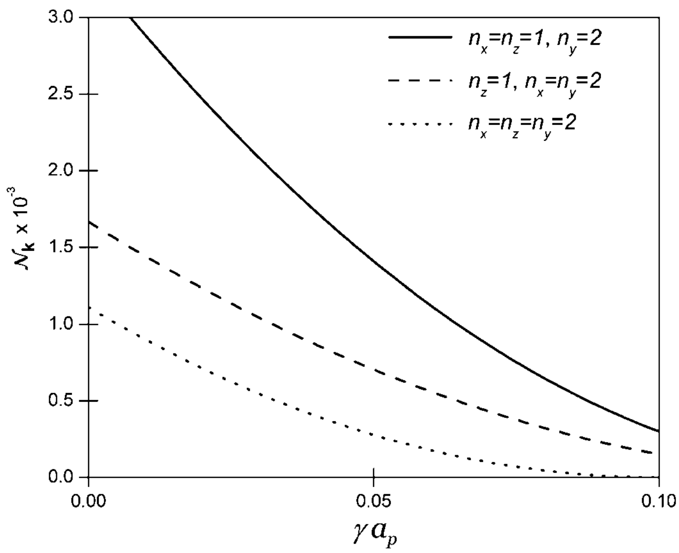

Figure 2. The mean number of particles created in the **k**th mode, $\mathcal{N}_{\mathbf{k}}/\tau_p^2$, plotted against γa_p, setting $\chi = 0$.

cavity, with one of its plane mirrors allowed to move, placed in a static gravitational field described by a diagonal metric. We first computed the mean number of particles created under an arbitrary law of motion of the cavity mirror, employing the Bogoliubov coefficients obtained by perturbative expansions. Next, the mean number is analyzed under the particular circumstances of an oscillatory motion of the mirror and a weak gravitational field described by the Schwarzschild metric. As already emphasized, our resonant results are restricted to the short-time approximation $\tau_p \ll 1$.

Our first-order result, that a constant gravitational field does not affect the mean number of particles created by the DCE, is in agreement with those in [7, 9] showing that a constant field does not change the static Casimir energy. The reason for this behavior is the fact that the physical results do not depend on the origin of the coordinate system [7, 9]. Therefore, the effect of the gravitational field appears only in a second- or higher order approximation, in which the spatial dependence of the metric arises.

Considering only the parametric resonance regime, our second-order result shows that the mean number of created particles is diminished by the gravitational field, again in agreement with the fact that the coupling with this field weakens the Casimir energy [7, 9]. Expressed differently, as the frequencies of the cavity scalar field are redshifted relative to their values in the absence of gravity, the mean number of created particles, proportional to the square of these frequencies, must decrease.

We emphasize that we have obtained, as expected, the same correction to the number of particle creation due to the gravitational field as that in [7, 9] for the Casimir energy. However, a worthwhile extension of the present work is to consider an arbitrary direction of the surface of the oscillating mirror with respect to the gravitational acceleration. Moreover, the backreaction of the particle creation rate on the gravitational field and vice versa is by itself an interesting and challenging problem to be studied. We also observe that the effect of the temperature, which is important for the experimental verification of the DCE, can be taken into account by considering a non-null scalar curvature in the cavity field equation.

We finally recall from section 1 that the possibility of the experimental observation of the force produced by vacuum fluctuations acting on a rigid Casimir cavity in a weak gravitational field was discussed in [8]. Although such an experimental test of the effects reported here is certainly much more challenging than the verification of the DCE itself, our entirely theoretical goal in the present work was to derive the dynamical counterpart of the result obtained in [7, 9].

Acknowledgments

We wish to express thanks for the support from CNPq and FAPESP, Brazilian agencies.

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