Breakdown of the adiabatic limit in low-dimensional gapless systems

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It is generally believed that a generic system can be reversibly transformed from one state to another by a sufficiently slow change of parameters. Microscopically, this belief is often justified using connections to the quantum adiabatic theorem stating that there are no transitions between different energy levels if the hamiltonian changes slowly in time. Here, we show that in fact the response to such a slow change can be non-trivial in low-dimensional gapless systems. We identify three generic regimes of the response: analytic, non-analytic and non-adiabatic, which are characterized by a different behaviour of the heating induced in the system with the ramp rate. In the last regime, the limits of the ramp rate going to zero and the system size going to infinity do not commute and the adiabatic process does not exist in the thermodynamic limit. We support our results with numerical and analytical calculations.

Adiabatic processes play a major role both in physics and technology. The adiabatic process in thermodynamics is formally defined as a process in which no heat is transferred to the system from the environment and thus the entropy is conserved[1]. Typically, such processes occur at timescales that are fast enough compared with the thermalization times with the environment but slow compared with some intrinsic timescales. In isolated systems with no environment, on the other hand, the definition of adiabaticity on the basis of entropy conservation is not useful, as strictly speaking the entropy of any isolated system is conserved because of Liouville's theorem[1]. Instead, the heating induced in the system can be characterized by the increase of its total energy due to the finite rate of change of parameters of the system (or the ramp rate). The adiabatic process is defined as a process in which the heating vanishes. If the process is cyclic, that is, the final and the initial values of the external parameters coincide, then the absence of heating implies that the energy of the systems does not change.

To be specific, assume that some external parameter κ is slowly driven from an initial value κ_A to the final one κ_B with a ramp rate δ. A simple argument can be made for the generic scaling of the energy added to the system due to finite δ. We assume that on the way the system does not undergo any discontinuous phase transitions (although second-order phase transitions are generally allowed). Then, the energy density (or the energy per unit volume) of the system in the final state, $\mathcal{E}_B$, will be a function of this parameter δ and we can expect that for small enough δ that we can expand $\mathcal{E}_B$ into the Taylor series:

$$\mathcal{E}_B(\delta) = \mathcal{E}_B(0) + \beta\delta^2, \qquad (1)$$

where $\mathcal{E}_B(0)$ is the energy of the state adiabatically connected to the initial state and β is a coefficient. In particular, if $\kappa_A = \kappa_B$, that is, if we deal with a cyclic process, then $\mathcal{E}_B(0) = \mathcal{E}_A$. On general grounds, we can argue that the linear term in δ is absent because to satisfy the second law of thermodynamics, the energy can only increase with δ and thus cannot be sensitive to its sign.

For integrable systems, we can define the adiabatic process as a process in which the density of excitations n_{ex} generated during the ramp vanishes with δ. Then, on general grounds we can expect that the analogue of equation (1) will be $n_{ex}(\delta) \approx n_{ex}(0) + \tilde{\beta}\delta^2$.

The adiabatic theorem in thermodynamics is intimately related to the adiabatic theorem in quantum mechanics, which states that under slow enough external perturbations there are no transitions between energy levels and thus there is no work done for any cyclic process. In principle, the quantum mechanical adiabatic theorem implies the thermodynamic adiabatic theorem; however, there are important subtleties involved[2]. In particular, in a generic many-particle system, the distance between energy levels is exponentially small and the transitions between them are unavoidable, unless we are interested in times possibly exceeding the lifetime of our Universe[2], so the adiabatic limit in the absolute sense can never be reached. An example where the quantum adiabatic theorem leads to the thermodynamic one is a system with a gap at zero temperature. Then, the ground state can be excited only through the so-called Landau–Zener mechanism, which gives the exponentially small probability of excitations. However, systems that have a gap are the exception rather than the rule. Indeed, most systems with broken continuous symmetries have gapless excitations over the ground state (Goldstone modes). Thus, solids have phonons (sound waves), ferromagnets and antiferromagnets have gapless magnon or spin-wave excitations, superfluids have gapless Bogoliubov excitations and so on. Even in gapped systems, for example superconductors, we always have a continuum spectrum above the gap, which is occupied by quasiparticles at finite temperatures.

Complete absence of the transitions is however a too restrictive requirement for the adiabaticity. Indeed, in thermodynamics the adiabatic theorem is understood in the sense that the increase in intensive quantities such as density of excited states or entropy scales to zero with δ. Then, we can generally argue that even though the transitions are unavoidable, the available phase space for creating excitations decreases as the ramp speed δ becomes smaller and we can still expect that equation (1) holds. This argument

shows that the quantum mechanical adiabatic theorem suggests the thermodynamic adiabatic theorem but does not prove it because it does not show that the order of limits $\delta \to 0$ and the system size $L \to \infty$ does not matter.

The reasoning leading to equation (1) based on the Taylor expansion is similar in spirit to the mean-field argument suggesting the existence of long-range order in systems with broken continuous symmetries. However, in low dimensions, either quantum[3] or thermal[4] fluctuations can change the picture and destroy the long-range order completely. An ultimate reason for this is that at low dimensions the density of the low-energy states is generically high[1]. The quantum or thermal fluctuations force excitations to occupy these low-energy states and if their density is sufficiently high, they can qualitatively change the nature of the equilibrium state. The same arguments can be applied to the adiabatic process we are interested in. Indeed, at slow ramps some low-energy excitations will always be created. In low dimensions, these excitations can significantly alter the behaviour of the system and in particular invalidate equation (1). In the remainder of the article, we will address this issue in detail.

The main conclusion of our work is that there are three possible generic regimes of a system's response to a slow process: analytic, non-analytic and non-adiabatic. In the analytic regime, we find that indeed the energy of a system and other thermodynamic quantities are analytic functions of the adiabaticity parameter δ and equation (1) is valid. This regime is generically realized in sufficiently high dimensions. In the non-analytic regime, we still have the adiabatic limit in the sense that there is a well-defined limit $\delta \to 0$, but the correction in δ to the energy density or other quantities behaves non-analytically:

$$\mathcal{E}_{\mathrm{B}}(\delta) \approx \mathcal{E}_{\mathrm{B}}(0) + \beta|\delta|^{\nu}, \tag{2}$$

where $\nu < 2$ is some power that depends on the dimensionality and on the universal details of the spectrum. Finally, in the non-adiabatic regime, we have

$$\mathcal{E}_{\mathrm{B}}(\delta) \approx \mathcal{E}_{\mathrm{B}}(0) + \beta|\delta|^{\nu} L^{\eta}, \tag{3}$$

where L is the system size and $\eta > 0$ is another exponent. In this regime, there is no adiabatic process in the thermodynamic limit. In other words, the limits $\delta \to 0$ and $L \to \infty$ do not commute. This is a striking conclusion, which states that no matter how slowly we try to drive the system, if the latter is sufficiently large, we will never reach the adiabatic limit. As we show both analytically and numerically, such a regime is realized in (but not limited to) the situation where we start from an ensemble of non-interacting bosons in one and two dimensions at finite temperature and slowly increase the interaction strength. We point out that the existence of the non-adiabatic regime is consistent with recent theoretical[5,6] and experimental[7] studies suggesting that there is no adiabatic limit in a particular problem of a Bardeen–Cooper–Schrieffer to Bose–Einstein-condensation crossover in cold atoms. The non-analytic regime was found in several papers in the context of crossing a quantum critical point[8–14]. Although in most of the studies this regime was associated with the Kibble–Zurek mechanism[15], we point out that the existence of the quantum phase transition is neither a necessary nor sufficient condition to have this non-analytic behaviour. In particular, in ref. 8, one of us showed that above critical dimensionality, the non-analytic scaling for the number of generated defects breaks down and we are back to the analytic regime. Conversely, we give explicit examples here where we can get the non-analytic regime without any phase transitions involved. We point out that we use the word generic in our classification. For example, we cannot exclude the possibility that in the analytic regime the leading coefficient in front of δ^2 vanishes for some reason and the expansion of $\mathcal{E}_{\mathrm{B}}(\delta)$ starts from a higher power of δ. However, we believe that this can happen only under very special circumstances. We also note that technically the non-adiabatic regime comes from the infrared divergence of the energy coming from a high population of low-energy modes. If this divergence is cut off by the system size, then we get a strong system-size dependence. However, in real systems, this divergence can be cut off by other long-distance scales coming from, for example, weak coupling to the environment, various relaxation processes and so on. In the analytic and non-analytic regimes, there are no infrared divergences and finite-size or relaxation effects are not important.

Apart from many applications in condensed-matter and atomic physics, our findings may be relevant to such diverse fields as quantum optics, quantum computing, in particular for adiabatic quantum computation[16]; inflationary cosmology, which has the phase of adiabatic evolution of gaussian scale-invariant quantum fluctuations in the slow-roll approximation[17]; cosmic microwave background radiation[18,19]; systems related to cold atoms in optical lattices and Bose–Einstein condensates in a trap. In all of these fields, the assumption of adiabaticity in slow processes is usually taken for granted, whereas our results suggest that this might not always be the case. We analyse some of these models in the Supplementary Information.

ANALYTIC TREATMENT

The existence of all three regimes can be illustrated using a simple quadratic hamiltonian:

$$\mathcal{H} = \sum_{q} \frac{\rho_s q^2}{2} |\phi_q|^2 + \frac{1}{2}\kappa_q |\Pi_q|^2, \tag{4}$$

where ϕ_q and Π_q are the coordinate and the conjugate momentum. We note that this hamiltonian describes a very wide class of gapless systems. Thus, in solids ϕ_q represents the phonon or the plasmon field, in ferromagnets and antiferromagnets ϕ_q describes magnons or spin-waves and in superfluids ϕ_q describes Bogoliubov's excitations. This list can easily be extended further. Depending on the system, the couplings ρ_s and κ_q have different meanings. For example, for superfluids ρ_s denotes the superfluid density and κ_q is related to the compressibility. Dependence of κ_q on q also varies for different systems. Thus, $\kappa_q = \mathrm{const}(q)$ in solids, antiferromagnets and superfluids and $\kappa_q \propto q^2$ in ferromagnets and non-interacting Bose systems. The reason the hamiltonian (4) is so generic and applicable to such different situations is that it describes Goldstone modes of systems with a broken continuous symmetry. We further choose $\kappa_q = \kappa + \lambda q^2$. This choice enables us to cover all of the situations mentioned above. In a superfluid, κ stands for the compressibility of the system and we will use this terminology here.

Let us now imagine that we slowly increase κ in time

$$\kappa(t) = \kappa_0 + \delta t, \tag{5}$$

where κ_0 is the initial value of the compressibility. In superfluids, this choice corresponds to increasing interaction strength between particles. Ramps of other parameters can also be considered. Here, we will focus on the positive sign of δ, that is, the situation where compressibility increases in time. It can be shown that the opposite process, where κ decreases, gives similar results up to unimportant numerical prefactors.

The evolution of the wavefunction (or more generally the density matrix) can be explicitly obtained for our non-interacting

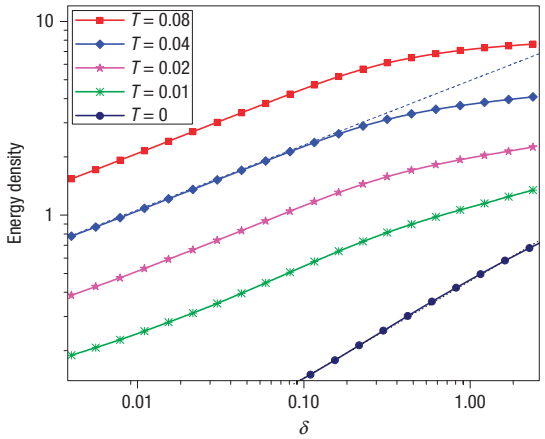

Figure 1 Dependence of the energy density added to a one-dimensional system of linear size $L = 128$ during the ramp on the parameter δ for different initial temperatures. The dashed and dotted lines are the fits to power laws $\Delta\mathcal{E} = A\delta^{1/3}$ and $\Delta\mathcal{E} = B\delta^{1/2}$, respectively. The dotted line is indistinguishable from the zero-temperature curve and the dashed line perfectly agrees with a finite-temperature curve at sufficiently small δ in agreement with predictions of equations (6) and (8). Note also that at small ramps the curves at different temperatures are equidistant, indicating that $\delta\mathcal{E} \propto T$, again in accord with equation (8).

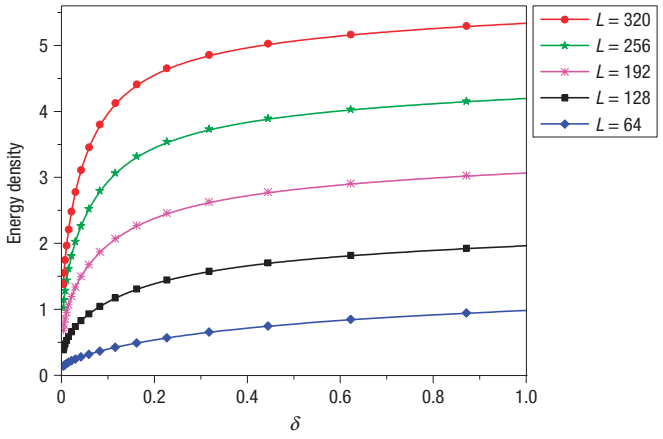

Figure 2 Dependence of $\Delta\mathcal{E}$ on δ for different sizes at fixed temperature $T = 0.02$. As we argued (see equation (8)), there is clearly no thermodynamic limit and the heating becomes more severe with the growth of the system size.

model (see the Methods section). Here, we will outline the main results. Let us first assume that the system is initially prepared in the ground state. Then its wavefunction factorizes to the product of gaussians corresponding to the ground state of a harmonic oscillator for each momentum q (see equation (11)). There are two different regimes of the system behaviour depending on whether the initial compressibility κ_0 is finite or zero with the crossover occurring at $\kappa_0^\star \approx \delta\sqrt{\lambda/\rho_s}$. For large initial compressibility, $\kappa_0 \gg \kappa_0^\star$, the response of the system belongs to the analytic regime in all three spatial dimensions, that is, the energy density behaves as $\mathcal{E} \propto \delta^2$ (with an extra logarithmic correction to this scaling in one dimension: $\Delta\mathcal{E} \propto \delta^2|\ln\delta|$). If $\kappa_0 \ll \kappa_0^\star$, then the non-analytic regime is realized in all three spatial dimensions and the energy density scales as (compare with equation (2))

$$\mathcal{E} \propto \frac{\delta^{(d+1)/4}}{(\rho_s\lambda^3)^{(d+1)/8}}. \tag{6}$$

Similarly, we can analyse the response of the system to the ramp if it is initially prepared at some finite temperature T (see the Methods section and Supplementary Information, Section IV). The results are again strongly sensitive to the initial compressibility. Thus, if κ_0 is large then in one dimension we get

$$\Delta\mathcal{E} = A\frac{|\delta|T\sqrt{\kappa_f}}{\sqrt{\rho_s}\kappa_0^2}. \tag{7}$$

In this case, obviously the non-analytic regime is realized. In two and three dimensions, $\Delta\mathcal{E} \propto \delta^2$ (with logarithmic corrections in two dimensions) and we have the analytic regime. If $\kappa_0 = 0$, then in one and two dimensions the energy density diverges with the system size:

$$\Delta\mathcal{E} = A_d\frac{T\sqrt{\kappa_f}}{\lambda\rho_s^{1/6}}\delta^{1/3}L^{7/3-d} \tag{8}$$

and the non-adiabatic regime is realized (compare with equation (3)). In three dimensions, there is no dependence on the system size and we find the non-analytic regime

$$\Delta\mathcal{E} = A_3\frac{T\sqrt{\kappa_f}}{\rho_s^{1/4}\lambda^{5/4}}\sqrt{\delta}. \tag{9}$$

If the system is non-integrable and is allowed to rethermalize at long times, then the excess in the energy $\Delta\mathcal{E}$ can be recalculated to the excess in entropy ΔS and other thermodynamic quantities using standard thermodynamic relations, for example, $\Delta S \approx (\partial S/\partial\mathcal{E})\Delta\mathcal{E}$.

NUMERICAL RESULTS: APPLICATION TO INTERACTING BOSONS

Although the previous analysis is formally exact, it directly applies only to a special case of an integrable harmonic system. Most real systems are non-integrable. However, at low energies, the harmonic approximation is often very good.

To study effects of non-integrability, we carry out numerical simulations using the Bose–Hubbard model on a square lattice described by the hamiltonian[3]:

$$\mathcal{H}_{bh} = -J\sum_{\langle ij\rangle}(a_i^\dagger a_j + a_j^\dagger a_i) + \frac{U(t)}{2}\sum_j a_j^\dagger a_j(a_j^\dagger a_j - 1), \tag{10}$$

where a_j and $a_j^\dagger$ are the bosonic annihilation and creation operators, J represents the tunnelling matrix element and U is the interaction strength. The sum in the first term is taken over the nearest-neighbour pairs. We take J to be time independent and the interaction to be increasing in time according to $U(t) = U_0\tanh(\delta t)$. For small enough interactions, $U_0 \ll Jn_0$, where n_0 is the mean number of atoms per lattice site, in the quadratic approximation the hamiltonian (10) maps to the Bogoliubov hamiltonian (4) with $\rho_s \approx 2Jn_0$, $\kappa \approx U$ and $\lambda \approx \rho_s/2n_0^2$. We emphasize that the Bose–Hubbard model is non-integrable in all dimensions as long as both J and U are finite. Therefore, after the ramp we expect that the system always relaxes to thermal equilibrium. To simulate dynamics of the system we use the semiclassical approach developed by one of us[20] (see Supplementary Information, Section V for details).

To avoid potential complications related to strong quantum effects, we choose the parameters of the system deep in the superfluid regime throughout the entire evolution:

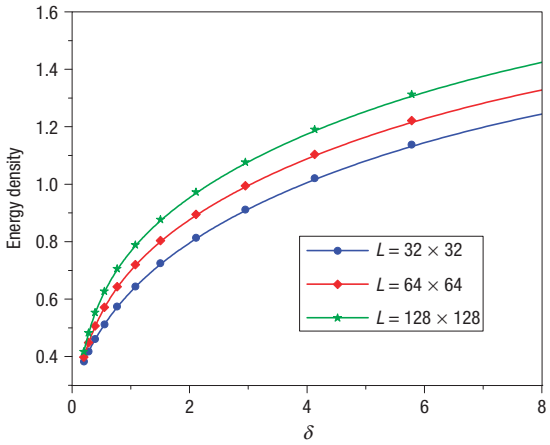

Figure 3 Dependence of $\Delta\mathcal{E}$ on δ in a two-dimensional system for two different sizes. Here, $T = 0.2$ and the other parameters are the same as in Fig. 2. The asymptotical behaviour of $\Delta\mathcal{E}$ at small δ agrees with the $\delta^{1/3}$ dependence predicted by equation (8). In addition, $\Delta\mathcal{E}$ slowly increases with the system size consistent with the analytic prediction $\Delta\mathcal{E} \propto L^{1/3}$.

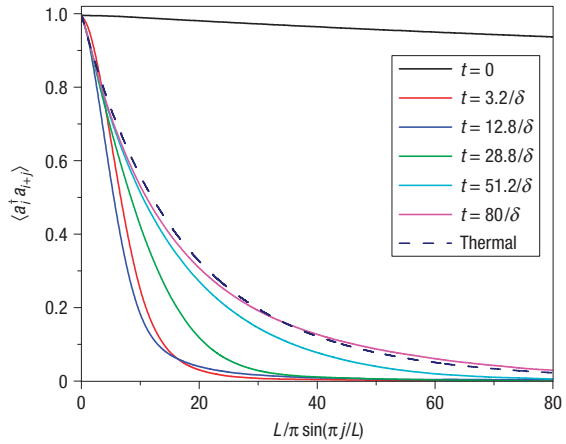

Figure 4 Correlation function $\langle a_i^\dagger a_{i+j}\rangle$ as a function of scaled distance $L/\pi\sin(\pi j/L)$ at different moments of time. Here, $\delta = 0.1$, $L = 256$ and the other parameters are the same as in Fig. 2. The dashed line is the equilibrium thermal result.

$n_0 = 20, J = 1, U_0 = 0.25$ so that the semiclassical parameter[21] $U_0/Jn_0 \sim 10^{-2}$ is very small. At the same time, the interaction energy per particle $U_0 n_0$ is larger than the typical kinetic energy J, implying that at long times $U(t) \approx U_0$ the system is strongly interacting. According to our theoretical expectations, the system is mostly excited while the healing length remains large, that is, when $U \ll U_0$. In this regime, $U(t) = U_0 \tanh \delta t \approx U_0 \delta t$ linearly increases with t; thus, we can directly compare numerical results with the analytical predictions of the previous section.

Figure 1 shows the dependence of the energy per site added to the one-dimensional system during the ramp as a function of the parameter δ for different temperatures T. Note that even at very low temperatures $T \sim 0.01J$ the behaviour of $\Delta\mathcal{E}$ is dominated by the thermal effects. This trend becomes even more apparent if we analyse the dependence of $\Delta\mathcal{E}$ on L (Fig. 2). In agreement with the analytic results, (8) $\Delta\mathcal{E}$ strongly grows with the system size and clearly there is no thermodynamic adiabatic limit. We checked that, as predicted by equation (6) at zero temperature, $\Delta\mathcal{E}$ is almost insensitive to L.

We also carried out numerical simulations in a two-dimensional system (Fig. 3). We used identical parameters to those in the one-dimensional case, except much higher temperature $T = 0.2$ and slightly smaller linear sizes. The reason we had to choose higher T is that at a given temperature thermal effects in two dimensions dominate the system behaviour at much larger linear sizes than in one dimension. We find that the results are again in good agreement with predictions of equation (8). In particular, we find that $\Delta\mathcal{E} \propto \delta^{1/3}$ at small δ and that $\Delta\mathcal{E}$ slowly grows with the system size consistent with $\Delta\mathcal{E} \propto L^{1/3}$.

To emphasize the non-integrability of the Bose–Hubbard model in Fig. 4, we plot correlation functions $\langle a_i^\dagger a_{i+j}\rangle$ at different moments of time. We use $\delta = 0.1$ and the other parameters are the same as in Fig. 1. Note that for $t \gtrsim 3/\delta$, the interaction essentially does not change in time and the energy is constant. Yet from the graph, it is obvious that correlation functions continue to evolve in time. At relatively short times, $t \lesssim 10/\delta$, the correlation functions decay very rapidly with distance in agreement with what we would obtain from solving the non-interacting harmonic problem (see Supplementary Information, Section V). At longer times, the shape of the correlation function saturates and agrees well with the thermal distribution (dashed line). We emphasize that the temperature $T = 4.8$ used to obtain the thermal result was not a fitting parameter, it was extracted from the total energy of the system shown in Fig. 1. The small deviation between the steady state and thermal distributions is probably attributed to using a harmonic approximation to find the equilibrium curve. Note that the steady-state temperature here, $T_f \sim 5$, is relatively high (compared with the Josephson energy $\Omega_J = \sqrt{2UJn_0} \sim 3$), yet the thermalization time is quite long, $\tau \sim 10^3$, which is very typical for Goldstone modes. As we emphasized earlier, the thermalization occurs at a fixed energy and does not affect our conclusions. This example also highlights the importance of using the energy dependence on δ and L for the classification of the response to the ramp.

SUMMARY AND OUTLOOK

Here, we analysed the response of low-dimensional gapless systems to slow ramps. We argued that in general there exist three possible regimes. The first, analytic, regime is mean-field like. There, we can apply simple arguments based on the analytic dependence of various thermodynamic observables on the ramp speed δ. This regime is typically realized at high dimensions. In the second, non-analytic, regime the energy, entropy and other quantities depend on δ in a non-analytic way: $\mathcal{E}_B(\delta) \approx \mathcal{E}_B(0) + \beta|\delta|^\nu$, where $\nu < 2$. The exponent ν depends on the universal critical exponents characterizing the gapless phase. Finally, in low dimensions, we can have a non-adiabatic regime. In this regime, the limits of $\delta \to 0$ and the system size $L \to \infty$ do not commute: $\mathcal{E}_B(\delta) \approx \mathcal{E}_B(0) + \beta|\delta|^\nu L^\eta$.

We showed how all of these three regimes can arise by analysing simple harmonic systems, which are described by quadratic phonon-like excitations. Despite this particular choice, we point out again that our hamiltonian (4) describes a wide class of phenomena such as phonons in solids, magnons in ferro- and antiferromagnets, Bogoliubov excitations in superfluids and so on. In addition, we carried out a careful numerical analysis of the interacting non-integrable Bose–Hubbard model and showed that our conclusions are robust and at least weak deviations from integrability do not alter them. We also analysed a number of other

interacting models and again obtained excellent agreement with the above classification (see the Supplementary Information).

An obvious outcome of our analysis is that it is necessary to be very careful when making statements about adiabaticity in low-dimensional systems. This can be important in many situations ranging from realizing proposals on adiabatic quantum computation and preparing interacting systems in a given state through slow ramps, to inflationary cosmology and black hole radiation. Perhaps cold atoms are the systems in which our results can be immediately tested in experiments. There, we have all of the necessary experimental tools such as isolation from the environment together with high tunability of parameters of the system and the possibility to carry out tuning in real time[22]. In addition, the effects of non-adiabaticity will probably show up in preparation of various strongly correlated low-dimensional states, especially those involving spin-exchange interactions. Most of the current theoretical and experimental proposals are based on the possibility to first cool atoms and then adiabatically tune the system to the desired regime. The coherent dynamic also becomes increasingly important in other fields such as nanophysics, quantum optics, cavity quantum electrodynamics and so on; we expect that our results should be relevant to many potential applications.

METHODS

ZERO TEMPERATURE

The hamiltonian (4) is quadratic and thus the time evolution can be found exactly. First, we look at the zero-temperature case. For quadratic hamiltonians, it is well known that if the initial wavefunction is gaussian, it will remain gaussian for an arbitrary time dependence of the parameters of the hamiltonian. The ground-state wavefunction is given by

$$\Psi(\{\phi_q\}) = \prod_q \frac{1}{(2\pi\sigma_{0,q})^{1/4}} \exp\left[-\frac{|\phi_q|^2}{4\sigma_{0,q}}\right], \tag{11}$$

where $\sigma_{0,q} = 1/(2q)\sqrt{\kappa_{0,q}/\rho_s}$. If κ changes with time, σ_q acquires time dependence:

$$i\frac{\mathrm{d}\sigma_q}{\mathrm{d}t} = 2\rho_s q^2\sigma_q^2 - \frac{1}{2}\kappa_q(t). \tag{12}$$

This is a Riccati equation, which can be explicitly solved through Airy functions. We give details of this solution in Supplementary Information, Section III.

It is straightforward to check that the number of excitations per mode q is related to σ_q through:

$$n_q = \frac{1}{2}\left[\frac{\sigma_q^{\mathrm{eff}}}{\sigma_q^{\mathrm{eq}}} - 1\right], \tag{13}$$

where $\sigma_q^{\mathrm{eff}} = 1/\Re(\sigma_q^{-1})$ and σ_q^{eq} is the equilibrium (ground state) value of σ in the final state. The asymptotical expressions for n_q in the limit of large and small q can be easily found from equation (13) and the solution of equation (12), in particular:

$$n_q \approx \frac{\delta^2}{64q^2\rho_s\kappa_{0,q}^3} \tag{14}$$

at $q\sqrt{\rho_s\kappa_{0,q}^3} \gg \delta$ and

$$n_q \approx \frac{\pi}{3^{2/3}\Gamma^2(1/3)} \frac{\delta^{1/3}}{q^{1/3}(\rho_s\kappa_{0,q}^3)^{1/6}} \tag{15}$$

in the opposite limit, where $\Gamma(x)$ stands for the Γ-function.

Once we know the complete wavefunction we can, in principle, find arbitrary observables such as various correlation functions, response to external probes, density of excitations and so on. For example, the energy density of the system is

$$\mathcal{E} \approx \frac{\sqrt{\rho_s\kappa_f}}{2} \int \frac{\mathrm{d}^d q}{(2\pi)^d} q\left(\frac{\sigma_q^{\mathrm{eff}}}{\sigma_q^{\mathrm{eq}}} - 1\right), \tag{16}$$

where κ_f is the final compressibility. The integral above converges in all dimensions leading to equation (6).

FINITE-TEMPERATURE CASE

We can generalize the previous results to the situation where the system is initially prepared at some finite temperature T. Instead of the wavefunction, we have to deal with the density matrix, but essentially the derivation is similar to the zero-temperature case. We present the details of the analysis in Supplementary Information, Section IV. The result of these calculations is very simple: $\sigma_q^{\mathrm{eff}}|_T = r_q\sigma_q^{\mathrm{eff}}|_{T=0}$, where $r_q = \coth[q\sqrt{\kappa_{0,q}\rho_s}/(2T)]$. In other words, we need to take the zero-temperature asymptotics for the width of the wavefunction and multiply them by r_q. At zero temperature, $r_q \equiv 1$ and we obviously reproduce the previous results. At high temperatures, $T \gg q\sqrt{\rho_s\kappa_{0,q}}$, we have $r_q \approx 2T/(q\sqrt{\rho_s\kappa_{0,q}})$ and thus σ_q^{eff} diverges at $q \to 0$ much faster than in the zero-temperature case. Let us observe that the number of excitations of a q mode is

$$n_q = \frac{1}{2}\left[\frac{\sigma_q^{\mathrm{eff}}}{\sigma_q^{\mathrm{eq}}} - 1\right] = r_q\, n_q\big|_{T=0} + \frac{1}{2}(r_q - 1). \tag{17}$$

The last term is the initial equilibrium occupation of the q mode. Thus, we see that by multiplying the zero temperature result for n_q by r_q we obtain the number of extra excitations in the q mode Δn_q generated during the ramp. Using equations (14), (15) and (17), we can easily compute the energy density (equations (7)–(9)) and other thermodynamic quantities at finite temperature.

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Author contributions

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