EUF

Exame Unificado das Pós-graduações em Física

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FORMULÁRIO

□

Constantes físicas

Velocidade da luz no vácuo	$c = 3{,}00\times10^{8}$ m/s
Constante de Planck	$h = 6{,}63\times10^{-34}$ J s $= 4{,}14\times10^{-15}$ eV s
	$\hbar = h/2\pi = 1{,}06\times10^{-34}$ J s $= 6{,}58\times10^{-16}$ eV s
	$hc \simeq 1240$ eV nm $= 1240$ MeV fm
	$\hbar c \simeq 200$ eV nm $= 200$ MeV fm
Constante de Wien	$W = 2{,}898\times10^{-3}$ m K
Permeabilidade magnética do vácuo	$\mu_0 = 4\pi\times10^{-7}$ N/A^2 $= 12{,}6\times10^{-7}$ N/A^2
Permissividade elétrica do vácuo	$\epsilon_0 = \dfrac{1}{\mu_0 c^2} = 8{,}85\times10^{-12}$ F/m
	$\dfrac{1}{4\pi\epsilon_0} = 8{,}99\times10^{9}$ Nm^2/C^2
Constante gravitacional	$G = 6{,}67\times10^{-11}$ N m^2/kg^2
Carga elementar	$e = 1{,}60\times10^{-19}$ C
Massa do elétron	$m_\mathrm{e} = 9{,}11\times10^{-31}$ kg $= 511$ keV/c^2
Comprimento de onda Compton	$\lambda_\mathrm{C} = 2{,}43\times10^{-12}$ m
Massa do próton	$m_\mathrm{p} = 1{,}673\times10^{-27}$ kg $= 938$ MeV/c^2
Massa do nêutron	$m_\mathrm{n} = 1{,}675\times10^{-27}$ kg $= 940$ MeV/c^2
Massa do dêuteron	$m_\mathrm{d} = 3{,}344\times10^{-27}$ kg $= 1876$ MeV/c^2
Massa da partícula α	$m_\alpha = 6{,}645\times10^{-27}$ kg $= 3727$ MeV/c^2
Constante de Rydberg	$R_H = 1{,}10\times10^{7}$ m^{-1} , $R_H hc = 13{,}6$ eV
Raio de Bohr	$a_0 = 5{,}29\times10^{-11}$ m
Constante de Avogadro	$N_\mathrm{A} = 6{,}02\times10^{23}$ mol^{-1}
Constante de Boltzmann	$k_\mathrm{B} = 1{,}38\times10^{-23}$ J/K $= 8{,}62\times10^{-5}$ eV/K
Constante universal dos gases	$R = 8{,}31$ J mol^{-1} K^{-1}
Constante de Stefan-Boltzmann	$\sigma = 5{,}67\times10^{-8}$ W m^{-2} K^{-4}

Raio do Sol	=	$6{,}96\times10^{8}$ m	Massa do Sol	=	$1{,}99\times10^{30}$ kg
Raio da Terra	=	$6{,}37\times10^{6}$ m	Massa da Terra	=	$5{,}98\times10^{24}$ kg
Distância Sol-Terra	=	$1{,}50\times10^{11}$ m			

1 J $= 10^{7}$ erg   1 eV $= 1{,}60\times10^{-19}$ J   1 Å $= 10^{-10}$ m   1 fm $= 10^{-15}$ m

Constantes numéricas

$\pi \cong 3{,}142$	$\ln 2 \cong 0{,}693$	$\cos(30°) = \sin(60°) = \sqrt{3}/2 \cong 0{,}866$
$\mathrm{e} \cong 2{,}718$	$\ln 3 \cong 1{,}099$	$\sin(30°) = \cos(60°) = 1/2$
$1/\mathrm{e} \cong 0{,}368$	$\ln 5 \cong 1{,}609$	$\mathrm{e}^2 \cong 7{,}39$ $\mathrm{e}^3 \cong 20{,}1$ $\mathrm{e}^4 \cong 54{,}6$
$\log_{10}\mathrm{e} \cong 0{,}434$	$\ln 10 \cong 2{,}303$	$\mathrm{e}^5 \cong 148$ $\mathrm{e}^6 \cong 403$

Mecânica Clássica

$$\mathbf{L} = m\mathbf{r}\times\mathbf{v} = \mathbf{r}\times\mathbf{p} \qquad \mathbf{N} = \mathbf{r}\times\mathbf{F} = \frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t}$$

$$\mathbf{r} = r\hat{\mathrm{e}}_r \qquad \mathbf{v} = \dot{r}\hat{\mathrm{e}}_r + r\dot{\theta}\hat{\mathrm{e}}_\theta \qquad \mathbf{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\hat{\mathrm{e}}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\mathrm{e}}_\theta$$

$$\mathbf{r} = \rho\hat{\mathrm{e}}_\rho + z\hat{\mathrm{e}}_z \qquad \mathbf{v} = \dot{\rho}\hat{\mathrm{e}}_\rho + \rho\dot{\varphi}\hat{\mathrm{e}}_\varphi + \dot{z}\hat{\mathrm{e}}_z \qquad \mathbf{a} = \left(\ddot{\rho} - \rho\dot{\varphi}^2\right)\hat{\mathrm{e}}_\rho + \left(\rho\ddot{\varphi} + 2\dot{\rho}\dot{\varphi}\right)\hat{\mathrm{e}}_\varphi + \ddot{z}\hat{\mathrm{e}}_z$$

$$\mathbf{r} = r\hat{\mathrm{e}}_r \qquad \begin{aligned}\mathbf{v} = {}& \dot{r}\hat{\mathrm{e}}_r + r\dot{\theta}\hat{\mathrm{e}}_\theta \\ & + r\dot{\varphi}\sin\theta\hat{\mathrm{e}}_\varphi\end{aligned} \qquad \begin{aligned}\mathbf{a} = {}& \left(\ddot{r} - r\dot{\theta}^2 - r\dot{\varphi}^2\sin^2\theta\right)\hat{\mathrm{e}}_r \\ & + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta} - r\dot{\varphi}^2\sin\theta\cos\theta\right)\hat{\mathrm{e}}_\theta \\ & + \left(r\ddot{\varphi}\sin\theta + 2\dot{r}\dot{\varphi}\sin\theta + 2r\dot{\theta}\dot{\varphi}\cos\theta\right)\hat{\mathrm{e}}_\varphi\end{aligned}$$

$$\frac{\mathrm{d}^2u}{\mathrm{d}\theta^2} + u = -\frac{m}{L^2u^2}F\left(1/u\right), \quad u = \frac{1}{r}; \qquad \left(\frac{\mathrm{d}u}{\mathrm{d}\theta}\right)^2 + u^2 = \frac{2m}{L^2}\left[E - V(1/u)\right]$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial L}{\partial\dot{q}_k}\right) - \frac{\partial L}{\partial q_k} = 0, \quad L = T - V \qquad \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial\dot{q}_k}\right) - \frac{\partial T}{\partial q_k} = Q_k \qquad p_k = \frac{\partial L}{\partial\dot{q}_k}$$

$$Q_k = \sum_{i=1}^{N} F_{ix}\frac{\partial x_i}{\partial q_k} + F_{iy}\frac{\partial y_i}{\partial q_k} + F_{iz}\frac{\partial z_i}{\partial q_k} \qquad Q_k = -\frac{\partial V}{\partial q_k}$$

$$\left(\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2}\right)_{\text{fixo}} = \left(\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2}\right)_{\text{rotação}} + 2\boldsymbol{\omega}\times\left(\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}\right)_{\text{rotação}} + \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) + \dot{\boldsymbol{\omega}}\times\mathbf{r}$$

$$H = \sum_{k=1}^{f} p_k\dot{q}_k - L; \qquad \dot{q}_k = \frac{\partial H}{\partial p_k}; \qquad \dot{p}_k = -\frac{\partial H}{\partial q_k}; \qquad \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}$$

$$E = T + V = \frac{1}{2}m|\mathbf{v}|^2 + V(\mathbf{r}) \qquad \mathbf{F} = -\nabla V(\mathbf{r})$$

$$E = \frac{1}{2}\left(m_1|\mathbf{v}_1|^2 + m_2|\mathbf{v}_2|^2\right) + V(|\mathbf{r}_1 - \mathbf{r}_2|) = \frac{M}{2}|\dot{\mathbf{R}}_{\text{cm}}|^2 + \frac{\mu}{2}|\dot{\mathbf{r}}|^2 + V(r)$$

$$= \frac{M}{2}|\dot{\mathbf{R}}_{\text{cm}}|^2 + \frac{1}{2}\mu\left(\dot{r}^2 + r^2\dot{\theta}^2\right) + V(r) = \frac{M}{2}|\dot{\mathbf{R}}_{\text{cm}}|^2 + \frac{1}{2}\mu\dot{r}^2 + \frac{l^2}{2mr^2} + V(r)$$

$$M = m_1 + m_2 \qquad \mu = \frac{m_1m_2}{m_1 + m_2} \qquad \mathbf{R}_{\text{cm}} = \frac{m_1\mathbf{r}_1 + m_2\mathbf{r}_2}{M} \qquad \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$$

$$\mathbf{L} = m_1\mathbf{r}_1\times\mathbf{v}_1 + m_2\mathbf{r}_2\times\mathbf{v}_2 = M\mathbf{R}_{\text{cm}}\times\dot{\mathbf{R}}_{\text{cm}} + \mu\mathbf{r}\times\dot{\mathbf{r}} = M\mathbf{R}_{\text{cm}}\times\dot{\mathbf{R}}_{\text{cm}} + \mathbf{l}$$

$$\mathbf{l} = \mu\mathbf{r}\times\dot{\mathbf{r}} \qquad |\mathbf{l}| = l = \mu r^2\dot{\theta}$$

Eletromagnetismo

$$\nabla\cdot\mathbf{E} = \rho/\epsilon_0 \qquad \nabla\cdot\mathbf{D} = \rho_F$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad \nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\cdot\mathbf{B} = 0 \qquad \nabla\cdot\mathbf{B} = 0$$

$$\nabla\times\mathbf{B} = \mu_0\mathbf{J} + \mu_0\epsilon_0\frac{\partial\mathbf{E}}{\partial t} \qquad \nabla\times\mathbf{H} = \mathbf{J}_F + \frac{\partial\mathbf{D}}{\partial t}$$

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v}\times\mathbf{B}) \qquad d\mathbf{F} = I d\mathbf{l}\times\mathbf{B}$$

$$\nabla\cdot\mathbf{P} = -\rho_P \qquad \mathbf{P}\cdot\hat{\mathbf{n}} = \sigma_P \qquad \rho = \rho_F + \rho_P$$

$$\nabla\times\mathbf{M} = \mathbf{J}_M \qquad \mathbf{M}\times\hat{\mathbf{n}} = \mathbf{K}_M \qquad \mathbf{J} = \mathbf{J}_F + \mathbf{J}_M + \frac{\partial\mathbf{P}}{\partial t}$$

$$\mathbf{D} = \epsilon_0\mathbf{E} + \mathbf{P} = \epsilon\mathbf{E} \qquad \mathbf{J}_F = \sigma\mathbf{E}$$

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu\mathbf{H}$$

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0}\frac{\hat{\mathrm{e}}_r}{r^2} \qquad V = \frac{q}{4\pi\epsilon_0}\frac{1}{r} \qquad \mathbf{E} = -\nabla V \qquad V = -\int \mathbf{E}\cdot d\mathbf{l}$$

$$\mathbf{F}_{2\to 1} = \frac{q_1 q_2}{4\pi\epsilon_0}\frac{(\mathbf{r}_1 - \mathbf{r}_2)}{|\mathbf{r}_1 - \mathbf{r}_2|^3} \qquad U_{12} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{|\mathbf{r}_1 - \mathbf{r}_2|}$$

$$V(r,\theta) = \sum_{l=0}^{\infty}\left[A_l r^l + \frac{B_l}{r^{(l+1)}}\right]P_l(\cos\theta) \qquad \mathbf{B}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{r}')\times(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}dV'$$

$$\mathbf{B} = \nabla\times\mathbf{A} \qquad \mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int\frac{\mathbf{J}(\mathbf{r}')dV'}{|\mathbf{r} - \mathbf{r}'|}$$

$$u = \frac{\epsilon_0}{2}\mathbf{E}\cdot\mathbf{E} + \frac{1}{2\mu_0}\mathbf{B}\cdot\mathbf{B} \qquad \mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B} \qquad P_{rad} = \frac{\langle S\rangle}{c} = \frac{I}{c}$$

$$E = \frac{Q^2}{2C} = \frac{CV^2}{2} \qquad Q = CV$$

Relatividade

$$\gamma = \frac{1}{\sqrt{1 - V^2/c^2}} \qquad x' = \gamma\,(x - Vt) \qquad t' = \gamma\left(t - Vx/c^2\right)$$

$$v'_x = \frac{v_x - V}{1 - Vv_x/c^2} \qquad v'_y = \frac{v_y}{\gamma\,(1 - Vv_x/c^2)} \qquad v'_z = \frac{v_z}{\gamma\,(1 - Vv_x/c^2)}$$

$$E = \gamma m_0 c^2 \qquad \mathbf{p} = \gamma m_0\mathbf{V} \qquad E = \sqrt{(pc)^2 + (m_0 c^2)^2}$$

Mecânica Quântica

$$i\hbar\,\frac{\partial\Psi(x,t)}{\partial t} = \frac{-\hbar^2}{2m}\frac{\partial^2\Psi(x,t)}{\partial x^2} + V(x)\Psi(x,t) \qquad \frac{-\hbar^2}{2m}\frac{\partial^2\psi(x)}{\partial x^2} + V(x)\psi(x) = E\psi(x)$$

$$p_x = \frac{\hbar}{i}\frac{\partial}{\partial x} \qquad [x, p_x] = i\hbar$$

$$\hat{a} = \sqrt{\frac{m\omega}{2\hbar}}\left(\hat{x} + i\frac{\hat{p}}{m\omega}\right) \qquad \hat{a}|n\rangle = \sqrt{n}|n-1\rangle\ , \qquad \hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

$$L_\pm = L_x \pm iL_y \qquad L_\pm Y_{\ell m}(\theta,\varphi) = \hbar\sqrt{l(l+1) - m(m\pm 1)}\ Y_{\ell m\pm 1}(\theta,\varphi)$$

$$L_z = x\,p_y - y\,p_x \qquad L_z = \frac{\hbar}{i}\frac{\partial}{\partial\varphi}\ , \qquad [L_x,L_y] = i\hbar L_z$$

$$E_n^{(1)} = \langle n|\delta H|n\rangle \qquad E_n^{(2)} = \sum_{m\neq n}\frac{|\langle m|\delta H|n\rangle|^2}{E_n^{(0)} - E_m^{(0)}}\ , \qquad \phi_n^{(1)} = \sum_{m\neq n}\frac{\langle m|\delta H|n\rangle}{E_n^{(0)} - E_m^{(0)}}\phi_m^{(0)}$$

$$\hat{\mathbf{S}} = \frac{\hbar}{2}\vec{\sigma} \qquad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\ ,\quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\ ,\quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\bar{\psi}(\vec{p}) = \frac{1}{(2\pi\hbar)^{3/2}}\int d^3r\, e^{-i\vec{p}\cdot\vec{r}/\hbar}\,\psi(\vec{r}) \qquad \psi(\vec{r}) = \frac{1}{(2\pi\hbar)^{3/2}}\int d^3p\, e^{i\vec{p}\cdot\vec{r}/\hbar}\,\bar{\psi}(\vec{p})$$

$$e^{\hat{A}} \equiv \sum_{n=0}^{+\infty}\frac{\hat{A}^n}{n!}$$

Física Moderna

$$p = \frac{h}{\lambda} \qquad E = h\nu = \frac{hc}{\lambda} \qquad E_n = -\frac{Z^2}{n^2}\,\frac{m_e e^4}{(4\pi\epsilon_0)^2 2\hbar^2} = -\frac{Z^2}{n^2}\,hcR_H = -Z^2\,\frac{13{,}6}{n^2}\mathrm{eV}$$

$$L = mvr = n\hbar \qquad a_0 = \frac{4\pi\epsilon_0\hbar^2}{m_e e^2} \qquad R_T = \sigma T^4 \qquad \lambda_{\max}T = W$$

$$\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\theta) \quad n\lambda = 2d\sin\theta_n \quad \Delta x\ \Delta p \geq \hbar/2 \qquad \Delta E\ \Delta t \geq \hbar/2$$

Termodinâmica e Mecânica Estatística

$$dU = dQ - dW \qquad dU = TdS - pdV + \mu dN$$

$$dF = -SdT - pdV + \mu dN \qquad dH = TdS + Vdp + \mu dN$$

$$dG = -SdT + Vdp + \mu dN \qquad d\Phi = -SdT - pdV - Nd\mu$$

$$F = U - TS \qquad G = F + pV$$

$$H = U + pV \qquad \Phi = F - \mu N$$

$$\left(\frac{\partial T}{\partial V}\right)_{S,N} = -\left(\frac{\partial p}{\partial S}\right)_{V,N} \qquad \left(\frac{\partial S}{\partial V}\right)_{T,N} = \left(\frac{\partial p}{\partial T}\right)_{V,N}$$

$$\left(\frac{\partial T}{\partial p}\right)_{S,N} = \left(\frac{\partial V}{\partial S}\right)_{p,N} \qquad \left(\frac{\partial S}{\partial p}\right)_{T,N} = -\left(\frac{\partial V}{\partial T}\right)_{p,N}$$

$$p = -\left(\frac{\partial F}{\partial V}\right)_{T,N} \qquad S = -\left(\frac{\partial F}{\partial T}\right)_{V,N} \qquad M = -\left(\frac{\partial F}{\partial h}\right)_{T,V,N}$$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_{V,N} = T\left(\frac{\partial S}{\partial T}\right)_{V,N} \qquad C_p = \left(\frac{\partial H}{\partial T}\right)_{p,N} = T\left(\frac{\partial S}{\partial T}\right)_{p,N}$$

Gás ideal: $pV = nRT$, $U = C_V T = nc_V T$,
Processo adiabático: $pV^\gamma = \text{const.}$, $\gamma = c_p/c_V = (c_V + R)/c_V$

$$S = k_B \ln[\Omega(E,N,V)] \qquad \frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{N,V}$$

$$Z_N(\beta,V) = \int \frac{\prod_i d^3p_i d^3r_i}{h^{3N}N!} e^{-\beta H[\{\mathbf{p}_i,\mathbf{r}_i\}]} \qquad Z_N(\beta,V) = \sum_n e^{-\beta E_n} \qquad (\beta = 1/k_B T)$$

$$F = -\frac{1}{\beta}\ln Z_N(\beta,V) \qquad U = -\left[\frac{\partial \ln Z_N(\beta,V)}{\partial \beta}\right]_{V,N}$$

$$\Xi(\beta,V,z) = \sum_{N=0}^{\infty} Z_N(\beta,V) z^N \ (z = e^{\beta\mu}) \quad \Phi = -\frac{1}{\beta}\ln \Xi(\beta,V,z) \quad \langle N\rangle = z\left[\frac{\partial \ln \Xi(\beta,V,z)}{\partial z}\right]_{\beta,V}$$

$$U = -\left[\frac{\partial \ln \Xi(\beta,V,z)}{\partial \beta}\right]_{V,z} \qquad f_{\mathrm{FD}} = \frac{1}{e^{\beta(\epsilon-\mu)}+1} \qquad f_{\mathrm{BE}} = \frac{1}{e^{\beta(\epsilon-\mu)}-1}$$

Resultados matemáticos

$$\int_{-\infty}^{\infty} x^{2n} \mathrm{e}^{-ax^2} \mathrm{d}x = \frac{1.3.5...(2n{+}1)}{(2n{+}1)2^n a^n} \left(\frac{\pi}{a}\right)^{\frac{1}{2}} \quad (n = 0,1,2,\ldots)$$

$$\sum_{k=0}^{\infty} x^k = \frac{1}{1-x} \quad (|x| < 1) \qquad\qquad \mathrm{e}^{i\theta} = \cos\theta + i\sin\theta$$

$$\int \frac{\mathrm{d}x}{\left(a^2+x^2\right)^{1/2}} = \ln\left(x + \sqrt{x^2+a^2}\right) \qquad\qquad \ln N! \cong N\ln N - N$$

$$\int \frac{\mathrm{d}x}{\left(a^2+x^2\right)^{3/2}} = \frac{x}{\left(a^2\sqrt{x^2+a^2}\right)} \qquad\qquad \int \frac{x^2\mathrm{d}x}{\left(a^2+x^2\right)^{3/2}} = \ln\left(x+\sqrt{x^2+a^2}\right) - \frac{x}{\sqrt{x^2+a^2}}$$

$$\int \frac{\mathrm{d}x}{1-x^2} = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right) \qquad\qquad \int \frac{\mathrm{d}x}{x(x-1)} = \ln(1-1/x)$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\,\arctan\frac{x}{a} \qquad\qquad \int \frac{x}{a^2+x^2}\,dx = \frac{1}{2}\,\ln(a^2+x^2)$$

$$\int_0^{\infty} \frac{z^{x-1}}{e^z+1}\,dz = \left(1-2^{1-x}\right)\Gamma(x)\,\zeta(x) \qquad (x>0)$$

$$\int_0^{\infty} \frac{z^{x-1}}{e^z-1}\,dz = \Gamma(x)\,\zeta(x) \qquad (x>1)$$

$$\Gamma(2) = 1 \qquad \Gamma(3) = 2 \qquad \Gamma(4) = 6 \qquad \Gamma(5) = 24 \qquad \Gamma(n) = (n-1)!$$

$$\zeta(2) = \frac{\pi^2}{6} \cong 1{,}645 \qquad \zeta(3) \cong 1{,}202 \qquad \zeta(4) = \frac{\pi^4}{90} \cong 1{,}082 \qquad \zeta(5) \cong 1{,}037$$

$$\int_{-\pi}^{\pi} \sin(mx)\ \sin(nx)\ dx = \pi\delta_{m,n} \qquad\qquad \int_{-\pi}^{\pi} \cos(mx)\ \cos(nx)\ dx = \pi\delta_{m,n}$$

$$\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \rho\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}z \qquad\qquad \mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = r^2\mathrm{d}r\sin\theta\,\mathrm{d}\theta\,\mathrm{d}\phi$$

$$Y_{0,0} = \sqrt{\frac{1}{4\pi}} \qquad\qquad Y_{1,0} = \sqrt{\frac{3}{4\pi}}\cos\theta \qquad\qquad Y_{1,\pm1} = \mp\sqrt{\frac{3}{8\pi}}\sin\theta\mathrm{e}^{\pm i\phi}$$

$$Y_{2,0} = \sqrt{\frac{5}{16\pi}}\left(3\cos^2\theta - 1\right) \qquad Y_{2,\pm1} = \mp\sqrt{\frac{15}{8\pi}}\sin\theta\cos\theta\mathrm{e}^{\pm i\phi} \qquad Y_{2,\pm2} = \mp\sqrt{\frac{15}{32\pi}}\sin^2\theta\mathrm{e}^{\pm 2i\phi}$$

$$P_0(x) = 1 \qquad\qquad P_1(x) = x \qquad\qquad P_2(x) = (3x^2-1)/2$$

$$\nabla\cdot(\nabla\times\mathbf{V}) = 0 \qquad\qquad \nabla\times\nabla f = 0$$

$$\nabla\times(\nabla\times\mathbf{V}) = \nabla(\nabla\cdot\mathbf{V}) - \nabla^2\mathbf{V}$$

$$\oint \mathbf{A}\cdot\mathrm{d}\mathbf{S} = \int (\nabla\cdot\mathbf{A})\,\mathrm{d}V \qquad\qquad \oint \mathbf{A}\cdot\mathrm{d}\mathbf{l} = \int (\nabla\times\mathbf{A})\cdot\mathrm{d}\mathbf{S}$$

Coordenadas cartesianas

$$\nabla\cdot\mathbf{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right)\hat{\mathrm{e}}_x + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right)\hat{\mathrm{e}}_y + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)\hat{\mathrm{e}}_z$$

$$\nabla f = \frac{\partial f}{\partial x}\hat{\mathrm{e}}_x + \frac{\partial f}{\partial y}\hat{\mathrm{e}}_y + \frac{\partial f}{\partial z}\hat{\mathrm{e}}_z \qquad \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Coordenadas cilíndricas

$$\nabla\cdot\mathbf{A} = \frac{1}{\rho}\frac{\partial(\rho A_\rho)}{\partial\rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial\varphi} + \frac{\partial A_z}{\partial z}$$

$$\nabla\times\mathbf{A} = \left[\frac{1}{\rho}\frac{\partial A_z}{\partial\varphi} - \frac{\partial A_\varphi}{\partial z}\right]\hat{\mathrm{e}}_\rho + \left[\frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial\rho}\right]\hat{\mathrm{e}}_\varphi + \left[\frac{1}{\rho}\frac{\partial(\rho A_\varphi)}{\partial\rho} - \frac{1}{\rho}\frac{\partial A_\rho}{\partial\varphi}\right]\hat{\mathrm{e}}_z$$

$$\nabla f = \frac{\partial f}{\partial\rho}\hat{\mathrm{e}}_\rho + \frac{1}{\rho}\frac{\partial f}{\partial\varphi}\hat{\mathrm{e}}_\varphi + \frac{\partial f}{\partial z}\hat{\mathrm{e}}_z \qquad \nabla^2 f = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial f}{\partial\rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 f}{\partial\varphi^2} + \frac{\partial^2 f}{\partial z^2}$$

Coordenadas esféricas

$$\nabla\cdot\mathbf{A} = \frac{1}{r^2}\frac{\partial(r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial(\sin\theta A_\theta)}{\partial\theta} + \frac{1}{r\sin\theta}\frac{\partial(A_\varphi)}{\partial\varphi}$$

$$\nabla\times\mathbf{A} = \left[\frac{1}{r\sin\theta}\frac{\partial(\sin\theta A_\varphi)}{\partial\theta} - \frac{1}{r\sin\theta}\frac{\partial A_\theta}{\partial\varphi}\right]\hat{\mathrm{e}}_r$$

$$+ \left[\frac{1}{r\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{1}{r}\frac{\partial(rA_\varphi)}{\partial r}\right]\hat{\mathrm{e}}_\theta + \left[\frac{1}{r}\frac{\partial(rA_\theta)}{\partial r} - \frac{1}{r}\frac{\partial A_r}{\partial\theta}\right]\hat{\mathrm{e}}_\varphi$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{\mathrm{e}}_r + \frac{1}{r}\frac{\partial f}{\partial\theta}\hat{\mathrm{e}}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial\varphi}\hat{\mathrm{e}}_\varphi$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial\varphi^2}$$