Supplementary material for "Detecting equilibrium and dynamical quantum phase transitions in Ising chains via out-of-time-ordered correlators"

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THE OTO CORRELATOR SIGNALS THE EQUILIBRIUM QPT AFTER A QUENCH FOR SHORT RANGE INTERACTING MODELS

For many short range interacting models, such as the TFI and ANNNI models, studied in the main text, the equilibrium and dynamical quantum phase transition (DQPT) points coincide. Therefore, it is a natural question to ask, which one of these is indeed signaled by the OTO correlator? To answer this question we turn to the 1D XY model in a magnetic field, which has been studied in Ref. 1, and was demonstrated that for certain parameter range DQPT can occur without crossing an equilibrium phase boundary, thus without an equilibrium QPT counterpart. Its Hamiltonian is

$$H_{XY}=\sum_{j=1}^{N-1}\frac{1+\gamma}{2}\sigma_j^x\sigma_{j+1}^x+\frac{1-\gamma}{2}\sigma_j^y\sigma_{j+1}^y-h\sum_{j=1}^{N}\sigma_j^z. \tag{S1}$$

For $\gamma>0$ and $h<1$, the spins are ordered in the x direction and quenching from an initial value (h_0,γ_0) to a final (h_1,γ_1) within the same equilibrium phase, DQPT show up for

$$2\gamma_0\gamma_1<1-h_0h_1-\sqrt{(h_0^2-1)(h_1^2-1)}, \tag{S2}$$

as was calculated in Ref. 1. Thus, by choosing $h_0=0$, $\gamma_0=0.3$ and e.g. $\gamma_1=0.2$, a DQPT occurs for $h_1>0.475$, though the equilibrum phase boundary is located at $h_1=1$. Following Ref. 1, we have calculated the Loschmidt overlap, i.e.

$$G(t)=\langle\Psi(h_0,\gamma_0)|\exp(itH(h_1,\gamma_1))|\Psi(h_0,\gamma_0)\rangle \tag{S3}$$

for $(h_0,\gamma_0)=(0,0.3)$ and $(h_1,\gamma_1)=(0.6,0.2)$. The rate function, $r(t)=-\lim_{N\to\infty}\frac{1}{N}\ln|G(t)|^2$ is shown in Fig. S1 together with the OTO correlator of the order parameter, σ_n^x using ED for N=20.

While cusps show up in the rate function, indicating the occurence of DQPT, the OTO correlator of the order parameter stays finite at late times, and only vanishes upon crossing the QPT, similarly to the TFI models.

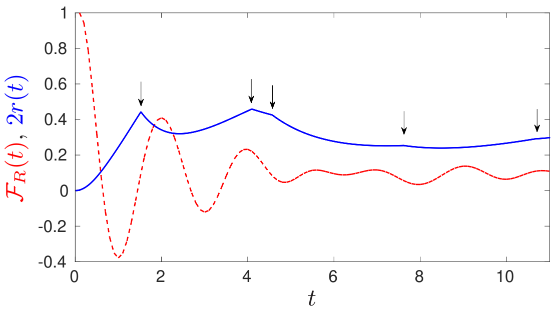

FIG. S1. The OTO correlator (red dashed line) from ED for $N=20$ and twice the rate function (blue solid line) from Ref. 1 for the XY model are plotted for a quench within the ordered phase with $(h_0,\gamma_0)=(0,0.3)$ and $(h_1,\gamma_1)=(0.6,0.2)$. While the rate function signals a DQPT (the non-analytic points are denoted by arrows from $\partial r(t)/\partial t$), the OTO correlator stays also finite at late times.

ORDER PARAMETER CORRELATION FUNCTION

We address the behaviour of the correlation function of the order parameter,

$$\chi(t)=\langle\mathcal{M}(t)\mathcal{M}\rangle \tag{S4}$$

for several variants of the transverse field Ising model both in equilibrium and after a quench from the fully polarized state. Note that for the latter case, the wavefunction is an eigenstate of the order parameter as $\mathcal{M}|\uparrow\uparrow\uparrow\dots\rangle=|\uparrow\uparrow\uparrow\dots\rangle$, thus the above correlation function measures simply the decay of the initial magnetization, i.e.

$$\chi(t)=\langle\dots\uparrow\uparrow\uparrow|\mathcal{M}(t)|\uparrow\uparrow\uparrow\dots\rangle \text{ after a quench.} \tag{S5}$$

In equilibrium, for large temporal separation, $\chi(t)$ measures the square of the magnetization and is finite/zero in the ordered/disordered phases, respectively[2]. In contrast, after a quench, it vanishes with time[3] since the quench heats up the system and no long range order is possible for 1D short range interacting models at finite temperatures, as we demonstrate below.

Let us start with the transverse field Ising model, Eq. (4) in the main text. The $\chi(t)$ correlation function in

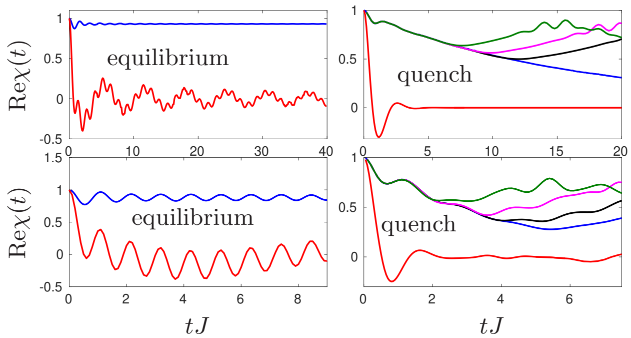

FIG. S2. The correlation function of the order parameter in equilibrium and after a quench is shown. The top panels depict the TFI with $N = 60$ from TEBD for $g/J = 0.5$ (blue) and 1.5 (red). For comparison, the ED data with $N = 22$ (black), 18 (magenta) and 14 (green) is also shown after a quench within the ordered phase. The bottom panels visualize the response of the ANNNI model from ED with $g/J = 1$ (blue) and 2 (red) with $N = 22$. The ED data with $N = 18$ (black), 14 (magenta) and 10 (green) is also shown after a quench within the ordered phase, paralleling closely to the behaviour of the TFI model.

equilibrium signals the ordering, reaching a finite steady state value in the ordered phase while vanishing in the disordered phase, as shown in Fig. S2. After a quench, however, the very same correlation function, measuring now $\langle \mathcal{M}(t) \rangle$ in the polarized state, vanishes identically in the long time limit. In particular, the numerical data for $g/J = 0.5$ after the quench is well fitted by $0.95 \exp(-0.056 Jt)$, in agreement with the expected exponential decay of the magnetization. The finite size data also follows this trend before the finite size effects kick in.

The ANNNI model produces qualitatively similar behaviour: while the steady state value of the equilibrium $\chi(t)$ indicates the ordered/disordered phase, it vanishes after the quench, therefore its steady state value cannot indicate the equilibrium quantum critical point and cannot serve as a putative order parameter for the DQPT. As we have demonstrated in the main text, the OTO correlator, on the other hand, fulfills this job perfectly.

Finally, for the sake of completeness, we also show $\chi(t)$ for the LMG model. This model has a finite temperature phase transition, thus the correlation function of the order parameter detects not only the equilibrium but also the DQPT, in which case it measures directly the magnetization. In this context, one does not gain much by studying the OTO correlator as simpler correlators contain already information about ordering, nevertheless to treat long range interacting models on equal footing as short range models, it is important to emphasize that the OTO correlator serves as a universal diagnostic tool for equilibrium and DQPT, unlike $\chi(t)$.

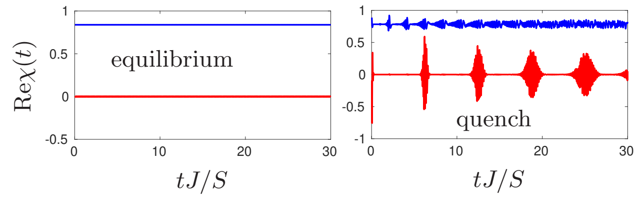

FIG. S3. The representative time evolution of the correlation function of the order parameter is shown for the LMG model in equilibrium (left panel) and after a quench (right panel) from the fully polarized state in the ordered phase with $g/J = 0.4$ (blue) and disordered phase with g/J=1.2 (red) for $N = 499$.

POLARIZED VS. UNPOLARIZED STATE

By numerically diagonalizing the transverse field Ising model in Eq. (4) using periodic boundary conditions, the ground state wavefunction is a linear superposition of the two symmetry broken ground states in the ordered phase, such that the magnetization vanishes for this wavefunction. By adding a small source field to a given site for the transverse field Ising model of the form $H' = -J'\sigma_1^z$ with $J' \ll J, g$, this additional term facilitates symmetry breaking and the expectation value of the magnetization using this wavefunction gives the actual order parameter. On the other hand, a tiny source field does not yield any magnetization in the disordered phase. The OTO correlator is insensitive whether we take the polarized ground state wavefunction or the superposition with the symmetry broken ground states with no magnetization, as shown in Fig. S4.

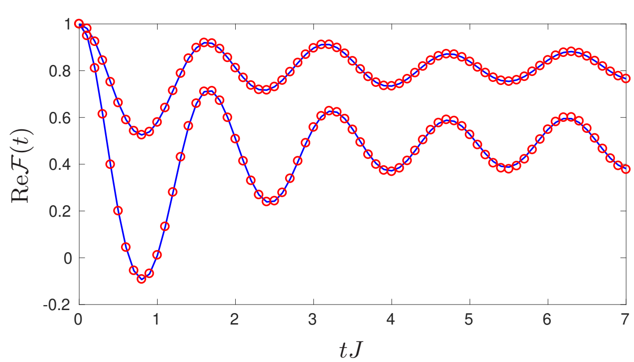

FIG. S4. The real part of OTO correlators for TFI with $N = 14$ from ED are shown in equilibrium in the ordered phase with $g/J = 0.5$ (top) and 0.8 (bottom) using an unpolarized ground state with $J' = 0$ (blue solid line) and a polarized ground state with $J' = J/100$ (red circles). The respective homogeneous magnetizations per site are 0.965 and 0.786 for $J' = J/100$ and zero for $J' = 0$. In spite of the completely different magnetizations, the OTO correlators are identical.

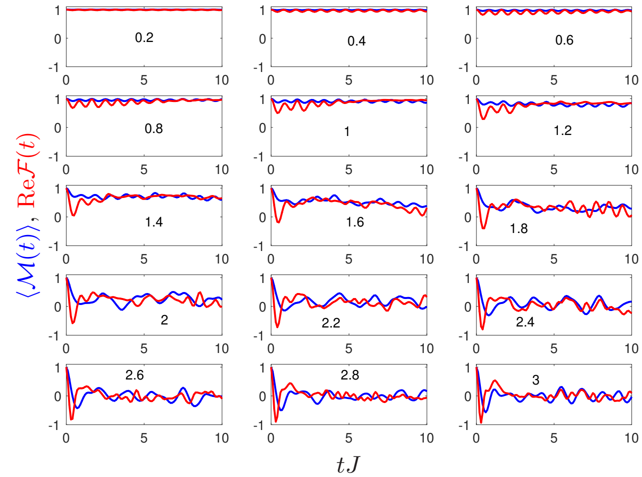

FIG. S5. The magnetization (blue) and the real part of OTO correlator (red) for 2D TFI on 4×4 square lattice from ED are shown after quench from a polarized state. The g/J ratio of the final Hamiltonian is indicated in each panel.

2D TFI MODEL

In this section, we elaborate on whether the OTO correlator signals the ground state or thermal phase transition. To this end, we investigate the two dimensional TFI model, which possesses a thermal as well as quantum phase transition and is non-integrable, thus it is expected to thermalize after a quench. Its magnetization after a quench from the polarized state indicates the dynamical phase transition, similarly to the LMG model in the main text, what we compare to the behaviour of the OTO correlator. Its Hamiltonian with periodic boundary condition is given by

$$H = -J \sum_{\langle R,R' \rangle} \sigma^z_R \sigma^z_{R'} + g \sum_R \sigma^x_R, \tag{S6}$$

where R and R' are lattice vectors for the 2D square lattice and $\langle R, R' \rangle$ denotes nearest neighbour pairs such, that each pair is condidered only once.

The model in Eq. (S6) exhibits an equilibrium quantum phase transition[4, 5] at $g/J \approx 3.04$ from a ferromagnetic state to a paramagnetic phase with increasing g. We focus on the time dependence of the magnetization, $\mathcal{M} = \sigma^z_R$, after a quench from a fully polarized state, as well as the corresponding OTO correlator for a 4×4 square lattice using ED. The results are plotted in Fig. S5. While the precise location of the DQPT cannot be determined unambiguously due to the small lattice size, it seems that the magnetization and the OTO correlator follow the same behaviour, thus both signal the DQPT transition equally well. The DQPT is located at around $g/J = 2 - 2.5$, and for small g, both the magnetization and the OTO correlator saturates to a finite, non-zero value, while they oscillate around zero for $g/J > 2.5$. This indicates that the OTO correlator in models with broken symmetry states at finite temperature signals the thermal and not the ground state phase diagram, though our results are far from being conclusive due to the small lattice size we consider.

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