Theory of Neutrino Oscillations

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Abstract — We present a derivation of the flavor neutrino states which describe neutrinos produced or detected in charged-current weak-interaction processes, including those operating in neutrino oscillation experiments. We also present a covariant derivation of the probability of neutrino oscillations which is consistent with the fact that flavor is Lorentz-invariant. Finally, we clarify the negative answers to three commonly asked questions: “Do charged leptons oscillate?”; “Is the standard phase wrong by a factor of 2?” “Are flavor neutrinos described by Fock states?”.

1 Introduction

The study of neutrino oscillations is a very important field of contemporary experimental ant theoretical research in high-energy physics. The main reason is that neutrino oscillations is a consequence of the existence of neutrino masses, as was discovered by Bruno Pontecorvo in the late 50’s [1, 2, 3]. However, neutrinos are massless in the Glashow-Salam-Weinberg Standard Model [4, 5, 6]. Hence, neutrino oscillations represents an open window on the physics beyond the Standard Model.

In view of such importance of neutrino oscillations, it is necessary to have a correct and clear understanding of all aspects of the theory of neutrino oscillations, in order to have a correct interpretation of experimental results. The theory of neutrino oscillations has been discussed in many papers [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36] and reviewed in [37, 38, 39, 40, 41, 42, 43, 44] (conference contributions and papers with other points of view are listed in Ref. [45]).

In this report we first review briefly the standard plane-wave theory of neutrino oscillations in Section 2. In Section 3 we discuss the problem of the definition of flavor neutrino states and in Section 4 we show that the flavor neutrino states are appropriate for the description of neutrinos produced or detected in charged-current weak-interaction processes. In Section 5 we present a covariant derivation of the probability of neutrino oscillations. In Section 6 we try to give clear answers to three questions which are often asked. Finally, we conclude in Section 7.

2 Standard Plane-Wave Theory

Neutrino oscillations are a consequence of neutrino mixing: the left-handed flavor neutrino fields $\nu_{\alpha L}$, with $\alpha = e, \mu, \tau$, are unitary linear combinations of the massive neutrino fields[1)] ν_{kL}, with $k = 1, 2, 3$,

$$\nu_{\alpha L} = \sum_{k=1}^{3} U_{\alpha k}\, \nu_{kL} \qquad (\alpha = e, \mu, \tau)\,, \tag{2.1}$$

where U is the unitary mixing matrix.

In the standard plane-wave theory of neutrino oscillations [7, 8, 9, 10, 37] it is assumed that the neutrinos created

[1)] For simplicity we consider the simplest case of three-neutrino mixing, neglecting the possible existence of additional sterile neutrino fields (see Refs. [46, 47, 48, 49]).

or detected together with a charged antilepton[2)] ℓ_α^+, with $\alpha = e, \mu, \tau$, are described by the flavor neutrino state

$$|\nu_\alpha\rangle = \sum_{k=1}^{3} U_{\alpha k}^* \, |\nu_k\rangle \, . \tag{2.2}$$

Since the massive neutrino states $|\nu_k\rangle$ have definite mass m_k and definite energy E_k, they evolve in time as plane waves:

$$i \, \frac{\partial}{\partial t} \, |\nu_k(t)\rangle = \mathscr{H} \, |\nu_k(t)\rangle = E_k \, |\nu_k(t)\rangle \quad \Longrightarrow \quad |\nu_k(t)\rangle = e^{-iE_k t} \, |\nu_k\rangle \, , \tag{2.3}$$

where $\mathscr{H}$ is the Hamiltonian operator and $|\nu_k\rangle = |\nu_k(t=0)\rangle$. The consequent time evolution of the flavor neutrino state (2.2) is given by

$$|\nu_\alpha(t)\rangle = \sum_{k=1}^{3} U_{\alpha k}^* \, e^{-iE_k t} \, |\nu_k\rangle = \sum_{\beta=e,\mu,\tau} \left(\sum_{k=1}^{3} U_{\alpha k}^* \, e^{-iE_k t} \, U_{\beta k} \right) |\nu_\beta\rangle \, , \tag{2.4}$$

which shows that if the mixing matrix U is different from unity (*i.e.* if there is neutrino mixing), for $t > 0$ the state $|\nu_\alpha(t)\rangle$, which has pure flavor α at $t = 0$, is a superposition of different flavors. The quantity in round parentheses in Eq. (2.4) is the amplitude of $\nu_\alpha \to \nu_\beta$ transitions at the time t after ν_α production, whose squared absolute value gives the probability of a $\nu_\alpha \to \nu_\beta$ transitions:

$$P_{\nu_\alpha \to \nu_\beta}(t) = |\langle \nu_\beta | \nu_\alpha(t) \rangle|^2 = \left| \sum_{k=1}^{3} U_{\alpha k}^* \, e^{-iE_k t} \, U_{\beta k} \right|^2 = \sum_{k,j=1}^{3} U_{\alpha k}^* \, U_{\beta k} \, U_{\alpha j} \, U_{\beta j}^* \, e^{-i(E_k - E_j)t} \, . \tag{2.5}$$

One can see that $P_{\nu_\alpha \to \nu_\beta}(t)$ depends on the energy differences $E_k - E_j$. In the standard theory of neutrino oscillations it is assumed that all massive neutrinos have the same momentum $\vec{p}$. Since detectable neutrinos are ultrarelativistic[3)], we have

$$E_k = \sqrt{\vec{p}^{\,2} + m_k^2} \simeq E + \frac{m_k^2}{2E} \quad \Longrightarrow \quad E_k - E_j = \frac{\Delta m_{kj}^2}{2E} \, , \tag{2.6}$$

where $E \equiv |\vec{p}|$ is the energy of a massless neutrino (or, in other words, the neutrino energy in the massless approximation), and $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$. In order to obtain a measurable flavor transition probability, it is necessary to convert the time t, which is not measured in neutrino oscillation experiments, in the known source-detector distance L. Considering ultrarelativistic neutrinos, we have $t \simeq L$, leading to the standard formula for the oscillation probability

$$P_{\nu_\alpha \to \nu_\beta}(L) = \sum_{k,j=1}^{3} U_{\alpha k}^* \, U_{\beta k} \, U_{\alpha j} \, U_{\beta j}^* \, \exp\left(-i \, \frac{\Delta m_{kj}^2 \, L}{2 \, E} \right) \, , \tag{2.7}$$

which is used in the analyses of the experimental data on neutrino oscillations in vacuum.

Summarizing, there are three main assumptions in the standard theory of neutrino oscillations:

(A1) Neutrinos produced or detected in charged-current weak interaction processes are described by the flavor states (2.2).

(A2) The massive neutrino states $|\nu_k\rangle$ in Eq. (2.2) have the same momentum.

(A3) The propagation time is equal to the distance L traveled by the neutrino between production and detection.

3 Flavor Neutrino States

The flavor neutrino states which describe neutrinos produced or detected in charged-current weak interaction processes can be derived in the framework of Quantum Field Theory [32, 36, 42]. The final state $|f\rangle$ in an interaction process is given by

$$|f\rangle = \boldsymbol{S} \, |i\rangle \, , \tag{3.1}$$

[2)] Since α is an index, we use the notation $\ell_e = e$, $\ell_\mu = \mu$, $\ell_\tau = \tau$.

[3)] It is known that neutrino masses are smaller than about one eV (see Refs. [48, 50]). Since only neutrinos with energy larger than about 100 keV can be detected (see the discussion in Ref. [32]), in oscillation experiments neutrinos are always ultrarelativistic.

where $|i\rangle$ is the initial state and $\boldsymbol{S}$ is the S-matrix operator. In general, the final state can be a superposition of orthogonal states,

$$|f\rangle = \sum_{k=1}^{3} \mathcal{A}_k \, |f_k\rangle \,, \tag{3.2}$$

with $\langle f_k | f_j \rangle = \delta_{kj}$. From Eq. (3.1) it follows that the amplitudes $\mathcal{A}_k$ are given by

$$\mathcal{A}_k = \langle f_k | f \rangle = \langle f_k | \, \boldsymbol{S} \, | i \rangle \,. \tag{3.3}$$

Turning now to the case of neutrinos, for definiteness let us consider neutrino production in the general decay process

$$P_I \to P_F + \ell_\alpha^+ + \nu_\alpha \,, \tag{3.4}$$

where P_I is the decaying particle, P_F is a decay product (which can be absent, for example in $\pi^+ \to \mu^+ + \nu_\mu$), and $\alpha = e, \mu, \tau$. Other production processes, as well as detection processes, can be treated in a similar way with straightforward modifications to the formalism. The final state of P_I decay,

$$|f\rangle = \boldsymbol{S} \, |P_I\rangle \,, \tag{3.5}$$

is given by

$$|f\rangle = N_\alpha \, |\nu_\alpha, \ell_\alpha^+, P_F\rangle + \ldots \,, \tag{3.6}$$

where N_α is a normalization coefficient and the dots represents other decay channels and the undecayed initial state P_I itself. The state $|\nu_\alpha, \ell_\alpha^+, P_F\rangle$ can be decomposed in a superposition of orthogonal states corresponding to different massive neutrinos:

$$|\nu_\alpha, \ell_\alpha^+, P_F\rangle = \frac{1}{N_\alpha} \sum_{k=1}^{3} \mathcal{A}_{\alpha k} \, |\nu_k, \ell_\alpha^+, P_F\rangle \,, \tag{3.7}$$

with the amplitudes $\mathcal{A}_{\alpha k}$ given by

$$\mathcal{A}_{\alpha k} = \langle \nu_k, \ell_\alpha^+, P_F | f \rangle = \langle \nu_k, \ell_\alpha^+, P_F | \, \boldsymbol{S} \, | P_I \rangle \,. \tag{3.8}$$

The normalization of the state (3.7) requires that

$$|N_\alpha|^2 = \sum_{k=1}^{3} |\mathcal{A}_{\alpha k}|^2 \,. \tag{3.9}$$

It is convenient to write Eq. (3.7) in the form

$$|P_F, \ell_\alpha^+\rangle \, |\nu_\alpha\rangle = |P_F, \ell_\alpha^+\rangle \left(\sum_{k=1}^{3} |\mathcal{A}_{\alpha k}|^2 \right)^{-1/2} \sum_{k=1}^{3} \mathcal{A}_{\alpha k} \, |\nu_k\rangle \,, \tag{3.10}$$

which shows that the final neutrino in the decay (3.4) is described by the flavor state

$$|\nu_\alpha\rangle = \left(\sum_{k=1}^{3} |\mathcal{A}_{\alpha k}|^2 \right)^{-1/2} \sum_{k=1}^{3} \mathcal{A}_{\alpha k} \, |\nu_k\rangle \,, \tag{3.11}$$

which is calculable through Eq. (3.8). This flavor state has the same structure as the standard one in Eq. (2.2). In order to see if the amplitudes $\mathcal{A}_{\alpha k} \left(\sum_{k=1}^{3} |\mathcal{A}_{\alpha k}|^2 \right)^{-1/2}$ agree with the standard ones given by $U_{\alpha k}^*$, we need to calculate the matrix elements in Eq. (3.8). In the case of the low-energy weak decay (3.4) the effective S-matrix operator can be approximated at first order in the perturbative expansion as

$$\boldsymbol{S} \simeq \boldsymbol{1} - i \int \mathrm{d}^4 x \, \mathscr{H}_{\mathrm{I}}^{\mathrm{CC}}(x) \,, \tag{3.12}$$

where $\mathscr{H}_{\mathrm{I}}^{\mathrm{CC}}(x)$ is the charged-current weak interaction Hamiltonian

$$\mathscr{H}_{\mathrm{I}}^{\mathrm{CC}}(x) = \frac{G_{\mathrm{F}}}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \overline{\nu_\alpha}(x) \, \gamma^\rho \left(1 - \gamma^5\right) \ell_\alpha(x) \, J_\rho^{P_I \to P_F}(x) + \text{H.c.}$$

$$= \frac{G_{\mathrm{F}}}{\sqrt{2}} \sum_{\alpha=e,\mu,\tau} \sum_{k=1}^{3} U_{\alpha k}^{*} \, \overline{\nu_k}(x) \, \gamma^{\rho} \left(1-\gamma^{5}\right) \ell_{\alpha}(x) \, J_{\rho}^{P_I \to P_F}(x) + \text{H.c.} \, . \tag{3.13}$$

The operator $J_{\rho}^{P_I \to P_F}(x)$ describes the transition $P_I \to P_F$. For the matrix element $\langle \nu_k, \ell_{\alpha}^{+}, P_F | \, \boldsymbol{S} \, | P_I \rangle$ we obtain

$$\mathcal{A}_{\alpha k} = \langle \nu_k, \ell_{\alpha}^{+}, P_F | \, \boldsymbol{S} \, | P_I \rangle = U_{\alpha k}^{*} \, \mathcal{M}_{\alpha k} \, , \tag{3.14}$$

with

$$\mathcal{M}_{\alpha k} = -i \, \frac{G_{\mathrm{F}}}{\sqrt{2}} \int \mathrm{d}^4 x \, \langle \nu_k, \ell_{\alpha}^{+}, P_F | \, \overline{\nu_k}(x) \, \gamma^{\rho} \left(1-\gamma^{5}\right) \ell_{\alpha}(x) \, J_{\rho}^{P_I \to P_F}(x) \, | P_I \rangle \, . \tag{3.15}$$

Hence, the flavor neutrino state (3.11) is given by

$$| \nu_{\alpha} \rangle = \sum_{k=1}^{3} \frac{\mathcal{M}_{\alpha k}}{\sqrt{\sum_{j=1}^{3} |U_{\alpha j}|^2 \, |\mathcal{M}_{\alpha j}|^2}} \, U_{\alpha k}^{*} \, | \nu_k \rangle \, . \tag{3.16}$$

We notice that the flavor neutrino state (3.16) has the structure of the standard flavor state (2.2), with the amplitudes of the massive neutrinos $|\nu_k\rangle$ proportional to the corresponding element $U_{\alpha k}^{*}$ of the mixing matrix. The additional coefficients $\mathcal{M}_{\alpha k} \left(\sum_{j=1}^{3} |U_{\alpha j}|^2 \, |\mathcal{M}_{\alpha j}|^2 \right)^{-1/2}$ disappear in the realistic case of ultrarelativistic neutrinos, because in this case the experiments are not sensitive to the dependence of $\mathcal{M}_{\alpha k}$ on the neutrino masses m_k, leading to the approximation

$$\mathcal{M}_{\alpha k} \simeq \mathcal{M}_{\alpha} \quad \Longrightarrow \quad \frac{\mathcal{M}_{\alpha k}}{\sqrt{\sum_{j=1}^{3} |U_{\alpha j}|^2 \, |\mathcal{M}_{\alpha j}|^2}} \simeq 1 \, , \tag{3.17}$$

where we have used the unitarity relation $\sum_{j=1}^{3} |U_{\alpha j}|^2 = 1$.

Hence, in the case of ultrarelativistic neutrinos we obtain the standard flavor neutrino states (2.2) in a rigorous way in the framework of Quantum Field Theory. Since in all neutrino oscillation experiments the approximation (3.17) is valid, in neutrino oscillations flavor neutrinos are correctly described by the standard flavor states (2.2), confirming the validity of the standard assumption (A1). Let us emphasize that the validity of this approximation is very important, because the standard flavor states (2.2) do not depend on the kinematics of the process in which the flavor neutrino is created or detected. Hence, it is possible to derive a general expression for the oscillation probability, as we will see in Section 5.

4 Neutrino Production and Detection

In this Section we show that the flavor states (3.11) are appropriate for the description of neutrinos produced or detected in charged-current weak interaction processes. We consider for simplicity only the decay process (3.4), but the same reasoning can be applied with straightforward modifications to any production or detection process.

It is well known [51, 52, 53, 54, 55] that the probability of the decay (3.4) is the sum of the decay probabilities in the different massive neutrinos ν_k weighted by the squared absolute value $|U_{\alpha k}|^2$ of the element of the mixing matrix that weights the contribution of ν_k to the charged-current weak interaction Hamiltonian (3.13). The reason is that the massive neutrinos have definite kinematical properties and constitute the possible orthogonal asymptotic states of the decay. In other words, each decay in a massive neutrino constitutes a possible decay channel. Hence, the probability of the decay (3.4) is given by

$$|\mathcal{A}_{\alpha}|^2 = \sum_{k=1}^{3} |U_{\alpha k}|^2 \, |\mathcal{M}_{\alpha k}|^2 \, , \tag{4.1}$$

with $\mathcal{M}_{\alpha k}$ given by Eq. (3.15).

Let us now check that the description of the neutrino ν_α produced in the decay (3.4) through the flavor state (3.11) gives the correct result (4.1). Indeed, the amplitude of the decay process (3.4) is given by

$$\mathcal{A}_\alpha = \langle \nu_\alpha, \ell_\alpha^+, P_F | \, \widehat{S} \, | P_I \rangle = \left(\sum_{k=1}^{3} |\mathcal{A}_{\alpha k}|^2 \right)^{-1/2} \sum_{k=1}^{3} \mathcal{A}_{\alpha k}^* \, \langle \nu_k, \ell_\alpha^+, P_F | \boldsymbol{S} | P_I \rangle = \left(\sum_{k=1}^{3} |\mathcal{A}_{\alpha k}|^2 \right)^{1/2} . \tag{4.2}$$

Therefore, the decay probability is given by an incoherent sum of the probabilities of production of different massive neutrinos: using Eq. (3.14) we obtain

$$|\mathcal{A}_\alpha|^2 = \sum_{k=1}^{3} |\mathcal{A}_{\alpha k}|^2 = |U_{\alpha k}|^2 \, |\mathcal{M}_{\alpha k}|^2 \, , \tag{4.3}$$

in agreement with Eq. (4.1). Thus, the coherent character of the flavor state (3.11) is correctly irrelevant for the decay rate.

It is important to understand that this result is very important for the interpretation of the data of neutrino oscillation experiments, because in the analysis of these data it is necessary not only to calculate the oscillation probability but also the neutrino production and detection rates. We have shown that both tasks can be accomplished in a consistent framework with the description of the produced and detected neutrinos through the flavor states (3.11).

5 Covariant Plane Wave Neutrino Oscillations

Since flavor is a Lorentz-invariant quantity (for example, an electron is an electron in any reference frame), the probability of neutrino oscillations is Lorentz invariant [29, 35]. Indeed, the standard oscillation probability formula (2.7) is Lorentz-invariant, as shown in Ref. [35]. However, the standard derivation of the oscillation probability reviewed in Section 2 is not formulated in a covariant way. In this Section we present a covariant derivation of the oscillation probability which shows explicitly its Lorentz-invariance [29, 36, 42, 56, 57, 58].

In the following derivation we consider ultrarelativistic neutrinos. Of course, neutrinos are not ultrarelativistic in all reference frames[4)]. However, since the oscillation probability is Lorentz-invariant, it can be evaluated in any frame and we can choose to evaluate it in a frame where the massive neutrinos are ultrarelativistic. Let us emphasize that this is not a peculiar choice, because there is a continuous infinite set of reference frames in which massive neutrinos are ultrarelativistic, which includes the one in which the detector is at rest. Indeed, we will always work with explicitly Lorentz-invariant expressions for the oscillation probability, because these expressions must be valid in the continuous infinite set of reference frames in which massive neutrinos are ultrarelativistic.

The amplitude of $\nu_\alpha \to \nu_\beta$ transitions at a distance L and after a time T from the production of the flavor neutrino ν_α is given by

$$A_{\alpha\beta}(L,T) = \langle \nu_\beta | e^{-i\boldsymbol{E}T + i\boldsymbol{P}L} | \nu_\alpha \rangle \, , \tag{5.1}$$

where $\boldsymbol{E}$ and $\boldsymbol{P}$ are, respectively, the energy and momentum operators. Since the massive neutrinos have definite masses and kinematical properties, using the flavor states (2.2), which are valid for ultrarelativistic neutrinos, we obtain

$$A_{\alpha\beta}(L,T) = \sum_{k=1}^{3} U_{\alpha k}^* \, U_{\beta k} \, e^{-iE_k T + i p_k L} \, , \tag{5.2}$$

with

$$E_k = \sqrt{p_k^2 + m_k^2} \, . \tag{5.3}$$

The oscillation amplitude (5.2) is explicitly Lorentz-invariant. Let us notice that we do not adopt the standard assumption (A2) of equal momentum for different massive neutrinos. Indeed, it can be shown that in general massive neutrinos do not have neither equal momenta nor equal energies [29, 30, 33, 33]. However, in spite of the unrealistic character of the standard assumption (A2) we will see that the standard expression for the oscillation probability is correct.

In oscillation experiments the neutrino propagation time T is not measured. As in the standard plane-wave theory reviewed in Section 2, for ultrarelativistic neutrinos it is possible to approximate $T \simeq L$ (assumption (A3)) in the phase in Eq. (5.2). The reason is that in reality neutrinos are described by wave packets [11, 12, 14, 16, 18, 19, 23,

[4)] We are grateful to Prof. D. V. Ahluwalia-Khalilova for asking by e-mail a stimulating question on this point.

24, 25, 28, 31, 32, 33, 36, 43], which are localized on the production process at the production time and propagate between the production and detection processes with a group velocity which is close to the velocity of light. If the massive neutrinos are ultrarelativistic and contribute coherently to the detection process, their wave packets overlap with the detection process for an interval of time $[T - \Delta T \,,\, T + \Delta T]$, with

$$T = \frac{L}{\overline{v}} \simeq L \left(1 + \frac{\overline{m^2}}{2E^2} \right) , \qquad \Delta T \sim \sigma_x \,, \tag{5.4}$$

where $\overline{v}$ is the average group velocity, $\overline{m^2}$ is the average of the squared neutrino masses, and σ_x is given by the spatial uncertainties of the production and detection processes summed in quadrature [25] (the spatial uncertainty of the production process determines the size of the massive neutrino wave packets). The correction $L\overline{m^2}/2E^2$ to $T = L$ in Eq. (5.4) can be neglected, because it gives corrections to the oscillation phases which are of higher order in the very small ratios m_k^2/E^2. The corrections due to $\Delta T \sim \sigma_x$ are also negligible, because in all realistic experiments σ_x is much smaller than the oscillation length $L_{kj}^{\rm osc} = 4\pi E/\Delta m_{kj}^2$, otherwise oscillations could not be observed [12, 14, 33, 43]. One can summarize these arguments by saying that the approximation $T \simeq L$ is correct because the phase of the oscillations is practically constant over the interval of time in which the massive neutrino wave packets overlap with the detection process.

Using the approximation $T \simeq L$ the phase in Eq. (5.2) becomes

$$-E_k T + p_k L \simeq - \left(E_k - p_k \right) L = - \frac{E_k^2 - p_k^2}{E_k + p_k} \, L = - \frac{m_k^2}{E_k + p_k} \, L \simeq - \frac{m_k^2}{2E} \, L \,, \tag{5.5}$$

where E is the neutrino energy neglecting mass contributions. Let us emphasize that both the left and right hand sides of Eq. (5.5) are Lorentz-invariant [35].

Inserting the expression (5.5) in Eq. (5.2) one can obtain in a straightforward way a probability of $\nu_\alpha \to \nu_\beta$ transitions in space which coincides with the standard one in Eq. (2.7). Therefore, the result of the covariant derivation of the oscillation probability confirms the correctness of the standard formula in Eq. (2.7).

It is important to notice that Eq. (5.5) shows that the phases of massive neutrinos relevant for the oscillations are independent from the particular values of the energies and momenta of different massive neutrinos [14, 29, 30, 33, 56], because of the relativistic dispersion relation (5.3). Hence, as remarked above, the standard assumption (A2) of equal momenta for different massive neutrinos, albeit unrealistic, is irrelevant for the correctness of the oscillation probability.

6 Questions

In this section we try to clarify the negative answers to three questions which are often asked.

6.1 Do Charged Leptons Oscillate? No!

In order to understand the negative answer to this question it is important to comprehend that the only characteristic which distinguishes different charged leptons is their mass. Thus, flavor is distinguished through the mass of the corresponding charged lepton or, in other words, flavor and mass coincide for charged leptons. Since for charged leptons there is no mismatch between flavor and mass, there cannot be flavor oscillations of charged leptons [29].

In Refs. [59, 60, 61] it has been claimed that the probability to detect a charged lepton oscillates in space-time. This claim has been refuted in Ref. [22]. A similar effect, called "Lambda oscillations", which has been claimed to exist [62, 63] for the Λ's produced together with a neutral kaon, as in the process $\pi^- + p \to \Lambda + K^0$, has been refuted in Refs. [26, 64]. Considering the pion decay process $\pi^- \to \mu^- + \bar\nu_\mu$, the authors of Refs. [59, 60, 61, 65] argued that, since the final state of the muon and antineutrino is entangled, if the probability to detect the antineutrino oscillates as a function of distance, also the probability to detect the muon must oscillate. However, it is well known that the probability to detect the antineutrino, irrespective of its flavor, does not oscillate. This property is usually called "conservation of probability" or "unitarity" and is represented mathematically by the general relation $\sum_\beta P_{\bar\nu_\mu \to \bar\nu_\beta} = 1$. Hence, this argument actually proves that the probability to detect a charged lepton does *not* oscillates in space-time!

6.2 Is the Standard Phase Wrong by a Factor of 2? No!

It as been claimed that the standard phase of neutrino oscillations in Eq. (2.7),

$$\Phi_{kj} = -\frac{\Delta m^2_{kj} \, L}{2 \, E} \, , \tag{6.1}$$

is wrong by a factor of two [66] or there is an ambiguity by a factor of two in the oscillation phase [67, 68, 69, 70]. A similar discrepancy by a factor of two in the phase of $K^0 - \bar{K}^0$ oscillations has been claimed in Refs. [62, 71, 72] and an ambiguity by a factor of two has been claimed in Ref. [73]. These claims, which have been refuted in Refs. [29, 31, 34, 64, 74, 75, 76, 77], stem from the following fallacious reasoning.

Different massive neutrinos propagate with different velocities

$$v_k = \frac{p_k}{E_k} \, , \tag{6.2}$$

where E_k and p_k are, respectively, the energy and momentum of the neutrino with mass m_k, related by the relativistic dispersion relation (5.3). According to the fallacious reasoning, the phases of the different massive neutrinos wave functions after a propagation distance L should take into account the different times of propagation of different massive neutrinos:

$$\tilde{\Phi}_k = p_k \, L - E_k \, t_k \, . \tag{6.3}$$

The propagation times are given by

$$t_k = \frac{L}{v_k} = \frac{E_k}{p_k} \, L \, , \tag{6.4}$$

which lead, in the relativistic approximation, to the phase difference

$$\Delta\tilde{\Phi}_{kj} \equiv \tilde{\Phi}_k - \tilde{\Phi}_j = -\frac{\Delta m^2_{kj} \, L}{E} \, . \tag{6.5}$$

This phase difference is twice of the standard one in Eq. (6.1).

Let us notice that in Eq. (6.3) we have considered the possibility of different energies and momenta for different massive neutrino wave functions, as we have done in the covariant derivation of the neutrino oscillation probability in Section 5. The authors of Refs. [67, 70, 73] claimed that a correct way to obtain the standard oscillation phase is to assume the same energy for the different massive neutrino wave functions. Apart from the fact that this is an unphysical assumption [29, 30, 33, 33], it is not true that the disagreement of a factor of two disappears assuming the same energy for the different massive neutrino wave functions, as clearly shown by the above derivation of Eq. (6.5), in which the energies of the different massive neutrino wave functions could have been taken to be equal [31, 78]. Indeed, even if the different massive neutrino wave functions have the same energy, the time contribution $-Et_k + Et_j$ to the phase difference $\Delta\tilde{\Phi}_{kj}$ does not disappear if $t_k \neq t_j$. This contribution has been missed in Refs. [67, 70, 73].

The mistake in the fallacious reasoning described above is due to a wrong use of the group velocity (6.2) in the phase (6.3), which cannot depend on the group velocity. The group velocity (6.2) is the velocity of the factor which modulates the amplitude of the wave packet of the corresponding massive neutrino. In neutrino oscillation experiments the envelopes of the wave packets of the different massive neutrinos propagate with the corresponding group velocity and take different times to cover the distance between the source and the detector. However, these different propagation times have no effect on the phases of the wave functions of the different massive neutrinos, whose interference generates the oscillations. Since the interference is a local effect, it must be calculated at the same time, as well as in the same point, for all the massive neutrino contributions [29, 31, 34, 64, 74, 75, 76, 77], as we have done in Section 5. Since only the amplitude of the wave function of each massive neutrino is determined by the wave packet envelope, the only possible effect of the different arrival times of the envelopes of the wave packets of different massive neutrinos is a reduction of the overlap of the different wave packets, leading to a decoherence effect [11, 12, 14, 16, 18, 19, 23, 24, 25, 28, 31, 32, 33, 36, 43]. For more details, see Ref. [31].

6.3 Are Flavor Neutrinos Described by Fock States? No!

It must be noted that the flavor state (2.2) is *not* a quantum of the flavor field ν_α [15]. Indeed, one can easily check that the flavor state (2.2) is not annihilated by the flavor field ν_α if the neutrino masses are taken into account. It is, however, possible to construct a Fock space for the quantized flavor fields [79, 80, 81, 82, 83, 84, 85]. Then, it is natural to ask if the flavor Fock states describe real neutrinos, produced and detected in charged-current weak interaction processes. Let us notice that, since the flavor Fock states are different from the standard flavor states (2.2) and the “exact” flavor states (3.11) that we have derived in the framework of Quantum Field Theory, if the answer to the question under discussion were positive, the theory of neutrino oscillations would have to be revised, as claimed in Refs. [79, 80, 81, 82, 83, 84, 85]. However, the answer is negative, as explained in Ref. [86], to which we direct the interested reader for the detailed proof. Here we mention only that the unphysical character of the flavor Fock states can be proved by *reductio ad absurdum*: the description of neutrinos created or detected in charged-current weak interaction processes through the flavor Fock states leads to unphysical results. Let us emphasize that this fact precludes the description of neutrinos in oscillation experiments through the flavor Fock states, because these neutrinos must be produced and detected in weak interaction processes. Hence, the flavor Fock states are ingenious mathematical constructs without relevance for the description of real neutrinos.

7 Conclusions

We have shown that the standard flavor states (2.2) used in the derivation of neutrino oscillations can be derived in the framework of Quantum Field Theory in the realistic case of ultrarelativistic neutrinos (Section 3). In Section 4 we have shown that the “exact” flavor states (3.11) are appropriate for the description of neutrinos produced or detected in charged-current weak-interaction processes, taking into account the neutrino masses. In Section 5 we have presented a covariant derivation of the probability of neutrino oscillations which is consistent with the fact that flavor is Lorentz-invariant. Finally, in Section 6 we have discussed the negative answers to three questions which are often asked. In conclusion, we would like to emphasize that the standard expression (2.7) for the probability of neutrino oscillations in vacuum has been proved to be correct in the framework of Quantum Field Theory and all contrary claims stem from some misunderstanding.

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