Universal decoherence due to gravitational time dilation

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The physics of low-energy quantum systems is usually studied without explicit consideration of the background spacetime. Phenomena inherent to quantum theory in curved spacetime, such as Hawking radiation, are typically assumed to be relevant only for extreme physical conditions: at high energies and in strong gravitational fields. Here we consider low-energy quantum mechanics in the presence of gravitational time dilation and show that the latter leads to the decoherence of quantum superpositions. Time dilation induces a universal coupling between the internal degrees of freedom and the centre of mass of a composite particle. The resulting correlations lead to decoherence in the particle position, even without any external environment. We also show that the weak time dilation on Earth is already sufficient to affect micrometre-scale objects. Gravity can therefore account for the emergence of classicality and this effect could in principle be tested in future matter-wave experiments.

One of the most striking features of quantum theory is the quantum superposition principle. It has been demonstrated in numerous experiments with diverse systems, such as neutrons[1], atoms[2] and even large molecules[3]. However, quantum superpositions are not observed on everyday, macroscopic scales. The origin of the quantum-to-classical transition is still an active field of research. A prominent role in this transition is commonly attributed to decoherence[4,5]: owing to interaction with an external environment, a particle gets entangled with its environment and loses its quantum coherence. Many specific models have been studied in which a particle interacts with its surroundings, such as a bath of phonons[6], photons[7,8], spins[9,10] and gravitational waves[11–13]. An alternative route to explain classicality is taken in so-called wavefunction collapse models, which postulate an inherent breakdown of the superposition principle at some scale without any external environment[14–16]. Such models are often inspired by general relativity, but they rely on a fundamental modification of quantum theory. In contrast, here we derive the existence of decoherence due to time dilation without any modification of quantum mechanics and which takes place even for isolated composite systems. We show that even the weak time dilation on Earth is already sufficient to decohere micro-scale quantum systems.

We consider standard quantum mechanics in the presence of time dilation, with the focus on gravitational time dilation which causes clocks to run slower near a massive object. In the Methods, we derive the Hamiltonian governing the quantum dynamics of a composite system on an arbitrary, static background spacetime (and show that the same result is obtained as a limit of a quantum field model). As we consider slowly moving particles and weak gravitational fields (that is, to lowest order in c^{-2}, where c is the speed of light), the results can also be obtained directly from the mass–energy equivalence[17]: any internal energy contributes to the total weight of a system and thus also couples to gravity. Given any particle of mass m and an arbitrary Hamiltonian H_0 that generates the time evolution of

its internal degrees of freedom, gravity couples to the total rest mass $m_{tot} = m + H_0/c^2$—that is, gravity also couples to internal energy. The interaction with the gravitational potential $\Phi(x)$ is therefore $m_{tot}\Phi(x) = m\Phi(x) + H_{int}$, where $H_{int} = \Phi(x)H_0/c^2$. This interaction term is just another formulation of gravitational time dilation (the same argument applies to inertial mass as well, so one recovers both the special relativistic and gravitational time dilation governed by $H_{int} = \Gamma(x,p)H_0/c^2$, with $\Gamma(x,p) = \Phi(x) - p^2/2m^2$). For example, if the particle is a simple harmonic oscillator with frequency ω, the above interaction with gravity effectively changes the frequency according to $\omega \to \omega(1 + \Phi(x)/c^2)$. This is the well-tested[18,19] gravitational redshift to lowest order in c^{-2}. When the energy is treated as a classical variable the time-dilation-induced interaction H_{int} yields only this frequency shift. However, in quantum mechanics the internal energy H_0 and the position x are quantized operators, thus time dilation causes an additional, purely quantum mechanical effect: entanglement between the internal degrees of freedom and the centre-of-mass position of the particle[20]. Even though the time dilation on Earth is very weak, it leads to a significant effect for composite quantum systems, as we will show below.

Before deriving in full generality the time evolution of the centre of mass of an arbitrary composite system subject to time dilation, we consider a simplified model where a particle has $N/3$ constituents that are independent three-dimensional harmonic oscillators. Such a model equivalently describes N internal harmonic modes of the particle. The internal Hamiltonian for this system is $H_0 = \sum_{i=1}^{N} \hbar\omega_i n_i$, where n_i are the number operators for the ith mode with frequency ω_i. The centre of mass (with x and p being its vertical position and momentum, respectively) of the whole system is subject to the gravitational potential $\Phi(x)$. For a homogeneous gravitational field in the x-direction we can approximate $\Phi(x) \approx gx$, where $g = 9.81 \text{ m s}^{-2}$ is the gravitational acceleration on earth. The total Hamiltonian of the system is therefore $H = H_{cm} + H_0 + H_{int}$, where H_{cm} is some Hamiltonian for the centre of mass of the particle

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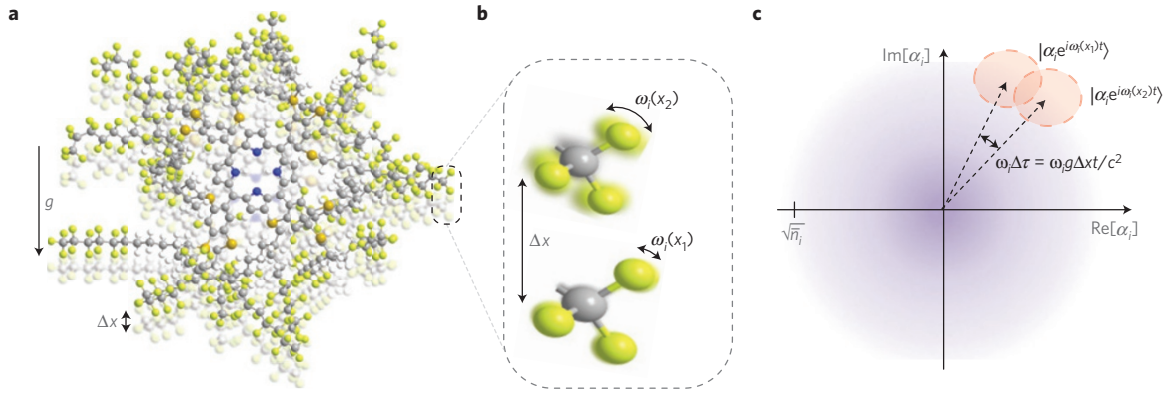
Figure 1 | Gravitational time dilation causes decoherence of composite quantum systems. a, Illustration of a TPPF20 molecule that has recently been used for matter-wave interference[3]. Here we illustrate a vertical superposition of size Δx in Earth's gravitational potential $\Phi(x) = gx$. **b**, The frequencies ω_i of internal oscillations are modified in the gravitational field—that is, $\omega_i \to \omega_i(x) = \omega_i(1 + gx/c^2)$—which correlates the internal states and the centre-of-mass position of the molecule. **c**, Phase-space representation of the ith constituent, which is in a thermal state with average occupation $\bar{n}_i \approx k_B T/\hbar\omega_i$. In the coherent-state representation of the internal states, the frequency of each coherent state depends on the position of the whole molecule and thus differs between the two superposed amplitudes by an amount $\omega_i \Delta\tau$. Even for small time dilations, this causes decoherence of the molecule with N constituents after a time τ_{dec}, given in equation (3).

and the gravitational time-dilation-induced interaction (to lowest order in c^{-2}) between position and internal energy is

$$H_{int} = \Phi(x)\frac{H_0}{c^2} = \hbar\frac{gx}{c^2}\left(\sum_{i=1}^{N}\omega_i n_i\right) \qquad (1)$$

To demonstrate decoherence, we first consider the case when the gravitational contribution to time dilation is dominant such that the velocity contributions can be neglected. A typical such case is a particle at rest in superposition of two vertically distinct positions x_1 and x_2 and a height difference $\Delta x = x_2 - x_1$. The centre of mass is in the state $|\psi_{cm}(0)\rangle = (1/\sqrt{2})(|x_1\rangle + |x_2\rangle)$. The internal degrees of freedom are in thermal equilibrium at local temperature T, thus each ith constituent is described by the thermal density matrix $\rho_i = (\pi\bar{n}_i)^{-1}\int d^2\alpha_i \exp(-|\alpha_i|^2/\bar{n}_i)|\alpha_i\rangle\langle\alpha_i|$, where we used the coherent-state representation with the average excitation $\bar{n}_i = (e^{\hbar\omega_i/k_B T} - 1)^{-1}$ and where k_B is the Boltzmann constant. The total initial state is thus given by $\rho(0) = |\psi_{cm}(0)\rangle\langle\psi_{cm}(0)| \otimes \prod_{i=1}^{N}\rho_i$. Gravitational time dilation now couples the centre-of-mass position of the system to the internal degrees of freedom ρ_i via the Hamiltonian in equation (1). The off-diagonal elements $\rho_{12} = \langle x_1|\rho|x_2\rangle = \rho_{21}^*$, which are responsible for quantum interference, evolve to $\rho_{12}(t) = (2\pi\bar{n}_i)^{-1}e^{img\Delta xt/\hbar}\prod_{i=1}^{N}\int d^2\alpha_i e^{-|\alpha_i|^2/\bar{n}_i}|\alpha_i e^{-i\omega_i(x_1)t}\rangle\langle\alpha_i e^{-i\omega_i(x_2)t}|$, where $\omega_i(x) = \omega_i(1 + gx/c^2)$. The frequencies of the internal oscillators depend on the position in the gravitational field, in accordance with gravitational time dilation (see also Fig. 1). To see decoherence of the centre of mass, we trace out the internal degrees of freedom. The quantum coherence can be quantified by the interferometric visibility $V(t) = 2|\rho_{cm}^{(12)}(t)| = 2|\prod_{i=1}^{N}Tr_i[\rho_{12}(t)]|$, which becomes $V(t) = |\prod_{i=1}^{N}[1 + \bar{n}_i(1 - e^{-i\omega_i tg\Delta x/c^2})]^{-1}|$. This expression can be simplified for the typical case $\omega_i tg\Delta x/c^2 \ll 1$. In the high-temperature limit we also have $\bar{n}_i \approx (k_B T/\hbar\omega_i)$, so that the frequency dependence completely drops out from the visibility. In this case, the reduction of quantum interference is given by $V(t) \approx (1 + (k_B Tg\Delta xt/\hbar c^2)^2)^{-N/2}$. For times $t^2 \ll N\tau_{dec}^2$ this can be written as

$$V(t) \approx e^{-(t/\tau_{dec})^2} \qquad (2)$$

where we defined the decoherence time

$$\tau_{dec} = \sqrt{\frac{2}{N}}\frac{\hbar c^2}{k_B Tg\Delta x} \qquad (3)$$

The above equation shows that gravitational time dilation causes superpositions of composite systems to decohere. The decoherence rate derived here scales linearly with the superposition size Δx, in contrast to other decoherence mechanisms, which typically show a quadratic scaling[21]. Also, decoherence due to gravitational time dilation depends on the number of oscillating internal states of the system, N. The suppression of quantum effects takes place even for completely isolated systems, provided that the superposition amplitudes acquire a sufficient proper time difference. In the high-temperature limit the frequencies of the internal oscillations drop out entirely from the final expression, therefore it is not necessary to have fast-evolving internal states. Note that the decoherence derived here depends on the constants $\hbar$, c, k_B and the gravitational acceleration g: it can therefore be considered a relativistic, thermodynamic and quantum mechanical effect.

The effect is very general and originates from the total proper time difference between superposed world lines. Quantum systems with internal degrees of freedom are affected on arbitrary spacetime metrics, as long as a proper time difference is accumulated. To highlight this, consider a particle moving in superposition along two arbitrary world lines with proper time difference $\Delta\tau$ (Fig. 2). The two superposed amplitudes can interfere when the world lines meet. Owing to time dilation, the internal energy is effectively altered by $H_0(1 + \Gamma/c^2)$, with $\Gamma(x, p) = \Phi(x) - p^2/2m^2$ (see also Methods). Each amplitude therefore evolves with $U(t) = \exp[-(i/\hbar)\int dt(H_{cm} + (1 + (\Gamma/c^2))H_0)]$ along the respective world lines. H_0 does not depend on x, p and t and, for clarity, we restrict the analysis to semi-classical paths—that is, constrained to have coordinates $\bar{x}_1(t)$, $\bar{p}_1(t)$ and $\bar{x}_2(t)$, $\bar{p}_2(t)$ along the two world lines, respectively (as in Fig. 2c). The interference visibility is then $V = |Tr[e^{-i/\hbar\int dt(1+\Gamma(\bar{x}_1,\bar{p}_1)/c^2)H_0}\rho_0 e^{i/\hbar\int dt(\Gamma(\bar{x}_2,\bar{p}_2)/c^2)H_0}]|$. As $d\tau = dt\sqrt{g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu} \approx dt(1 + \Gamma/c^2)$, the interference visibility is simply

$$V = |\langle e^{-iH_0\Delta\tau/\hbar}\rangle| \qquad (4)$$

where $\Delta\tau = \tau_1 - \tau_2$ is the proper time difference between the two world lines and the expectation value is taken with respect to the initial state. This result is manifestly coordinate invariant and shows that decoherence occurs if a proper time difference is present, and if the internal states are not eigenstates of internal energy. Equation (3) is recovered as a special case of the general formula above (expanding to lowest order in $\Delta\tau$, assuming N internal

a $\quad d\tau^2 = (1 + 2\phi(x))dt^2 - dx^2$

b $\quad d\tau^2 = dt'^2 - dx'^2$

c $\quad d\tau^2 = -g_{\mu\nu}dx^\mu dx^\nu$
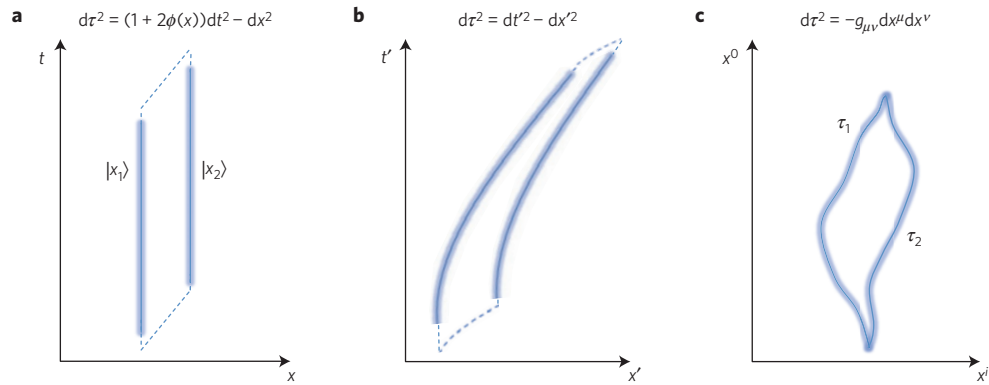
Figure 2 | A composite particle in superposition will decohere owing to time dilation. The figure shows two superposed world lines in different situations and spacetimes (here $c=1$). **a**, A particle in superposition at two different fixed heights above the earth, as considered in the main text (the dashed lines represent arbitrary small non-stationary contributions necessary to perform an interferometric experiment). The centre of mass will decohere after a time τ_{dec} as given in equation (3). In general, the full evolution of the centre of mass is given by equation (5). **b**, A particle undergoing uniform acceleration g in flat spacetime will experience the same time dilation and thus the same decoherence as in case **a**. Equivalently, the diagram describes the previous situation from the point of view of a freely falling observer. **c**, A composite particle with internal Hamiltonian H_0 in an arbitrary spacetime will decohere if the two superposed trajectories differ in proper time. The visibility of quantum interference V reduces depending on the proper time difference $\Delta\tau$, see equation (4).

harmonic oscillators such that $\Delta E_0^2 = \langle H_0^2 \rangle - \langle H_0 \rangle^2 \approx N k_B^2 T^2$ and neglecting the p-dependent term yields equation (3)). For the special case of a pure two-level system, equation (4) also reproduces the effect discussed in ref. 20, which can be interpreted as due to the which-way information acquired by a clock. In contrast, which-way information is never available for thermal states (a more detailed discussion can be found in the Supplementary Information). Equation (4) shows, however, that time dilation affects any state that is not an eigenstate of H_0. The coupling is universal, which follows directly from the universality of time dilation. Thus the decoherence is as universal as time dilation itself, in the sense that all composite quantum systems are affected, independently of the nature and kind of their internal energy H_0.

We now consider the full time evolution in the presence of time dilation that takes any arbitrary internal Hamiltonian H_0 and centre-of-mass Hamiltonian H_{cm} into account. To this end, we derive a master equation that describes the quantum dynamics of a composite system on a background spacetime to lowest non-vanishing order in c^{-2} (see Methods for details). The resulting time evolution of the centre of mass in the presence of special relativistic and gravitational time dilation is

$$\dot{\rho}_{cm}(t) = -\frac{i}{\hbar}\left[H_{cm} + \frac{\bar{E}_0}{c^2}\Gamma(x,p), \rho_{cm}(t) \right]$$
$$- \left(\frac{\Delta E_0}{\hbar c^2}\right)^2 \int_0^t ds\,[\Gamma(x,p),[e^{-iH_{cm}s/\hbar}\Gamma(x,p)e^{iH_{cm}s/\hbar}, \rho_{cm}(t)]]\quad(5)$$

where $\Gamma(x,p) = \Phi(x) - p^2/2m^2$. The first term describes the unitary evolution of the centre of mass due to an arbitrary Hamiltonian H_{cm}, which is completely general and can also include external interactions (as, for example, those necessary for keeping the particle in superposition or realizing an interference experiment) as well as relativistic corrections to the centre-of-mass dynamics. The correction dependent on $\bar{E}_0 = \langle H_0 \rangle$ stems from the relativistic contribution to the mass. The second term causes the suppression of off-diagonal elements of the density matrix and is responsible for the decoherence. It is proportional to $\Delta E_0^2 = \langle H_0^2 \rangle - \langle H_0 \rangle^2$, the fluctuations in internal energy, or equivalently the heat capacity $C_v = \Delta E_0^2/k_B T^2$ (the high-temperature limit of the model that we used previously corresponds to the Einstein solid model). The integral captures the fact that decoherence depends on the overall

acquired proper time difference during a particle's evolution. For stationary particles, and if the centre-of-mass Hamiltonian H_{cm} does not induce significant changes to the off-diagonal elements on the decoherence timescale, the master equation becomes approximately

$$\dot{\rho}_{cm}(t) \approx -\frac{i}{\hbar}\left[\tilde{H}_{cm} + \left(m + \frac{\bar{E}_0}{c^2}\right)gx, \rho_{cm}(t) \right]$$
$$- \left(\frac{\Delta E_0 g}{\hbar c^2}\right)^2 t[x,[x,\rho_{cm}(t)]]\quad(6)$$

In the unitary part we have separated for clarity the Newtonian gravitational potential (that is, $H_{cm} = \tilde{H}_{cm} + mgx$): it is evident that the potential couples to an effective total mass $m_{tot} = m + \bar{E}_0/c^2$ that includes the average internal energy, which becomes $\bar{E}_0 = \langle H_0 \rangle \approx N k_B T$ for the previously considered model with N thermal internal harmonic oscillators. This is in accordance with the notion of heat in general relativity (in Einstein's words[22]: 'a piece of iron weighs more when red-hot than when cool'), the relation to the Tolman effect is briefly discussed in the Supplementary Information. The non-unitary part now depends only on the stationary x-contributions (see also Fig. 2a). The decoherence timescale is found from the solution to equation (6), which for the off-diagonal terms $\rho_{cm}^{(12)}$ is approximately (to order $O(\hbar^{-2})$): $\rho_{cm}^{(12)}(t) \sim \rho_{cm}^{(12)}(0)\,e^{-(t/\tau_{dec})^2}$, with $\tau_{dec} = \sqrt{2}\hbar c^2/(\Delta E_0 g \Delta x)$. The loss of visibility thus agrees with equation (2) and the decoherence timescale reduces to equation (3) for the specific model used previously. The master equation due to gravitational time dilation, equation (6), is similar in form to other master equations typically studied in the field of decoherence[4,21], but does not include any dissipative term. Thus time dilation provides naturally an 'ideal' master equation for decoherence that suppresses off-diagonal terms in the position basis for stationary particles. For non-stationary systems, decoherence is governed by equation (5) and the pointer basis derives from a combination of x and p. Position and momentum are therefore naturally driven into becoming classical variables. The evolution in the presence of gravitational time dilation is inherently non-Markovian, because the overall acquired proper time difference is crucial. This results in a Gaussian decay (rather than an exponential decay as in Markovian models) of the off-diagonal elements and the decoherence time depends directly on the fluctuations in internal energy. This again highlights the interplay between

thermodynamics, relativity and quantum theory that is relevant for this effect.

To estimate the strength of the decoherence due to time dilation, we make use of equation (3) and consider a human-scale macroscopic system at room temperature. Assuming that the system has Avogadro's number of constituent particles which oscillate, we set $N \sim 10^{23}$, which amounts to a gram-scale system. For a superposition size of $\Delta x = 10^{-6}$ m, the decoherence time (3) becomes

$$\tau_{dec} \approx 10^{-3} \, s$$

Remarkably, even though the gravitational time dilation is very weak, its resulting decoherence is already substantial on human scales and not just for astrophysical objects. Macroscopic objects completely decohere on Earth on a short timescale owing to gravitational time dilation. In contrast to other decoherence mechanisms, this effect cannot be shielded and decoherence will occur whenever there is time dilation between superposed amplitudes. But, as any other decoherence in quantum theory, the effect is in principle reversible: as is apparent from equation (4), 'revivals' of coherence will occur after a sufficiently large proper time difference is accumulated, dependent on the frequencies of the internal degrees of freedom. However, the corresponding recurrence times typically scale exponentially with the size of the system[23], thus time dilation can cause decoherence which is irreversible 'for all practical purposes' if interfering paths have a proper time difference. This will be increasingly difficult to control in experiments with large, composite systems, and is simply unavoidable for systems not under experimental control.

From the perspective of quantum theory, decoherence due to time dilation is fully analogous to any other decoherence source. The loss of coherence takes place because of correlations with degrees of freedom that are not accessible—here, internal degrees of freedom of the composite system. The unique aspect of the effect described here is that the correlations are induced by relativistic time dilation, and would not take place in Newtonian gravity. Thus, to understand this effect it is necessary to invoke quantum theory and time dilation stemming from Earth's gravitational field. The phenomenon arises already in weak, stationary spacetimes and decoheres composite systems into the position basis, even if they are isolated from any external environment. It is thus of a different nature from that of decoherence due to scattering with gravitational waves[11–13]. Importantly, the time-dilation-induced decoherence is entirely within the framework of quantum mechanics and classical general relativity. No free 'model parameters' enter and unitarity is preserved on a fundamental level. This is in stark contrast to hypothetical models where gravity leads to spontaneous collapse of the wavefunction and that require a breakdown of unitarity[14–16] or include stochastic fluctuations of the metric[24]. Our results show that general relativity can account for the suppression of quantum behaviour for macroscopic objects without introducing any modifications to quantum mechanics or to general relativity.

We now discuss a possible direct experimental verification of the derived decoherence mechanism. The gravitational time dilation is well tested in classical physics[18,19], but the quantized Hamiltonian (1) has not yet been studied experimentally. In particular, an experiment to study the induced quantum entanglement of internal degrees of freedom with the centre-of-mass mode, first proposed in ref. 20, has not yet been realized. To confirm this quantum mechanical interaction one can use controllable internal states in matter-wave interferometry[20], or use Shapiro-delay in single photon interference[25] (a related effect for entangled photon pairs was also discussed in ref. 26). Such an experimental verification of the quantum Hamiltonian (1) would be a strong indication for the presence of the decoherence described here. To test directly the decoherence due to time dilation it is necessary to bring relatively complex
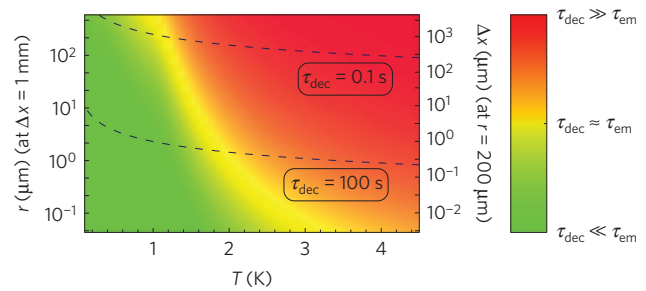
Figure 3 | Decoherence due to gravitational time dilation as compared to decoherence due to emission of thermal radiation for sapphire microspheres. In the green region time dilation is the dominant decoherence mechanism. The left axis shows various sphere radii r (corresponding to particle numbers $N = 10^7$ to $N = 10^{18}$) for a fixed superposition size Δx, whereas the right axis shows various superposition sizes for a fixed particle radius. The dashed lines correspond to the respective time dilation decoherence time scales as in equation (3). Sapphire was chosen for its low emission at microwave frequencies.

systems into superposition. This can in principle be achieved with molecule interferometry[3,8], cooled microspheres[27,28] or with micro-mechanical mirrors[29]. The latter, however, is expected to be limited to very small separations only (of the order of 1 pm) and is therefore less suitable. To see decoherence caused by time dilation, other decoherence mechanisms will need to be suppressed: The scattering with surrounding molecules and with thermal radiation requires such an experiment to be performed at liquid helium temperatures and in ultrahigh vacuum[7]. Furthermore, the emission and absorption of thermal radiation by the system[8] will be a competing decoherence source. For the parameter regime studied here, emission of radiation is expected to be the dominant decoherence effect, with the decoherence time[4,7] $\tau_{em} = (\int dk k^2 c \, g(k) \sigma_{eff}(k) \Delta x^2)^{-1}$, where $g(k)$ is the mode density of the wavevectors k, and $\sigma_{eff}(k)$ the effective scattering cross section. To see the time-dilation-induced decoherence, we require that the decoherence due to emission of radiation is weaker than due to time dilation—that is, $\tau_{dec} \lesssim \tau_{em}$. To get quantitative estimates, we rely on the harmonic oscillator model introduced previously and show in Fig. 3 the parameter regime where time-dilation-induced decoherence can in principle be distinguished from decoherence due to thermal emission, focusing on micro-scale particles at cryogenic temperatures (sapphire was chosen owing to its low microwave emission at low temperatures[30]). The emission of radiation can be further suppressed if the mode density is reduced, which can ease the restrictions on temperature. However, we note that the simple model for the composition of the system, necessary to estimate the time dilation decoherence rate, is very crude and at low temperatures we expect the model to break down. Given a specific system, the time dilation decoherence can be estimated more accurately by measurement of the internal energy fluctuations through the heat capacity. Although an experiment to measure decoherence due to proper time is very challenging, the rapid developments in controlling large quantum systems[3,27,28] for quantum metrology and for testing wavefunction collapse models[16,29] will inevitably come to the regime where the time-dilation-induced decoherence predicted here will be of importance. In the long run, experiments on Earth will have to be specifically designed to avoid this gravitational effect on quantum coherence. As a final remark, we note that owing to the universality of time dilation, all dynamical processes contribute to this decoherence, even those that are typically experimentally inaccessible (such as nuclear dynamics, which has not been taken into account in our treatment). Thus time dilation decoherence could also serve as a tool to indirectly study dynamical processes within composite systems.

Methods

Methods and any associated references are available in the online version of the paper.

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Author contributions

I.P., M.Z., F.C. and Č.B. contributed to all aspects of the research, with the leading input from I.P.

Additional information

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Competing financial interests

The authors declare no competing financial interests.

Methods

Hamiltonian for gravitational time dilation. We present a Hamiltonian formalism useful to describe the dynamics of low-energy quantum systems with internal degrees of freedom subject to time dilation. We further show an explicit example of a quantum field in curved spacetime whose dynamics reduces to the same Hamiltonian in the appropriate limit.

We consider the time evolution of a composite quantum system in the low-energy limit on a generic spacetime, described by a metric $g_{\mu\nu}$ with signature $(-+++)$. We restrict the treatment to static metrics with $\partial_0 g_{\mu\nu} = 0$ and $g_{0i} = 0$, where Latin indices refer to the spatial 3-components. The typical systems we consider are low-energy quantum systems (such as atoms, molecules, nanospheres, and so on) with internal energy levels (such as electronic, rotational, vibrational) in a weak gravitational field and under small accelerations. For such systems, one can assume that the relative distances between their constituents are sufficiently small, such that variations of the metric over their extension can be neglected. In this case one can assign a single position degree of freedom to the centre of mass of the system, which in the classical limit describes a single world line. In other words, we consider the limit in which the system can be effectively considered as 'point-like' with internal degrees of freedom. This is directly analogous to the notion of ideal clocks in relativity, which measure time along a well-localized world line.

The rest energy $E_{\rm rest}$ of the system is defined as the invariant quantity

$$p_\mu p^\mu = g^{\mu\nu} p_\mu p_\nu = -(E_{\rm rest}/c)^2 \qquad (7)$$

corresponding to the total mass–energy as measured by a comoving observer. Here p_μ is the system's total 4-momentum in arbitrary coordinates (we restrict to coordinates which keep the stated assumptions for the metric). The dynamics of the system can be described in terms of the evolution with respect to an arbitrary time coordinate t. The generator of the coordinate-time translations follows from equation (7) and is given by the Hamiltonian

$$H = cp_0 = \sqrt{-g_{00}(E_{\rm rest}^2 + c^2 g^{ij} p_i p_j)} \qquad (8)$$

If the particle is at rest with respect to a static observer, the energy is $H = \sqrt{-g_{00}}E_{\rm rest}$. This 'redshift factor' is also sometimes expressed in terms of the time-like Killing vector k^μ as $\sqrt{-k_\mu k^\mu} = \sqrt{-g_{00}}$. The rest energy $E_{\rm rest}$ is the total Hamiltonian of the system in its local comoving frame. The static rest mass contribution mc^2 can be explicitly separated, and the remaining part is just the Hamiltonian of the internal degrees of freedom, which we denote by H_0. The full dynamics of internal and external degrees of freedom is thus governed by the total Hamiltonian (8), with

$$E_{\rm rest} = mc^2 + H_0 \qquad (9)$$

Relativistically, there is no distinction between 'rest mass' and 'rest energy'. In fact, the largest contribution to the rest mass m of the systems we consider, for example, a molecule, is already given by binding energies between atoms, nucleons, quarks and all other constituents (at an even more fundamental level, masses of fundamental particles are reducible to interaction energies with the Higgs field, according to the standard model of particle physics). The natural choice for the split (9) is dictated by the energy scale: if some degrees of freedom are 'frozen', their contribution to the rest energy can be incorporated in the mass term. The split between mass and internal energy (9) is thus merely conventional, and amounts to a choice of the zero-energy state of the internal degrees of freedom. The general expression (9) can also be derived from the action of a particle in the comoving frame, in which the time coordinate coincides with the proper time: $S = \int L_{\rm rest} d\tau$, where $L_{\rm rest} = L_{\rm rest}(q_i, q_i')$ is the Lagrangian describing the internal degrees of freedom with coordinates q_i and $q_i' = dq_i/d\tau$. Changing to the lab frame this expression becomes $S = \int L_{\rm rest}(q_i, \dot{q}_i t') \dot{\tau} dt$, with $\dot{\tau} = d\tau/dt = \sqrt{g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}$ and $t' = dt/d\tau$. The Legendre transform yields the Hamiltonian for the system, which gives exactly the expression (8) with

$$E_{\rm rest} = \frac{\partial L_{\rm rest}}{\partial q_i'} q_i' - L_{\rm rest} \qquad (10)$$

If the internal dynamics is irrelevant, we simply have $L_{\rm rest} = -mc^2$ and $E_{\rm rest} = mc^2$. But in general any arbitrary internal dynamics governed by $L_{\rm rest}$ gives rise to the total energy as in equation (8) with $E_{\rm rest}$ as in equation (10). For example, two masses on a spring (with total mass m, reduced mass μ and spring constant k) in the comoving frame of the centre of mass are described by $L_{\rm rest} = \mu q'^2/2 - kq'^2/2 - mc^2$, where q is the relative degree of freedom of the two masses. This gives $E_{\rm rest} = mc^2 + \mu q'^2/2 + kq^2/2 = mc^2 + H_0$, where H_0 describes the dynamical part of the internal degrees of freedom.

To obtain the quantum equations of motion one can replace the 4-momenta in the full expression (7) with covariant derivatives, which leads to a modified Klein–Gordon equation with $E_{\rm rest}$ as the invariant total mass (a similar derivation can be applied to the Dirac equation to describe particles with spin). For energies small compared to $E_{\rm rest}$ (such that particle creation and other quantum field effects are negligible), the Klein–Gordon field is treated as a particle in first quantization, and the low-energy Schrödinger evolution is obtained. Specifically, on a

post-Newtonian background metric with $g_{00} = -(1 + 2\Phi(x)/c^2 + 2\Phi^2(x)/c^4)$ and $g_{ij} = \delta_{ij}(1 - 2\Phi(x)/c^2)$, where δ_{ij} is the Kronecker delta, the Hamiltonian for the dynamics of a point particle is obtained following ref. 31 (with the addition of internal degrees of freedom in $E_{\rm rest}$), resulting in

$$H = \bar{m}_r c^2 + \frac{p^2}{2\bar{m}_r} + \bar{m}_r \Phi(x) - \frac{p^4}{8\bar{m}_r^3 c^2} + \frac{\bar{m}_r \Phi^2(x)}{2c^2}$$
$$+ \frac{3}{2\bar{m}_r c^2}\left(\Phi(x)p^2 + [p\Phi(x)]p + \frac{1}{2}[p^2\Phi(x)]\right) \qquad (11)$$

where $[p\Phi]$ acts only on the potential and we introduced $\bar{m}_r := E_{\rm rest}/c^2$ to keep the expansion to order c^{-2} explicit (in deriving the Hamiltonian there is an ambiguity in the ordering of the $p\Phi(x)$ terms, but which does not affect the results for time dilation). For an eigenstate $|E_i\rangle$ of the internal Hamiltonian H_0 the rest energy in the Hamiltonian (11) can be treated as a parameter $\bar{m}_r = m + E_i/c^2$. For arbitrary internal states and owing to the linearity of quantum mechanics, $E_{\rm rest}$ has to be treated as an operator acting on the internal degrees of freedom according to (9). The Hamiltonian (11) thus describes the full quantum dynamics of the system, including internal and external degrees of freedom. Expanding the result to first order in H_0/mc^2, we find

$$H = H_{\rm cm} + H_0\left(1 + \frac{\Phi(x)}{c^2} - \frac{p^2}{2mc^2}\right) = H_{\rm cm} + H_0\left(1 + \frac{\Gamma(x,p)}{c^2}\right) \qquad (12)$$

where $H_{\rm cm}$ includes all terms acting on the centre of mass to this order of approximation (and can also include any other interaction, such as the electromagnetic interaction[31,32]). The term $\Gamma(x,p) = \Phi(x) - p^2/2m^2$ captures the time dilation, which effectively shifts the internal energy. An alternative route to get equation (12) is to directly expand equation (8) in orders of c^{-2} and canonically quantize the result, which yields the same Schrödinger equation with relativistic corrections. The term proportional to $\Phi(x)$ stems from $\sqrt{-k_\mu k^\mu} = \sqrt{-g_{00}}$, whereas the p^2-term stems from the spatial g_{ij} components of the metric. The former captures gravitational time dilation and the latter is the velocity-dependent special relativistic time dilation. In total, equation (12) describes the special and general relativistic corrections to the dynamics of a quantum system with internal degrees of freedom, to lowest order in c^{-2}. Note that the coupling between internal and external degrees of freedom is completely independent of the nature and kinds of interactions involved in the internal dynamics H_0. This is a consequence of the universality of time dilation, which affects all kinds of clocks, irrespectively of their specific construction.

The description above is well suited for low-energy particles on a background spacetime. The same results can also be obtained from a field theory description and, to highlight this, we consider an explicit example in which the effective Hamiltonian is derived starting from a quantum field. We consider a Klein–Gordon field of mass m, in a state approximately localized in some region of space. The position of the region of space represents the 'centre of mass', whereas the field describes the 'internal degrees of freedom' of our system. The dynamics of the field is given by the action

$$S = -\frac{1}{2}\int d^4 x \sqrt{-g}\left[g^{\mu\nu}\frac{\partial\phi}{\partial x^\mu}\frac{\partial\phi}{\partial x^\nu} + m^2\phi^2\right] \qquad (13)$$

where g is the determinant of $g_{\mu\nu}$ and we use units with $c = 1$. To simplify the discussion, we consider a system at a fixed height above Earth. The effect already appears by expanding the metric at the first order: $g_{ij} = \delta_{ij}$, $g_{00} = -[1 + 2\Phi(x)]$, where $\Phi(x)$ is the Newtonian potential. In this case the determinant is $g = g_{00}$ and $g^{00} = 1/g_{00}$, so we can rewrite equation (13) as

$$S = -\frac{1}{2}\int d^4 x \sqrt{-g_{00}}\left[\frac{(\partial_t\phi)^2}{g_{00}} + |\nabla\phi|^2 + m^2\phi^2\right]$$

Once the coordinates are fixed, we can write $S = \int dt L$ and single out the Lagrangian

$$L = \frac{1}{2}\int d^3 x\left[\frac{(\partial_t\phi)^2}{\sqrt{-g_{00}}} - \sqrt{-g_{00}}(|\nabla\phi|^2 + m^2\phi^2)\right]$$

To pass to the Hamiltonian picture, we need the conjugate momenta

$$\pi(x) := \frac{\delta L}{\delta\partial_t\phi(x)} = \frac{1}{\sqrt{-g_{00}}}\partial_t\phi(x)$$

The Hamiltonian in these coordinates is given by $H = \int d^3 x\, \pi(x)\partial_t\phi(x) - L$. Substituting $\partial_t\phi(x) = \sqrt{-g_{00}}\pi(x)$, we get

$$H = \frac{1}{2}\int d^3 x \sqrt{-g_{00}}[\pi(x)^2 + (|\nabla\phi|^2 + m^2\phi^2)] \qquad (14)$$

We are in particular interested in a system of small size sitting at a fixed space coordinate $\bar{x}$ above Earth. This can be modelled by confining the field to a small box of volume V around $\bar{x}$, so that the space integral (14) can be restricted to that

volume. If the potential is approximately constant within the volume, $\Phi(x) \approx \Phi(\bar{x})$ for $x \in V$, we can take it out of the integral and obtain

$$H \approx \sqrt{-g_{00}(\bar{x})} H_0 = \sqrt{1 + 2\Phi(\bar{x})} H_0 \qquad (15)$$

where the rest Hamiltonian is given by $H_0 = (1/2)\int_V d^3x[\pi(x)^2 + |\nabla\phi|^2 + m^2\phi^2]$, which is just the usual Klein–Gordon Hamiltonian in a finite volume in Minkowski spacetime. The factor in front of H_0 in equation (15) is responsible for the gravitational redshift: all energies, when measured according to coordinate time, are rescaled with respect to those measured locally at the position of the system. We can see this explicitly by considering a cubic box of side $l = V^{1/3}$, for which the rest Hamiltonian can be diagonalized as

$$H_0 = \sum_{\mathbf{k}} E_{\mathbf{k}} a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}}$$

where, restoring units, $E_{\mathbf{k}} = c\sqrt{\mathbf{k}^2 + m^2 c^2}$, $k_j = (2\pi\hbar/l)n_j$ for $n_j \in \mathbb{N}$, $j = 1, 2, 3$ and $a_{\mathbf{k}}^{\dagger}$ ($a_{\mathbf{k}}$) creates (annihilates) a boson with momentum $\mathbf{k}$ (neglecting the constant vacuum energy). For a particle at a distance x above Earth, we can set $\sqrt{-g_{00}(x)} \approx 1 + \Phi(x)/c^2 \approx 1 + gx/c^2$, where g is the gravitational acceleration. The Hamiltonian (15) thus becomes

$$H \approx \left(1 + \frac{gx}{c^2}\right) \sum_{\mathbf{k}} E_{\mathbf{k}} a_{\mathbf{k}}^{\dagger} a_{\mathbf{k}} \qquad (16)$$

Equation 16 shows the same coupling between position and internal energy as described in the main text (equation (1)). It is valid for a system at rest in the chosen coordinate system—that is, with vanishing external momentum. The contribution of the external momentum is recovered by moving to an arbitrary coordinate system, yielding the coupling (12).

Master equation due to gravitational time dilation. Here we derive an equation of motion for the centre of mass of a composite quantum system in the presence of time dilation. We keep the composition, the centre-of-mass Hamiltonian H_{cm} and the relativistic time dilation completely general. The overall Hamiltonian of the system is $H_{tot} = H_{cm} + H_0 + H_{int}$, where H_0 governs the evolution of the internal constituents and $H_{int} = H_0 \Gamma(x,p)/c^2$ captures the time-dilation-induced coupling between internal degrees of freedom and the centre of mass to lowest order in c^{-2}. Γ is a function of the centre-of-mass position x and momentum p to which the internal states couple owing to special relativistic and general relativistic time dilation (for the Schwarzschild metric in the weak-field limit we have $\Gamma(x,p) = \Phi(x) - p^2/2m^2$). We start with the von Neumann equation for the full state $\dot{\rho} = -i/\hbar[H_{tot}, \rho]$ and write $H_{tot} = H + H_{int}$, where $H = H_{cm} + H_0$. We change frame to primed coordinates, which we define through $\rho'(t) = e^{it(H+h)/\hbar} \rho(t) e^{-it(H+h)/\hbar}$, where $h = h(x,p) = \prod_{i=1}^{N} \text{Tr}_i[H_{int}\rho_i(0)] = \Gamma(x,p)\bar{E}_0/c^2$ with the average internal energy $\bar{E}_0$. The resulting von Neumann equation is

$$\dot{\rho}'(t) = \frac{i}{\hbar}[H'(t) + h'(t), \rho'(t)] - \frac{i}{\hbar}[H'(t) + H'_{int}(t), \rho'(t)]$$
$$= -\frac{i}{\hbar}[H'_{int}(t) - h'(t), \rho'(t)]$$

where $h'(t) = h(x'(t), p'(t))$. The formal solution $\rho'(t) = \rho'(0) - (i/\hbar)\int_0^t ds[H'_{int}(s) - h'(s), \rho'(s)]$ is used in the equation above, which yields the integro-differential equation

$$\dot{\rho}'(t) = -\frac{i}{\hbar}[H'_{int}(t) - h'(t), \rho'(0)]$$
$$- \frac{1}{\hbar^2}\int_0^t ds[H'_{int}(t) - h'(t), [H'_{int}(s) - h'(s), \rho'(s)]]$$

We can now trace over the internal degrees of freedom. The state is initially uncorrelated $\rho(0) = \rho_{cm}(0) \otimes \prod_i^N \rho_i(0)$ and we take the Born approximation, keeping only terms to second order in H_{int}. In this case $\rho'(s)$ can be replaced under the integral by $\rho'_{cm}(s) \otimes \rho_i(0)$ and the master equation for the centre of mass becomes

$$\dot{\rho}'_{cm}(t) = \prod_{i=1}^{N} \text{Tr}_i[\dot{\rho}'(t)]$$
$$\approx -\frac{1}{\hbar^2}\prod_{i=1}^{N}\int_0^t ds\,\text{Tr}_i\{[H'_{int}(t) - h'(t), [H'_{int}(s) - h'(s), \rho'(s)]]\}$$
$$= -\left(\frac{1}{\hbar c^2}\right)^2 \prod_{i=1}^{N}\int_0^t ds\,\text{Tr}_i\{(H_0 - \bar{E}_0)^2[\Gamma'(t), [\Gamma'(s), \rho'(s)]]\}$$
$$= -\left(\frac{\Delta E_0}{\hbar c^2}\right)^2 \int_0^t ds[\Gamma'(t), [\Gamma'(s), \rho'_{cm}(s)]]$$

Here we used the notation $\Delta E_0^2 = \prod_{i=1}^{N} \text{Tr}_i\{(H_0 - \bar{E}_0)^2\} = \langle H_0^2\rangle - \langle H_0\rangle^2$ for the energy fluctuations of the internal states and $\Gamma'(s) = \Gamma(x'(s), p'(s))$. Changing back to the Schrödinger picture, and introducing $s \rightarrow t - s$ we obtain the integro-differential equation:

$$\dot{\rho}_{cm}(t) = -\frac{i}{\hbar}\left[H_{cm} + \Gamma(x,p)\frac{\bar{E}_0}{c^2}, \rho_{cm}(t)\right]$$
$$- \left(\frac{\Delta E_0}{\hbar c^2}\right)^2 \int_0^t ds[\Gamma(x,p), [\Gamma(x,p), \rho_{cm}(t-s)]|_s] \qquad (17)$$

where $[\Gamma, \rho_{cm}]|_s = e^{-isH_{cm}/\hbar}[\Gamma, \rho_{cm}]e^{isH_{cm}/\hbar}$. Equation 17 is the general equation of motion for a composite particle of arbitrary composition that undergoes time dilation. The decoherence of its off-diagonal elements is governed by its internal energy spread ΔE_0 and by the metric-dependent coupling Γ. The former is $\Delta E_0^2 \approx N(k_B T)^2$ in the high-temperature limit for $N/3$ non-interacting internal harmonic oscillators and the latter is $\Gamma = \Phi(x) \approx gx$ for stationary particles in the homogeneous weak-field limit of the Schwarzschild metric.

References

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