

## Appendix

In basic electricity consumption optimization problems, a binary variable  $x_{m,t}$  represents the operating status of adjustable appliance  $m$  at time  $t$ . Adjustable appliances are categorized into shiftable appliances, interruptible appliances (e.g., electric vehicles), and thermostatically controlled appliances (e.g., air conditioners). For shiftable appliances, the start time can be adjusted within a specified period, but once started, they must run for a fixed duration. This is expressed by the following constraints:

$$\sum_{i=t}^{t+T_m^r} x_{m,i} \geq s_{m,t} \cdot T_m^r \quad (1)$$

$$U_m^{\min} \cdot T_m^r \leq \sum_{t=1}^{T^{\text{total}}} x_{m,t} \leq U_m^{\max} \cdot T_m^r \quad (2)$$

where  $s_{m,t}$  is an auxiliary variable indicating appliance start,  $T^{\text{total}}$  is the total number of optimization periods,  $T_m^r$ ,  $U_m^{\min}$ , and  $U_m^{\max}$  represent the operating duration, minimum daily usage, and maximum daily usage of appliance  $m$ , respectively.

For electric vehicles, the battery state of charge must exceed a threshold  $\text{soc}^{\text{set}}$  before the departure time  $T^{\text{ev}}$ . The change in battery state of charge  $\text{soc}_t$  is expressed as follows<sup>1</sup>.

$$\text{soc}_t = \text{soc}_{t-1} + (x_{\text{ev},t} \cdot P^{\text{ev}} \cdot \eta^{\text{ev}} \cdot \Delta t) / b^{\text{max}} \quad (3)$$

where  $P^{\text{ev}}$ ,  $\eta^{\text{ev}}$ , and  $b^{\text{max}}$  represent the charging power, charging efficiency, and maximum battery capacity, respectively.

For air conditioners, the operating status is related to indoor and outdoor temperatures ( $TP_t^{\text{in}}$ ,  $TP_t^{\text{out}}$ ) based on the Equivalent Thermal Parameter (ETP) model, as shown below:

$$\begin{aligned} TP_t^{\text{in}} &= e^{-\Delta t/RC} \cdot TP_{t-1}^{\text{in}} + R \cdot (e^{-\Delta t/RC} - 1) \cdot P^{\text{ac}} \cdot x_{\text{ac},t-1} \\ &+ (1 - e^{-\Delta t/RC}) \cdot TP_{t-1}^{\text{out}} \end{aligned} \quad (4)$$

where  $R$  and  $C$  are the thermal resistance and thermal capacitance of the house, and  $P^{\text{ac}}$  is the air conditioner's power.

Electricity consumption optimization may aim to minimize costs, maintain indoor temperature preferences, meet appliance usage preferences, and reduce electric vehicle charging losses<sup>2</sup>. These objectives are combined into a single minimization target using weighted coefficients  $w$ , as shown below:

$$\begin{aligned} \text{obj} &= w_1 \cdot \sum_{m=1}^M \sum_{t=1}^{T^{\text{total}}} x_{m,t} \cdot P_m \cdot \Delta t \cdot \delta_t \\ &+ w_2 \cdot \sum_{t=1}^{T^{\text{total}}} 2 \cdot \left| TP_t^{\text{in}} - \frac{TP_{\text{max}}^{\text{in}} + TP_{\text{min}}^{\text{in}}}{2} \right| / (TP_{\text{max}}^{\text{in}} - TP_{\text{min}}^{\text{in}}) \\ &+ w_3 \cdot \sum_{m=1}^M \sum_{t=1}^{T^{\text{total}}} x_{m,t} \cdot PR_{m,t} + w_4 \cdot \sum_{t=1}^{T^{\text{total}}} c_t^{\text{ev}} \cdot \beta^{\text{ev}} \end{aligned} \quad (5)$$

where  $M$  is the total number of adjustable appliances,  $\delta$  is the electricity price, and  $TP_{\text{max}}^{\text{in}}$  and  $TP_{\text{min}}^{\text{in}}$  are the maximum and minimum indoor temperature setpoints. Besides,  $PR_{m,t}$  is the user's preference weight for appliance  $m$  at time  $t$ ,  $c_t^{\text{ev}}$  and  $\beta^{\text{ev}}$  are the charging state indicator and loss coefficient for the electric vehicle, respectively.

The above integer programming optimization problem can be solved with solvers like Gurobi<sup>3</sup> and Scipy<sup>4</sup>.

## References

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- 2 Peng, B. *et al.* Integrating non-intrusive load monitoring based on graph-to-point learning into a self-adaptive home energy management system. *Int. J. Electr. Power Energy Syst.* **154**, 109442 (2023).
- 3 Gurobi. *Gurobi Optimizer: Mathematical Optimization Software*, <<https://www.gurobi.com/>> (2025).
- 4 Community, T. S. *SciPy: Open-source scientific computing tools for Python*, <<https://scipy.org/>> (2025).