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ROSSBY WAVE PROPAGATION IN A BAROTROPIC ATMOSPHERE

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ABSTRACT

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The ideas of ray tracing from geometrical optics and wave propagation in a slowly varying medium are applied to Rossby waves propagating in a barotropic atmosphere.

The propagation of low-frequency Rossby waves in a zonally symmetric basic state is compared with that for stationary waves presented by Hoskins and Karoly (1981). These ideas are then used to study the propagation of Rossby waves in a basic state with zonally varying middle latitude or low latitude jets. Conditions which allow cross-equatorial wave propagation are presented. For a zonally varying middle latitude jet, there is weak wave convergence in regions of decreasing jet speed. However, this is not sufficient to explain the enhanced wave amplitude found in numerical-model experiments using a zonally varying basic state.

1. INTRODUCTION

A recent review of the theory of stationary eddies in the extra-tropical troposphere by Held (1983) has shown that the far-field response to large-scale forcing in the atmosphere is dominated by the external Rossby wave, in agreement with the numerical model solutions of Hoskins and Karoly (1981; hereafter referred to as HK). This is supported by the observed barotropic vertical structure of the planetary-scale atmospheric teleconnections found by Wallace and Gutzler (1981). This suggests that a barotropic model may be used to study the horizontal propagation of planetary waves in the atmosphere.

In HK, the ideas of ray tracing from geometrical optics and wave propagation in a slowly varying medium were used to study stationary planetary wave propagation in a zonally symmetric barotropic atmosphere. Good agreement was found between the wavetrains obtained using this ray

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tracing, the forced stationary wave solutions from a baroclinic numerical model and some of the teleconnections in Wallace and Gutzler (1981), particularly the Pacific–North American pattern of anomalies in the upper troposphere. In this paper, the ray tracing of barotropic Rossby waves used in HK is applied to the propagation of low-frequency waves in a zonally symmetric basic state and to the propagation of stationary waves in a zonally varying basic state.

The dispersion of low-frequency waves from an initial disturbance or from transient forcing makes a contribution to a time-mean anomaly pattern and its importance depends on the relative amplitudes of the stationary and low-frequency wave components. It is, therefore, of interest to consider any differences between the propagation of low-frequency and stationary waves. Mekki and McKenzie (1977) obtained ray solutions for a number of situations of Rossby waves propagating through a meridionally varying flow in the atmosphere but they did not try to relate their solutions to observations. Schopf et al. (1981) have recently used these ideas to investigate the dispersion of low-frequency Rossby waves in the ocean.

The use of a zonally symmetric basic state in HK is a crude representation of the time-mean flow in the troposphere. Recent model experiments have shown that zonal variations of the basic state may be important in determining the planetary wave response. The forced stationary wave solutions of a barotropic numerical model with a zonally varying basic state obtained by Webster and Holton (1982) suggest that cross-equatorial propagation, which is normally blocked by equatorial easterlies, is possible if a region of westerly wind exists in the zonal mean equatorial easterlies. The results obtained by Simmons (1982) from a similar model show an increased wave response in a zonally varying basic state compared to that when a zonally symmetric basic state is used. These two effects are investigated using two-dimensional ray tracing of stationary waves in a zonally varying basic state.

2. RAY THEORY

As in HK, solutions of the non-divergent barotropic vorticity equation on the sphere are considered and again we use a Mercator projection of the sphere,

$$x = a\lambda, \; y = a \ln[(1 + \sin\theta)/\cos\theta] \tag{1}$$

where λ is longitude, θ latitude and a the radius of the Earth. Then the Laplacian is

$$\nabla^2 = \frac{1}{\cos^2\theta}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) = \frac{\nabla_M^2}{\cos^2\theta}$$

and the velocity in the Mercator projection is

$$\mathbf{v}_M = (u_M, v_M) = \mathbf{v}/\cos\theta$$

The equation for the horizontal streamfunction ψ takes the form

$$\left(\frac{\partial}{\partial t} + u_M\frac{\partial}{\partial x} + v_M\frac{\partial}{\partial y}\right)\left(\frac{\nabla_M^2\psi}{\cos^2\theta}\right) + \frac{2\Omega\cos^2\theta}{a}\left(\frac{1}{\cos^2\theta}\frac{\partial\psi}{\partial x}\right) = F \tag{2}$$

where Ω is the Earth's rotation rate and F is the forcing of the mean flow.

2.1. *Zonally symmetric flow*

When (2) is linearized about a zonally symmetric basic state with zonal flow $\bar{u}_M(y)$, the equation for the perturbation streamfunction ψ' becomes (HK; eq. 5.9)

$$\left(\frac{\partial}{\partial t} + \bar{u}_M\frac{\partial}{\partial x}\right)\nabla_M^2\psi' + \beta_M\frac{\partial\psi'}{\partial x} = 0 \tag{3}$$

where

$$\beta_M = 2\Omega\cos^2\theta/a - \frac{\partial}{\partial y}\left[\frac{1}{\cos^2\theta}\frac{\partial}{\partial y}(\cos^2\theta\bar{u}_M)\right]$$

is $\cos\theta$ times the meridional gradient of the absolute vorticity. Assuming that there is a separation in scale between the mean flow and the perturbations, we look for a solution of (3) using the WKB form (as in Young and Rhines, 1980),

$$\psi' = A(X, Y, T)\exp[i\epsilon^{-1}\phi(X, Y, T)] \tag{4}$$

where $(X, Y, T) = \epsilon(x, y, t)$ and

$$\epsilon = \frac{\text{length scale of the perturbations}}{\text{length scale of the mean flow}} \ll 1$$

Substituting (4) into (3) and equating like powers of ϵ leads to a series of differential equations for the wave phase ϕ and amplitude A.

The zeroth-order equation gives the dispersion relation,

$$\omega = \bar{u}_M k - \beta_M k/K^2 \tag{5}$$

where the wavenumber $\mathbf{K} = (k, l)$ and the frequency ω are defined to be the derivatives of the phase

$$(k, l, \omega) = (\phi_X, \phi_Y, -\phi_T)$$

Expressions for the group velocity $\mathbf{c}_g = (u_g, v_g)$ are obtained from the dispersion relation using kinematic wave theory (Whitham, 1960) or from the first-order equation for wave amplitude, giving

$$u_g = \omega/k + 2\beta_M k^2/K^4, \quad v_g = 2\beta_M kl/K^4 \tag{6}$$

The WKB solution is valid provided that $(1/A)\nabla^2 A \ll K^2$ and $\nabla \cdot \mathbf{K} \ll K^2$.

Since the basic state is zonally symmetric and independent of time, ω and k are constant. The meridional wavenumber l varies with position to satisfy the dispersion relation. A wave ray is defined to be the integral path of the group velocity and shows the propagation of wave activity.

The simplification of the ray equations for stationary (zero frequency) waves has been given in HK. It was shown there that an important quantity for stationary-wave propagation is the stationary wavenumber, $K_s = (k^2 + l^2)^{1/2} = (\beta_M/\bar{u}_M)^{1/2}$. For waves with non-zero frequency, the difference between the flow speed and the phase speed of the wave takes the role of the flow speed for stationary waves. The total wavenumber then becomes $K_\omega = [\beta_M/(\bar{u}_M - \omega/k)]^{1/2}$. A critical line exists where the flow speed equals the phase speed of the wave and, in a linear dissipative model, it acts as a wave absorber. Waves with easterly phase speed have a critical line in easterly wind. Since the phase speed is inversely proportional to k, low-frequency wave solutions approach the stationary-wave solutions for increasing k. The direction of propagation is not given by the ratio of the meridional and zonal wavenumbers for low-frequency waves and westward propagation is possible.

2.2. *Zonally varying flow*

Returning to (2), this equation is linearized about a time-mean basic state, with streamfunction $\bar{\psi}(x, y)$, which is a function of longitude as well as latitude. The equation for the perturbation streamfunction can then be written

$$\left(\frac{\partial}{\partial t} + \bar{u}_M \frac{\partial}{\partial x} + \bar{v}_M \frac{\partial}{\partial y}\right)\nabla_M^2 \psi' + \bar{q}_y \psi'_x + \bar{q}_x \psi'_y = 0 \tag{7}$$

where $\bar{q} = \nabla_M^2 \bar{\psi}/\cos^2\theta + 2\Omega \sin\theta$ is the absolute vorticity on the sphere and $\bar{q}_y = \beta_M$. Again, solutions of the WKB form (4) are obtained and the dispersion relation is

$$\omega = \bar{u}_M k + \bar{v}_M l + (\bar{q}_x l - \bar{q}_y k)/K^2 \tag{8}$$

The group velocity is then

$$u_g = \bar{u}_M + \left[(k^2 - l^2)\bar{q}_y - 2kl\,\bar{q}_x\right]/K^4 \tag{9}$$

and

$$v_g = \bar{v}_M + \left[2kl\,\bar{q}_y + (k^2 - l^2)\bar{q}_x\right]/K^4$$

These expressions are simple extensions of those for the zonally symmetric flow, including advection by the meridional flow and derivatives in the zonal direction.

Since the basic state varies with latitude and longitude, both k and l vary with position along a ray and are given, from kinematic wave theory, by

$$\frac{\mathrm{d}_g k}{\mathrm{d}t} = -\frac{\partial \omega}{\partial x} = -k\bar{u}_x - l\bar{v}_x + (\bar{q}_{xy}k - \bar{q}_{xx}l)/K^4 \tag{10}$$

and

$$\frac{\mathrm{d}_g l}{\mathrm{d}t} = -\frac{\partial \omega}{\partial y} = -k\bar{u}_y - l\bar{v}_y + (\bar{q}_{yy}k - \bar{q}_{xy}l)/K^4$$

where $\mathrm{d}_g/\mathrm{d}t = \partial/\partial t + \mathbf{c}_g \cdot \nabla$ is the material derivative moving with the group velocity. Expressions for the wave action and energy can be obtained after some algebra, as in Young and Rhines (1980), but they cannot be used to obtain a simple expression for the wave amplitude, as was possible in HK. The WKB solution is again valid provided that $(1/A)\nabla^2 A \ll K^2$ and $\nabla \cdot \mathbf{K} \ll K^2$. The validity of the solution is not tested using these expressions but rather by comparison with appropriate numerical-model solutions.

It should be noted that, for strict application of this ray theory, the zonal periodicity of the sphere should be dropped. Then the zonal wavenumber is a continuous variable and rays for integer values should be taken as representative of zonal wavenumbers in a band centred on that value. The application of these ideas to the sphere is valid only when propagation around the whole sphere is not possible due to dissipation, wave absorption at a critical line or some other limitation on the extent of propagation.

3. LOW-FREQUENCY WAVE PROPAGATION

To compare the propagation of low-frequency waves with that for stationary waves, the ray equations for a zonally symmetric flow from the previous section are integrated. The basic states are spherical harmonic representations of the global zonal mean 300 mb zonal flow for the seasons December–February (DJF) and June–August (JJA) from Newell et al. (1972). The flows and the associated stationary wavenumber profiles are shown in Fig. 1. A slight smoothing of the flows has removed any small-scale meridional variations in the stationary wavenumber.

The differential equations for the ray and wavenumber variation are integrated using a fourth-order Runge–Kutta scheme for given initial position and wavenumber. The initial wavenumber has integer zonal wavenumber from one to six and both positive and negative meridional wavenumber so both poleward and equatorward propagation are possible. The rays and propagation speeds for sources at 30°N and 30°S in the DJF flow are shown in Fig. 2 for both stationary and low-frequency waves. The frequencies of the waves are chosen to be representative of those which could contribute to

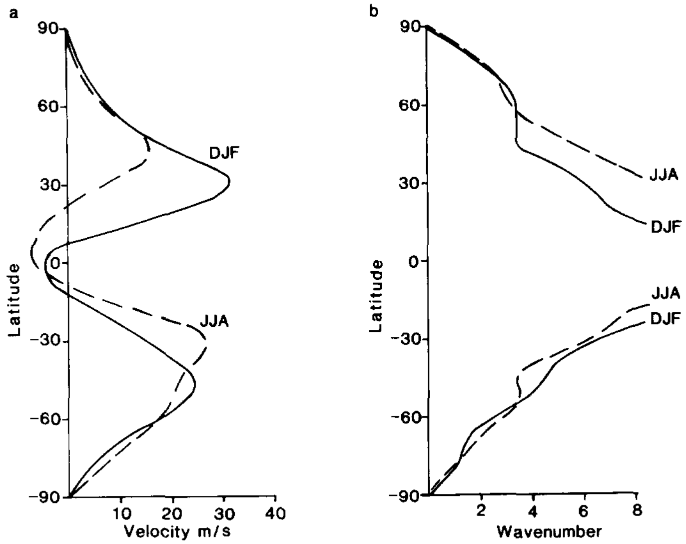


Fig. 1. (a) Zonal flow for December–February (DJF) and June–August (JJA) used for the zonally symmetric basic state. (b) Stationary wavenumber profiles determined from the zonal flow.

5-day mean anomalies. The low-frequency waves have periods of 50 and 20 days, corresponding to easterly phase speeds for zonal wavenumber one of 9.3 and 23.2 m s^{-1}, respectively. The phase speeds for higher zonal wavenumbers can be found be dividing the wavenumber one phase speed by the zonal wavenumber. The rays are terminated close to a critical line or after 15 days propagation, based on the observed dissipative time scale for planetary waves in the troposphere (Lau, 1979).

The rays for stationary waves in the Northern Hemisphere (NH) are the same as in HK, except that a rectangular latitude–longitude plot is used. In the Southern Hemisphere (SH) summer, the pattern of wave propagation is much the same, with zonal wavenumbers two to four following similar paths, but not propagating as far into high latitudes as wavenumber one. The speed of propagation is faster in the SH than in the NH because of the stronger flow in the SH.

For the 50-day period waves in Fig. 2(b), the rays for higher zonal wavenumbers are changed only slightly from those for stationary waves, with reduced eastward propagation and small westward propagation close to the critical line. The rays for zonal wavenumbers one and two have larger changes, with pronounced westward propagation at low latitude and increased propagation speed. The wavenumber one easterly phase speed is larger than the easterly jet maximum so the propagation of wavenumber one

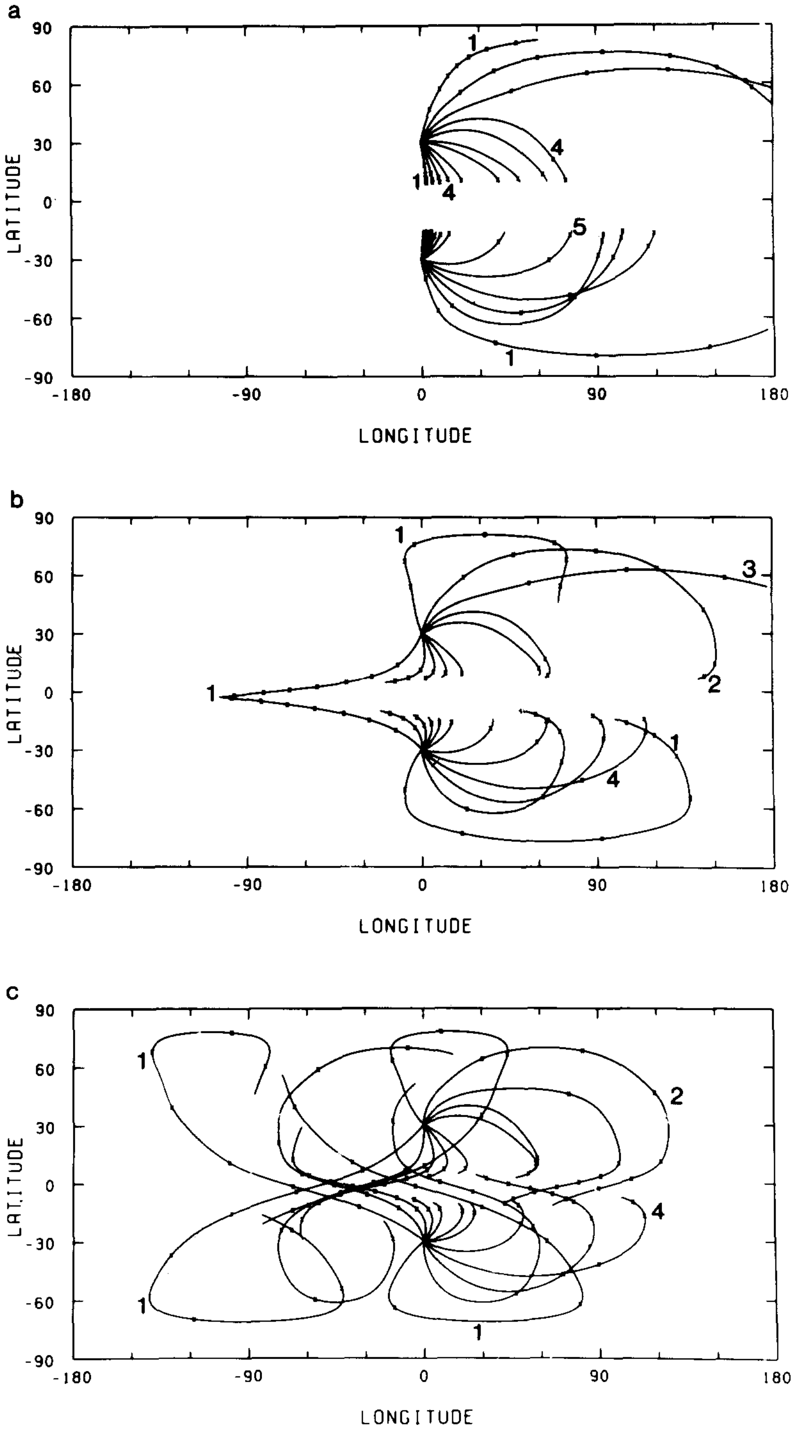


Fig. 2. Rays and propagation speed, shown by crosses at 2-day time intervals, for sources at 30°N and 30°S in the DJF flow. Some of the zonal wavenumbers associated with the rays are indicated. (a) Stationary waves; (b) 50-day; and (c) 20-day period waves with easterly phase speed.

is not limited by critical layer absorption. The phase structure of these low-frequency waves is much the same as for the stationary waves.

For the 20-day period waves, the higher wavenumbers have greater meridional propagation into the equatorial easterlies and large westward propagation in the easterlies. Wavenumbers one, two and three have easterly phase speeds larger than the easterly jet maximum and high propagation speeds so they propagate freely between the hemispheres, being reflected at

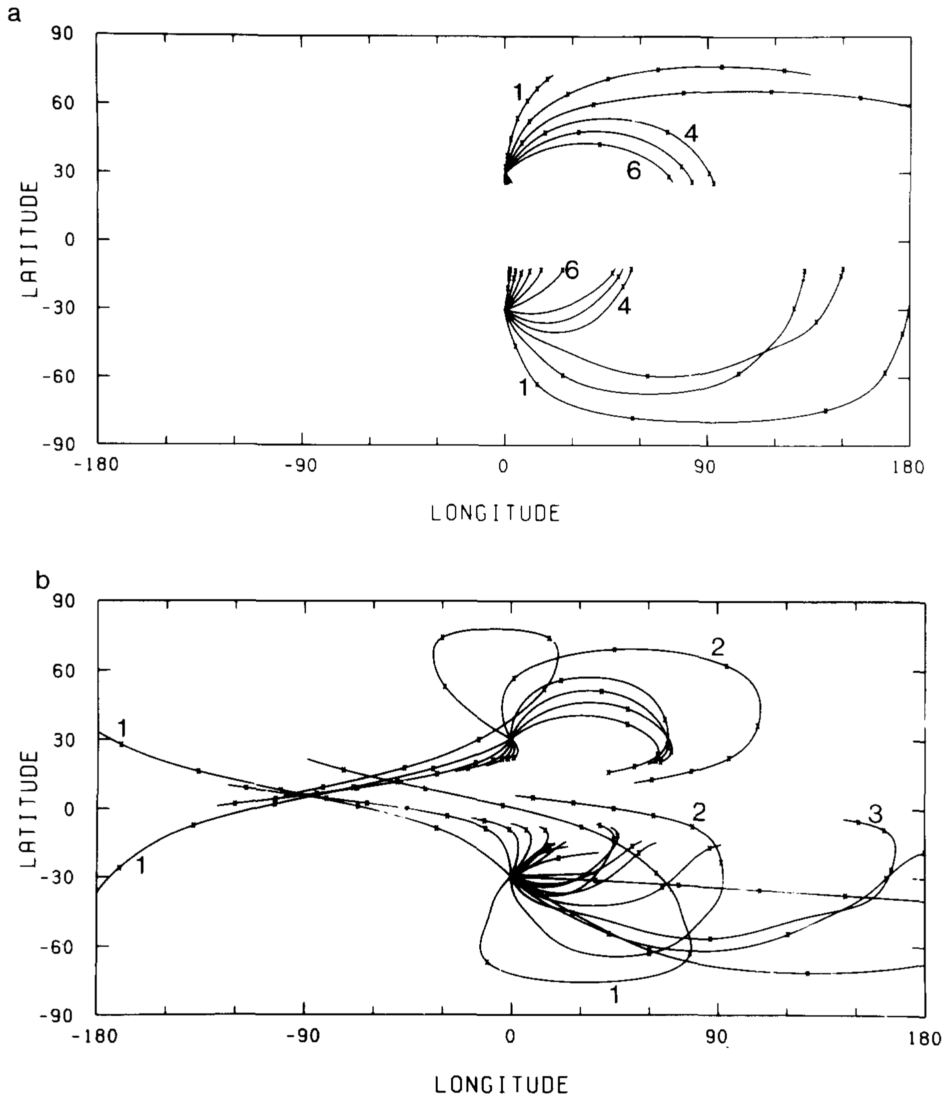


Fig. 3. As in Fig. 2 but for the JJA flow. (a) Stationary waves and (b) 20-day period waves with easterly phase speed.

high latitudes. If the dissipation is not large, these waves can propagate around the sphere and interact with the wave source. In this situation, the rays are difficult to interpret as wave propagation from a local wave source and a resonant or normal mode wave solution may be more appropriate. For higher frequency (shorter period) waves, more of the higher wavenumbers have phase speeds larger than the easterly jet maximum and propagate between the hemispheres so the ray solution is inappropriate unless the dissipation is very strong.

The equatorial easterlies in the JJA flow are larger than in the DJF flow, so waves need larger easterly phase speeds before they can propagate between the hemispheres. The rays for stationary and 20-day period waves in the JJA flow are shown in Fig. 3. In the NH summer, the flow is weaker than in the winter so the propagation speeds are smaller and the phase variation is larger. In the SH winter, the flow is stronger so the reverse occurs. The pattern of propagation for stationary waves is generally the same for the two flows, with the largest difference being a greater split between the low and high wavenumbers in the SH winter than in the summer.

For the 20-day period waves in the JJA flow, only zonal wavenumbers one and two have easterly phase speeds larger than the easterly jet maximum but they do not propagate strongly into the opposite hemisphere as the westward propagation is large. For the higher wavenumbers, the difference between the propagation of the low-frequency and stationary waves is smaller in the JJA flow than in the DJF flow.

Low-frequency waves with westerly phase speeds have slightly increased poleward propagation but the critical line is in westerly wind and is located poleward of that for stationary waves. Thus, the latitude band in which wave propagation can occur is reduced compared to that for stationary or easterly phase speed waves.

Hoskins et al. (1977) studied energy dispersion on the sphere using a linearised barotropic numerical model. They found that, for realistic zonal flows with equatorial easterlies, large-scale transient waves cross the equator but small-scale transients and the stationary response are trapped poleward of the easterlies, in agreement with these ray solutions. However, their solutions for transient forcing do not show the westward propagation for zonal wavenumbers one and two shown by the rays. This may be due to the wave frequencies chosen for the rays, since higher-frequency waves have faster meridional propagation and reduced westward propagation.

4. ZONALLY VARYING EASTERLY JET

The ray equations for a zonally varying flow from Section 2.2 are integrated for a basic state with a zonally varying easterly jet at the equator.

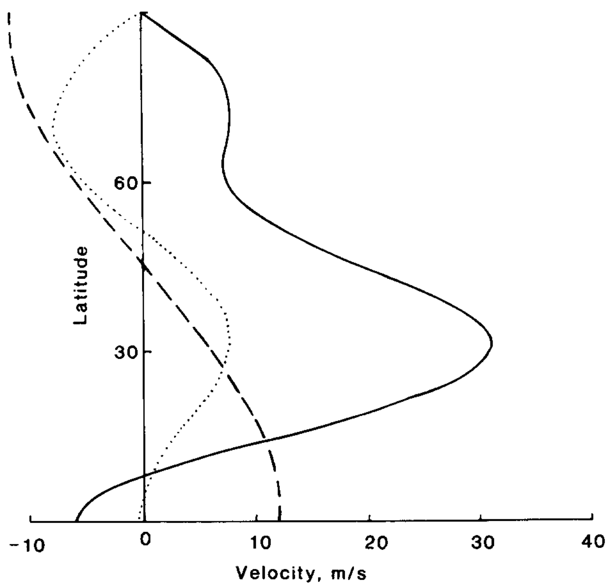
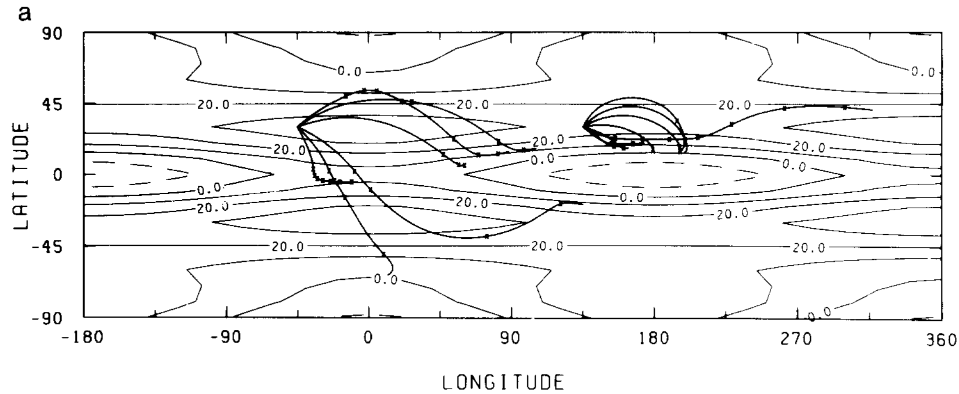


Fig. 4. Zonal mean zonal flow (solid) and amplitude of the zonal wavenumber one (dashed) and zonal wavenumber two (dotted) components for the zonally varying basic state.

The flow is a simple representation of the observed flow in the tropical upper troposphere from Newell et al. (1972). This is obtained from a spherical harmonic representation of the basic state relative vorticity, using four symmetric components for the zonal mean and a single symmetric zonal wavenumber one component for the zonal variation. The zonal mean flow is a smoothed representation of the Northern Hemisphere winter 300 mb flow, having a mid-latitude westerly jet of 31 m s^{-1} and equatorial easterlies of 6 m s^{-1}. The zonal mean flow profile is shown in Fig. 4 together with the amplitude of the zonal wavenumber one flow variation, which is 12 m s^{-1} at the equator and has a zero at 45°. The full flow has zonal mean easterlies at the equator but the equatorial jet varies from 18 m s^{-1} easterlies to 6 m s^{-1} westerlies.

The differential equations for the ray and wavenumber (9) and (10) are integrated using the same procedure as in the previous section. Stationary waves with zonal wavenumbers three and higher are considered so that there is some scale separation between the mean flow variation and the waves. The rays are terminated at a critical line or after 15 days propagation as before. The rays for two sources at 30°N are shown in Fig. 5, together with the zonal flow.

For the source poleward of the tropical easterlies, the wave propagation is very similar to that for stationary waves in a zonally symmetric basic state,



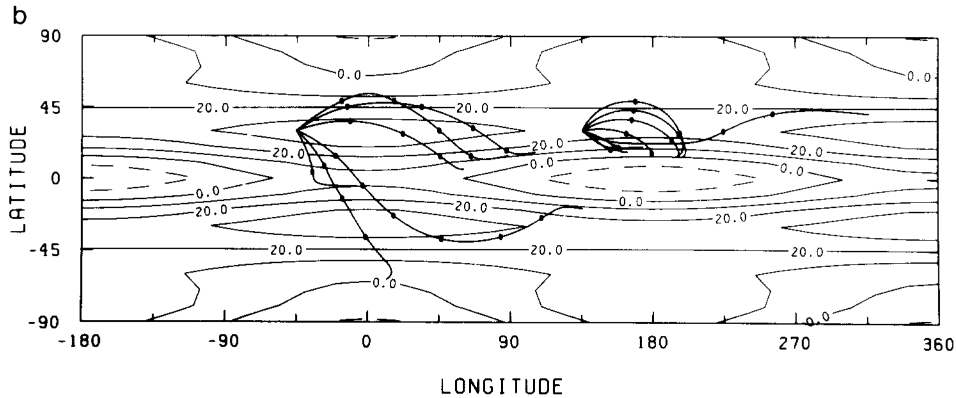


Fig. 5. Rays for stationary waves in the basic state with zonally varying low-latitude easterlies from sources at 30°N for zonal wavenumbers $\geqslant 3$. Contours show the zonal flow, with contour interval of $10\ \mathrm{m\ s^{-1}}$ and dashed contours for easterly flow. (a) Propagation time marked by crosses at 2-day time intervals. (b) Phase variation marked by circles at 180° intervals. If all wavelengths give an extremum at the source, the circles indicate the positions of successive extrema of opposite sign.

shown in Fig. 2(a). The propagation is affected very little by the zonal variation and the equatorward propagation is terminated at the critical line at low latitude. For the source poleward of the low-latitude westerlies, the equatorward propagating waves do not encounter a critical line and propagate into the opposite hemisphere. The speed of propagation, shown in Fig. 5(a) is small in the weak westerlies at low latitude so dissipation is likely to be important there. The phase variation along the rays, shown in Fig. 5(b), is similar to that for the zonally symmetric flow poleward of the source but shows several amplitude maxima in the opposite hemisphere for the propagation through the westerly duct.

These solutions are in good agreement with the numerical-model solutions of Webster and Holton (1982), showing interhemispheric propagation when the source is in the same longitude region as a westerly wind duct in the low-latitude easterlies. The positions of the maxima shown in Fig. 5(b) agree with the positions of the maxima shown in Fig. 12 of Webster and Holton (1982). Since the zonal mean wind field used in the model is weaker than that used here, the wave propagation speeds in the model are smaller and dissipation is important. Ray solutions have been obtained with different amplitudes for the zonally varying part of the flow but the cross-equatorial propagation does not show much sensitivity to this, as long as a region of equatorial westerlies still exists.

5. ZONALLY VARYING MID-LATITUDE JET

To provide a representation of the observed Northern Hemisphere winter tropospheric jet, the zonal mean basic state from the previous section is combined with a simple zonal wavenumber two flow variation to give a zonally varying westerly jet. The amplitude of the wavenumber two flow variation, shown in Fig. 4, is about 7 m s^{-1} at 30° and -8 m s^{-1} at 70°, with zeros at the equator, 51° and the pole. The mid-latitude westerly jet varies from 24 m s^{-1} to 38 m s^{-1}, consistent with the observed zonal variation in Newell et al. (1972).

The ray equations are integrated as before. To see the effect of the zonal variation of the jet on wave propagation, poleward propagation of stationary waves with wavenumbers three and higher from sources at 15°N and 45° longitude intervals is considered. Equatorward propagation is not considered as it is terminated close to the source by the low-latitude critical line. The rays in Fig. 6(a) show that the wave propagation is similar to that for a zonally symmetric flow, with little effect from the jet variations. There is a tendency for increased zonal propagation in the regions of increasing westerlies and reduced propagation in the decreasing westerlies so there is slight wave convergence from the jet maximum to the jet minimum. This can be seen more clearly in a similar basic state, for which the wavenumber two flow component has doubled amplitude, so the westerly jet varies from 16 m s^{-1} to 46 m s^{-1}. The rays for this basic state in Fig. 5(b) show wave convergence into the low-latitude region from the jet maximum to the jet minimum.

This is consistent with the large amplitude response at the jet minimum found by Simmons (1982) for low-latitude forcing in a barotropic numerical model using the zonally varying NH January 300 mb flow. However, the solutions obtained by Simmons have maximum amplitude near 40° and have larger zonal and poleward propagation than in the ray solutions. It is necessary to consider the propagation of smaller zonal wavenumbers to

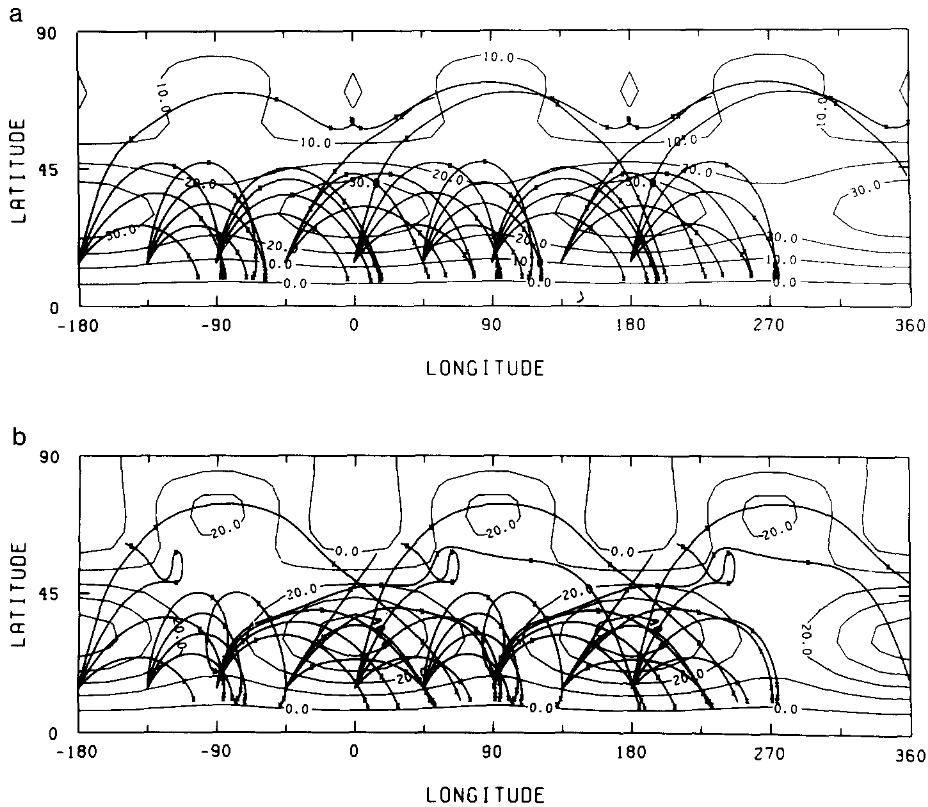


Fig. 6. Rays for stationary waves in the basic state with zonally varying mid-latitude jet from sources at 15°N and 45° longitude intervals. Only poleward rays for zonal wavenumbers $\geqslant 3$ are shown. Contours show the zonal flow as in Fig. 5. (a) Basic state with zonal wavenumber two flow amplitude shown in Fig. 4. (b) Basic state with doubled amplitude for zonal wavenumber two flow component.

obtain increased zonal and poleward propagation of the rays, but the ray theory is then invalid as there is no scale separation between the flow variation and the waves. Although there is an indication of convergence in the rays, it does not seem possible to explain the large amplitude enhancement of the model response in a zonally varying mid-latitude jet using these ray solutions.

The simple mean flow used for this case does not have the small length-scale variations of the observed mean flow so the ray solutions, which require scale separation between the waves and the mean flow, are more likely to be valid for the simple flow. However, the differences between the ray solutions and Simmons' solutions indicate that the ray solutions are not appropriate for this case.

6. DISCUSSION

The ray tracing technique described in HK and in this paper provides a simple and inexpensive method for studying the propagation of forced planetary waves in different basic states. The agreement between the rays and the numerical solutions of linearised barotropic models shows that the essential characteristics of barotropic Rossby wave propagation is retained by the rays. However, before this technique can be applied with confidence to more realistic situations, its sensitivity and limitations must be investigated. This study has extended the ray tracing technique to allow for non-zero frequency waves and zonally varying basic states.

In general, the differences between the propagation of stationary and very-low-frequency waves are small and decrease with increasing zonal wavenumber. The major difference is the change in position or even the existence of a critical line, which can allow propagation of waves with easterly phase speed into the opposite hemisphere. Higher frequency waves propagate between the hemispheres, trapped between the poles so that, if the dissipation is not strong, propagation around the sphere is possible and a resonant or normal mode solution may be more useful than the rays. For short time periods, the propagation from a local source still resembles that for stationary waves. The 5-day lagged anomaly correlation patterns in Lau (1981) and the transient wave solutions in Hoskins et al. (1977) agree better with the stationary wave ray solutions than with the transient wave rays. This suggests that the wave response to slowly time varying forcing appears as stationary waves with varying amplitude rather than as low-frequency waves. Thus the stationary wave rays may be more relevant to wave propagation in the atmosphere than the low-frequency wave rays.

For stationary waves and a zonally varying easterly jet, both the ray solutions and the barotropic model solutions obtained by Webster and Holton (1982) show cross-equatorial stationary wave propagation through a westerly wind duct in the zonal mean easterlies. The model solutions are sensitive to the magnitude of the westerly wind in the duct but, because a basic state with stronger zonal flow is used for the ray solutions, they are not as sensitive. An observational study of cross-equatorial anomaly correlations might show the importance of this effect in the atmosphere.

The limitations of this technique are highlighted in the basic state with zonally varying mid-latitude jet. It is not possible using the ray tracing to obtain much indication of increased wave response in a zonally varying basic state compared to that in a zonally symmetric basic state. The model solutions indicate that small zonal wavenumbers, for which the ray solutions are not valid, may be important in the increased response.

This study has shown that two-dimensional ray tracing can be used to give ideas on the speed and direction of horizontal wave dispersion from a local source in a number of basic states relevant to the atmosphere. However, care must be taken with the interpretation and application of quantitative details from the ray solutions to the atmosphere.

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