

# Linear Regression

## Useful Formulas

- Pseudoinverse of matrix  $\Phi$ :  $\Phi_p = (\Phi^T \Phi)^{-1} \Phi^T$

## Questions

1. What is the role of basis functions in linear regression?
2. Can an algorithm doing linear regression learn only linear functions of the inputs?
3. When can we solve the linear regression problem exactly (with 0 error)? Why is it not a good idea to do so?
4. What is the error we want to minimize when doing linear regression?
5. What is the least-squares solution? How is it affected by outliers?
6. How can we find the least-squares solution when there are too many points to compute the pseudoinverse efficiently?
7. What are the bias and the variance for a supervised learning problem?
8. What is the link between the error on the validation set increasing with training, and the bias/variance decomposition?
9. Given the dataset:  $\langle -1, -0.5 \rangle, \langle 0, 1.1 \rangle, \langle 1, 3.8 \rangle, \langle 2, 8.8 \rangle$ , find the least-squares solution for the function:  $y(x, \mathbf{w}) = w_0 + w_1 x$
10. 10. Given the dataset:  $\langle -1, 0.78 \rangle, \langle 0, 1 \rangle, \langle 1, 1.22 \rangle, \langle 2, 1.52 \rangle$ , find the least-squares solution for  $y(x, \mathbf{w}) = w_0 + w_1 e^{\frac{(x+1)^2}{20}}$
11. Given the dataset:  $\langle -1, 1.6 \rangle, \langle 0, 0.95 \rangle, \langle 1, 1.2 \rangle, \langle 2, 1.9 \rangle$ , find the least-squares solution for the function:  $y(x, \mathbf{w}) = w_0 + w_1 \frac{1}{1 + e^{-(x+1)}}$