Fake Pevatrons

Elena Amato

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1 Maximum proton energy during the evolution of a SNR

The maximum energy of a SNR can be found by solving the following equation (see Cardillo, Amato, Blasi 2015):

$$\Psi(E_{\max}) = \frac{2e}{(4-m_i)5cE_0}\xi_{CR}v_s(R)^2\sqrt{4\pi\rho(R)R^2} \tag{1}$$

where e is the electron charge, c is the speed of light, R is the current radius of the SNR, v_s the current shock velocity, ρ the mass density of the ambient medium upstream of the shock, E_0 the minimum energy of the particles in the distribution, ξ_{CR} the fraction of energy that goes into cosmic rays, $m_i = 0, 2$ depending on whether the remnant is expanding in the ISM ($i = 1$, type Ia explosions) or in the progenitor wind ($i = 2$, core-collapse SNe expanding in a density profile $\propto r^{-2}$). The function Ψ carries information about the spectrum of accelerated particles, assumed in the form $N(E) \propto E^{-2-\beta}$, with:

$$\Psi(E_{\max}) = \begin{cases} (E_{\max}/E_0)\ln(E_{\max}/E_0) & \beta = 0 \\ \frac{1+\beta}{\beta}\left(\frac{E_{\max}}{E_0}\right)^{1+\beta}\left[1-\left(\frac{E_0}{E_{\max}}\right)^{\beta}\right] & \beta \neq 0 \end{cases} . \tag{2}$$

In order to solve Eq. 1 the additional pieces of information needed are:

$$R(t) = R_{\rm ST}\left[\left(\frac{t}{t_{\rm ST}}\right)^{a\lambda_{\rm ED}} + \left(\frac{t}{t_{\rm ST}}\right)^{a\lambda_{\rm ST}}\right] \tag{3}$$

and

$$v_s(t) = \frac{R_{\rm ST}}{t_{\rm ST}} \left(\frac{R}{R_{\rm ST}}\right)^{1-a} \left[\lambda_{\rm ED} \left(\frac{t}{t_{\rm ST}}\right)^{a\lambda_{\rm ED}-1} + \lambda_{\rm ST} \left(\frac{t}{t_{\rm ST}}\right)^{a\lambda_{\rm ST}-1}\right] \tag{4}$$

where a is a smoothing parameter, fitted by Martina as $a = -5$, $R_{\rm ST}$ and $t_{\rm ST}$ are the radius and age of the remnant at the transition between the ejecta dominated and Sedov-Taylor phase of expansion and $\lambda_{\rm ED}$ and $\lambda_{\rm ST}$ are the power indices describing the remnant expansion during the ejecta-dominated and Sedov-Taylor phases respectively. One has:

$$\lambda_{\rm ED} = \frac{k_i - 3}{k_i - m_i} \quad \text{and} \quad \lambda_{\rm ST} = \frac{2}{5 - m_i} \tag{5}$$

where k_i is the power-law index describing the ejecta density profile, with

$$k_i = \begin{cases} 7 & \text{if} \quad i = 1 \\ 9 & \text{if} \quad i = 2 \end{cases} . \tag{6}$$

The final missing ingredient to be able to solve the above set of equations is the definition of the Sedov-Taylor time and radius for the two types of explosions. We define $R_{\rm ST}$ the size of the remnant at the time when it has swept up a mass equal to that of the ejecta. In the case of type Ia explosions this means:

$$R_{\rm ST,1} = \left(\frac{3M_{\rm ej}}{4\pi\rho_{\rm ISM}}\right)^{1/3} = 2\text{pc} \, \left(\frac{M_{\rm ej}}{M_\odot}\right)^{1/3} \left(\frac{n_{\rm ISM}}{\text{cm}^{-3}}\right)^{-1/3} \tag{7}$$

while for core collapse events:

$$R_{\rm ST,2} = \frac{M_{\rm ej} v_w}{\dot{M}} = 1\text{pc} \, \left(\frac{M_{\rm ej}}{M_\odot}\right) \left(\frac{v_w}{10 \text{ km s}^{-1}}\right) \left(\frac{\dot{M}}{10^{-5} M_\odot \text{yr}^{-1}}\right)^{-1} . \tag{8}$$

The corresponding times $t_{\rm ST,(1,2)}$ can be found as:

$$t_{\rm ST,i} = \left[R_{\rm ST,i} \left(\frac{B_i}{A_i}\right)^{\frac{1}{k_i - m_i}}\right]^{\frac{k_i - m_i}{k_i - 3}} \tag{9}$$

with

$$A_i = \frac{1}{4\pi k_i} \frac{[10(k_i - 5) E_{\rm SN}]^{\frac{k_i - 3}{2}}}{[3(k_i - 3) M_{\rm ej}]^{\frac{k_i - 5}{2}}} \tag{10}$$

and

$$B_i = \begin{cases} \rho_{\rm ISM} & \text{if } \; i=1 \\ \frac{\dot{M}}{4\pi v_w} & \text{if } \; i=2 \end{cases} \tag{11}$$

We obtain:

$$t_{\rm ST,1} = 235 \text{ yr} \left(\frac{M_{\rm ej}}{M_\odot}\right)^{\frac{5}{6}} \left(\frac{n_{\rm ISM}}{\rm cm^{-3}}\right)^{-\frac{1}{3}} \left(\frac{E_{\rm SN}}{10^{51}\text{erg s}^{-1}}\right)^{-\frac{1}{2}} \tag{12}$$

and

$$t_{\rm ST,2} = 84,5 \text{ yr} \left(\frac{M_{\rm ej}}{M_\odot}\right)^{\frac{3}{2}} \left(\frac{\dot{M}}{10^{-5}\ M_\odot \text{ yr}^{-1}}\right)^{-1} \left(\frac{v_w}{10 \text{ km s}^{-1}}\right) \left(\frac{E_{\rm SN}}{10^{51}\text{erg s}^{-1}}\right)^{-\frac{1}{2}} . \tag{13}$$

The values of Sedov-Taylor times and radii from Eqs. 7,8,12,13 can be used in Eqs. 3 and 4 to compute the shock radius and velocity at any age and at that point one can finally solve Eq. 1 to compute the maximum proton energy at the time of interest. The evolution with time of $E_{\rm max}$ is plotted in Fig. 1 for both types of explosions and standard values of the parameters as specified above.

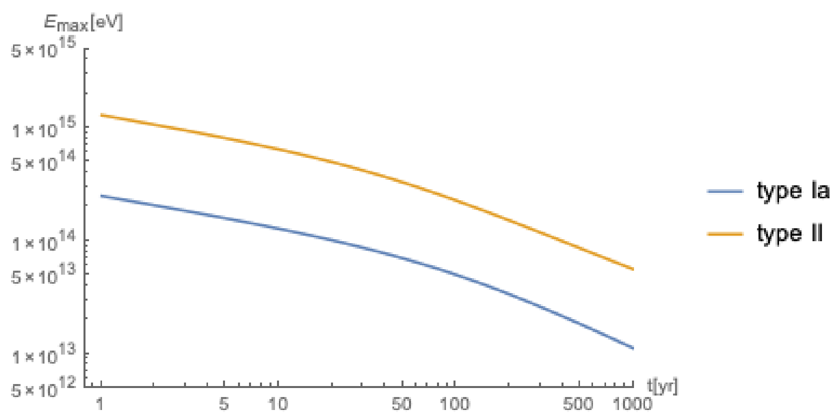

Figura 1: Time evolution of the maximum proton energy for both types of explosions and standard values of the parameters as used above.

2 Amplified magnetic field

The strength of the amplified magnetic field can be computed at all times by assuming that the non-resonant instability reaches saturation upstream of the supernova shock:

$$\frac{\delta B^2}{4\pi} = \frac{v_s}{c}\xi_{\rm CR}\rho v_s^2 \ . \tag{14}$$

The caveat is that the remnant be younger than about 10^3 yr. Of course this applies that everything that is written in these pages. The magnetic field strength downstream of the shock, which is what is relevant for particle energy losses, can then be estimated as:

$$\delta B = \sqrt{11}\sqrt{\frac{4\pi}{c}\xi_{\rm CR}\rho(t)v_s(t)^3} = 1\, mG\, \left(\frac{\xi_{\rm CR}}{0.1}\right)^{\frac{1}{2}}\left(\frac{\rho(t)}{1\ {\rm cm}^{-3}}\right)^{\frac{1}{2}}\left(\frac{v_s(t)}{10^4\ {\rm km\ s}^{-1}}\right)^{\frac{3}{2}} . \tag{15}$$

where the factor $\sqrt{1}$ assumes compression of a completely tangled magnetic field, such as one expects based on simulations of fully developed non-resonant instability. The time evolution of the downstream magnetic field is plotted in Fig. 2 for the two types of explosion and for the usual reference values of the parameters.

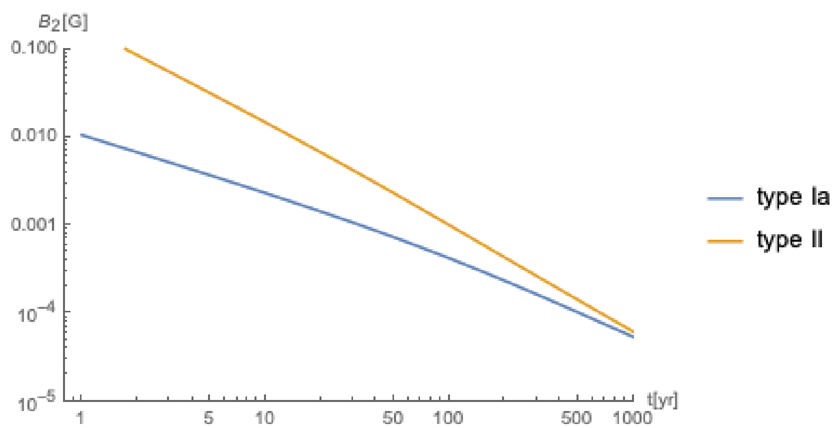

Figura 2: Time evolution of the downstream magnetic field for both types of explosions and standard values of the parameters as used above.

3 Maximum energy of electrons

At this point one can check whether the maximum energy of electrons will be limited by radiative losses, or, as for protons, by the growth time of the instability. Usually the maximum energy is determined by comparing the acceleration with the minimum between the system age and the radiative timescale:

$$t_{\rm acc}(E) \leq \min(t, t_{\rm rad}(E)) \ . \tag{16}$$

In this case, however, there is another time-scale that is critical, namely the growth-time of the instability that produces the field. This effect must be taken into account by making sure that the maximum particle energy does not exceed the maximum energy achievable by the system, namely $E_{\rm max,p}$, the maximum proton energy. We should then determine the maximum electron energy as:

$$E_{\rm max,el} = \min(E_{\rm max,p}, E_{\rm max,rad}) \ , \tag{17}$$

with

$$E_{\rm max,rad} = m_e c^2 \ \frac{v_s(t)}{c} \sqrt{\frac{18\pi e}{\sigma_T \delta B(t)}} \ . \tag{18}$$

What the dominant limiting factor is depends on the remnant type and evolutionary stage. Fig. 3 shows the relative importance of losses and growth time of the instability in determining $E_{\rm max,el}$ in the two types of remnants.

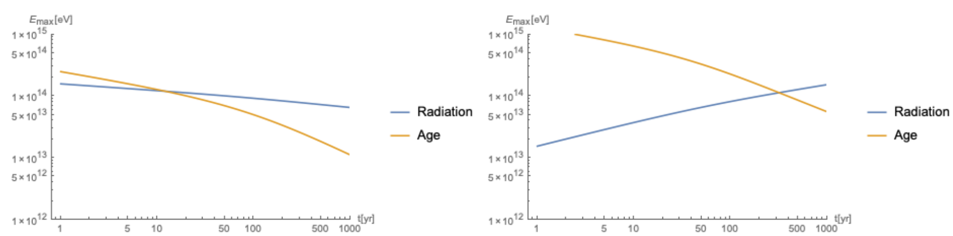

Figura 3: Maximum electron energy as determined by radiation losses and growth time of the magnetic field. The left panel is for type Ia explosions and the right one for type II.

Finally, in Fig. 3 I plot the maximum electron energy as a function of time for standard values of the parameters and for both types of explosion:

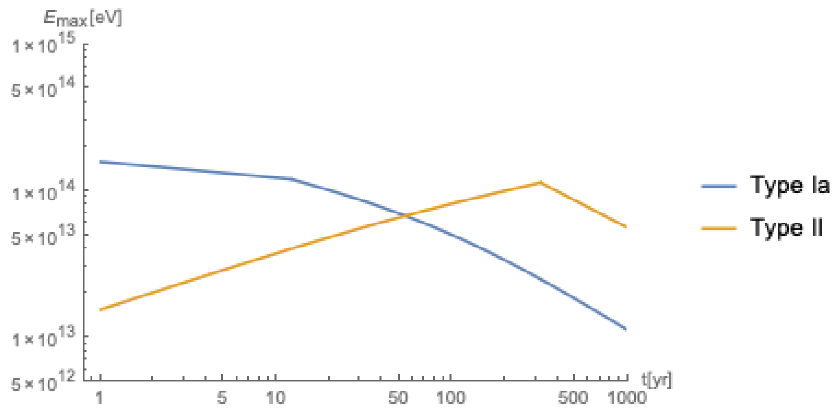

Figura 4: Maximum electron energy as a function of time for both kinds of explosions and standard parameters.

4 Maximum electron energy in Pulsar Wind Nebulae

While the mechanism providing particle acceleration in these sources is not known, the maximum achievable energy cannot exceed the maximum potential drop available in the pulsar magnetosphere, namely:

$$E_{\rm max,PSR} = e\sqrt{\frac{\dot{E}_{\rm PSR}}{c}} = 1.7\times10^{15}\ {\rm eV}\left(\frac{\dot{E}}{10^{36}\ {\rm erg\ s^{-1}}}\right)^{\frac{1}{2}}\ . \qquad (19)$$

One side note is that this is really the extreme maximum, while the most efficient accelerator we know, the Crab Nebula, only achieves a fraction of order 10% of it.