Estimating the maximum particle energy in SNRs and PWNe

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1 Maximum proton energy during the evolution of a SNR

The maximum energy of a SNR can be found by solving the following equation (see Cardillo et al (2015)):

$$\Psi(E_{\max}) = \frac{2e}{(4-m_i)5cE_0}\xi_{CR}v_s(R)^2\sqrt{4\pi\rho(R)R^2} \tag{1}$$

where e is the electron charge, c is the speed of light, R is the current radius of the SNR, v_s the current shock velocity, ρ the mass density of the ambient medium upstream of the shock, E_0 the minimum energy of the particles in the distribution, ξ_{CR} the fraction of energy that goes into cosmic rays, $m_i = 0, 2$ depending on whether the remnant is expanding in the ISM ($i = 1$, type Ia explosions) or in the progenitor wind ($i = 2$, core-collapse SNe expanding in a density profile $\propto r^{-2}$). The function Ψ carries information about the spectrum of accelerated particles, assumed in the form $N(E) \propto E^{-2-\beta}$, with:

$$\Psi(E_{\max}) = \begin{cases} (E_{\max}/E_0)\ln(E_{\max}/E_0) & \beta = 0 \\ \\ \frac{1+\beta}{\beta}\left(\frac{E_{\max}}{E_0}\right)^{1+\beta}\left[1-\left(\frac{E_0}{E_{\max}}\right)^{\beta}\right] & \beta \neq 0 \end{cases} . \tag{2}$$

Eq. 1 allows for an analytical solution independently of the value of β, if one assumes that $E_0 \approx m_pc^2$, the proton rest energy, and $E_{\max} >> E_0$.

Having in mind $E_{\max} \approx 10^{14} - 10^{15}$eV, one has:

$$\Psi(E_{\max}) \approx \begin{cases} \Lambda \ln(E_{\max}/E_0) & \beta = 0 \\ \\ \frac{1+\beta}{\beta}\left(\frac{E_{\max}}{E_0}\right)^{1+\beta} & \beta \neq 0 \end{cases}, \tag{3}$$

where $\Lambda = \ln(E_{\max}/E_0) \approx 12 - 14$.

In order to solve Eq. 1 the additional pieces of information needed are:

$$R(t) = R_{\rm ST}\left[\left(\frac{t}{t_{\rm ST}}\right)^{a\lambda_{\rm ED}} + \left(\frac{t}{t_{\rm ST}}\right)^{a\lambda_{\rm ST}}\right]^{\frac{1}{a}} \tag{4}$$

and

$$v_s(t) = \frac{R_{\rm ST}}{t_{\rm ST}}\left(\frac{R}{R_{\rm ST}}\right)^{1-a}\left[\lambda_{\rm ED}\left(\frac{t}{t_{\rm ST}}\right)^{a\lambda_{\rm ED}-1} + \lambda_{\rm ST}\left(\frac{t}{t_{\rm ST}}\right)^{a\lambda_{\rm ST}-1}\right] \tag{5}$$

where a is a smoothing parameter, fitted by Cardillo et al (2015)as $a = -5$, $R_{\rm ST}$ and $t_{\rm ST}$ are the radius and age of the remnant at the transition between the ejecta dominated and Sedov-Taylor phase of expansion and $\lambda_{\rm ED}$ and $\lambda_{\rm ST}$ are the power indices describing the remnant expansion during the ejecta-dominated and Sedov-Taylor phases respectively. One has:

$$\lambda_{\rm ED} = \frac{k_i - 3}{k_i - m_i} \quad \text{and} \quad \lambda_{\rm ST} = \frac{2}{5 - m_i} \tag{6}$$

where k_i is the power-law index describing the ejecta density profile, with

$$k_i = \begin{cases} 7 & \text{if} \quad i = 1 \\ 9 & \text{if} \quad i = 2 \end{cases}. \tag{7}$$

The final missing ingredient to be able to solve the above set of equations is the definition of the Sedov-Taylor time and radius for the two types of explosions. We define $R_{\rm ST}$ the size of the remnant at the time when it has swept up a mass equal to that of the ejecta. In the case of type Ia explosions this means:

$$R_{\rm ST,1} = \left(\frac{3M_{\rm ej}}{4\pi\rho_{\rm ISM}}\right)^{1/3} = 2\text{pc}\,\left(\frac{M_{\rm ej}}{M_\odot}\right)^{1/3}\left(\frac{n_{\rm ISM}}{\text{cm}^{-3}}\right)^{-1/3} \tag{8}$$

while for core collapse events:

$$R_{\rm ST,2} = \frac{M_{\rm ej} v_w}{\dot{M}} = 1{\rm pc} \left(\frac{M_{\rm ej}}{M_\odot}\right) \left(\frac{v_w}{10 \text{ km s}^{-1}}\right) \left(\frac{\dot{M}}{10^{-5} M_\odot {\rm yr}^{-1}}\right)^{-1} . \tag{9}$$

The corresponding times $t_{\rm ST,(1,2)}$ can be found as:

$$t_{\rm ST,i} = \left[R_{\rm ST,i} \left(\frac{B_i}{A_i}\right)^{\frac{1}{k_i - m_i}}\right]^{\frac{k_i - m_i}{k_i - 3}} \tag{10}$$

with

$$A_i = \frac{1}{4\pi k_i} \frac{[10(k_i - 5) E_{\rm SN}]^{\frac{k_i - 3}{2}}}{[3(k_i - 3) M_{\rm ej}]^{\frac{k_i - 5}{2}}} \tag{11}$$

and

$$B_i = \begin{cases} \rho_{\rm ISM} & \text{if } \ i = 1 \\ \frac{\dot{M}}{4\pi v_w} & \text{if } \ i = 2 \end{cases} \tag{12}$$

We obtain:

$$t_{\rm ST,1} = 235 \text{ yr} \left(\frac{M_{\rm ej}}{M_\odot}\right)^{\frac{5}{6}} \left(\frac{n_{\rm ISM}}{{\rm cm}^{-3}}\right)^{-\frac{1}{3}} \left(\frac{E_{\rm SN}}{10^{51} {\rm erg \ s}^{-1}}\right)^{-\frac{1}{2}} \tag{13}$$

and

$$t_{\rm ST,2} = 84,5 \text{ yr} \left(\frac{M_{\rm ej}}{M_\odot}\right)^{\frac{3}{2}} \left(\frac{\dot{M}}{10^{-5} \ M_\odot \ {\rm yr}^{-1}}\right)^{-1} \left(\frac{v_w}{10 \text{ km s}^{-1}}\right) \left(\frac{E_{\rm SN}}{10^{51} {\rm erg \ s}^{-1}}\right)^{-\frac{1}{2}} . \tag{14}$$

The values of Sedov-Taylor times and radii from Eqs. 8,9,13,14 can be used in Eqs. 4 and 5 to compute the shock radius and velocity at any age and at that point one can finally solve Eq. 1 to compute the maximum proton energy at the time of interest. The evolution with time of $E_{\rm max}$ is plotted in Fig. 1 for both types of explosions and standard values of the parameters as specified above, with in addition $\beta = 0$ and $\xi_{\rm CR} = 0.1$.

2 Amplified magnetic field

The strength of the amplified magnetic field can be computed at all times by assuming that the non-resonant instability reaches saturation upstream

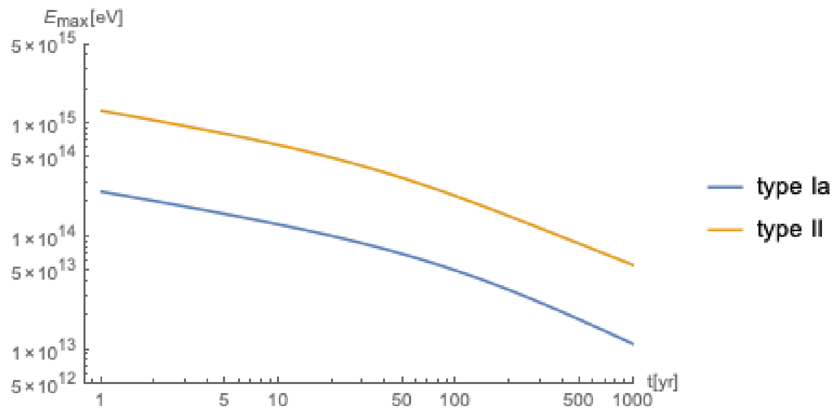

Figura 1: Time evolution of the maximum proton energy for both types of explosions and standard values of the parameters as used above.

of the supernova shock:

$$\frac{\delta B^2}{4\pi} = \frac{v_s}{c}\xi_{\rm CR}\rho v_s^2 \ . \tag{15}$$

The caveat is that the remnant be younger than $few \ \times 10^3$ **yr.** Of course this applies to everything that is written in these pages. The magnetic field strength downstream of the shock, which is what is relevant for particle energy losses, can then be estimated as:

$$\delta B = \sqrt{11}\sqrt{\frac{4\pi}{c}\xi_{\rm CR}\rho(t)v_s(t)^3} = 1\, mG \, \left(\frac{\xi_{\rm CR}}{0.1}\right)^{\frac{1}{2}} \left(\frac{\rho(t)}{1 \ {\rm cm}^{-3}}\right)^{\frac{1}{2}} \left(\frac{v_s(t)}{10^4 \ {\rm km \ s}^{-1}}\right)^{\frac{3}{2}} . \tag{16}$$

where the factor $\sqrt{1}$ assumes compression of a completely tangled magnetic field, such as one expects based on simulations of fully developed non-resonant instability. The time evolution of the downstream magnetic field is plotted in Fig. 2 for the two types of explosion and for the usual reference values of the parameters.

3 Maximum energy of electrons

At this point one can check whether the maximum energy of electrons will be limited by radiative losses, or, as for protons, by the growth time of the

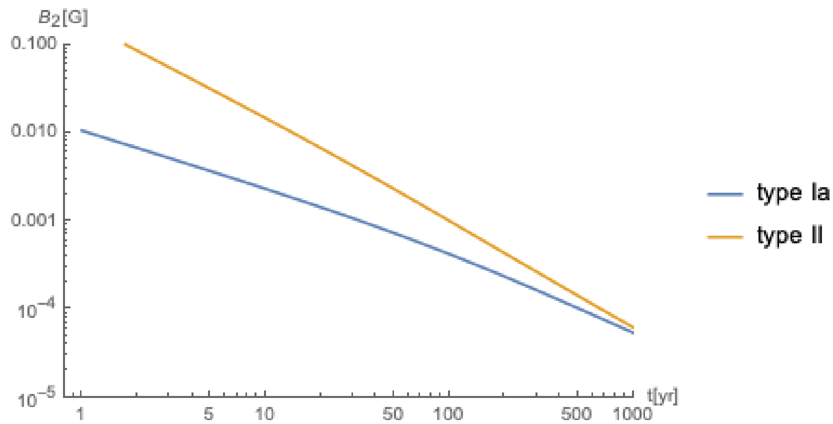

Figura 2: Time evolution of the downstream magnetic field for both types of explosions and standard values of the parameters as used above.

instability. Usually the maximum energy is determined by comparing the acceleration with the minimum between the system age and the radiative timescale:

$$t_{\mathrm{acc}}(E) \leq \min(t, t_{\mathrm{rad}}(E)) \ . \tag{17}$$

In this case, however, there is another time-scale that is critical, namely the growth-time of the instability that produces the field. This effect must be taken into account by making sure that the maximum particle energy does not exceed the maximum energy achievable by the system, namely $E_{\mathrm{max,p}}$, the maximum proton energy. We should then determine the maximum electron energy as:

$$E_{\mathrm{max,el}} = \min(E_{\mathrm{max,p}}, E_{\mathrm{max,rad}}) \ , \tag{18}$$

with $E_{\mathrm{max,rad}}$ determined by equating the acceleration time to the synchrotron loss time. The former reads:

$$t_{\mathrm{acc}}(E) \approx \frac{3}{u_1 - u_2} \frac{D_2}{u_2} = \frac{16}{3} \frac{cE}{v_s(t)^2 e B_2(t)} \ , \tag{19}$$

where D is the diffusion coefficient, assumed to be Bohm like, u is the fluid speed, and the subscripts $\{1, 2\}$ indicate quantities upstream and downstream of the shock respectively.

For the synchrotron loss time we have:

$$t_{\rm sync}(E,t) = \frac{9\pi (m_e\, c^2)^2}{\sigma_T c B_2(t)^2 E}\ , \tag{20}$$

where we are considering losses in an isotropic magnetic field ($B_\perp = (2/3)B_{\rm tot}$).

The final result is:

$$E_{\rm max,rad} = m_e c^2\ \frac{v_s(t)}{c} \sqrt{\frac{27\pi}{16} \frac{e}{\sigma_T \delta B(t)}}\ . \tag{21}$$

What the dominant limiting factor is depends on the remnant type and evolutionary stage. Fig. 3 shows the relative importance of losses and growth time of the instability in determining $E_{\rm max,el}$ in the two types of remnants.

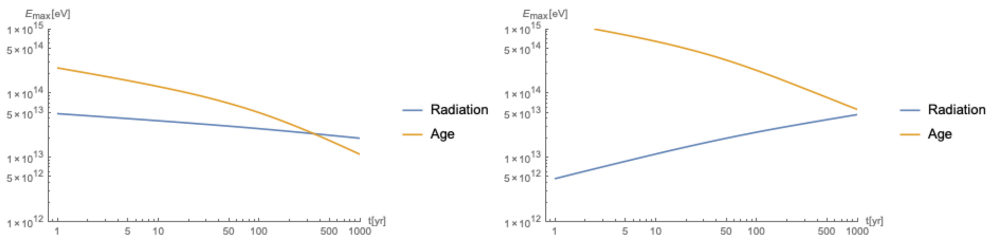

Figura 3: Maximum electron energy as determined by radiation losses and growth time of the magnetic field. The left panel is for type Ia explosions and the right one for type II.

Finally, in Fig. 4 I plot the maximum electron energy as a function of time for standard values of the parameters and for both types of explosion:

4 Maximum electron energy in Pulsar Wind Nebulae

4.1 Pulsar potential drop

While the mechanism providing particle acceleration in these sources is not known, the maximum achievable energy cannot exceed the maximum poten-

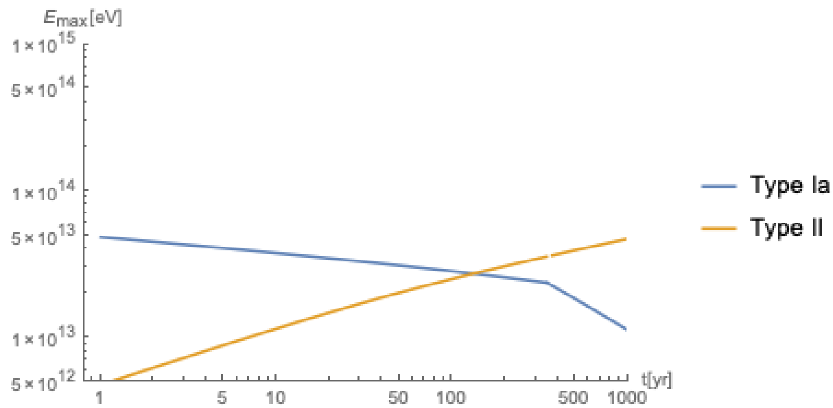

Figura 4: Maximum electron energy as a function of time for both kinds of explosions and standard parameters.

tial drop available in the pulsar magnetosphere, namely:

$$E_{\rm max,PSR} = e\sqrt{\frac{\dot{E}_{\rm PSR}}{c}} = 1.7 \times 10^{15}\ {\rm eV} \left(\frac{\dot{E}}{10^{36}\ {\rm erg\ s^{-1}}}\right)^{\frac{1}{2}} . \tag{22}$$

4.2 Radiation limited acceleration

A more refined estimate however is obtained by taking into account the fact that even particles that achieve a considerable fraction of the entire potential drop in the magnetosphere will largely loose their energy adiabatically when they become part of the wind. Actual acceleration will then occur mostly at the wind termination shock, where again radiation losses are the primary limit.

In order to compute the maximum electron energy from shock acceleration we need an estimate of the magnetic field at the pulsar wind termination shock, $B_{\rm TS}$. This requires some assumptions, which will be different in the two cases of a pulsar still inside its parent supernova remnant and a bow shock nebula.

4.2.1 PWN inside its parent SNR

The simplest (and still reasonable) way of estimating $B_{\rm TS}$ is by assuming that the field is uniform throughout the nebula and carries a fraction $\eta \approx 0.5$ of the total energy. Let us consider a pulsar losing rotational energy according to:

$$\dot{E}(t) = \frac{\dot{E}_0}{(1+t/\tau)^q} \ . \tag{23}$$

As reference values for τ and q we will consider in the following $\tau = 10^3$ yr and $q = 2.5$. For given τ and q one can derive $\dot{E}_0$ from the current value of the spin down power and estimated age of the system.

The only other piece of information that is needed to derive $B_{\rm TS}$ as outlined above is the pulsar wind nebula size, for which we use the parametrization:

$$R_{\rm PWN}(t) = R_* \frac{(t/\tau)^{\frac{6}{5}}}{\left(1+\frac{t}{2\tau}\right)^{\frac{1}{5}}} \tag{24}$$

with

$$R_* = \frac{3}{2} E_{\rm SN}^{\frac{3}{10}} \dot{E}_0^{\frac{1}{5}} \tau^{\frac{6}{5}} M_{\rm ej}^{-\frac{1}{2}} \ . \tag{25}$$

For standard parameters we have:

$$R_* = 3 \ {\rm pc} \ \left(\frac{E_{\rm SN}}{10^{51} \ {\rm erg}}\right)^{\frac{3}{10}} \ \left(\frac{\dot{E}_0}{10^{38} \ {\rm erg/s}}\right)^{\frac{1}{5}} \left(\frac{\tau}{10^3 \ {\rm yr}}\right)^{\frac{6}{5}} \left(\frac{M_{\rm ej}}{1 \ M_\odot}\right)^{-\frac{1}{2}} \tag{26}$$

and as one can easily see, the dependence of $R_{\rm PWN}$ on the exact value of the various system parameters is not very strong.

For the magnetic pressure within the nebula, one finds:

$$W_B(t) = \eta W_{\rm tot}(t) = \eta \left(\frac{4\pi}{3} R_{\rm PWN}(t)^4\right)^{-1} \int_0^t dt' \ \dot{E}(t') \ R_{\rm PWN}(t') \ , \tag{27}$$

and then we can then write the nebular magnetic field as:

$$B_{\rm TS}(t) = \sqrt{8 \ \pi \ W_B(t)} \ . \tag{28}$$

An excellent analytical approximation for $W_{\rm tot}(t)$ is provided by Bucciantini et al (2004):

$$W_{\rm tot}(t) \approx \frac{\dot{E}_0^{2/5} \ M_{\rm ej}^{3/2}}{E_{\rm SN}^{9/10} \ \tau^{13/5}} \ \frac{0.01}{\left(1+\frac{2t}{3\tau}\right)^{7/5} \ (t/\tau)^{13/5}} \ , \tag{29}$$

which leads to a magnetic field:

$$B_{\rm TS}(t) \approx 3.4 \times 10^{-5}{\rm G} \left(\frac{\dot{E}_0}{10^{38}\ {\rm erg/s}}\right)^{\frac{1}{5}} \left(\frac{M_{\rm ej}}{M_\odot}\right)^{\frac{3}{4}} \left(\frac{E_{\rm SN}}{10^{51}\ {\rm erg/s}}\right)^{-\frac{9}{20}} \times \tag{30}$$

$$\left(\frac{\eta}{0.5}\right)^{\frac{1}{2}} \times \left(\frac{t}{\rm kyr}\right)^{-\frac{13}{10}} \left(1+\frac{2t}{3\tau}\right)^{-\frac{7}{10}}$$

The magnetic field at the termination shock as a function of time is plotted in Fig. 5.

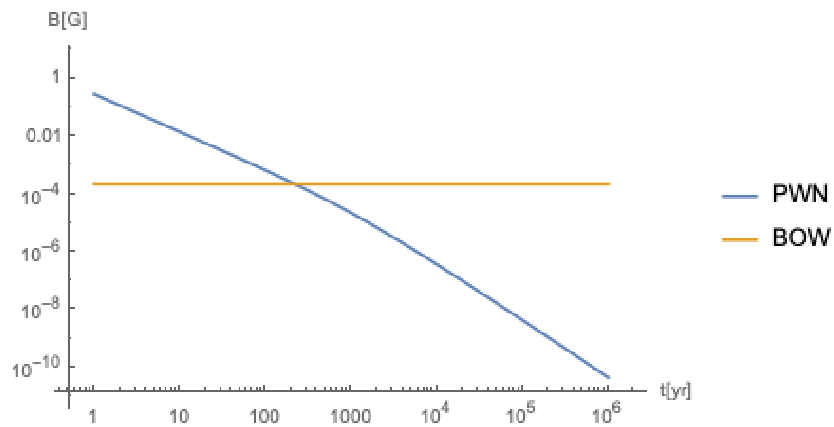

Figura 5: Time evolution of the magnetic field in PWNe and field estimate for a bow shock nebula for standard values of the parameters. The estimate of the field in the Bow Shock phase (see Sec. 4.2.2) is also plotted for comparison.

The radiation limited particle energy is easily written, based on the synchrotron loss time, computed in Eq. 20, and the acceleration time, which now we write as:

$$t_{\rm acc,PWN} = \frac{E}{dE/dt} = \frac{E}{\zeta eBc} \tag{31}$$

where we have used the fact that the termination shock is relativistic and written the accelerating electric field as a fraction ζ of the magnetic field, with $\zeta \leq 1$. In the following we assume acceleration at the maximum rate

$\zeta = 1$. The maximum achievable energy is then:

$$E_{\rm max,TS}(t) = m_e\, c^2 \sqrt{\frac{9\pi\ e}{\sigma_T B(t)}} \tag{32}$$

The limit on energy imposed by Eq. 32 is plotted as a function of the system age in Fig. 7, where we also compare it with the decreasing pulsar potential drop and with the radiation limited acceleration in systems where the pulsar has exited the parent SNR.

4.2.2 Bow shock PWN

Due to the high average proper motion of the pulsar population, these objects will leave the parent SNR at a time that can be roughly estimated as:

$$v_{\rm PSR} t_{\rm BOW} = R_{\rm SNR}(t_{\rm BOW}) = R_{\rm ST,2} \left(\frac{t_{\rm BOW}}{t_{\rm ST,2}}\right)^{\frac{2}{5}} \ , \tag{33}$$

where $v_{\rm PSR}$ is the pulsar velocity.

Using the expressions for $t_{\rm ST,2}$ and $R_{\rm ST,2}$ in Eqs. 9 and 14, we can solve Eq. 33 and obtain:

$$t_{\rm BOW} = 2.5 \times 10^4 \ {\rm yr} \left(\frac{M_{\rm ej}}{1\ M_\odot}\right)^{\frac{2}{3}} \left(\frac{\dot{M}}{10^{-5}\ M_\odot {\rm yr}^{-1}}\right)^{-1} \left(\frac{v_w}{10\ {\rm km\ s}^{-1}}\right) \times \tag{34}$$

$$\times \left(\frac{E_{\rm SN}}{10^{51}{\rm erg\ s}^{-1}}\right)^{\frac{1}{3}} \left(\frac{v_{\rm PSR}}{400\ {\rm km\ s}^{-1}}\right)^{-\frac{5}{3}} .$$

After $t_{\rm BOW}$ the magnetic field at the PWN termination shock can be computed based on balance between the ISM ram pressure, $\rho_{\rm ISM} v_{\rm PSR}^2$, and the pressure in the nebula, a fraction η of which is stil assumed to be in the form of magnetic fields.

One finds:

$$B_{\rm Bow}(t) = \sqrt{8\pi\eta\rho_{\rm ISM}}\ v_{\rm PSR} = 2 \times 10^{-4} {\rm G} \left(\frac{\eta}{0.5}\right)^{\frac{1}{2}} \left(\frac{n_{\rm ISM}}{1\ {\rm cm}^{-3}}\right)^{\frac{1}{2}} \left(\frac{v_{\rm PSR}}{400\ {\rm km/s}}\right) . \tag{35}$$

Finally, in Fig. 6, the 3 different limits on the maximum achievable energy in a PWN are plotted as a function of time. It is clear from the Figure that the green line is basically never relevant, since for standard values of the parameters, it would apply to a phase when the potential drop imposes a stronger limitation (this would deserve a more ample discussion, and rephrasing the potential drop in terms of a limit on the system size, but the conclusion is unaltered). The final result for the maximum energy is shown

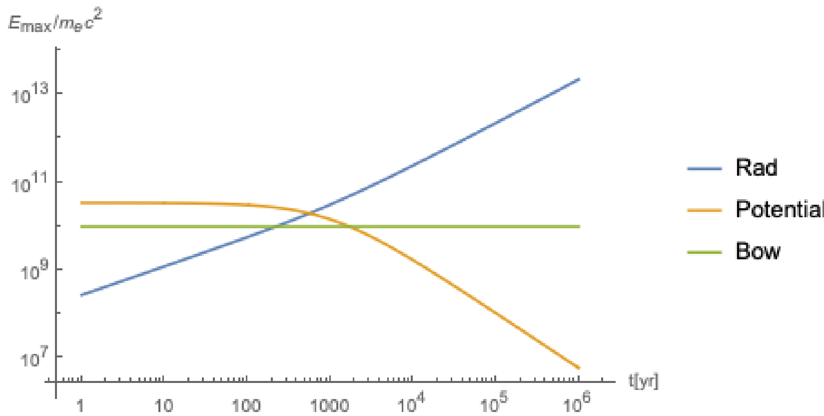

Figura 6: Maximum electron Lorentz factor as a function of time as determined by the pulsar potential drop and by radiation limited acceleration in confined PWNe and in bow shock PWNe.

in Fig. 7, from where one sees that for young objects the actual limitation comes from radiative losses, while for older ones from the pulsar potential drop.

Riferimenti bibliografici

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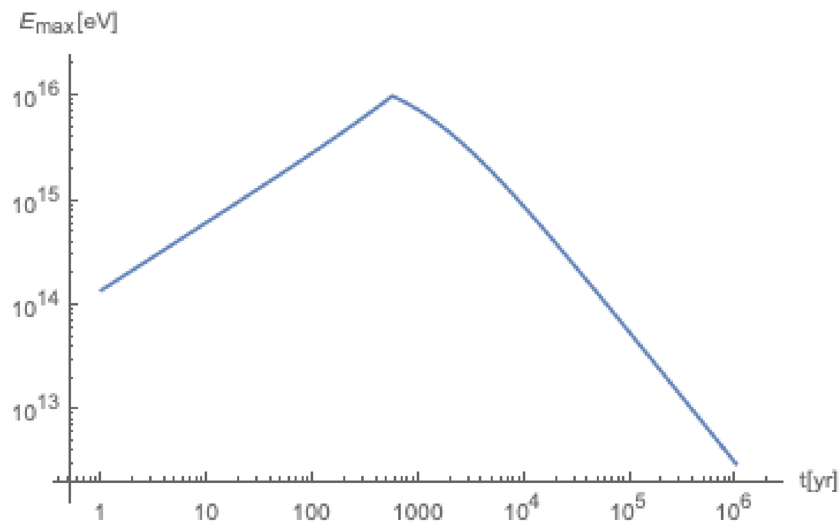

Figura 7: Maximum electron energy in PWNe as a function of time for standard values of the parameters.