${}^{B}_{A}T = \begin{bmatrix} {}^{B}_{A}R & -{}^{B}_{A}R\,{}^{A}O_B \\ 0 & 1 \end{bmatrix}$ X：横滚，Y：俯仰，Z：偏摆。基本旋转矩阵：d

$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$ $R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$

Z-X-Y 欧拉角：${}^{A}_{D}R = R_z(\alpha)R_y(\beta)R_x(\gamma)$

$$R_{Z'Y'X'}(\alpha,\beta,\gamma) = \begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & c\gamma & -s\gamma \\ 0 & s\gamma & c\gamma \end{bmatrix}$$

$$= \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & -s\alpha s\beta s\gamma + c\alpha c\gamma & -s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix}$$

Z-Y-Z 欧拉角：${}^{A}_{D}R = R_z(\alpha)R_y(\beta)R_z(\gamma)$

$$R_{Z'Y'Z'}(\alpha,\beta,\gamma) = \begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix}\begin{bmatrix} c\gamma & -s\gamma & 0 \\ s\gamma & c\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\alpha c\beta c\gamma - s\alpha s\gamma & -c\alpha c\beta s\gamma - s\alpha c\gamma & c\alpha s\beta \\ s\alpha c\beta c\gamma + c\alpha s\gamma & -s\alpha c\beta s\gamma + c\alpha c\gamma & s\alpha s\beta \\ -s\beta c\gamma & s\beta s\gamma & c\beta \end{bmatrix}$$

X-Y-Z 固定角：$R_{XYZ}(\gamma,\beta,\alpha) = \begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 0 & c\gamma & -s\gamma \\ 0 & s\gamma & c\gamma \end{bmatrix}$

$$= \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & -s\alpha s\beta s\gamma + c\alpha c\gamma & -s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix}$$

右乘联体左乘基。命题：$R_z(\pm\pi+\alpha)R_y(\pm\pi-\beta)R_x(\pm\pi+\gamma) = R_z(\alpha)R_y(\beta)R_x(\gamma)$

等效轴角：以单位向量${}^{A}K = [k_x \quad k_y \quad k_z]^T$ 表示旋转轴，旋转角为θ

$$R_K(\theta) = \begin{bmatrix} k_x^2 v\theta + c\theta & k_x k_y v\theta - k_z s\theta & k_x k_z v\theta + k_y s\theta \\ k_x k_y v\theta + k_z s\theta & k_y^2 v\theta + c\theta & k_y k_z v\theta - k_x s\theta \\ k_x k_z v\theta - k_y s\theta & k_y k_z v\theta + k_x s\theta & k_z^2 v\theta + c\theta \end{bmatrix}$$

欧拉参数：$\eta = \cos\frac{\theta}{2}$ ，$\varepsilon = \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{bmatrix} = \begin{bmatrix} k_x \sin\frac{\theta}{2} \\ k_y \sin\frac{\theta}{2} \\ k_z \sin\frac{\theta}{2} \end{bmatrix}$ ，满足约束$\eta^2 + \varepsilon_1^2 + \varepsilon_2^2 + \varepsilon_3^2 = 1$

$$R_\varepsilon(\eta) = \begin{bmatrix} 2(\eta^2+\varepsilon_1^2)-1 & 2(\varepsilon_1\varepsilon_2 - \eta\varepsilon_3) & 2(\varepsilon_1\varepsilon_3 + \eta\varepsilon_2) \\ 2(\varepsilon_1\varepsilon_2 + \eta\varepsilon_3) & 2(\eta^2+\varepsilon_2^2)-1 & 2(\varepsilon_2\varepsilon_3 - \eta\varepsilon_1) \\ 2(\varepsilon_1\varepsilon_3 - \eta\varepsilon_2) & 2(\varepsilon_2\varepsilon_3 + \eta\varepsilon_1) & 2(\eta^2+\varepsilon_3^2)-1 \end{bmatrix}$$

Grassmann 积：$\begin{bmatrix} x \\ y \end{bmatrix} \oplus \begin{bmatrix} w \\ z \end{bmatrix} = \begin{bmatrix} xw - yz \\ xz + yw \end{bmatrix}$,R^2中描述二维向量的旋转，在R^4中定义：

$$\begin{bmatrix} \eta \\ \varepsilon \end{bmatrix} \oplus \begin{bmatrix} \xi \\ \delta \end{bmatrix} = \begin{bmatrix} \eta\xi - \varepsilon^T\delta \\ \eta\delta + \xi\varepsilon + \varepsilon\times\delta \end{bmatrix} = \begin{bmatrix} \eta & -\varepsilon_1 & -\varepsilon_2 & -\varepsilon_3 \\ \varepsilon_1 & \eta & -\varepsilon_3 & \varepsilon_2 \\ \varepsilon_2 & \varepsilon_3 & \eta & -\varepsilon_1 \\ \varepsilon_3 & -\varepsilon_2 & \varepsilon_1 & \eta \end{bmatrix}\begin{bmatrix} \xi \\ \delta_1 \\ \delta_2 \\ \delta_3 \end{bmatrix}$$

$$= \begin{bmatrix} \eta\xi - \varepsilon_1\delta_1 - \varepsilon_2\delta_2 - \varepsilon_3\delta_3 \\ \eta\delta_1 + \varepsilon_1\xi + \varepsilon_2\delta_3 - \varepsilon_3\delta_2 \\ \eta\delta_2 - \varepsilon_1\delta_3 + \varepsilon_2\xi + \varepsilon_3\delta_1 \\ \eta\delta_3 + \varepsilon_1\delta_2 - \varepsilon_2\delta_1 + \varepsilon_3\xi \end{bmatrix}$$

记${}^{B}P = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$，${}^{A}P = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$，${}^{A}P = {}^{A}_{B}R\,{}^{B}P$，${}^{A}_{B}R = R_\varepsilon(\eta)$，有：

$$ix_2 + jy_2 + kz_2 = (\eta + i\varepsilon_1 + j\varepsilon_2 + k\varepsilon_3)(ix_1 + iy_1 + iz_1)(\eta + i\varepsilon_1 + j\varepsilon_2 + k\varepsilon_3)^*$$

$${}^{i-1}_{i}T = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & a_{i-1} \\ \sin\theta_i\cos\alpha_{i-1} & \cos\theta_i\cos\alpha_{i-1} & -\sin\alpha_{i-1} & -\sin\alpha_{i-1}\,d_i \\ \sin\theta_i\sin\alpha_{i-1} & \cos\theta_i\sin\alpha_{i-1} & \cos\alpha_{i-1} & \cos\alpha_{i-1}\,d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

D-H 参数表：$i \quad \alpha_{i-1} \quad a_{i-1} \quad d_i \quad \theta_i$

逆运动学：已知工具坐标系相对于工作台坐标系的期望位置和姿态，计算一系列满足期望要求的关节角

化简为多项式：$u = \tan\frac{\theta}{2}, \cos\theta = \frac{1-u^2}{1+u^2}, \sin\theta = \frac{2u}{1+u^2}$

三轴相交的 PIEPER 解法。

Q 是空间中动点，{A}和{B}是动坐标系，求${}^{A}V_Q$与${}^{B}V_Q$的关系

$${}^{A}V_Q = {}^{A}V_{BORG} + {}^{A}_{B}\dot{R}\,{}^{B}Q + {}^{A}_{B}R\,{}^{B}V_Q = {}^{A}V_{BORG} + {}^{A}_{B}R\,{}^{B}V_Q + {}^{A}\Omega_B \times {}^{A}_{B}R\,{}^{B}Q$$

角速度向量${}^{A}\Omega_B = \begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix}$ ，角速度矩阵${}^{A}_{B}S = \begin{bmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{bmatrix}$

$${}^{A}\Omega_C = {}^{A}\Omega_B + {}^{A}_{B}R\,{}^{B}\Omega_C$$

连杆间的速度传递：当关节 i+1 是旋转关节时：

$${}^{i+1}\omega_{i+1} = {}^{i+1}_{i}R\,{}^{i}\omega_i + \dot{\theta}_i\,{}^{i+1}\hat{Z}_{i+1}$$
$${}^{i+1}v_{i+1} = {}^{i+1}_{i}R\left({}^{i}v_i + {}^{i}\omega_i \times {}^{i}P_{i+1}\right)$$

当关节 i+1 是移动关节时

$${}^{i+1}\omega_{i+1} = {}^{i+1}_{i}R\,{}^{i}\omega_i$$
$${}^{i+1}v_{i+1} = {}^{i+1}_{i}R\left({}^{i}v_i + {}^{i}\omega_i \times {}^{i}P_{i+1}\right) + \dot{d}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}$$

雅克比：

$$J(\Theta) = \begin{bmatrix} \hat{Z}_1 \times (P_N - P_1) & \hat{Z}_2 \times (P_N - P_2) & \cdots & \hat{Z}_{N-1} \times (P_N - P_{N-1}) & 0 \\ \hat{Z}_1 & \hat{Z}_2 & \cdots & \hat{Z}_{N-1} & \hat{Z}_N \end{bmatrix}$$

$$v_N = J(\Theta)\dot{\Theta}$$

若关心 i 中的笛卡尔速度矢量

$$\begin{bmatrix} {}^{i}v_N \\ {}^{i}\omega_N \end{bmatrix} = \begin{bmatrix} {}^{i}_{0}R & 0 \\ 0 & {}^{i}_{0}R \end{bmatrix} J(\Theta)\dot{\Theta},\quad 记\,{}^{i}J(\Theta) = \begin{bmatrix} {}^{i}_{0}R & 0 \\ 0 & {}^{i}_{0}R \end{bmatrix} J(\Theta)$$

连杆间静力传递：向内迭代

$${}^{i}f_i = {}^{i}_{i+1}R\,{}^{i+1}f_{i+1},\quad {}^{i}n_i = {}^{i}_{i+1}R\,{}^{i+1}n_{i+1} + {}^{i}P_{i+1} \times {}^{i}f_i$$

力域中的雅克比：$\mathcal{F}$是末端作用于外部的 6X1 维笛卡尔力-力矩矢量

τ是 6X1 维关节力矩矢量，有$\tau = J^T\mathcal{F}$

雅可比的转置将作用于操作臂的笛卡尔力映射成等效关节力矩

线加速度公式：

$${}^{A}\dot{V}_Q = {}^{A}\dot{V}_{BORG} + {}^{A}_{B}R\,{}^{B}\dot{V}_Q + 2\,{}^{A}\Omega_B \times {}^{A}_{B}R\,{}^{B}V_Q + {}^{A}\dot{\Omega}_B \times {}^{A}_{B}R\,{}^{B}Q$$
$$+ {}^{A}\Omega_B \times ({}^{A}\Omega_B \times {}^{A}_{B}R\,{}^{B}Q)$$

当${}^{B}Q$是常数，化简为：

$${}^{A}\dot{V}_Q = {}^{A}\dot{V}_{BORG} + {}^{A}\dot{\Omega}_B \times {}^{A}_{B}R\,{}^{B}Q + {}^{A}\Omega_B \times ({}^{A}\Omega_B \times {}^{A}_{B}R\,{}^{B}Q)$$

角加速度公式：

$${}^{A}\dot{\Omega}_C = {}^{A}\dot{\Omega}_B + {}^{A}_{B}R\,{}^{B}\dot{\Omega}_C + {}^{A}\Omega_B \times {}^{A}_{B}R\,{}^{B}\Omega_C$$

惯性张量：${}^{A}I = \begin{pmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ I_{xz} & -I_{yz} & I_{zz} \end{pmatrix}$，$I_{xx}$：惯性矩，$I_{xy}$：惯性基

$$I_{xx} = \iiint_V (y^2 + z^2)\rho\, dv,\quad I_{xy} = \iiint_V xy\rho\, dv$$

平行移轴定理：{C}是以刚体质心为原点的坐标系，{A}为任意平移后的坐标系。

$P_C = \begin{bmatrix} x_C \\ y_C \\ z_C \end{bmatrix}$表示刚体质心在坐标系{A}中的位置

$${}^{A}I_{xx} = {}^{C}I_{xx} + m(y_C^2 + z_C^2) \qquad {}^{A}I_{xy} = {}^{C}I_{xy} - m x_C y_C$$
$${}^{A}I = {}^{C}I + m[P_C^T P_C I_3 - P_C P_C^T]$$

牛顿方程：$F = m\dot{v}_c$ ，欧拉方程：${}^{C}N = {}^{C}I\,{}^{C}\dot{\omega} + {}^{C}\omega \times {}^{C}I\,{}^{C}\omega$

第 i+1 个关节为旋转关节：

$${}^{i+1}\dot{\omega}_{i+1} = {}^{i+1}_{i}R\,{}^{i}\dot{\omega}_i + \ddot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1} + {}^{i+1}_{i}R\,{}^{i}\omega_i \times \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}$$
$${}^{i+1}\dot{v}_{i+1} = {}^{i+1}_{i}R\left[{}^{i}\dot{v}_i + {}^{i}\dot{\omega}_i \times {}^{i}P_{i+1} + {}^{i}\omega_i \times \left({}^{i}\omega_i \times {}^{i}P_{i+1}\right)\right]$$

第 i+1 个关节为移动关节：

$${}^{i+1}\dot{\omega}_{i+1} = {}^{i+1}_{i}R\,{}^{i}\dot{\omega}_i$$
$${}^{i+1}\dot{v}_{i+1} = {}^{i+1}_{i}R\left[{}^{i}\dot{v}_i + {}^{i}\dot{\omega}_i \times {}^{i}P_{i+1} + {}^{i}\omega_i \times \left({}^{i}\omega_i \times {}^{i}P_{i+1}\right)\right] + \ddot{d}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}$$
$$+ 2\,{}^{i+1}\omega_{i+1} \times \dot{d}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}$$

连杆质心线加速度：

$${}^{i}\dot{v}_{C_i} = {}^{i}\dot{\omega}_i \times {}^{i}P_{C_i} + {}^{i}\omega_i \times \left({}^{i}\omega_i \times {}^{i}P_{C_i}\right) + {}^{i}\dot{v}_i$$

牛顿欧拉方程：

外推：

$${}^{i+1}\omega_{i+1} = {}^{i+1}_{i}R\,{}^{i}\omega_i + \dot{\theta}_i\,{}^{i+1}\hat{Z}_{i+1}$$
$${}^{i+1}\dot{\omega}_{i+1} = {}^{i+1}_{i}R\,{}^{i}\dot{\omega}_i + \ddot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1} + {}^{i+1}_{i}R\,{}^{i}\omega_i \times \dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1}$$
$${}^{i+1}\dot{v}_{i+1} = {}^{i+1}_{i}R\left[{}^{i}\dot{v}_i + {}^{i}\dot{\omega}_i \times {}^{i}P_{i+1} + {}^{i}\omega_i \times \left({}^{i}\omega_i \times {}^{i}P_{i+1}\right)\right]$$

${}^i\dot{v}_{C_i} = {}^i\dot{\omega}_i \times {}^iP_{C_i} + {}^i\omega_i \times ({}^i\omega_i \times {}^iP_{C_i}) + {}^i\dot{v}_i$

${}^{i+1}F_{i+1} = m_{i+1}{}^i\dot{v}_{C_i}$

${}^{i+1}N_{i+1} = {}^{C_{i+1}}I_{i+1}{}^{i+1}\dot{\omega}_{i+1} + {}^{i+1}\omega_{i+1} \times {}^{C_{i+1}}I_{i+1}{}^{i+1}\omega_{i+1}$

内推：${}^{n+1}f_{n+1} = 0,\ {}^{n+1}n_{n+1} = 0$

${}^if_i = {}^i_{i+1}R\,{}^{i+1}f_{i+1} + {}^iF_i$

${}^in_i = {}^iN_i + {}^i_{i+1}R\,{}^{i+1}n_{i+1} + {}^iP_{C_i} \times {}^iF_i + {}^iP_{i+1} \times {}^i_{i+1}R\,{}^{i+1}f_{i+1}$

$\tau_i = {}^in_i^T\,{}^i\hat{Z}_i$

状态空间方程：$\tau = M(\Theta)\ddot{\Theta} + V(\Theta,\dot{\Theta}) + G(\Theta)$

位形空间方程：$\tau = M(\Theta)\ddot{\Theta} + B(\Theta)(\dot{\Theta}\dot{\Theta}) + C(\Theta)(\dot{\Theta}^2) + G(\Theta)$

拉格朗日方程：$L = K - P$，L 是拉格朗日函数，K 是系统动能，P 是系统势能

$$F_i = \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}_i}\right) - \frac{\partial L}{\partial x_i} \qquad T_i = \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}_i}\right) - \frac{\partial L}{\partial \theta_i}$$

F 是线运动中所有外力的和，T 是转动中所有外力矩之和，X 是系统变量

操作臂动能表达式：$k_i = \frac{1}{2}m_i v_{C_i}^T v_{C_i} + \frac{1}{2}{}^i\omega_i^T\,{}^{C_i}I_i\,{}^i\omega_i$

操作臂动能之和：$k = \sum_{i=1}^n k_i$，操作臂动能可描述为关节位置和速度的标量函数：

$$k(\Theta,\dot{\Theta}) = \frac{1}{2}\dot{\Theta}^T M(\Theta)\dot{\Theta}$$

连杆 i 的势能可表示为：$u_i = -m_i{}^\circ g^T\,{}^\circ P_{C_i} + u_{ref_i}$

${}^\circ g$是 3X1 的重力矢量，${}^\circ P_{C_i}$是连杆 i 质心的位置，u_{ref_i}是使u_i的最小值为零的常数

操作臂总势能：$u = \sum_{i=1}^n u_i$

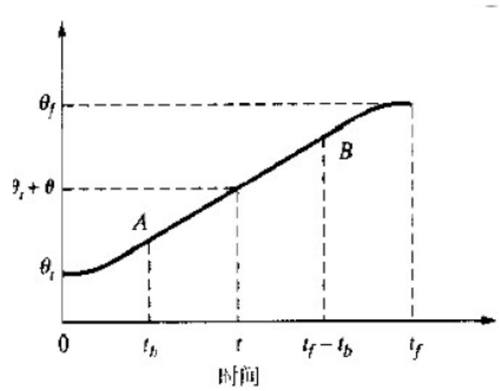
操作臂运动方程：$\frac{d}{dt}\frac{\partial L}{\partial \dot{\Theta}} - \frac{\partial L}{\partial \Theta} = \tau$，$\tau$是 nX1 驱动力矩矢量

三次多项式：$\theta(t) = a_0 + a_1t + a_2t^2 + a_3t^3$，求解需要四个约束条件

$$\theta(0) = \theta_0, \qquad \theta(t_f) = \theta_f, \qquad \dot{\theta}(0) = 0, \qquad \dot{\theta}(t_f) = 0$$

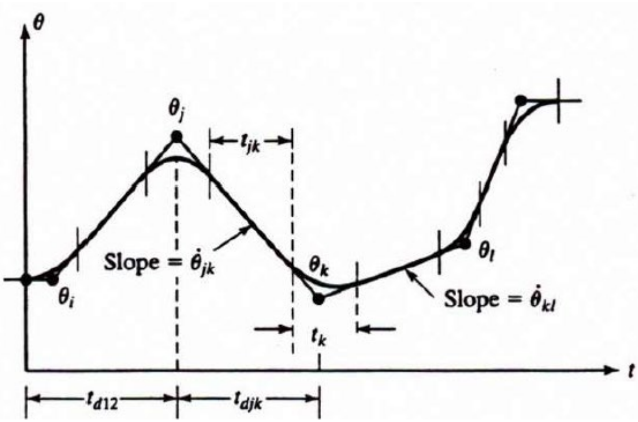
三次样条函数（具有中间点的三次多项式）：先用直线段把中间点连接起来。如果这些直线的斜率在中间点处改变符号，则把速度选定为零；如果这些直线的斜率没有改变符号，则选取两斜率的平均值作为该点的速度

抛物线拟合：



$\ddot{\theta}t_b = \frac{\theta_h-\theta_b}{t_h-t_b}$，其中$\theta_b = \theta_0 + \frac{1}{2}\ddot{\theta}t_b^2$，再考虑$t_h = \frac{t_f}{2}$，$\theta_h = \frac{\theta_0+\theta_f}{2}$，有：

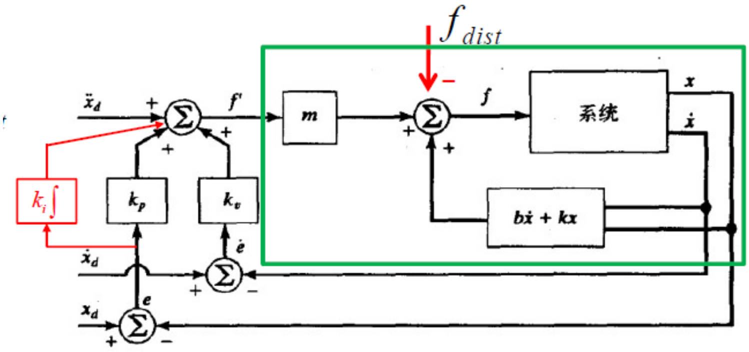
$$\ddot{\theta}t_b^2 - \ddot{\theta}t_ft_b + (\theta_f - \theta_0) = 0$$



$\dot{\theta}_{jk} = \frac{\theta_j-\theta_k}{t_{djk}}$，$\ddot{\theta}_k = SGN(\dot{\theta}_{kl} - \dot{\theta}_{jk})|\ddot{\theta}_k|$，$t_k = \frac{\dot{\theta}_{kl}-\dot{\theta}_{jk}}{\ddot{\theta}_k}$，$t_{jk} = t_{djk} - \frac{1}{2}t_j - \frac{1}{2}t_k$

操作臂的线性控制：$m\ddot{x} + b\dot{x} + kx = f = \alpha f' + \beta$

取$\beta = b\dot{x} + kx$，$\alpha = m$，设计控制律$f' = -k_v\dot{x} - k_px$



加入积分项可以消除恒定干扰的稳定误差：$f' = \ddot{x}_d + k_v\dot{e} + k_pe + k_i\int e\,dt$

电枢电路的动态方程：$l_a\frac{di_a}{dt} + r_ai_a = v_a - k_e\frac{d\theta_m}{dt}$，$v_a$：电枢电压，$l_a$：电枢电感，$r_a$：电枢电阻，$k_e$是电机反电势常数。

电机减速器：升矩$\tau_b = \eta\tau_a$，降速：$\dot{\theta} = \dot{\theta}_m/\eta$

电机转子力矩平衡：$\tau_m - \tau_a = I_m\ddot{\theta}_m + b_m\dot{\theta}_m$，负载力矩平衡：$\tau_b - \tau_g = I\ddot{\theta} + b\dot{\theta}$

有：$\tau_m = \left(I_m + \frac{I}{\eta^2}\right)\ddot{\theta}_m + \left(b_m + \frac{b}{\eta^2}\right)\dot{\theta}_m + \frac{\tau_g}{\eta}$

I_m是电机转子惯量，b_m是电机转子轴承的粘滞摩擦系，I 是负载惯量，b 是负载轴承的粘滞摩擦系数，τ_g是干扰力矩

负载侧：$\tau = (I + \eta^2I_m)\ddot{\theta} + (b + \eta^2)\dot{\theta} + \tau_g$

自然约束是特定接触条件下自然形成的，期望运动无关。

质量-弹簧系统：$f = mk_e^{-1}\ddot{f}_e + f_e + f_{dist}$

控制律中不含干扰力f_{dist}，则实际控制律为：$f = mk_e^{-1}[\ddot{f}_d + k_{vf}\dot{e}_f + k_{pf}e_f] + f_e$，

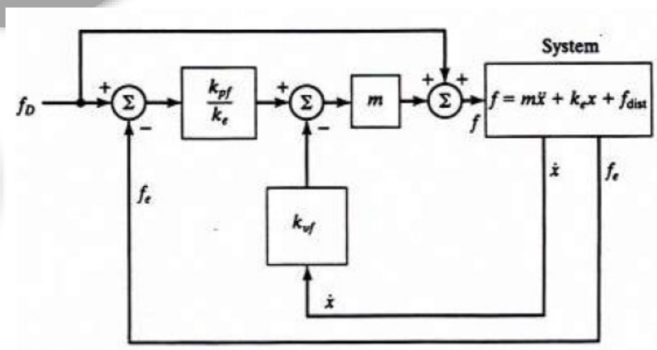
稳态误差：$e_f = \frac{f_{dist}}{\alpha}$，$\alpha = mk_e^{-1}k_{pf}$

用f_d代替$f_e + f_{dist}$，控制律为$f = mk_e^{-1}[\ddot{f}_d + k_{vf}\dot{e}_f + k_{pf}e_f] + f_d$

稳态误差$e_f = \frac{f_{dist}}{1+\alpha}$，$\alpha = mk_e^{-1}k_{pf}$

实际中，力轨迹一般为常值，力检测的噪声很大，无法使用数值为分，用$\dot{f}_e = k_e\dot{x}$代替

实用的力控制：$f = m[k_{pf}k_e^{-1}e_f - k_{vf}\dot{x}] + f_d$



笛卡尔空间解耦控制：$F = M_\chi(\Theta)\ddot{\chi} + V_\chi(\Theta,\dot{\Theta}) + G_\chi(\Theta)$

降低位置增益的主动柔顺控制方法：机械臂末端有一定柔顺性，即在笛卡尔六自由度有一定刚度：$F = K_{px}\delta x$，K_{px}是 6X6 对角阵，表示三个移动刚度和三个旋转刚度。

由雅克比：$\delta x = J(\Theta)\delta\Theta$，$\tau = J^T(\Theta)F$，$F = K_{px}J(\Theta)\delta\Theta$

有：$\tau = J^T(\Theta)K_{px}J(\Theta)\delta\Theta$

一个简单的通过位置增益同时进行位置控制和柔顺控制的控制律：

$$\tau = J^T(\Theta)K_{px}J(\Theta)E + K_v\dot{E}$$

坐标系的标准命名：基坐标系{B}，固定坐标系{S}，腕部坐标系{W}，工具坐标系{T}，目标坐标系{G}