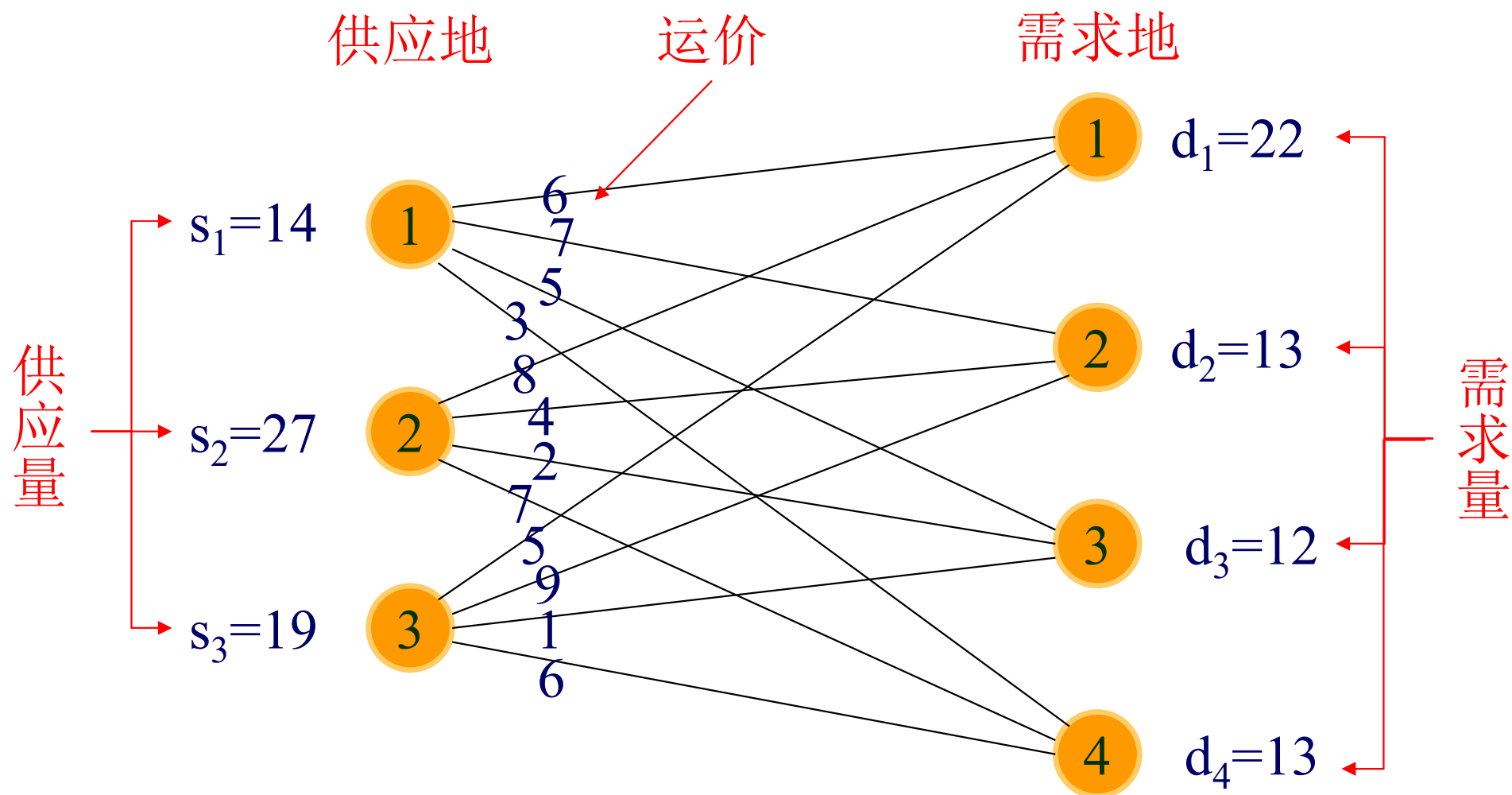


第三章 运输规划简介

- 运输问题及其数学模型
- 产销平衡问题的表上作业法
- 产销不平衡问题
- 有转运问题
- 应用举例



问题提出



数学模型

$$\begin{array}{lcl}
 \min & z = & 6x_{11} + 7x_{12} + 5x_{13} + 3x_{14} + 8x_{21} + 4x_{22} + 2x_{23} + 7x_{24} + 5x_{31} + 9x_{32} + 10x_{33} + 6x_{34} \\
 s.t. & & \\
 & x_{11} + x_{12} + x_{13} + x_{14} & = 14 \\
 & & x_{21} + x_{22} + x_{23} + x_{24} = 27 \\
 & & & x_{31} + x_{32} + x_{33} + x_{34} = 19 \\
 & x_{11} & + x_{21} + x_{31} = 22 \\
 & & x_{12} + x_{22} + x_{32} = 13 \\
 & & & x_{13} + x_{23} + x_{33} = 12 \\
 & & & & x_{14} + x_{24} + x_{34} = 13 \\
 & x_{11} & x_{12} & x_{13} & x_{14} & x_{21} & x_{22} & x_{23} & x_{24} & x_{31} & x_{32} & x_{33} & x_{34} & = & 0
 \end{array}$$

产销平衡问题的一般模型

$$\min z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_{j=1}^n x_{ij} = a_i$$

$$\sum_{i=1}^m x_{ij} = b_j$$

$$x_{ij} \geq 0 \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n$$

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j$$

产销平衡约束

问题的特点

1 .A矩阵稀疏

$$P_{ij}=[0,\dots,0,1,0,\dots,0,1,0,\dots,0]^T$$

2 .基变量只有 $m+n-1$ 个

第 i 个

第 $m+j$ 个

3 .一定存在（有界）最优解

$$x_{ij} = \frac{a_i b_j}{\sum_{i=1}^m a_i} = \frac{a_i b_j}{\sum_{j=1}^n b_j}$$

可行解

运输表上的单纯形法

销地 产地	B_1	B_2	\dots	B_n	产量
A_1	$c_{11} x_{11}$	$c_{12} x_{12}$	\dots	$c_{1n} x_{1n}$	a_1
A_2	$c_{21} x_{21}$	$c_{22} x_{22}$	\dots	$c_{2n} x_{2n}$	a_2
\vdots	\vdots	\vdots	\dots	\vdots	\vdots
A_m	$c_{m1} x_{m1}$	$c_{m2} x_{m2}$	\dots	$c_{mn} x_{mn}$	a_m
销量	b_1	b_2	\dots	b_n	

初始基可行解

销地 产地	1	2	3	4	产量
1	6	7	5	3	14
2	8	4	2	7	27
3	5	9	10	6	19
销量	22	13	12	13	

Diagram illustrating the initial basic feasible solution (Initial Basic Feasible Solution) for a transportation problem. The table shows the supply (产量) and demand (销量) for four destinations (销地) and three origins (产地).

The initial solution is marked by red and blue lines, indicating the allocation of units to specific cells. The allocations are shown in red and blue numbers, with green arrows indicating the flow of units.

Key allocations and flows:

- From Origin 1 to Destination 1: 14 units (Red arrow from cell (1,1) to cell (2,1)).
- From Origin 2 to Destination 1: 8 units (Green arrow from cell (2,1) to cell (3,1)).
- From Origin 2 to Destination 2: 13 units (Green arrow from cell (2,2) to cell (3,2)).
- From Origin 2 to Destination 3: 6 units (Green arrow from cell (2,3) to cell (3,3)).
- From Origin 3 to Destination 4: 13 units (Green arrow from cell (3,4) to cell (2,4)).

The total supply is 14 + 27 + 19 = 60, and the total demand is 22 + 13 + 12 + 13 = 60, indicating a balanced problem.

最优性检验

销地 产地	1	2	3	4	产量
1	6 14	7 5	5 5	3 7	14
2	8 8	4 13	2 6	7 9	27
3	5 -11	9 -3	10 6	6 13	19
销量	22	13	12	13	

$$\sigma_{32} = c_{32} - c_{22} + c_{23} - c_{33} = 9 - 4 + 2 - 10 = -3$$

位势法

■ 对偶问题:

$$\max w = \sum_{i=1}^m a_i u_i + \sum_{j=1}^n b_j v_j$$

$$\text{s.t.} \quad u_i + v_j \leq c_{ij} \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n$$

$$u_i, v_j \text{ free}$$

$$\text{检验数:} \quad \sigma_{ij} = c_{ij} - \mathbf{y}^T \mathbf{p}_{ij} = c_{ij} - (u_i + v_j)$$

基变量部分: $u_i + v_j = c_{ij}$ 对偶解不唯一, 称为位势

等价性

问题：位势法所得的检验数是否唯一？

$$\sigma_{32} = c_{32} - c_{22} + c_{23} - c_{33}$$

$$= c_{32} - (u_2 + v_2) + (u_2 + v_3) - (u_3 + v_3)$$

$$= c_{32} - (u_3 + v_2)$$

解的改进

销地 产地	1	2	3	4	产量
1	6 14	7 5	5 5	3 7	14
2	8 8	4 13	2 6	7 9	27
3	5 -11	9 -3	10 6	6 13	19
销量	22	13	12	13	

改进后运输表

销地 产地	1	2	3	4	产量
1	6	7	5	3	14
2	8	4	2	7	27
3	5	9	10	6	19
销量	22	13	12	13	

	14	5	5	-4	
2	2	13	12	-2	
3	6	8	11	13	

最终运输表

<div>销地</div> <div>产地</div>	1	2	3	4	产量
1	<div>6</div> <div>1</div>	<div>7</div> <div>5</div>	<div>5</div> <div>5</div>	<div>3</div> <div>13</div>	14
2	<div>8</div> <div>2</div>	<div>4</div> <div>13</div>	<div>2</div> <div>12</div>	<div>7</div> <div>2</div>	27
3	<div>5</div> <div>19</div>	<div>9</div> <div>8</div>	<div>10</div> <div>11</div>	<div>6</div> <div>4</div>	19
销量	22	13	12	13	

产销不平衡问题的数学模型

$$\sum_{i=1}^m a_i > \sum_{j=1}^n b_j \quad \text{产大于销问题}$$

$$\min z = \sum_{i=1}^m \sum_{j=1}^n c_{ij} x_{ij}$$

$$\text{s.t.} \quad \sum_{j=1}^n x_{ij} \leq a_i$$

$$\sum_{i=1}^m x_{ij} = b_j$$

$$x_{ij} \geq 0 \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n$$

标准化

思路：化为平衡问题

方法：增加一个假象的销地 $n+1$,

$$\min z = \sum_{i=1}^m \sum_{j=1}^{n+1} c_{ij} x_{ij} \quad c_{i,n+1} = 0 \quad i = 1, \dots, m$$

$$\text{s.t.} \quad \sum_{j=1}^{n+1} x_{ij} = a_i$$

$$\sum_{i=1}^m x_{ij} = b_j \quad b_{n+1} = \sum_{i=1}^m a_i - \sum_{j=1}^n b_j$$

$$x_{ij} \geq 0 \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n+1$$

产小于销问题

思路：如果产小于销问题如何处理？

方法：增加一个假象的产地 $m+1$,

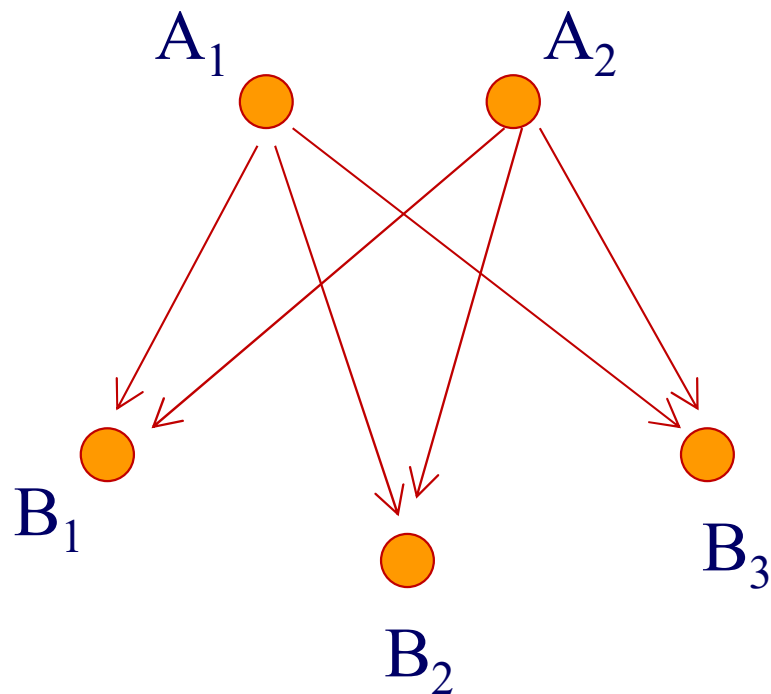
$$\min z = \sum_{i=1}^{m+1} \sum_{j=1}^n c_{ij} x_{ij} \quad c_{m+1,j} = 0 \quad i = j, \dots, n$$

$$\text{s.t.} \quad \sum_{j=1}^{n+1} x_{ij} = a_i \quad a_{m+1} = \sum_{j=1}^n b_j - \sum_{i=1}^m a_i$$

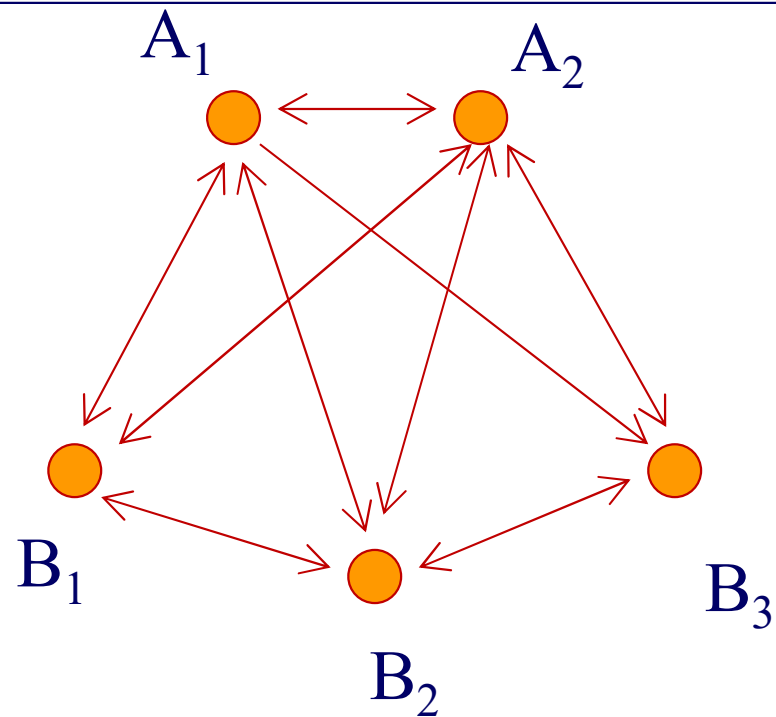
$$\sum_{i=1}^m x_{ij} = b_j$$

$$x_{ij} \geq 0 \quad i = 1, 2, \dots, m; \quad j = 1, 2, \dots, n+1$$

有转运问题



无转运问题



有转运问题

新增“产地”： $a_{m+j}=0 \quad j=1,2,\dots,n$

新增“销地”： $b_i=0 \quad i=1,2,\dots,m$

有转运问题的数学模型

$$\min z = \sum_{i=1}^{m+n} \sum_{\substack{j=1 \\ i \neq j}}^{m+n} c_{ij} x_{ij} + \sum_{i=1}^{m+n} c_i t_i$$

$$\text{s.t.} \quad \sum_{j=1, j \neq i}^{m+n} x_{ij} = a_i + t_i$$

$$\sum_{i=1, i \neq j}^{m+n} x_{ij} = b_j + t_j$$

$$x_{ij} \geq 0 \quad i, j = 1, 2, \dots, m+n; i \neq j$$

$$t_i \geq 0 \quad i = 1, 2, \dots, m+n \quad \text{第} i \text{个节点的转运量}$$

产销平衡问题

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j = Q$$

问题：能否取 $x_{ii}=t_i$ ？

问题：能否取 $x_{ii}=-t_i$ ？

有转运问题的标准化

$$\min z = \sum_{i=1}^{m+n} \sum_{j=1}^{m+n} c_{ij} x_{ij} + \sum_{i=1}^{m+n} c_i Q$$

$$\text{s.t.} \quad \sum_{j=1}^{m+n} x_{ij} = a_i + Q \quad i = 1, 2, \dots, m+n$$

$$\sum_{i=1}^{m+n} x_{ij} = b_j + Q \quad j = 1, 2, \dots, m+n$$

$$x_{ij} \geq 0 \quad i, j = 1, 2, \dots, m+n$$

$$t_i \geq 0 \quad i = 1, 2, \dots, m+n \quad \text{第} i \text{个节点的转运量}$$

产销平衡问题

$$\sum_{i=1}^m a_i = \sum_{j=1}^n b_j = Q$$

$$c_{ii} \triangleq -c_i$$

$$x_{ii} \triangleq Q - t_i$$