答案

一、填空

1. $\frac{1}{2}$，$\frac{3}{4}$，$\frac{1}{3}(ts+1)+\frac{1}{4}(t+s)$

2. $\frac{1}{12}e^{-3|t|}+\frac{1}{4}e^{-|t|}$，$0$

3. 0.06，0.385，0.36，0.08

4. $2e^{-2}$，$\frac{9}{2}e^{-3}$

二、

(1) 1；(2) $\frac{1}{2\sqrt{\pi}}e^{-\frac{x^2}{4}}$，(3) 0.0456

(4) 0.5；(5) 0

三、

(1) 5；(2) $e^{-5}-e^{-10}$ (4) $\frac{24}{61}$

四、(题目应该有误为 $B\sin(t-A\pi)$)

(1) $-\frac{1}{\pi}\cos t$，$\frac{1}{4}\cos(s-t)$；(2) $\frac{\pi}{4}(\delta(w-1)+\delta(w+1))$

(3) 0，$\frac{\pi^2}{2}\cos 2$；(4) 不是

五

(1) $\{1,4\}$ 开，$\{2,3\}$ 闭，$\{5,6,7\}$ 闭

(2) $d_1=d_4=d_5=d_6=d_7=1$，$d_2=d_3=2$，1,4 暂留，剩下正常返

(3) $\mu_2=\mu_3=2$，$\mu_5=7$，$\mu_6=\frac{7}{4}$，$\mu_7=\frac{7}{2}$

(4) $\frac{3}{4}$ (5) 0，$\frac{3}{7}$

过程：

一.填空题

1(1) $P\{X(1)\le 1\} = P\{A+B\le 1\} = F_A(A\le 1-B) = \int_0^1\int_0^{1-b} da\,db = \frac{1}{2}$

(2) $P\{X(1)\le 1 \mid X(0)\le 0.5\} = P\{A+B\le 1 \mid B\le \frac{1}{2}\} = P\{A+B\le 1, B\le\frac{1}{2}\}/P\{B\le\frac{1}{2}\}$

$= 2\int_0^{\frac{1}{2}}\int_0^{1-b} da\,db = \frac{3}{4}$

(3) $R_X(t,s) = E[X(t)X(s)] = E(A^2)ts + E(A)E(B)(t+s) + E(B^2)$

$= \frac{1}{3}(ts+1) + \frac{1}{4}(t+s)$

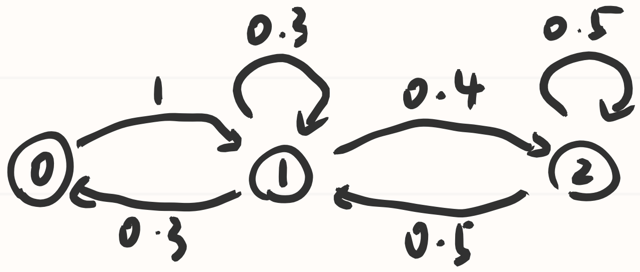
2(1) $S_X(w) = \frac{1}{2(w^2+9)} + \frac{1}{2(w^2+1)}$

$\because \frac{2a}{w^2+a^2} \xrightarrow{F^{-1}} e^{-a|t|}$

$\therefore R_X(\tau) = F^{-1}(S_X(w)) = \frac{1}{12}e^{-3|t|} + \frac{1}{4}e^{-1|t|}$

(2) $\lim_{\tau\to\infty} R_X(\tau) = 0 = \mu_X^2 \Rightarrow \mu_X = 0$

3



(1) $f_{22}^{(3)} = P_{21}P_{11}P_{12} = 0.06$

(2) $P^2 = \begin{bmatrix} 0.3 & 0.3 & 0.4 \\ 0.09 & 0.59 & 0.32 \\ 0.15 & 0.4 & 0.45 \end{bmatrix}$ $P_{22}^{(4)} = P_{20}P_{02}^{(2)} + P_{21}P_{12}^{(2)} + P_{22}P_{22}^{(2)} = 0.385$

(3) $P\{X_2=2\} = P\{X_0=0\}P_{02}^{(2)} + P\{X_0=1\}P_{12}^{(2)} = 0.36$

(4) $P\{X_0=1, X_1=2, X_3=2\} = P\{X_0=1\} P_{12}^{(2)} P_{22} = 0.08$

4 (1) $P\{N(2)=2 \mid N(1)=1\} = P\{N(2)-N(1)=1\} = \frac{[2(2-1)]^1}{1!} e^{-2} = 2e^{-2}$

(2) $P\{N(3)=3 \mid N(1)=1\} = P\{N(3)-N(1)=2\} = \frac{[2+1]^2}{2!} e^{-3} = \frac{9}{2} e^{-3}$

二.

(1) $Cov(B(1), B(2)) = \min\{1,2\} = 1$

(2) $B(2) \sim N(0,2)$ $\quad \therefore f_{B(2)}(x) = \frac{1}{2\sqrt{\pi}} e^{-\frac{x^2}{4}}$

(3) $P\{\min_{1\le t\le 2} B(t) < -1 \mid B(1)=1\}$

由于布朗运动的对称性

原式 $= P\{\max_{1\le t\le 2} B(t) > 1 \mid B(1) = -1\}$

$= P\{\max_{0\le t\le 1} B(t) > 2\}$

$= P\{T_2 < 1\} = 2P\{B(1) > 2\} = 2(1-\Phi(2))$

$= 0.0456$

(4) $P\{B(1) \ge 1 \mid B(2) = 2\} = P\{\tilde{B}(1) \ge 1 \mid 2\tilde{B}(\frac{1}{2}) = 2\}$

$= P\{\tilde{B}(1) - \tilde{B}(\frac{1}{2}) \ge 0\} = 0.5$

(5) $E(B(2)B(4)B(6)) = E\{B(2)[B(4)-B(2)][B(6)-B(4)]\}$

$- E[B^2(2)B(4)] + E[B^2(2)B(6)] + E[B(2)B^2(4)]$

$= -E\{B^2(2)[B(4)-B(2)+B(2)]\} + E\{B^2(2)[B(6)-B(2)+B(2)]\}$
$+ E\{B(2)[B(4)-B(2)+B(2)]^2\}$
$= -E(B^3(2)) + E(B^3(2)) + E(B^3(2))$
$= E(B^3(2)) = \int_{-\infty}^{+\infty} x^3 \frac{1}{2\sqrt{\pi}} e^{-\frac{x^2}{4}} dx = 0$

(注. $B(2), B(4)-B(2), B(6)-B(4)$ 相互独立且 $\mu_B = 0$)

三. 设 $N_1(t)$ 为 $[0,t]$ 时间内钓到鲫鱼的条数, $\lambda_1 = 3$
$N_2(t)$ 鳊鱼 $\lambda_2 = 2$
$N(t)$ 鱼

(1) $\lambda = \lambda_1 + \lambda_2 = 5$

(2) $P(N(1)=0, N(2)-N(1) \geq 1) = e^{-5} - e^{-10}$

(4) $P(N_1(1)=1 \mid N(2) \leq 2) = \dfrac{P(N_1(1)=1, N(2) \leq 2)}{P(N(2) \leq 2)}$

$= P(N_1(1)=1)[P(N_1(2)-N_1(1)=1, N_2(2)=0) + P(N_1(2)-N_1(1)=0, N_2(2)=1)$
$+ P(N_1(2)-N_1(1)=0\ N_2(2)=0]/P(N(2) \leq 2)$
$= \frac{24}{61}$

四.

(1) $\mu_X(t) = E(B\sin(t-A\pi))$
$= \frac{1}{2}\int_0^1 \sin(t-a\pi) da$
$= -\frac{1}{\pi}\cos t$

$$R_X(s,t)=E[B^2\sin(s-A\pi)\sin(t-A\pi)]$$
$$=\frac{1}{2}E\left[-\frac{1}{2}\left(\cos(s+t-2\pi A)-\cos(s-t)\right)\right]$$
$$=\frac{1}{4}\cos(s-t)$$

$$R_X(t,t+\tau)=\frac{1}{4}\cos\tau=R_X(\tau)$$

$\therefore$ 是宽平稳过程

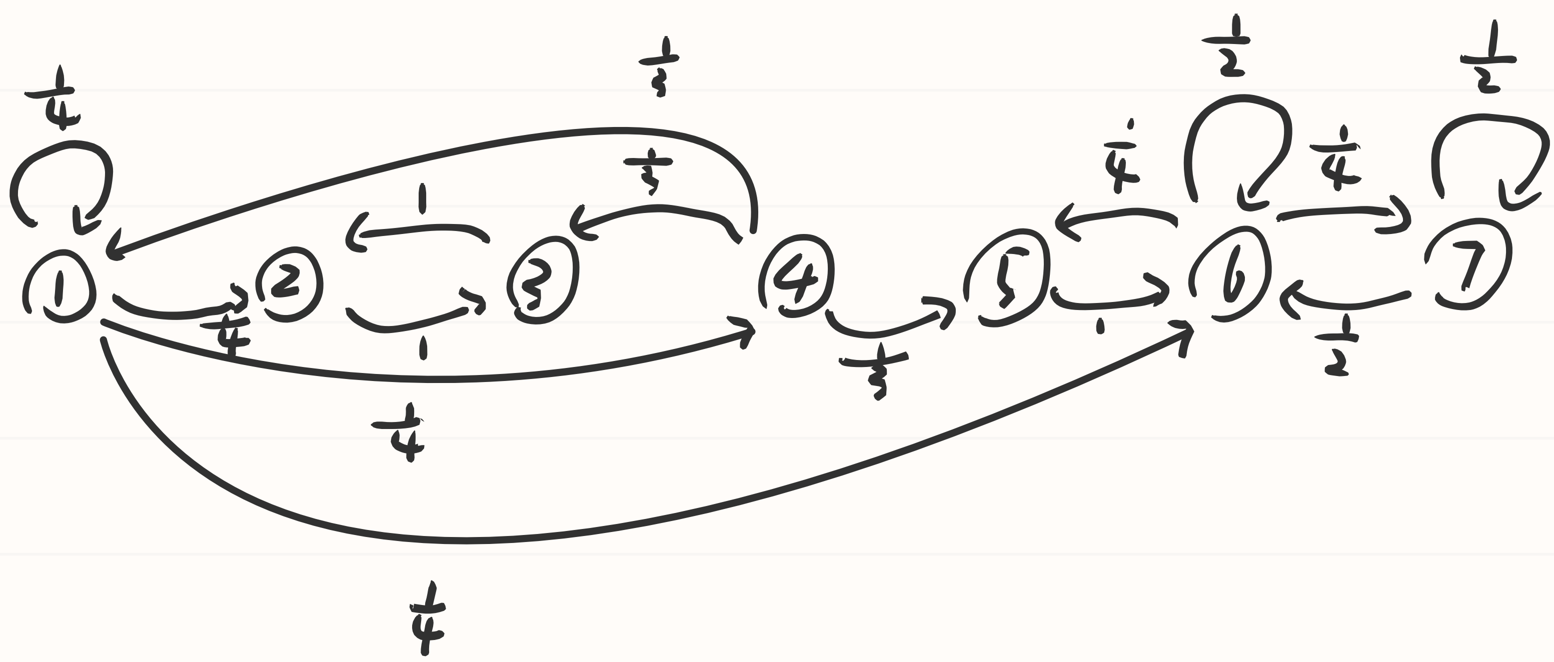
(2) $S_X(w)=F(R_X(\tau))=\frac{\pi}{4}\left(\delta(w-1)+\delta(w+1)\right)$

(3) $\langle X(t)\rangle=\lim\limits_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}B\sin(t-A\pi)\,dt$
$$=\lim_{T\to\infty}\frac{1}{2T}\left(-B\cos(T-A\pi)+B\cos(T+A\pi)\right)=0$$

$$\langle X(t)X(t+\tau)\rangle=\lim_{T\to\infty}\frac{1}{2T}\int_{-T}^{T}B^2\sin(t-A\pi)\sin(t+\tau-A\pi)\,dt$$
$$=\lim_{T\to\infty}-\frac{B^2}{4T}\left(\frac{1}{2}\sin(2T+\tau-2A\pi)-\frac{1}{2}\sin(-2T+\tau-2A\pi)-2T\cos\tau\right)$$
$$=\frac{B^2}{2}\cos\tau$$

(4) 不是

五.



(1) $\{1,4\}$，$\{2,3\}$，$\{5,6,7\}$

不闭　　闭　　闭

(2) $d_1=d_4=1$, $d_2=d_3=2$ $d_5=d_6=d_7=1$

1,4 暂留 2,3,5,6,7 正常返

(3) $\begin{cases}\pi_2+\pi_3=1\\ \pi_2=\pi_3\end{cases} \Rightarrow \begin{cases}\pi_2=0.5\\ \pi_3=0.5\end{cases} \Rightarrow \begin{cases}\mu_2=2\\ \mu_3=2\end{cases}$

$\begin{cases}\pi_5+\pi_6+\pi_7=1\\ \pi_5=\frac{1}{4}\pi_6\\ \pi_6=\pi_5+\frac{1}{2}\pi_6+\frac{1}{2}\pi_7\end{cases} \Rightarrow \begin{cases}\pi_5=\frac{1}{7}\\ \pi_6=\frac{4}{7}\\ \pi_7=\frac{2}{7}\end{cases} \Rightarrow \begin{cases}\mu_5=7\\ \mu_6=\frac{7}{4}\\ \mu_7=\frac{7}{2}\end{cases}$

(4) 设 h 为从1出发进 $\{5,6,7\}$ 的概率

$h=\frac{1}{4}+\frac{1}{4}\times\frac{1}{3}+\frac{1}{4}\times\frac{1}{3}h \Rightarrow h=\frac{1}{2}$

$\therefore P=\frac{1}{2}+\frac{1}{2}h=\frac{3}{4}$

(5) $\lim\limits_{n\to+\infty} P(X_n=1)=0$

$\lim\limits_{n\to+\infty} P(X_n=6)=P\{T_{\{5,6,7\}}<T_{\{2,3\}}\}\cdot\pi_6=\frac{3}{7}$