**第一章 绪论：**

**1.定义:** ①Robotics is concerned with the study of those machines that can replace human beings in the execution of a task, as regards both physical activity and decision making.

②A goal oriented machine that can sense, plan and act.③A robot is a reprogrammable multifunctional manipulator designed to move material, parts, tools, or specialized devices through variable programmed motions for the performance of a variety of tasks.

**2.** Robot first appeared in 1920 science fiction play “Rossum‘s Universal Robots”.

**3. Three laws of robotics:** ①A robot may not injure a human being or, through inaction, allow a human being to come to harm. ②A robot must obey the orders given by human beings, except when such orders would conflict with the first law. ③A robot must protect its own existence, as long as such protection does not conflict with the first or second law.

**4.** **UAV**: Unmanned aerial vehicle; **AUV**: Autonomous underwater vehicle.

**5.** Subclasses of robotics includes manufacturing robot, service robots, filed robot and humanoid robot.

**6.** A manufacturing robot is typically an arm-type manipulator, often placed on fixed bases and perform repetitive tasks within local work cells. **优点:** high speed and high precision.

**7.** Cooperative robots can work with people togther. Human class, human-scale task in a human environment.

**8.** Field and service robots face specific and significant challenges: ①Filed robot must operate and move in a complex, cluttered and changing environment. ②These robot must operate safely in the presence of people.

**第二章 ：Representing Position and Orientation**

**1.** Point P's position can be described by a coordinate vector. The vector represents its displacement from the reference coordinate system.

**2.** We assume the object is rigid and attach a coordinate frame {B} to the object. **Rigid:** constituent points maintain a constant relative position w.r.t. {B}. Objects can be robots, cameras, workpieces, obstacles and paths.

**3.** The position and orientation of an object’s coordinate frame is referred to as its **pose**.

**4. AξB** is pose of frame {B} w.r.t. frame {A}, where {A} is reference coordinate frame, {B} is frame being described.

**5. Ap= AξB · Bp, Aξc=****AξB** ⊕ **Bξc，Ap=( AξB** ⊕ **Bξc)· cp**. Relative poses ξ can be composed or compounded.

**6.** A directed graph is a graph that is a set of vertices (nodes: pose) connected by edges(relative pose), where the edges have a direction associated with them.

**7. ξ ⊕ 0=ξ，ξ 0=ξ，ξ1⊕ξ2 ≠ ξ2⊕ξ1 (composition is not commutative).**

**8.** ξ can be any mathematical object that supports the algebra described above and is suited to the problem at hand. Homogeneous transformations (齐次变换), orthonormal rotation matrices (正交旋转矩阵) and quaternions (四元数).**9. 旋转矩阵**：VRB=[cosθ -sinθ; sinθ cosθ] describes how points are transformed from frame {B} to frame {V} when the frame is rotated. **性质**：①Rotation matrix is orthonormal. ②The inverse of R is the same as the transpose. ③Rotation matrix's determinant is +1.

**10. Matrix Exponential: R=e[θ]x, [θ]x=[0 -θ; θ 0] (Skew-symmetric matrix)**.

**11. 齐次变换**：[Ax, Ay, 1]T=[ cosθ -sinθ x; sinθ cosθ y; 0 0 1][Bx, By, 1] T，Ap=ATB Bp.

**12. 欧拉旋转定理**：Any two independent orthonormal coordinate frames can be related

by a sequence of rotations (not more than three) about coordinate axes, where no two successive rotations may be about the same axis.

**13.三维旋转表示方法**：①Orthonormal rotation matrices. ②Euler and Cardan angles. ③Rotation axis and angle.

**14.**[Ax, Ay, Az]T=ARB [Bx, By, Bz] T=[xA·xB xA·yB xA·zB；yA·xB yA·yB yA·zB；zA·xB zA·yB zA·zB] [Bx, By, Bz] T。ARB的每一列代表 {B} 的坐标轴在{A} 中的方向，每一行代表 {A} 的坐标轴在 {B} 中的方向。

**15.** **绕xyz旋转矩阵**：；

；.

**6.欧拉角和卡尔丹角：**There are two classes of rotation sequence: Eulerian and Cardanian. ①Eulerian: XYX, XZX, YXY, YZY, ZXZ, ZYZ; ②Cardanian：XYZ, XZY, YZX, YXZ, ZXY, ZYX. All these sequences are called Euler angles.

**17.** R=Rz(𝝓)Ry(𝜽)Rz(𝝍)，ZYZ sequence is commonly used in aeronautics and mechanical dynamics，The Euler angles are the 3-vector 𝚪 = (𝝓, 𝜽, 𝝍).

**18.** R=Rz(θy)Ry(θp)Rx(θr)，RPY sequence allows all angles to have arbitrary sign but it has a singularity when θp=±π/2. θy: yaw angle (偏航) θp: pitch angle (俯仰) θr: roll angle (横滚).

**19.** For camera we can use optical(摄像头方向) axis as z-axis，left side of camera as x-axis. For mobile robot we can use gravitational acceleration vector (measured with accelerometers) as the z-axis heading direction (measured with an electronic compass) as the x-axis.

**20.** An orthonormal rotation matrix always have one real eigenvalue at λ=1 and a complex pair λ=cosθ±sinθ，v (rotation axis) is the eigenvector corresponding to eigenvalue λ=1.

**21.** R=I3×3+sinθ S(v) + (1-cosθ)(vvT-I3×3) ，S(v)=[0 -vz vy；vz 0 -vx；-vy vx 0].

**22.** R=e^[]xθ，where θ is rotation angle, is unit-vector paralle to rotation axis. ω=[ωx, ωy ωz]T，**[ω]x=[0 -ωz ωy；ωz 0 -ωx；-ωy ωx 0]**; **(三维反对称矩阵)**

**23.** T1T2=[R1 t1；0 1][R2 t2；0 1]=[R1R2 t1+R1t2；0 1]；T-1=[R t；0 1] -1=[RT -RTt；0 1]

**第三章 ：Time and Motion**

**1.** ARB(t)=e^[]xθ(t)，ARB= []x ARB= ARB []x，Aω defines instantaneous axis and rate of rotation.

**2**. [Av]x=A[v]xAT.

**3**.，**AvB**=，AωB=Aω

**4. 空间速度**： **AvB** = ( **AvB**, **AωB**)，this is the instantaneous velocity of frame {B} w.r.t. {A}.

**5**. **空间速度变换**： Av=[ ARB 03×3；03×3 ARB]Bv，{A}为世界坐标系，{B}为运动物体坐标系。

**6.** **空间速度变换**：cv=[ cRB [CtB]x cRB；03×3 cRB]Bv=Ad(cξB) Bv，{C}是运动物体上的另一个坐标系。

**7. 增量旋转**：RB(t+δt)=RB(t) BRΔ=RB(t)+δt= RB(t)+δtRB(t)[Bω]x，BRΔ=δt[Bω]x+I3×3

**8. 增量运动**：**ξ2 = ξ1****⊕ ξΔ，** **ξΔ ~ TΔ =** [**RΔ tΔ；0**1×3 **1**]= [[**ΔR**]x+**I**3×3 **Δt；0**1×3 **1**]，

**Δξ =**(**Δt，ΔR)** 是个6维向量。

**9. 欧拉运动方程**：**BJ B+Bω ×** (**BJBω**)=**Bτ**，τ是力矩.

**10. 力旋量wrench**：**W**=(fx, fy, fz, mx, my, mz)，由力和转矩构成的6维向量。

**11.** cW=[ cRB [CtB]x cRB；03×3 cRB]T BW=Ad(cξB)T BW

**12. 惯性参考系**：a reference frame that is not accelerating or rotating。**惯性力**：fictitious force

**13. 路径**：A locus in space that leads from an initial pose to a final pose

**14. 轨迹**：A path with specified timing

**15. tpoly**: For most of the time, the velocity is far less than the maximum. The mean velocity is only ~52% of the peak.

**16. lspb**: comprises a **L**inear **S**egment with **P**arabolic **B**lends. Too high or too low a value for the maximum velocity will result in an infeasible trajectory.

**17. Cartesian motion**: a smooth path between two poses in SE(3) which involves change in position + orientation.

**18.惯性导航系统**：Inertial navigation system (INS) measures its accelerations and angular velocities and integrates them over time to estimate velocity, orientation and position. INS has no external inputs, it well suited to applications such as submarine, spacecraft and missile guidance.

**19.** Two common conventions have x-, y-, z-axes respectively parallel to north-east-down (NED) directions (北东地) and east-north-up (ENU) directions (东北天).

**20. 陀螺仪**：A triaxial gyroscope measures angular velocity vector Bω# in body frame {B}.

**21**. ， **~** e^[Bω#]xδt为增量位姿，代表估计的位姿。

**22. 加速度计**：Accelerometers are sensors that measure acceleration，Even when not moving they sense the acceleration due to gravity，Gravitational acceleration is a function of Earth’s material and our distance from Earth’s center. 赤道处半径大，g较小。

**23.** d=m(a+g)/k，a#=a+g，0g=[0 0 g]T，Bg=(0ξB) 0g=[-gsinθp gcosθpsinθr gcosθpcosθr]T=Ba#=[ax, ay, az]T，其中 0ξB=Rz(θy)Ry(θp)Rx(θr).

**24.** **,** 是物体坐标系{B}观测到的加速度,是世界坐标系中物体加速度。

**25. 磁力计**：At any point on the Earth magnetic flux (磁通量) lines can be considered a vector m，the length of the vector, magnetic field intensity(磁场强度B), is measured by a magnetometer in units of Tesla (T).**26.** We describe vector 𝒎’s direction in terms of two angles: declination (磁偏角D) and Inclination (磁倾角I).

**27.** 0**m=**B[cosI 0 sinI]T**, Bm**=(**0ξB**) 0**m，Bm**#=[mx my mz]T，arctan[cosθp(mzsinθr – mycosθr)]/(mx+BsinIsinθp )，**tn=**-D.

**28.** x#=sx+b+ε，𝑠 can be determined by some calibration(标定) procedure，bias b that varies over time is often called sensor drift，Scale factor and bias are typically both a function of temperature. Bias varies with time and temperature.

**29. 显式互补滤波器**：=e^[Bω#(k)-(k)+kpσp(k)]xδt，，，**,** 是惯性系中已知的向量信号(如g).

**第四章：Mobile Robot Vehicles**

**1.** one of the most important functions of a mobile robot is to move to some place.

**2. automated guided vehicles (AGVs)**：use fixed infrastructure for guidance.

**3.** Unmanned aerial vehicles **(UAVs)**；Autonomous underwater vehicles **(AUVs)**；Autonomous surface vehicles **(ASVs)**.

**4**. The lines of no motion intersect at Instantaneous Center of Rotation **(ICR)**.

**5.** **角速度:** **, 转弯半径:**，γ是steering angle.

**6.** The angles of the steered wheels γL and γR should be very slightly different, this is achieved by Ackerman steering mechanism.

**7.** Driven wheels rotate at different speeds on corners. Differential gearbox is required between motor and driven wheels.

**8.** The rate of change of heading is referred to as turn rate, heading rate or yaw rate.

**9.** **自行车模型**：.

**10.** Bicyle vehicle cannot change its orientation when it is not moving.

**11. 移动到指定点**：，，，. Operator Ө returns the angular difference lie within [−𝜋, 𝜋).

**12. 跟随直线ax+by+c=0**：， ，，，**.**

**13. 跟随轨迹：，，，，**为保持的距离。

**14. 移动到指定位姿 (x\*, y\*, θ\*)**：**，，，，，**其中**，**即目标在前方**，**控制器**：，，**

**，**稳定条件**：，**若目标在后方，把控制器的v和γ取负，若要运动到任意姿态(x\*, y\*, θ\*)，进行变换：**，，，**

**15.差速转向: ,,,**前进速度

**16. {B}→{B’}：，，**其中 {B’} 是 {B} 沿着自己的x轴平移a个单位得到。现在的x和y是小车上其他点在世界坐标系中的坐标。

**17. 飞行与地面机器异同**：①They have 6 DoF and their configuration 𝑞 ∈ 𝒞, where 𝒞 ⊂ℝ3 × 𝕊1 × 𝕊1 × 𝕊1；②They are actuated by forces, that is their motion model is expressed in terms of forces and torques rather than velocities. We use a dynamic rather than a kinematic model.

**18. 推力公式**：**，**b>0称为Lift constant，depends on：①air density；②cube of the rotor blade(浆片) radius；③number of blades；④chord length of the blade.

**19. 平移动力学**：，其中总推力T=Σ Ti，Bv为空气阻力

**20. 旋转力矩**：①Rolling torque：；②Pitching(俯仰)：，d为旋臂长度；③Reaction(反作用)/yaw(偏航)：

**21.**

**22. 俯仰角控制器**：τy\*=Kτ,p (θp\* -θp# )+Kτ,d (p\*-p#)，其中θp#是实际俯仰角，可用陀螺仪测量

**23.** ，，

控制**，**，由下面得到。

**24. 移动到指定位置**：0v\*=Kp(0p\*-0p#)，将速度变换到{B’}中：

**第五章：Navigation**

**1. Reactive navigation：**①heading towards a light;②following a white line on the ground;③moving through a maze by following a wall;④vacuuming room by following random path

**2. Map-based navigation**：①requires a map of the environment；②requires that the robot’s position is always known.

**3.** Robotic vacuum cleaners use only random motion and information from contact sensors to perform a complex task.

**4.** To estimate the gradient requires >= 2 measurements of the field.

**5.** Multiple behaviors and the ability to switch between them leads to an approach known as behavior-based robotics.

**6.** Bugs use simple automata, bug includes a state machineand other logic in between the sensor and the motors, Automata has memory which our earlier Braitenberg vehicle lacked.

**7. 反应式导航假设：**①The robot operates in a grid world and occupies one grid cell；②The robot does not have any nonholonomic constraints and can move to any of its eight neighboring grid cells；③It is able to determine its position on the plane；④The robot can only sense its goal and whether adjacent cells are occupied.

**8. Bug2算法：**①It moves along a straight line towards its goal；②If it encounters an obstacle it moves around the obstacle (always counter-clockwise or clockwise)；③It continues until it encounters a point that lies along its original line that is closer to the goal than where it first encountered the obstacle. **9.** The world is treated as a grid of cells，We use ①0: an unoccupied cell or free space where the robot can drive ②1: an occupied or non-drivable cell.

**10.** A cell size 1 × 1 m requires just 125 kbyte km–2.

**11. 基于地图导航假设**：①②③反应式导航前三点；④The robot is able to use the map to compute the path it will take.**12. Distance Transform**：Wherever robot starts, it moves to neighboring cell has **smallest** distance to goal. Once it is created, it can plan path from **any initial point** to that goal.

**13. D\***：D\* generalizes the occupancy grid to a cost map. The costs assigned to cells may depend on the characteristics of the vehiclethe. The key feature of D\* is that it **supports incremental replanning.** D\* is able to efficiently **change cost map during mission.** It ensures continuity of motion even if the plan has changed.**14. D\***：Every cell is a graph vertex and has：①a state t ∈ {NEW, OPEN, CLOSED}；②a cost (a distance to the goal)；③a link to the neighboring cell that is closest to the goal.

**15. D\***：Initially every cell is in the NEW state. The cost of the goal cell [1,3] is zero and its state is OPEN. The cost of reaching cells that are neighbors of an OPEN cell is computed. These cells in turn are set to OPEN and the original cell is removed from the open list and becomes CLOSED.**16.** D\* and distance transform does **not support** a changing goal.**17.** In roadmap methods, query can include both start and goal positions，The planning phase provides analysis that supports changing starting points and changing goals.

**18.** The roadmap need only be computed once. It can then be used like the train network to get us from any start location to any goal location.

**19.** Thinning or skeletonization, like the distance transform, is a computationally expensive iterative algorithm.

**20**. Probabilistic methods sparsely sample the world map.**21. PRM(probabilistic roadmap)**：The planning phase finds N random points that lie in free space. A network is created with a minimal number of disjoint components and no cycles. A network is created with a minimal number of disjoint components and no cycles. we can provide start and goal to the query phase.

**22. PRM的问题**：①The underlying random sampling of the free space means that adifferent graph is created every time the planner is run, resulting in different paths and path lengths；②The planner can fail by creating a network consisting of disjointcomponents；③Long narrow gaps between obstacles are unlikely to be exploited ( Probability of randomly choosing points that lie along such gaps is very low ).**23. Lattice Planner**：Each path is an arc which requires a constant steering wheel setting. Arc radius is chosen so that at the end of each arc the robot’s heading direction is some multiple of π/2. While the paths appear smooth and continuous the curvature(曲率) is in fact discontinuous. At some nodes the steering wheel angle would have to change instantaneously from hard left to hard right.

**24. RRT(Rapidly-Exploring Random Tree)**：The limiting factor is the combinatoric explosion in the number of possible poses. In the random tree, the paths have a good coverage of the configuration space (x, y, θ). So the algorithm is known as rapidly exploring.

**第六章：Location**

**1.** Kalman filter is an optimal estimator for the case where process and measurement noise are **zero-mean** Gaussian noise.

**2. 卡尔曼滤波器特点：**①It is optimal, but only if the noise is truly Gaussian with zero mean and time invariant parameters；②It is recursive, the output of one iteration is the input to the next；③It is asynchronous.

**3. 线性系统**：；；

其中，高斯噪声.

**4. 预测方程**: ；

，This is an open-loop step.

**5. 新息innovation**：

**6. 卡尔曼增益：,**

**新息方差：**

**7. 更新方程**：，

**8. 非线性系统**：；

**9. 预测方程：；**

**10.新息：**

**11.卡尔曼增益：**

**12.更新方程：；**

**13：**measurements are subject to both systematic and random error.

**14.** It is useful to describe the robot’s position in terms of a probability density function (PDF) over all possible positions of the robot.

**15. Odometer(里程计)** : a sensor that measures distance travelled and sometimes also change in heading direction.

**16.** At each time step, Kalman filter tells us two things: ①The best estimate of where the robot is；②How certain we are about that estimate.

**17. Dead reckoning建模：**里程计输出，噪音，，状态方程

**18. Dead reckoning估计位姿：**，其中

；

**19.** Uncertainty in position grows without bound using dead-reckoning alone.

**20.传感器模型**：，传感器误差，其中xv是机器人位姿，xi是路标位置，β是方位角(bearing angle).

**21. 传感器模型线性化**：，

其中,

**22. 新息**：

**23. 创建地图的假设：**The robot has ideal localization.

**24. 创建地图的状态向量**：，状态向量包含了M(M≤ N)个已经被观测到的路标的坐标。

**25. 创建地图预测方程**：，

**26. 拓展状态向量**：**，**

**，，**其中和是传感器的输出，，

**27. 传感器模型局部线性化：，**

**，**

**28. SLAM**：Simultaneous localization and mapping；

**CML**：Concurrent mapping and localization

**29. SLAM状态向量**：

**30. SLAM的Yz**：**，**其中

**31. SLAM的Hx：，**

**第七章：Robot Arm Kinematics**

**1. 平面关节型机器人**：SCARA (Selective Compliance Assembly Robot Arm) is rigid in the vertical direction and compliant in the horizontal plane. **桁架式机器人:** A gantry robot has one or two degrees of freedom of motion along overhead rails.

**2. Kinematics** is the branch of mechanics that studies the motion of a body, or a system of bodies, without consideration given to its mass or the forces acting on it.

**3.** Each joint has one DoF, either translational (a sliding or prismatic joint) or rotational (a revolute joint).

**4. R**：转动副；**P**：移动副

**5. 2-joint planner特点**：①Most end-effector positions can be reached with two different joint angle vectors；②The robot can position end-effector at any point within its reach but we cannot specify an arbitrary orientation.

**6.** Truly useful robots have a task space 𝒯 ⊂ 𝐒𝐄(3) enabling arbitrary pose of {𝐸}. This requires a robot with dim 𝒞 ≥ dim 𝒯. It can be achieved by a robot with six or more joints.

**7. Puma560**：

**8.** Denavit-Hartenberg (DH) notation is a systematic way of describing the geometry of a serial chain of links and joints. Link 𝑗 connects joint 𝑗 to joint 𝑗 + 1, Joint 𝑗 connects link 𝑗 − 1 to link 𝑗 and moves them relative to each other.

**9.** Relationship between two LINK coordinate frames would ordinarily entail 6 parameters (3 for translation and 3 for rotation). For DH notation there are only 4 parameters.

**10. 两个约束：**① Axis 𝑥𝑗 intersects 𝑧𝑗−1 ( 𝑥𝑗∥𝑧𝑗−1)；②Axis 𝑥𝑗 is perpendicular to 𝑧𝑗−1 ( 𝑥𝑗⊥𝑧𝑗−1)**11. DH Parameters**：①关节角：Joint angle ；②连杆偏移：Link offset ；③连杆长度(常数)：Link length ；④连杆扭转角(常数)：Link twist

**12. 从连杆坐标系 {j-1} 变换到 {j}**：j-1ξj (θj, dj, aj, αj)=Rz(θj) ⊕ Tz(dj) ⊕ Tx(aj) ⊕ Rx(αj)

**13. Revolute joint:** ①𝜃𝑗: joint variable；②𝑑𝑗: constant；

**Prismatic joint**: ①𝑑𝑗: joint variable；②𝜃𝑗: constant；③α𝑗 = 0

**14.** For an 𝑁-joint robot, generalized joint coordinates 𝑞 ∈ 𝒞，𝒞 ⊂ ℝ𝑁 is called joint space or configuration space.

**15.** Joint coordinates are also referred to as pose of the manipulator. It is different to pose of the end-effector which is a Cartesian pose 𝜉 ∈ 𝐒𝐄(3)

**16. N连杆机械臂位姿**：最后一个关节相对于基座：**，**

末端执行器相对世界坐标系

**17**. In general the solution to the function q=K-1(ξ) is not unique.**18. 数值解法**：q\*=argmin ||K(q) ξ\* ||，初值q0决定解.

**19. 球形腕:** A necessary condition for a closed-form solution of a 6-axis robot is a spherical wrist mechanism. Spherical wrists have three axes of rotation. They are orthogonal and intersect at a common point. The robot end-effector pose, position and an orientation, is defined at the center of the wrist. The wrist axes intersect at a common point they cause zero translation. The position of the end-effector is a function only of the first three joints.

**20.** **Puma560多解**：In general there are 2×2×2=8 sets of joint coordinates that give the same end-effector pose：left- and right-handed、elbow up or down、wrist flipped or not.

**21. Puma560无解**：Due to mechanical limits on joint angles and possible collisions between links not all eight solutions are physically achievable.

**22. Puma560奇异位姿**：It has a wrist singularity when q5=0 and the axes of joints 4 and 6 become aligned.

**23. 关节空间运动**：A joint-space trajectory is formed by smoothly interpolating between joint configurations q1 and q2.（可以避免奇异位姿）

**第八章：Manipulator Velocity**

**1.** The joint rate of change and the end-effector velocity are related by the manipulator Jacobian matrix.

**2.** **，**This relationship is sometimes referred to as the instantaneous forward kinematics.

**3. 世界坐标系下：** ，0𝑱(𝑞) ∈ ℝ6×𝑁 is manipulator Jacobian or geometric Jacobian.

**4. 机械臂雅克比矩阵**：① Each row corresponds to a Cartesian degree of freedom；②Each column corresponds to a joint

**5. 末端执行器坐标系下**：

**6. 解析雅克比矩阵：**Analytical Jacobians are those where the rotational velocity is expressed in a representation other than angular velocity, commonly in terms of triple-angle rates.

**7.** , 其中

**8. ，**，当俯仰角θp=±π/2时，A奇异；t is referred to as a representational singularity.

**9.** 如果将 写作**，**

则

**10. J(q) 奇异条件：**①the robot is at maximum reach；②when one or more axes become aligned resulting in the loss of degrees of freedom (gimbal lock problem)**11.** At the Puma’s ready pose the Jacobian is singular.

**12.** However if one or more radii are very small this indicates that the endeffector cannot achieve velocity in the directions corresponding to those small radii.

**13. 可操作性Manipulability：，**it describes how spherical the ellipsoid is, for instance the ratio of the smallest to the largest radius.

**14. 分解速率运动控制**：**，**，

闭环控制：

**15. 阻尼逆矩阵：，**𝜆 is a small constant added to the diagonal which places a floor under the determinant，𝜆𝐼 will introduces some error in，the closed-loop resolved-rate motion scheme would minimize this error.

**16. 伪逆pseudo inverse**：**，**Another approach is to delete from the Jacobian all those columns that are linearly dependent on other columns.