QSite 2024 Classiq Open Challenge Project Proposal

Team Name: *Quantum Duckies*

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Abstract

Short description of our plan

1 Introduction

In "A Quantum Algorithm for Solving Linear Differential Equations: Theory and Experiment", [1] Tao Xin, et al. demonstrate the capability of quantum algorithms to provide exponential speedups over classical methods. We intend to use this paper's algorithm and Classiq's end-to-end quantum software platform to solve a harmonic oscillator equation with frequency $\omega = 1$:

$$y'' + \omega^2 y = 0, y(0) = 1, y'(0) = 1 \tag{1}$$

First, we solve it by hand to investigate expected outcomes:

$$y'' + \omega^2 y = 0 \text{ hence the characteristic equation is } r^2 + \omega^2 r^0 = 0$$
$$= r^2 + \omega^2 = 0 \implies r = \pm i\omega \text{ (two imaginary roots)}$$
$$\implies y(t) = c_1 \cos(\omega t) + c_2 \sin(\omega t) \text{ is the general solution}$$
$$y(0) = c_1 \cos(0) + c_2 \sin(0) = 1 \implies c_1 = 1$$
$$y'(0) = -c_1\omega \sin(\omega t) + c_2\omega \cos(\omega t) = 1 \implies c_2 = 1/\omega$$

Therefore,

$$y(t) = \cos(\omega t) + \frac{1}{\omega}\sin(\omega t) \tag{2}$$

We observe that $y(t)$ describes the displacement of a system experiencing simple harmonic motion (SHM). Hence, we have the following relations for such a system:

$$E(t) = KE(t) + PE(t) \tag{3}$$

$$KE(t) = \frac{1}{2}m(y'(t))^2 \tag{4}$$

$$PE(t) = \frac{1}{2}m\omega^2(y(t))^2 \tag{5}$$

Additionally, we can expand $KE(t)$ and $PE(t)$ in terms of y using Equation 2.

$$KE(t) = \frac{1}{2}m\left(\omega^2 \sin^2(\omega t) - 2\omega \sin(\omega t)\cos(\omega t) + \cos^2(\omega t)\right) \tag{6}$$

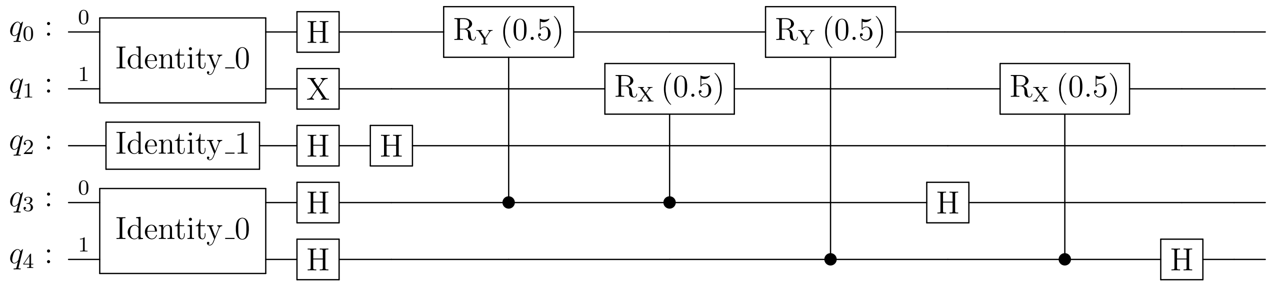
$$PE(t) = \frac{1}{2}m\omega^2 \left(\cos^2(\omega t) + 2\frac{1}{\omega}\cos(\omega t)\sin(\omega t) + \frac{1}{\omega^2}\sin^2(\omega t) \right) \tag{7}$$

Using Equations 6 and 7 we can evaluate the kinetic and potential energy in the time interval $[0, 1]$, depending on the values of m and ω. Now that this Classical context has been established, we can turn to Quantum Methods.

2 Methods

3 Progress

Our proposal culminates in the following diagram:



4 Conclusion

References

[1] Tao Xin, Shijie Wei, Jianlian Cui, Junxiang Xiao, Iñigo Arrazola, Lucas Lamata, Xiangyu Kong, Dawei Lu, Enrique Solano, and Guilu Long. Quantum algorithm for solving linear differential equations: Theory and experiment. *Phys. Rev. A*, 101:032307, Mar 2020.