Spectral Algebraics: Audible Geometry via E8-Inspired Signal Synthesis and 3D Visualization

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November 2025

Abstract

This work explores the intersection of algebraic structure and sensory experience by mapping properties of the E8 Lie group onto audio signals and interactive visualizations. Algorithmically generated waveforms—structured by E8 matrices and root patterns—are subjected to detailed time–frequency analysis, yielding amplitude and phase matrices rendered in three dimensions. By encoding instantaneous phase as color and amplitude as spatial topology, we reveal polytope-like formations whose symmetries follow directly from the mathematics. The result is a new form of "audible geometry," in which listeners and viewers encounter mathematical abstraction as immersive phenomena. All code and data are provided for replication and further exploration.

Methods

All source material was generated algorithmically in Python using custom routines that assemble waveforms from arrays of base frequencies. Each array was modulated through ternary logic rules and harmonic stacking schemes referencing E8 lattice algebra. Signals were rendered at 44.1 kHz with 32-bit floating-point precision to preserve fine time–frequency detail. Short-time Fourier transforms (STFT) were computed via `scipy.signal.stft` with Hann windowing (4096-sample window, 50% overlap) to balance temporal and spectral resolution. The resulting spectrogram matrices (time × frequency × amplitude) were mapped into three dimensions, encoding amplitude as the z-axis and instantaneous phase as HSV color. Three-dimensional renderings were produced using `plotly` and `matplotlib`, allowing interactive rotation to reveal polytope-like symmetries

through algorithmic or user-driven matrix transformations. All scripts and data, including `sonify_triality.py` to generate `.wav` files, are hosted at https://github.com/QSOLKCB/QEC.

Results and Discussion

The resulting sonograms and interactive renderings reveal emergent geometric order arising from the embedded E8 algebra. Spectral structures align with the high-dimensional symmetries defined in the synthesis algorithm, generating polytope-like formations across the frequency–time–amplitude continuum. Amplitude surfaces fold along discernible trajectories, while phase mappings exhibit rotational invariance and discrete discontinuities—audible analogs of geometric operations such as reflection, projection, and permutation [1, 2, 3].

These sonic geometries are not aesthetic embellishments but direct manifestations of the signal's algebraic scaffolding. As the listener navigates interlocking spectra and polychromatic phase arcs, the geometry is both seen and heard: each modulation rule translates into an audible event, and the topology of E8 inscribes itself in harmonic and spatial form alike [4, 5, 6, 7, 8, 9].

This synthesis of mathematics and perception suggests that complex algebraic structures can be rendered experientially. By transforming symmetry into a sensory domain, *Spectral Algebraics* positions geometry not as an abstraction behind the sound, but as the sound itself—a new axis of musical meaning where structure and perception converge [10, 11, 12, 13].

Figures

Acknowledgments

This research was developed within the QSOL Audio Geometry Series using open-source scientific tools including Python, SciPy, Plotly, and Matplotlib. The author thanks collaborators across the QSOL-IMC network for their contributions to the conceptual and technical development of *Spectral Algebraics*.

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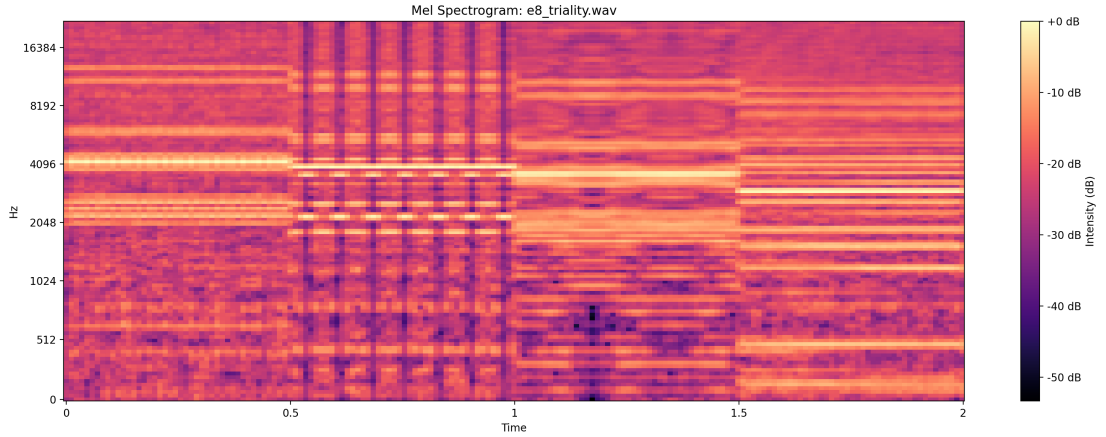

Figure 1: **Figure 1.** Time–frequency spectrogram of the E8-inspired waveform, extracted from *e8_triality.wav*. Amplitude is mapped to brightness, revealing unfolding spectral symmetry and periodic accentuations as direct consequences of the underlying algebraic structure. Notable are the folding and reflection patterns, which correspond to shifts in E8-derived synthesis parameters.

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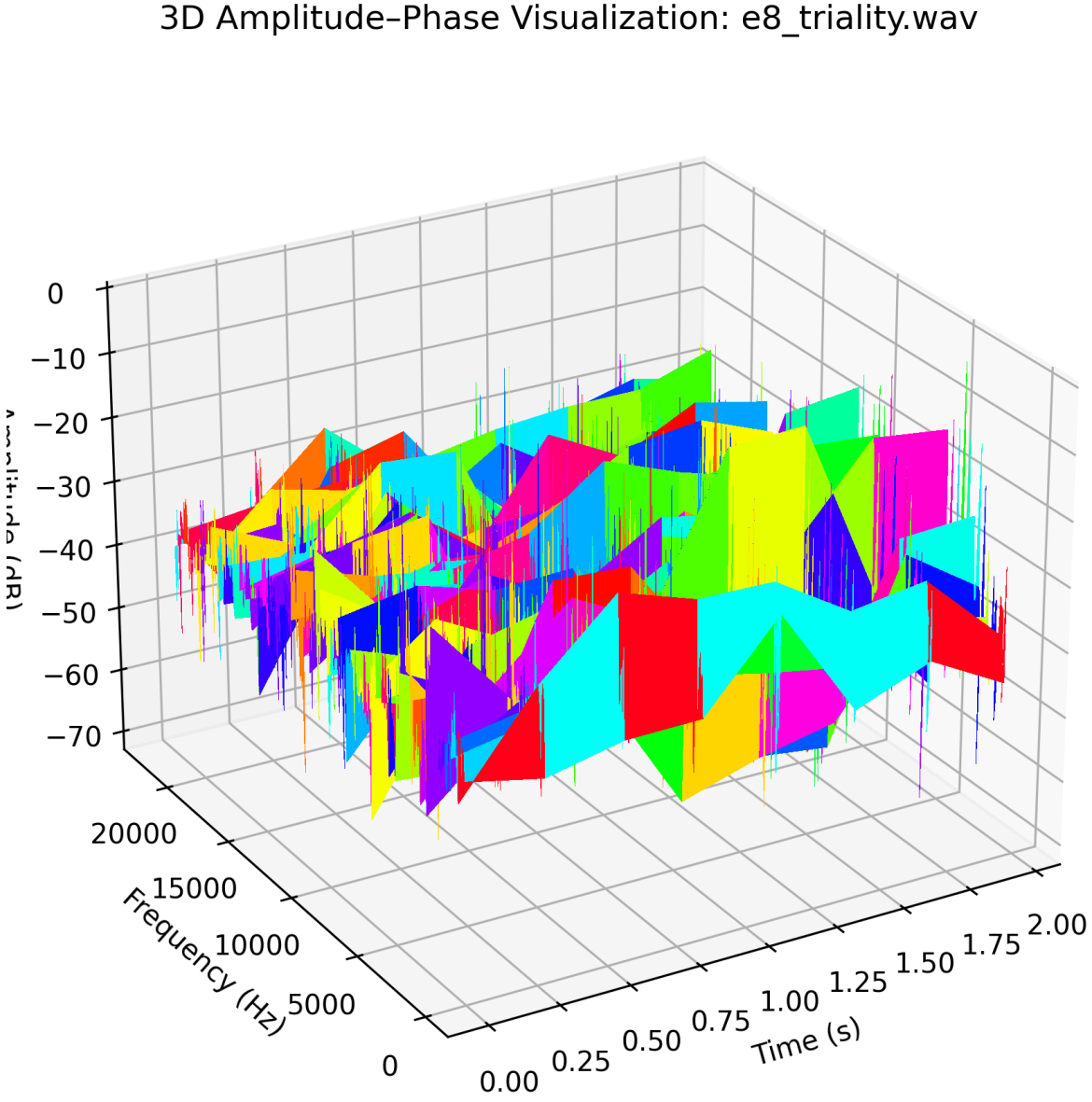

Figure 2: **Figure 2.** Three-dimensional visualization of amplitude and instantaneous phase for the E8-generated signal. The z-axis encodes magnitude, while color maps phase according to the HSV wheel. Polytope-like formations and recurring rotational symmetries emerge, making the abstract structure of E8 perceptible as a tangible, navigable geometry—here, the projection reveals the eightfold rotational symmetry characteristic of E8's Coxeter plane.

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