Solutions of Introduction to Algorithms: A Creative Approach

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Chapter 1

Mathematical Induction

1.1 Counting Regions in the Plane

A set of lines in the plane is said to be in **general position** if no two lines are parallel and no three lines intersect at a common point.

Guess: Adding one more line to $n-1$ lines in general position in the plane **increases** the number of regions by n. In other words $T(n) = T(n-1) + n$.

The base cases is trivial

- $T(0) = 1$
- $T(1) = T(0) + 1 = 2$
- $T(2) = T(1) + 2 = 2 + 2 = 4$
- $T(3) = T(2) + 3 = 4 + 3 = 7$

So we assume $T(n)$ is correct, now we want to prove $T(n+1)$ is also correct. Let's remove line n^{th}. According to induction hypothesis Adding line $(n+1)^{th}$ add n new regions. If we add line n^{th} again, it intersect with line $(n+1)^{th}$ at exactly one point p. This point is located in region R.

In the absence of line n^{th}, line $(n+1)^{th}$ adds only one new region when it passes R. But in presence of line n^{th}, it adds 2 new regions when it passes R. For other regions line $(n+1)^{th}$ adds $n-1$ new regions with or without the presence of line n^{th}. So line $(n+1)^{th}$ adds $n-1+2 = n+1$ new regions when n^{th} is presented.

So instead of proving the number of regions by adding a new line, we proved how many new regions are added when we have line $(n+1)^{th}$. So It's easy to prove the number of regions. Starting with one line we have $2+2+3+4+\cdots+n = 1 + 1 + 2 + \cdots + n = 1 + \frac{n \times (n+1)}{2}$.