

# Online Contests Solutions

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# Chapter 1

## LeetCode

### 1.1 Medium

#### 1.1.1 Longest Substring Without Repeating Characters

Find the length of the longest substring without repeating characters. For example in "abcabcbb" the answer is "abc".

We can use mathematical induction. Assume  $s[i..j-1]$  is a valid substring (it doesn't have any repetitive characters). Now we want to add  $s[j]$  to it. If  $s[j]$  is not in  $s[i..j-1]$ , then we can easily add  $s[j]$  to the list and claim  $s[i..j]$  is also valid. Otherwise there is exactly one  $i \leq k \leq j-1$  that  $s[k] = s[j]$ .

Because of  $s[j]$ , we cannot extend  $s[i..j-1]$  anymore, but we may be able to extend  $s[k+1..j]$ . The running time of the following implementation is  $O(n)$ :

```
int lengthOfLongestSubstring(string s) {
    int res = 0;
    unordered_map<char, int> mp;
    for (int i = 0, j = 0; i < s.length() && j < s.length(); ++j)
    {
        auto iter = mp.find(s[j]);
        if (iter != mp.end())
        {
            //Note that after mp.erase, iter will be invalid!
            //So we store the index in k
            int k = iter->second;
            for (; i <= k; ++i)
                mp.erase(s[i]);
        }
        mp[s[j]] = j;
        res = max(res, j - i + 1);
    }
    return res;
}
```

### 1.1.2 Longest Palindromic Substring

Given a string  $s$ , return the longest palindromic substring in  $s$ . For another variation of this problem refer to 3.1.1.

Assume  $S[i..j] = s_i s_{i+1} \dots s_j$ . We define  $S'[i'..j'] = s_{i'} s_{i'+1} \dots s_{j'}$  in such a way  $(s_j = s_{i'}) \wedge (s_{j-1} = s_{i'+1}) \wedge \dots \wedge (s_i = s_{j'})$ . Let's Assume string  $P$  is a palindromic string and  $c$  is an arbitrary character. We can break  $P$  into:

$$P[i..j] = \begin{cases} S[i..m]S'[m+1..j] & (j-i+1) \text{ is even} \\ S[i..m-1]cS'[m+1..j] & (j-i+1) \text{ is odd} \end{cases}$$

It's obvious that  $m = \lfloor \frac{i+j}{2} \rfloor$ . To solve this problem we need to find the place of  $m$  that can be  $0 \leq m < n$ .  $n$  is the length of the string. The running time of this algorithm is  $O(n^2)$ :

```
string longestPalindrome(string s) {
    int len = 0;
    int index = -1;

    const auto findLongest = [&](int l, int r)
    {
        for (; l >= 0 && r < s.length() && s[l] == s[r]; --l, ++r)
        {
            if ((r - l + 1) > len)
            {
                len = r - l + 1;
                index = l;
            }
        }
    };

    for (int m = 0; m < s.length(); ++m)
    {
        //Check for S S_r (length is even)
        findLongest(m, m + 1);
        //Check for S c S_r (length is odd)
        findLongest(m, m);
    }
    if (index == -1)
        return "";
    return s.substr(index, len);
}
```

### 1.1.3 Container With Most Water

You are given an integer array *height* of length  $n$ . There are  $n$  vertical lines drawn such that the two endpoints of  $i^{th}$  line are  $(i, 0)$  and  $(i, height[i])$ . Find two lines that together with the x-axis form a container, such that the container contains the most water. You may not slant the container.

We use mathematical induction. Suppose we know the answer for the number of lines less than  $n$ . Now we consider  $n$  lines. We consider the first and  $n^{th}$  lines and call them  $l_1$  and  $l_n$ :

1.  $height[1] < height[n]$ : We find the best solution for  $l_1$ . Because for  $2 \leq k \leq n-1$ , the container  $l_1$  and  $l_k$  cannot have more water than  $l_1$  and  $l_n$ . We can safely remove  $l_1$  and use mathematical induction to find the optimal solution. Then we need to compare that solution with  $l_1$  and  $l_n$  and choose the maximum
2.  $height[1] > height[n]$ : We find the best solution for  $l_n$ , like above we can safely remove  $l_n$  and find the best solution and compare it with  $l_1$  and  $l_n$
3.  $height[1] = height[n]$  We find the best solution for both  $l_1$  and  $l_n$ . We can safely remove both of them and find the optimal solution and compare it with  $l_1$  and  $l_n$ .

The running time of this algorithm is  $O(n)$ .

```
int maxArea(vector<int>& height) {
    int left = 0, right = height.size() - 1;
    int res = 0;
    while (left < right)
    {
        if (height[left] < height[right])
        {
            res = max(res, (right - left) * height[left]);
            ++left;
        }
        else if (height[left] > height[right])
        {
            res = max(res, (right - left) * height[right]);
            --right;
        }
        else
        {
            res = max (res, (right - left) * height[right]);
            ++left;
            --right;
        }
    }
    return res;
}
```

#### 1.1.4 Two Sum II - Input Array Is Sorted

Given a sorted array, return two indices  $i, j$  such that  $numbers[i] + numbers[j] = target \wedge i < j$ . The indices start from 1.

For similar questions, refer to 1.1.5. For a general solution refer to 1.1.6

We use mathematical induction. We assume we know the solution for all sorted arrays with length less than  $n$ . Now consider a sorted array with length  $n$ :

1.  $numbers[0] + numbers[n-1] < target$ : Since the array is sorted,  $numbers[n-1]$  is the maximum and  $numbers[0]$  is the minimum. So  $numbers[0]$  cannot be in the solution. We use mathematical induction to get the answer from  $numbers[1..n-1]$

2.  $numbers[0] + numbers[n - 1] > target$ : With a similar argument we can say  $numbers[n - 1]$  cannot be in the solution. So we use the induction to solve  $numbers[0..n - 2]$
3.  $number[0] + numbers[n - 1] == target$ : We have the solution.

```
vector<int> twoSum(vector<int>& numbers, int target) {
    int left = 0, right = numbers.size() - 1;
    while (left < right)
    {
        const auto sum = numbers[left] + numbers[right];
        if (sum < target)
            ++left;
        else if (sum > target)
            --right;
        else
            return {left + 1, right + 1};
    }
    throw 1;
}
```

### 1.1.5 3Sum

Given an integer array *nums*, return all triplets  $[nums[i], nums[j], nums[k]]$  such that  $i \neq j \wedge i \neq k \wedge j \neq k$ . Notice that the solution set must not contain **duplicate triplets**.

This is the extension of 1.1.4. For general solution refer to 1.1.6

The tricky part here is how to avoid duplicates. We assume *nums* is sorted, then we use mathematical induction. We assume we know how to solve it for less than *n* sorted numbers. Now we want to solve *n* sorted numbers. We consider the first element *nums*[0]. Now consider *nums*[*j*] such that  $j = k + 1$ . We define *k* as  $\{nums[0], nums[1], \dots, nums[k]\}$  such that  $nums[m] = nums[m + 1]$  for  $0 \leq m \leq k - 1$ . We use mathematical induction to find the answers for *nums*[*j*..*n* - 1].

Note that we cannot use induction for subproblems *nums*[*m*..*n* - 1] for  $1 \leq m \leq k$  because it's possible they have triplets that start with *nums*[0]. As mentioned in problem statement, duplicates are not allowed.

Then we need to find the answers that contain *nums*[0]. For doing that we should find set *S*:

$$S = \{ \{p, q\} : 1 \leq p < q \leq n - 1 \wedge nums[p] + nums[q] = -nums[0] \}$$

Note that *S* shouldn't have any duplicates. Also it's important that we should find the pairs in *nums*[1..*n* - 1] (not in *nums*[*j*..*n* - 1]. For example [0, 0, 0] is a valid triplet). This is **two sum II problem** (refer to 1.1.4), but we should handle duplicates. There are two solutions to implement two sum:

```
vector<vector<int>>> threeSum(vector<int>& nums) {
    vector<vector<int>>> res;
```



```

sort(nums.begin(), nums.end()); //O(nlogn)
unordered_set<int> visited;
for (int i = 0; i < nums.size() && nums[i] <= 0; ++i)
{
    // To find subproblem nums[j..n - 1]
    //note that i points to the first repetitive character
    if (i > 0 && nums[i] == nums[i - 1])
        continue;
    visited.clear();
    for (int j = i + 1; j < nums.size(); ++j)
    {
        //make sure j points to the last repetitive character:
        if (j + 1 < nums.size() && nums[j] == nums[j + 1])
        {
            // to handle cases like [0, 0, 0]
            visited.insert(nums[j]);
            continue;
        }
        auto target = -nums[i] - nums[j];
        if (visited.count(target) > 0)
            res.push_back({nums[i], target, nums[j]});
        visited.insert(nums[j]);
    }
}
return res;
}

```

The second solution:

```

vector<vector<int>> threeSum(vector<int>& nums) {
    vector<vector<int>> res;

    sort(nums.begin(), nums.end()); //O(nlogn)
    for (int i = 0; i + 1 < nums.size() && nums[i] <= 0; ++i)
    {
        // To find subproblem nums[j..n - 1]:
        if (i > 0 && nums[i] == nums[i - 1])
            continue;
        int target = -nums[i];
        int left = i + 1, right = nums.size() - 1;
        while (left < right)
        {
            auto sum = nums[left] + nums[right];
            if (sum < target)
                ++left;
            else if (sum > target)
                --right;
            else
            {
                res.push_back({nums[i], nums[left], nums[right]});
                ++left;
                while (left < right && nums[left] == nums[left - 1])
                    ++left;
                --right;
            }
        }
    }
}

```

```

    return res;
}

```

### 1.1.6 4sum

Given an array *nums* of *n* integers, return an array of all the **unique** quadruplets [*nums*[*a*], *nums*[*b*], *nums*[*c*], *nums*[*d*]] such that  $a < b < c < d$  and  $nums[a] + nums[b] + nums[c] + nums[d] = target$ .

The base case for this solution is 2sum. For more information refer to 1.1.4. It's a good idea to look at 3sum in 1.1.5

We can solve it similar to 3sum (for more information refer to 1.1.5). We suppose the array is sorted. We use mathematical induction, so we know how to solve it for less than *n* sorted numbers. Now we consider *n* sorted numbers. We choose the first two numbers *nums*[0] and *nums*[1]. We choose subproblem *nums*[*j*..*n* - 1] such that  $nums[m] = nums[0] \vee nums[m] = nums[1]$  for  $2 \leq m \leq j - 1$ . As explained in section 1.1.5, this is required to avoid duplication. We use mathematical induction to find quadruplets in *nums*[*j*..*n* - 1]. Then for finding quadruplets that contain *nums*[0] and *nums*[1], we use 2sum (refer to section 1.1.4) algorithm on *nums*[2..*n* - 1] (not *nums*[*j*..*n* - 1]). For example [0, 0, 0, 0] is a valid quadruplet if *target* = 0).

```

vector<vector<int>>> fourSum(vector<int>& nums, int target) {
    vector<vector<int>>> res;
    const auto size = nums.size();
    sort(nums.begin(), nums.end());
    for (int i = 0; (i + 3) < size; ++i)
    {
        if (i > 0 && nums[i] == nums[i - 1])
            continue;
        for (int j = i + 1; (j + 2) < size; ++j)
        {
            if (j > i + 1 && nums[j] == nums[j - 1])
                continue;
            int t = target - nums[i] - nums[j];
            int left = j + 1;
            int right = size - 1;
            while (left < right)
            {
                int sum = nums[left] + nums[right];
                if (sum < t)
                    ++left;
                else if (sum > t)
                    --right;
                else
                {
                    res.push_back({nums[i], nums[j], nums[left], nums[right]});
                    ++left;
                    while (nums[left] == nums[left - 1] && left < right)
                        ++left;
                    --right;
                }
            }
        }
    }
}

```

```

    }
  }
}
return res;
}

```

To solve *ksum* problem, we need  $k-2$  for loops to find  $[nums[0], nums[1], \dots, nums[k-3]]$  and we use 2sum algorithm to find  $nums[k-2]$  and  $nums[k-1]$ . We can use a recursive function to solve the problem dynamically during the runtime.

```

vector<vector<int>> twosum(vector<int>& nums, int left, int target)
{
    int right = nums.size() - 1;
    vector<vector<int>> res;
    while (left < right)
    {
        auto sum = nums[left] + nums[right];
        if (sum < target)
            ++left;
        else if (sum > target)
            --right;
        else
        {
            res.push_back({nums[left], nums[right]});
            ++left;
            while (nums[left] == nums[left - 1] && left < right)
                ++left;
            --right;
        }
    }
    return res;
}

vector<vector<int>> ksum(vector<int>& nums, int start, int k, int target)
{
    vector<vector<int>> res;
    if (start == nums.size())
        return res;
    int average = target / k;
    //nums[start] * k > target
    if (nums[start] > average)
        return res;
    //nums.back() * k < target
    if (nums.back() < average)
        return res;
    if (k == 2)
        return twosum(nums, start, target);

    for (int i = start; i < nums.size(); ++i)
    {
        if (i > start && nums[i] == nums[i - 1])
            continue;
        auto partial = ksum(nums, i + 1, k - 1, target - nums[i]);
        for (auto& val : partial)
        {
            val.insert(val.begin(), nums[i]);
            res.push_back(val);
        }
    }
}

```

```

    }
}
return res;
}

vector<vector<int>> fourSum(vector<int>& nums, int target) {
    sort(nums.begin(), nums.end());
    return ksum(nums, 0, 4, target);
}

```

### 1.1.7 Generate Parentheses

Given  $n$  pairs of parentheses, write a function to generate all combinations of well-formed parentheses.

We use mathematical induction. Let's say  $S_n$  is the set of all valid  $n$  pairs. We assume we know how to generate  $S_k$  for  $k < n$ . It's obvious that  $S_1 = \{ "()" \}$  and  $S_0 = \{ "" \}$ . Now we consider  $S_n$ . let's assume  $p \in S_i$  and  $q \in S_{n-1-i}$  for  $0 \leq i \leq n-1$ . It's obvious that  $"(p)q"$  is a valid parentheses with  $n$  pairs ( $i + n - 1 - i + 1 = n$ ). We define

$$(S_i) = \{ (p) : p \in S_i \}$$

Also we define **Cartesian product** as follow ( $pq$  means the concatenation of strings  $p$  and  $q$ ):

$$S_i \times S_j = \{ pq : p \in S_i \wedge q \in S_j \}$$

So we can use the induction to generate  $S_n$  as follow:

$$S_n = \bigcup_{i=0}^{n-1} (S_i) \times S_{n-1-i}$$

```

vector<string> generateParenthesis(int n) {
    vector<string> res;
    if (n == 0)
        return {" "};
    for (int i = 0; i < n; ++i)
    {
        auto set_left = generateParenthesis(i);
        auto set_right = generateParenthesis(n - i - 1);
        for (const auto& p : set_left)
        {
            string left_str = "(" + p + ")";
            for (const auto& q : set_right)
                res.push_back(left_str + q);
        }
    }
    return res;
}

```

### 1.1.8 Divide Two Integers

Divide two integers **without** using multiplication, division, and mod operator. We can only use **32-bit signed integers**.

We know that the range of a 32-bit signed integer is  $[-2^{31}, 2^{31} - 1]$ . We should convert negative numbers to positive ones. The tricky part is how to handle  $-2^{31}$ . Because we cannot fit it inside a positive 32-bit integer and we are not allowed to use *unsigned* variables.

#### Method 1

We use mathematical induction. Suppose  $a = bq + r$ . We define  $div(a, b) = q$  and  $mod(a, b) = r$ . Suppose we know how to calculate  $div$  and  $mod$  for less than  $a$ . Now we want to calculate for  $a$ :

$$div(a, b) = \begin{cases} 2 \times div(k, b) + div(2 \times mod(k, b), b) & a = 2k \\ 2 \times div(k, b) + div(2 \times mod(k, b) + 1, b) & a = 2k + 1 \\ 0 & a < b \\ 1 & a = b \\ a & b = 1 \\ base_{div}(a, b) & k \leq b \end{cases}$$

$$mod(a, b) = \begin{cases} mod(2 \times mod(k, b), b) & a = 2k \\ mod(2 \times mod(k, b) + 1, b) & a = 2k + 1 \\ a & a < b \\ 0 & a = b \vee b = 1 \\ base_{mod}(a, b) & k \leq b \end{cases}$$

For calculating  $base$  we use a straightforward approach. Since we cannot use multiplication and division, we can use  $2 \times a = a + a$  and we use shift operator to do division by 2 ( $a \gg 1$ ).

Let's consider  $a = -2^{31}$ . Let's assume  $q = div(2^{31}, |b|)$  and  $r = mod(2^{31}, |b|)$ :

$$|div(-2^{31}, |b|)| = \begin{cases} q & r + 1 < |b| \\ q + 1 & r + 1 \geq |b| \end{cases}$$

#### Method2

This is easier to implement. We assume  $a = bq + r$ . We define  $div(a, b) = q$ . Like previous method, suppose we know how to calculate  $div$  for less than  $a$ . Now we want to calculate for  $a$ . Suppose  $b \times 2^k \leq a < b \times 2^{k+1}$ . So we can say  $a = b \times 2^k + r$ . If  $r < b$  we find the answer; otherwise we use the induction to find  $r = bq' + r'$ . So the answer is:

$$\begin{aligned} a &= b \times 2^k + r \\ &= b \times 2^k + bq' + r' \\ &= b \times (2^k + q') + r' \end{aligned}$$

We can summarize it as:

$$\text{div}(a, b) = \begin{cases} 2^k + \text{div}(a - b \times 2^k, b) & a \geq b \\ 0 & a < b \end{cases}$$

Let's consider  $a = -2^{31}$ . We cannot store it as positive in a 32-bit signed integer. Let's assume  $a = bq + r$ :

$$\begin{aligned} & b(q-1) + r \\ &= bq + r - b \\ &= a - b \\ \implies & a - b = b(q-1) + r \end{aligned}$$

So  $\text{div}(|2^{31}|, |b|) = \text{div}(|2^{31}| - |b|, |b|) + 1$ :

```
int div(int a, int b)
{
    if (a < b)
        return 0;
    const int half_of_a = a >> 1;
    int q = 1, bq = b;
    for (; bq <= half_of_a; bq <= 1, q <= 1);
    return q + div(a - bq, b);
}

int divide(int a, int b) {
    static const int min_int = numeric_limits<int>::min();
    static const int max_int = numeric_limits<int>::max();
    int sign = 1;
    int res = 0;

    if (b == min_int)
        return a == min_int ? 1 : 0;
    if (b == 1)
        return a;
    if (b == -1)
        return a == min_int ? max_int : -a;

    if (b < 0)
        sign = -sign, b = -b;
    if (a == min_int)
        a += b, res = 1;
    if (a < 0)
        sign = -sign, a = -a;

    res += div(a, b);

    return res * sign;
}
```

### 1.1.9 Next Permutation

Implement `std::next_permutation` for  $nums[0..n-1]$ ! For example if  $nums = [1, 3, 2]$ , the answer is  $[2, 1, 3]$

Suppose  $p$  and  $q$  are two permutations of  $nums[0..n-1]$ . If  $p < q$ , then we have a  $0 \leq i < n$  such that  $p[k] = q[k]$  for  $0 \leq k < i$  and  $p[i] < q[i]$ . For solving this question we should find the smallest  $q$  that is greater than  $p$ .

Assume  $i$  is the maximum index such that  $nums[i] < nums[i+1]$  for  $0 \leq i \leq n-2$ . If such  $i$  does not exist it means we have the last permutation. So we should return the first one. Since  $i$  is the maximum index, it implies  $nums[k] \geq nums[k+1]$  for all  $i+1 \leq k \leq n-2$ . In other words,  $nums[i+1..n-1]$  is in non-ascending order.

Let's assume  $j \geq i+1$  is the maximum index such that  $nums[i] < nums[j]$ . Since  $nums[i+1..n-1]$  is in non-ascending order, that means  $nums[j]$  is the smallest element that is bigger than  $nums[i]$  in that range. We should swap  $nums[i]$  and  $nums[j]$  and reverse  $nums[i+1..n-1]$  to make it in non-descending order. This is the smallest permutation greater than  $nums$ .

To summarize we break  $nums[0..n-1]$  into  $A = nums[0..i]$  and  $B = nums[i+1..n-1]$ . The former is in non-descending and the latter in non-ascending order. We define  $B' = \{nums[j] \in B : nums[j] > nums[i]\}$ . We find the smallest element in  $B'$  and swap it with  $nums[i]$ . Then we should sort  $B$  in non-descending order.

The implementation is based on [possible implementation](#) of `next_permutation`. To understand how `reverse_iterator::base` (`i.base()`) works, refer to [A.3](#).

```
void nextPermutation(vector<int>& nums) {
    auto i = is_sorted_until(nums.rbegin(), nums.rend());
    if (i != nums.rend())
    {
        auto j = upper_bound(nums.rbegin(), i, *i);
        iter_swap(i, j);
    }
    //https://en.cppreference.com/w/cpp/iterator/reverse_iterator/base
    reverse(i.base(), nums.end());
}
```

### 1.1.10 Search in Rotated Sorted Array

There is an integer array  $nums$  sorted in ascending order (with distinct values). For example let's assume we have  $[0, 1, 2, 4, 5, 6, 7]$ . We might rotate at pivot index 3 so it becomes  $[4, 5, 6, 7, 0, 1, 2]$ . Write an  $O(\log n)$  algorithm to find the index of *target*.

In the non-rotated versions, the minimum number starts at index 0. We use the following algorithm to find the minimum index in the rotated one. We assume  $middle = \lfloor \frac{left+right}{2} \rfloor$

$$f(left, right) = \begin{cases} f(middle + 1, right) & nums[middle] > nums[right] \\ f(left, middle) & nums[middle] < nums[right] \\ left & left = right \end{cases}$$

Let's say  $k = f(0, n-1)$ . Then we have two ascending lists  $A = nums[0..k-1]$

and  $B = \text{nums}[k..n-1]$ . Based on *target* value we choose  $A$  or  $B$  to do a binary search to see whether *target* is in the list.

```
int search(vector<int>& nums, int target) {
    int left = 0;
    const int n = nums.size();
    int right = n - 1;
    int k;
    while (left <= right)
    {
        if (left == right)
        {
            k = left;
            break;
        }
        int middle = (left + right) / 2;
        if (nums[middle] > target)
            left = middle + 1;
        else
            right = middle;
    }
    if (target >= nums[k] && target <= nums[n - 1])
        left = k, right = n - 1;
    else
        left = 0, right = k - 1;
    while (left <= right)
    {
        if (left == right)
            return nums[left] == target ? left : -1;
        int middle = (left + right) / 2;
        if (nums[middle] < target)
            left = middle + 1;
        else
            right = middle;
    }
    return -1;
}
```

### 1.1.11 Find First and Last Position of Element in Sorted Array

Given an array of integers *nums* sorted in non-decreasing order, find the starting and ending position of a given *target* value.

We define  $f(i, j)$  that has the index of left-most *target* in  $\text{nums}[i..j]$ . On the other hand we define  $g(i, j)$  that keeps the index of the right-most *target* in  $\text{nums}[i..j]$ .

Let's assume *mid* is index of the middle element. Both  $f$  and  $g$  acts similarly when  $\text{nums}[\text{mid}] < \text{target}$  or  $\text{nums}[\text{mid}] > \text{target}$ . Let's assume  $\text{nums}[\text{mid}] = \text{target}$ . Since  $f$  should choose the left-most target, it tries to find the answer in  $\text{nums}[0..\text{mid}]$ . There is a possibility that other elements with value *target* exists in  $\text{nums}[0..\text{mid}]$ . On the other hand,  $g$  should find the rightmost *target*. So it chooses  $\text{nums}[\text{mid}..n-1]$ .



We assume  $mid_f = \lfloor \frac{left+right}{2} \rfloor$ :

$$f(left, right) = \begin{cases} f(mid_f + 1, right) & \text{nums}[mid_f] < target \\ f(left, mid_f) & \text{nums}[mid_f] \geq target \\ left & left = right \wedge \text{nums}[left] = target \\ -1 & \text{otherwise} \end{cases}$$

We define  $mid_c = \lceil \frac{left+right}{2} \rceil$ :

$$g(left, right) = \begin{cases} g(mid_c, right) & \text{nums}[mid_c] \leq target \\ g(left, mid_c - 1) & \text{nums}[mid_c] > target \\ left & left = right \wedge \text{nums}[left] = target \\ -1 & \text{otherwise} \end{cases}$$

Finding  $mid$  becomes tricky when the size of sub-problem is 2. Let's say we are at  $nums[i..i+1]$ . We know  $mid_f = \lfloor \frac{i+i+1}{2} \rfloor = i$  and  $mid_c = \lceil \frac{i+i+1}{2} \rceil = i+1$ . For  $f$  choosing  $mid_c$  leads to an infinite loop and for  $g$  it's  $mid_f$ :

$$f(i, i+1) = \begin{cases} f(mid_f + 1, i+1) = f(i+1, i+1) & \text{nums}[mid_f] < target \\ f(i, mid_f) = f(i, i) & \text{nums}[mid_f] \geq target \end{cases}$$

$$f(i, i+1) = \begin{cases} f(mid_c + 1, i+1) = f(i+2, i+1) & \text{nums}[mid_c] < target \\ f(i, mid_c) = f(i, i+1) & \text{nums}[mid_c] \geq target \end{cases}$$

Also we can think that  $f$  wants to find the leftmost  $target$ , so it makes sense to choose  $mid_f$ , but  $g$  wants to find the rightmost  $target$  so it should choose  $mid_c$ . Now we can use  $f(0, n-1)$  and  $g(0, n-1)$  to find the solution:

```
vector<int> searchRange(vector<int>& nums, int target) {
    int n = nums.size();
    int left = 0, right = n - 1;

    while (left < right)
    {
        int mid = (left + right) / 2;
        if (nums[mid] < target)
            left = mid + 1;
        else
            right = mid;
    }
    if (left > right || nums[left] != target)
        return {-1, -1};
    vector<int> res(1, left);
    right = n - 1;
    while (left < right)
    {
        int mid = (left + right + 1) / 2;
        if (nums[mid] <= target)
            left = mid;
        else
            right = mid - 1;
    }
    res.push_back(left);
    return res;
}
```

```
        right = mid - 1;
    }
    res.push_back(left);
    return res;
}
```

### 1.1.12 Valid Sudoku

```
bool isValidSudoku(vector<vector<char>>& board) {
    array<array<bool, 10>, 9> row_seen{}, col_seen{};
    array<array<array<bool, 10>, 3>, 3> box_seen{};

    for (int r = 0; r < 9; ++r)
    {
        for (int c = 0; c < 9; ++c)
        {
            if (board[r][c] == '.')
                continue;
            int d = board[r][c] - '0';
            int box_row = r / 3;
            int box_col = c / 3;
            auto& rs = row_seen[r][d];
            auto& cs = col_seen[c][d];
            auto& bs = box_seen[box_row][box_col][d];
            if (rs || cs || bs)
                return false;
            rs = cs = bs = true;
        }
    }
    return true;
}
```

## Chapter 2

# HackerRank

### 2.1 New Year Chaos

We define  $index_i$  as the current index for person  $i$ . For example if we have 1, 2, 3, 4 and 4 bribes 3, the queue looks like 1, 2, 4, 3. So  $index_4 = 3$ . Since no body can bribe more than 2 times,  $index_i \geq i - 2$  for  $1 \leq i \leq n$ . Consider person  $n$ . No body can bribe that person. So  $n - 2 \leq index_n \leq n$ . After we retruned that person to his actual place we can consider  $n - 1$ . So we have  $n - 3 \leq index_{n-1} \leq n - 1$  (note that at this moment  $index_n = n$ ).

```
void minimumBribes(vector<int> q) {  
  
    const auto& n = q.size();  
    int res = 0;  
    for (int num = n; num > 0; --num)  
    {  
        for (int i = max(0, num - 3); i < num - 1; ++i)  
        {  
            if (q[i] == num)  
            {  
                ++res;  
                swap(q[i], q[i + 1]);  
            }  
        }  
        if (q[num - 1] != num)  
        {  
            cout << "Too chaotic" << endl;  
            return;  
        }  
    }  
    cout << res << endl;  
}
```

## 2.2 Minimum Swaps 2

Note that this solution is based on **Selection Sort** in which the number of swaps are minimum. According to Wikipedia: "One thing which distinguishes selection sort from other sorting algorithms is that it makes the minimum possible number of swaps,  $n - 1$  in the worst case.". Although Selection sort has minimum number of swaps among all sorts algorithms, it has  $O(n^2)$  comparisons. Since the final result is  $\{1, 2, \dots, n\}$ , it's like we have the set in sorted order so we can bypass comparisons and use Selection Sort advantage which is the minimum number of swaps.

We define  $index_i$  as the current index of number  $i$ . Suppose we have  $n$  numbers, so  $1 \leq index_i \leq n$ . The goal is to have  $index_i = i$ . Without losing generality suppose  $i < j \wedge index_i = j$ . There are two cases to consider:

1. If  $index_j = i$ , then by swapping  $arr_i$  and  $arr_j$ , we put both  $i$  and  $j$  in their corresponding positions.
2. If  $index_j = k \wedge k \neq i \wedge k \neq j$ . In this case by swapping  $arr_i$  and  $arr_j$  we only put  $i$  in its corresponding position. So we need to do an extra swap to put  $j$  in its correct position.

We can start from  $i = 1$  to  $i = n$  and make sure  $i$  is in correct position; otherwise we perform a swap. In each iteration we fix the position of one or two numbers. A good example is  $\{4, 3, 2, 1\}$ .

```
int minimumSwaps(vector<int> arr) {
    const auto& n = arr.size();
    vector<int> index(n + 1);

    for (int i = 0; i < n; ++i)
        index[arr[i]] = i;
    int cnt = 0;
    for (int num = 1; num <= n; ++num)
    {
        if (index[num] != num - 1)
        {
            ++cnt;
            index[arr[num - 1]] = index[num];
            swap(arr[index[num]], arr[num - 1]);
            index[num] = num - 1;
        }
    }
    return cnt;
}
```

## 2.3 Count Triplets

We use dynamic programming to solve it. For mathematical induction we define  $cnt[num][n]$  like this:

$$cnt[a_{i_1}][0] = |\{a_{i_0} \in arr \mid a_{i_1} = a_{i_0} \times r \wedge i_1 < i_2\}|$$

$$cnt[a_{i_2}][1] = |\{(a_{i_0}, a_{i_1}) \in arr \times arr \mid a_{i_k} = a_{i_{k-1}} \times r \wedge i_{k-1} < i_k \text{ for } 1 \leq k \leq 2\}|$$

So the final answer is:

$$\sum_{n \in arr} cnt[n][1]$$

Then for each number  $n$  we have

$$cnt[n \times r][0] = cnt[n \times r][0] + 1$$

$$cnt[n \times r][1] = cnt[n \times r][1] + cnt[n][0]$$

Since  $r = 1$ , the order of assignments are very important.

```

long countTriplets(vector<long> arr, long r) {
    const auto n = arr.size();
    unordered_map<long, array<long, 2>> cnt;
    //cnt[a[j]][0] = |\{a[i]\}| in which i < j and
    //                a[j] = a[i] * r
    //cnt[a[k]][1] = |\{a[i], a[j]\}| in which
    //                i < j < k and
    //a[k] = a[j] * r and a[j] = a[i] * r

    long res = 0;
    for (const auto& num : arr)
    {
        res += cnt[num][1];
        const auto next = num * r;
        cnt[next][1] += cnt[num][0];
        ++cnt[next][0];
    }
    return res;
}

```

## 2.4 Fraudulent Activity Notifications

Basically we want a  $O(n \log n)$  algorithm to find median of a sequence, when we removed the first element and add another one. So we need two binary search trees. In the first one the maximum element is the median itself and in the second one the minimum element is the second median in case of  $d = 2k$  or a value greater than median when  $d = 2k + 1$ . So if  $d = 2k$  both of these binary search trees always have  $k$  element. When  $d = 2k + 1$ , the first one always has  $k + 1$  elements and the second one has  $k$  elements. Let's call them *lessEqual* and *greaterEqual*.

If both removing element and new element belong to the same tree, nothing extra is required. So we only need to remove one element and add the new one. If the removing element is from *lessEqual*, we must remove the minimum

element from *greaterEqual* and add it to *lessEqual*. If the removing element is from *greaterEqual*, we must remove the maximum element from *lessEqual* and add it to *greaterEqual*. By doing that the maximum element is *lessEqual* is median. In case of  $d = 2k$ , the minimum element in *greaterEqual* is the second median. The running time of this algorithm is  $O(n \log n)$ .

```
int activityNotifications(vector<int> expenditure, int d)
{
    multiset<int, greater<int>> lessEqual;
    multiset<int> greaterEqual;

    vector<int> init(d);
    copy(expenditure.begin(), expenditure.begin() + d,
        init.begin());
    sort(init.begin(), init.end());

    const bool isEven = (d & 1) == 0;

    int medianIndex = (d - 1) / 2;
    int i;
    for (i = 0; i <= medianIndex; ++i)
        lessEqual.insert(init[i]);
    for (; i < d; ++i)
        greaterEqual.insert(init[i]);

    int res = 0;
    for (int i = d; i < expenditure.size(); ++i)
    {
        const int median1 = *lessEqual.begin();
        if (isEven)
        {
            const int median2 = *greaterEqual.begin();
            if (expenditure[i] >= (median1 + median2))
                ++res;
        }
        else
        {
            if (expenditure[i] >= 2 * median1)
                ++res;
        }

        const auto removed = expenditure[i - d];

        if (removed <= median1 &&
            expenditure[i] <= median1)
        {
            lessEqual.erase(lessEqual.find(removed));
            lessEqual.insert(expenditure[i]);
        }
        else if (removed > median1 &&
            expenditure[i] > median1)
        {
            greaterEqual.erase(greaterEqual.find(removed));
            greaterEqual.insert(expenditure[i]);
        }
        else if (removed <= median1)
        {

```

```

        //For handling d=1, it should first:
        greaterEqual.insert(expenditure[i]);
        lessEqual.erase(lessEqual.find(removed));
        lessEqual.insert(*greaterEqual.begin());
        greaterEqual.erase(greaterEqual.begin());
    }
    else
    {
        //For handling d=1, it should be first:
        lessEqual.insert(expenditure[i]);
        greaterEqual.erase(greaterEqual.find(removed));
        greaterEqual.insert(*lessEqual.begin());
        lessEqual.erase(lessEqual.begin());
    }
}
return res;
}

```

## 2.5 Merge Sort: Counting Inversions

### Problem Statement

We can solve it using merge sort. Suppose we have  $arr[left..right]$ . We break it into two subproblem  $arr[left..mid]$  and  $arr[mid + 1..right]$ . Both of them are sorted. According to merge sort algorithm  $left \leq i \leq mid$  and  $mid + 1 \leq j \leq right$ . In other words we already put  $arr[left..i - 1]$  and  $arr[mid + 1..j - 1]$  into their correct positions. So when  $arr[j] < arr[i]$ , it means  $arr[j] < arr[i] \leq arr[x]$  in which  $i + 1 \leq x \leq mid$ . So we need to have  $mid - i + 1$  swaps.

```

long mergeSort(vector<int>& arr, int leftIndex,
               int rightIndex)
{
    if (leftIndex == rightIndex)
        return 0;

    long res = 0;
    int midIndex = (leftIndex + rightIndex) / 2;
    res = mergeSort(arr, leftIndex, midIndex);
    res += mergeSort(arr, midIndex + 1, rightIndex);

    vector<int> sorted(rightIndex - leftIndex + 1);
    int i, j, k;
    for (i = leftIndex, j = midIndex + 1, k = 0;
         i <= midIndex && j <= rightIndex;)
    {
        if (arr[i] <= arr[j])
            sorted[k++] = arr[i++];
        else
        {
            res += midIndex - i + 1;
            sorted[k++] = arr[j++];
        }
    }
}

```

```
    if (i <= midIndex)
        copy(arr.begin() + i,
            arr.begin() + midIndex + 1,
            sorted.begin() + k);
    else
        copy(arr.begin() + j ,
            arr.begin() + rightIndex + 1,
            sorted.begin() + k);
    copy(sorted.begin(), sorted.end(),
        arr.begin() + leftIndex);
    return res;
}
// Complete the countInversions function below.
long countInversions(vector<int> arr) {
    return mergeSort(arr, 0, arr.size() - 1);
}
```



## Chapter 3

# TopCoder

### 3.1 SRM 428

#### 3.1.1 ThePalindrome

For another variation refer to [1.1.2](#). We want to add the minimum number of characters to the end of string to make it a palindrome. The straightforward approach is to try add the first  $i$  characters in reverse for all  $0 \leq i \leq n - 1$  in which  $n$  is the length of string. So we start from  $i = 0$  and check whether the string is palindrome. If it's not we check for  $i = 1$  and so on. The running time of this algorithm is  $O(n^2)$ .

```
bool isPalindrome(const string& str)
{
    int left = 0, right = str.length() - 1;
    for (; left < right && str[left] == str[right];
        ++left, --right);
    return left >= right;
}

int find(string s)
{
    for (int i = 0; i < s.length(); ++i)
    {
        string tmp = s + string(s.rend() - i, s.rend());
        if (isPalindrome(tmp))
            return tmp.length();
    }
    throw 1;
}
```

Since  $n \leq 50$ , this algorithm is fast. We can make it  $O(n \log_2 n)$  if we use binary search tree to find the minimum  $i$ .

There is another approach. Let's assume we have string  $S = s_1 s_2 \dots s_n$ . We define  $S' = s_n \dots s_2 s_1$ . Suppose we can write string  $S$  as  $QP$ . In other words,  $S$  is the concatenation of two strings  $Q$  and  $P$ . We assume  $P$  is palindrome but

$Q$  is not. We can make  $S$  palindrome if we convert  $QP$  to  $QPQ'$  we call this new String  $Z$ .  $Z$  is palindrome because if we reverse it we have:

$$\begin{aligned} Z &: QPQ' \\ Z' &: QP'Q' \end{aligned}$$

Since we want the length of  $Q$  be minimum, we must find the maximal  $P$ :

```
bool isPalindrome(const string& str, int start)
{
    int left = start, right = str.length() - 1;
    for (; left < right && str[left] == str[right];
        ++left, --right);
    return left >= right;
}

int find(string s)
{
    for (int i = 0; i < s.length(); ++i)
    {
        if (isPalindrome(s, i))
            return s.length() + i;
    }
    throw 1;
}
```

As the previous implementation the running time is  $O(n^2)$  but it's easy to convert it to  $O(n \log_2 n)$  using binary search.

This implementation has a unique feature. We can convert it to an  $O(n)$  algorithm using **KMP algorithm**. Suppose  $S = QP$  where  $P$  is a palindrome. We want to find palidnrome postfix  $P$  which its length is maximum among all palindrome post-fixes. We need to run KMP pre-compute calculation on  $S' = P'Q'$ . Then we run KMP algorithm as if we want to find whether  $S'$  is a substring of  $S$ . Suppose we use  $i$  as an index for  $S$  and  $j$  as an index for  $S'$ . The algorithm start with  $i = 0$  and ends when  $i = \text{len}(S)$  in which  $S'[0..j - 1]$  is  $P'$  or  $P$  (since it's palindrome).

```
vector<int> calculateNext(const string& B)
{
    vector<int> next(B.length());
    next[0] = -2;
    next[1] = -1;
    int i, j;
    for (i = 2; i < B.length(); ++i)
    {
        j = next[i - 1] + 1;
        for (; j >= 0 && B[j] != B[i - 1]; j = next[j] + 1);
        next[i] = j;
    }
    return next;
}

int find(string A)
{
    const string B = string(A.rbegin(), A.rend());
```

```
const auto next = calculateNext(B);
int i, j;
for (i = 0, j = 0; i < A.length() && j < B.length();)
{
    if (A[i] == B[j])
        ++i, ++j;
    else if ((j = next[j] + 1) < 0)
    {
        //Since B.front() == A.back(), it's impossible
        //i == A.length() here:
        ++i, j = 0;
    }
}
int palindromeLen = j;
return A.length() + A.length() - palindromeLen;
}
```



# Appendix A

## C++ refresher

### A.1 Inserting into a container

The following code uses most insert operations:

```
#include <iostream>
#include <vector>
#include <deque>
#include <iterator>
#include <algorithm>

template<typename Container>
void print(const Container& c)
{
    static int id = 1;
    std::cout << id++ << ": ";
    std::for_each(c.begin(), c.end(), [](const auto& val) {
        std::cout << val << " ";
    });
    std::cout << std::endl;
}

int main()
{
    std::vector<int> base{0, 1, 2, 3, 4};
    auto v(base);
    std::vector<int> v2{100, 200, 300};

    auto i = std::next(v.begin()); // v.begin() + 1
    i = v.insert(i, v2[0]);
    i = v.insert(i, v2[1]);
    i = v.insert(i, v2[2]);
    print(v); // 1: 0 300 200 100 1 2 3 4
    // Note that after
    // v.insert(i, v[0]);
    // i becomes invalid. So this is undefined behavior:
    // v.insert(i, v[1]);
    v = base;
    i = std::next(v.begin()); // v.begin() + 1
```

```

i = v.insert(i, v2.begin(), v2.end());
print(v); // 2: 0 100 200 300 1 2 3 4

v = base;
std::copy(v2.begin(), v2.end(),
          std::inserter(v, std::next(v.begin(), 1)));
print(v); // 3: 0 100 200 300 1 2 3 4

v = base;
std::copy(v2.begin(), v2.end(), std::back_inserter(v));
print(v); // 4: 0 1 2 3 4 100 200 300

//std::vector doesn't have push_front:
std::deque<int> d(base.begin(), base.end());
std::copy(v2.begin(), v2.end(), std::front_inserter(d));
print(d); // 1: 300 200 100 0 1 2 3 4
}

```

## A.2 std::vector iterators

It uses [LegacyRandomAccessIterator](#). For more information refer to [Iterator library](#).

```

#include <iostream>
#include <algorithm>
#include <iterator>
#include <vector>

template<typename Container>
void print(const Container& c)
{
    const auto m_print = [](const auto& val)
    {
        std::cout << val << ' ';
    };
    std::for_each(c.begin(), c.end(), m_print);
    std::cout << std::endl;
}

//my implementation of std::reverse
template<typename Container>
void my_reverse(Container& c)
{
    auto left = c.begin();
    auto right = std::prev(c.end());
    for (; left < right; ++left, --right)
        std::iter_swap(left, right);
}

template<typename Container>
void my_reverse2(Container& c)
{
    auto left = 0;
    auto right = std::distance(c.begin(), std::prev(c.end()));
    for (; left < right; ++left, --right)

```

```

    std::swap(c[left], c[right]);
}

int main()
{
    std::vector<int> v{0, 1, 2, 3, 4};
    my_reverse(v);
    print(v); // 4 3 2 1 0
    my_reverse2(v);
    print(v); // 0 1 2 3 4
}

```

### A.3 `std::reverse_iterator`

To understand how `reverse_iterator::base` works, see the following snippet. For more information refer to `std::reverse_iterator`.

```

int main()
{
    using iter = std::vector<int>::iterator;
    using r_iter = std::vector<int>::reverse_iterator;

    std::vector<int> v {0, 1, 2, 3, 4};
    r_iter res = std::find(v.rbegin(), v.rend(), 2);
    iter base = res.base();
    assert(*res == 2 && *base == 3);
}

```

### A.4 `std::mismatch`

```

#include <iostream>
#include <algorithm>
#include <string>
#include <cassert>
#include <vector>
#include <iterator>

bool is_palindrome(const std::string& str)
{
    auto res = std::mismatch(str.begin(), str.end(),
                             str.rbegin(), str.rend());
    return res.first == str.end();
}

int main()
{
    assert(is_palindrome("aba"));
    assert(is_palindrome("abba"));
    assert(!is_palindrome("abcd"));

    std::vector<int> v1{1, 2, 3, 4};
    std::vector<int> v2{1, 2};
}

```

```

    auto res = std::mismatch(v1.begin(), v1.end(),
                             v2.begin(), v2.end());

    //C++14 and above:
    assert(res.first == std::next(v1.begin(), v2.size()) &&
           res.second == v2.end());
}

```

## A.5 Boyer–Moore string-search algorithm

Note that unlike [KMP](#), the worst case is still  $O(mn)$ . It's implemented since [C++17](#). Another variation is [Boyer–Moore–Horspool algorithm](#).

```

#include <iostream>
#include <algorithm>
#include <functional> //std::boyer_moore_searcher
#include <string>
#include <cassert>
#include <iterator>

int main()
{
    std::string s1 = "This is a test string";
    std::string s2 = "test";
    std::string before = "This is a ";
    auto res = std::search(s1.begin(), s1.end(),
                           std::boyer_moore_searcher(s2.begin(), s2.end()));
    assert(res == std::next(s1.begin(), before.length()));
}

```

## A.6 std::array

Note that if you don't use `std::array`, both `std::array` and c-style array are not initialized when you are using primitive type. So they can have arbitrary values.

```

#include <iostream>
#include <array>
#include <algorithm>
#include <cassert>
#include <cstring>

int main()
{
    // a1 members are not initialized:
    // std::array<int, 7> a;
    // c_a members are not initialized:
    // int c_a[7];
    // a2 members are initialized to default:
    std::array<int, 7> a1{};
    // c_a1 members are initialized to default:
    int c_a1[7] {};
    for (auto i = 0; i < a1.size(); ++i)
        assert(a1[i] == 0 && c_a1[i] == 0);
}

```



```

a1.fill(7);
std::fill(c_a1, c_a1 + 7, 7);
for (auto i = 0; i < a1.size(); ++i)
    assert(a1[i] == 7 && c_a1[i] == 7);

std::array<std::array<int, 2>, 3> a2{};
int c_a2[3][2] = {{1, 2}, {3, 4}};

std::memcpy(a2.data(), c_a2, sizeof(c_a2));
for (auto i = 0; i < a2.size(); ++i)
    for (auto j = 0; j < a2[0].size(); ++j)
        assert(a2[i][j] == c_a2[i][j]);

assert(a2.size() == 3);
assert(a2[0].size() == 2);

a2.fill({4, 5});
for (const auto& val : a2)
    assert(val[0] == 4 && val[1] == 5);
std::array<int, 3> a3 {1, 2};
assert(a3[0] == 1 && a3[1] == 2 && a3[2] == 0);
}

```

## A.7 Initializing arrays with `std::fill`, `memcpy` and `memset`

Note that `memset` fills every byte in the buffer. If it's an array of elements bigger than 1-byte (e.g. array of integers), it does make sense to use it just for 0 or -1 (in a 32-bit signed integer -1 = 0xffffffff).

Unless it's an array of elements of 1-byte (e.g. `char`), it's wrong to use `sizeof` for `std::fill`. Note that the first two parameters of `std::fill` are two pointers. One points to the first element and the other to one after the last. On the other hand we should use `sizeof` for `memcpy` and `memset`.

```

#include <iostream>
#include <array>
#include <algorithm>
#include <cassert>
#include <cstring>
#include <iterator>

int main()
{
    std::array<int, 3> a1{};
    assert(sizeof(a1) == sizeof(int) * 3);
    std::fill(a1.begin(), a1.end(), -20);
    for (const auto& val : a1)
        assert(val == -20);

    std::array<std::array<int, 2>, 3> a2{};
    int c_a2[3][2] = {{1, 2}, {3, 4}};
}

```

```

std::fill(&a2[0][0], &a2[1][0], 7);
std::fill(&a2[1][0], &a2[2][0], 8);
std::fill_n(&a2[2][0], 2, 9);
for (int i = 0; i < a2.size(); ++i)
    if (i == 0)
        assert(a2[i][0] == 7 && a2[i][1] == 7);
    else if (i == 1)
        assert(a2[i][0] == 8 && a2[i][1] == 8);
    else
        assert(a2[i][0] == 9 && a2[i][1] == 9);

std::fill(a2.begin()->begin(),
          std::next(a2.begin())->begin(),
          10);
std::fill(std::next(a2.begin())->begin(),
          std::next(a2.begin(), 2)->begin(),
          11);
std::fill(std::next(a2.begin(), 2)->begin(),
          std::prev(a2.end())->end(),
          12);
for (int i = 0; i < a2.size(); ++i)
    if (i == 0)
        assert(a2[i][0] == 10 && a2[i][1] == 10);
    else if (i == 1)
        assert(a2[i][0] == 11 && a2[i][1] == 11);
    else
        assert(a2[i][0] == 12 && a2[i][1] == 12);

assert(sizeof(a2) == sizeof(int) * 2 * 3);
// Note that &a2[0] is a pointer to std::array<int, 2>
std::fill(&a2[0][0], &a2[0][0] + 2 * 3, 15);
for (const auto& val : a2)
    assert(val[0] == 15 && val[1] == 15);

std::memcpy(a2.data(), c_a2, sizeof(c_a2));
for (auto i = 0; i < a2.size(); ++i)
    for (auto j = 0; j < a2[0].size(); ++j)
        assert(a2[i][j] == c_a2[i][j]);

std::fill_n(a2.begin()->begin(), 2 * 3, 16);
for (const auto& val : a2)
    assert(val[0] == 16 && val[1] == 16);

std::copy_n(&c_a2[0][0], 2 * 3, a2.begin()->begin());
for (auto i = 0; i < a2.size(); ++i)
    for (auto j = 0; j < a2[0].size(); ++j)
        assert(a2[i][j] == c_a2[i][j]);

std::memset(a2.data(), -1, sizeof(a2));
for (const auto& val : a2)
    assert(val[0] == -1 && val[1] == -1);

std::memset(a2.data(), 1, sizeof(a2));
for (const auto& val : a2)
    assert(val[0] == 0x01010101 && val[1] == 0x01010101);
}

```