Online Contests Solutions

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Chapter 1

LeetCode

1.1 Medium

1.1.1 Longest Substring Without Repeating Characters

Find the length of the longest substring without repeating characters. For example in "abcabcbb" the answer is "abc".

We can use mathematical induction. Assume s[i..j-1] is a valid substring (it doesn't have any repetitive characters). Now we want to add s[j] to it. If s[j] is not in s[i..j-1], then we can easily add s[j] to the list and claim s[i..j] is also valid. Otherwise there is exactly one $i \le k \le j-1$ that s[k] = s[j].

Because of s[j], we cannot extend s[i..j-1] anymore, but we may be able to extend s[k+1..j]. The running time of the following implementation is O(n):

```
int lengthOfLongestSubstring(string s) {
  int res = 0;
  unordered_map<char, int> mp;
  for (int i = 0, j = 0; i < s.length() && j < s.length(); ++j)
  {
    auto iter = mp.find(s[j]);
    if (iter != mp.end())
    {
        //Note that after mp.erase, iter will be invalid!
        //So we store the index in k
        int k = iter->second;
        for (; i <= k; ++i)
            mp.erase(s[i]);
    }
    mp[s[j]] = j;
    res = max(res, j - i + 1);
    }
    return res;
}</pre>
```

1.1.2 Longest Palindromic Substring

Given a string s, return the longest palindromic substring in s. For another variation of this problem refer to 3.1.1.

Assume $S[i..j] = s_i s_{i+1} \dots s_j$. We define $S'[i'..j'] = s_{i'} s_{i'+1} \dots s_{j'}$ in such a way $(s_j = s_{i'}) \wedge (s_{j-1} = s_{i'+1}) \wedge \dots \wedge (s_i = s_{j'})$. Let's Assume string P is a palindromic string and c is an arbitrary character. We can break P into:

$$P[i..j] = \begin{cases} S[i..m]S'[m+1..j] & (j-i+1) \text{ is even} \\ S[i..m-1]cS'[m+1..j] & (j-i+1) \text{ is odd} \end{cases}$$

It's obvious that $m = \lfloor \frac{i+j}{2} \rfloor$. To solve this problem we need to find the place of m that can be $0 \le m < n$. n is the length of the string. The running time of this algorithm is $O(n^2)$:

```
string longestPalindrome(string s) {
 int len = 0;
  int index = -1;
  const auto findLongest = [&](int 1, int r)
    for (; 1 >= 0 && r < s.length() && s[1] == s[r]; --1, ++r)
      if ((r - 1 + 1) > len)
       len = r - 1 + 1;
        index = 1;
 };
 for (int m = 0; m < s.length(); ++m)
    //Check for S S_r (length is even)
    findLongest(m, m + 1);
    //Check for S c S_r (length is odd)
   findLongest(m, m);
 if (index == -1)
    return "";
  return s.substr(index, len);
```

1.1.3 Container With Most Water

You are given an integer array hieght of length n. There are n vertical lines drawn such that the two endpoints of i^{th} line are (i,0) and (i,height[i]). Find two lines that together with the x-axis form a container, such that the container contains the most water. You may not slant the container.

We use mathematical induction. Suppose we know the answer for the number of lines less than n. Now we consider n lines. We consider the first and n^{th} lines and call them l_1 and l_n :

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1. height[1] < height[n]: We find the best solution for l_1 . Because for $2 \le k \le n-1$, the container l_1 and l_k cannot have more water than l_1 and l_n . We can safely remove l_1 and use mathematical induction to find the optimal solution. Then we need to compare that solution with l_1 and l_n and choose the maximum

- 2. height[1] > height[n]: We find the best solution for l_n , like above we can safely remove l_n and find the best solution and compare it with l_1 and l_n
- 3. height[1] = height[n] We find the best solution for both l_1 and l_n . We can safely remove both of them and find the optimal solution and compare it with l_1 and l_n .

The running time of this algorithm is O(n).

```
int maxArea(vector<int>& height) {
 int left = 0, right = height.size() - 1;
 int res = 0;
 while (left < right)
    if (height[left] < height[right])</pre>
     res = max(res, (right - left) * height[left]);
      ++left;
    else if (height[left] > height[right])
      res = max(res, (right - left) * height[right]);
      --right;
    }
    else
      res = max (res, (right - left) * height[right]);
      ++left:
       -right;
 }
 return res;
```

1.1.4 Two Sum II - Input Array Is Sorted

Given a sorted array, return two indices i, j such that $numbers[i] + numbers[j] = target <math>\land i < j$. The indices start from 1.

For similar questions, refer to 1.1.5. For a general solution refer to 1.1.6

We use mathematical induction. We assume we know the solution for all sorted arrays with length less than n. Now consider a sorted array with length n:

1. numbers[0]+numbers[n-1] < target: Since the array is sorted, numbers[n-1] is the maximum and numbers[0] is the minimum. So numbers[0] cannot be in the solution. We use mathematical induction to get the answer from numbers[1..n-1]

- 2. numbers[0] + numbers[n-1] > target: With a similar argument we can say numbers[n-1] cannot be in the solution. So we use the induction to solve numbers[0..n-2]
- 3. number[0] + numbers[n-1] == target: We have the solution.

```
vector<int> twoSum(vector<int>& numbers, int target) {
  int left = 0, right = numbers.size() - 1;
  while (left < right)
  {
    const auto sum = numbers[left] + numbers[right];
    if (sum < target)
        ++left;
    else if ( sum > target)
        --right;
    else
        return {left + 1, right + 1};
    }
    throw 1;
}
```

1.1.5 3Sum

Given an integer array nums, return all triplets [nums[i], nums[j], nums[k]] such that $i \neq j \land i \neq k \land j \neq k$. Notice that the solution set must not contain duplicate triplets.

This is the extension of 1.1.4. For general solution refer to 1.1.6

The tricky part here is how to avoid duplicates. We assume nums is sorted, then we use mathematical induction. We assume we know how to solve it for less than n sorted numbers. Now we want to solve n sorted numbers. We consider the first element nums[0]. Now consider nums[j] such that j=k+1. We define k as $\{nums[0], nums[1], \ldots, nums[k]\}$ such that nums[m] = nums[m+1] for $0 \le m \le k-1$. We use mathematical induction to find the answers for nums[j.n-1].

Note that we cannot use induction for subproblems nums[m..n-1] for $1 \le m \le k$ because it's possible they have triplets that start with nums[0]. As mentioned in problem statement, duplicates are not allowed.

Then we need to find the answers that contain nums[0]. For doing that we should find set S:

```
S = \{ \{p, q\} : 1 \le p < q \le n - 1 \land nums[p] + nums[q] = -nums[0] \}
```

Note that S shouldn't have any duplicates. Also it's important that we should find the pairs in nums[1..n-1] (not in nums[j..n-1]. For example [0,0,0] is a valid triplet). This is two sum II problem (refer to 1.1.4), but we should handle duplicates. There are two solutions to implement two sum:

```
vector<vector<int>> threeSum(vector<int>& nums) {
  vector<vector<int>> res;
```

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```
sort(nums.begin(), nums.end()); //O(nlogn)
unordered_set<int> visited;
for (int i = 0; i < nums.size() && nums[i] <= 0; ++i)</pre>
  // To find subprolem nums[j..n - 1]
 //note that i points to the first repetitive character
 if (i > 0 && nums[i] == nums[i - 1])
    continue;
  visited.clear();
  for (int j = i + 1; j < nums.size(); ++j)</pre>
    //make sure j points to the last repetitive character:
    if (j + 1 < nums.size() && nums[j] == nums[j + 1])</pre>
      visited.insert(nums[j]);
      continue;
    auto target = -nums[i] - nums[j];
    if (visited.count(target) > 0)
     res.push_back({nums[i], target, nums[j]});
    visited.insert(nums[j]);
 }
return res;
```

The second solution:

```
vector<vector<int>> threeSum(vector<int>& nums) {
 vector<vector<int>> res;
 sort(nums.begin(), nums.end()); //O(nlogn)
 for (int i = 0; i + 1 < nums.size() && nums[i] <= 0; ++i)
    // To find subprolem nums[j..n - 1]:
   if (i > 0 && nums[i] == nums[i - 1])
   int target = -nums[i];
   int left = i + 1, right = nums.size() - 1;
   while (left < right)
     auto sum = nums[left] + nums[right];
     if (sum < target)
       ++left;
      else if (sum > target)
        --right;
      else
       res.push_back({nums[i], nums[left], nums[right]});
       ++left;
       while (left < right && nums[left] == nums[left - 1])</pre>
         ++left;
        --right;
     }
   }
 }
 return res;
```

}

1.1.6 4sum

Given an array nums of n integers, return an array of all the **unique** quadruplets [nums[a], nums[b], nums[c], nums[d]] such that a < b < c < d and nums[a] + nums[b] + nums[c] + nums[d] = target.

The base case for this solution is 2sum. For more information refer to 1.1.4. It's a good idea to look at 3sum in 1.1.5

We can solve it similar to 3sum (for more information refer to 1.1.5). We suppose the array is sorted. We use mathematical induction, so we know how to solve it for less than n sorted numbers. Now we consider n sorted numbers. We choose the first two numbers nums[0] and nums[1]. We choose subproblem nums[j..n-1] such that $nums[m] = nums[0] \vee nums[m] = nums[1]$ for $2 \leq m \leq j-1$. As explained in section 1.1.5, this is required to avoid duplication. We use mathematical induction to find quadruplets in nums[j..n-1]. Then for finding quadruplets that contain nums[0] and nums[1], we use 2sum (refer to section 1.1.4) algorithm on nums[2..n-1] (not nums[j..n-1]. For example [0,0,0,0] is a valid quadruplet if target=0).

```
vector<vector<int>> fourSum(vector<int>& nums, int target) {
  vector<vector<int>> res;
 const auto size = nums.size();
 sort(nums.begin(), nums.end());
 for (int i = 0; (i + 3) < size; ++i)
   if (i > 0 && nums[i] == nums[i - 1])
      continue:
   for (int j = i + 1; (j + 2) < size; ++j)
      if (j > i + 1 && nums[j] == nums[j - 1])
       continue;
      int t = target - nums[i] - nums[j];
      int left = j + 1;
      int right = size - 1;
      while (left < right)
        int sum = nums[left] + nums[right];
        if (sum < t)
          ++left;
        else if (sum > t)
          --right;
        else
          res.push_back({nums[i], nums[j], nums[left], nums[right]})
          ++left;
          while (nums[left] == nums[left - 1] && left < right)</pre>
            ++left;
          --right;
```

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```
}
}
return res;
}
```

To solve ksum problem, we need k-2 for loops to find $[nums[0], nums[1], \ldots, nums[k-3]]$ and we use 2sum algorithm to find nums[k-2] and nums[k-1]. We can use a recursive function to solve the problem dynamically during the runtime.

```
vector<vector<int>> twosum(vector<int>& nums, int left, int target)
 int right = nums.size() -1;
 vector<vector<int>> res;
 while (left < right)
    auto sum = nums[left] + nums[right];
   if (sum < target)</pre>
     ++left;
    else if (sum > target)
      --right;
    else
     res.push_back({nums[left], nums[right]});
      while (nums[left] == nums[left - 1] && left < right)</pre>
       ++left;
      --right;
   }
 }
 return res;
vector<vector<int>> ksum(vector<int>& nums, int start, int k, int
    target)
 vector<vector<int>> res;
 if (start == nums.size())
   return res;
 int average = target / k;
  //nums[start] * k > target
 if (nums[start] > average)
   return res;
  //nums.back() * k < target
 if (nums.back() < average)</pre>
   return res;
 if (k == 2)
   return twosum(nums, start, target);
 for (int i = start; i < nums.size(); ++i)</pre>
   if (i > start && nums[i] == nums[i - 1])
     continue;
    auto partial = ksum(nums, i + 1, k - 1, target - nums[i]);
   for (auto& val : partial)
      val.insert(val.begin(), nums[i]);
     res.push_back(val);
```

```
}
return res;
}

vector<vector<int>> fourSum(vector<int>& nums, int target) {
  sort(nums.begin(), nums.end());
  return ksum(nums, 0, 4, target);
}
```

Chapter 2

HackerRank

2.1 New Year Chaos

We define $index_i$ as the current index for person i. For example if we have 1,2,3,4 and 4 bribes 3, the queue looks like 1,2,4,3. So $index_4=3$. Since no body can bribe more than 2 times, $index_i \geq i-2$ for $1 \leq i \leq n$. Consider person n. No body can bribe that person. So $n-2 \leq index_n \leq n$. After we retruned that person to his actual place we can consider n-1. So we have $n-3 \leq index_{n-1} \leq n-1$ (note that at this moment $index_n=n$).

2.2 Minimum Swaps 2

Note that this solution is based on Selection Sort in which the number of swaps are minimum. According to Wikipedia: "One thing which distinguishes selection sort from other sorting algorithms is that it makes the minimum possible number of swaps, n-1 in the worst case." Altourh Selection sort has minimum number of swaps among all sorts agorithms, it has $O(n^2)$ comparisons. Since the final result is $\{1,2,\ldots,n\}$, it's like we have the set in sorted order so we can bypass comparisons and use Selection Sort advantage which is the minimum number of swaps.

We define $index_i$ as the current index of number i. Suppose we have n numbers, so $1 \le index_i \le n$. The goal is to have $index_i = i$. Without losing generality suppose $i < j \land index_i = j$. There are two cases to consider:

- 1. If $index_j = i$, then by swapping arr_i and arr_j , we put both i and j in their corresponding positions.
- 2. If $index_j = k \land k \neq i \land k \neq j$. In this case by swapping arr_i and arr_j we only put i in its corresponding position. So we need to do an extra swap to put j in its correct position.

We can start from i = 1 to i = n and make sure i is in correct position; otherwise we perform a swap. In each iteration we fix the position of one or two numbers. A good example is $\{4, 3, 2, 1\}$.

2.3 Count Triplets

We use dynamic programming to solve it. For mathematical induciton we define cnt[num][n] like this:

$$cnt[a_{i_1}][0] = |\{a_{i_0} \in arr \mid a_{i_1} = a_{i_0} \times r \land i_1 < i_2\}|$$

$$cnt[a_{i_2}][1] = |\{(a_{i_0}, a_{i_1}) \in arr \times arr \mid a_{i_k} = a_{i_{k-1}} \times r \land i_{k-1} < i_k \text{ for } 1 \le k \le 2\}|$$

So the final answer is:

$$\sum_{n \in arr} cnt[n][1]$$

Then for each number n we have

$$cnt[n \times r][0] = cnt[n \times r][0] + 1$$
$$cnt[n \times r][1] = cnt[n \times r][1] + cnt[n][0]$$

Since r = 1, the order of assignments are very important.

```
long countTriplets(vector<long> arr, long r) {
    const auto n = arr.size();
    unordered_map<long, array<long, 2>> cnt;
    //cnt[a[j]][0] = |\{a[i]\}| in which i < j and
                     a[j] = a[i] * r
    //cnt[a[k]][1] = /{a[i], a[j]}/ in which
                     i < j < k and
    //a[k] = a[j] * r and a[j] = a[i] * r
    long res = 0;
    for (const auto& num : arr)
        res += cnt[num][1];
        const auto next = num * r;
        cnt[next][1] += cnt[num][0];
        ++cnt[next][0];
    }
    return res;
}
```

2.4 Fraudulent Activity Notifications

Basically we want a O(nlogn) algorithm to find median of a sequuence, when we removed the first element and add another one. So we need two binary search trees. In the first one the maximum element is the median itself and in the secon one the minimum element is the second median in case of d=2k or a value greater than median when d=2k+1. So if d=2k both of these binary search trees always have k element. When d=2k+1, the first one always has k+1 elements and the second one has k elements. Let's call them lessEqual and greaterEqual.

If both removing element and new element belong to the same tree, nothing extra is required. So we only need to remove one element and add the new one. If the removing element is from lessEqual, we must remove the minimum

element from greaterEqual and add it to lessEqual. If the removing element is from greaterEqual, we must remove the maximum element from lessEqual and add it to greaterEqual. By doing that the maximum element is lessEqual is median. In case of d=2k, the minimum element in greaterEqual is the second median. The running time of this algorithm is O(nlogn).

```
int activityNotifications(vector<int> expenditure, int d)
   multiset<int, greater<int>> lessEqual;
   multiset<int> greaterEqual;
   vector<int> init(d);
    copy(expenditure.begin(), expenditure.begin() + d,
         init.begin());
    sort(init.begin(), init.end());
   const bool isEven = (d & 1) == 0;
   int medianIndex = (d - 1) / 2;
   for (i = 0; i <= medianIndex; ++i)</pre>
       lessEqual.insert(init[i]);
   for (; i < d; ++i)
        greaterEqual.insert(init[i]);
   int res = 0;
   for (int i = d; i < expenditure.size(); ++i)</pre>
        const int median1 = *lessEqual.begin();
        if (isEven)
            const int median2 = *greaterEqual.begin();
            if (expenditure[i] >= (median1 + median2))
                ++res;
        }
        else
        {
            if (expenditure[i] >= 2 * median1)
                ++res;
        }
        const auto removed = expenditure[i - d];
        if (removed <= median1 &&
            expenditure[i] <= median1)</pre>
            lessEqual.erase(lessEqual.find(removed));
            lessEqual.insert(expenditure[i]);
        }
        else if (removed > median1 &&
                 expenditure[i] > median1)
          greaterEqual.erase(greaterEqual.find(removed));
          greaterEqual.insert(expenditure[i]);
        else if ( removed <= median1)</pre>
```

```
//For handling d=1, it should first:
    greaterEqual.insert(expenditure[i]);
    lessEqual.erase(lessEqual.find(removed));
    lessEqual.insert(*greaterEqual.begin());
    greaterEqual.erase(greaterEqual.begin());
}
else
{
    //For handling d=1, it should be first:
    lessEqual.insert(expenditure[i]);
    greaterEqual.erase(greaterEqual.find(removed));
    greaterEqual.insert(*lessEqual.begin());
    lessEqual.erase(lessEqual.begin());
}
return res;
}
```

2.5 Merge Sort: Counting Inversions

Problem Statement

We can solve it using merge sort. Suppose we have arr[left..right]. We break it into two subproblem arr[left..mid] and arr[mid+1..right]. Both of them are sorted. According to merge sort algorithm $left \leq i \leq mid$ and $mid+1 \leq j \leq right$. In other words we already put arr[left..i-1] and arr[mid+1..j-1] into their correct positions. So when arr[j] < arr[i], it means $arr[j] < arr[i] \leq arr[x]$ in which $i+1 \leq x \leq mid$. So we need to have mid-i+1 swaps.

```
long mergeSort(vector<int>& arr, int leftIndex,
               int rightIndex)
    if (leftIndex == rightIndex)
        return 0:
   long res = 0;
   int midIndex = (leftIndex + rightIndex) / 2;
   res = mergeSort(arr, leftIndex, midIndex);
   res += mergeSort(arr, midIndex + 1, rightIndex);
    vector<int> sorted(rightIndex - leftIndex + 1);
    int i, j, k;
    for (i = leftIndex, j = midIndex + 1, k = 0;
         i <= midIndex && j <= rightIndex;)</pre>
        if (arr[i] <= arr[j])</pre>
            sorted[k++] = arr[i++];
        else
            res += midIndex - i + 1;
            sorted[k++] = arr[j++];
    }
```

Chapter 3

TopCoder

3.1 SRM 428

3.1.1 The Palindrome

For another variation refer to 1.1.2. We want to add the minimum number of characters to the end of string to make it a palindrome. The straightforward approach is to try add the first i characters in reverse for all $0 \le i \le n-1$ in which n is the length of string. So we start from i=0 and check whether the string is palindrome. If it's not we check for i=1 and so on. The running time of this algorithm is $O(n^2)$.

Since $n \leq 50$, this algorithm is fast. We can make it $O(n \log_2 n)$ if we use binary search tree to find the minimum i.

There is another approach. Let's assume we have string $S = s_1 s_2 \dots s_n$. We define $S' = s_n \dots s_2 s_1$. Suppose we can write string S as QP. In other words, S is the concatenation of two strings Q and P. We assume P is palindrome but

Q is not. We can make S palindrome if we convert QP to QPQ' we call this new String Z. Z is palindrome because if we reverse it we have:

Z: QPQ'Z': QP'Q'

Since we want the length of Q be minimum, we must find the maximal P:

As the previous implementation the running time is $O(n^2)$ but it's easy to convert it to $O(n \log_2 n)$ using binary search.

This implementation has a unique feature. We can convert it to an O(n) algorithm using KMP algorithm. Suppose S=QP where P is a palindrome. We want to find palidnrome postfix P which its length is maximum among all palindrome post-fixes. We need to run KMP pre-compute calculation on S'=P'Q'. Then we run KMP algorithm as if we want to find whether S' is a substring of S. Suppose we use i as an index for S and j as an index for S'. The algorithm start with i=0 and ends when i=len(S) in which S'[0..j-1] is P' or P (since it's palindrome).

```
vector<int> calculateNext(const string& B)
{
  vector<int> next(B.length());
  next[0] = -2;
  next[1] = -1;
  int i, j;
  for (i = 2; i < B.length(); ++i)
  {
      j = next[i - 1] + 1;
      for (; j >= 0 && B[j] != B[i - 1]; j = next[j] + 1);
      next[i] = j;
  }
  return next;
}

int find(string A)
{
  const string B = string(A.rbegin(), A.rend());
```

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```
const auto next = calculateNext(B);
int i, j;
for (i = 0, j = 0; i < A.length() && j < B.length();)
{
   if (A[i] == B[j])
        ++i, ++j;
   else if ((j = next[j] + 1) < 0)
   {
        //Since B.front() == A.back(), it's impossible
        //i == A.length() here:
        ++i, j = 0;
        j = 0;
   }
}
int palindromeLen = j;
return A.length() + A.length() - palindromeLen;
}</pre>
```