Online Contests Solutions

Saman Saadi

Contents

1	Lee	tCode	1
	1.1	Medium	1
		1.1.1 Longest Substring Without Repeating Characters	1
		1.1.2 Longest Palindromic Substring	2
		1.1.3 Container With Most Water	2
		1.1.4 Two Sum II - Input Array Is Sorted	3
		1.1.5 3Sum	4
		1.1.6 4sum	6
		1.1.7 Generate Parentheses	8
		1.1.8 Divide Two Integers	9
		1.1.9 Next Permutation	10
		1.1.10 Search in Rotated Sorted Array	11
		1.1.11 Find First and Last Position of Element in Sorted Array .	12
		1.1.12 Valid Sudoku	14
		1.1.13 Multiply Strings	14
		1.1.14 Jump Game	15
		1.1.15 Jump Game II	15
		1.1.16 Rotate Image	16
		1.1.17 Spiral Matrix	17
2	Hac	kerRank	19
	2.1	New Year Chaos	19
	2.2	Minimum Swaps 2	20
	2.3	Count Triplets	20
	2.4	Fraudulent Activity Notifications	21
	2.5	Merge Sort: Counting Inversions	23
3	Top	Coder	25
	3.1		25
			25
Α	C+-	+ refresher	29
			29
		·	29

iv CONTENTS

A.2	Associative containers	29
	A.2.1 std::set	29
A.3	Inserting into a container	30
A.4	std::vector iterators	31
A.5	std::reverse_iterator	32
A.6	std::mismatch	32
A.7	Boyer–Moore string-search algorithm	33
A.8	std::array	33
A.9	Initializing arrays with std::fill, memcpy and memset	34

Chapter 1

LeetCode

1.1 Medium

1.1.1 Longest Substring Without Repeating Characters

Find the length of the longest substring without repeating characters. For example in "abcabcbb" the answer is "abc".

We can use mathematical induction. Assume s[i..j-1] is a valid substring (it doesn't have any repetitive characters). Now we want to add s[j] to it. If s[j] is not in s[i..j-1], then we can easily add s[j] to the list and claim s[i..j] is also valid. Otherwise there is exactly one $i \le k \le j-1$ that s[k] = s[j].

Because of s[j], we cannot extend s[i..j-1] anymore, but we may be able to extend s[k+1..j]. The running time of the following implementation is O(n):

```
int lengthOfLongestSubstring(string s) {
  int res = 0;
  unordered_map<char, int> mp;
  for (int i = 0, j = 0; i < s.length() && j < s.length(); ++j)
  {
    auto iter = mp.find(s[j]);
    if (iter != mp.end())
    {
        //Note that after mp.erase, iter will be invalid!
        //So we store the index in k
        int k = iter->second;
        for (; i <= k; ++i)
            mp.erase(s[i]);
    }
    mp[s[j]] = j;
    res = max(res, j - i + 1);
    }
    return res;
}</pre>
```

1.1.2 Longest Palindromic Substring

Given a string s, return the longest palindromic substring in s. For another variation of this problem refer to 3.1.1.

Assume $S[i..j] = s_i s_{i+1} \dots s_j$. We define $S'[i'..j'] = s_{i'} s_{i'+1} \dots s_{j'}$ in such a way $(s_j = s_{i'}) \wedge (s_{j-1} = s_{i'+1}) \wedge \dots \wedge (s_i = s_{j'})$. Let's Assume string P is a palindromic string and c is an arbitrary character. We can break P into:

$$P[i..j] = \begin{cases} S[i..m]S'[m+1..j] & (j-i+1) \text{ is even} \\ S[i..m-1]cS'[m+1..j] & (j-i+1) \text{ is odd} \end{cases}$$

It's obvious that $m = \lfloor \frac{i+j}{2} \rfloor$. To solve this problem we need to find the place of m that can be $0 \le m < n$. n is the length of the string. The running time of this algorithm is $O(n^2)$:

```
string longestPalindrome(string s) {
 int len = 0;
  int index = -1;
  const auto findLongest = [&](int 1, int r)
    for (; 1 >= 0 && r < s.length() && s[1] == s[r]; --1, ++r)
      if ((r - 1 + 1) > len)
       len = r - 1 + 1;
        index = 1;
 };
 for (int m = 0; m < s.length(); ++m)
    //Check for S S_r (length is even)
    findLongest(m, m + 1);
    //Check for S c S_r (length is odd)
   findLongest(m, m);
 if (index == -1)
    return "";
  return s.substr(index, len);
```

1.1.3 Container With Most Water

You are given an integer array hieght of length n. There are n vertical lines drawn such that the two endpoints of i^{th} line are (i,0) and (i,height[i]). Find two lines that together with the x-axis form a container, such that the container contains the most water. You may not slant the container.

We use mathematical induction. Suppose we know the answer for the number of lines less than n. Now we consider n lines. We consider the first and n^{th} lines and call them l_1 and l_n :

1. height[1] < height[n]: We find the best solution for l_1 . Because for $2 \le k \le n-1$, the container l_1 and l_k cannot have more water than l_1 and l_n . We can safely remove l_1 and use mathematical induction to find the optimal solution. Then we need to compare that solution with l_1 and l_n and choose the maximum

- 2. height[1] > height[n]: We find the best solution for l_n , like above we can safely remove l_n and find the best solution and compare it with l_1 and l_n
- 3. height[1] = height[n] We find the best solution for both l_1 and l_n . We can safely remove both of them and find the optimal solution and compare it with l_1 and l_n .

The running time of this algorithm is O(n).

```
int maxArea(vector<int>& height) {
 int left = 0, right = height.size() - 1;
 int res = 0;
 while (left < right)
    if (height[left] < height[right])</pre>
     res = max(res, (right - left) * height[left]);
      ++left;
    else if (height[left] > height[right])
      res = max(res, (right - left) * height[right]);
      --right;
    }
    else
      res = max (res, (right - left) * height[right]);
      ++left:
       -right;
 }
 return res;
```

1.1.4 Two Sum II - Input Array Is Sorted

Given a sorted array, return two indices i, j such that $numbers[i] + numbers[j] = target <math>\land i < j$. The indices start from 1.

For similar questions, refer to 1.1.5. For a general solution refer to 1.1.6

We use mathematical induction. We assume we know the solution for all sorted arrays with length less than n. Now consider a sorted array with length n:

1. numbers[0]+numbers[n-1] < target: Since the array is sorted, numbers[n-1] is the maximum and numbers[0] is the minimum. So numbers[0] cannot be in the solution. We use mathematical induction to get the answer from numbers[1..n-1]

- 2. numbers[0] + numbers[n-1] > target: With a similar argument we can say numbers[n-1] cannot be in the solution. So we use the induction to solve numbers[0..n-2]
- 3. number[0] + numbers[n-1] == target: We have the solution.

```
vector<int> twoSum(vector<int>& numbers, int target) {
  int left = 0, right = numbers.size() - 1;
  while (left < right)
  {
    const auto sum = numbers[left] + numbers[right];
    if (sum < target)
        ++left;
    else if ( sum > target)
        --right;
    else
        return {left + 1, right + 1};
    }
    throw 1;
}
```

1.1.5 3Sum

Given an integer array nums, return all triplets [nums[i], nums[j], nums[k]] such that $i \neq j \land i \neq k \land j \neq k$. Notice that the solution set must not contain duplicate triplets.

This is the extension of 1.1.4. For general solution refer to 1.1.6

The tricky part here is how to avoid duplicates. We assume nums is sorted, then we use mathematical induction. We assume we know how to solve it for less than n sorted numbers. Now we want to solve n sorted numbers. We consider the first element nums[0]. Now consider nums[j] such that j=k+1. We define k as $\{nums[0], nums[1], \ldots, nums[k]\}$ such that nums[m] = nums[m+1] for $0 \le m \le k-1$. We use mathematical induction to find the answers for nums[j.n-1].

Note that we cannot use induction for subproblems nums[m..n-1] for $1 \le m \le k$ because it's possible they have triplets that start with nums[0]. As mentioned in problem statement, duplicates are not allowed.

Then we need to find the answers that contain nums[0]. For doing that we should find set S:

```
S = \{ \{p, q\} : 1 \le p < q \le n - 1 \land nums[p] + nums[q] = -nums[0] \}
```

Note that S shouldn't have any duplicates. Also it's important that we should find the pairs in nums[1..n-1] (not in nums[j..n-1]. For example [0,0,0] is a valid triplet). This is two sum II problem (refer to 1.1.4), but we should handle duplicates. There are two solutions to implement two sum:

```
vector<vector<int>> threeSum(vector<int>& nums) {
  vector<vector<int>> res;
```

```
sort(nums.begin(), nums.end()); //O(nlogn)
unordered_set<int> visited;
for (int i = 0; i < nums.size() && nums[i] <= 0; ++i)</pre>
  // To find subprolem nums[j..n - 1]
 //note that i points to the first repetitive character
 if (i > 0 && nums[i] == nums[i - 1])
    continue;
  visited.clear();
  for (int j = i + 1; j < nums.size(); ++j)</pre>
    //make sure j points to the last repetitive character:
    if (j + 1 < nums.size() && nums[j] == nums[j + 1])</pre>
      // to handle cases like [0, 0, 0]
      visited.insert(nums[j]);
      continue;
    auto target = -nums[i] - nums[j];
    if (visited.count(target) > 0)
     res.push_back({nums[i], target, nums[j]});
    visited.insert(nums[j]);
}
return res;
```

The second solution:

```
vector<vector<int>> threeSum(vector<int>& nums) {
 vector<vector<int>> res;
 sort(nums.begin(), nums.end()); //O(nlogn)
 for (int i = 0; i + 1 < nums.size() && nums[i] <= 0; ++i)
   // To find subprolem nums[j..n - 1]:
   if (i > 0 && nums[i] == nums[i - 1])
     continue;
   int target = -nums[i];
   int left = i + 1, right = nums.size() - 1;
   while (left < right)
     auto sum = nums[left] + nums[right];
     if (sum < target)</pre>
       ++left;
      else if (sum > target)
       --right;
      else
       res.push_back({nums[i], nums[left], nums[right]});
       while (left < right && nums[left] == nums[left - 1])</pre>
         ++left;
       --right;
     }
   }
 }
```

```
return res;
}
```

1.1.6 4sum

Given an array nums of n integers, return an array of all the **unique** quadruplets [nums[a], nums[b], nums[c], nums[d]] such that a < b < c < d and nums[a] + nums[b] + nums[c] + nums[d] = target.

The base case for this solution is 2sum. For more information refer to 1.1.4. It's a good idea to look at 3sum in 1.1.5

We can solve it similar to 3sum (for more information refer to 1.1.5). We suppose the array is sorted. We use mathematical induction, so we know how to solve it for less than n sorted numbers. Now we consider n sorted numbers. We choose the first two numbers nums[0] and nums[1]. We choose subproblem nums[j..n-1] such that $nums[m] = nums[0] \vee nums[m] = nums[1]$ for $2 \leq m \leq j-1$. As explained in section 1.1.5, this is required to avoid duplication. We use mathematical induction to find quadruplets in nums[j..n-1]. Then for finding quadruplets that contain nums[0] and nums[1], we use 2sum (refer to section 1.1.4) algorithm on nums[2..n-1] (not nums[j..n-1]. For example [0,0,0,0] is a valid quadruplet if target=0).

```
vector<vector<int>> fourSum(vector<int>& nums, int target) {
 vector<vector<int>> res:
 const auto size = nums.size();
 sort(nums.begin(), nums.end());
 for (int i = 0; (i + 3) < size; ++i)
    if (i > 0 && nums[i] == nums[i - 1])
      continue;
   for (int j = i + 1; (j + 2) < size; ++j)
      if (j > i + 1 && nums[j] == nums[j - 1])
       continue:
      int t = target - nums[i] - nums[j];
      int left = j + 1;
      int right = size - 1;
      while (left < right)
        int sum = nums[left] + nums[right];
        if (sum < t)
          ++left;
        else if (sum > t)
          --right:
        else
          res.push_back({nums[i], nums[j], nums[left], nums[right]})
          ++left;
          while (nums[left] == nums[left - 1] && left < right)</pre>
            ++left;
           -right;
```

```
}
}
return res;
}
```

To solve ksum problem, we need k-2 for loops to find $[nums[0], nums[1], \ldots, nums[k-3]]$ and we use 2sum algorithm to find nums[k-2] and nums[k-1]. We can use a recursive function to solve the problem dynamically during the runtime.

```
vector<vector<int>> twosum(vector<int>& nums, int left, int target)
 int right = nums.size() -1;
 vector<vector<int>> res;
 while (left < right)
   auto sum = nums[left] + nums[right];
   if (sum < target)</pre>
     ++left:
    else if (sum > target)
     --right;
    else
     res.push_back({nums[left], nums[right]});
     while (nums[left] == nums[left - 1] && left < right)</pre>
      --right;
   }
 }
 return res;
vector<vector<int>> ksum(vector<int>& nums, int start, int k, int
    target)
 vector<vector<int>> res;
 if (start == nums.size())
   return res;
 int average = target / k;
  //nums[start] * k > target
 if (nums[start] > average)
    return res;
  //nums.back() * k < target
 if (nums.back() < average)</pre>
   return res;
 if (k == 2)
   return twosum(nums, start, target);
 for (int i = start; i < nums.size(); ++i)</pre>
   if (i > start && nums[i] == nums[i - 1])
      continue;
    auto partial = ksum(nums, i + 1, k - 1, target - nums[i]);
    for (auto& val : partial)
      val.insert(val.begin(), nums[i]);
     res.push_back(val);
```

```
}
}
return res;
}

vector<vector<int>> fourSum(vector<int>& nums, int target) {
  sort(nums.begin(), nums.end());
  return ksum(nums, 0, 4, target);
}
```

1.1.7 Generate Parentheses

Given n pairs of parentheses, write a function to generate all combinations of well-formed parentheses.

We use mathematical induction. Let's say S_n is the set of all valid n pairs. We assume we know how to generate S_k for k < n. It's obvious that $S_1 = \{"()"\}$ and $S_0 = \{""\}$. Now we consider S_n . let's assume $p \in S_i$ and $q \in S_{n-1-i}$ for $0 \le i \le n-1$. It's obvious that "(p)q" is a valid parentheses with n pairs (i+n-1-i+1=n). We define

$$(S_i) = \{(p) : p \in S_i\}$$

Also we define Cartesian product as follow (pq means the concatenation of strings p and q):

$$S_i \times S_j = \{ pq : p \in S_i \land q \in S_j \}$$

So we can use the induction to generate S_n as follow:

$$S_n = \bigcup_{i=0}^{n-1} (S_i) \times S_{n-1-i}$$

```
vector<string> generateParenthesis(int n) {
  vector<string> res;
  if (n == 0)
    return {""};
  for (int i = 0; i < n; ++i)
  {
    auto set_left = generateParenthesis(i);
    auto set_right = generateParenthesis(n - i - 1);
    for (const auto& p : set_left)
    {
       string left_str = "(" + p + ")";
       for (const auto& q : set_right)
            res.push_back(left_str + q);
      }
    }
    return res;
}</pre>
```

1.1.8 Divide Two Integers

Divide two integers without using multiplication, division, and mod operator. We can only use **32-bit signed integers**.

We know that the range of a 32-bit signed integer is $[-2^{31}, 2^{31} - 1]$. We should convert negative numbers to positive ones. The tricky part is how to handle -2^{31} . Because we cannot fit it inside a positive 32-bit integer and we are not allowed to use *unsigned* variables.

Method 1

We use mathematical induction. Suppose a = bq + r. We define div(a, b) = q and mod(a, b) = r. Suppose we know how to calculate div and mod for less than a. Now we want to calculate for a:

$$div(a,b) = \begin{cases} 2 \times div(k,b) + div(2 \times mod(k,b),b) & a = 2k \\ 2 \times div(k,b) + div(2 \times mod(k,b) + 1,b) & a = 2k + 1 \\ 0 & a < b \\ 1 & a = b \\ a & b = 1 \\ base_{div}(a,b) & k \le b \end{cases}$$

$$mod(a,b) = \begin{cases} mod(2 \times mod(k,b),b) & a = 2k \\ mod(2 \times mod(k,b) + 1,b) & a = 2k + 1 \\ a & a < b \\ 0 & a = b \lor b = 1 \\ base_{mod}(a,b) & k \le b \end{cases}$$

For calculating *base* we use a straightforward approach. Since we cannot use multiplication and division, we can use $2 \times a = a + a$ and we use shift operator to do division by $2 \ (a >> 1)$.

Let's consider $a=-2^{31}$. Let's assume $q=div(2^{31},|b|)$ and $r=mod(2^{31},|b|)$:

$$|div(-2^{31}, |b|)| = \begin{cases} q & r+1 < |b| \\ q+1 & r+1 \ge |b| \end{cases}$$

Method2

This is easier to implement. We assume a = bq + r. We define div(a, b) = q. Like previous method, suppose we know how to calculate div for less than a. Now we want to calculate for a. Suppose $b \times 2^k \le a < b \times 2^{k+1}$. So we can say $a = b \times 2^k + r$. If r < b we find the answer; otherwise we use the induction to find r = bq' + r'. So the answer is:

$$a = b \times 2^{k} + r$$
$$= b \times 2^{k} + bq' + r'$$
$$= b \times (2^{k} + q') + r'$$

We can summarize it as:

$$div(a,b) = \begin{cases} 2^k + div(a-b \times 2^k, b) & a \ge b \\ 0 & a < b \end{cases}$$

Let's consider $a = -2^{31}$. We cannot store it as positive in a 32-bit signed integer. Let's assume a = bq + r:

$$b(q-1) + r$$

$$= bq + r - b$$

$$= a - b$$

$$\implies a - b = b(q-1) + r$$

So $div(|2^{31}|, |b|) = div(|2^{31}| - |b|, |b|) + 1$:

```
int div(int a, int b)
{
 if (a < b)
   return 0;
 const int half_of_a = a >> 1;
 int q = 1, bq = b;
 for (; bq <= half_of_a; bq <<= 1, q <<= 1);
 return q + div(a - bq, b);
int divide(int a, int b) {
 static const int min_int = numeric_limits<int>::min();
 static const int max_int = numeric_limits<int>::max();
 int sign = 1;
 int res = 0;
  if (b == min_int)
   return a == min_int ? 1 : 0;
  if (b == 1)
    return a;
 if (b == -1)
    return a == min_int ? max_int : -a;
 if (b < 0)
    sign = -sign, b = -b;
  if (a == min_int)
   a += b, res = 1;
  if (a < 0)
    sign = -sign, a = -a;
 res += div(a, b);
  return res * sign;
```

1.1.9 Next Permutation

Implement std::next_permutation for nums[0..n-1]! For example if nums = [1,3,2], the answer is [2,1,3]

Suppose p and q are two permutations of nums[0..n-1]. If p < q, then we have a $0 \le i < n$ such that p[k] = q[k] for $0 \le k < i$ and p[i] < q[i]. For solving this question we should find the smallest q that is greater than p.

Assume i is the maximum index such that nums[i] < nums[i+1] for $0 \le i \le n-2$. If such i does not exist it means we have the last permutation. So we should return the first one. Since i is the maximum index, it implies $nums[k] \ge nums[k+1]$ for all $i+1 \le k \le n-2$. In other words, nums[i+1..n-1] is in non-ascending order.

Let's assume $j \geq i+1$ is the maximum index such that nums[i] < nums[j]. Since nums[i+1..n-1] is in non-ascending order, that means nums[j] is the smallest element that is bigger than nums[i] in that range. We should swap nums[i] and nums[j] and reverse nums[i+1..n-1] to make it in non-descending order. This is the smallest permutation greater than nums.

To summarize we break nums[0..n-1] into A = nums[0..i] and B = nums[i+1..n-1]. The former is in non-descending and the latter in non-ascending order. We define $B' = \{nums[j] \in B : nums[j] > nums[i]\}$. We find the smallest element in B' and swap it with nums[i]. Then we should sort B in non-descending order.

The implementation is based on possible implementation of next_permutation. To understand how reverse iterator::base (i.base()) works, refer to A.5.

```
void nextPermutation(vector<int>& nums) {
   auto i = is_sorted_until(nums.rbegin(), nums.rend());
   if (i != nums.rend())
   {
      auto j = upper_bound(nums.rbegin(), i, *i);
      iter_swap(i, j);
   }
   //https://en.cppreference.com/w/cpp/iterator/reverse_iterator/base
   reverse(i.base(), nums.end());
}
```

1.1.10 Search in Rotated Sorted Array

There is an integer array nums sorted in ascending order (with distinct values). For example let's assume we have [0, 1, 2, 4, 5, 6, 7]. We might rotate at pivot index 3 so it becomes [4, 5, 6, 7, 0, 1, 2]. Write an $O(\log n)$ algorithm to find the index of target.

In the non-rotated versions, the minimum number starts at index 0. We use the following algorithm to find the minimum index in the rotated one. We assume $middle = \lfloor \frac{left + right}{2} \rfloor$

$$f(left, right) = \begin{cases} f(middle + 1, right) & nums[middle] > nums[right] \\ f(left, middle) & nums[middle] < nums[right] \\ left & left = right \end{cases}$$

Let's say k = f(0, n - 1). Then we have two ascending lists A = nums[0..k - 1]

and B = nums[k..n-1]. Based on target value we choose A or B to do a binary search to see whether target is in the list.

```
int search(vector<int>& nums, int target) {
 int left = 0;
  const int n = nums.size();
 int right = n - 1;
 int k;
 while (left <= right)
    if (left == right)
     k = left;
      break;
    int middle = (left + right) / 2;
    if (nums[middle] > nums[right])
     left = middle + 1;
      right = middle;
 if (target >= nums[k] && target <= nums[n - 1])</pre>
   left = k, right = n - 1;
   left = 0, right = k - 1;
 while (left <= right)
    if (left == right)
     return nums[left] == target ? left : -1;
   int middle = (left + right) / 2;
    if (nums[middle] < target)</pre>
      left = middle + 1;
      right = middle;
 }
 return -1;
7
```

1.1.11 Find First and Last Position of Element in Sorted Array

Given an array of integers nums sorted in non-decreasing order, find the starting and ending position of a given target value.

We define f(i, j) that has the index of left-most target in nums[i...j]. On the other hand we define g(i, j) that keeps the index of the right-most target in nums[i...j].

Let's assume mid is index of the middle element. Both f and g acts similarly when nums[mid] < target or nums[mid] > target. Let's assume nums[mid] = target. Since f should choose the left-most target, it tries to find the answer in nums[0..mid]. There is a possibly that other elements with value target exists in nums[0..mid]. On the other hand, g should find the rightmost target. So it chooses nums[mid..n-1].

We assume $mid_f = \lfloor \frac{left + right}{2} \rfloor$:

$$f(left, right) = \begin{cases} f(mid_f + 1, right) & nums[mid_f] < target \\ f(left, mid_f) & nums[mid_f] \geq target \\ left & left = right \land nums[left] = target \\ -1 & otherwise \end{cases}$$

We define $mid_c = \lceil \frac{left + right}{2} \rceil$:

$$g(left,right) = \begin{cases} g(mid_c,right) & nums[mid_c] \leq target \\ g(left,mid_c-1) & nums[mid_c] > target \\ left & left = right \land nums[left] = target \\ -1 & otherwise \end{cases}$$

Finding mid becomes tricky when the size of sub-problem is 2. Let's say we are at nums[i..i+1]. We know $mid_f = \lfloor \frac{i+i+1}{2} \rfloor = i$ and $mid_c = \lceil \frac{i+i+1}{2} \rceil = i+1$. For f choosing mid_c leads to an infinite loop and for g it's mid_f :

$$\begin{split} f(i,i+1) &= \begin{cases} f(mid_f+1,i+1) = f(i+1,i+1) & nums[mid_f] < target \\ f(i,mid_f) = f(i,i) & nums[mid_f] \geq target \end{cases} \\ f(i,i+1) &= \begin{cases} f(mid_c+1,i+1) = f(i+2,i+1) & nums[mid_c] < target \\ f(i,mid_c) = f(i,i+1) & nums[mid_c] \geq target \end{cases} \end{split}$$

Also we can think that f wants to find the leftmost target, so it makes sense to choose mid_f , but g wants to find the rightmost target so it should choose mid_c . Now we can use f(0, n-1) and g(0, n-1) to find the solution:

```
vector<int> searchRange(vector<int>& nums, int target) {
 int n = nums.size();
 int left = 0, right = n - 1;
 while (left < right)
    int mid = (left + right) / 2;
   if (nums[mid] < target)</pre>
     left = mid + 1;
      right = mid;
 if (left > right || nums[left] != target)
   return {-1, -1};
 vector<int> res(1, left);
 right = n - 1;
 while (left < right)
   int mid = (left + right + 1) / 2;
   if (nums[mid] <= target)</pre>
     left = mid;
    else
```

```
right = mid - 1;
}
res.push_back(left);
return res;
}
```

1.1.12 Valid Sudoku

```
bool isValidSudoku(vector<vector<char>>& board) {
  array<array<bool, 10>, 9> row_seen{}, col_seen{};
  array<array<br/>tool, 10>,3>,3> box_seen{};
 for (int r = 0; r < 9; ++r)
    for (int c = 0: c < 9: ++c)
      if (board[r][c] == '.')
        continue;
      int d = board[r][c] - '0';
      int box_row = r / 3;
      int box_col = c / 3;
      auto& rs = row_seen[r][d];
      auto& cs = col_seen[c][d];
      auto& bs = box_seen[box_row][box_col][d];
      if (rs || cs || bs)
        return false;
      rs = cs = bs = true;
 }
 return true;
}
```

1.1.13 Multiply Strings

Given two non-negative integers num1 and num2 represented as strings, return the product of num1 and num2, also represented as a string.

Let's assume the size of num1 is n1 and the size of num2 is n2. The result has at most n1+n2 digits. We use mathematical induction. We assume we know how to solve the problem for every num1 of any size and num2 for sizes less than n2. Now we want to calculate the result when num2 has size n2. We use mathematical induction to calculate num1[0..n1-1] and num2[1..n2-1]. Let's assume the answer is in res[1..n1+n2-1]. We assume res[0]=0. Now we use the induction to calculate the result of num1[0..n1-1] and num2[0..n2-1]. For $0 \le i \le n1-1$ we have:

$$\begin{split} res[i+1] &= (res[i+1] + num1[i] \times num2[0]) \bmod 10 \\ res[i] &= res[i] + \lfloor \frac{res[i+1] + num1[i] \times num2[0]}{10} \rfloor \end{split}$$

Note that res[i] keeps the carry. It's possible it goes above 9 but after the algorithm finishes, it has the correct digit.

1.1.14 **Jump Game**

You are given an integer array nums. You are initially positioned at the array's first index, and each element in the array represents your maximum jump length at that position. Return true if you can reach the last index, or false otherwise.

For explanation refer to 1.1.15.

```
bool canJump(vector<int>& nums) {
  int u;
  int upper_index = nums[0];
  for (u = 1; u <= upper_index && u < nums.size(); ++u)
    upper_index = max(upper_index, u + nums[u]);
  return upper_index >= nums.size() - 1;
}
```

1.1.15 Jump Game II

Given an array of non-negative integers nums, you are initially positioned at the first index of the array. Each element in the array represents your maximum jump length at that position. Your goal is to reach the last index in the minimum number of jumps. You can assume that you can always reach the last index.

This is one application of BFS. The graph is created in a special way that it's easy to implement it in O(V) instead of O(V + E).

If we run a BFS on this graph, we will have a BFS tree. The level of the root (index 0) is 0. The level of the children of the root are 1 and so on. The answer is to find the level of the last element in a BFS tree.

We define $upper_index[i]$ as the rightmost index of level i:

- level 0 starts at $upper_index[0]$ and ends at $upper_index[0]$ ($upper_index[0] = 0$)
- level 1 starts at $upper_index[0] + 1$ and ends at $upper_index[1]$

• level i > 0 starts at $upper_index[i-1] + 1$ and ends at $upper_index[i]$

We know $upper_index[0] = 0$. For $i \ge 1$ we assume $upper_index[i-1] + 1 \le j \le upper_index[i]$, then we have:

$$upper_index[i+1] = max(nums[j])$$

Since $upper_index$ increases for next levels, we can assume $0 \le j \le upper_index[i]$. So for caluclating $upper_index[i+1]$ we just need to know $upper_index[i]$. That means we can avoid an array and store them in two variables $upper_index$ and $next_upper_index$:

```
int jump(vector<int>& nums) {
  int upper_index = 0, next_upper_index = 0;
  int d = 0;
  for (int u = 0; u < nums.size(); ++u)
  {
    if (u > upper_index)
    {
        ++d;
        upper_index = next_upper_index;
    }
    next_upper_index = max(next_upper_index, u + nums[u]);
  }
  return d;
}
```

1.1.16 Rotate Image

You are given an $n \times n$ 2D matrix representing an image, rotate the image **in-place** by 90 degrees (clockwise)

Let's consider a 3×3 image and rotate it 90 degrees:

Let's assume the image is m and the rotated image is r, then we have:

$$m[i][j] = r[n-1-j][i]$$

For solving this problem first we do a transpose and then a reflect:

$$m[i][j] = m'[j][i] = r[n-1-j][i]$$

```
void rotate(vector<vector<int>>& matrix) {
  const auto n = matrix.size();
  for (int i = 0; i < n; ++i)
     for (int j = i; j < n; ++j)
        swap(matrix[i][j], matrix[j][i]);
  const int half = n / 2;
  for (int i = 0; i < n; ++i)
     for (int j = 0; j < half; ++j)
        swap(matrix[i][j], matrix[i][n - 1 - j]);
}</pre>
```

1.1.17 Spiral Matrix

Given an $m \times n$ matrix, return all elements of the matrix in spiral order. This is an implementation question:

```
int dir[4][2] = {{0, 1}, {1, 0}, {0, -1}, {-1, 0}};
vector<int> spiralOrder(vector<vector<int>>& matrix) {
 int m = matrix.size();
  int n = matrix[0].size();
  int up = 0, bottom = m - 1, left = 0, right = n - 1;
  int cells = 0;
  int r = 0, c = -1;
  vector<int> res;
  while (cells < m * n)
    for (int i = 0; i < 4; ++i)
      int nr = r + dir[i][0];
      int nc = c + dir[i][1];
      while (nr <= bottom && nr >= up && nc <= right && nc >= left)
       ++cells;
       r = nr, c = nc;
        res.push_back(matrix[r][c]);
        nr += dir[i][0];
        nc += dir[i][1];
      if (i == 0)
        ++up;
      else if (i == 1)
        --right;
      else if (i == 2)
        --bottom;
      else
        ++left;
    }
  }
  return res;
}
```

Chapter 2

HackerRank

2.1 New Year Chaos

We define $index_i$ as the current index for person i. For example if we have 1,2,3,4 and 4 bribes 3, the queue looks like 1,2,4,3. So $index_4=3$. Since no body can bribe more than 2 times, $index_i \geq i-2$ for $1 \leq i \leq n$. Consider person n. No body can bribe that person. So $n-2 \leq index_n \leq n$. After we retruned that person to his actual place we can consider n-1. So we have $n-3 \leq index_{n-1} \leq n-1$ (note that at this moment $index_n=n$).

2.2 Minimum Swaps 2

Note that this solution is based on Selection Sort in which the number of swaps are minimum. According to Wikipedia: "One thing which distinguishes selection sort from other sorting algorithms is that it makes the minimum possible number of swaps, n-1 in the worst case." Altourh Selection sort has minimum number of swaps among all sorts agorithms, it has $O(n^2)$ comparisons. Since the final result is $\{1,2,\ldots,n\}$, it's like we have the set in sorted order so we can bypass comparisons and use Selection Sort advantage which is the minimum number of swaps.

We define $index_i$ as the current index of number i. Suppose we have n numbers, so $1 \le index_i \le n$. The goal is to have $index_i = i$. Without losing generality suppose $i < j \land index_i = j$. There are two cases to consider:

- 1. If $index_j = i$, then by swapping arr_i and arr_j , we put both i and j in their corresponding positions.
- 2. If $index_j = k \land k \neq i \land k \neq j$. In this case by swapping arr_i and arr_j we only put i in its corresponding position. So we need to do an extra swap to put j in its correct position.

We can start from i = 1 to i = n and make sure i is in correct position; otherwise we perform a swap. In each iteration we fix the position of one or two numbers. A good example is $\{4, 3, 2, 1\}$.

2.3 Count Triplets

We use dynamic programming to solve it. For mathematical induciton we define cnt[num][n] like this:

$$cnt[a_{i_1}][0] = |\{a_{i_0} \in arr \mid a_{i_1} = a_{i_0} \times r \land i_1 < i_2\}|$$

$$cnt[a_{i_2}][1] = |\{(a_{i_0}, a_{i_1}) \in arr \times arr \mid a_{i_k} = a_{i_{k-1}} \times r \land i_{k-1} < i_k \text{ for } 1 \le k \le 2\}|$$

So the final answer is:

$$\sum_{n \in arr} cnt[n][1]$$

Then for each number n we have

$$cnt[n \times r][0] = cnt[n \times r][0] + 1$$

$$cnt[n \times r][1] = cnt[n \times r][1] + cnt[n][0]$$

Since r = 1, the order of assignments are very important.

```
long countTriplets(vector<long> arr, long r) {
    const auto n = arr.size();
    unordered_map<long, array<long, 2>> cnt;
    //cnt[a[j]][0] = |\{a[i]\}| in which i < j and
                     a[j] = a[i] * r
    //cnt[a[k]][1] = /{a[i], a[j]}/ in which
                     i < j < k and
    //a[k] = a[j] * r and a[j] = a[i] * r
    long res = 0;
    for (const auto& num : arr)
        res += cnt[num][1];
        const auto next = num * r;
        cnt[next][1] += cnt[num][0];
        ++cnt[next][0];
    }
    return res;
}
```

2.4 Fraudulent Activity Notifications

Basically we want a O(nlogn) algorithm to find median of a sequuence, when we removed the first element and add another one. So we need two binary search trees. In the first one the maximum element is the median itself and in the secon one the minimum element is the second median in case of d=2k or a value greater than median when d=2k+1. So if d=2k both of these binary search trees always have k element. When d=2k+1, the first one always has k+1 elements and the second one has k elements. Let's call them lessEqual and greaterEqual.

If both removing element and new element belong to the same tree, nothing extra is required. So we only need to remove one element and add the new one. If the removing element is from lessEqual, we must remove the minimum

element from greaterEqual and add it to lessEqual. If the removing element is from greaterEqual, we must remove the maximum element from lessEqual and add it to greaterEqual. By doing that the maximum element is lessEqual is median. In case of d=2k, the minimum element in greaterEqual is the second median. The running time of this algorithm is O(nlogn).

```
int activityNotifications(vector<int> expenditure, int d)
   multiset<int, greater<int>> lessEqual;
   multiset<int> greaterEqual;
   vector<int> init(d);
    copy(expenditure.begin(), expenditure.begin() + d,
         init.begin());
    sort(init.begin(), init.end());
   const bool isEven = (d & 1) == 0;
   int medianIndex = (d - 1) / 2;
   for (i = 0; i <= medianIndex; ++i)</pre>
       lessEqual.insert(init[i]);
   for (; i < d; ++i)
        greaterEqual.insert(init[i]);
   int res = 0;
   for (int i = d; i < expenditure.size(); ++i)</pre>
        const int median1 = *lessEqual.begin();
        if (isEven)
            const int median2 = *greaterEqual.begin();
            if (expenditure[i] >= (median1 + median2))
                ++res;
        }
        else
        {
            if (expenditure[i] >= 2 * median1)
                ++res;
        }
        const auto removed = expenditure[i - d];
        if (removed <= median1 &&
            expenditure[i] <= median1)</pre>
            lessEqual.erase(lessEqual.find(removed));
            lessEqual.insert(expenditure[i]);
        }
        else if (removed > median1 &&
                 expenditure[i] > median1)
          greaterEqual.erase(greaterEqual.find(removed));
          greaterEqual.insert(expenditure[i]);
        else if ( removed <= median1)</pre>
```

```
//For handling d=1, it should first:
    greaterEqual.insert(expenditure[i]);
    lessEqual.erase(lessEqual.find(removed));
    lessEqual.insert(*greaterEqual.begin());
    greaterEqual.erase(greaterEqual.begin());
}
else
{
    //For handling d=1, it should be first:
    lessEqual.insert(expenditure[i]);
    greaterEqual.erase(greaterEqual.find(removed));
    greaterEqual.insert(*lessEqual.begin());
    lessEqual.erase(lessEqual.begin());
}
return res;
}
```

2.5 Merge Sort: Counting Inversions

Problem Statement

We can solve it using merge sort. Suppose we have arr[left..right]. We break it into two subproblem arr[left..mid] and arr[mid+1..right]. Both of them are sorted. According to merge sort algorithm $left \leq i \leq mid$ and $mid+1 \leq j \leq right$. In other words we already put arr[left..i-1] and arr[mid+1..j-1] into their correct positions. So when arr[j] < arr[i], it means $arr[j] < arr[i] \leq arr[x]$ in which $i+1 \leq x \leq mid$. So we need to have mid-i+1 swaps.

```
long mergeSort(vector<int>& arr, int leftIndex,
               int rightIndex)
    if (leftIndex == rightIndex)
        return 0:
   long res = 0;
   int midIndex = (leftIndex + rightIndex) / 2;
   res = mergeSort(arr, leftIndex, midIndex);
   res += mergeSort(arr, midIndex + 1, rightIndex);
    vector<int> sorted(rightIndex - leftIndex + 1);
    int i, j, k;
    for (i = leftIndex, j = midIndex + 1, k = 0;
         i <= midIndex && j <= rightIndex;)</pre>
        if (arr[i] <= arr[j])</pre>
            sorted[k++] = arr[i++];
        else
            res += midIndex - i + 1;
            sorted[k++] = arr[j++];
    }
```

Chapter 3

TopCoder

3.1 SRM 428

3.1.1 The Palindrome

For another variation refer to 1.1.2. We want to add the minimum number of characters to the end of string to make it a palindrome. The straightforward approach is to try add the first i characters in reverse for all $0 \le i \le n-1$ in which n is the length of string. So we start from i=0 and check whether the string is palindrome. If it's not we check for i=1 and so on. The running time of this algorithm is $O(n^2)$.

Since $n \le 50$, this algorithm is fast. We can make it $O(n \log_2 n)$ if we use binary search tree to find the minimum i.

There is another approach. Let's assume we have string $S = s_1 s_2 \dots s_n$. We define $S' = s_n \dots s_2 s_1$. Suppose we can write string S as QP. In other words, S is the concatenation of two strings Q and P. We assume P is palindrome but

Q is not. We can make S palindrome if we convert QP to QPQ' we call this new String Z. Z is palindrome because if we reverse it we have:

Z: QPQ'Z': QP'Q'

Since we want the length of Q be minimum, we must find the maximal P:

As the previous implementation the running time is $O(n^2)$ but it's easy to convert it to $O(n \log_2 n)$ using binary search.

This implementation has a unique feature. We can convert it to an O(n) algorithm using KMP algorithm. Suppose S = QP where P is a palindrome. We want to find palidnrome postfix P which its length is maximum among all palindrome post-fixes. We need to run KMP pre-compute calculation on S' = P'Q'. Then we run KMP algorithm as if we want to find whether S' is a substring of S. Suppose we use i as an index for S and j as an index for S'. The algorithm start with i = 0 and ends when i = len(S) in which S'[0..j-1] is P' or P (since it's palindrome).

```
vector<int> calculateNext(const string& B)
{
  vector<int> next(B.length());
  next[0] = -2;
  next[1] = -1;
  int i, j;
  for (i = 2; i < B.length(); ++i)
  {
      j = next[i - 1] + 1;
      for (; j >= 0 && B[j] != B[i - 1]; j = next[j] + 1);
      next[i] = j;
  }
  return next;
}

int find(string A)
{
  const string B = string(A.rbegin(), A.rend());
```

3.1. SRM 428 27

```
const auto next = calculateNext(B);
int i, j;
for (i = 0, j = 0; i < A.length() && j < B.length();)
{
   if (A[i] == B[j])
        ++i, ++j;
   else if ((j = next[j] + 1) < 0)
   {
        //Since B.front() == A.back(), it's impossible
        //i == A.length() here:
        ++i, j = 0;
        j = 0;
   }
}
int palindromeLen = j;
return A.length() + A.length() - palindromeLen;
}</pre>
```

Appendix A

C++ refresher

A.1 std::tuple

A.1.1 std::tie

Creates a tuple of lyalue references to its arguments or instances of std::ignore

A.2 Associative containers

A.2.1 std::set

```
#include <iostream>
#include <algorithm>
#include <cassert>
#include <set>
#include <tuple>
#include <array>
int main()
  std::set<int> s({1, 2, 3, 4, 5, 6, 7});
  assert(s.count(7) == 1);
  assert(s.count(8) == 0);
  bool result;
  std::set<int>::iterator iter;
  std::tie(iter, result) = s.insert(8);
  assert(result == true && *iter == 8 && s.count(8) == 1);
  std::tie(std::ignore, result) = s.insert(9);
  assert(result = true && s.count(9) == 1);
  std::array<int, 6> a = {10, 11, 12, 13, 14, 15};
  s.insert(a.begin(), a.end());
  assert(s.count(10) == 1);
  iter = s.find(13);
  assert(iter != s.end());
```

```
iter = s.lower_bound(4);
assert(*iter == 4);
iter = s.upper_bound(4);
assert(*iter == 5);

s = {1, 2, 4, 5};
iter = s.lower_bound(3);
assert(*iter == 4);
iter = s.upper_bound(3);
assert(*iter == 4);
}
```

A.3 Inserting into a container

The following code uses most insert operations:

```
#include <iostream>
#include <vector>
#include <deque>
#include <iterator>
#include <algorithm>
template < typename Container >
void print(const Container& c)
 static int id = 1;
 std::cout << id++ << ": ";
 std::for_each(c.begin(), c.end(), [](const auto& val) {
                std::cout << val << ' ';
                });
 std::cout << std::endl;</pre>
}
int main()
 std::vector<int> base{0, 1, 2, 3, 4};
 auto v(base);
 std::vector<int> v2{100, 200, 300};
 auto i = std::next(v.begin()); // v.begin() + 1
 i = v.insert(i, v2[0]);
 i = v.insert(i, v2[1]);
 i = v.insert(i, v2[2]);
print(v); // 1: 0 300 200 100 1 2 3 4
 // Note that after
 // v.insert(i, v[0]);
 // i becomes invalid. So this is undefined behavior:
 // v.insert(i, v[1]);
 v = base;
 i = std::next(v.begin()); // v.begin() + 1
 i = v.insert(i, v2.begin(), v2.end());
 print(v); // 2: 0 100 200 300 1 2 3 4
 v = base;
```

A.4 std::vector iterators

It uses LegacyRandomAccessIterator. For more information refer to Iterator library.

```
#include <iostream>
#include <algorithm>
#include <iterator>
#include <vector>
template < typename Container >
void print(const Container& c)
  const auto m_print = [](const auto& val)
  {
   std::cout << val << ' ';
 std::for_each(c.begin(), c.end(), m_print);
  std::cout << std::endl;</pre>
//my implementation of std::reverse
template < typename Container >
void my_reverse(Container& c)
  auto left = c.begin();
  auto right = std::prev(c.end());
  for (; left < right; ++left, --right)</pre>
    std::iter_swap(left, right);
template < typename Container >
void my_reverse2(Container& c)
  auto left = 0;
  auto right = std::distance(c.begin(), std::prev(c.end()));
  for (; left < right; ++left, --right)</pre>
    std::swap(c[left], c[right]);
int main()
```

```
{
    std::vector<int> v{0, 1, 2, 3, 4};
    my_reverse(v);
    print(v); // 4 3 2 1 0
    my_reverse2(v);
    print(v); // 0 1 2 3 4
}
```

A.5 std::reverse_iterator

To understand how reverse_iterator::base works, see the following snippet. For more information refer to std::reverse_iterator.

```
int main()
{
  using iter = std::vector<int>::iterator;
  using r_iter = std::vector<int>::reverse_iterator;

std::vector<int> v {0, 1, 2, 3, 4};
  r_iter res = std::find(v.rbegin(), v.rend(), 2);
  iter base = res.base();
  assert(*res == 2 && *base == 3);
}
```

A.6 std::mismatch

```
#include <iostream>
#include <algorithm>
#include <string>
#include <cassert>
#include <vector>
#include <iterator>
bool is_palindrome(const std::string& str)
 auto res = std::mismatch(str.begin(), str.end(),
                           str.rbegin(), str.rend());
 return res.first == str.end();
}
int main()
 assert(is_palindrome("aba"));
 assert(is_palindrome("abba"));
 assert(!is_palindrome("abcd"));
 std::vector<int> v1{1, 2, 3, 4};
 std::vector<int> v2{1, 2};
 auto res = std::mismatch(v1.begin(), v1.end(),
                           v2.begin(), v2.end());
  //C++14 and above:
 assert(res.first == std::next(v1.begin(), v2.size()) &&
```

```
res.second == v2.end());
}
```

A.7 Boyer–Moore string-search algorithm

Note that unlike KMP, the worst case is still O(mn). It's implemented since C++17. Another variation is Boyer-Moore-Horspool algorithm.

A.8 std::array

Note that if you don't use, both std::array and c-style array are not initialized when you are using primitive type. So they can have arbitrary values.

```
#include <iostream>
#include <array>
#include <algorithm>
#include <cassert>
#include <cstring>
int main()
 // a1 members are not initialized:
  // std::array<int, 7> a;
  // c_a members are not initialized:
  // int c_a[7];
  //a2 members are initialized to default:
  std::array<int, 7> a1{};
  // c_a1 members are initiliazed to default:
  int c_a1[7] {};
  for (auto i = 0; i < a1.size(); ++i)</pre>
    assert(a1[i] == 0 && c_a1[i] == 0);
  a1.fill(7);
  std::fill(c_a1, c_a1 + 7, 7);
  for (auto i = 0; i < a1.size(); ++i)</pre>
```

```
assert(a1[i] == 7 && c_a1[i] == 7);

std::array<std::array<int, 2>, 3> a2{};
int c_a2[3][2] = {{1, 2}, {3, 4}};

std::memcpy(a2.data(), c_a2, sizeof(c_a2));
for (auto i = 0; i < a2.size(); ++i)
    for (auto j = 0; j < a2[0].size(); ++j)
        assert(a2[i][j] == c_a2[i][j]);

assert(a2.size() == 3);
assert(a2[0].size() == 2);

a2.fill({4, 5});
for (const auto& val : a2)
    assert(val[0] == 4 && val[1] == 5);
std::array<int, 3> a3 {1, 2};
assert(a3[0] == 1 && a3[1] == 2 && a3[2] == 0);
}
```

A.9 Initializing arrays with std::fill, memcpy and memset

Note that memset fills every byte in the buffer. If it's an array of elements bigger than 1-byte (e.g. array of integers), it does make sense to use it just for 0 or -1 (in a 32-bit signed integer -1 = 0xffffffff).

Unless it's an array of elements of 1-byte (e.g. char), it's wrong to use sizeof for std::fill. Note that the first two parameters of std::fill are two pointers. One points to the first element and the other to one after the last. On the other hand we should use sizeof for memcpy and memset.

```
#include <iostream>
#include <array>
#include <algorithm>
#include <cassert>
#include <cstring>
#include <iterator>
int main()
 std::array<int, 3> a1{};
 assert(sizeof(a1) == sizeof(int) * 3);
  std::fill(a1.begin(), a1.end(), -20);
 for (const auto& val : a1)
    assert(val == -20);
 std::array<std::array<int, 2>, 3> a2{};
 int c_a2[3][2] = {{1, 2}, {3, 4}};
 std::fill(&a2[0][0], &a2[1][0], 7);
 std::fill(&a2[1][0], &a2[2][0], 8);
  std::fill_n(&a2[2][0], 2, 9);
 for (int i = 0; i < a2.size(); ++i)
```

```
if (i == 0)
     assert(a2[i][0] == 7 && a2[i][1] == 7);
    else if (i == 1)
      assert(a2[i][0] == 8 && a2[i][1] == 8);
      assert(a2[i][0] == 9 && a2[i][1] == 9);
  std::fill(a2.begin()->begin(),
            std::next(a2.begin())->begin(),
  std::fill(std::next(a2.begin())->begin(),
            std::next(a2.begin(), 2)->begin(),
            11);
  std::fill(std::next(a2.begin(), 2)->begin(),
            std::prev(a2.end())->end(),
            12);
  for (int i = 0; i < a2.size(); ++i)
    if (i == 0)
      assert(a2[i][0] == 10 && a2[i][1] == 10);
    else if (i == 1)
     assert(a2[i][0] == 11 && a2[i][1] == 11);
      assert(a2[i][0] == 12 && a2[i][1] == 12);
  assert(sizeof(a2) == sizeof(int) * 2 * 3);
  // Note that &a2[0] is a pointer to std::array<int, 2>
  std::fill(&a2[0][0], &a2[0][0] + 2 * 3, 15);
  for (const auto& val : a2)
   assert(val[0] == 15 && val[1] == 15);
  std::memcpy(a2.data(), c_a2, sizeof(c_a2));
  for (auto i = 0; i < a2.size(); ++i)</pre>
   for (auto j = 0; j < a2[0].size(); ++j)
      assert(a2[i][j] == c_a2[i][j]);
  std::fill_n(a2.begin()->begin(), 2 * 3, 16);
  for (const auto& val : a2)
   assert(val[0] == 16 && val[1] == 16);
  std::copy_n(\&c_a2[0][0], 2 * 3, a2.begin()->begin());
  for (auto i = 0; i < a2.size(); ++i)
   for (auto j = 0; j < a2[0].size(); ++j)
      assert(a2[i][j] == c_a2[i][j]);
  std::memset(a2.data(), -1, sizeof(a2));
  for (const auto& val : a2)
    assert(val[0] == -1 && val[1] == -1);
  std::memset(a2.data(), 1, sizeof(a2));
  for (const auto& val : a2)
    assert(val[0] == 0x01010101 && val[1] == 0x01010101);
}
```