Superconductivity: Josephson junctions

Objective: This module introduces the fundamental concepts of quantum computation, covering qubits and the physical systems necessary to their practical implementation. Quantum computing requires maintaining quantum effects in large systems, which presents a challenge even with current materials or quantum circuits developed through modern manufacturing techniques.

Activity with the interface
https://qtechedu.centralesupelec.fr/EN/ex4.html

Section 1

1) What potential and Hamiltonian describes the harmonic oscillator?

2) What realistic superconducting system is described by a harmonic oscillator?

3) What are the energy levels of harmonic oscillator? How are they spaced?

4) What is the (absolute and relative) anharmonicity and how it is defined?

5) What is the anharmonicity for harmonic oscillator?

6) The washboard potential is an example of an anharmonic potential. Why is that?

7) How the energy levels of the washboard potential system are spaces?

8) The washboard potential can be approximated by a harmonic oscillator. How good is that approximation? What are its limits? When it gets poor?

9) How anharmonicities depend on the parameters of the system?

Section 2

1) What is the Josephson junction as a physical system?

2) What are Cooper pairs?

3) What is interpretation of the tunneling Hamiltonian?

$$H_J = \frac{-E_J}{2} \sum_{n \in \mathbb{Z}} |N-1\rangle\langle N| + |N+1\rangle\langle N| \quad .$$

4) Why Josephson junction also have a capacitance (kinetic) term in the Hamiltonian? How that relates to the physical structure of the junction?

5) What is DC Josephson effect?

6) What is a current-biased Josephson junction?

7) What is a Cooper-pair box and a transmon? How do they differ?

8) Is the Cooper-pair box solved in number operator representation or in the flux representation? Why?

9) How the energy levels of the Cooper-pair box depend on the biasing current? When the system is a good qubit, and when it is in a good preparation state?

10) What happens to the Cooper-pair box when tunneling parameter grows?

11) How the anharmonicities depend on the biasing current and the ratio of parameters E_J/E_C ? When the system is protected from current noise?

12) How transmon qubit can be realized as a physical system?

13) Is the transmon solved in number operator representation or in the flux representation? Why?

14) Is it a harmonic or anharmonic system?

15) How the anharmonicities depend on the biasing current and the ratio of parameters E_J/E_C ? When the system is protected from current noise?

Computational exercises after finishing the notebook:

1) The Cooper-pair box/transmon and the phase qubit are not the only physical realizations of qubits. The flux qubit is described by a Hamiltonian

$$H=4E_C\widehat{n}^2+\frac{1}{2L}\left(\frac{\Phi_0}{2\pi}\right)\widehat{\phi}^2-E_J\cos\left(\hat{\phi}-\phi_{ext}\right) \quad ,$$

where $\Phi_0=\frac{h}{2e}$ and E_C, L, E_J and ϕ_{ext} are parameters of the system. ϕ_{ext} is an externally applied flux. Usually, the flux qubit occupies the region of the parameter space where \$ $10 E_C < E_J < 10^3 E_C$ (i.e. the strong tunneling regime).

2) Plot the potential of the flux qubit as a function of the external flux. What are the two regimes that one can notice (hint: try $2\pi \times integer$ and $2\pi \times half-integer$ flux)? What are the consequences for the ground state? Which is the good *preparation state* and which is a good *qubit state*?

3) Given that the flux qubit operates in the strong tunneling regime, should it be solved in the number operator representation or the flux representation (compare with the transmon)?

4) Use the flux representation and the provided DVR solver to obtain the lowest energy wave functions and their energies.

5) Graph the energies of the states. Are they harmonic or anharmonic?

6) Graph the anharmonicity coefficients for flux qubit. How they behave? Do you think that flux qubit is sensitive to charge noise?