Trapped-Ions Oxford/Hub virtual device

VQD setup

Set the main directory as the current directory

In[217]:=

```
SetDirectory[NotebookDirectory[]];
```

Load the QuESTLink package
One may also use the off-line questlink.m file, change it to the location of the local file

In[218]:=

```
Import["https://qtechtheory.org/questlink.m"]
```

This will download a binary file **quest_link** if there is no such file found
Otherwise, use a locally-compiled that called **quest_link***

In[219]:=

```
(* Search for existing files that match the pattern "quest_link*" *)
With[{questLinkFiles = Sort@FileNames["quest_link*", {NotebookDirectory[]}]}
 ,
 If[Length[questLinkFiles] > 0,
   (* If one or more matching files are found,
   use the first one alphabetically *)
   Print["Using the existing link file: ", First@questLinkFiles];
   CreateLocalQuESTEnv[First@questLinkFiles];
   ,
   (*If no matching files are found, download the link file*)
   Print["No link file found, download quest_link"];
   CreateDownloadedQuESTEnv[];
  ];
]
```

Load the **VQD** package; must be loaded after QuESTlink is loaded

In[220]:=

```
Get["../vqd.wl"]
```

Set the default configuration of the virtual ion traps

frequency unit: ***MHz***
time unit: ***μs***

In[221]:=

```
Options[TrappedIonOxford] = {
  (* the name of trap nodes name together and the number of ions on each node *)
  Nodes → <|"Alice" → 4, "Bob" → 4|>
  ,
  (* the T1 time, exponential decay *)
  T1 → <|"Alice" → 3 * 10^9, "Bob" → 3 * 10^9  |>
  ,
  (* the T2* time, Gaussian decay *)
  T2s → <|"Alice" → 10^5, "Bob" → 10^5  |>
  ,
  (* Duration for moving operations: Split, Combine, and physical swap *)
  DurMove →
   <|
    "Alice" → <|Shutl → 25, Splz → 50, Comb → 50, SWAPLoc → 10 |>,
    "Bob" → <|Shutl → 25, Splz → 50, Comb → 50, SWAPLoc → 10 |>
    |>
  ,
  (* fidelity and duration of initialisation on each qubit;
  the initialisation is done simultaneously on all ions *)
  FidInit → <|"Alice" → 0.9999, "Bob" → 0.9998|>
  ,
  DurInit → <|"Alice" → 20, "Bob" → 20|>
  ,
  (* readout duration *)
  DurRead → <|"Alice" → 50, "Bob" → 50|>
  ,
  (* Symmetric bit-flip eror during readout  *)
  ProbBFRead → <|"Alice" → 10^-3, "Bob" → 10^-3|>
  ,
  (*Fidelity of single x- and y- rotations;
  z-rotation is instaneous (noiseless, virtual)*)
  FidSingleXY → <|"Alice" → 0.99999, "Bob" → 0.99999|>
```

```
 ,
 (*fraction of depolarising:dephasing noise of the x- and y- rotations *)
 EFSingleXY → <|"Alice" → {1, 0}, "Bob" → {1, 0}|>
 ,
 (* Rabi frequency on single rotations *)
 RabiFreq → <|"Alice" → 10, "Bob" → 10 |>
 ,
 (* Frequency of CZ operation *)
 FreqCZ → <|"Alice" → 0.1, "Bob" → 0.1|>
 ,
 (* Fidelity of CZ operation *)
 FidCZ → <|"Alice" → 0.999, "Bob" → 0.999|>
 ,
 (* fraction of two-
  qubit depolarising:dephasing error after entanglement distillation *)
 EFCZ → <|"Alice" → {0.1, 0.9}, "Bob" → {0.1, 0.9}|>
 ,
 (* rate of heralded remote entanglement generation *)
 FreqEnt → 0.1
 ,
 (* fidelity of the raw bell pair *)
 FidEnt → 0.95
 ,
 (* fraction of noise on the obtained raw bell pair,
 2-qubit depolarising:dephasing *)
 EFEnt → {0.1, 0.9}
 ,
 (* Switch on/off the standard passive noise: decays T1 and T2*  *)
 StdPassiveNoise → True
};
```

Elementary guide

Native gates

Operators

Initialisation and readout

$\text{Init}_{1,2,\ldots,n}$[node], Read_q[node]

Single-qubit gates

Rx_q[node, θ], Ry_q[node, θ], Rz_q[node, θ]

Two-qubit gates
$CZ_{q1,q2}$[node]

Remote gates (create a remote Bell-pair)
Ent[node1, node2]

Physical moves/shuttling
$SWAPLoc_{q1,q2}$[node], $Splz_{q1,q2}$[node, zone_destination], $Comb_{q1,q2}$[node], $Comb_{q1,q2}$[node, zone_destination]

others: doing nothing
$Wait_q$[duration]

Zone and allowed operations
Zone 1 : Shutl, Init, Read, Splz, Comb, SWAPLoc
Zone 2 : Shutl, Init, Read, Splz, Comb, SWAPLoc, Rx, Ry, Rz, CZ
Zone 3 : Shutl, Init, Read, Splz, Comb, SWAPLoc, Rx, Ry, Rz, CZ
Zone 4 : Shutl, Ent

Basic operations to create remote entangled pair

Convenient modules

In[222]:=

```
(* Transformation to the Bell basis for plotting *)
mat2BellBasis[m_] := With[
  {p = (1 / √2) * {{1, 1, 0, 0}, {0, 0, 1, -1}, {0, 0, 1, 1}, {1, -1, 0, 0}},
   pinv = (1 / √2) * {{1, 0, 0, 1}, {1, 0, 0, -1}, {0, 1, 1, 0}, {0, -1, 1, 0}}},
  pinv.m.p]
```

In[223]:=

```
(* Plot options *)
chartbell[label_ : ""] := {ImageSize → 200,
   BarSpacing → 0.1`, ColorFunction → Function[{height}, getcolor[height]],
   ChartElementFunction → "Cube", ChartStyle → EdgeForm[Thick],
   PlotTheme → "Business", Ticks → {{{1, "Φ+"}, {2, "Φ-"}, {3, "Ψ+"}, {4, "Ψ-"}},
     {{1, "Φ+"}, {2, "Φ-"}, {3, "Ψ+"}, {4, "Ψ-"}}, Automatic},
   TicksStyle → Directive[Bold, 14, FontFamily → "FreeSerif"],
   Epilog → Inset[Style[label, 16, Bold, FontFamily → "Serif"], ImageScaled[{.3, .9}]],
   PlotRange → {{0.5, 4.5}, {0.5, 4.5}, {0, 1}},
   LabelingFunction → (Placed[Rotate[
         Which[
          # ≥ 0.5, Style[NumberForm[#, 5], 13, GrayLevel[0.82]],
          (# ≥ 0.001) && (# < 0.5), Style[NumberForm[#, 2], 10, Black],
          True, Style[ScientificForm[#, 2], 10, Black]
         ]
         , 90 Degree], If[#1 ≥ 0.5, Center, Above]] &),
   ViewAngle → All,
   FaceGrids → None,
   BoxRatios → {1, 1, 0.6}, Axes → {True, True, False}
  };
getcolor[height_] := (If[height ≤ 0.5,
   ColorData["IslandColors"][1 + 0.1 Log10@height], ColorData["RedBlueTones"][height]])
```

Demo

Shuttle the ions around and perform an entanglement : initial and final configuration is shown.

This will show the total scheduling, noise operation, and the final form in the simulation.
Set the **MapQubits->False** and **ReplaceAliases->True** if you intended to do the density matrix simulation.
InsertCircuitNoise will update the state of **dev**

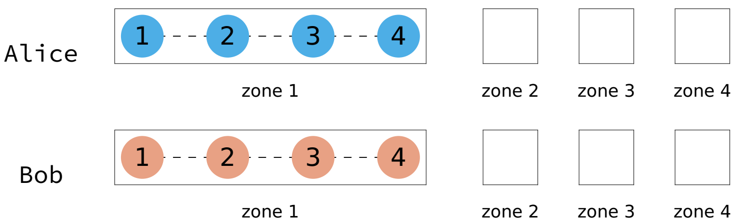
In[225]:=

```
dev = TrappedIonOxford[];
```

In[226]:=

```
dev[ShowNodes]
```

Out[226]=



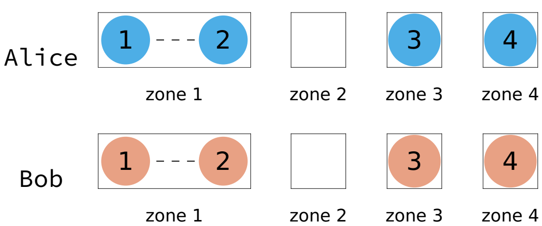
In[227]:=

```
circ = {Init_{1,2,3,4}["Alice"], Splz_{2,3}["Alice"], Splz_{2,3}["Bob"],
    Shutl_{4,3}["Alice", 3], Shutl_{4,3}["Bob", 3], Splz_{3,4}["Alice"], Splz_{3,4}["Bob"],
    Shutl_{4}["Alice", 4], Shutl_{4}["Bob", 4], Ent_{4,4}["Alice", "Bob"]};
```

In[228]:=

```
circ2 = CircTrappedIons[circ, dev, MapQubits → False];
noisycirc = InsertCircuitNoise[circ2, dev, ReplaceAliases → True];
dev[ShowNodes]
```

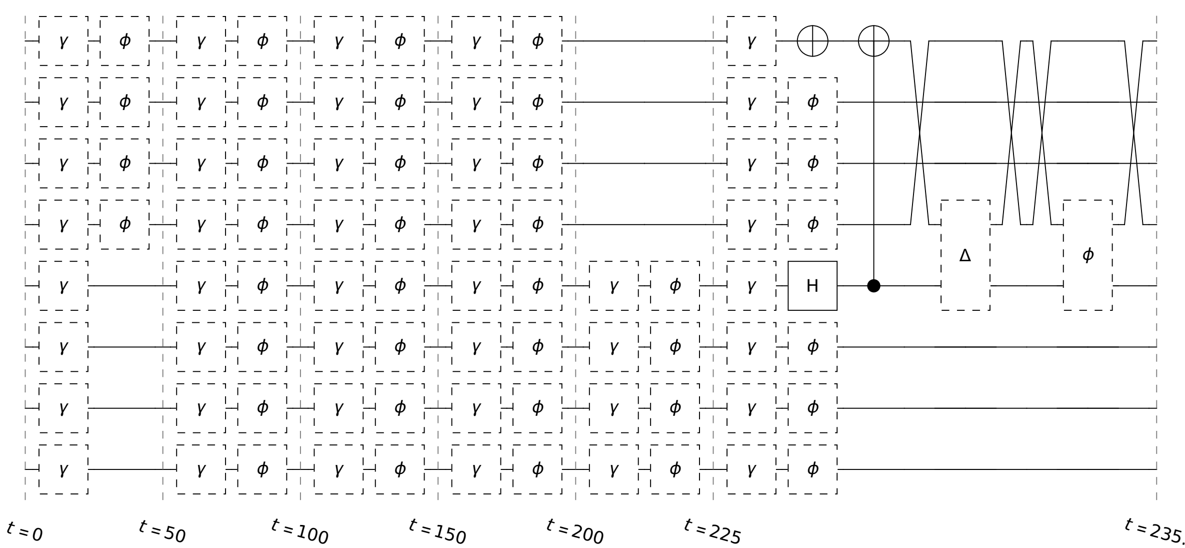
Out[230]=



In[231]:=

```
(* the actual circuit+noise that is run on the simulation *)
DrawCircuit[noisycirc, 8]
```

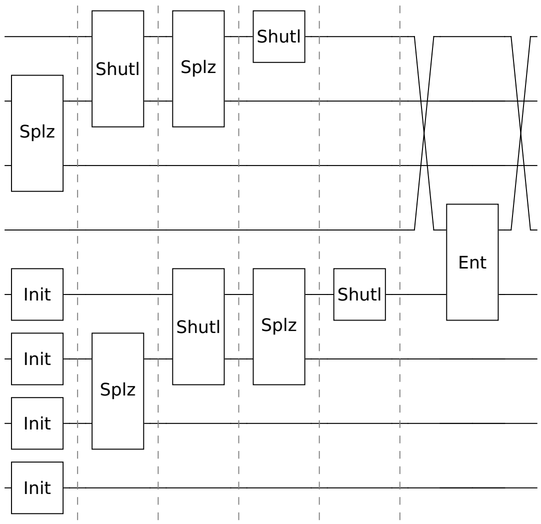
Out[231]=



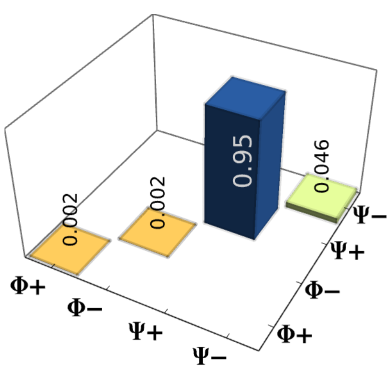
Check the arrangement of the circuit in the total density matrix. Here is completely serial within a node.

It does not update the device state, thus we cannot check the final position of ions.
The first half (from below) belongs to Alice and another half belongs to Bob

In[232]:=
```
DrawCircuit@CircTrappedIons[circ, dev, MapQubits → True]
```
Out[232]=



In[233]:=
```
DestroyAllQuregs[];
ρ = CreateDensityQureg[8];
```
In[235]:=
```
ApplyCircuit[ρ, ExtractCircuit@noisycirc];
PlotDensityMatrix[
 mat2BellBasis@PartialTrace[ρ, 0, 1, 2, 4, 5, 6], Sequence @@ chartbell[""]]
```
Out[236]=



Paper supplement

4-ions entanglement distillation on each node for up to 3 rounds

0 = dephasing distillation

1 = bitflip distillation

The modules

In[237]:=

```
(* Bit-Flip distillation operation: The CNOT equivalent *)
cx = {Ry1[-π / 2], C0[Z1], Ry1[π / 2]};
(* Phase-flip distillation: Alice and Bob *)
pfa = {Ry0[-π / 2], C0[Z1], Rx1[π], Ry1[-π / 2]};
pfb = {Ry0[π / 2], C0[Z1], Ry1[π / 2]};
```

In[240]:=

```
heraldout::usage = "heraldout[outputs]. Check
    if all outputs 00 or 11 in all measurement outcomes.";
heraldout[out_] := With[{fout = Flatten@out},
  If[Length@fout > 1, And @@ (Equal @@@ Partition[fout, 2]), True]]
```

In[242]:=

```
distcirc[p_, q_] := <|
   (*dephasing distillation sequence*)
   0 → Sequence @@ {Ryp["Alice", π / 2], Ryp["Bob", π / 2], CZp,q["Alice"],
      CZp,q["Bob"], Ryq["Alice", π / 2], Rxp["Alice", π], Ryq["Bob", π / 2]},
   (*bitflip distillation sequence*)
   1 → Sequence @@ {Ryq["Alice", -π / 2], Ryq["Bob", -π / 2],
      CZp,q["Alice"], CZp,q["Bob"], Ryq["Alice", π / 2], Ryq["Bob", π / 2]}
   |>;

(*
Distillation on 4 ions on each nodes, up to 3 rounds of distillation
*)
DistCircTrappedIons4::usage =
  "DistillationOnTrappedIons4[sequence]. Distillation on 4-ions
    of two nodes. Works up to 3 rounds. Sequence contains 0
    (dephasing) or 1 (bit-flip). Qubit 4 is always the final qubit.";
DistCircTrappedIons4[sequence_ : {}] := Module[{circ, nrounds},
   nrounds = Length@sequence;
   (* raw entangled pairs *)
   circ = {Splz3,4["Alice"], Splz3,4["Bob"],
     Shutl4["Alice", 4], Shutl4["Bob", 4], Ent4,4["Alice", "Bob"]};
   If[nrounds ≥ 1,
    circ = Join[circ, {Shutl4["Alice", 1], Shutl4["Bob", 1], Comb3,4["Alice"],
       Comb3,4["Bob"], SWAPLoc3,4["Alice"], SWAPLoc3,4["Bob"], Splz4,3["Alice"],
```

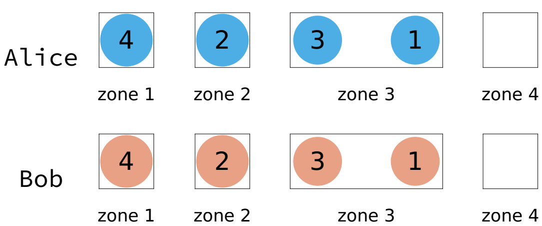
```
      Splz_{4,3}["Bob"], Shutl_3["Alice", 4], Shutl_3["Bob", 4], Ent_{3,3}["Alice", "Bob"],
      Splz_{2,4}["Alice"], Splz_{2,4}["Bob"], Shutl_4["Alice", 2], Shutl_4["Bob", 2],
      Shutl_3["Alice", 2], Shutl_3["Bob", 2], Comb_{4,3}["Alice"], Comb_{4,3}["Bob"],
      distcirc[4, 3][sequence[[1]]], Splz_{4,3}["Alice"], Splz_{4,3}["Bob"],
      Shutl_3["Alice", 3], Shutl_3["Bob", 3], Read_3["Alice"], Read_3["Bob"]
     }
   ]
];
If[nrounds ≥ 2,
  circ = Join[circ, {Shutl_3["Alice", 4], Shutl_3["Bob", 4], Ent_{3,3}["Alice", "Bob"],
      Splz_{1,2}["Alice"], Splz_{1,2}["Bob"], Shutl_{1,2}["Alice", 2], Shutl_{1,2}["Bob", 2],
      Comb_{2,4}["Alice"], Comb_{2,4}["Bob"], SWAPLoc_{2,4}["Alice"], SWAPLoc_{2,4}["Bob"],
      Shutl_3["Alice", 2], Shutl_3["Bob", 2], Comb_{2,3}["Alice"], Comb_{2,3}["Bob"],
      SWAPLoc_{2,3}["Alice"], SWAPLoc_{2,3}["Bob"], Splz_{4,3}["Alice"], Splz_{4,3}["Bob"],
      Shutl_{3,2}["Alice", 3], Shutl_{2,3}["Bob", 3], Splz_{3,2}["Alice"], Splz_{3,2}["Bob"],
      Shutl_2["Alice", 4], Shutl_2["Bob", 4], Ent_{2,2}["Alice", "Bob"],
      Shutl_2["Alice", 3], Shutl_2["Bob", 3], Comb_{3,2}["Alice"], Comb_{3,2}["Bob"],
      distcirc[3, 2][sequence[[1]]], Splz_{3,2}["Alice"], Splz_{3,2}["Bob"], Shutl_3["Alice", 2],
      Shutl_3["Bob", 2], Read_2["Alice"], Read_2["Bob"], Comb_{4,3}["Alice"],
      Comb_{4,3}["Bob"], distcirc[4, 3][sequence[[2]]], Splz_{4,3}["Alice"], Splz_{4,3}["Bob"],
      Shutl_{1,4}["Alice", 1], Shutl_{1,4}["Bob", 1], Read_3["Alice"], Read_3["Bob"]}
   ]
];
If[nrounds ≥ 3,
  circ =
   Join[circ, {Comb_{1,4}["Alice"], Comb_{1,4}["Bob"], SWAPLoc_{1,4}["Alice"], SWAPLoc_{1,4}["Bob"],
      Shutl_2["Alice", 4], Shutl_2["Bob", 4], Ent_{2,2}["Alice", "Bob"], Shutl_2["Alice", 2],
      Shutl_2["Bob", 2], Comb_{3,2}["Alice"], Comb_{3,2}["Bob"], SWAPLoc_{3,2}["Alice"],
      SWAPLoc_{3,2}["Bob"], Splz_{2,3}["Alice"], Splz_{2,3}["Bob"], Shutl_3["Alice", 4],
      Shutl_3["Bob", 4], Ent_{3,3}["Alice", "Bob"] , Shutl_3["Alice", 2], Shutl_3["Bob", 2],
      Comb_{2,3}["Alice"], Comb_{2,3}["Bob"], distcirc[2, 3][sequence[[1]]], Splz_{2,3}["Alice"],
      Splz_{2,3}["Bob"], Shutl_3["Alice", 3], Shutl_3["Bob", 3], Read_3["Alice"], Read_3["Bob"],
      Shutl_3["Alice", 4], Shutl_3["Bob", 4], Ent_{3,3}["Alice", "Bob"], Splz_{4,1}["Alice"],
      Splz_{4,1}["Bob"], Shutl_1["Alice", 2], Shutl_1["Bob", 2], Comb_{1,2}["Alice"],
      Comb_{1,2}["Bob"], SWAPLoc_{1,2}["Alice"], SWAPLoc_{1,2}["Bob"], Splz_{2,1}["Alice"],
      Splz_{2,1}["Bob"], Shutl_1["Alice", 3], Shutl_1["Bob", 3], Shutl_3["Alice", 3],
      Shutl_3["Bob", 3], Comb_{1,3}["Alice"], Comb_{1,3}["Bob"], SWAPLoc_{1,3}["Alice"],
      SWAPLoc_{1,3}["Bob"], Splz_{3,1}["Alice"], Splz_{3,1}["Bob"], Shutl_1["Alice", 4],
```

```
            Shutl_1["Bob", 4], Ent_{1,1}["Alice", "Bob"], Shutl_1["Alice", 3], Shutl_1["Bob", 3],
            Comb_{3,1}["Alice"], Comb_{3,1}["Bob"], distcirc[3, 1][sequence[[1]]], Splz_{3,1}["Alice"],
            Splz_{3,1}["Bob"], Shutl_3["Alice", 2], Shutl_3["Bob", 2], Read_1["Alice"],
            Read_1["Bob"], Comb_{2,3}["Alice"], Comb_{2,3}["Bob"], distcirc[2, 3][sequence[[2]]],
            Splz_{2,3}["Alice"], Splz_{2,3}["Bob"], Shutl_2["Alice", 1], Shutl_2["Bob", 1],
            Read_3["Alice"], Read_3["Bob"], Shutl_3["Alice", 3], Shutl_3["Bob", 3],
            Comb_{4,2}["Alice"], Comb_{4,2}["Bob"], Shutl_{4,2}["Alice", 2], Shutl_{4,2}["Bob", 2],
            distcirc[4, 2][sequence[[3]]], Splz_{4,2}["Alice"], Splz_{4,2}["Bob"],
            Shutl_4["Alice", 1], Shutl_4["Bob", 1], Read_2["Alice"], Read_2["Bob"]
           }
          ]
        ];
        circ
      ];
```

In[245]:=

```
circ = DistCircTrappedIons4[{0, 0, 0}];
dev = TrappedIonOxford[];
circ2 = CircTrappedIons[circ, dev, MapQubits → False];
noisycirc = InsertCircuitNoise[circ2, dev, ReplaceAliases → True];
dev[ShowNodes]
```

Out[249]=



Play around with the parameters

In[250]:=

```
TIonDist::usage = "TIonDist[ρ,sequence:{},device_option:{}].
    Distillation simulation on trapped ions.";
TIonDist[ρ_, sequence_ : {}, devoptions_ : {}] := Module[
  {dev, circ, noisysch, noisycirc, nions},
  dev = TrappedIonOxford[Sequence @@ devoptions];
  nions = Min[Length /@ Flatten /@ Values@Values@dev[Nodes]];
  circ = DistCircTrappedIons4[sequence];
  noisysch = InsertCircuitNoise[
    CircTrappedIons[circ, dev, MapQubits → False], dev, ReplaceAliases → True];
  noisycirc = ExtractCircuit@noisysch;
  While[Not@heraldout@ApplyCircuit[ρ, noisycirc]];
  noisysch
 ]
seqlabel[l_List] := With[{conv = <|0 → "ph", 1 → "bf"|>},
  If[Length@l > 0, StringRiffle[conv /@ l, ","], "raw"]]
```

In[253]:=

```
DestroyAllQuregs[];
ρ8 = CreateDensityQureg[8];
```

In[255]:=

```
plotρ8[label_, raster_ : False] :=
 With[{plot = PlotDensityMatrix[Chop@mat2BellBasis[PartialTrace[ρ8, 0, 1, 2, 4, 5, 6]],
     ViewPoint → {π, -2 π, π / 2}, Sequence @@ chartbell[label]]},
  If[raster,
   Rasterize[plot, RasterSize → Full],
   raster
  ]
 ]
```

In[256]:=

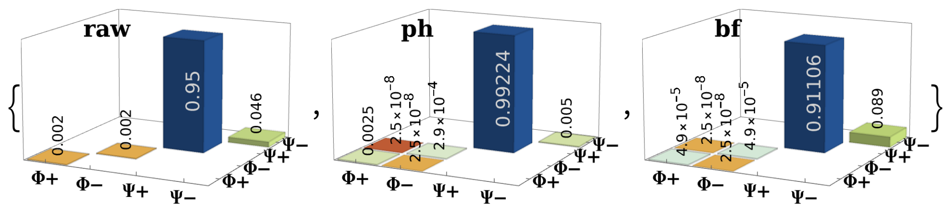
```
devoptions = {};
```

In[257]:=

```
Print["Round 0-1"]
round1 = {};
plot1 = (AppendTo[round1, {TIonDist[ρ8, #, devoptions], seqlabel@#}];
    plotρ8[seqlabel@#, True]) & /@ {{}, {0}, {1}}
```

Round 0-1

Out[259]=

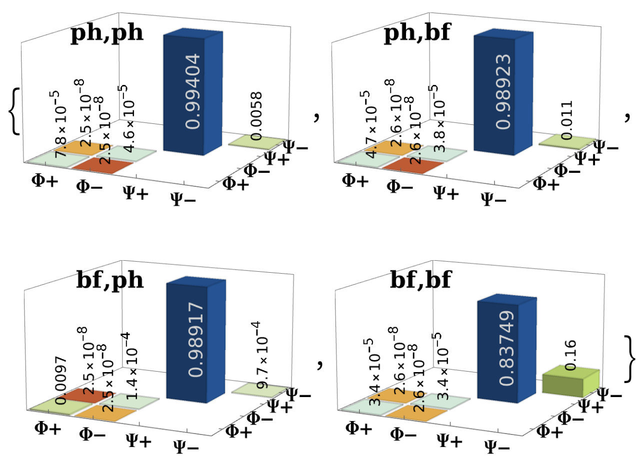


In[260]:=

```
Print["Round 2"]
round2 = {};
plot2 = (AppendTo[round2, {TIonDist[ρ8, #, devoptions], seqlabel@#}];
    plotρ8[seqlabel@#, True]) & /@ {{0, 0}, {0, 1}, {1, 0}, {1, 1}}
```

Round 2

Out[262]=

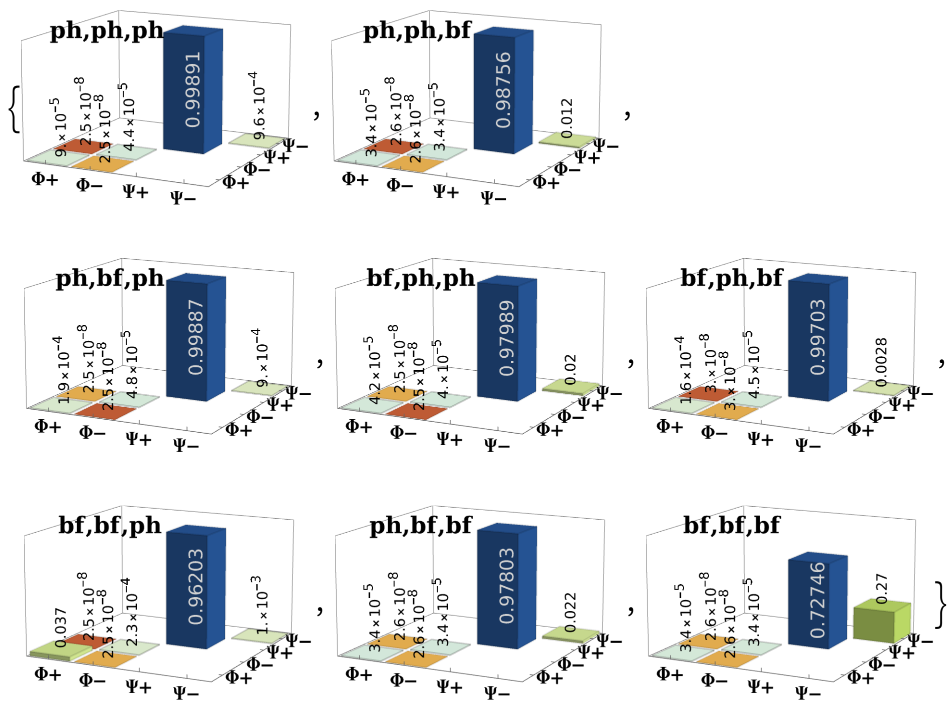


In[263]:=

```
Print["Round 3"]
round3 = {};
plot3 = (AppendTo[round3, {TIonDist[ρ8, #, devoptions], seqlabel@#}];
    plotρ8[seqlabel@#, True]) & /@
  {{0, 0, 0}, {0, 0, 1}, {0, 1, 0}, {1, 0, 0}, {1, 0, 1}, {1, 1, 0}, {0, 1, 1}, {1, 1, 1}}
```

Round 3

Out[265]=

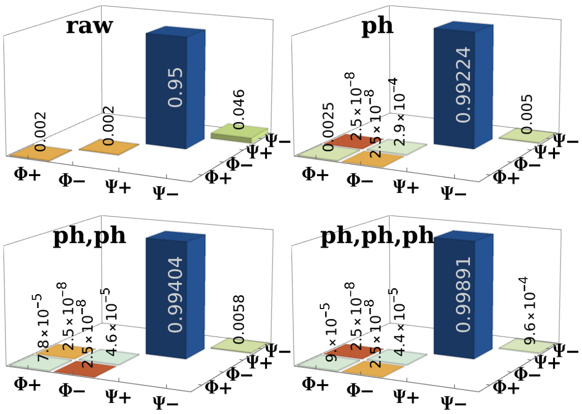


The best-two strategy for the virtual device

In[266]:=

```
Grid[{{plot1[[1]], plot1[[2]]}, {plot2[[1]], plot3[[1]]}}, Spacings → 0.]
(*
Export["dist_best.pdf",%]
*)
```

Out[266]=



In[267]:=

```
(*Varying the parameters and take the best fidelity among all trials up to 3-
 rounds of distillation*)
fullseq = {{}, {0}, {1}, {0, 0}, {0, 1}, {1, 0}, {1, 1}, {0, 0, 0},
   {0, 0, 1}, {0, 1, 0}, {1, 0, 0}, {1, 0, 1}, {1, 1, 0}, {0, 1, 1}, {1, 1, 1}};
varyParams[ρ_, func_, vars_, sequences_ : fullseq] :=
 Module[{newfid, fid, bestfid, results},
  Table[
        results = {seqlabel@#, (TIonDist[ρ, #, func[fid]];
         (mat2BellBasis@PartialTrace[ρ, 0, 1, 2, 4, 5, 6])[[3, 3]] // Re)} & /@ sequences;
       bestfid = First@results[[Ordering[results[[All, 2]], -1]]];
       {fid, Sequence @@ bestfid}
   , {fid, vars}]
 ]
```

In[269]:=

```
fids = N[(1 - 1.5^-#)] & /@ Range[6, 35, 0.5]
```

Out[269]=

```
{0.912209, 0.928319, 0.941472, 0.952212, 0.960982, 0.968142, 0.973988, 0.978761,
 0.982658, 0.985841, 0.988439, 0.99056, 0.992293, 0.993707, 0.994862, 0.995805,
 0.996575, 0.997203, 0.997716, 0.998135, 0.998478, 0.998757, 0.998985,
 0.999171, 0.999323, 0.999448, 0.999549, 0.999632, 0.999699, 0.999754, 0.9998,
 0.999836, 0.999866, 0.999891, 0.999911, 0.999927, 0.999941, 0.999951,
 0.99996, 0.999968, 0.999974, 0.999978, 0.999982, 0.999986, 0.999988,
 0.99999, 0.999992, 0.999994, 0.999995, 0.999996, 0.999997, 0.999997,
 0.999998, 0.999998, 0.999998, 0.999999, 0.999999, 0.999999, 0.999999}
```

In[270]:=

```
fsxy = {FidSingleXY → <|"Alice" → #, "Bob" → #|>} &;
fcz = {FidCZ → <|"Alice" → #, "Bob" → #|>} &;
fent = {FidEnt → #} &;
fsxycz =
  {FidSingleXY → <|"Alice" → #, "Bob" → #|>, FidCZ → <|"Alice" → #, "Bob" → #|>} &;
fsxyent = {FidSingleXY → <|"Alice" → #, "Bob" → #|>, FidEnt → #} &;
fczent = {FidCZ → <|"Alice" → #, "Bob" → #|>, FidEnt → #} &;
```

```
ressxy = varyParams[ρ8, fsxy, fids];
rescz = varyParams[ρ8, fcz, fids];
resent = varyParams[ρ8, fent, fids];
rsxycz = varyParams[ρ8, fsxycz, fids];
rsxyent = varyParams[ρ8, fsxyent, fids];
rczent = varyParams[ρ8, fczent, fids];
fidvars = {"Rx,Ry" -> ressxy, "CZ" -> rescz, "Ent" -> resent, "Rx,Ry,CZ" -> rsxycz, "Rx,Ry,Ent" -> rsxyent,
"CZ,Ent" -> rczent};
```

In[276]:=

```
(* load pre-run data, otherwise run the code above to reproduce the data *)
fidvars << "supplement/DistillationOnTrappedIons/fidvars.mx";
```

In[277]:=

```
fidvars // Keys
```

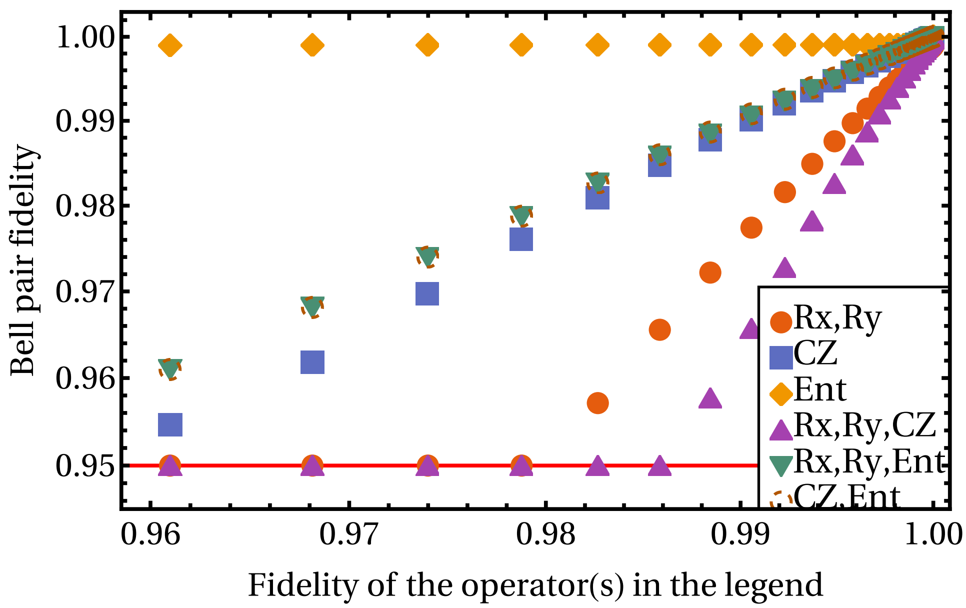
Out[277]=

```
{Rx,Ry, CZ, Ent, Rx,Ry,CZ, Rx,Ry,Ent, CZ,Ent}
```

In[278]:=

```
fplot1 = ListLogLogPlot[#[[All, {1, 3}]][[5 ;;]] & /@ Values[fidvars],
  PlotRange → All, Frame → True, FrameStyle → Directive[Black, Thick],
  PlotMarkers → {Automatic, 12}, BaseStyle → {17, FontFamily → "Sans Serif"},
  AspectRatio → 0.6, PlotTheme → "Scientific",
  FrameLabel → {"Fidelity of the operator(s) in the legend", "Bell pair fidelity"},
  ImageSize → 500, Joined → Join[ConstantArray[False, Length@fidvars],
    ConstantArray[True, Length@fidvars]], PlotStyle → Dashed, PlotLegends →
   Placed[LineLegend[{"Rx,Ry", "CZ", "Ent", "Rx,Ry,CZ", "Rx,Ry,Ent", "CZ,Ent"},
     Spacings → 0., LegendFunction → (Framed[#, FrameStyle → (Antialiasing → False),
         FrameMargins → 0, Background → White] &)], {0.89, 0.2}],
  GridLines → {{}, {0.95}}, GridLinesStyle → {{{Thick, Red}}, {}}]
```

Out[278]=



In[279]:=

```
(*Export["tions_vars.pdf",fplot1]*)
```

Timing and time profiling on each operations

In[280]:=

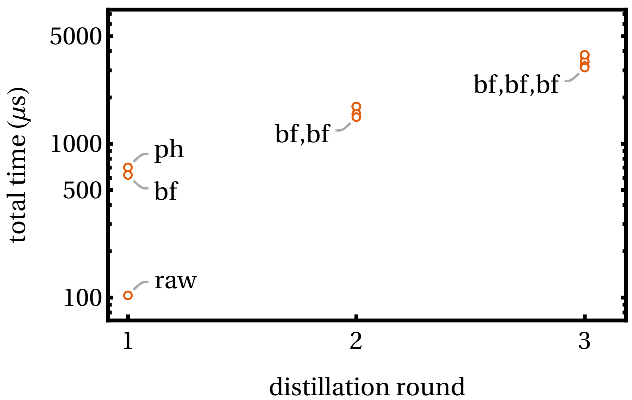
```
totaltime = Join[
   {1, Last[#[[1]][[All, 1]]]} → #[[2]] & /@ round1,
   {2, Last[#[[1]][[All, 1]]]} → #[[2]] & /@ round2,
   {3, Last[#[[1]][[All, 1]]]} → #[[2]] & /@ round3
  ];
```

Plot all the total time for all possible configuration of the distillationsf or up to 3 rounds

In[281]:=

```
ListPlot[totaltime, Frame → True, PlotTheme → "Scientific",
 FrameLabel → {"distillation round", "total time (μs)"},
 ScalingFunctions → "Log", FrameTicks → {{Automatic, Automatic}, {{1, 2, 3}, None}},
 FrameStyle → Directive[Black, Thick], BaseStyle → {13, FontFamily → "Sans Serif"},
 PlotMarkers → {"OpenMarkers", Small}, ImageSize → 330, AspectRatio → 0.6]
(*Export["tions_totaltime.pdf",%]*)
```

Out[281]=



In[282]:=

```
totaltime
```

Out[282]=

```
{{1, 103.301} → raw, {1, 701.989} → ph, {1, 626.989} → bf,
 {2, 1746.91} → ph,ph, {2, 1746.91} → ph,bf, {2, 1565.8} → bf,ph,
 {2, 1490.8} → bf,bf, {3, 3765.84} → ph,ph,ph, {3, 3715.84} → ph,ph,bf,
 {3, 3787.1} → ph,bf,ph, {3, 3424.89} → bf,ph,ph, {3, 3424.89} → bf,ph,bf,
 {3, 3203.79} → bf,bf,ph, {3, 3787.1} → ph,bf,bf, {3, 3128.79} → bf,bf,bf}
```

Create time profile on each operator

In[283]:=

```
SetAttributes[getTime, HoldAll]
getTime[gate_, lopt_] := Module[{g, node, moves, sgate, θ, opt = Association[lopt]},
  {g, node} = gate /. gg_[n_, ___] :→ {gg, n};
  moves = {Splz, Comb, SWAPLoc, Shutl};
  sgate = {Rx, Ry};
  Which[
   MemberQ[moves, g],
   opt[DurMove][node][g]
   ,
   MemberQ[sgate, g],
   θ = gate /. {_[_, t_] :→ t};
   Abs[θ] / opt[RabiFreq][node]
   ,
   g === CZ,
   π / opt[FreqCZ][node]
   ,
   g === Ent,
   π / opt[FreqEnt],
   g === Init,
   opt[DurInit][node],
   g === Read,
   opt[DurRead][node],
   True,
   0
  ]
 ]
```

In[285]:=

```
timeProfile[circ_, opt_, keys_ : {}] := Module[{time, gid, nodes, node, tkeys},
   nodes = Keys[<|opt|>[Nodes]];
   tkeys = If[Length@keys < 1, DeleteDuplicates[circ /. g__[n_, ___] :> g], keys];
   time = <|# → AssociationThread[tkeys → 0] & /@ nodes|>;
   Table[
    {gid, node} = gate /. g__[n_, ___] :> {g, n};
    If[gid === Ent,
     time[#][gid] += getTime[gate, opt] & /@ (gate /. Ent_[n1_, n2_] :> {n1, n2})
     ,
     time[node][gid] += getTime[gate, opt]
    ];
    , {gate, circ}];
   Association@Table[
     n → KeyDrop[time[n], Wait],
     {n, nodes}]
  ]
```

In[286]:=

```
countProfile[circ_, opt_, keys_ : {}] := Module[{count, gid, nodes, node, tkeys},
   nodes = Keys[<|opt|>[Nodes]];
   tkeys = If[Length@keys < 1, DeleteDuplicates[circ /. g__[n_, ___] :> g], keys];
   count = <|# → AssociationThread[tkeys → 0] & /@ nodes|>;
   Table[
    {gid, node} = gate /. g__[n_, ___] :> {g, n};
    If[gid === Ent,
     count[#][gid] += 1 & /@ (gate /. Ent_[n1_, n2_] :> {n1, n2}),
     count[node][gid] += 1
    ];
    , {gate, circ}];
   Association@Table[n → KeyDrop[count[n], Wait], {n, nodes}]
  ]
```

In[287]:=

```
timeprofileplot[node_, data_, keys_, title_, legend_ : True, opt_ : {}] :=
   BarChart[data[[All, 2]][[All, node]],
    ChartLabels → {Rotate[#, -π / 3] & /@ data[[All, 1]], None}, Sequence @@ opt,
    Frame → True, FrameStyle → Directive[Black, Thick], ChartLayout → "Percentile",
    ChartStyle → "ThermometerColors", ImageSize → 330, BarSpacing → {1, 0.3},
    BaseStyle → {12, FontFamily → "Times"}, FrameLabel → {{None, None}, {None, title}},
    If[legend, ChartLegends → Placed[keys, Right], Sequence @@ {}], AspectRatio → 1];
```

In[288]:=

```
keys = {Splz, Comb, Shutl, SWAPLoc, Ent, CZ, Ry, Rx, Read};
```

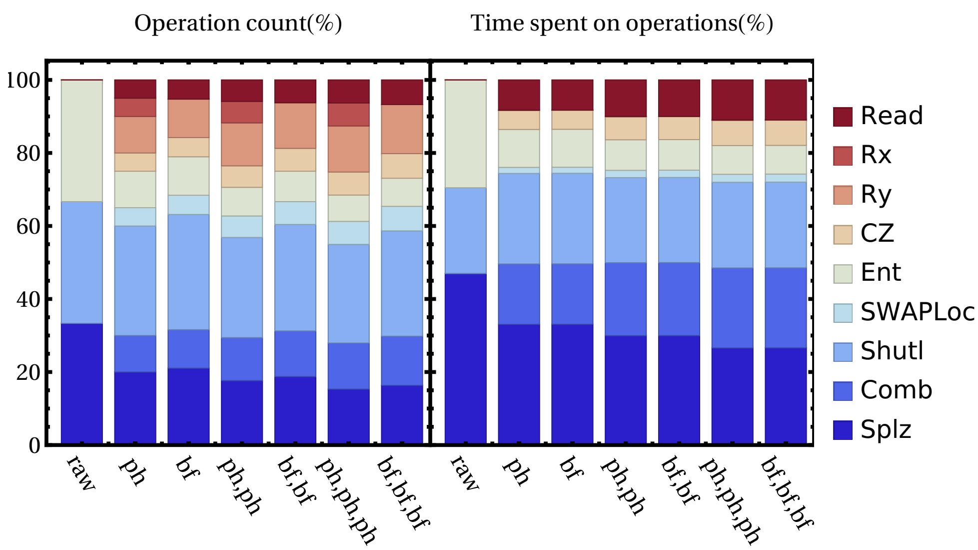
In[289]:=

```
datatime = {seqlabel[#],
      timeProfile[DistCircTrappedIons4[#], Options@TrappedIonOxford, keys]} & /@
   {{}, {0}, {1}, {0, 0}, {1, 1}, {0, 0, 0}, {1, 1, 1}};
datacount = {seqlabel[#],
      countProfile[DistCircTrappedIons4[#], Options@TrappedIonOxford, keys]} & /@
   {{}, {0}, {1}, {0, 0}, {1, 1}, {0, 0, 0}, {1, 1, 1}};
plot1 = timeprofileplot["Alice", datacount, keys, "Operation count(%)",
   False, {ImagePadding → {{20, 0}, {55, 30}}, ImageSize → 220}];
plot2 = timeprofileplot["Alice", datatime, keys, "Time spent on operations(%)",
   True, {ImagePadding → {{0, 0}, {55, 30}}, ImageSize → 200}];
```

In[293]:=

```
Grid[{{plot1, plot2}}, Spacings → 0]
(*Export["tions_timeprofile.pdf",%]*)
```

Out[293]=



Print the step-by-step distillation process + animation

In[294]:=

```
AnimateDistillation::usage = "AnimateDistillation[sequence, device_options]";
AnimateDistillation[sequence_ : {}, devoptions_ : {}] := Module[
  {dev, circ, ccirc, show = {}, format, noisy, fulltime, step = 1, nions},
  format[zones_, noise_, t_, s_] := Framed[
    Row@{Column[{"(" <> ToString[s] <> ")  t=" <> ToString[t, TraditionalForm] <> "μs",
        Rasterize[zones, RasterSize → 500, ImageSize → 200]},
       Center], Pane[noise, 280]}];
  dev = TrappedIonOxford[Sequence @@ devoptions];

  nions = Min[Length /@ Flatten /@ Values@Values@dev[Nodes]];
  circ = DistCircTrappedIons4[sequence];
  ccirc = CircTrappedIons[circ, dev, MapQubits → False];
   fulltime = ToString[N@#, FormatType → TraditionalForm] & /@
    (InsertCircuitNoise[ccirc, TrappedIonOxford[Sequence @@ devoptions]][[All, 1]] +
      OptionValue[TrappedIonOxford, DurInit]["Alice"]);
  AppendTo[show, format[dev[ShowNodes],
    DrawCircuit[{Init# & /@ Range[0, 5], Damp#[1.] & /@ Range[0, 5]}], "0", step++]];
  Table[
   noisy = InsertCircuitNoise[ccirc[[c]], dev];
   AppendTo[show,
    format[dev[ShowNodes],
     DrawCircuit[Flatten@noisy[[All, 2]], dev[NumTotalQubits]], fulltime[[c]], step++]
   ]
   , {c, Length@ccirc}];
  show
 ]
```

In[296]:=

```
(* Print distillation step by step *)
PrintDistillation[sequence_ : {}, devoptions_ : {}] := Module[
  {dev, circ, ccirc, show = {}, format, noisy, fulltime, step = 0, nions},
  format[zones_, c_, t_, s_] :=
   {Column[{"(" <> ToString[s, FormatType → TraditionalForm] <> ")t=" <> t <> "μs",
      StringRiffle[ToString[#, FormatType → TraditionalForm] & /@ c]}, Left],
    Rasterize[zones, RasterSize → Full, ImageSize → 150]};
  dev = TrappedIonOxford[Sequence @@ devoptions];

  nions = Min[Length /@ Flatten /@ Values@Values@dev[Nodes]];
  circ = DistCircTrappedIons4[sequence];
  ccirc = CircTrappedIons[circ, dev, MapQubits → False];
  fulltime = ToString[N[#], FormatType → TraditionalForm] & /@
    (InsertCircuitNoise[ccirc, TrappedIonOxford[Sequence @@ devoptions]][[All, 1]] +
      OptionValue[TrappedIonOxford, DurInit]["Alice"]);
  AppendTo[show, format[dev[ShowNodes], {"Initialisation"}, "0", step++]];
  Table[
   noisy = InsertCircuitNoise[ccirc[[c]], dev];
   AppendTo[show,
    format[dev[ShowNodes],
     If[Length@ccirc[[c]] < 1, "End", ccirc[[c]]], fulltime[[c]], step++]
   ]
   , {c, Length@ccirc}];
  show
 ]
```

Use the code below to render details of the distillation steps into GIF animation

```
picts = AnimateDistillation[{0, 0, 0}];
Export["distillation.gif", picts, AnimationRepetitions -> 1, "DisplayDurations" -> ConstantArray[1, Length@picts], RasterSize -> 600];
Import["distillation.gif", "AnimatedImage"]
```

In[297]:=

```
steps = PrintDistillation[{0, 0, 0}];
```

In[298]:=

```
steps // Length
```

Out[298]=

```
112
```

In[299]:=

```
(* uncomment to produce the steps shown in the appendix *)
(*Export["distillation_steps1.pdf",TableForm@steps⟦;;19⟧]
 Export["distillation_steps2.pdf",TableForm@steps⟦20;;38⟧]
 Export["distillation_steps3.pdf",TableForm@steps⟦39;;54⟧]
 Export["distillation_steps4.pdf",TableForm@steps⟦55;;75⟧]
 Export["distillation_steps5.pdf",TableForm@steps⟦76;;93⟧]
 Export["distillation_steps6.pdf",TableForm@steps⟦94;;⟧]*)
```