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ORIGINAL RESEARCH

Base belief function: an efficient method of conflict management

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Abstract

Dempster–Shafer evidence theory is widely used in many applications such as decision making and pattern recognition. However, Dempster's combination rule often produces results that do not reflect the actual distribution of belief when collected evidence highly conflicts each other. In this paper, a base belief function is proposed to modify the classical basic probability assignment before combination in closed-world. Base belief function focuses on making combination result intuitive especially when evidences highly conflict each other. Compared to other methods, the combination result produced by proposed method is logical and consistent with real world with less computational complexity and better performance. The advantage of base belief function is that it can avoid high conflicts between evidences and is especially suitable for the situation where the evidences appear in sequence. Several numerical examples as well as experiments using real data sets from the UCI machine learning repository for classification are employed to verify the rationality of the proposed method.

Keywords Dempster–Shafer evidence theory · Belief function · Conflicting evidence · Base belief function

1 Introduction

How to measure the uncertainty has attracted much attention over the last few years (Klir and Folger 1988; Yao 2001; Bloch et al. 2001; Borgonovo 2008). Many methods have been proposed to address this issue, such as Bayesian probability theory, Dempster–Shafer evidence theory (Dempster 1967; Rota 1977), fuzzy sets (Xiao 2018; Zheng and Deng 2018; Zhang et al. 2017; Han and Deng 2018), AHP (Zhou et al. 2018; Han and Deng 2018), Z-number (Kang et al. 2018a, b), D numbers (Fan et al. 2016; Xiao 2018; Deng and Deng 2018; Bian et al. 2018; Mo and Deng 2018), the construction of network (Yin and Deng 2018; Li et al. 2018; Bian and Deng 2018) and so on. Among these methods, Dempster–Shafer evidence theory is the core thought.

Dempster–Shafer evidence theory (D–S theory) is an efficient method to solve problems such as decision making (Utkin 2009; Chao et al. 2015), risk evaluation (Kabir et al. 2015; Liu et al. 2014; Jiang et al. 2016), pattern recognition (Fabre and Dherete 2003; Wang 2008; Zhu et al. 2017; Lin et al. 2016; Gong et al. 2018; Wang and Liu 2017) and other intelligent fusion processes (Fabre et al. 2001; Fabre and Briottet 2002; Ma et al. 2016; Leung et al. 2017; Talavera et al. 2013; Wu 2017; Fernandes and Bala 2017; Perez et al. 2016; Gruyer et al. 2016; Han and Deng 2018; Yin and Deng 2018) under uncertainty circumstances. For instance, a weighted averaging combination role is presented for multi-sensor data fusion in fault diagnosis (Jiang et al. 2016) based on D–S theory. In Zhu et al. (2017), D–S theory and fuzzy neural network are utilized to improve the reliability of recognizing fatigue driving. A novel approach based on extended D–S theory is proposed to estimate the accident probability of dangerous goods transportation (Leung et al. 2017).

One of the most valuable aspects of D–S theory is that reasoning or decision making can be carried out with incomplete or conflicting pieces of evidence without prior information. However, since sometimes the combination outcome can be counterintuitive when the evidences highly conflict each other, how to manage conflicts is a major problem especially during the fusion of many information sources. Many solutions have been addressed in the past few years (Yang and Dong-Ling 2013; Lefevre et al. 2002; Murphy 2000; Voorbraak 1988; Dubois and Prade 1992; Fabre et al.

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2001; Yager 1987; Smets 1990; Peida et al. 2013; Dubois and Prade 1988; Deng 2015; Yong et al. 2004; Shafer 2016; Jousselme et al. 2001; Liu 2006; Florea et al. 2009; Yager 2014; Jiroušek and Shenoy 2018; Xiao 2017, 2018, 2019; Liu et al. 2017; Chen et al. 2017; Jafari et al. 2017; Jiang et al. 2017; Ye et al. 2017; Wang et al. 2017; Li and Deng 2018; Zhang and Deng 2018). These solutions can roughly be divided into two categories, either rebuild the combination rules or modify the data model.

For the first kind, take Yager (1987), Dubois and Prade (1992) and Lefevre et al. (2002) as examples. Yager points out that since we cannot make reasonable decision, we need to remove the normalization factor and put it in to the unknown field (Yager 1987). Dubois and Prade further develop Yager's thought and present a combination operator that is better adapted and more specific than Yager's thought (Dubois and Prade 1992). Lefevre defines a formalism to describe a family of combination operators and develop a generic framework in order to unify several classical rules of combination (Lefevre et al. 2002). He takes weighing factors into consideration which based on the use of training set and the minimization of an error criterion.

For the second kind, discounted coefficient can be used to preprocess mass function of evidences, which is a common data modification thought to manage conflict. Take (Jousselme et al. 2001), (Murphy 2000), and (Yong et al. 2004; Deng 2015) as examples. Jousselme introduces a principled distance between two basic probability assignments based on a quantification of the similarity between sets. Murphy advocates an average method to use Dempster's rules multiple times in place of normalization. Deng et al. further develop Murphy's thought, using weighed average in terms of the similarity of evidences.

Nevertheless, both these two methodologies have some open issues. For the first kind, it is impracticable to assign every weighing factors when hundreds of thousands of evidences appear. The amount of computation grows exponentially when there are many subsets in the frame of discernment. Meanwhile, so far few of the modified combination rules still maintain associative. Lacking such significant property will cause trouble if the combination sequence is unclear. Haenni has listed these shortcomings before (Haenni 2002). For the second kind, accumulating amount of data and calculating data's similarity or correlation are required. Such methods increase the computing time, which are incompetent in case of high real-time requirements. To address these limitations, a new base belief function is presented in this paper.

The motivation of this study is to develop a new conflicting management method which keeps the desirable properties of Dempster's combination rule but with less computational load than so called data modification method. The significance of this paper is to propose a novel approach, in which we define base belief function to modify the classical BPA and get an intuitive combination result. One of the most crucial point is that this proposed method can eliminate the situation where evidences totally contradict with each other. As a result, sequential fusion can be achieved, which is suitable for high real-time updating system such as military system.

In this paper, we have reviewed the literature and summarized existing management conflicting methods and shortcomings above. The remainder of this paper is organized as follows. We first briefly review some basic concepts, and analyze the defect of Dempster's combination rule as well as the existing solutions in Sect. 2. In Sect. 3, we define base belief function and propose a new rule to modify the basic probability assignment which can get a reasonable combination result by using D–S theory. Numerical examples are used to test the correctness of base belief function. Classification experiments are presented in Sect. 4 to illustrate the effectiveness of base belief function. Conclusions are given in Sect. 5.

2 Preliminaries

In this section, some preliminaries are briefly introduced.

2.1 Dempster–Shafer evidence theory

D–S theory (Dempster 1967), presented by Dempster and further developed by Rota (1977), is often regarded as an extension of the Bayesian theory of probabilities. D–S theory can manage to handle uncertainties caused by unknown prior probabilities while the subjective Bayes cannot. Since it focuses on the question of uncertainty, it has been applied by many fields (Utkin 2009; Kabir et al. 2015; Gong et al. 2018; Yin and Deng 2018). Some definitions and theories are addressed as follows.

Let Ω be a nonempty finite set of N possible values which are mutually exclusive. Ω is called the frame of discernment and defined as

$$\Omega = \{H_1, H_2, \dots, H_N\} \tag{1}$$

The power of set composed with 2^N propositions of Ω is defined:

$$\begin{aligned}2^{\Omega} = &\{\phi, \{H_1\}, \{H_2\}, \dots, \{H_N\}, \{H_1 \cup H_2\}, \dots,\\ &\{H_1 \cup H_2 \cup \dots \cup H_i\}, \dots, \Omega\}\end{aligned} \tag{2}$$

For Ω, a basic probability assignment (BPA) (also called mass function) is a mapping m: $2^{\Omega} \rightarrow [0, 1]$, which satisfies the following properties:

$$m(\phi) = 0 \tag{3}$$

$$\sum_{A \in 2^{\Omega}} m(A) = 1 \tag{4}$$

If $A \subset \Omega$ and $A \neq \phi$, then the mass function $m(A)$ represents the possibility of evidence A that supports the claim. The bigger $m(A)$ is, the stronger evidence supports hypothesis A. Any subset $A \subseteq \Omega$ such as $m(A) > 0$ is called a focal element of m.

For Ω, a belief function from a BPA m is defined as

$$bel : 2^{\Omega} \rightarrow [0, 1] \tag{5}$$

$$bel(A) = \sum_{B \subseteq A} m(B) \tag{6}$$

The quantity $bel(A)$ can be interpreted as a measure of one's belief that hypothesis A is true. In particular, $bel(\phi) = 0$ and $bel(\Omega) = 1$

Due to the difference source, we can get different BPAs from the same evidence. Dempster proposed to use orthogonal sum to combine these BPAs, which is commutative and associative. Given n rules, based on independent evidence, the orthogonal sum of their mass functions computes the degree of belief for the combined rules.

$$m = m_1 \bigoplus m_2 \bigoplus m_3 \bigoplus \cdots \bigoplus m_n \tag{7}$$

where m_i is mass function of each rule.

Assume there are n BPAs indicated by $m_1, m_2, \ldots, m_n$. The combination rules are as follows:

$$\begin{cases} m(\phi) = 0 \\ m(A) = k \cdot \sum_{\cap A_i = A} \prod_{1 \leq j \leq n} m_i(A_i) \end{cases} \tag{8}$$

where k is a normalization factor:

$$k^{-1} = \sum_{\cap A_i \neq \phi} \prod_{1 \leq i \leq n} m_i(A_i) \tag{9}$$

To be specific, suppose there are two BPAs indicated by m_1 and m_2, the focal elements are B_i and C_i, then the combination rules are simplified as

$$m(A) = \begin{cases} 0 & A = \emptyset; \\ \frac{1}{1-K} \sum_{B \cap C = A} m_1(B) m_2(C) & A \neq \emptyset, \end{cases} \tag{10}$$

where k measures the degree of conflict between m_1 and m_2

$$k = \sum_{B \cap C = \phi} m_1(B) m_2(C) \tag{11}$$

$k = 0$ means m_1 is consistent with m_2. $k = 1$ means m_1 totally contradict with m_2, that is, two sources strongly support different hypotheses which are not compatible.

2.2 Existing conflicting management

Example 1 (Zadeh 1986) Suppose that the FOD is $\Omega = \{a, b, c\}$ and two BPAs are given as

$m_1(a) = 0.99, m_1(a, b) = 0.01$

$m_2(b) = 0.01, m_1(c) = 0.99$

Such example can illustrate the problem of classical Dempster's rule clearly.

1. A minority opinion can be magnified to 100% certainty. Using Dempster's combination rule, it is surprising that the combination result $m(b) = 1$, and the rest of mass functions are all zero. It is impossible because both sources have little support on $\{b\}$. Source 1 strongly supports $\{a\}$, while source 2 strongly supports $\{c\}$. Just because they both give a little support on $\{b\}$, this little support is infinitely magnified.
2. The absoluteness of data leads to the irreversibility of the combination result. If the mass function of one hypothesis become zero, this hypothesis will be zero forever, which is also unfathomable (Murphy 2000). A small number of wrong data may cause erroneous results.

As a result, the dependency of data accuracy and the lack of robustness are the main limitations of D–S theory. For more discussion, please refer to famous Zadeh paradox (Zadeh 1986).

As mentioned in Sect. 1, there are two types of approaches to manage conflicting evidence: (1) rebuilding new combination rules; (2) modifying the data model. The differences and shortcomings have also been briefly discussed. Here we take one typical method from each type to illustrate the disadvantages of each method in detail.

2.2.1 Yager's method: rebuilding new combination rules

Yager (1987) believes that since we cannot make a proper decision on the conflicting evidences, we should classify them into unknown field. He proposes a modified combination rule that eliminate the normalization factor $1/(1-k)$ and assign k to $m(\Omega)$. For Example 1, the combination result of Yager's method is $m(A) = m(C) = 0$, $m(B) = 0.0001$, $m(\Omega) = 0.9999$. It can be seen that $\{b\}$ has little support before combination, and it still has little support after combination, while the probability of almost one are distributed to Ω. It is suggested that the conflicting evidences are absolutely denied after combination. We can deduce that although multiple sources offer evidences, since they conflict each other, we still know little about them.

The disadvantage is that illogical result may be produced if there are more than two sources of evidences. Since such rule completely negates conflicting evidences, inaccuracy data has a serious influence on combination result. The entire system may not work normally due to a few sensors' error.

2.2.2 Deng et al.'s method: modifying the data model

Based on Murphy's approach, the idea of Deng et al.'s method is that the importance of each body of evidence may be different (Yong et al. 2004). They propose a similarity measure to calculate the distance between two evidences as weighing factor. Each evidence has a different impact on the combination result and the impact is also affected by other evidences. After obtain all weighting factors, one can use the classical Dempster's rule to combine the weighted average of the mass functions $n-1$ times to get the combination result.

The advantage is that it has better performance of convergence to handle conflicts. The disadvantage is that the proposed weighted average is more complex than the classical Dempster's rule, which is not a high-efficiency way with big data in real time application system. Also, the commutativity and associativity properties are also not satisfied, which means the order of combination needs to be consider separately. Besides, if one evidence is inaccuracy and repeated several times, it is also likely to get illogical result.

3 The proposed method

In this section, we define a base belief function to modify the basic probability assignment. We further propose its applicable conditions to get an efficient method in conflict management and discuss the computational complexity compared to the alternative techniques.

3.1 Base belief function

Our assumption is in a closed-world. Let Ω be a set of N possible values which are mutually exclusive. The power set of Ω is 2^{Ω}, in which the number of elements is 2^N. When the frame of discernment is complete, $m(\phi)=0$. Thus, we define base belief function m_b as

$$m_b(A_i)=\frac{1}{2^N-1} \tag{12}$$

where A_i is every subset in Ω except for the empty set ϕ.

Then, we can use different methods to generate BPAs according to the evidences we get. Assume the BPAs is indicated by m, we use m_b to modify m by calculating the arithmetic mean:

$$m'(A_i)=\frac{m_b(A_i)+m(A_i)}{2} \tag{13}$$

The aim of base belief function is to give each subset in the frame of discernment an equal possibility before they generate BPAs. Consider there are three identical balls of different colors and at least one is in an opaque bag. If we do not get other clues, then we can get seven kinds of situation which has the same possibilities. Base belief function is based on such thoughts. A BPA represents the degree of one source supports the hypotheses in a situation at a given time. But before every source appears, our belief in every situation must be equal. Base belief function is equivalent to these original possibility.

Then, when a source comes, we use base belief function to modify the classical BPA. If $m(A_i)>m_b(A_i)$, then $m'(A_i)>m_b(A_i)$, which means the coming source strengthens the original possibility. If $m(A_i)<m_b(A_i)$, then $m'(A_i)<m_b(A_i)$, which means the original possibility is weakened.

The most advantage of base belief function is that it eliminates the situation where evidences totally contradict with each other. Evidences which highly conflict often occurs when some of the probability masses are zero, and base belief function can avoid such situation perfectly. Even if all the sources weaken one hypothesis, the modified BPA of this hypothesis will be close to zero, but it will never be zero. Such method negates the absoluteness in real world and give every BPA an error-tolerate rate. In other words, the inaccuracy of sources will not have a decisive influence on the final combination result, especially when we need to handle a large amount of data. Even if the evidences available strongly support hypothesis A and oppose hypothesis B now, since we do not collect all the evidences, and we may never collect all the evidences in most situations in reality, possibility still exists that A is wrong and B is right.

Inevitably, this proposed method exchanges time for accurate and intuitive combination result, which requires a huge amount of computation complexity. The condition where the calculation load can be properly reduced is introduced in Sect. 3.3.

3.2 Numerical examples of using base belief function

First, we give two simple and extreme examples to verify the rightness of the proposed thought.

Example 2 Suppose that the FOD is $\Omega=\{a,b\}$ and two BPAs are given as

$m_1(a)=1, m_1(b)=0, m_1(a,b)=0$

$m_2(a)=1, m_2(b)=0, m_2(a,b)=0$

We first get the base belief function according to formula [12]:

$m_b(a) = m_b(b) = m_b(a,b) = \frac{1}{3}$

Then, we modify two BPAs based on formula [13]:

$m_i'(a) = \frac{\frac{1}{3}+1}{2} = 0.6667, m_i'(b) = m_i'(a,b) = \frac{\frac{1}{3}}{2} = 0.1667$

where $i = 1, 2$

Using Dempster's combination rules, we can get the result:

$m(a) = 0.8571, m(b) = 0.1071, m(a,b) = 0.0357$

According to the result, we can analyze that $\{a\}$ is strongly supported by the original source, which is consistent with the reality. However, we still concede that $\{b\}$ is likely to become true, though the possibility is very small.

Example 3 Suppose that the FOD is $\Omega = \{a, b\}$ and two BPAs are given as

$$m_1(a) = 1, m_1(b) = 0, m_1(a,b) = 0$$
$$m_2(a) = 0, m_2(b) = 1, m_2(a,b) = 0$$

The base belief function is

$m_b(a) = m_b(b) = m_b(a,b) = \frac{1}{3}$

After modify two BPAs, we can get the combination result using D–S theory:

$m(a) = m(b) = 0.4737, m(a,b) = 0.0526$

This example indicates that $\{a\}$ and $\{b\}$ have equal possibility to be true, which reflects the reality correctly because the sources given above are complete conflict.

In Sect. 2.2, we have given an example and several reasons for the possibility of causing conflicts. In this subsection, we still use the same example to test the effect of base belief function as follows.

Example 4 (*The same as Example 1*) Suppose that the FOD is $\Omega = \{a, b, c\}$ and two BPAs are given as

$$m_1(a) = 0.99, m_1(a,b) = 0.01$$
$$m_2(b) = 0.01, m_2(c) = 0.99$$

We get the base belief function:

$m_b(a) = m_b(b) = m_b(c) = m_b(a,b) = m_b(a,c) = m_b(b,c) = m_b(a,b,c) = \frac{1}{7}$

Then, we modify two BPAs and get the final result:

$m(a) = m(c) = 0.3957, m(b) = 0.1012, m(a,b) = 0.0337, m(a,c) = m(b,c) = 0.0322, m(a,b,c) = 0.0107$

As shown in Table 1, the combination result of the proposed method is more reasonable than the classical Dempster's combination result in Sect. 2.2. $m(b) = 0.1012$ is more rational than $m(b) = 1$. Though $m(b)$ is much greater than its original mass function, it is still smaller than $\frac{1}{7}$, which means $\{b\}$ does not have too much support. $m(a) = m(c) = 0.3957$ is logical because $\{a\}$ and $\{c\}$ do get powerfully support from original sources.

3.3 Applicable conditions

Example 5 Suppose that the FOD is $\Omega = \{a, b, c\}$ and two BPAs are given as

$$m_1(a) = 0.9, m_1(a,b,c) = 0.1$$
$$m_2(c) = 0.9, m_2(a,b,c) = 0.1$$

The results of two combination rules are shown in Table 2.

Example 6 Suppose that the FOD is $\Omega = \{a, b, c\}$ and two BPAs are given as

$$m_1(a) = 0.9, m_1(b) = 0.05, m_1(c) = 0.05$$
$$m_2(a) = 0.05, m_2(b) = 0.05, m_2(c) = 0.9$$

The results of two combination rules are shown in Table 3.

As can be seen from Tables 2 and 3, when all the mass functions of the single sets of two evidences are nonzero or all the mass functions of the complete set of two evidences are nonzero, the results of these two rules have little differences. Examples 5 and 6 give strong supports on $\{a\}$ and $\{c\}$, respectively. The results of two combination rules are both plausible, giving equal possibilities to $\{a\}$ and $\{c\}$ which are far more than $\{b\}$. In order to decreases computational complexity, there is no need to use base belief function when these two situations appear.

Table 1 Results of two combination rules of Example 4

	$m(a)$	$m(b)$	$m(c)$	$m(a,b)$	$m(a,c)$	$m(b,c)$	$m(a,b,c)$
Classical Dempster's rule	0	1	0	0	0	0	0
Proposed rule	0.3957	0.1012	0.3957	0.0337	0.0322	0.0322	0.0107

Table 2 Results of two combination rules of Example 5

	$m(a)$	$m(b)$	$m(c)$	$m(a,b)$	$m(a,c)$	$m(b,c)$	$m(a,b,c)$
Classical Dempster's rule	0.4737	0	0.4737	0	0	0	0.0526
Proposed rule	0.3756	0.0976	0.3756	0.0413	0.0413	0.0413	0.0271

Table 3 Results of two combination rules of Example 6

	$m(a)$	$m(b)$	$m(c)$	$m(a, b)$	$m(a, c)$	$m(b, c)$	$m(a, b, c)$
Classical Dempster's rule	0.4865	0.0270	0.4865	0	0	0	0.0526
Proposed rule	0.3876	0.1222	0.3876	0.0308	0.0308	0.0308	0.0103

One special case needs to emphasis. If all the mass function of the single sets of one source are nonzero, and the mass function of the complete set of the other source is nonzero, base belief function is still needed. Let us cite another example to illustrate our thought.

Example 7 Suppose that the FOD is $\Omega = \{a, b, c\}$ and two BPAs are given as

$m_1(a) = 0.9, m_1(a, b, c) = 0.1$

$m_2(a) = 0.05, m_2(b) = 0.05, m_2(c) = 0.9$

The results by two combination rules are shown in Table 4.

It can be easily seen that the result from our proposed method is more logical than that of classical Dempster's rule. Source 1 strongly supports $\{a\}$, while source 2 gives little support on $\{a\}$ and conversely supports $\{c\}$. Although little support on $\{b\}$ is generally accepted by two sources, the combination result that gives equal support to $\{a\}$ and $\{c\}$ is more logical.

More things can be observed and discussed. For Examples 5 and 6, the complete set of the combination result is nonzero even if we use classical Dempster's rule. In such situations, two combination rules have roughly the same effect. However, for Example 7 the complete set of the combination result is zero, and two results have distinctive differences. Thus, we can roughly conclude that in situations where conflict exists, complete set can be seen as the attenuator of the conflict. Every time we generate BPA, lots of conflicts can be avoided if mass functions of the complete set are nonzero.

3.4 Procedure of the proposed method using base belief function

Step 1 Generate basic belief function Whether evidence is a real-time updating application system or a data set, each attribute is considered an independent source of information. Using different kinds of method to generate BPAs of each attribute is the priority of the procedure. The approach of generating BPA depends on its circumstances. The first two BPAs are chosen into the next step.

Step 2 Judge conditions

- Condition 1: All the mass functions of the single sets of two BPAs are nonzero
- Condition 2: All the mass functions of the complete set of two BPAs are nonzero
- Condition 3: Not belong to Condition 1 and Condition 2

We have to figure out which condition these two BPAs belong to. The aim of this step is to minimize the amount of calculation.

Step 3 Use appropriate combination rule According to the aforementioned Examples 5 and 6, if two BPAs belong to Condition 1 or Condition 2, only classical Dempster's rule is required to combine two sources. However, if two BPAs belong to Condition 3, we need to use base belief function and generate a modified BPA from each independent source of information. The modified BPA can be used into Dempster's combination rule. Finally, a combination result is obtained and the number of all BPAs is reduced by one. Repeating Steps 1–3, we can gain the final result when there is only one BPA left. The only difference between first procedure and repeating procedure is that in repeating procedure, one of the two BPAs is the combination result from last procedure. Figure 1 shows the flow chart of the proposed method.

Such procedure preserves the precious properties of classical Dempster's rule such as associativity and commutativity. Another advantage of the proposed method is that sequential fusion can be achieved. We can combine the evidences according to the order of their arrival rather than considering their combination order, which is critical

Table 4 Results of two combination rules of Example 7

	$m(a)$	$m(b)$	$m(c)$	$m(a, b)$	$m(a, c)$	$m(b, c)$	$m(a, b, c)$
Classical Dempster's rule	0.3448	0.0345	0.6207	0	0	0	0
Proposed rule	0.3791	0.1110	0.3846	0.0362	0.0362	0.0362	0.0166

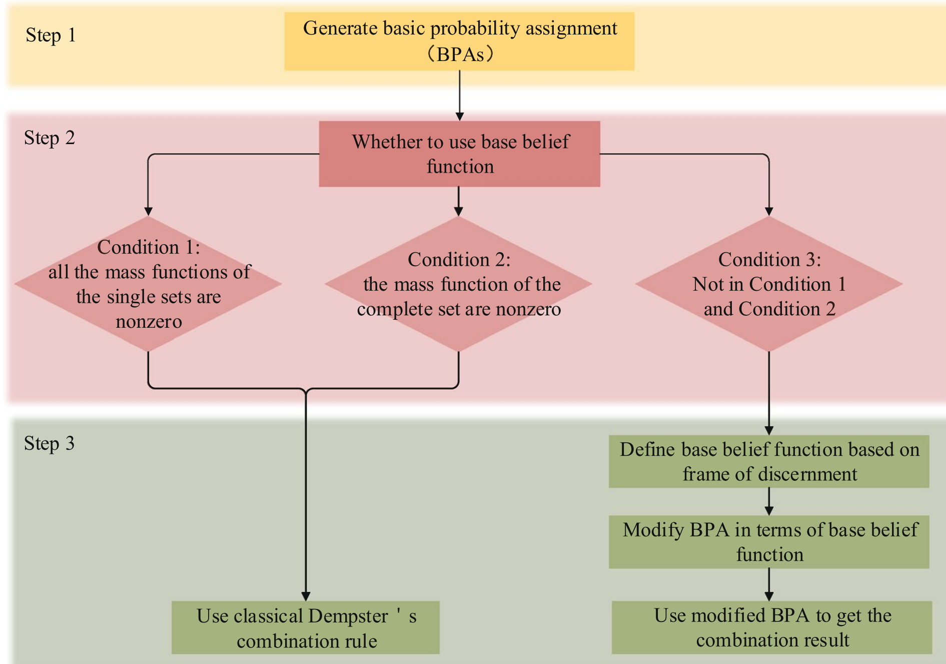

Fig. 1 Flow chart

Table 5 Triangular fuzzy numbers of four attributes

	a	*b*	*c*
SL	(4.30, 5.03, 5.80)	(4.90, 6.01, 7.10)	(5.40, 6.68, 7.90)
SW	(3.00, 3.42, 4.40)	(2.20, 2.77, 3.20)	(2.50, 2.97, 3.80)
PL	(1.20, 1.46, 1.70)	(2.20, 4.26, 5.00)	(4.80, 5.55, 6.90)
PW	(0.10, 0.23, 0.40)	(1.00, 1.30, 1.60)	(1.50, 2.03, 2.50)

importance especially in military system. The only thing we need to concern is whether the coming evidence belongs to one of these two condition mentioned above. If so, combining directly with the generated result; if not, using base belief function to modify BPA before combination.

4 Experiments

Two classification experiments using real data set have been carried out to evaluate the effectiveness of base belief function in this section. The real data sets are from the UCI machine learning repository (Bache and Lichman 2013).

4.1 Experiment 1 (Iris data set classification)

There are three species (*Setosa*(a), *Versicolor*(b), *Virginica*(c)) in Iris data set with four attributes and each species contains 50 instances. We randomly select 40 instances from each species and generate the triangular fuzzy numbers (Klir and Yuan 1996) of four attributes which is shown in Table 5.

The remainder ten instances are regarded as test sets. We randomly choose one instance from species *Setosa* of the test sets and generate its BPA. The result is shown in Table 6. The four attributes of this instance are (5.3, 3.5, 1.3, 0.2).

Then we use the proposed method to get the final result. To be specific, the mass function of attribute *PL* and *PW* need to be modified with the base belief function, for they do not satisfy either of the particular conditions. Table 7 shows modified BPAs.

The results by different combination rules are shown in Table 8.

As can be seen from Table 8, all these rules can recognize that the test instance is likely to be Setosa, which conform to the actual situation. Although the conclusion of the classical Dempster's rule seems more obvious, it exists the risk of conflicting because $m(c) = 0$. If there are more attributes

Table 6 BPAs of four attributes

	$m(a)$	$m(b)$	$m(c)$	$m(a, b)$	$m(a, c)$	$m(b, c)$	$m(a, b, c)$
SL	0.3337	0.3165	0.2816	0.0307	0.0052	0.0272	0.0052
SW	0.3164	0.2501	0.2732	0.0304	0.0481	0.0515	0.0304
PL	0.6699	0.3258	0	0	0	0.0043	0
PW	0.6996	0.2778	0	0	0	0.0226	0

Table 7 BPAs of four attributes using base belief function

	$m(a)$	$m(b)$	$m(c)$	$m(a, b)$	$m(a, c)$	$m(b, c)$	$m(a, b, c)$
SL	0.3337	0.3165	0.2816	0.0307	0.0052	0.0272	0.0052
SW	0.3164	0.2501	0.2732	0.0304	0.0481	0.0515	0.0304
PL	0.4064	0.2343	0.0714	0.0714	0.0714	0.0736	0.0714
PW	0.4213	0.2103	0.0714	0.0714	0.0714	0.0827	0.0714

Table 8 Results of different combination rules of Iris experiment

	$m(a)$	$m(b)$	$m(c)$	$m(\Omega)$
Classical Dempster's rule	0.8457	0.1543	0	0
Yager's rule (Yager 1987)	0.5337	0.1484	0	0.3180
Deng et al.'s rule (Yong et al. 2004)	0.8533	0.1361	0.0119	0
Proposed rule	0.6232	0.2671	0.1083	0

and $m(a) = 0$, unreasonable result will be produced. For the sake of accuracy and insurance, the proposed method and Deng et al.'s rule are much better. In the next subsection, the advantage of base belief function will be much more distinct.

4.2 Experiment 2 (wine data set classification)

In this experiment, we use wine data set classification to demonstrate the application width of base belief function. The wine data set consists of three different varieties of wine with 13 attributes. We randomly choose 45 elements from each wine as training samples and the remaining elements in each variety as test samples. Then we choose one instance, an extreme example, from variety a and generate its BPAs. The 13 attributes of this test instance are (13.24, 3.98, 2.29, 17.5, 103, 2.64, 2.63, 0.32, 1.66, 4.36, 0.82, 3, 680). The process of test is referred to in Sect. 4.1. For simplicity, only the obtained BPAs and the final combination result are shown in Tables 9 and 10, respectively.

As is shown in Table 10, the combination result of classical Dempster's rule illustrates that the test instance absolutely belongs to variety b. The result of Yager's method suggests that there are possibility that the test instance is variety b and we still have little knowledge. The result of Deng et al.'s rule also suggests that the test instance likely to be variety b. Only our proposed method illustrates that there is a bit tiny possibility that the test instance belongs to variety a than b, which is consistent with the actual situation. Obviously, the proposed method is much better than other combination rules in this experiment. The reason for the

Table 9 BPAs of 13 attributes

	$m(a)$	$m(b)$	$m(c)$	$m(a, b)$	$m(a, c)$	$m(b, c)$	$m(a, b, c)$
alcohol	0.1304	0.1160	0.2752	0.1160	0.1304	0.1160	0.1160
malic acid	0.3082	0.1681	0.0889	0.1681	0.0889	0.0889	0.0889
ash	0.1413	0.1994	0.1295	0.1413	0.1295	0.1295	0.1295
alcalinity of ash	0.3966	0.3017	0	0.3017	0	0	0
magnesium	0.2252	0.2372	0.0781	0.2252	0.0781	0.0781	0.0781
total phenols	0.2599	0.2642	0.0540	0.2599	0.0540	0.0540	0.0540
flavanoids	0.2885	0.4230	0	0.2885	0	0	0
nonflavanoid phenols	0.1840	0.1776	0.1152	0.1776	0.1152	0.1152	0.1152
proanthocyanins	0.1301	0.2030	0.1383	0.1301	0.1301	0.1383	0.1301
color intensity	0.2123	0.2849	0.0726	0.2123	0.0726	0.0726	0.0726
hue	0	0.2921	0.4158	0	0	0.2921	0
$OD280/OD315$ *of diluted wines*	0.3314	0.3372	0	0.3314	0	0	0
proline	0	0.3106	0.3788	0	0	0.3106	0

Table 10 Results of different combination rules of Wine experiment

	$m(a)$	$m(b)$	$m(c)$	$m(a, b)$	$m(a, c)$	$m(b, c)$	$m(\Omega)$
Classical Dempster's rule	0	1	0	0	0	0	0
Yager's rule (Yager 1987)	0	0.4371	0.1237	0	0	0.1014	0.3370
Deng et al.'s rule (Yong et al. 2004)	0.1914	0.7820	0.0010	0	0	0	0
Proposed rule	0.4997	0.4569	0.0434	0	0	0	0

huge divergence of these combination results is that highly conflicts are exist in generating BPAs, which verifies the conflicting management of base belief function. From this experiment, we can conclude that the performance of our proposed method is better than Deng et al.'s method in mass data situation.

5 Conclusion

When combining conflicting beliefs utilizing Dempster's rules, the results are often counterintuitive. Of all the alternative approaches that address the problem, they either change the combination rules or modify the data model. In this paper, the proposed method defines base belief function based on the frame of discernment, making mass functions of every subset nonzero to avoid the problem of classical Dempster's rule, which is a fundamental solution to manage conflicts. Furthermore, as long as the mass functions of the single sets or the complete set are nonzero, the combination outcome will be logical. Such method preserves some desirable properties such as commutativity and associativity.

The proposed method has some limitations, though. Base belief function can only be used in a closed-world. In an open world, there are many possibilities that cause conflicts, either incomplete frame of discernment or unreliable sensors. It is unwise to solve the conflict in one general way. Also, the effect of proposed method is not obvious when data set is rather small. Since it gives fault tolerance to every sensor, final results are obvious correct only if mass data sets appear.

Besides, computational load is another important thing needs to be considered. It is well known that time consuming of classical Dempster's combination rule is already large, and the computational load of our proposed method is no less than that of classical Dempster's rule. Assume that there are n exclusive values in the frame of discernment and time complexity of classical Dempster's rule is ω. The time complexity of our proposed method is no greater than $\omega + 3\mathcal{O}(n)$. Rebuilding rules means either consider different circumstances or define weighing factors, implying the increase of time complexity. Modifying data means consider the relationship between any two sources, thus $\mathcal{O}(n^2)$ is inevitable. Compared to these alternative approaches, time complexity of combination-rebuilt rules such as Yager's rule is no less than ω, time complexity of data-modified rules such as Deng et al.'s rule is roughly $\omega + \mathcal{O}(n^2) + 3\mathcal{O}(n)$. Based on accuracy of proposed method in mass data situation, such computational complexity can be tolerated. In conclusion, our proposed method is a not simple but effective method.

Our proposed method can be used in decision-making or pattern recognition under strong uncertainty circumstance, especially suitable for mass data situation such as multi-agent systems or military system. In these situations, highly conflicting evidences often occur due to numerous information sources and aberrant measurements, which make the classical Dempster's rule impossible. Using base belief function to modify BPAs, although the combination result may not obvious in small data sets because of conflicting, it will gradually approach the correct answer due to big data sets. Consequently, we can combine the evidences according to their arrival time without considering the combination order, which is perfectly suitable for high real-time updated application system.

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