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A new base function in basic probability assignment for conflict management

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Abstract

To address highly conflicting evidence combinations, a new base function is proposed to alleviate conflicts that exist in pieces of evidence provided before the fusion of them to get intuitive results from the combination. The proposed method assigns a corresponding value to each proposition according to its importance. Single subset propositions are considered more crucial than multiple ones, which intends to reduce uncertainties existing in the frame of discernment so that indicative results of combination can be obtained. More than that, to avoid a considerable deviation from the modified mass to the original ones, an operation of average is carried out twice to achieve this effect. The proposed conflicting management method not only has the advantage of eliminating conflicts among evidence but also the ability to produce intuitive results. Several numerical examples and experiments using datasets are illustrated to verify the accuracy and correctness of the proposed method in processing highly conflicting information.

Keywords Dempster-Shafer evidence theory · Base function · Conflict management · Basic probability assignment

1 Introduction

How to acquire useful information from multi-source evidences is an open issue, which has attracted much attention from a lot of researchers. Many mathematical theories have been proposed to solve problems occur in the process of combination of different evidences. For example, Dempster-Shafer theory and its extension [1–6] provides some advanced techniques to give a proper description of the information offered, belief function [7–11] is designed to manage uncertainty contained in information, optimization theory [12, 13] improves the process of disposing evidences from different sources, fuzzy sets theory [14–17] indicates a more detained situation of separate propositions and evidences to help extract useful information from uncertainty, Z-numbers [18–22] is a further improvement of the probability distribution to ensure accuracy in the system of judgement, D-numbers [23, 24] generalize the system of judgement to adapt to more complicated conditions, soft sets [25, 26] offers a view from a completely dimension to treat the process of managing multi-source information, complex mass function [27–32] extends the traditional evidence system into the field of complex numbers and some other meaningful works. Among them, a groundbreaking work of complex function is the generalization of Dempster-Shafer evidence theory [32] which expands the traditional system of judgement into the complex field. When probabilities of events can not be directly observed, the events is uncertain and the probability of them to take place can be introduced to a complex environment to better present a different level of uncertainties. An enlightenment obtained from related works about complex mass function is that a fuzzy environment can be simplified by introducing a distinctive form of elements contained in a frame of discernment and can be also be provided a basic base function to abate conflicts among evidence from the start of managing evidences. More than that, these methods are also applied in many situations of real world including but not limit to risk evaluation [33–35], target recognition [36–39], multi-factor decision making [40–44], uncertainty measure [45–48] so on [49–54]. All of the works provide their specific solution to extract truly meaningful part of information offered and the most enlightening point is that all of the elements which

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are deviant can not bring negative effects on the results of combination, which means conflicting or extreme ones are disposed appropriately through different kinds of methods.

Among them, Dempster-Shafer evidence theory is a representative modality of traditional bayesian theory of probability and meets a weaker condition than traditional bayesian theory, which manifests its superiority in handling uncertain information from different sources and attracts focuses of many scholars from different fields. Without doubt, Dempster-Shafer evidence theory has been applied in real world under separate complex circumstances. However, D-S evidence theory is powerless in indicating the most valuable information when facing a situation that highly conflicting evidences are supposed to be fused efficiently and correctly.

On account of the failure of classic Dempster's combination rule in managing highly conflicting evidences, lots of researchers have come up with a variety of ideas to improve the performance of their combination theory which is based on the classic D-S evidence theory. In brief, these theory can be roughly divided into two categories. One is reasonably distributing the values of basic probability in highly conflicting section, namely modifying the rule of combination. The other is modifying evidences which is offered in information sources. In the first kind of category, Lefevre *et.al*'s method [55] and Yager's method [56] are the representative methods which amends classic combination rules. Lefevre invents a completely new regulation by referring the principles proposed in the classic rules of combination and creates a formalism to describe family of operators. Besides, Yager's method cancels the step of normalization in classic combination theory and distributes the remaining mass to uncharted fields to indicate the conflicting level among evidences instead of distributing the value of conflicting parts to every proposition inexplicably. In the second kind of category, typical examples of the methods which adjusts the mass of each proposition in the frame of discernment are Murphy's average method [57] and Deng *et.al*'s improved weighted method [58]. Murphy chooses to average every evidence to get the mean of the evidences and theses evidences are taken into consideration without discrimination. In order to realize the convergence of this method, the refined data should be combined n-1 times to get final values. Besides, Deng *et.al*'s improved weighted method [58] introduces a weight factor based on distances between evidences, combination of evidences puts more focus on certain propositions instead of treating every evidence equally. Except for the two kinds of categories mentioned above, the loss function can be utilized to measure the degree of loss of a given entity of evidences when disposing different information to retain an enough high accuracy in extracting the truly useful key points for decision making. And the trend of the change of loss can also be a prominent tool in observing current situations of evidences in analysing errors appeared in process of combining multi-source information. Besides, a technique of optimization can be also added into the process of managing conflicts to alleviate the parts of no use in handling multi-source information to reduce wastage of data. And a step of optimization can simplify the process of disposing information to decrease time consumption by reducing the extend of complexity of procedures. All in all, the two useful methods can be utilized to provide a more complete and valid foundation in constructing a system of combining information from different sources.

All kinds of methods successfully solve some problems appear in process of combination. However, these methods also has the unavoidable drawbacks. For example, in the first category, the correctness of new combination rule is difficult to justify its validity and rationality. It is possible that classic Dempster's combination rule is already ideal and sources of inaccurate results may come from errors in the generation of evidences. And in the second category, the process of producing weight factors or generating average numerical values may have an exponential increase in time consumption when the number of evidences rises in a very high speed.

Therefore, the main motivation of this study is to introduce a reasonable base mass to every proposition to avoid irrational and counter-intuitive results produced in the process of combination. With less time consumption and complexity in computing, the new base function can still achieve satisfying results of combination and adopts classic combination methods to enhance its validity and correctness. All in all, in order to better illustrate advantages of the proposed method, a simple list is provided which is given as:

(1) A completely new base function is designed for propositions which owns extreme values to adapt them to better conform to the actual situations.
(2) The new base function is inclined to single propositions instead of treating all of the propositions in the same way to reduce the degree of uncertainty of the whole frame of discernment.
(3) Compared with the previously proposed base function, the one proposed in this paper takes the carnality of propositions into consideration to properly change the mass distributed to specific propositions.
(4) The new base function effectively handles conflicts of evidences to provide an intuitive results of combination of evidences offered.

In this paper, the rest parts is constructed as follows. In Section 2, some fundamental concepts of evidence theory and previous methods are introduced, such as the definition of frames of discernment. In Section 3, the proposed method is defined and clearly explained. In Section 4, some numerical examples and UCI datasets are used as tested data to verify the new method's validity and correctness. Conclusions are provided in Section 5. Besides, the brief information of references are given in Table 1.

1.1 Some discussions about conflict management and consensus modelling

In the field of conflict management, it is supposed to alleviate conflicts among evidences in the process of combination. The methods can be divided into different categories, the most representative ones are disposing evidences before combination and improve the rule of combination. In this paper, a kind of new base function is proposed to reduce the conflicts contained in evidences to avoid producing counter-intuitive results and the process of modification is comparatively simple and does not take lots of time to be calculated. However, compared with the consensus modelling, this kind of method dose not take relations of evidences into consideration, which may affects the whole situation of evidences provided and have side-effects on the combination of evidences. Some agreements and threshold value must be satisfied in the consensus modelling and then the next step can be carried out, which indicates that every step of management of information is under some strict regulations to ensure enough accuracy of information modified to be retained. But the two general methods have some similarities which lie in the goal of producing reasonable and correct descriptions and predictions on the base of actual situations to formulate appropriate policies for making decisions. Besides, in general, exceptions in evidences are properly solved and the elements which is abnormal are also deleted or altered in a reasonable manner to create a self-consistent system of management of information from different sources.

2 Preliminaries

How to properly combine evidences [61–65] and produce rational decisions [66–70] are still open issues to be solved. In the work of Xiao [62], a multi-layered system of judgement is introduced by utilizing similarity measure, support degree and credibility degree of evidences. Besides, in another passage of Xiao [66], a novel evidential correlation coefficient (ECC) for belief functions to measure the degree of conflict is introduced to provide a convenient tool to manage conflicts in different information sources to better produce intuitive and reasonable decisions. Both of the works propose a distinctive method to serve as a powerful tool in decision making. The former passage allows different standards of judgement to avoid errors which may occur in the process of disposing evidences obtained from different information sources. And with respect to the latter one, the degree of conflict is measured directly to provide a specially designed weight for every piece of evidence. In this part, some fundamental theory and a former solution in managing conflicts are generally introduced.

2.1 Dempster-Shafer evidence theory

Let Θ become a non-empty and finite set which consists of N elements and elements contained in this set are mutually exclusive. Besides, set Θ is named as the frame of discernment. Therefore, Θ is defined as [2, 3]:

$$\Theta = \{H_1, H_2, H_3, H_4, ...H_N\} \tag{1}$$

On the base of the frame of discernment, the power set of Θ which has 2^N different propositions is defined as [2, 3]:

$$\{\emptyset, \{H_1\}, \{H_2\}, ...\{H_N\}, ..., \{H_1 \cup H_2\}, \{H_1 \cup H_3\}, ..., \Theta\} \tag{2}$$

For set Θ, a basic probability assignment (BPA, also named as mass function) is a mapping m which is thus defined as [2, 3]:

$$m: \quad 2^{\Theta} \rightarrow [0, 1] \tag{3}$$

More than that, the function of the frame of discernment also satisfies the following properties [2, 3]:

$$m(\emptyset) = 0 \tag{4}$$

$$\sum_{A \in 2^{\Theta}} m(A) = 1 \tag{5}$$

According to the definition mentioned above, it is concluded that if proposition $A \subset \Theta$ and is not equal to $\emptyset$, then the basic probability assignment $m(A)$ represents the probability of evidence which called A that support the proposition. Obviously, when the value of $m(A)$ gets bigger, it is estimated that the confidence level of proposition A rise higher. Vice versa.

Because the evidences come from different sources, the internal information needed to be gained properly. Dempster adopts the orthogonal sum as a method to combine different BPAs to get degrees of proposition's belief, which is also commutative and associative. The rule is defined as [2]:

$$m(A) = \frac{1}{1-k} * \sum_{A_i \cap B_j \cap C_k \cap ... = A} m_1(A_i) * m_2(B_j) * m_3(C_k) * ... \tag{6}$$

The normalization coefficient k indicates the conflict among evidences, the formula to calculate the value of k is defined as [2]:

$$k = \sum_{A_i \cap B_j \cap C_k \cap ... = \emptyset} m_1(A_i) * m_2(B_j) * m_3(C_k) * ... \tag{7}$$

Table 1 Methods in handling different situations

Approach of combination	Base algorithm/method	Key features
Extended D-S evidence theory [1, 4–6]	Negation, Paradox	Generalized structure of D-S evidence theory
Belief function [7–11]	Correlation, Probability transformation	Obtaining proper results of combination
Optimization theory [12, 13]	Optimization based on evidence theory	Producing more reasonable judgements on situations
Fuzzy sets theory [14–17]	Divergence measure, Linguistic approach	Multiple attribute decision making
Z-numbers [18–22]	D-S evidence theory utilizing Z-numbers	Uncertainty measure, Application in expert system
D-numbers [23, 24]	Entropy and belief interval using D-numbers	Uncertainty measure, Multiple attribute decision making
Soft sets [25, 26]	Cross entropy, Association rule	Decision making
Complex mass function [27–32]	Complex environment	Generalized complex decision system
Risk evaluation [33–35]	Disposing utilizing failure mode	Information fusion, Effects analysis
Target recognition [36–39]	Information redistribution, elbow method	Determine number of targets, Evidence combination
Multi-factor decision making [40–44]	Distributed preference, Evidential reasoning	MADM, EEG
Uncertainty measure [45–48]	Distance measure, Information volume	Pattern classification
Hybrid combination method [49, 51, 53, 54, 59, 60]	Vector, OWA	Alarm system design, Evidence combination

2.2 Yager's combination rules

What Yager intended to propose is that if evidences conflict with each other highly, then the mass should not be distributed to any propositions which has already existed in the frame of discernment. Moreover, due to the unavailability of classic Dempster evidence theory in the situation that the conflict of information sources is very high, Yager considers that the conflicting part is useless and should be discarded. Therefore, he proposes that the normalization factor should be removed and the conflicting part is supposed to distributed to unknown field, which means his method completely ignores and denies the role of conflicting part in indicating truly useful information from complex evidences. The combination rules are defined as [56]:

$$m(\emptyset) = 0 \tag{8}$$

$$m(A) = \sum_{A_i \cap B_j \cap C_k \cap \ldots = A} m_1(A_i) * m_2(B_j) * m_3(C_k) * \ldots \tag{9}$$

$$k = \sum_{A_i \cap B_j \cap C_k \cap \ldots = \phi} m_1(A_i) * m_2(B_j) * m_3(C_k) * \ldots \tag{10}$$

$$m(X) = \sum_{A_i \cap B_j \cap C_k \cap \ldots = X} m_1(A_i) * m_2(B_j) * m_3(C_k) * \ldots + k \tag{11}$$

Nevertheless, the method dose not truly solve the core problem which occurs in processing conflicting data. The central obstacle is that how to manage conflicting part and utilize them to properly indicate primary objects. But this method avoids disposing argumentative portion and places it to a variable which is unrelated in the fusion of different evidences. A simple example can show and demonstrate the futility of Yager's method in handling highly conflicting evidences. For example, $m_1(A) = 0.99, m_1(B) = 0.01$; $m_2(C) = 0.99, m_2(B) = 0.01$ is a typical example that the evidences are conflicting with each other which is properly designed and this kind of phenomenon is observed and proposed by Zadeh [71]. By using Yager's method, $m(A) = 0 = m(C), m(B) = 0.0001, m(X) = 0.9999$ is obtained. It can be easily clarified that evidence 1 and evidence 2 support proposition A and B respectively, but the result of combination strangely allocates zero mass to each of them. More than that, on the contrary, proposition B which has very low support in both evidence is distributed to the maximum mass, which is irrational and counter-intuitive. The value of $m(X)$ is just abandoned but only represent conflicts among evidences and it is of no use in gaining precise indicators of certain important propositions. As a consequence, researchers still can not benefit from Yager's method and know little about actual facts.

2.3 Two existing method to dispose conflicts

In order to solve the problem appears in the process of combination of highly conflicting evidences, two methods have been proposed to solve the conundrum.

2.3.1 New base basic probability assignment

A straightforward basic probability is allocated to every proposition which exists in the frame of discernment to alleviate clash among evidences. The base belief function is defined as [72]:

$$m_{Nbase}(A_i) = \frac{1}{N}, \quad if\ A_i\ is\ a\ single\ subset\ event \tag{12}$$

$$m_{Nbase}(A_i) = 0, \quad if\ A_i\ is\ not\ a\ single\ \ subset\ event \tag{13}$$

$$m(A) = \frac{m_{Nori}(A) + m_{Nbase}(A)}{2} \tag{14}$$

The method adopts classic Dempster's combination rule to combine evidences n-1 times to get the final indicator for certain proposition. However, the modulation in the values of propositions is very crude, so a deviation from the practical truth might be easily observed.

Note: $m_{Nori}(A)$ is the original mass function of new base basic probability assignment. N denotes the number of the mutually exclusive propositions in frame of discernment.

2.3.2 Base belief function

The second method also adopts the pattern which alters the values of all propositions, and the distributed value of the base changes with the number of propositions exist in the power set of the frame of discernment [73]:

$$m_{Bbase}(A) = \frac{1}{2^N - 1} \tag{15}$$

$$m(A) = \frac{m_{Bori}(A) + m_{Bbase}(A)}{2} \tag{16}$$

After adjusting the value of every proposition, the orthogonal sum in Dempster combination rule is used to combine evidences n-1 times to obtain an eventual result. A drawback of this function is that although a proposition dose not exist in a frame of discernment, an identical value is allocated to every proposition indiscriminately, which may cause unnecessary deviation during the process of combination.

Note: $m_{Bori}(A)$ is the original mass function of base belief function. N denotes the number of the mutually exclusive propositions in frame of discernment.

3 Proposed method

In this section, a new method to abate conflicts among evidences and to reduce the loss may occur in the process of modifying data from the very beginning of disposing information. Like what have been defined in Dempster-Shafer evidence theory, there is a set Θ which contains N elements and a power set of Θ which has $2^N - 1$ elements.

Definition 1 The new base function is defined as:

$$m_{base}(A) = \frac{2^{N+1-|A|}}{\sum_{B\in 2^{\Omega}} 2^{N+1-|B|}} \tag{17}$$

N is the number of single subsets. If evidence 1 has a mass for proposition A, then no matter what the value of proposition of A in evidence 2 is, the mass in evidence 2 is required to get average sum with its base.

Definition 2 The operation of getting modified BPA is defined as:

$$m_{temp1}(A) = \frac{m_{base}(A) + m(A)}{2} \tag{18}$$

And average the basic mass with original mass to get the new mass of the proposition.

First, if evidence 1 has a mass for single subset proposition A, then no matter what the value of proposition of A in evidence 2 is or whether the proposition A exists in another evidence's frame of discernment, the mass of that proposition in evidence 2 is required to get an average sum with its base.

Second, if the cardinal number of evidences is very big, then only conflicting evidences are processed to reconcile conflicts among evidences. Because in this case, except highly conflicting evidences, other ones can already reflect actual conditions well.

Third, if the mass of a proposition in all of evidences is 0, then no operation is going to carried out on this proposition.

Definition 3 The operation of averaging is defined as:

$$m_{temp2}(A) = \frac{m_{temp1}(A) + m(A)}{2} \tag{19}$$

$$m_{final}(A) = \frac{m_{temp2}(A) + m(A)}{2} \tag{20}$$

Forth, the new base is calculated with the original mass of the proposition to get a average value. This operation is carried out twice to minimize errors which may appear in the operation of distributing basic mass.

$$m(A) = \frac{1}{1-k} * \sum_{A_i \cap B_j \cap C_k \cap \ldots = A} m_1(A_i) * m_2(B_j) * m_3(C_k) * \ldots \tag{21}$$

In the last, the improved BPAs are combined n-1 times to get the final mass which indicates the probabilities of propositions [2].

Algorithm 1 Detailed procedure of new base function.

Input: The values of every proposition and the corresponding frame of discernment.
Output: The combined values corresponding to propositions of evidences provided
If The carnality number of evidences is very big **then**
Only extreme evidences are disposed

If Any evidence has a value for proposition A which is non-zero and extreme **then**

Generate an identical base for proposition existing in the frame of discernment which is needed to be disposed.

$$base \leftarrow \frac{2^{N+1-|A|}}{\sum_{Z\in 2^{\Theta}} 2^{N+1-|Z|}}$$

else

Do not regard proposition A as a conflicting element and not dispose the piece of evidence due to the value of proposition A.

end

If A certain proposition is taken into the process of disposing in a piece of evidence **then**

Calculate the average of the base and original mass to get adjusted values of all propositions which are needed to be managed in all of the evidences provided which are going to be further altered in the next step.

$$base_{new} \leftarrow \frac{m_{ori}(A)+base}{2}$$

else

the proposition is not expected to be disposed and whether some evidences are handled is not according to the situation of the proposition.

end

Average the new mass with original values twice to get the modified value for every proposition.

$$m(A) \leftarrow \frac{\frac{base_{new}+m_{ori}(A)}{2}+m_{ori}(A)}{2}$$

Combine the modified evidences n-1 times utilizing Dempster's rule of combination to obtain the results of combination of evidences.

else
Some conflicting evidences are provided, all of the evidences are disposed
end

4 Numerical examples and discussions

4.1 Example 1

In this example, a totally conflicting evidence is offered to signify the advantage of proposed method in handling conflicts.

Assume that the frame of discernment is $\Theta = \{A,B\}$, and two basic probability assignments are given as:

$$m_1(A) = 1, m_1(B) = 0, m_1(A, B) = 0$$

$$m_2(A) = 0, m_2(B) = 1, m_2(A, B) = 0$$

This is the result of Dempster's combination rule: [2]:

$$k = m_1(A) * m_2(B) = 1,$$

Because the value of k is equal to 1, the step of normalization in the classic Dempster's combination rule can not be carried out.
This is the result of Yager's method:

$$k = m_1(A) * m_2(B) = 1,$$

$$m(X) = k = 1$$

Yager's method indicates that the conflict in evidences is very high, so all of the value is allocated to unknown field. However, it is not useful in manifesting the probabilities of propositions.

This is the result of new base basic probability assignment [72]:

$$\begin{aligned} m_{Nbase}(A) &= \frac{1}{N} = \frac{1}{2} \\ m_1(A) &= \frac{m_{1ori(A)} + m_{Nbase}}{2} = \frac{3}{4} \\ m_1(B) &= \frac{m_{1ori(B)} + m_{Nbase}}{2} = \frac{1}{4} \\ m_2(A) &= \frac{m_{2ori(A)} + m_{Nbase}}{2} = \frac{1}{4} \\ m_2(B) &= \frac{m_{2ori(B)} + m_{Nbase}}{2} = \frac{3}{4} \end{aligned}$$

Then, calculate the eventual mass using Dempster's combination rule:

$$m(A) = 0.5$$

$$m(B) = 0.5$$

In this case, the base basic assignment successfully avoids abnormality caused by conflicts between evidences in the process of combination. And the results produced is perfectly consistent with practical situations.

According to the definition of the algorithm 1, the proposition $\{A, B\}$ is exactly 0 which is not supposed to be taken into consideration and the process of disposing extreme values. This is the result of base belief function [73]:

$$m_{Bbase}(A) = \frac{1}{2^N - 1} = \frac{1}{3}$$
$$m_1(A) = \frac{m_{1ori(A)} + m_{Bbase}}{2} = \frac{2}{3}$$
$$m_1(B) = \frac{m_{1ori(B)} + m_{Bbase}}{2} = \frac{1}{6}$$
$$m_1(A, B) = \frac{m_{1ori(A,B)} + m_{Bbase}}{2} = \frac{1}{6}$$
$$m_2(A) = \frac{m_{2ori(A)} + m_{Bbase}}{2} = \frac{1}{6}$$
$$m_2(B) = \frac{m_{2ori(B)} + m_{Bbase}}{2} = \frac{2}{3}$$
$$m_2(A, B) = \frac{m_{2ori(A,B)} + m_{Bbase}}{2} = \frac{1}{6}$$

Then, calculate the eventual mass using Dempster's combination rule:

$$m(A) = 0.4736$$
$$m(B) = 0.4736$$
$$m(A, B) = 0.0526$$

Base belief function avoids abnormality caused by conflicts between evidences in the process of combination. But there is a lot of improvements of its accuracy in indicating prime propositions can be made.

This is result of proposed method:

$$m_{base}(A) = \frac{2^{N+1-|A|}}{\sum_{Z\in 2^\Theta} 2^{N+1-|Z|}} = 0.5$$
$$m_{base}(B) = \frac{2^{N+1-|B|}}{\sum_{Z\in 2^\Theta} 2^{N+1-|Z|}} = 0.5$$
$$m_1(A) = \frac{\frac{\frac{m_{base}(A)+m_{ori(A)}}{2} + m_{ori(A)}}{2} + m_{ori(A)}}{2} = 0.9375$$
$$m_1(B) = \frac{\frac{\frac{m_{base}(B)+m_{ori(B)}}{2} + m_{ori(B)}}{2} + m_{ori(B)}}{2} = 0.0625$$
$$m_2(A) = \frac{\frac{\frac{m_{base}(A)+m_{ori(A)}}{2} + m_{ori(A)}}{2} + m_{ori(A)}}{2} = 0.0625$$
$$m_2(B) = \frac{\frac{\frac{m_{base}(B)+m_{ori(B)}}{2} + m_{ori(B)}}{2} + m_{ori(B)}}{2} = 0.9375$$

Then, calculate the eventual mass using Dempster's combination rule:

$$m(A) = 0.5$$
$$m(B) = 0.5$$

An evaluation of this example: In the first and second piece of evidence, the propositions A and B contained in evidences are distributed a mass of 1 respectively. As for other propositions, their values are all 0. Clearly, without other interferences from other events, the range of the values of propositions A and B can be restricted to two accurate values which are 0.5 and 0.5.

Discussions about the results expected of combination: In this example, classic Dempster's combination rule can not be carried out. Besides, the value of k generated in Yager's method successfully indicate a complete conflict among evidences but can not manifest clear probabilities of propositions. Two previous methods have settled the problem caused by conflicts in evidences. In new base basic assignment, its performance in this example is great. A mass of 0.5 is distributed to proposition A and B, which is consistent with the descriptions of evidences on propositions. And base belief function has fewer accuracy than base basic assignment in disposing this case. Obviously, the new base function also has a good performance in the fusion of highly conflicting evidences and loses no accuracy in the process of combination. For example, in general, evidence 1 and evidence 2 support proposition A and proposition B. It is supposed that both of the propositions should get equally mass which is expected to approach or equal to a mass of 0.5. The proposed method has an identical results of combinations to intuitive estimates. More than that, proposed method manage to reduce the mass of proposition A, B at a large range, which also briefly conforms to the practical facts and is better than the result of combination of base belief function. All in all, the proposed method performs very well in example 1.

4.2 Example 2

This example is used to test the effectiveness of proposed method in the situation that classic Dempster's combination rule may produce irrational and counter-intuitive results. Recall the phenomenon of conflicts in evidences proposed by Zadeh [71], an example is designed to manifest this kind of conflicts.

Assume that the frame of discernment is $\Theta = \{a,b,c\}$, and two basic probability assignments are given as:

$$m_1(A) = 0.99, m_1(B) = 0.01, m_1(C) = 0$$
$$m_2(A) = 0, m_2(B) = 0.01, m_2(C) = 0.99$$

The results combined by classic Dempster's combination rule, Yager's method, base basic probability assignment, base belief function and proposed method are presented in Table 2.

An evaluation of this example: According the description of the actual situations provided by the evidences, the propositions A and C are given a mass of 0.99 respectively

Table 2 Combination results of example 2

	m(A)	*m(B)*	*m(C)*	*m(A,B)*	*m(A,C)*	*m(B,C)*	*m(A,B,C)*	k	m(X)
Dempster's combination rule [2]	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.9999	0.00
Yager's method [56]	0.00	0.0001	0.00	0.00	0.00	0.00	0.00	0.9999	0.9999
New base basic assignment [72]	0.4410	0.1178	0.4410	0.00	0.00	0.00	0.00	0.7499	0.00
Base belief function [73]	0.3948	0.1028	0.3948	0.03225	0.03225	0.03225	0.01075	0.52548	0.00
Proposed method	0.4837	0.0325	0.4837	0.00	0.00	0.00	0.00	0.9217	0.00

and a mass of 0.01 of proposition B is given in the two evidences likewise. Similar to the last example, propositions A and C are supposed to be allocated a same value and the values of them are very close to 0.5. Besides, for the proposition B, a minor value which ranges from 0.05 to 0.01 is supposed to be distributed to this proposition.

Discussions about the results expected of combination: This is a representative example of highly conflicting evidences in which classic Dempester's combination rule produces irrational and counter-intuitive results. In Dempster's combination rule, a mass of 1 is allocated to proposition B, which is very baffling. Both of evidence 1 and evidence 2 has a very low support on proposition B in which proposition B has only a mass of 0.01. On the contrary, it can be concluded that evidence 1 completely support proposition A and evidence 2 totally agrees with proposition C. In general, the result of combination is supposed to support proposition A and C and both of the proposition should have a very high and same belief level instead of distributing all of the mass to a proposition which has very low support in evidences. And the results of combination of Yager's method are also puzzling. Although Yager's method does not allocate a too big mass to proposition to proposition B, proposition B still has the biggest mass among the propositions existed in frame of discernment, which is still unacceptable. Moreover, proposition A and B gets a mass of 0 and it dose not conforms to the practical situation. A mass of 0.9999 is distributed to $m(X)$ to indicate the high conflicts among evidences, but the $k's$ value is of no use in generating mass of indicator. Besides, the new base basic assignment and base belief function reconcile the conflicts which exist in evidences, which is satisfying by and large. However, the drawback of the two methods is that the accuracy in indicating probability is not satisfying. The results of proposition A and B produced by this two methods are 0.4410, 0.4410 and 0.3948, 0.3948, which is far from the expectation that proposition A and B should get a mass which is near 0.5. Compared to these two previous, the proposed method not only solve conflicts among evidences but also improve the accuracy of combined mass. Final mass allocated to proposition A and B both is 0.4837, which is similar to subjective judgements on the mass of proposition A and B. More than that, the mass of proposition B in the two previous methods are 0.1178 and 0.1028 which is much larger than the mass of 0.0325 produced in proposed method. The mass of proposition B has in evidences 1 and 2 also support the combined result in proposed method, which is only 0.01. In sum, this classic example illustrates the advantage of proposed method in managing conflicts and proves its high accuracy in indicating probabilities of propositions.

4.3 Example 3

This example is used to test the correctness of the proposed method in indicating propositions' probability when the mass of propositions in one evidence has a mass of zero.

$$m_1(A) = 0.5, m_1(B) = 0.45, m_1(A, B, C) = 0.05$$

$$m_2(A) = 0, m_2(B) = 0.9, m_2(C) = 0.1$$

The results combined by classic Dempster's combination rule, Yager's method, base basic probability assignment, base belief function and proposed method are presented in Table 3.

An evaluation of this example: The two evidences provided by sensors have given proposition A a total value of 0.5 and proposition B a total value of 1.35. Besides, with respect to propositions A, B, C and C, they are given a value of 0.05 and 0.1. Therefore, proposition B should be allocated the biggest value which manifest the most probability of it to take place, which means a value range from 0.8 to 0.9 should be distributed to it. For proposition A, due to the lack of support to it, the value of the proposition is reasonable to range from 0.05 to 0.15 to manifest the large contrast of it in the two evidences. With respect to other propositions, their value can be categorized as a useless part of the results combined which should be allocated a mass less than 0.01.

Discussions about the results expected of combination: In this example, Dempster's combination rule successfully indicates the precise proposition B, which is rational and satisfying. However, the mass of proposition A generated by

Table 3 Combination results of example 3

	m(A)	*m(B)*	*m(C)*	*m(A,B)*	*m(A,C)*	*m(B,C)*	*m(A,B,C)*	k	m(X)
Dempster's combination rule [2]	0.00	0.9890	0.0109	0.00	0.00	0.00	0.00	0.545	0.00
Yager's method [56]	0.00	0.45	0.0050	0.00	0.00	0.00	0.00	0.545	0.545
New base basic assignment [72]	0.2180	0.7684	0.0134	0.00	0.00	0.00	0.00	0.5987	0.00
Base belief function [73]	0.2006	0.5947	0.1065	0.0287	0.0287	0.0287	0.0116	0.4064	0.00
Proposed method	0.0509	0.9325	0.00009	0.01651	0.00	0.00	0.00	0.5492	0.00

Dempster's combination rule is set to 0, which is unacceptable. It can be easily told that evidence 1 support proposition A. If the result of combination is completely opposed to proposition A, then it obviously lacks of credibility, which may ignore some small but relatively important information in the process of combination. Besides, Yager's method detects conflicts between evidences and distribute a mass of 0.545 to $m(X)$. But it still cannot avoid the drawback that the mass of proposition A is set to 0, which also dose not conforms to the actual situation. And a key point is that the results of combination in Yager's method is much smaller than the ones in other methods, which may manifest its availability in indicating the probability of propositions is very limited. Except these two classic methods, some improved methods' performance is also meritorious. Their combined results allocate a mass to proposition A, which accords practical situation. However, these two methods have two disadvantages, namely improper distribution to proposition A and unreasonable allocation to other propositions. The only role of the mass of proposition A is to indicate that proposition A still has a probability to happen, but evidence 2 totally disagree with proposition A. Therefore, the mass of proposition A in combined results should be very small instead of stripping quite a part of proposition $A's$ mass, which might cause an unnecessary deviation in indicating the most important objects. The combined mass of proposition B in new base basic assignment and base belief function are 0.7684 and 0.5947, which is much less than the one in classic Dempster's combination rule. And results of proposed method is 0. 9325, which is much closer to the value in Dempster's combination rule. Meanwhile, combined results also indicate proposition A has a probability to happen but reduce its level of importance to highlight proposition $B's$ significance. More than that, base belief function distributes some mass to some propositions which even do not exist in frame of discernment, which is not a wise operation in allocating mass. Besides, proposition $C's$ mass is also worth discussing. In base belief function, a mass of 0.1065 is allocated to proposition C. However, in evidences, only evidence 2 has a supported mass of 0.1 on proposition C, a dramatic rise in combined result of proposition C is irrational. In new base basic assignment, the mass of proposition C is roughly bigger than 0.01. Compared to the main propositions, the importance of proposition C can be ignored. Evidence 1 indicates proposition C is very uncertain and evidence 2 gives only a mass of 0.1 to proposition C. All of these results show that very little mass is expected to be retained to show the existence of proposition C but most of the mass should be distributed to more important proposition to help recognize valuable information, which is exactly what the proposed method attempts to achieve.

4.4 Example 4

This example also verifies proposed method's correctness in handling conflicting evidences and accuracy in the appropriate distribution of every proposition which exists in evidences. This example is similar to the third one but the difference lies in that one proposition does not have relatively big mass among evidences in the same time.

$m_1(A) = 0.9, m_1(B) = 0, m_1(A, B, C) = 0.1$

$m_2(A) = 0.1, m_2(B) = 0.05, m_2(C) = 0.85$

The results combined by classic Dempster's combination rule, Yager's method, base basic probability assignment, base belief function and proposed method are presented in Table 4.

An evaluation of this example: In the two piece of evidences, a total value of proposition A is 1 and the one for propositions C is 0.85. Therefore, the value of propositions A in the combined results is more than the value of proposition C. The former one possesses a value bigger than 0.5 and the latter one is far less than 0.5 which ranges from 0.3 to 0.4 due to the indirect support of it in the first piece of evidence.

Discussions about the results expected of combination: Dempster's combination rule produces relatively satisfying results in this example. However, one flaw of classic Dempster's combination rule is that the distance between the values of prime proposition and secondary proposition is not very distinct. If observers cannot easily make a decision, then all the results produced are useless. Therefore, how to rightly expand the distance between evidences but not lose

Table 4 Combination results of example 4

	m(A)	*m(B)*	*m(C)*	*m(A,B)*	*m(A,C)*	*m(B,C)*	*m(A,B,C)*	k	m(X)
Dempster's combination rule [2]	0.5263	0.0263	0.4473	0.00	0.00	0.00	0.00	0.81	0.00
Yager's method [56]	0.1	0.0050	0.085	0.00	0.00	0.00	0.00	0.81	0.81
New base basic assignment [72]	0.6510	0.2303	0.1185	0.00	0.00	0.00	0.00	0.7504	0.00
Base belief function [73]	0.4080	0.1086	0.3607	0.0354	0.0354	0.03547	0.0162	0.4672	0.00
Proposed method	0.5673	0.0623	0.3703	0.00	0.00	0.00	0.00	0.7849	0.00

accuracy in indicating probabilities of propositions is very important. This point is going to be further discussed in the last of this paragraph. And the value of k which is generated in Yager's method indicates conflicts between evidences but its combined results cannot give a clear indicator on the most possible proposition. However, when judging the results combined by new base basic assignment and base belief function, the situation is different. Although new base basic assignment generates the biggest mass of proposition A, the mass of proposition C become too small, which is not similar to real circumstances. In evidence 2, proposition C has a mass of 0.85, which is a very high belief level. The distribution of mass is supposed to be much bigger than 0.1185. And the result of base belief function is even worse than the result produced in classic Dempster's combination rule. The mass of proposition A is much lower than any other ones, which is not a good indicator on propositions' selection. Besides, some propositions which do not exist in the frame of discernment are distributed a mass, which is not reasonable. Proposed method solves the problems mentioned above. It appropriately enlarge the distance between prime and secondary proposition to be helpful in decision making but not lose the accuracy in reflecting actual circumstances. It even has a higher indicative mass than classic Dempster's combination rule in a relatively mild situation.

4.5 Experiment on UCI dataset

Simple examples provided above illustrate the superiority of proposed method in managing highly conflicting evidences. However, the validity of rationality of the proposed method in real situations need to be further checked. Two datasets from UCI (University of California Irvine) are used to test correctness of proposed method in practical applications, namely iris and wine dataset [73].

4.5.1 Experiment on iris dataset

Three categories are in iris dataset and they have four attributes. Every category has 50 instances. Each category also has 40 samples, 120 samples in total are randomly selected to generate triangular fuzzy numbers. In the remaining samples, one of the samples is chosen to generate its BPAs, which are used to be combined to get eventual mass and they are given in Table 5. After the process of generating BPAs, some mass of propositions is exactly 0, which means the data can be utilized to test correctness of proposed method in managing highly conflicting evidences. Modified data of proposed method is shown in Table 8. And the modified data of other methods is shown in Tables 6 and 7 respectively.

4.5.2 Analysis about combined results of five methods

From the Table 9, it can be concluded that classic Dempster's combination rule has a good effect on indicating prime propositions, like proposition A and B. However, proposition C does not get its deserved mass due to the conflicts among evidences. A mass of 0.0001 for proposition C is too small considering BPAs offered in iris dataset. And the value of k which is generated in Yager's method indicates high conflicts among evidences. However, the mass of proposition C is still too small to manifest its deserved probability not to mention the combined result of Yager's method is not intuitive enough. New base basic assignment and base belief function successfully alleviate conflicts among evidences. However, both of them have

Table 5 BPAs generated by using IRIS dataset [73]

	m(A)	*m(B)*	*m(C)*	*m(A,B)*	*m(A,C)*	*m(B,C)*	*m(A,B,C)*
Sepal length	0.3337	0.3165	0.2816	0.0307	0.0052	0.0272	0.0052
Sepal width	0.3164	0.2501	0.2732	0.0304	0.0481	0.0515	0.0304
Petal length	0.6699	0.3258	0.0	0.0	0.0	0.0043	0.0
Petal width	0.6996	0.2778	0.0	0.0	0.0	0.0226	0.0

Table 6 Modified data of new base basic assignment

	m(A)	m(B)	m(C)	m(A,B)	m(A,C)	m(B,C)	m(A,B,C)
Sepal length	0.3335	0.3249	0.3074	0.0153	0.0026	0.0136	0.0026
Sepal width	0.3248	0.2917	0.3032	0.0152	0.0240	0.0257	0.0152
Petal length	0.5016	0.3295	0.1666	0.00	0.00	0.0021	0.00
Petal width	0.5164	0.3055	0.1666	0.00	0.00	0.0113	0.00

some defects. In new base basic assignment, the mass of proposition A is much less than classic Dempster's combination rule, which is not satisfying. Besides, the distributed to proposition C does not conforms to actual situation. Although proposition C has a mass which is more than 0.2 in two evidences, the other two evidences completely oppose proposition C. Therefore, what should be achieved is that the mass of proposition C should much bigger than 0 but should be controlled in a reasonable level to help distinguish truly valuable information. If the allocation of mass is distributed to these unimportant proposition, the modification might hamper detecting useful information. Similar problems appear in the combined results of base belief function. More than that, the mass of proposition A produced by base belief function much smaller than the one in classic Dempster's combination rule, which illustrates that the effectiveness of base belief function in indicating prime propositions is not reasonable enough. On the contrary, the proposed method solves all the problems mentioned above. The accuracy of indicating certain propositions is retained and distribution to corresponding propositions is reasonable and rational.

4.5.3 Experiment on wine dataset

In this dataset, some values of propositions are also 0. Therefore, this dataset can be used in test proposed method's correctness and validity in handling conflicts. Wine dataset has 3 species which has 13 attributes respectively. 45 elements are randomly selected as training samples to generate triangular fuzzy numbers. In the remaining elements, one of them is chosen to produce BPAs which are given in Table 10. All of the modified data of proposed method is shown in Table 11. And different combined results are present in Table 12.

4.5.4 Analysis about combined results of five methods

In this experiment, the results produced by classic Dempster's combination rule is too absolute. What can be inferred from the evidences is that proposition A should also be allocated to some mass. The most important point is that if there is one more evidences are opposed to a certain proposition, then this conflict should be partially accepted. So, the mass of proposition A should be controlled in a very low level to manifest non-supports from some evidences, which conforms to actual situation. In new base basic assignment, proposition A is allocated to a mass of 0.1, but the mass of proposition B is only 0.889. Compared to Dempster's combination rule, this mass is too small to indicate the prime proposition. Besides, the non-supports from evidence Hue and Proline are not shown in the final results of combination, which is not satisfying. Also, the base belief function even distributes a bigger mass to proposition A than proposition B, which is unacceptable. Because the supports to proposition B is consistent and the average mass of proposition B is about 0.25 which is not a low level of belief compared to other propositions. Therefore, the mass of combination of proposition B should be the biggest among evidences. And meanwhile, the mass of proposition A should not be set to 0. A very small number of mass can be allocated to proposition A to present its existence to avoid loss any tiny pieces of information. In sum, the distribution of mass in base belief function is irrational and does not reflect the most valuable information contained in evidences. For proposed method, it satisfies all the features mentioned above. It has the same high level of belief mass like Dempster's combination rule to indicate the prime propositions. In the same time, a small portion of mass is allocated to proposition A to manifest its probability to happen instead of setting the

Table 7 Modified data of base belief function

	m(A)	m(B)	m(C)	m(A,B)	m(A,C)	m(B,C)	m(A,B,C)
Sepal length	0.2382	0.2296	0.2122	0.0867	0.0740	0.0850	0.0740
Sepal width	0.2296	0.1964	0.2080	0.0866	0.0954	0.0971	0.0866
Petal length	0.4063	0.2343	0.0714	0.0714	0.0714	0.0735	0.0714
Petal width	0.4212	0.2103	0.0714	0.0714	0.0714	0.0827	0.0714

Table 8 Modified data of proposed method

	m(A)	*m(B)*	*m(C)*	*m(A,B)*	*m(A,C)*	*m(B,C)*	*m(A,B,C)*
Sepal length	0.3183	0.3032	0.2727	0.0400	0.0177	0.0369	0.0111
Sepal width	0.3031	0.2451	0.2653	0.0397	0.0552	0.0582	0.0331
Petal length	0.6124	0.3113	0.0263	0.0131	0.0131	0.0169	0.0065
Petal width	0.6384	0.2693	0.0263	0.0131	0.0131	0.0329	0.0065

Table 9 Combination results of iris dataset

	m(A)	*m(B)*	*m(C)*	*m(A,B)*	*m(A,C)*	*m(B,C)*	*m(A,B,C)*	m(X)
Dempster's combination rule [2]	0.8454	0.1544	0.0001	0.00	0.00	2.9197e-06	0.00	0.00
Yager's method [56]	0.0746	0.0136	1.2241e-05	0.00	0.00	2.5787e-07	0.00	0.9117
New base basic assignment [72]	0.6777	0.2539	0.0703	0.00	0.00	3.1395e-07	0.00	0.00
Base belief function [73]	0.5994	0.2767	0.1133	0.0034	0.0033	0.0040	0.0002	0.00
Proposed method	0.8093	0.1845	0.0077	1.6074e-05	1.0925e-05	4.5452e-05	1.7862e-07	0.00

Table 10 BPAs Generated By Using Wine Dataset [73]

	m(a)	*m(b)*	*m(c)*	*m(a,b)*	*m(a,c)*	*m(b,c)*	*m(a,b,c)*
Alcohol	0.1304	0.1160	0.2752	0.1160	0.1304	0.1160	0.1160
Malic acid	0.3082	0.1681	0.0889	0.1681	0.0889	0.0889	0.0889
Ash	0.1413	0.1994	0.1295	0.1413	0.1295	0.1295	0.1295
Alcalinity of ash	0.3966	0.3017	0.0	0.3017	0.0	0.0	0.0
Magnesium	0.2252	0.2372	0.0781	0.2252	0.0781	0.0781	0.0781
Total phenols	0.2599	0.2642	0.0540	0.2599	0.0540	0.0540	0.0540
Flavanoids	0.2885	0.4230	0.0	0.2885	0.0	0.0	0.0
Nonflavanoid phenols	0.1840	0.1776	0.1152	0.1776	0.1152	0.1152	0.1152
Proanthocyanins	0.1301	0.2030	0.1383	0.1301	0.1301	0.1383	0.1301
Color intensity	0.2123	0.2849	0.0727	0.2123	0.0726	0.0726	0.0726
Hue	0.0	0.2921	0.4158	0.0	0.0	0.2921	0.0
OD280/OD315 of siluted wines	0.3314	0.3372	0.0	0.3314	0.0	0.0	0.0
Proline	0.0	0.3106	0.3788	0.0	0.0	0.3106	0.0

Table 11 Modified data in proposed method

	m(a)	*m(b)*	*m(c)*	*m(a,b)*	*m(a,c)*	*m(b,c)*	*m(a,b,c)*
Alcalinity of ash	0.3733	0.2904	0.0264	0.2774	0.0131	0.0131	0.0065
Flavanoids	0.2788	0.3965	0.0264	0.2656	0.0131	0.0131	0.0065
Hue	0.0264	0.2820	0.3902	0.0131	0.0131	0.2688	0.0065
OD280/OD315 of diluted wines	0.3162	0.3213	0.0263	0.3031	0.0131	0.0131	0.0065
Proline	0.0265	0.2981	0.3578	0.0131	0.0131	0.2849	0.0065

Table 12 Combination results of wine dataset

	m(A)	*m(B)*	*m(C)*	*m(A,B)*	*m(A,C)*	*m(B,C)*	*m(A,B,C)*	m(X)
Dempster's combination rule [2]	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00
Yager's method [56]	0.00	0.0013	0.00	0.00	0.00	0.00	0.00	0.9986
New base basic assignment [72]	0.1091	0.8891	0.0073	0.00	0.00	0.00	0.00	0.00
Base belief function [73]	0.4997	0.4569	0.0434	0.00	0.00	0.00	0.00	0.00
Proposed method	0.0086	0.9913	4.644e-05	2.6941e-07	4.252e-12	8.625e-10	6.319e-17	0.00

mass of proposition A to 0, which cannot conform to real situations. All in all, proposed method performs well in presenting practical information contained in evidences and also keep a high level of mass of indicator, which inherits advantage of Dempster's combination rule and successfully disposes conflicts among evidences.

5 Conclusion

When evidences conflicts with each other highly, classic Dempster's combination rule may produce irrational and counter-intuitive results. The new base function is proposed to reconcile conflicts among evidences and partially accept extreme mass of certain propositions if there are one more evidences are opposed to the propositions, which means the conflicts should be regarded as a way of acquiring information instead of ignoring them. This is the core idea of the new base function. It strives to eliminate the effects brought by conflicts and defer to actual conditions to distribute proper mass to corresponding propositions. Of course, how to truly distribute a reasonable mass to every proposition instead of modifying data unreasonably is still an open issue to be solved.

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