



Article

An Extended Base Belief Function in Dempster-Shafer Evidence Theory and Its Application in Conflict Data Fusion

Dingyi Gan, Bin Yang and Yongchuan Tang *

School of Big Data and Software Engineering, Chongqing University, Chongqing 401331, China; 20181668@cqu.edu.cn (D.G.); 20181665@cqu.edu.cn (B.Y.)

* Correspondence: tangyongchuan@cqu.edu.cn

Received: 19 October 2020; Accepted: 25 November 2020; Published: 1 December 2020



Abstract: The Dempster–Shafer evidence theory has been widely applied in the field of information fusion. However, when the collected evidence data are highly conflicting, the Dempster combination rule (DCR) fails to produce intuitive results most of the time. In order to solve this problem, the base belief function is proposed to modify the basic probability assignment (BPA) in the exhaustive frame of discernment (FOD). However, in the non-exhaustive FOD, the mass function value of the empty set is nonzero, which makes the base belief function no longer applicable. In this paper, considering the influence of the size of the FOD and the mass function value of the empty set, a new belief function named the extended base belief function (EBBF) is proposed. This method can modify the BPA in the non-exhaustive FOD and obtain intuitive fusion results by taking into account the characteristics of the non-exhaustive FOD. In addition, the EBBF can degenerate into the base belief function in the exhaustive FOD. At the same time, by calculating the belief entropy of the modified BPA, we find that the value of belief entropy is higher than before. Belief entropy is used to measure the uncertainty of information, which can show the conflict more intuitively. The increase of the value of entropy belief is the consequence of conflict. This paper also designs an improved conflict data management method based on the EBBF to verify the rationality and effectiveness of the proposed method.

Keywords: Dempster–Shafer (D-S) evidence theory; belief function; non-exhaustive frame of discernment (FOD); base belief function; conflicting evidence; generalized combination rule

1. Introduction

With the development of technology in computers, the Internet, and other related fields, information fusion technology, which was born in the military field [1], has been running through every corner of people's production and lives [2]. The realization of information fusion needs to deal with a lot of uncertain information. The existing theoretical tools for dealing with uncertain information include probability theory [3], fuzzy set theory [4], Dempster–Shafer evidence theory [5,6], information entropy theory [7,8], and so on. Dempster–Shafer evidence theory is a widely used and typical tool for uncertain information processing and data fusion, and it is widely used in various fields, including uncertainty reasoning [9,10], target identification [11], controller design [12,13], industrial production safety [14], classification [15,16], and so on [17]. At the same time, Reference [18] proposes the Transferable Belief Model (TBM) conjunctive rule, which is the unnormalized version of the Dempster combination rule (DCR). Both of the two combination rules are commutative and associative, and they both assume the combined items of evidence to be distinct [19]. However, when the collected evidence data are highly conflicting, the DCR often fails to produce intuitive results. In order to solve this problem, scholars have proposed many solutions, among which the base belief

Mathematics 2020, 8, 2137 2 of 19

function [20] based on the size of the frame of discernment (FOD) is proposed. This method can eliminate the high conflict in evidence by modifying the basic probability assignment (BPA) in the exhaustive FOD, thus producing intuitive results. It is suitable for military systems and other real-time update systems [20]. At the same time, the base belief function has been widely used and extended since it was proposed, such as in [21,22]. However, the base belief function has some limitations. It does not consider the uncertain information caused by the incomplete FOD, so it can only be used in the exhaustive FOD.

The generalized combination rule (GCR) in the non-exhaustive FOD has the same problem as the classical DCR. When facing highly conflicting evidence, it often produces a combination result that is contrary to intuition. Moreover, under the assumption of non-exhaustive FOD, the sources of uncertain information are more complex. Among them, the uncertain information represented by the nonzero mass function of empty set and the possible incompleteness of the FOD [23] is ignored by the base belief function. In order to solve the above problems, this paper extends the base belief function and proposes a method to modify the BPA in the non-exhaustive FOD. This method not only inherits the original characteristics of the base belief function, but also takes into account the value of the nonzero empty set mass function, which makes it possible to modify the BPA in the open non-exhausted FOD. This method can not only be applied to non-exhaustive FOD, but can also be reduced to the base belief function in the exhaustive FOD. Moreover, we further find that the entropy increases significantly after the BPA is modified by the proposed method. Belief entropy is used to measure the uncertainty of information, which can show the conflict more intuitively. The increase of the value of entropy belief is the consequence of conflict. This paper also proposes a conflict data management method based on an extended base belief function (EBBF), and verifies the feasibility and effectiveness of the proposed method by analyzing some examples. The steps of the conflict data management method are as follows: Firstly, we calculate the value of the EBBF, then modify the corresponding BPA with the proposed modification method, and finally use the DCR or GCR for data fusion to obtain reasonable results.

The rest of this paper is as follows. Section 2 introduces the related work for background knowledge. Section 3 introduces the preliminary knowledge. Section 4 proposes the EBBF and some of its characteristics. Section 5 introduces the conflict management method based on the EBBF and gives some examples and applications to verify the effectiveness of the method. Moreover, in Section 5, the differences between the two combination rules mentioned above are also compared and discussed. Section 6 draws the conclusion of this paper.

2. Related Work

In the field of information fusion and conflict data management, scholars have adopted various methods to manage conflict data [24]. The first category is to rebuild the combination rules. Many researchers try to rebuild the combination rule to deal with highly conflicting data [23,25,26]. Among them, Yager pointed out that it is necessary to remove the normalization factor and put it into the unknown domain to obtain a reasonable decision [27]. Then, based on Yager's idea, Dubois and Prade proposed a more specific combination operator [28]. Lefevre et al. developed a general framework to unify several classical combination rules, taking into account the use of training sets and the weighting factor minimization of error criteria [29]. Su et al. proposed a new rule to combine bodies of evidence from the perspective of evidence's dependence and independence [30].

The second category is the preprocessing of the mass function of evidence [31,32]. BPA preprocessing can effectively eliminate the completely contradictory phenomena between evidences, which can avoid the non-intuitive fusion results caused by evidence conflict [20]. The negation of the BPA was also proposed to address uncertain information in the evidence [33,34]. The strategy in this paper is to preprocess the BPA. At present, the main research methods of BPA pretreatment are as follows.

Mathematics 2020, 8, 2137 3 of 19

2.1. BPA Preprocessing Methods Based on Fuzzy Sets

In the case of using the prior knowledge generation method for samples, Xu et al. generated a nested BPA using a probability density function [35]. This type of BPA is superior to the BPA with singletons to some degree because it causes less conflict. However, in reality, modern industry and technology are developing rapidly. Therefore, it is very likely that there are some unknown element categories outside the previously established FOD [23,36]. In order to solve this kind of problem, Zhang and Deng proposed a boundary point determination method based on triangular fuzzy numbers [37]. By determining the average value, standard deviation, extreme value, and triangle member function of each attribute, and using the intersection point of a test sample and the above model, a nested BPA function was constructed in [37], which can assign values to empty sets. This method is suitable for both exhaustive FOD and non-exhaustive FOD, requires less prior data, and is driven by data, so it can reduce subjective uncertainty. In addition, many scholars have proposed uncertainty measurement methods based on fuzzy sets, such as [38–41].

2.2. BPA Preprocessing Methods with Belief Entropy

In the field of information fusion, many scholars have proposed the belief entropy to measure the information uncertainty [42–46]. The weight factor calculated by the belief entropy can be used to modify the conflicting data [47]. Shannon entropy [48] is applicable to the measurement of uncertain information in a probabilistic framework, and has been widely recognized and extended to many fields, such as network entropy in complex networks [49] and gene amplification analysis in the bioinformatics field [50]. However, Shannon entropy can not be directly applied to the uncertain information measurement of the mass function in evidence theory. In order to solve this problem, many uncertainty measures in evidence theory have been proposed, including ambiguity measurement [51], Deng entropy [52], and so on [53,54]. Deng entropy [52] considers the uncertain information carried by the mass function. Deng entropy can not only degenerate into Shannon entropy under certain conditions, but also gives a reasonable measurement in many complex environments [48]. Moreover, Deng entropy has been used in many practical applications, such as fault diagnosis [55], decision making [56], and sensor data fusion [34,57]. However, Deng entropy does not take into account the size of the FOD, which is also an important source of uncertain information. The lack of such information may lead to the reduction of the effectiveness of information processing and even the inability to deal with some uncertain information effectively. Based on the size of the FOD, Tang et al. proposed the weighted Deng entropy [58], which lost the probability consistency that Deng entropy satisfied, but modeled more uncertain information in the evidence body. After the BPA is modified by this method, the information loss is effectively reduced and more reasonable fusion results can be obtained. However, Deng entropy does not verify the required properties for this type of measurement and presents some undesirable behaviors [59] in some cases. Although some scholars have proposed a modified Deng entropy [58,60], the work in [61] has proved that these modifications still could not meet most of the necessary mathematical properties, and they presented most of the behavioral defects existing in Deng entropy. Therefore, Deng entropy and the corresponding modification should be cautiously employed in practical applications.

2.3. BPA Preprocessing Methods Using a Base Belief Function

For the method of rebuilding the combination rule, Haenni proposed in the paper [62] that when there is too much evidence, it is unrealistic to assign each weight factor, and when there are many subsets in the FOD, the calculation amount increases exponentially. For modifying the mass function, it needs to record the amount of data and calculate the similarity or correlation of data, which increases the calculation time, so it is difficult to implement in the case of high real-time requirements. In view of the above limitation, Reference [20] proposes a new base belief function. This method maintains the good characteristics of the DCR, and the computational complexity is low. Moreover, the method

Mathematics 2020, 8, 2137 4 of 19

can eliminate the complete contradiction between evidences. Based on the base belief function, many methods have been proposed by other scholars, such as that of [22,63], which can solve the problem that the DCR can not obtain the intuitive results when applied to highly conflicting data. However, the above methods are not suitable for the non-exhaustive FOD. The method proposed in this paper is an extension of the base belief function to make it applicable for the non-exhaustive FOD.

3. Preliminaries

Some preliminaries are briefly introduced in this section, including Dempster–Shafer (D-S) evidence theory [5,6], the base belief function [20], the DCR [5,6], the generalized combination rule [23], and so on.

3.1. Classical Dempster-Shafer Evidence Theory

Definition 1. The frame of discernment Ω is defined as a finite non-empty set containing N mutually exclusive events, and its specific expression is as follows:

$$\Omega = \{\Theta_1, \Theta_2, ..., \Theta_i, ..., \Theta_N\}. \tag{1}$$

Definition 2. For Ω , a basic probability assignment (BPA) (or mass function) is a mapping $m: 2^{\Omega} \to [0,1]$, which satisfies the following properties:

$$m(\emptyset) = 0, \sum_{A \in \Omega} m(A) = 1.$$
 (2)

If m(A) > 0, the subset A is called a focal element, and m(A) > 0 is the mass function value of proposition subset A.

Definition 3. *In D-S evidence theory, the DCR can fuse two independent mass functions, m_1 and m_2:*

$$m(A) = (m_1 \oplus m_2)(A) = \frac{1}{1 - K} \sum_{B \cap C = A} m_1(B) m_2(C),$$
 (3)

where k is a normalization factor defined as follows:

$$k = \sum_{B \cap C = \emptyset} m_1(B) m_2(C). \tag{4}$$

It is worth noting that the classical definitions of the D-S evidence theory are defined and used in the exhaustive FOD.

Definition 4. In the non-exhaustive FOD hypothesis, the DCR is extended by Deng in [52]. The intersection of an empty set and an empty set is still an empty set, which satisfies the condition $\emptyset_1 \cap \emptyset_2 = \emptyset$. Given two BPAs $(m_1 \text{ and } m_2)$, the generalized combination rule (GCR) is defined as follows:

$$m(A) = \frac{(1 - m(\emptyset)) \sum_{B \cap C = A} m_1(B) \cdot m_2(C)}{1 - K},$$
(5)

$$K = \sum_{B \cap C = \emptyset} m_1(B) m_2(C), \tag{6}$$

$$m(\emptyset) = m_1(\emptyset) \cdot m_2(\emptyset), \tag{7}$$

$$m(\emptyset) = 1$$
 if and only if $K = 1$. (8)

Mathematics **2020**, *8*, 2137 5 of 19

3.2. Normalization and TBM Conjunctive Rule

Definition 5. A BPA m is said to be subnormal if \emptyset is a focal set $(m(\emptyset) \neq 0)$, which can be transformed by the normlization operation defined as follows [19]:

$$m(A) = k * m(A)$$
 if and only if $A \neq \emptyset$ (9)

$$else \ m(A) = 0 \tag{10}$$

for all $A \subseteq \Omega$, with $k = (1 - m(\emptyset))^{-1}$.

Definition 6. The TBM conjunctive rule is noted with \odot ; let m_1 and m_2 be two BPAs, and let m_1 and m_2 be the result of the combination by \odot , as follows [19]:

$$m_{1 \odot 2}(A) = \sum_{B \cap C = A} m_1(B) m_2(C)$$
(11)

for all $A \subseteq \Omega$.

3.3. Base Belief Function

Definition 7. As a data conflict management tool, the base belief function is proposed in [20], and the base belief function is defined as:

$$m_b(A_i) = \frac{1}{2^N - 1},\tag{12}$$

where A_i is an arbitrary subset of Ω , N is the size of the FOD, and then the original BPA is modified by the following formula:

$$m'(A_i) = \frac{m_b(A_i) + m(A_i)}{2}.$$
 (13)

The base belief function can effectively eliminate the absolute conflict between data, and is suitable for large real-time update systems, such as military systems.

3.4. The Extension of Deng Entropy in the Non-Exhaustive FOD Assumption

Definition 8. The extension of Deng entropy [52] is an entropy measurement method, which extends from the exhaustive FOD to the non-exhaustive FOD, and its definition is as follows [64]:

$$E_{ebeow} = -\sum_{A \subseteq X} m(A) \log_2 \frac{m(A)}{\left(2^{(|A| + \lceil m(\emptyset)|X| \rceil)} - 1\right)},\tag{14}$$

in which |A| represents the number of elements contained in proposition A, X represents the FOD, and |X| represents the potential of the FOD, representing the number of exactly known elements in the FOD. [] is an upper-limit function (CEILING function), which involves rounding up the independent variable, that is, to an integer not less than the independent variable; for example: [0.2] = 1.

4. The Extended Base Belief Function

In this section, we propose a belief distribution function in the non-exhaustive FOD based on the base belief function, which uses the value of the natural constant *e* and the empty set mass function as a characteristic factor; this function is used to modify BPA. Meanwhile, this method is also applicable to the exhaustive FOD.

Mathematics 2020, 8, 2137 6 of 19

It is assumed that the FOD A contains N mutually exclusive elements; then, A has 2^N subsets, and in the non-exhaustive FOD, the value of the empty set mass function is not 0. Based on the above assumptions, the extended basic belief assignment function is defined as follows:

$$n_{eb}(R_i) = \frac{1}{2^N - e^{m(\emptyset)}},\tag{15}$$

where R_i represents an arbitrary subset of FOD, N is the size of the FOD, and $m(\emptyset)$ is the mass function value of the empty set.

Using the extended base belief distribution function $n_{eb}(R_i)$, the BPA of each piece of evidence is modified, and the modification method is defined as follows:

$$m'(R_i) = \frac{n_{eb}(R_i) + m(R_i)}{1 + \frac{\left[2^N + m(\emptyset) - 1\right]}{2^N - e^{m(\emptyset)}}},$$
(16)

where $m(R_i)$ is the value of the BPA of all the evidence. $\lceil \rceil$ is an upper-limit function (CEILING function), which involves rounding up the independent variable, that is, to an integer not less than the independent variable; for example: $\lceil 0.2 \rceil = 1$.

The purpose of the $n_{eb}(R_i)$ calculation is to assign equal probability to each subset of the FOD before the BPA is generated. Before every source of evidence appears, our belief in every situation must be equal. $n_{eb}(R_i)$ is equivalent to these original possibilities, which can eliminate the phenomenon of complete contradiction between evidences. Even if all sources of evidence weaken a hypothesis, the revised BPA will be close to but never zero. This method denies the absoluteness of evidence, that is, even if all the existing evidence is against a subset A, but we have not found all the evidence, there is still the possibility that A is right. At the same time, the nonzero mass value of empty set is added to the $n_{eb}(R_i)$ and fusion process as the characteristic factor. That is, in the non-exhaustive FOD, the information uncertainty caused by the nonzero empty set mass function is taken into account in the formula so that the method can be used in the non-exhaustive FOD and the modified BPA still meets the basic property that the sum of mass function values of all subsets is 1.

5. Application of the Extended Base Belief Function

In this section, we propose a conflict data management method based on the EBBF and give some examples and related applications to verify its rationality and effectiveness.

5.1. The Conflict Data Management Method Based on the EBBF

This section proposes a conflict data management method based on the EBBF to verify the applicability and effectiveness of the EBBF in the field of information fusion. Figure 1 designs the framework of the conflict data management method based on the EBBF, and the detailed steps are as follows:

Step 1: In the non-exhaustive FOD, there is a lot of uncertain information in practical applications. In order to model the uncertain information systematically, in the framework of Dempster–Shafer evidence theory, the first step is to use the BPA to model the uncertain information.

Step 2: Calculating the value of the EBBF: When the data are highly conflicting, a reasonable and effective method is needed to preprocess the BPA before further processing the data. This method uses the EBBF to modify the BPA. The corresponding calculation formula of extended base belief function (BBF) is as follows:

$$n_{eb}(R_i) = \frac{1}{2^N - e^{m(\emptyset)}}. (17)$$

Mathematics 2020, 8, 2137 7 of 19

Step 3: According to the calculated EBBF value, the BPA of each group is modified, and the modification formula is as follows:

$$m'(R_i) = \frac{n_{eb}(R_i) + m(R_i)}{1 + \frac{\left[2^N + m(\emptyset) - 1\right]}{2^N - e^{m(\emptyset)}}}.$$
(18)

Step 4: According to whether the empty set mass function is 0, the DCR or the GCR is selected for data fusion. According to the (n-1) DCR, the combination result of each proposition can be obtained as follows:

$$m(A) = (((m_1(A) \oplus m_2(A)) \oplus m_3(A))... \oplus m_{n-1}(A)) \oplus m_n(A), n \ge 2.$$
 (19)

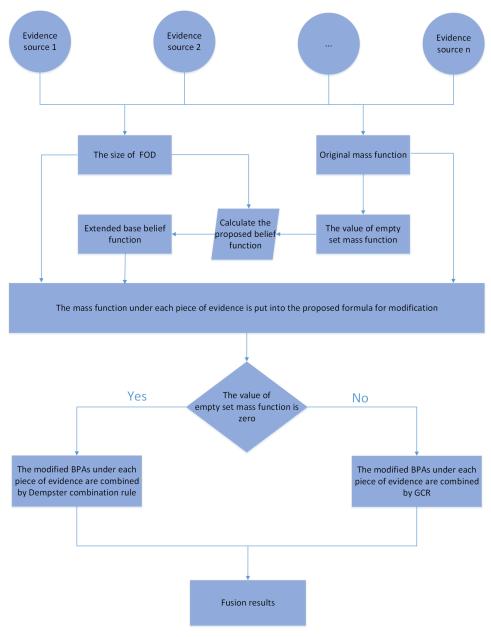


Figure 1. Conflict data management method framework based on the extended base belief function (EBBF).

Mathematics 2020, 8, 2137 8 of 19

5.2. Illustrative Example

Example 1. In the classic example that the FOD is $X = \{a, b, c\}$, the two BPAs are given as:

$$m_1(\lbrace a \rbrace) = 0.99, m_1(\lbrace c \rbrace) = 0.01, m_1(\lbrace \emptyset \rbrace) = 0$$
 (20)

$$m_2(\{b\}) = 0.99, m_2(\{c\}) = 0.01, m_2(\{\emptyset\}) = 0.$$
 (21)

In this example, the mass function of the empty set is 0, indicating that the mass function is allocated in the exhaustive FOD. The EBBF value is calculated as follows:

$$n_{eb}(R_i) = \frac{1}{2^3 - e^0} = 0.1429 = m_b(A_i).$$
 (22)

It is exactly the same as the value calculated with the base belief function. The BPA of the two groups is modified accordingly, and the results are shown in Table 1.

Table 1. Modified basic probability assignment (BPA) with the proposed method in Example 1.

BPA	$m(\{a\})$	$m(\{b\})$	$m(\{c\})$	$m(\{a,b\})$	$m(\{a,c\})$	$m(\{b,c\})$	$m(\{a,b,c\})$	$m(\{\emptyset\})$
$m_1(R_i)$	0.5664	0.0714	0.0764	0.0714	0.0714	0.0714	0.0714	0.0714
$m_2(R_i)$	0.0714	0.5664	0.0764	0.0714	0.0714	0.0714	0.0714	0.0714

Finally, the results of data fusion are shown in Figure 2. After consideration and research, the data fusion method we adopted here is the DCR instead of the TBM conjunctive rule. According to the formulas of the two methods, the difference between the two methods can be analyzed as follows: Both the TBM conjunctive rule and the DCR mainly consider the influence of intersection, but the TBM conjunctive rule is not normalized, which makes the sum of belief values less than 1. Obviously, such a result is counter-intuitive, so the TBM conjunctive rule is not suitable for the examples in this paper.

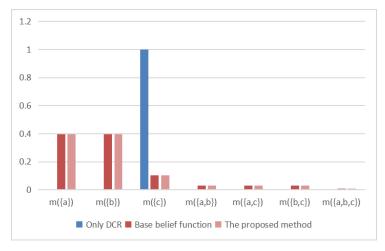


Figure 2. Fusion results of the two methods in Example 2.

The fusion results show that the DCR rule cannot produce logical results in this example, but both the method in this paper and the base belief function method can produce intuitive results. It can also be seen that under the exhaustive FOD assumption, the method in this paper can be reduced to base belief function.

Example 2. Suppose that the FOD is $X = \{a, b\}$, and two BPAs are given as:

$$m_1(\{a\}) = 0.9, m_1(\{\emptyset\}) = 0.1$$
 (23)

Mathematics **2020**, *8*, 2137 9 of 19

$$m_2(\{a\}) = 0.9, m_2(\{\emptyset\}) = 0.1.$$
 (24)

At this point, $m(\emptyset) \neq 0$, showing that the FOD is incomplete, and this example is under the non-exhaustive FOD assumption. The calculation process is shown in Figure 1.

Step 1: From the FOD $X = \{a, b\}$, we can know that the potential of the FOD is: N = 2.

Step 2: Bring the empty set mass function value and *N* into the formula and calculate the EBBF value as follows:

$$n_{eb}(R_i) = \frac{1}{2^2 - e^{0.1}} = 0.3454. \tag{25}$$

Step 3: Use the value of the belief function to modify the BPA of each piece of evidence and calculate as follows:

$$m_1(\{a\}) = \frac{0.3454 + 0.9}{1 + \frac{\left[2^2 + 0.1 - 1\right]}{2^2 - e^{0.1}}} = 0.5229$$
(26)

$$m_1(\{b\}) = m_1(\{a,b\}) = \frac{0.3454 + 0}{1 + \frac{\left[2^2 + 0.1 - 1\right]}{2^2 - \rho^{0.1}}} = 0.1450$$
 (27)

$$m_1(\{\emptyset\}) = \frac{0.3454 + 0.1}{1 + \frac{\left[2^2 + 0.1 - 1\right]}{2^2 - \rho 0.1}} = 0.1870$$
(28)

$$m_2(\{a\}) = \frac{0.3454 + 0.9}{1 + \frac{\lceil 2^2 + 0.1 - 1 \rceil}{2^2 - e^{0.1}}} = 0.5229$$
 (29)

$$m_2(\{b\}) = m_2(\{a,b\}) = \frac{0.3454 + 0}{1 + \frac{[2^2 + 0.1 - 1]}{2^2 - e^{0.1}}} = 0.1450$$
 (30)

$$m_2(\{\emptyset\}) = \frac{0.3454 + 0.1}{1 + \frac{\left[2^2 + 0.1 - 1\right]}{2^2 - \rho^{0.1}}} = 0.1870.$$
 (31)

Step 4: Judge whether the empty set mass function is 0 and select a different combination rule. In this example, the mass function value of the empty set is not 0, so the GCR is used to fuse the BPAs of two sets of evidence, and the following results are obtained.

$$m({a}) = 0.8053, m({b}) = 0.1195,$$
 (32)

$$m({a,b}) = 0.0398, m({\emptyset}) = 0.0350.$$
 (33)

The result shows a high support for proposition a, which is in line with the actual situation of the evidence. However, at the same time, the results also give b a certain small degree of support, which shows that we have not collected all the evidence; there is still the possibility that b is right, avoiding the data being too absolute.

Example 3. *In the FOD* $X = \{a, b\}$ *, there are two sets of highly conflicting BPAs, which are as follows:*

$$m_1(\{a\}) = 0.9, m_1(\{\emptyset\}) = 0.1$$
 (34)

$$m_2(\{b\}) = 0.9, m_2(\{\emptyset\}) = 0.1.$$
 (35)

Using the proposed method to calculate the belief function value, we can get:

$$n_{eb}(R_i) = 0.3454. (36)$$

Then, $n_{eb}(R_i)$ is used to modify the BPA value of each piece of evidence, as shown in Table 2.

Mathematics 2020, 8, 2137 10 of 19

Table 2. BPA modified with the prope	osed method in Example 3.
---	---------------------------

BPA	$m(\{a\})$	$m(\{b\})$	$m(\{a,b\})$	$m(\{\emptyset\})$
$m_1(R_i)$	0.5229	0.1450	0.1450	0.1870
$m_2(R_i)$	0.1450	0.5229	0.1450	0.1870

The fusion results obtained by the GCR are shown in Table 3. The data show that the proposed method gives the same degree of support for a and b in two sets of completely conflicting data, which is the correct answer in the intuition. It is proved that the proposed method can still give a good result in accordance with the facts in highly conflicting data.

Table 3. Fusion results in Example 3.

BPA	$m(\{a\})$	$m(\{b\})$	$m(\{a,b\})$	$m(\{\emptyset\})$
The proposed method	0.4548	0.4548	0.0554	0.0350

Example 4. *In the FOD* $X = \{a, b, c\}$ *, the BPAs are as follows:*

$$m_1(\{a\}) = 0.9, m_1(\{a,b\}) = 0.01, m_1(\{\emptyset\}) = 0.09$$
 (37)

$$m_2(\{b\}) = 0.01, m_2(\{c\}) = 0.91, m_2(\{\emptyset\}) = 0.08.$$
 (38)

Using the proposed method to calculate the values of the EBBF, we can get:

$$m_1: n_{eh}(R_i)_1 = 0.1448, m_2: n_{eh}(R_i)_2 = 0.1446.$$
 (39)

According to the values of the two EBBFs, the BPA of the two groups was modified accordingly, and the results are shown in Table 4.

Table 4. BPA modified with the proposed method in Example 4.

BPA	$m(\{a\})$	$m(\{b\})$	$m(\{c\})$	$m(\{a,b\})$	$m(\{a,c\})$	$m(\{b,c\})$	$m(\{a,b,c\})$	$m(\{\emptyset\})$
$m_1(R_i)$ $m_2(R_i)$	0.4841	0.0671	0.0671	0.0717	0.0671	0.0671	0.0671	0.1088
	0.0670	0.0717	0.4890	0.0670	0.0670	0.0670	0.0670	0.1041

Then, the GCR was used for fusion to obtain the result. At the same time, the BPA from the evidence in the table was fused directly with the GCR, and the results of the two methods were compared, as shown in Table 5 and Figure 3.

In this case, the two sets of evidence give high support to a and c, respectively, but the fusion result using only the GCR assigns the highest support to proposition b, which is obviously unreasonable. However, after the BPA was modified with the proposed method in this paper, the fusion results obtained by the GCR were highly supportive of both a and c. Moreover, as the support for c in the second piece of evidence was slightly greater than that in the first piece of evidence, the fusion results also gave a slight advantage to c. This indicates that in the case of highly conflicting data, the right intuitive answer cannot be obtained simply by using the GCR, while reasonable and intuitive results can be obtained by using the method in this paper.

The above examples verify that the proposed method is compatible with the base belief function in the exhaustive FOD and verify the feasibility and effectiveness of the proposed method. The following examples discuss some other properties of the proposed method.

Mathematics 2020, 8, 2137 11 of 19

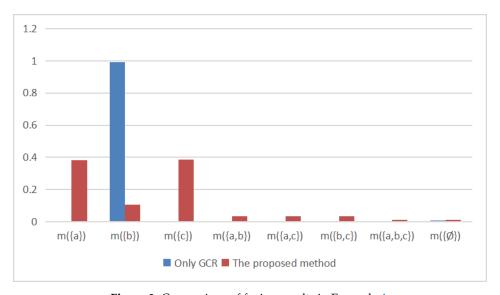


Figure 3. Comparison of fusion results in Example 4.

Table 5. Results of the two combination methods in Example 4.

Fusion Methods	$m(\{a\})$	$m(\{b\})$	$m(\{c\})$	$m(\{a,b\})$	$m(\{a,c\})$	$m(\{b,c\})$	$m(\{a,b,c\})$	$m(\{\emptyset\})$
Only GCR	0.0000	0.9928	0.0000	0.0000	0.0000	0.0000	0.0000	0.0072
The proposed method	0.3831	0.1061	0.3851	0.0354	0.0338	0.0338	0.0113	0.0113

Example 5. *In the FOD* $X = \{a, b, c\}$ *, the BPAs are as follows:*

$$m_1({a}) = 0.7, m_1({b}) = 0.1, m_1({c}) = 0.1, m_1({\emptyset}) = 0.1$$
 (40)

$$m_2({a}) = 0.1, m_2({b}) = 0.1, m_2({c}) = 0.7, m_2({\emptyset}) = 0.1.$$
 (41)

Using the proposed method to calculate the value of the EBBFs, we can get:

$$n_{eb}(R_i)_1 = n_{eb}(R_i)_2 = \frac{1}{2^3 - e^{0.1}} = 0.1454.$$
 (42)

Then, the BPA of each piece of evidence was modified by using the value of the calculated belief function, and the results are shown in Table 6.

Table 6. BPA modified with the proposed method in Example 5.

BPA	$m(\{a\})$	$m(\{b\})$	$m(\{c\})$	$m(\{a,b\})$	$m(\{a,c\})$	$m(\{b,c\})$	$m(\{a,b,c\})$	$m(\{\emptyset\})$
$m_1(R_i)$	0.3912	0.1134	0.1134	0.0671	0.0671	0.0671	0.0671	0.1134
$m_2(R_i)$	0.1134	0.1134	0.3912	0.0671	0.0671	0.0671	0.0671	0.1134

Then, the GCR was used for fusion to obtain the result. At the same time, the BPA from the evidence in the table was fused directly with the GCR, and the results of the two methods were compared, as shown in Table 7 and Figure 4.

Mathematics 2020, 8, 2137 12 of 19

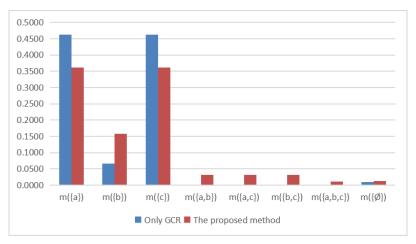


Figure 4. Comparison of the fusion results in Example 5.

Table 7. Results of the two combination methods in Example 5.

Fusion Methods	$m(\{a\})$	$m(\{b\})$	$m(\{c\})$	$m(\{a,b\})$	$m(\{a,c\})$	$m(\{b,c\})$	$m(\{a,b,c\})$	$m(\{\emptyset\})$
Only GCR	0.4620	0.0660	0.4620	0.0000	0.0000	0.0000	0.0000	0.0100
The proposed method	0.3620	0.1578	0.3620	0.0316	0.0316	0.0316	0.0105	0.0129

According to Table 7, when the mass function of each single element subset in the FOD is not 0, the fusion results obtained by using the method in this paper to modify the BPA and by using only the GCR both reflect that a and c have equally high confidence. We further calculated the entropy values of the original evidence mass function and the modified mass function. Here, we adopted the extended Deng entropy (EBEOW) [64], which can calculate the entropy value in the non-exhaustive FOD. According to the calculation, the extended Deng entropy value of the original BPAs is:

$$E_{eheow}(m)_1 = 2.9237. (43)$$

The extended Deng entropy of the BPAs modified by the proposed method in this paper is as follows:

$$E_{ebeow}(m)_2 = 3.8089. (44)$$

Obviously, the entropy corresponding to the fusion results of the proposed method increases significantly, and the increase in entropy indicates that the belief assignment is more dispersed and the uncertainty is increased, which is the consequence of the conflict. In general, when the proposition of a single element subset is not zero and the data are highly conflicting, the method proposed in this paper assigns part of the belief to other multi-subset elements, thus reducing the risk.

Example 6. Suppose that the FOD is $X = \{a, b, c\}$ and two BPAs are given as:

$$m_1(\{a\}) = 0.9, m_1(\{a, b, c\}) = 0.05, m_1(\{\emptyset\}) = 0.05$$
 (45)

$$m_2(\{b\}) = 0.9, m_2(\{a, b, c\}) = 0.05, m_2(\{\emptyset\}) = 0.05.$$
 (46)

According to the proposed method, the value of the belief function corresponding to evidence m_1 and m_2 is calculated as follows:

$$n_{eb}(R_i)_1 = n_{eb}(R_i)_2 = 0.1439.$$
 (47)

Then, the BPA of each piece of evidence was modified by using the value of the calculated belief function, and the results are shown in Table 8.

Mathematics 2020, 8, 2137 13 of 19

BPA	$m(\{a\})$	$m(\{b\})$	$m(\{c\})$	$m(\{a,b\})$	$m(\{a,c\})$	$m(\{b,c\})$	$m(\{a,b,c\})$	$m(\{\emptyset\})$
$m_1(R_i)$	0.4852	0.0669	0.0669	0.0669	0.0669	0.0669	0.0901	0.0901
$m_2(R_i)$	0.0669	0.4852	0.0669	0.0669	0.0669	0.0669	0.0901	0.0901

Table 8. BPA modified with the proposed method in Example 6.

Finally, the fusion results obtained with the GCR and the fusion results with the non-modified BPA obtained by the GCR are shown in Table 9 and Figure 5.

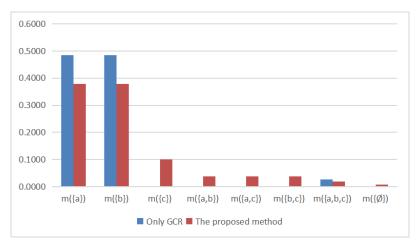


Figure 5. Comparison of the fusion results in Example 6.

Table 9. Results of the two combination methods in Example 6.

Fusion Methods	$m(\{a\})$	$m(\{b\})$	$m(\{c\})$	$m(\{a,b\})$	$m(\{a,c\})$	$m(\{b,c\})$	$m(\{a,b,c\})$	$m(\{\emptyset\})$
Only GCR	0.4853	0.4853	0.0000	0.0000	0.0000	0.0000	0.0270	0.0025
The proposed method	0.3796	0.3796	0.0998	0.0380	0.0380	0.0380	0.0187	0.0081

It can be seen from the results in Figure 5 that when all the mass functions of a complete set are nonzero, the fusion results obtained by using the proposed method of modifying the BPA in advance and by using only the GCR both reflect that a and b have equally high belief, which is an intuitive result. The extended belief entropies of the original evidence mass function and the modified mass function are further calculated. According to the calculation, the expanded belief entropy of the BPAs is $E_{ebeow}(m)_1 = 2.8185$, and the extended belief entropy of the BPAs modified by the proposed method is $E_{ebeow}(m)_2 = 3.8488$. The entropy value corresponding to the fusion result of this method obviously increases. Therefore, when the mass function of the whole element subset of each evidence is not 0 and the data are highly conflicting, the proposed method allocates part of the belief degree to other sub-propositions, thus reducing the risk.

Example 7. *In the FOD X* = $\{a, b, c\}$, the BPAs are as follows:

$$m_1(\lbrace a \rbrace) = 0.8, m_1(\lbrace b \rbrace) = 0.05, m_1(\lbrace c \rbrace) = 0.05, m_1(\lbrace \emptyset \rbrace) = 0.1$$
 (48)

$$m_2(\{b\}) = 0.8, m_2(\{a, b, c\}) = 0.1, m_2(\{\emptyset\}) = 0.1.$$
 (49)

Using the proposed method to calculate the belief function value, we can get:

$$n_{eb}(R_i)_1 = n_{eb}(R_i)_2 = 0.1450.$$
 (50)

Then, the GCR was used for fusion to obtain the result. At the same time, the BPA from the evidence in the table was fused directly with the GCR, and the results of the two methods were compared, as shown in Table 10 and Figure 6.

Mathematics 2020, 8, 2137 14 of 19

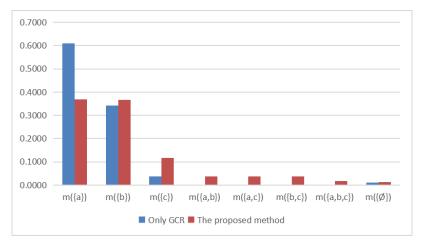


Figure 6. Comparison of the fusion results in Example 7.

Table 10. Results of the two combination methods in Example 7.

Fusion Methods	$m(\{a\})$	$m(\{b\})$	$m(\{c\})$	$m(\{a,b\})$	$m(\{a,c\})$	$m(\{b,c\})$	$m(\{a,b,c\})$	$m(\{\emptyset\})$
Only GCR	0.6092	0.3427	0.0381	0.0000	0.0000	0.0000	0.0000	0.0100
The proposed method	0.3699	0.3669	0.1176	0.0384	0.0384	0.0384	0.0176	0.0129

When the complete subset mass function of one set of evidence is nonzero, the value of all single element subset mass functions of the other set of evidence is nonzero, and the data are highly conflicting, the fusion results using only the GCR only give a a high confidence degree, while the fusion results using the method in this paper give a and b high support degrees. Obviously, the results of the proposed method are more in line with the objective reality.

5.3. Application to Artificial Data

In order to verify the availability and effectiveness of the conflict data management method proposed in this section, an example in [65] is used for example analysis, and the calculation results are compared with other methods.

Considering the problem of target recognition, it is assumed that three potential targets are represented as a, b, and c respectively. According to the conflict data management method based on the EBBF proposed in this section, the first step is to model uncertain information evidence. The reports of five sensors are modeled by the BPA, and the results are shown in Table 11. Table 11 slightly modifies the data in [65], causing an expansion from exhaustive FOD to non-exhaustive FOD. Intuitive analysis found that the report from the second sensor is inconsistent and conflicting with the other four sensors' reports. Moreover, the other four sensors reported that a was the most likely to be a potential target because its mass function value was the largest and the confidence level was the highest.

Table 11. BPAs of the application example. $W(\{x\}) = W(\{x\}) =$

BPA	$m(\{a\})$	$m(\{b\})$	$m(\{c\})$	$m(\{a,c\})$	$m(\{\emptyset\})$
1st sensor report: $m_1(\cdot)$	0.41	0.29	0.2	0	0.1
2nd sensor report: $m_2(\cdot)$	0	0.9	0.05	0	0.05
3rd sensor report: $m_3(\cdot)$	0.58	0.07	0	0.15	0.2
4th sensor report: $m_4(\cdot)$	0.55	0.1	0	0.15	0.2
5th sensor report: $m_5(\cdot)$	0.6	0.1	0.0	0.2	0.1

Based on the data in Table 11, the EBBF values of each group of evidence were calculated as follows:

$$n_{eb}(R_i)_1 = 0.1450, (51)$$

Mathematics 2020, 8, 2137 15 of 19

$$n_{eb}(R_i)_2 = 0.1439,$$
 (52)

$$n_{eb}(R_i)_3 = 0.1475, (53)$$

$$n_{eb}(R_i)_4 = 0.1475, (54)$$

$$n_{eb}(R_i)_5 = 0.1450. (55)$$

According to the EBBF values, the BPAs were modified, and the modified data of all BPAs in Table 11 are shown in Table 12.

Table 12. BPAs in the application after modification with the proposed method.

BPA	$m(\{a\})$	$m(\{b\})$	$m(\{c\})$	$m(\{a,b\})$	$m(\{a,c\})$	$m(\{b,c\})$	$m(\{a,b,c\})$	$m(\{\emptyset\})$
$m_1(\cdot)$	0.2569	0.2041	0.1597	0.0671	0.0671	0.0671	0.0671	0.1134
$m_2(\cdot)$	0.0669	0.4852	0.0901	0.0669	0.0669	0.0669	0.0669	0.0901
$m_3(\cdot)$	0.3337	0.0998	0.0677	0.0677	0.1365	0.0677	0.0667	0.1594
$m_4(\cdot)$	0.3199	0.1135	0.0677	0.0677	0.1365	0.0677	0.0667	0.1594
$m_5(\cdot)$	0.3449	0.1134	0.0671	0.0671	0.1597	0.0671	0.0671	0.1134

Data fusion using the generalized combination rule is as follows:

$$m(A) = (((m_1(A) \oplus m_2(A)) \oplus m_3(A)) \oplus m_4(A)) \oplus m_5(A) = 0.6840.$$
 (56)

Then, the GCR was used for fusion to obtain the result. At the same time, the BPAs from the evidence in the table were fused directly with the GCR, and the results of the two methods were compared, as shown in Table 13 and Figure 7.

The fusion results show that using the method proposed in this section, it can be concluded that a is the identified target, which is consistent with the intuitive analysis results. However, the result of GCR fusion without the EBBF shows that b is the identified target, while the possibility that a is the target is 0, which is obviously not an intuitive result. Through the above comparison, we can clearly see that the data fusion results verify the effectiveness of the method proposed in this section, which can be used well in conflict data management in practical engineering.

Table 13. Results of the two combination methods in the application to artificial data.

Fusion Methods	$m(\{a\})$	$m(\{b\})$	$m(\{c\})$	$m(\{a,b\})$	$m(\{a,c\})$	$m(\{b,c\})$	$m(\{a,b,c\})$	$m(\{\emptyset\})$
Only GCR	0.0000	0.8024	0.1976	0.0000	0.0000	0.0000	0.0000	2.0000×10^{-5}
The proposed method	0.6840	0.2025	0.1021	0.0011	0.0041	0.0011	3.3896×10^{-5}	2.9439×10^{-5}

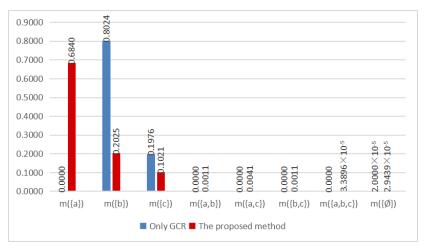


Figure 7. Comparison of the fusion results of the application.

Mathematics 2020, 8, 2137 16 of 19

5.4. Application to Classification

In order to verify the effectiveness of the proposed conflict data management method, a real data set for classification from the University of California Irvine (UCI) Machine Learning Repository was adopted in this application. In the iris data set of UCI, there are three types of irises: Setosa (*a*), Versicolor (*b*), and Virginia (*c*), each containing 50 samples. Each sample contains four attributes, namely calax length (SL), calax width (SW), petal length (PL), and petal width (PW). A group of data under non-exhaustive FOD, shown in Table 14, was obtained after the BPA generation process in [63].

Attribute	$m(\{a\})$	$m(\{b\})$	$m(\{c\})$	$m(\{a,b\})$	$m(\{a,c\})$	$m(\{b,c\})$	$m(\{a,b,c\})$	$m(\{\emptyset\})$
SL	0.6800	0.0000	0.0000	0.0610	0.0000	0.0000	0.1330	0.1260
SW	0.5030	0.0000	0.0000	0.0000	0.0100	0.0000	0.0000	0.4870
PL	0.9200	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0800
PW	0.8650	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.1350

Table 14. Data modeling results for irises with a mass function.

Using the proposed method to calculate the values of the EBBFs, we can get:

$$n_{eb}(R_i)_{SL} = 0.1457, n_{eb}(R_i)_{SW} = 0.1569, n_{eb}(R_i)_{PL} = 0.1446, n_{eb}(R_i)_{PW} = 0.1459.$$
 (57)

Then, the GCR was used for fusion to obtain the result. At the same time, the BPAs from the evidence in the table were fused directly with the GCR, and the results of the two methods were compared, as shown in Table 15.

Table 15. Results of the two combination methods in application to classification.

Fusion Methods	$m(\{a\})$	$m(\{b\})$	$m(\{c\})$	$m(\{a,b\})$	$m(\{a,c\})$	$m(\{b,c\})$	$m(\{a,b,c\})$	$m(\{\emptyset\})$
The proposed method	0.9207	0.0363	0.0339	0.0031	0.0027	0.0027	0.0002	0.0005

The fusion results are shown in Table 15. The results show that this method has good fusion performance. In the non-exhaustive FOD, the correct class *a* is still given a very high belief value, which is the intuitive result. The experiment shows that this method is also reasonable and effective in the real world.

6. Conclusions

In the exhaustive FOD, when the DCR is used to fuse highly conflicting data, the result is often contrary to intuition. In order to solve this problem, some scholars put forward the base belief function to modify the BPA to eliminate the absoluteness brought by conflict. Under the non-exhaustive FOD hypothesis, the generalized combination rule has the same problems as the DCR, but the base belief function cannot be applied to the non-exhaustive FOD. In this paper, an extended basic belief function based on the base belief function and a corresponding modified BPA method are proposed. This method is not only compatible with the base belief function and can be used for conflict data management in the exhaustive FOD, but can also be effectively applied in the non-exhaustive FOD. After modifying the BPA with the EBBF, the absoluteness brought by highly conflicting data can be effectively eliminated. This method not only considers the potential of the FOD, but also considers the uncertain information sources that are not taken into account by the base belief function and other existing methods, including the uncertain information brought by the nonzero mass function of empty set and the incomplete FOD. In order to verify the application of the proposed method in practice, this paper also designs a conflict data management method based on the EBBF and discusses and verifies the feasibility and effectiveness of the method through some examples and applications.

However, the method proposed in this paper still has some open issues that are worth discussing. The first is the problem of computational complexity. The method proposed in this paper has as large

Mathematics **2020**, *8*, 2137

of an amount of calculation as the GCR, but it can be applied to more situations than the GCR. In short, the method proposed in this paper is not simple, but it is effective. Then, since this method is proposed under the condition of incomplete information, and in the non-exhaustive FOD, there are many sources of uncertain information, the symbolic empty set mass function is used in this method. In subsequent improvements, other uncertain information sources can be considered.

Author Contributions: Conceptualization, B.Y., D.G., and Y.T.; Data curation, B.Y.; Formal analysis; Methodology, B.Y., D.G., and Y.T.; Project administration, Y.T.; Software, B.Y. and D.G.; Validation, D.G., and Y.T.; Visualization, B.Y. and D.G.; Writing—original draft, B.Y. and D.G.; Writing—review and editing, Y.T. All authors have read and agreed to the published version of the manuscript.

Funding: The work is partially supported by the SRTP of Chongqing University (Grant No. 02250051071001).

Conflicts of Interest: The authors declare no conflict of interest.

References

- 1. White, F.E. A model for data fusion. In Proceedings of the 1st National Symposium on Sensor Fusion, Orlando, FL, USA, 5–8 April 1988; Volume 2, pp. 149–158.
- 2. Li, S.; Tang, Y. A Simple Framework of Smart Geriatric Nursing considering Health Big Data and User Profile. *Comput. Math. Methods Med.* **2020**, 2020, 5013249. [CrossRef]
- 3. Feller, W. An Introduction to Probability Theory and Its Applications; John Wiley & Sons: Hoboken, NJ, USA, 2008.
- 4. Zadeh, L.A. Fuzzy sets. Inf. Control 1965, 8, 338–353. [CrossRef]
- 5. Dempster, A.P. Upper and Lower Probabilities Induced by a Multivalued Mapping. *Ann. Math. Stat.* **1967**, 38, 325–339. [CrossRef]
- 6. Shafer, G. A Mathematical Theory of Evidence; Princeton University Press: Princeton, NJ, USA, 1976; Volume 42.
- 7. Liang, J.; Shi, Z.; Li, D.; Wierman, M.J. Information entropy, rough entropy and knowledge granulation in incomplete information systems. *Int. J. Gen. Syst.* **2006**, *35*, 641–654. [CrossRef]
- 8. Paninski, L. Estimation of Entropy and Mutual Information. Neural Comput. 2014, 15, 1191–1253. [CrossRef]
- 9. Song, Y.; Wang, X.; Wu, W.; Quan, W.; Huang, W. Evidence combination based on credibility and non-specificity. *Pattern Anal. Appl.* **2018**, 21, 167–180. [CrossRef]
- 10. Fu, C.; Chin, K.S. Robust evidential reasoning approach with unknown attribute weights. *Knowl. Based Syst.* **2014**, *59*, 9–20. [CrossRef]
- 11. Li, Y.; Chen, J.; Ye, F.; Liu, D. The improvement of DS evidence theory and its application in IR/MMW target recognition. *J. Sens.* **2016**, *2016*. [CrossRef]
- 12. Tang, Y.; Zhou, D.; Jiang, W. A new fuzzy-evidential controller for stabilization of the planar inverted pendulum system. *PLoS ONE* **2016**, *11*, e0160416. [CrossRef]
- 13. Yager, R.R.; Filev, D.P. Including probabilistic uncertainty in fuzzy logic controller modeling using Dempster-Shafer theory. *IEEE Trans. Syst. Man Cybern.* **1995**, *25*, 1221–1230. [CrossRef]
- 14. Xu, X.; Li, S.; Song, X.; Wen, C.; Xu, D. The optimal design of industrial alarm systems based on evidence theory. *Control Eng. Pract.* **2016**, *46*, 142–156. [CrossRef]
- 15. Liu, Z.G.; Liu, Y.; Dezert, J.; Cuzzolin, F. Evidence combination based on credal belief redistribution for pattern classification. *IEEE Trans. Fuzzy Syst.* **2020**, *28*, 618–631. [CrossRef]
- 16. Liu, Z.G.; Pan, Q.; Dezert, J.; Martin, A. Combination of Classifiers with Optimal Weight Based on Evidential Reasoning. *IEEE Trans. Fuzzy Syst.* **2018**, *26*, 1217–1230. [CrossRef]
- 17. Song, Y.; Wang, X.; Zhu, J.; Lei, L. Sensor dynamic reliability evaluation based on evidence theory and intuitionistic fuzzy sets. *Appl. Intell.* **2018**, *48*, 3950–3962. [CrossRef]
- 18. Smets, P. The combination of evidence in the transferable belief model. *IEEE Trans. Pattern Anal. Mach. Intell.* **1990**, 12, 447–458. [CrossRef]
- 19. Denœux, T. Conjunctive and disjunctive combination of belief functions induced by nondistinct bodies of evidence. *Artif. Intell.* **2008**, 172, 234–264. [CrossRef]
- 20. Wang, Y.; Zhang, K.; Deng, Y. Base belief function: An efficient method of conflict management. *J. Ambient. Intell. Humaniz. Comput.* **2019**, *10*, 3427–3437. [CrossRef]

Mathematics 2020, 8, 2137 18 of 19

21. Ni, S.; Lei, Y.; Tang, Y. Improved Base Belief Function-Based Conflict Data Fusion Approach Considering Belief Entropy in the Evidence Theory. *Entropy* **2020**, 22, 801. [CrossRef]

- 22. Li, R.; Li, H.; Tang, Y. An improved method to manage conflict data using elementary belief assignment function in the evidence theory. *IEEE Access* **2020**, *8*, 37926–37932. [CrossRef]
- 23. Deng, Y. Generalized evidence theory. Appl. Intell. 2015, 43, 530–543. [CrossRef]
- 24. Martin, A. About conflict in the theory of belief functions. In *Belief Functions: Theory and Applications*; Springer: Berlin, Germany, 2012; pp. 161–168.
- 25. Pichon, F.; Destercke, S.; Burger, T. A consistency-specificity trade-off to select source behavior in information fusion. *IEEE Trans. Cybern.* **2014**, *45*, 598–609. [CrossRef] [PubMed]
- 26. Jiang, W.; Zhan, J. A modified combination rule in generalized evidence theory. *Appl. Intell.* **2017**, *46*, 630–640. [CrossRef]
- 27. Yager, R.R. On the Dempster-Shafer framework and new combination rules. *Inf. Sci.* **1987**, 41, 93–137. [CrossRef]
- 28. Dubois, D.; Prade, H. Combination of fuzzy information in the framework of possibility theory. *Data Fusion Robot. Mach. Intell.* **1992**, 12, 481–505.
- 29. Lefevre, E.; Colot, O.; Vannoorenberghe, P. Belief function combination and conflict management. *Inf. Fusion* **2002**, *3*, 149–162. [CrossRef]
- 30. Su, X.; Li, L.; Qian, H.; Mahadevan, S.; Deng, Y. A new rule to combine dependent bodies of evidence. *Soft Comput.* **2019**, 23, 9793–9799. [CrossRef]
- 31. Zhang, W.; Deng, Y. Combining conflicting evidence using the DEMATEL method. *Soft Comput.* **2019**, 23, 8207–8216. [CrossRef]
- 32. Martin, A. Conflict management in information fusion with belief functions. In *Information Quality in Information Fusion and Decision Making*; Springer: Berlin, Germany, 2019; pp. 79–97.
- 33. Wu, D.; Liu, Z.; Tang, Y. A new classification method based on the negation of a basic probability assignment in the evidence theory. *Eng. Appl. Artif. Intell.* **2020**, *96*, 103985. [CrossRef]
- 34. Chen, Y.; Tang, Y.; Lei, Y. An improved data fusion method based on weighted belief entropy considering the negation of basic probability assignment. *J. Math.* **2020**, 2020, 1594967. [CrossRef]
- 35. Xu, P.; Deng, Y.; Su, X.; Mahadevan, S. A new method to determine basic probability assignment from training data. *Knowl. Based Syst.* **2013**, *46*, 69–80. [CrossRef]
- 36. Smets, P.; Kennes, R. The transferable belief model. Artif. Intell. 1994, 66, 191–234. [CrossRef]
- 37. Zhang, J.; Deng, Y. A method to determine basic probability assignment in the open world and its application in data fusion and classification. *Appl. Intell.* **2017**, *46*, 934–951. [CrossRef]
- 38. Xiao, F. A hybrid fuzzy soft sets decision making method in medical diagnosis. *IEEE Access* **2018**, 6, 25300–25312. [CrossRef]
- 39. Zheng, H.; Deng, Y. Evaluation method based on fuzzy relations between Dempster–Shafer belief structure. *Int. J. Intell. Syst.* **2018**, *33*, 1343–1363. [CrossRef]
- 40. Zhang, R.; Ashuri, B.; Deng, Y. A novel method for forecasting time series based on fuzzy logic and visibility graph. *Adv. Data Anal. Classif.* **2017**, *11*, 759–783. [CrossRef]
- 41. Han, Y.; Deng, Y. An enhanced fuzzy evidential DEMATEL method with its application to identify critical success factors. *Soft Comput.* **2018**, 22, 5073–5090. [CrossRef]
- 42. Yager, R.R. Entropy and specificity in a mathematical theory of evidence. *Int. J. Gen. Syst.* **1983**, *9*, 249–260. [CrossRef]
- 43. Hohle, U. Entropy with respect to plausibility measures. In Proceedings of the 12th IEEE International Symposium on Multiple-Valued Logic, Paris, France, 23–25 May 1982.
- 44. Klir, G.J.; Ramer, A. Uncertainty in the Dempster-Shafer theory: A critical re-examination. *Int. J. Gen. Syst.* **1990**, *18*, 155–166. [CrossRef]
- 45. Klir, G.J.; Parviz, B. A note on the measure of discord. In *Uncertainty in Artificial Intelligence*; Elsevier: Amsterdam, The Netherlands, 1992; pp. 138–141.
- 46. George, T.; Pal, N.R. Quantification of conflict in Dempster-Shafer framework: A new approach. *Int. J. Gen. Syst.* **1996**, 24, 407–423. [CrossRef]
- 47. Xiao, F. Multi-sensor data fusion based on the belief divergence measure of evidences and the belief entropy. *Inf. Fusion* **2019**, *46*, 23–32. [CrossRef]

Mathematics 2020, 8, 2137 19 of 19

48. Shannon, C.E. A mathematical theory of communication. *ACM Sigmobile Mob. Comput. Commun. Rev.* **2001**, 5, 3–55. [CrossRef]

- 49. Cao, S.; Dehmer, M. Degree-based entropies of networks revisited. *Appl. Math. Comput.* **2015**, 261, 141–147. [CrossRef]
- 50. Xie, W.; Schultz, M.D.; Lister, R.; Hou, Z.; Rajagopal, N.; Ray, P.; Whitaker, J.W.; Tian, S.; Hawkins, R.D.; Leung, D.; et al. Epigenomic analysis of multilineage differentiation of human embryonic stem cells. *Cell* **2013**, *153*, 1134–1148. [CrossRef] [PubMed]
- 51. Wu, D.; Tang, Y. An improved failure mode and effects analysis method based on uncertainty measure in the evidence theory. *Qual. Reliab. Eng. Int.* **2020**, *36*, 1786–1807. [CrossRef]
- 52. Deng, Y. Deng entropy. Chaos Solitons Fractals 2016, 91, 549–553. [CrossRef]
- 53. Jiroušek, R.; Shenoy, P.P. A new definition of entropy of belief functions in the Dempster–Shafer theory. *Int. J. Approx. Reason.* **2018**, 92, 49–65. [CrossRef]
- 54. Jiroušek, R.; Shenoy, P.P. On properties of a new decomposable entropy of Dempster-Shafer belief functions. *Int. J. Approx. Reason.* **2020**, 119, 260–279. [CrossRef]
- 55. Yuan, K.; Xiao, F.; Fei, L.; Kang, B.; Deng, Y. Conflict management based on belief function entropy in sensor fusion. *SpringerPlus* **2016**, *5*, 638. [CrossRef]
- 56. Zhou, D.; Tang, Y.; Jiang, W. An improved belief entropy and its application in decision-making. *Complexity* **2017**, 2017, 4359195. [CrossRef]
- 57. Yang, B.; Gan, D.; Tang, Y.; Lei, Y. Incomplete Information Management Using an Improved Belief Entropy in Dempster-Shafer Evidence Theory. *Entropy* **2020**, *22*, 993. [CrossRef]
- 58. Tang, Y.; Zhou, D.; Xu, S.; He, Z. A weighted belief entropy-based uncertainty measure for multi-sensor data fusion. *Sensors* **2017**, *17*, 928. [CrossRef] [PubMed]
- 59. Abellán, J. Analyzing properties of Deng entropy in the theory of evidence. *Chaos Solitons Fractals* **2017**, 95, 195–199. [CrossRef]
- 60. Zhou, D.; Tang, Y.; Jiang, W. A modified belief entropy in Dempster-Shafer framework. *PLoS ONE* **2017**, 12, e0176832. [CrossRef] [PubMed]
- 61. Moral-García, S.; Abellán, J. Critique of modified Deng entropies under the evidence theory. *Chaos Solitons Fractals* **2020**, 140, 110112. [CrossRef]
- 62. Haenni, R. Are alternatives to Dempster's rule of combination real alternatives?: Comments on 'About the belief function combination and the conflict management problem'—Lefevre et al. *Inf. Fusion* **2002**, 3, 237–239. [CrossRef]
- 63. Jing, M.; Tang, Y. A new base basic probability assignment approach for conflict data fusion in the evidence theory. *Appl. Intell.* **2020**. [CrossRef]
- 64. Tang, Y.; Zhou, D.; Chan, F.T.S. An extension to Deng's entropy in the open world assumption with an application in sensor data fusion. *Sensors* **2018**, *18*, 1902. [CrossRef]
- 65. Deng, Y.; Shi, W.; Zhu, Z.; Liu, Q. Combining belief functions based on distance of evidence. *Decis. Support Syst.* **2004**, *38*, 489–493.

Publisher's Note: MDPI stays neutral with regard to jurisdictional claims in published maps and institutional affiliations.



© 2020 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).