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A new base basic probability assignment approach for conflict data fusion in the evidence theory

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Abstract

Dempster-Shafer evidence theory (D-S theory) is applied to process uncertain information in different scenarios. However, traditional Dempster combination rule may produce counterintuitive results while dealing with highly conflicting data. Inspired by a perspective of constructing base belief function for conflicting data processing in D-S theory, a new base basic probability assignment (bBPA) method is proposed to process the potential conflict before data fusion. Instead of assigning initial belief on the whole power set space, the new method assigns the base belief to basic events in the frame of discernment. Consequently, the bBPA is consistent with the classical probability theory. Several numerical examples are adopted to verify the reliability and accuracy of the method in processing highly conflicting data. The data sets in the University of California Irvine (UCI) Machine Learning Repository are used to verity the availability of the new method in classification problem. Experimental result shows that the new method has some superiority in dealing with highly conflicting data.

Keywords Dempster-Shafer evidencve theory · Basic probability assignment · Conflict management · Conflicting data fusion

1 Introduction

Dempster-Shafer evidence theory (D-S theory) [1, 2] has been adopted to process uncertain information in many domains such as classification [3, 4], clustering [5–7], fault diagnosis [8, 9], knowledge-based system [10, 11], medical diagnosis [12], sensor data fusion [13], decision making [14, 15], risk analysis [16–19], and so on [20]. Many efforts have been given to address the open issues in D-S theory. First of all, the generation of basic probability assignment (BPA) is the base of applying D-S theory [21]. Secondly, the fusion of conflicting data is a hot topic in both theory and practical domains [22, 23]. D-S theory may produce counterintuitive results while dealing with highly conflicting data. Thirdly, decision making based on mass function [24, 25]. The generated BPA cannot directly provide the probability of occurrences. How to transform the BPA to probability is a big problem [26]. Fourthly, the problem of high computational complexity [27, 28]. The computational complexity of Dempster combination rule is high. After the space is expanded, the number of the events in the power set increases exponentially and the computational complexity is high. Finally, the function for evidence evaluation [29], e.g. belief entropy-based methods for uncertainty measure [30–32]. This paper focuses on conflicting data fusion.

In D-S theory, a variety of methods are adopted for uncertain information and conflicting data management [33]. The first class is to modify the combination rule. Many researchers try to modify the combination rule in D-S theory to handle highly conflicting data [34–36]. In [37], a new rule based on the concept of joint belief distribution is proposed. These methods may lose the mathematical nature of the Dempster combination rule and fails to meet the associative and commutativity laws, which make it difficult to be widely applied in engineering. The second class is to modify the conflicting data before data fusion [22, 38]. In [39], the belief entropy is adopted to preprocess the conflicting data before data fusion. The uncertainty of evidence is modeled before evidence combination in [23].

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Evidential reasoning (ER) is also developed in D-S theory for uncertain information modeling and processing [40–42]. It enhances Dempster rule by determining how to combine multiple completely reliable evidence and addresses the conflicting data well. A general ER algorithm has been proposed in [43] which extends the original ER algorithm. In [44], ER is adopted to design a new multi-attribute decision making method considering incomplete interval value. ER model for discrete belief structure is proposed in [45]. It solves the problem of non-conflicting discrete evidence combination and overcomes the counterintuitive results of combining internal or external conflicting evidence. A method was proposed to solve the problems of weight over-bounding and reliability-dependence of ER in [46]. Many recognition frameworks based on ER have been proposed for practical applications. In [47], a new fuzzy multi attribute group decision making method based on intuitionistic fuzzy sets and ER was proposed. In [48], a new weighted combination method using ER for multiple classifiers working with different features of pattern was proposed. The ER algorithm is also used in many other domains, such as online safety assessment [49] and decision support system [50]. The recognition framework in ER has a good performance for the system with high accuracy requirement. To address conflicting data in uncertain information, some methods try to assign the initial belief to the events in the frame of discernment (FOD). A new strategy considering the initial belief degree of a proposition in the power set of FOD is proposed in [51], but it may cause a decentralized belief assignment. A decentralized belief assignment increases the belief degree of multi subset propositions. It causes the confidence to be assigned to meaningless sets and is not helpful for uncertain information fusion and decision making. In addition, the mutual exclusion events may not occur at the same time from the perspective of probability theory. Thus, assigning belief on the power set space averagely may be not efficient. To address this issue, a new base basic probability assignment (bBPA) method is proposed in this paper.

The bBPA is proposed to deal with uncertain information processing. In bBPA, the initial belief is averagely assigned among basic events in FOD. The initial belief assigned among basic events can introduce prior probability information to the element. In an unknown situation, the average distribution of belief will maximize the entropy. The principle of maximum entropy shows that when the entropy is the largest, the possible loss will be the smallest. Therefore, the average distribution of initial belief among elements brings in reasonable prior information. This strategy completes the deficiency of BPA that only depends on a single observation. By fusing the prior probability information, the bBPA performs better in the fusion of conflicting and non-conflicting data.

Compared to the strategy in [51], the motivation and contribution of bBPA are as follows. Firstly, in practical applications, each mutually exclusive event is independent with each other. There may be only one element at a time. Multi-subset events represent the uncertainty in occurrence of different elements. Secondly, bBPA has a lower computational complexity because it does not assign initial belief on the entire power set space of FOD, which can be a merit in real-time system. Thirdly, the bBPA avoids assigning initial belief on the entire power set space, which will decrease the belief loss on the multiple subsets and can be helpful for decision making. Fourthly, the bBPA can be well explained from the perspective of Bayesian theory, and it can integrate the prior probability information. So, the bBPA cannot only reflect the single observation result, but also the prior information. Numerical examples and two experiments on UCI Machine Learning Repository are used to verify the effectiveness and rationality of the new method.

The remainder of this paper is organized as follows. The preliminaries are introduced in Section 2. In Section 3, the base basic probability assignment method for conflicting data is proposed as well as a variety of numerical examples. In Section 4, the UCI Machine Learning Repository is adopted to verify the effectiveness of the new method in classification problem. Conclusions are given in Section 5.

2 Preliminaries

2.1 Dempster-Shafer evidence theory

D-S theory is proposed by Dempster [1] and Shafer [2]. Some basic elements are as follows.

Define a set that consists N events, each event is independent and mutually exclusive with others. The set of the N events is called the frame of discernment (FOD) and can be indicated as follows:

$$\Omega = \{H_1, H_2, H_3, \ldots, H_N\} . \tag{1}$$

The power set of Ω, which is composed with 2^N propositions, can be denoted as follows:

$$2^\Omega = \{\emptyset, \{H_1\}, \{H_2\}, ..., \{H_N\}, \{H_1, H_2\}, ..., \{H_1, H_2, ..., H_i\}, ..., \Omega\} . \tag{2}$$

A mass function is a mapping m from 2^Ω to [0, 1], defined as:

$$m : 2^\Omega \to [0, 1] , \tag{3}$$

which satisfies:

$$\sum_{A\in 2^{\Omega}} m(A) = 1, \tag{4}$$

$$m(\emptyset) = 0. \tag{5}$$

A mass function is also called a basic probability assignment (BPA) which represents the possibility of evidence A that supports the claim. In a real system, BPAs may come from different sensors, Dempster combination rule for BPAs is defined as follows:

$$m(A) = \begin{cases} \frac{1}{1-K} \sum\limits_{A=B\cap C} \mathrm{m}_1(B)m_2(C), & A \neq \emptyset \\ 0, & A = \emptyset \end{cases} \tag{6}$$

where k is a normalization factor defined as:

$$K = \sum_{B\cap C=\emptyset} m_1(B)m_2(C). \tag{7}$$

$K = 0$ means that two groups of BPAs are identical. $K = 1$ means that the two groups of BPAs are completely conflicting. In this case, Dempster combination rule cannot be used directly for evidence fusion. For multiple sets of BPA, this formula is commutative and associative.

2.2 Conflict management with base belief function

Dempster combination rule may produce counterintuitive results while dealing with highly conflicting data.

Example 1 (Zadeh 1986 [52]) Suppose the FOD is $\Omega = \{\mathrm{a}, b, c\}$ and two BPAs are given as follows:

$m_1(a) = 0.99, m_1(a, b) = 0.01,$

$m_2(b) = 0.01, m_2(c) = 0.99.$

The result of data fusion with Dempster combination rule is $m(b) = 1$, which is counterintuitive. Intuitively, the event b is supported with little belief, but the belief on b is magnified with Dempster combination rule.

To address this problem, in [51], the base belief function is proposed to modify the BPA before data fusion. Let Ω be a set of N possible values which are mutually exclusive. The power set of Ω is 2^{Ω}, in which the number of elements is 2^N. If the FOD is complete, $m(\emptyset) = 0$. Thus, the base belief function m_b is defined as follows [51]:

$$m_b(A_i) = \frac{1}{2^N - 1}, \tag{8}$$

where A_i is the subset in Ω except for the empty set $\emptyset$. Then m_b is adopted to modify the initial BPA m by calculating the arithmetic mean [51]:

$$m'(A_i) = \frac{m_b(A_i) + m(A_i)}{2}. \tag{9}$$

The base belief function assigns each subset in the power set an equal initial belief. It should be noted that the base belief function assigns the belief to the entire power set space of FOD.

3 The new base basic probability assignment for evidence modification

3.1 New base basic probability assignment

In a closed-world assumption, let Ω be a set of N possible values which are mutually exclusive. $\Omega = \{A_1, A_2, A_3, ...A_i, \ldots, A_N\}$. The power set of Ω is 2^{Ω}, in which the number of elements is 2^N. As an improved approach of the base belief function in [51], the new base basic probability assignment (bBPA), denoted as $m_b(A_i)$, is defined as follows:

$$m_b(A_i) = \begin{cases} \frac{1}{N}, & \text{if } A_i \ is \text{a single subset event,} \\ 0, & \text{if } A_i \text{ is not a single subset event.} \end{cases} \tag{10}$$

The new bBPA $m_b(A_i)$ is designed to modify the original BPAs for decreasing the potential conflict among different bodies of evidence. The modified mass function can be calculated and presented as follows:

$$m'(A_i) = \frac{m_b(A_i) + m(A_i)}{2}. \tag{11}$$

Instead of assigning base belief among the whole power set space with (8), the new bBPA distributes the base belief averagely on the basic mutual exclusive element, which is consistent with the classical probability theory.

According to classical probability theory, the mutually exclusive events cannot happen at the same time. Therefore, the bBPA averagely distributes the confidence to the basic mutually exclusive element in FOD. Wang et al. use the 'color balls in an opaque bag' as an example to explain the assignment scheme of the base belief in [51]. In detail, there are three balls with different colors and one opaque bag, and, at least one ball is in the bag. We assume that, each time, one can pick up a ball randomly from the bag. Each ball can be selected with an equal probability at a time. The mass value is averagely distributed among each basic event. In a closed-world assumption, each event is independent with each other. Only one event happens at a time. This prior information can be modeled with the new bBPA in (11).

The bBPA assigns the initial belief averagely to the basic event which is single subset element. Apart from the empty set, there are N basic events in a closed system. Thus, the bBPA assigned to each single subset element is $1/N$ and the bBPA for the non-single subset proposition is 0. It can be explained as that an initial prior probability is assigned to each basic element for overcoming the drawback of Dempster combination rule in fusing conflicting data.

3.2 Compatibility with Bayesian theory

In Bayesian theory, the posterior probability is proportional to the prior probability. New data can be obtained from valuable information to modify the prior probability. The bBPA assigns the initial belief averagely to basic events, which means a combination of prior probability information and observation information. In an unknown situation, the average distribution of belief will maximize the entropy. The principle of maximum entropy shows that when the entropy is the largest, the possible loss will be the smallest. Therefore, the bBPA embodies the principle of maximum entropy. Furthermore, if we have a certain understanding of the unknown event, which corresponds to the prior probability information of the event, we can use the prior probability to update the initial probability distribution. Specifically, for each basic event, (10) can be updated as:

$$m_b(A_i) = \begin{cases} P(A_i), & \text{if } A_i \text{ is a single subset event,} \\ 0, & \text{if } A_i \text{ is not a single subset event,} \end{cases} \tag{12}$$

where $P(A_i)$ represents a priori probability of A_i. Note that the sum of the prior probabilities of basic events should equal to 1, denoted as follows:

$$\sum_{i=1}^{N} P(A_i) = 1. \tag{13}$$

According to (12), we combine BPA with prior information, which will be helpful for information fusion and decision making under uncertainty. In general, we consider the entire system. At the beginning, we have no information about the occurrence probability of basic event in the system. Considering the principle of maximum entropy, we use bBPA to modify BPA. Once we have enough priori information about the event, (12) can be used to introduce prior information to BPA. In case the prior information has a strong interference with the results, we can set a threshold of the prior probability based on the specific problem. An example of this process will be introduced in Section 4. The following examples will explain and analyze the advantages of using bBPA under the principle of maximum entropy in dealing with highly conflicting data, where bBPA can be regarded as the prior information in the form of Bayesian probability.

3.3 Numerical examples

3.3.1 Conflicting data fusion

The following numerical examples are proposed to verify the effectiveness of the bBPA method in dealing with highly conflicting data. The result is compared with the method in [51]. First of all, recall the classic conflicting data fusion case proposed by Zadeh [52].

Example 2 Define that an FOD is $\Omega = \{a, b, c\}$, two BPAs are given as follows:

$$m_1(a) = 0.99, m_1(a, b) = 0.01,$$

$$m_2(b) = 0.01, m_2(c) = 0.99.$$

With the base belief function m_b in (8), the following base belief functions are constructed:

$$\begin{aligned} m_b(a) &= m_b(b) = m_b(c) = m_b(a, b) = m_b(a, c) \\ &= m_b(b, c) = m_b(a, b, c) = \frac{1}{7}. \end{aligned}$$

Consequently, the fusion result of the conflicting data after base belief function-based modification is as follows:

$$m(a) = m(c) = 0.3957, m(b) = 0.1012, m(a, b) = 0.0337,$$
$$m(a, c) = m(b, c) = 0.0322, m(a, b, c) = 0.0107.$$

Based on the new method in (10), the proposed bBPA function for this case should be given as follows:

$$m_b(a) = m_b(b) = m_b(c) = \frac{1}{3}.$$

According to (11), the result of the bBPA-based evidence modification is:

$$m_1'(a) = \frac{m_1(a)+m_b(a)}{2} = 0.6617, m_1'(b) = \frac{m_1(b)+m_b(b)}{2} = 0.1667,$$
$$m_1'(c) = \frac{m_1(c)+m_b(c)}{2} = 0.1667, m_1'(a, b) = \frac{m_1(a,b)+m_b(a,b)}{2} = 0.005.$$

$$m_2'(a) = \frac{m_2(a)+m_b(a)}{2} = 0.1667, m_2'(b) = \frac{m_2(b)+m_b(b)}{2} = 0.1717,$$
$$m_2'(c) = \frac{m_2(c)+m_b(c)}{2} = 0.6617.$$

Consequently, with Dempster combination rule, the modified conflicting data can be fused as follows:

$$m(a) = 0.4429, m(b) = 0.1175, m(c) = 0.4396.$$

In both of the original BPAs, the belief of $\{a\}$ is the same as that of $\{c\}$. However, since the first evidence assigns a belief on $\{a, b\}$, the proposition $\{a\}$ may have a higher uncertain belief than the proposition $\{c\}$. From this perspective, the new bBPA-based fusion results are more reasonable in this case because it catches more uncertainty than the original method. In addition, there is no belief on the uncertain events such as $\{a, b\}$, and $\{a, b, c\}$, which is helpful for a centralized belief assignment. The new method can address the uncertain information more accurately by catching more uncertainty in the BPAs. Above all, the new bBPA-based method contributes to belief convergence for single subset proposition, which is helpful for decision making in practical applications.

Example 3 Suppose that the FOD is $\Omega = \{a, b\}$ and two BPAs are given as:

$$m_1(a) = 1, \quad m_1(b) = 0, \quad m_1(a, b) = 0,$$
$$m_2(a) = 0, \quad m_2(b) = 1, \quad m_2(a, b) = 0.$$

With the base belief function m_b in (8), the following base belief functions are constructed:

$$m_b(a) = m_b(b) = m_b(a, b) = \frac{1}{3}.$$

The modified BPA can be fused by using Dempster combination rule and the result is as follows:

$$m(a) = m(b) = 0.4737, \; m(a, b) = 0.0526.$$

With the proposed method in (10), the new bBPA for BPA modification should be:

$$m_b(a) = m_b(b) = \frac{1}{2}, m_b(a, b) = 0.$$

With (11), the result of the bBPA-based modification is:

$$m'_1(a) = \frac{m_1(a)+m_b(a)}{2} = 0.75, m'_1(b) = \frac{m_1(b)+m_b(b)}{2} = 0.25, m'_1(a, b) = \frac{m_1(a,b)+m_b(a,b)}{2} = 0.$$

$$m'_2(a) = \frac{m_2(a)+m_b(a)}{2} = 0.25, m'_2(b) = \frac{m_2(b)+m_b(b)}{2} = 0.75, m'_2(a, b) = \frac{m_2(a,b)+m_b(a,b)}{2} = 0.$$

With Dempster combination rule, the modified conflicting data can be fused as follows:

$$m(a) = m(b) = 0.5.$$

The results show that $\{a\}$ and $\{b\}$ each accounts for a belief of 50%, which is logical. At the same time, compared to the old method, the bBPA does not assign beliefs to $\{a, b\}$.

Example 4 Suppose that the FOD is $\Omega = \{a, b, c\}$ and two BPAs are given as

$$m_1(a) = 0.9 \qquad m_1(a, b, c) = 0.1,$$
$$m_2(c) = 0.9 \qquad m_2(a, b, c) = 0.1.$$

The results of the proposed method in comparison with the old method are shown in Table 1. More belief is assigned to the propositions $\{a\}$ and $\{c\}$, which is superior to the old method and helpful for decision making.

Example 5 A special example where there is no single subset element.

$$m_1(b, c) = 0.8 \qquad m_1(a, c) = 0.2,$$
$$m_2(a, c) = 0.8 \qquad m_2(b, c) = 0.2.$$

With (10), the new bBPA for BPA modification should be:

$$m_b(a) = m_b(b) = m_b(c) = \frac{1}{3}.$$

Using (11), the result of the bBPA-based modification is:

$$m'_1(a) = m'_1(b) = m'_1(c) = \frac{0 + \frac{1}{3}}{2} = \frac{1}{6}, m'_1(b, c) = \frac{0.8 + 0}{2} = 0.4, m'_1(a, c) = \frac{0.2 + 0}{2} = 0.1.$$

$$m'_2(a) = m'_2(b) = m'_2(c) = \frac{0 + \frac{1}{3}}{2} = \frac{1}{6}, m'_2(a, c) = \frac{0.8 + 0}{2} = 0.4, m'_2(b, c) = \frac{0.2 + 0}{2} = 0.1.$$

With Dempster combination rule, the modified conflicting data can be fused as follows:

$$m(a) = 0.1667, m(b) = 0.1667, m(c) = 0.5467, m(b, c) = m(a, c) = 0.06.$$

Basing on the new method, a high degree of belief has been given to the element $\{c\}$ which appears in each proposition of the two groups of BPAs. At the same time, it has given a certain weight to both $\{a\}$ and $\{b\}$ to make the results have a convergence result on single focal element.

3.3.2 Data fusion without conflict

To show the superiority of the proposed method in comparison with the original method, some examples for data fusion without conflict among different pieces of evidence are designed as follows.

Example 6 Suppose that an FOD is $\Omega = \{a, b\}$, two BPAs are given as follows:

$$m_1(a) = 1, \quad m_1(b) = 0, \quad m_1(a, b) = 0,$$
$$m_2(a) = 1, \quad m_2(b) = 0, \quad m_2(a, b) = 0.$$

Table 1 Results of two combination rules of Example5

Fusion method	m(a)	m(b)	m(c)	m(a,b)	m(a,c)	m(b,c)	m(a,b,c)
Method in [51]	0.3756	0.0976	0.3756	0.0413	0.0413	0.0413	0.0271
Proposed bBPA	0.4291	0.1343	0.4291	0	0	0	0.0076

According to the base belief function in (8),

$$m_b(a) = m_b(b) = m_b(a, b) = \frac{1}{3}.$$

Then, the modification of the two BPAs based on (9) is shown as follows:

$$m_i'(a) = \frac{\frac{1}{3}+1}{2} = 0.6667, m_i'(b) = m_i'(a, b) = \frac{\frac{1}{3}}{2} = 0.1667,$$

where $i = 1, 2$. The modified BPA can be fused by Dempster combination rule and the result is as follows:

$$m(a) = 0.8571, m(b) = 0.1071, m(a, b) = 0.0357.$$

According to the fusion result, the belief on $\{a\}$ and $\{b\}$ are 85.71% and 10.71% respectively. It should be noted that a belief of 3.57% is assigned to the proposition $\{a, b\}$, which weakens the belief on single subset and is not helpful for decision making. Although the belief on the $\{a, b\}$ is small in this case, however, each tiny piece of evidence which can be seen as valuable information is important in decision making.

Applying the new bBPA to the solve the same data fusion case.

With (10), the new bBPA for BPA modification should be:

$$m_b(a) = m_b(b) = \frac{1}{2}, m_b(a, b) = 0.$$

Then, based on (11), the result of the bBPA-based modification is:

$$m_i'(a) = \frac{\frac{1}{2}+1}{2} = 0.75, m_i'(b) = \frac{\frac{1}{2}+0}{2} = 0.25, m_i'(a, b) = \frac{0+0}{2} = 0.$$

where $i = 1, 2$. Finally, with Dempster combination rule, the bBPA-based fusion result is:

$$m(a) = 0.9, m(b) = 0.1.$$

As can be seen from the results, the bBPA-based method has a higher belief degree on $\{a\}$ in comparison with the method with (8)-(9). The bBPA-based method has no belief loss on the proposition $\{a, b\}$, which means less information loss. The result shows that even for data fusion without conflict among different BPAs, the proposed method can get a better fusion result than the old one.

Example 7 Suppose that an FOD is $\Omega = \{a, b\}$, a number of BPAs are given as follows:

$$m_i(a) = 1, \quad m_i(b) = 0, \quad m_i(a, b) = 0,$$

where i represents the sequential number of BPAs for data fusion (i=1, 2, 3...). This is a special case, experts give the same assessment. Applying the new bBPA to the solve this data fusion case. With (10), the new bBPA for BPA modification should be:

$$m_b(a) = m_b(b) = \frac{1}{2}, m_b(a, b) = 0.$$

Then, based on (11), the result of the bBPA-based modification is:

$$m_i'(a) = \frac{\frac{1}{2}+1}{2} = 0.75, m_i'(b) = \frac{\frac{1}{2}+0}{2} = 0.25, m_i'(a, b) = \frac{0+0}{2} = 0.$$

Finally, with Dempster combination rule, the bBPA-based fusion result can be calculated. The results are shown in the Fig. 1.

It can be seen from Fig. 1, as more and more pieces of evidence are fused, $m(a)$ and $m(b)$ rapidly develop to the two extremes. Once four pieces of evidence of data are fused, the value of $m(a)$ is close to 1 and the value of $m(b)$ is close to 0. The bBPA shows a good performance in dealing with non-conflicting data.

Fig. 1 The result of Example 7

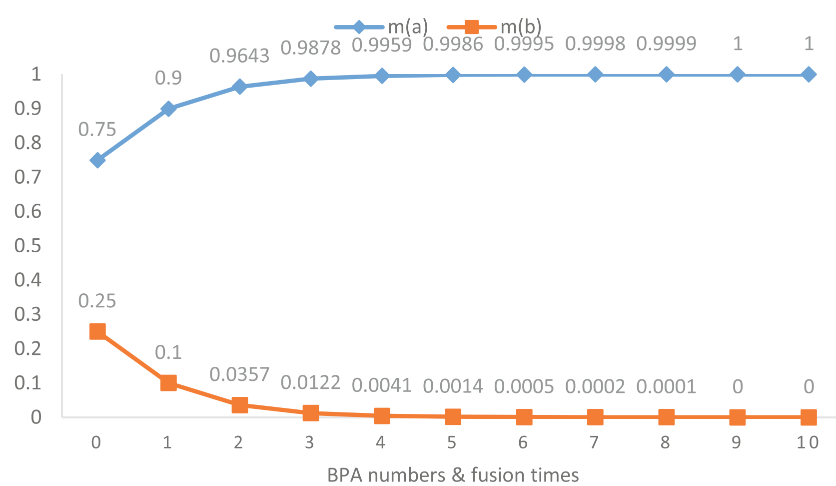
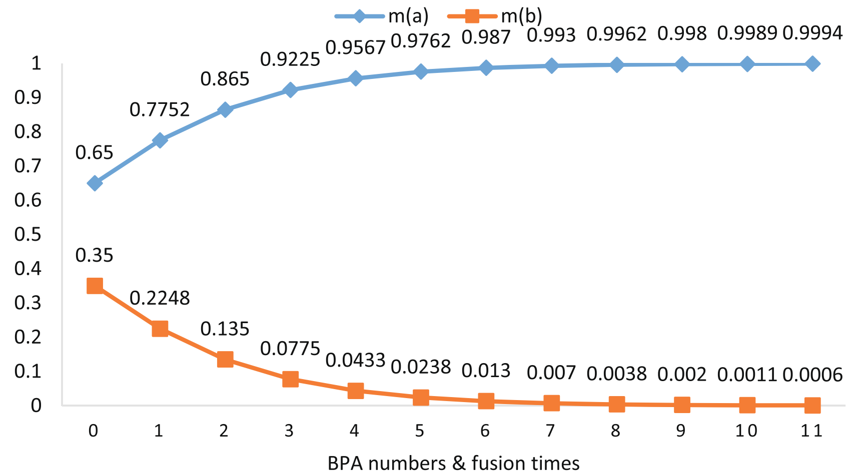


Fig. 2 The result of Example 8



Example 8 Suppose that an FOD is $\Omega = \{a, b\}$, a number of BPAs are given as follows:

$$m_i(a) = 0.8, \quad m_i(b) = 0.2, \quad m_i(a, b) = 0,$$

where i represents the sequential number of BPAs for data fusion (i=1, 2, 3...).

In this case, experts provide the same BPA. With our method, the experimental results are shown in Fig. 2. The value of $m(a)$ quickly converge to 1 and the value of $m(b)$ quickly converge to 0. The bBPA can integrate the opinions from different experts, which makes the credibility of a being much higher than that of b. The result is reasonable when the credibility of each expert is the same.

Fig. 3 Flowchart of decision making system built with bBPA

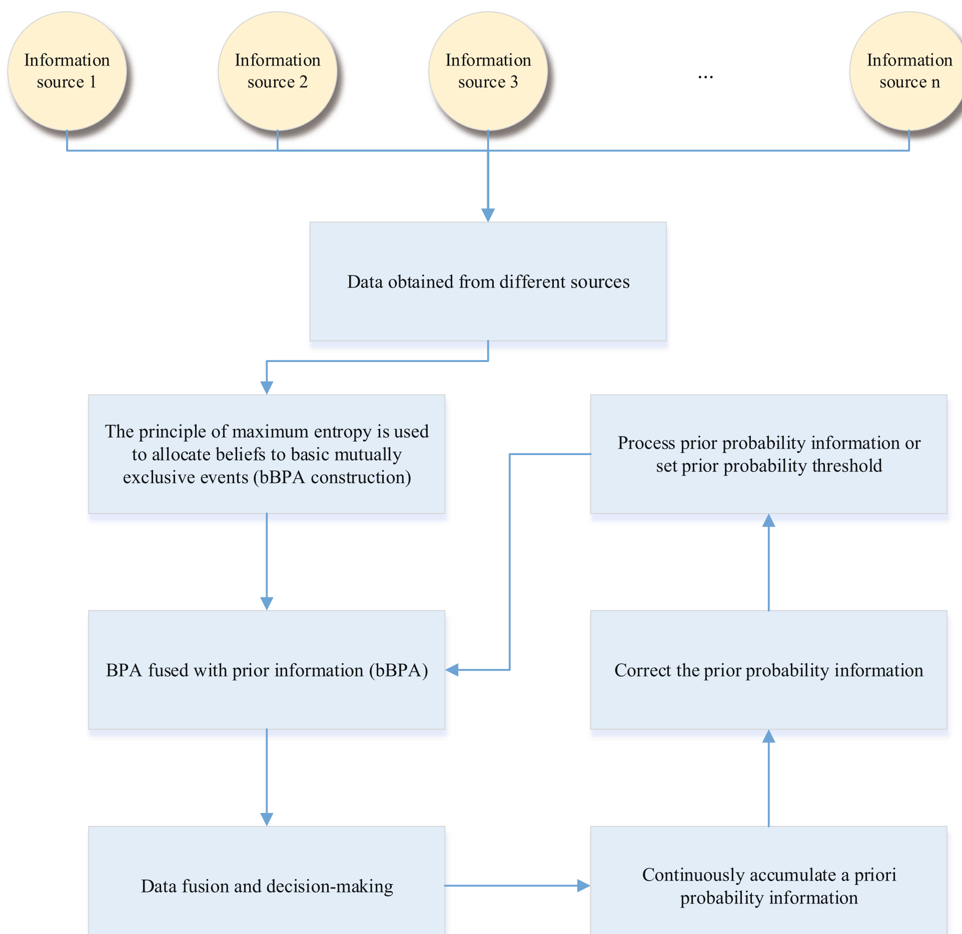


Table 2 BPAs of four attributes

Attribute	m(a)	m(b)	m(c)	m(a,b)	m(a,c)	m(b,c)	m(a,b,c)
SL	0.3337	0.3165	0.2816	0.0307	0.0052	0.0272	0.0052
SW	0.3164	0.2501	0.2732	0.0304	0.0481	0.0515	0.0304
PL	0.6699	0.3258	0	0	0	0.0043	0
PW	0.6996	0.2778	0	0	0	0.0226	0

3.3.3 bBPA with prior information

In this section, some examples are used to introduce the method under the Bayesian principle. The base of bBPA is consistent with Bayesian probability.

Example 9 Suppose that an FOD is $\Omega = \{a, b\}$, two BPAs are given as follows:

$$m_1(a) = 0.8, \quad m_1(b) = 0.2, \quad m_1(a, b) = 0,$$

$$m_2(a) = 0.2, \quad m_2(b) = 0.8, \quad m_2(a, b) = 0.$$

Meanwhile, we assume that the credibility assigned to a is 70% and the credibility of b is 30%, denoted as:

$$P(a) = 0.7, \quad P(b) = 0.3.$$

With Dempster combination rule and without bBPA, we will get the fusion result:

$$m(a) = 0.5, \quad m(b) = 0.5.$$

Based on (12), the new bBPA coming from prior probability for BPA modification should be:

$$m_b(a) = P(a) = 0.7, m_b(b) = P(b) = 0.3, m_b(a, b) = 0.$$

Consequently, based on (11), the result of the bBPA-based modification is:

$$m_1'(a) = \frac{m_1(a)+m_b(a)}{2} = 0.75, m_1'(b) = \frac{m_1(b)+m_b(b)}{2} = 0.25,$$
$$m_2'(a) = \frac{m_2(a)+m_b(a)}{2} = 0.45, m_2'(b) = \frac{m_2(b)+m_b(b)}{2} = 0.55,$$

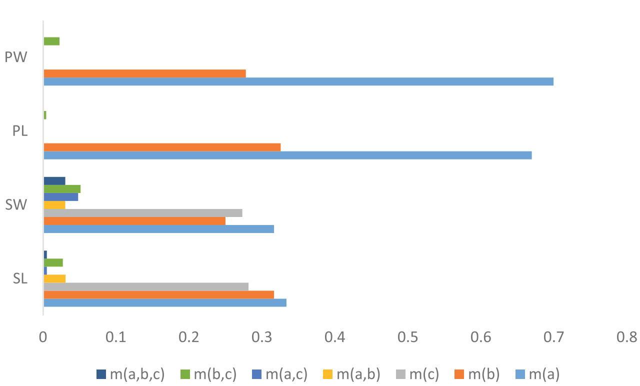


Fig. 4 Distribution of BPA in Iris classification

Finally, with Dempster combination rule, the modified conflicting data can be fused as follows:

$$m(a) = 0.7105, m(b) = 0.2895.$$

With the support of the prior information, there is a higher belief level for the proposition a. This can be explained than the history record has a positive effective on choosing a.

Example 10 Define that an FOD is $\Omega = \{a, b, c\}$, two BPAs are given as follows:

$$m_1(a) = 0.99, m_1(a, b) = 0.01,$$

$$m_2(b) = 0.01, m_2(c) = 0.99.$$

Recall this classic example again, here, we have a priori information provided. The prior probability of the event is given as follows:

$$P(a) = 0.3, P(b) = 0.1, P(c) = 0.6.$$

Using (12) and (11), the result of the bBPA-based modification is:

$$m_1'(a) = 0.645, m_1'(b) = 0.05, m_1'(c) = 0.3, m_1'(a, b) = 0.005,$$
$$m_2'(a) = 0.15, m_2'(b) = 0.055, m_2'(c) = 0.795.$$

Consequently, with Dempster combination rule, the modified conflicting data can be fused as follows:

$$m(a) = 0.2876, m(b) = 0.0089, m(c) = 0.7035.$$

It can be seen that on the basis of prior probability, the proposed method can also handle conflicting data effectively. The prior information can be integrated to the designed method.

4 Application in classification problem

In this section, we use the data set from the UCI Machine Learning Repository to verify the effectiveness of the new bBPA method in comparison with the original method in [51].

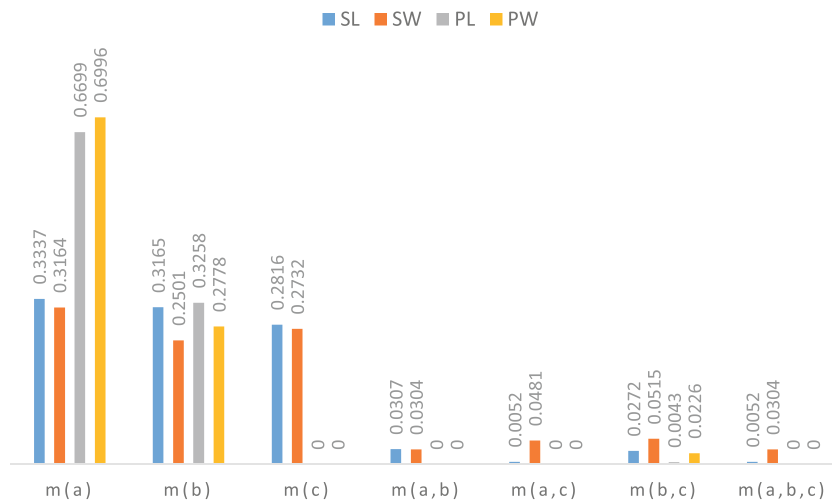


Fig. 5 Belief on different species

Table 3 Results of different data fusion methods for Iris classification experiment

Fusion method	m(a)	m(b)	m(c)	m(a,b)	m(a,c)	m(b,c)	m(a,b,c)
Method in [51]	0.6245	0.2667	0.1086	0.0004	0.0002	0.0006	0
Proposed bBPA	0.6777	0.2539	0.0703	0	0	0	0

Table 4 BPAs of 13 attributes

Attribute	m(a)	m(b)	m(c)	m(a,b)	m(a,c)	m(b,c)	m(a,b,c)
alcohol	0.1304	0.116	0.2752	0.116	0.1304	0.116	0.116
malic acid	0.3082	0.1681	0.0889	0.1681	0.0889	0.0889	0.0889
ash	0.1413	0.1994	0.1295	0.1413	0.1295	0.1295	0.1295
alcalinity of ash	0.3966	0.3017	0	0.3017	0	0	0
magnesium	0.2252	0.2372	0.0781	0.2252	0.0781	0.0781	0.0781
total phenols	0.2599	0.2642	0.054	0.2599	0.054	0.054	0.054
flavanoids	0.2885	0.423	0	0.2885	0	0	0
nonflavanoid phenols	0.1804	0.1776	0.1152	0.1776	0.1152	0.1152	0.1152
proanthocyanins	0.1301	0.203	0.1383	0.1301	0.1301	0.1383	0.1301
color intensity	0.2123	0.2849	0.0726	0.2123	0.0726	0.0726	0.0726
hue	0	0.2921	0.4158	0	0	0.2921	0
OD280/OD315	0.3314	0.3372	0	0.3314	0	0	0
proline	0	0.3106	0.3788	0	0	0.3106	0

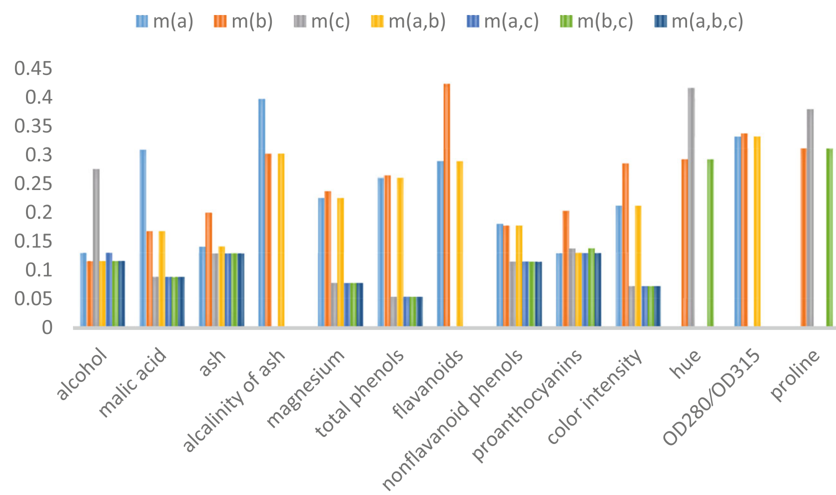


Fig. 6 Distribution of BPA in experiment 2

4.1 The new bBPA-based data fusion method

The flowchart of the data fusion method based on the new bBPA method is shown in Fig. 3.

Step 1 The information source generates a series of BPAs that may contain conflicting data.

Step 2 The new bBPA is constructed according to the number of basic events where the base belief is assigned to the mutual exclusion events.

Step 3 Modify the original BPAs with the bBPA to preprocess conflicting data.

Step 4 Data fusion using Dempster combination rule.

Step 5 Decision making based on data fusion results in classification.

Step 6 Update the prior probability information according to the accumulated probability information.

Step 7 Set a threshold for the prior probability so that it will not be too large.

Step 8 Use the prior information to process the following BPAs.

4.2 Experiment 1: Iris data set classification

There are three species ($Setosa(a)$, $Versicolor(b)$, $Virginica(c)$) in Iris data set with four attributes (SL, SW, PL, and PW) and each species contains 50 instances. The

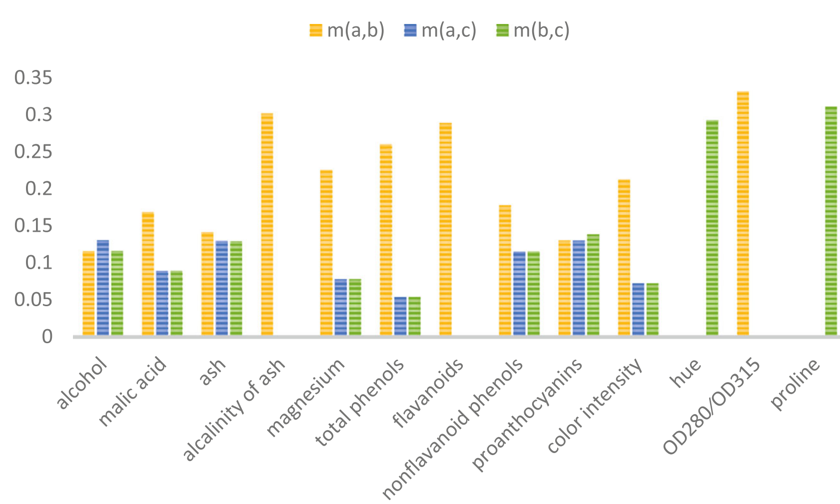


Fig. 7 Distribution of subset with two elements in experiment 2

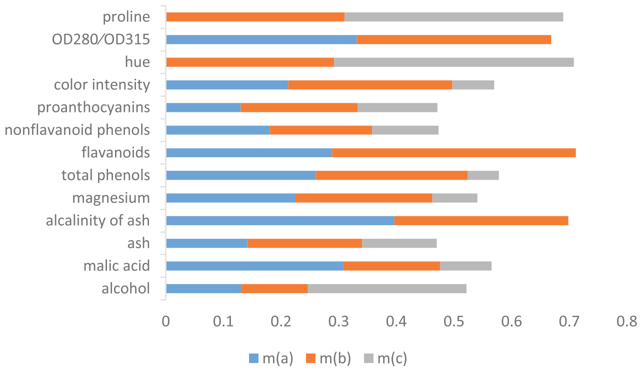


Fig. 8 Distribution of single subset of BPA in experiment 2

BPAs generated in [51] is adopted, where Wang et al. randomly select 40 instances from each species and generate the triangular fuzzy numbers [53] of four attributes. The remainder 10 instances are the test sets. Wang et al. randomly choose one instance from the species Setosa (a) as the test sets and generate the BPAs. The result is shown in Table 2. The distribution of BPA is shown in Figs. 4 and 5.

Figure 4 clearly shows the distribution of BPA generated by different attributes. Different colors represent different events. In Fig. 5, the distribution of BPA generated by different attributes is shown from another perspective. Different colors represent different attributes.

In Fig. 4, the attributes SL and SW of the flower are given a similar judgment on the species $\{a\}$, $\{b\}$ and $\{c\}$, but the attributes PL and PW gives the species $\{a\}$ and the species $\{b\}$ with a higher belief support. In Fig. 5, it can be clearly seen that the four different attributes all have the highest belief degree for the species $\{a\}$. Thus, it is reasonable to believe that the species Setosa is more likely to be the recognized type of flower. Finally, comparing the results using the method in [51] and the proposed bBPA method, the results with different data preprocessing methods are shown in Table 3.

As can be seen from Table 3, all the methods can recognize that the test instance is likely to be the species Setosa ($\{a\}$) with a belief of more than 60%, which conforms to the actual situation. The new bBPA method gives a higher belief degree on species $\{a\}$. In addition, the proposed method has no belief lose on the proposition with multiple species, which shows the superiority of bBPA in addressing classification problem.

4.3 Experiment 2: Wine data set classification

The classification experiment of wine data set is adopted to verify the new bBPA method. The wine data set includes 3 different varieties of wine (denoted as $\{a\}$, $\{b\}$ and $\{c\}$) with 13 attributes. The generated BPAs adopted from [51] are shown in Table 4. Figure 6 shows the distribution of BPA generated by 13 different attributes.

According to Fig. 6, it cannot be clearly figured out all the internal relations among different varieties, but it can be figured out that the belief degree for the varieties $\{a\}$ and $\{b\}$ is higher than the variety $\{c\}$. Intuitively, it can be assumed that the varieties $\{a\}$ and $\{b\}$ are more likely to be the potential recognized variety compared to the variety $\{c\}$.

In Fig. 7, only the subset $\{a, c\}$, $\{b, c\}$, and $\{a, b\}$ are kept. It can be clearly seen that in most attributes, the subset $\{a, b\}$ has a higher belief degree. In Fig. 8, another part of the attributes are hidden, only the subset $\{a\}$, $\{b\}$, and $\{c\}$ are kept. It can be seen that most of the attributes provide a higher belief degree on $\{a\}$ and $\{b\}$. Thus, compared to $\{c\}$, the types $\{a\}$ and $\{b\}$ will be more likely to be the potential recognized variety.

Figure 9 shows the distribution of BPA for the varieties $\{a\}$ and $\{b\}$, where the BPAs of $\{c\}$, $\{a, b\}$, and $\{a, b, c\}$ are hidden because they are not the key issue for the potential recognized variety. It can clearly seen that, among the 13 attributes, the variety $\{b\}$ has a higher belief degree than the variety $\{a\}$, while the $\{b, c\}$ has a higher belief degree than $\{a, c\}$. Thus, $\{b\}$ is more likely to be the recognized variety in the problem.

Finally, the result of data fusion is shown in Table 5. All three methods assign a higher belief degree on the variety $\{b\}$. 9 out of 13 BPAs considers that $\{b\}$ has a higher belief than $\{a\}$. Data fusion result shows that $\{c\}$ is not the recognized variety. Some belief is assigned to $\{a\}$, which is consistent with practical application. Therefore, the new bBPA method is reasonable and efficient.

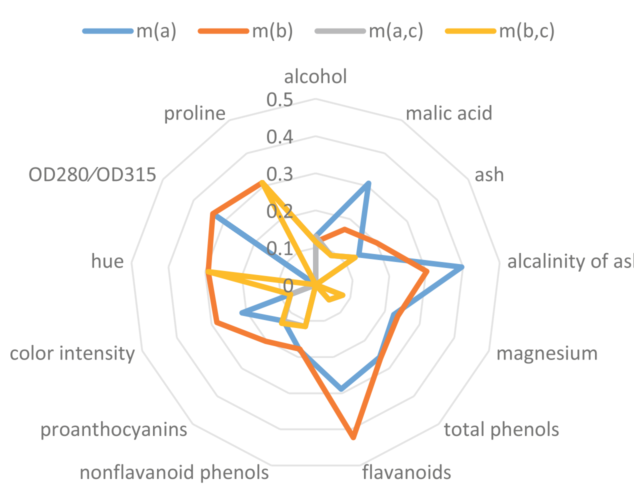


Fig. 9 Distribution of support for events by different evidence

Table 5 Results of different combination rules of Wine experiment

Fusion method	m(a)	m(b)	m(c)	m(a,b)	m(a,c)	m(b,c)	m(a,b,c)
Classical Dempster rule	0	1	0	0	0	0	0
Old base belief function	0.1729	0.8216	0	0.0043	0	0	0
Proposed method	0.1091	0.8891	0.0073	0	0	0	0

5 Conclusions

Traditional Dempster combination rule may produce counter-intuitive results while dealing with highly conflicting data. The base belief function can be a method to solve this problem. In this paper, a new base belief function method named bBPA is proposed. The highlights of bBPA are as follows. First of all, it has a lower computational complexity because bBPA does not need to assign base belief on the entire power set space, which can be a merit in real-time system. Secondly, bBPA distributes the belief averagely on the basic events, which is consistent with the classical probability theory. Finally, bBPA can express and fuse prior information and avoid increasing the belief on the multiple subset which is not helpful for decision making.

The bBPA is based on the basic event. The rest of the uncertain events in the power set of FOD may not be helpful for decision making. Thus, we assign initial base belief only on the basic event. Prior information can be modeled in data fusion with bBPA. Existing experience from experts can be adopted to deal with conflicting information fusion. In addition, the bBPA also reduces the belief degree of proposition with multiple elements, which makes the bBPA-based method to be more effective in decision making problems. Examples and experiments show that the proposed method can deal with conflicting data fusion effectively.

The following work may focus on applying the proposed method in decision making of practical problems. More importantly, at present, how to assign the belief of a proposition with multiple elements to a single subset event is still an open issue. The following work of bBPA-based decision making method should take into consideration of the issue.

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