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Problem Set 1

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1.

Final Output

Only Card II (one card)must be turned to verify the Proposition

Workings

Requisite propositional understandings

$P \Rightarrow Q \Leftrightarrow \neg Q \Rightarrow \neg P$

If we turn Card II we can reach a sufficient condition wherein we can prove or disprove the proposition ($P \Rightarrow Q \Leftrightarrow \neg Q \Rightarrow \neg P$)

We need not turn over the other cards due to the following reasons as we have reached a sufficient condition to prove or disprove the proposition.

2.

Statement 1 (Salazar): Rowena is from the Gryffindor family
Statement 2 (Rowena): Salazar and I are from different families

Given Propositions

(X) is Gryffindor ⇒ (X) is Truthful
(Y) is Slytherin ⇒ (Y) is a Liar

Final Output

Both Rowena and Salazar are from Slytherin and hence both of them lie, which is the only case in which the above propositional logic holds true

Workings Total Possibilities table

	Salazar tells the Truth	Salazar lies
Rowena	**#1 NOT POSSIBLE**	**#2 NOT POSSIBLE**

tells the truth	Salazar tells the truth ⇒ Salazar is a Gryffindor Salazar tells the truth ⇒ Rowena is a Gryffindor Rowena is a Gryffindor ⇒ Salazar and her are from different families The third proposition disallows the existence of the first two propositions	Salazar lies ⇒ Salazar is a Slytherin Salazar lies ⇒ Rowena is not Gryffindor (is Slytherin) Rowena is a Slytherin ⇒ Rowena lies Proposition 3 states that if Salazar lies Rowena lies too but in this scenario, Rowena is truthful thus disallowing this case
Rowena lies	**#3 NOT POSSIBLE** Salazar tells the truth ⇒ Salazar is a Gryffindor Salazar tells the truth ⇒ Rowena is a Gryffindor Rowena is a Gryffindor ⇒ she is truthful Proposition 3 implies that Rowena cannot lie.	**#4 POSSIBLE** Salazar lies ⇒ Salazar is a Slytherin Salazar lies ⇒ Rowena is not Gryffindor (is Slytherin) Rowena is a Slytherin ⇒ Rowena lies Rowena lies ⇒ Rowena and Salazar are from the same family This case is the only possible case in which propositional logic holds true

3.

a)

Choice Set of Ron = { (Floo Powder) , (Magical Purse) , (Portkey) }
Choice Set of Draco = { (Floo Powder) , (Magical Purse) , (Portkey) , {Floo Powder, Portkey} , {Floo Powder, Magical Purse} , {Portkey,Magical Purse}}

Reason:

Let the choice equation for the different combinations of the products be

$M \geq 10x+15y+15z$,

Where x,y,z are units of the Floo powder, Magical purse, Portkey. M is the Budget.

Assuming the shop only has one of each product the choice set of each of the wizards is all possible combination of the goods they can purchase.

b)

Choice Set of Ron A= { {Floo Powder} , {Magical Purse} , {Portkey} }
Choice Set of Draco B= { {Floo Powder} , {Magical Purse} , {Portkey} , {Floo Powder, Portkey} , {Floo Powder, Magical Purse} , {Portkey,Magical Purse}}

$A \wedge B$= { (Floo Powder) , (Magical Purse) , (Portkey) } = A
$\Rightarrow A \subseteq B$

Reason: Intersection is the “logical and” operator thus it is a set of all the elements common in A “and” B since the intersection of A and B outputs A it implies that A is a Subset of B

c)

Choice Set of Ron A= { {Floo Powder} , {Magical Purse} , {Portkey} }
Choice Set of Draco B= { {Floo Powder} , {Magical Purse} , {Portkey} , {Floo Powder, Portkey} , {Floo Powder, Magical Purse} , {Portkey,Magical Purse}}

$L = \{ (x,y), x \in A \text{ and } y \in B \text{ st } x \neq y , x \nmid y \}$

Conditions exist so that we are cognizant of the fact that the shop only has 1 unit of each item

4.

a) No,

{Lion, Eagle, Phoenix} $\not\subseteq$ A, as the elements Lion and Eagle do not feature in the Set A.

The correct notation would be { {Lion, Eagle} , Phoenix} $\subseteq$ A

b) Yes,

{Phoenix, {Lion, Eagle}, Hippogriff, Thestral} $\subseteq$ A, as all the elements of the set A are present in the above set and the order of the elements does not matter in the set

c) No,

{Lion, Eagle} $\subseteq$ A, implies that the elements Lion and Eagle individually occur in the set A, but it is the element “{Lion, Eagle}” that is in set A, thus the correct notation would be {{Lion, Eagle}} $\subseteq$ A

d) No,

To be the subset of a set the primary requirement is that one must be a set, thus the correct notation would be {Hippogriff} $\subseteq$ A

5.

a) No,

{(Seamus, Slytherin),(Gregory, Gryffindor),(Cho, Hufflepuff)}, the set is a relation but not a function, for a relation to be a function each element of the Domain should be mapped to an element in the Co-domain.

b) No,

{(Seamus, Slytherin),(Gregory, Gryffindor),(Cho, Hufflepuff),(Seamus, Ravenclaw)}, is a relation but not a fuction as in a fuction one element from the Domain cannot be mapped to two elements in the Co-domain

c) Yes,

{(Seamus, Slytherin), (Gregory, Gryffindor), (Cho, Hufflepuff)} is an anti-symmetric relation, but it is vacuously Anti-Symmetric, a relation is Anti-symmetric if for every (x,y) and (y,x) existing in a set x=y, but if (x,y) exists and (y,x) doesn't exist the condition is vacuously true.

d) No,

(Seamus, Slytherin),(Gregory, Gryffindor) is not a relation, as a relation is defined as a non-empty set of cartesian products of a set, and the above given is not a set in the first place.