Kalman Filter for Quadcopter Position Hold

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February 29, 2024

1 Problem statement

The system dynamics is described by Newton's second law of motion. Let z be the altitude of the quadcopter, g denotes the acceleration due to gravity and a is the acceleration that results from the forces of the propellers. Then, the system dynamics is as simple as Equation (**??**)

$$\ddot{z} = a - g. \tag{1}$$

The acceleration, a, is an affine function of the thrust reference signal (which comes from the RC); it is

$$a = \alpha\tau + \beta, \tag{2}$$

where α and β are coefficients to be estimated; we can obtain *a priori* estimates offline and update them online using measurements while flying (using the Kalman filter).

@Peter,

1. Copy here the system dynamics from Chandra's report (we don't need Section 2.2.3)
2. Write down what sensors we use and what their characteristics are (level of noise, presence of outliers, whether the sensors are biased, update frequencies)

and we'll take it from there.

2 Altitude Dynamics

The altitude dynamics of a quadcopter are defined within a global coordinate system, crucial for maintaining a predetermined altitude from the Earth's surface. The model that describes these dynamics is based on fundamental principles, delineated as follows:

The rate of change of the quadcopter's altitude, represented as $\dot{z}_t$, is the result of the vertical acceleration $a^z_{T_t}$ produced by the drone's motors at a given time minus the gravitational acceleration, g. This equation is continuous in time and is expressed as,

$$\dot{z}_t = a^z_{T,t} - g \tag{3}$$

Here, $a^z_{T_t}$ signifies the upward acceleration generated by the propulsion at time t, measured in meters per second squared. The constant g denotes the acceleration due to Earth's gravity, also in meters per second squared. The altitude z_t represents the drone's center of mass's vertical position at time t, measured in meters.

Additionally, the drone's vertical velocity v_{z_t} and vertical acceleration a_{z_t} are defined by the rate of altitude change $\dot{z}_t$ and the rate of vertical acceleration change $\dot{a}^z_{T_t} - g$, respectively. The term $\dot{a}^z_{T_t}$ is derived from the quadcopter's upward thrust and serves as the system's input, while g is considered a constant input in the opposite direction.

Let $y^z_t = z_t$ be the output equation of the system. Then the corresponding state-space representation is,

$$\begin{bmatrix} v_{v,t} \\ a^z_t \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z \\ v_{v,t} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} a^z_{T,t} \\ -g \end{bmatrix}, \tag{4}$$

for the barometer sensor's output, denoted by y_{barom}, is described by the equation,

$$y_{barom} = z + d^{bar} + e_{barom} \tag{5}$$

where,

$$d^{bar}_{t+1} = d^{bar}_t + w^{d^{bar}}_t \tag{6}$$

The bias (d^{bar}) should stay consistent throughout readings, the second reading of the bias should be equal to the first allowing for some additional noise/offest.
for the GPS sensor's output, denoted by y_{gps}, is described by the equation,

$$y^{\text{gps}} = z + e^{\text{gps}} \tag{7a}$$
$$= \begin{bmatrix} 1 & 0 \end{bmatrix} x + e^{\text{gps}} \tag{7b}$$

The Time-of-Flight (ToF) sensor's output, denoted by y_{ToF}, is described by the equation,

$$y^{\text{ToF}} = z + d^{\text{ToF}} + e_{\text{ToF}} \tag{8}$$

Throught all sensors y represents the output from the sensor. The variable z signifies the quadcopter's altitude, which is the measurement for all sensors. The term e encapsulates the measurement noise or errors associated with the sensors. This noise term, e, encompasses various factors such as sensor inaccuracies, the impact of environmental conditions on sensor performance, and any systematic bias that might be inherent in the sensor's readings.

Equation (**??**) describes the continuous-time altitude dynamics of the quadcopter. The discretization of the altitude dynamics of the system (**??**) with a sampling frequency of Ts using the zero-order hold technique is,

$$\begin{bmatrix} z_{t+1} \\ v_{z,t+1} \end{bmatrix} = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_t \\ v_{s,t} \end{bmatrix} + \begin{bmatrix} 1/2T^2{}_s & 1/2T^2{}_s \\ T_s & T_s \end{bmatrix} \begin{bmatrix} a^z_{T,t} \\ -g \end{bmatrix}, \tag{9}$$

2.1 State Vector Definition

The state vector x_t is defined as,

$$x_t = \begin{bmatrix} z_t & e_t & \alpha_t & \beta_t & d_t{}^{\text{bar}} & d_t{}^{\text{ToF}} & \end{bmatrix} \tag{10}$$

2.2 State Transition Equation

$$x_{t+1} = A_t x_t + W_t \tag{11}$$

2.3 Measurement Model

The measurement model is defined as

$$y_t = C_t x_t + e_t \tag{12}$$

$$y_t = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} x_t + e_t \tag{13}$$

We may need to estimate α or β for simplisity.

3 Estimator Design

A sentence

$$\begin{aligned} z_{t+1} =& z_t + T_s v_t^z + \frac{1}{2} T_s^2 (\alpha_t \tau_t + \beta_t) + w_t^z (\text{notInUse}) & (14) \\ z_{t+1} =& z_t + T_s v_t + w_t^z & (15) \\ v_{t+1}^z =& v_t^z + T_s (\alpha_t \tau_t + \beta_t) + w_t^v & (16) \\ \alpha_{t+1} =& \alpha_t + w_t^\alpha, & (17) \\ \beta_{t+1} =& \beta_t + w_t^\beta & (18) \\ & & (19) \end{aligned}$$

Consider using quation 14 for grater accuracy

where,

$$a_t = \alpha_t \tau_t + \beta_t \tag{20}$$

We define w_t^z and w_t^v as the process noise elements. w_t^z is distributed normally with zero mean and variance σ_z^2, expressed as $w_t^z \sim \mathcal{N}(0, \sigma_z^2)$, and similarly, w_t^v follows a normal distribution with $w_t^v \sim \mathcal{N}(0, \sigma_v^2)$. Additionally, w_t^α and w_t^β represent white noise processes with distributions $w_t^\alpha \sim \mathcal{N}(0, T_s \sigma_\alpha^2)$ and $w_t^\beta \sim \mathcal{N}(0, T_s \sigma_\beta^2)$ respectively. The system's state needing estimation is denoted by $x_t = (z_t, v_t^z, \alpha_t, \beta_t)$, with the system's dynamic model being describable as follows,

$$x_{t+1} = A_t x_t + w_t^z, \tag{21}$$

where the matrix A_t is defined as,

$$A_t = \begin{bmatrix} 1 & T_s & T_s \tau_t & T_s \\ 0 & 1 & T_s \tau_t & T_s \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{22}$$

4 Relationship between throttle signal and lift

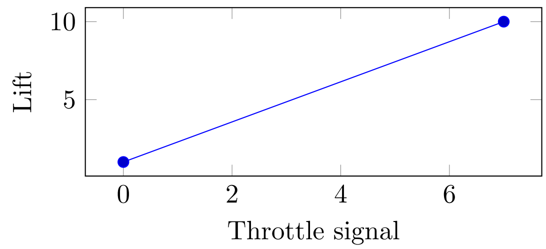

Figure 1:

This graph represents the relationship between throttle signal From our radio, and how this effects the lifing force on the quad. (please note that this graph is not to scale) This relationship is catagorised by the equation,

$$\alpha_t \tau_t + \beta_t$$