Kalman Filter for Quadcopter Position Hold

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1 Problem statement

The system dynamics is described by Newton's second law of motion. Let z be the altitude of the quadcopter, g denotes the acceleration due to gravity and a is the acceleration that results from the forces of the propellers. Then, the system dynamics is as simple as Equation (**??**)

$$\ddot{z} = a - g. \tag{1}$$

The acceleration, a, is an affine function of the thrust reference signal (which comes from the RC); it is

$$a = \alpha\tau + \beta, \tag{2}$$

where α and β are coefficients to be estimated; we can obtain *a priori* estimates offline and update them online using measurements while flying (using the Kalman filter).

@Peter,

1. Copy here the system dynamics from Chandra's report (we don't need Section 2.2.3)
2. Write down what sensors we use and what their characteristics are (level of noise, presence of outliers, whether the sensors are biased, update frequencies)

and we'll take it from there.

2 Altitude Dynamics

The altitude dynamics of a quadcopter are defined within a global coordinate system, crucial for maintaining a predetermined altitude from the Earth's surface. The model that describes these dynamics is based on fundamental principles, delineated as follows:

The rate of change of the quadcopter's altitude, represented as $\dot{z}_t$, is the result of the vertical acceleration $a^z_{T_t}$ produced by the drone's motors at a given time minus the gravitational acceleration, g. This equation is continuous in time and is expressed as,

$$\dot{z}_t = a^z_{T,t} - g \tag{3}$$

Here, $a^z_{T_t}$ signifies the upward acceleration generated by the propulsion at time t, measured in meters per second squared. The constant g denotes the acceleration due to Earth's gravity, also in meters per second squared. The altitude z_t represents the drone's center of mass's vertical position at time t, measured in meters.

Additionally, the drone's vertical velocity v_{z_t} and vertical acceleration a_{z_t} are defined by the rate of altitude change $\dot{z}_t$ and the rate of vertical acceleration change $\dot{a}^z_{T_t} - g$, respectively. The term $\dot{a}^z_{T_t}$ is derived from the quadcopter's upward thrust and serves as the system's input, while g is considered a constant input in the opposite direction.

Let $y^z_t = z_t$ be the output equation of the system. Then the corresponding state-space representation is,

$$\begin{bmatrix} v_{v,t} \\ a^z_t \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} z \\ v_{v,t} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} a^z_{T,t} \\ -g \end{bmatrix}, \tag{4}$$

for the barometer sensor's output, denoted by y_{barom}, is described by the equation,

$$y_{barom} = z + d^{bar} + e_{barom} \tag{5}$$

where,

$$d^{bar}_{t+1} = d^{bar}_t + w^{d^{bar}}_t \tag{6}$$

The bias (d^{bar}) should stay consistent throughout readings, the second reading of the bias should be equal to the first allowing for some additional noise/offest.
for the GPS sensor's output, denoted by y_{gps}, is described by the equation,

$$y^{\text{gps}} = z + e^{\text{gps}} \tag{7a}$$
$$= \begin{bmatrix} 1 & 0 \end{bmatrix} x + e^{\text{gps}} \tag{7b}$$

The Time-of-Flight (ToF) sensor's output, denoted by y_{ToF}, is described by the equation,

$$y^{\text{ToF}} = z + d^{\text{ToF}} + e_{\text{ToF}} \tag{8}$$

Throught all sensors y represents the output from the sensor. The variable z signifies the quadcopter's altitude, which is the measurement for all sensors. The term e encapsulates the measurement noise or errors associated with the sensors. This noise term, e, encompasses various factors such as sensor inaccuracies, the impact of environmental conditions on sensor performance, and any systematic bias that might be inherent in the sensor's readings.

Equation (**??**) describes the continuous-time altitude dynamics of the quadcopter. The discretization of the altitude dynamics of the system (**??**) with a sampling frequency of Ts using the zero-order hold technique is,

$$\begin{bmatrix} z_{t+1} \\ v_{z,t+1} \end{bmatrix} = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_t \\ v_{s,t} \end{bmatrix} + \begin{bmatrix} 1/2T^2{}_s & 1/2T^2{}_s \\ T_s & T_s \end{bmatrix} \begin{bmatrix} a^z_{T,t} \\ -g \end{bmatrix}, \tag{9}$$

2.1 State Vector Definition

The state vector x_t is defined as,

$$x_t = \begin{bmatrix} z_t & e_t & \alpha_t & \beta_t & {d_t}^{\text{bar}} & {d_t}^{\text{ToF}} \end{bmatrix} \tag{10}$$

2.2 State Transition Equation

$$x_{t+1} = A_t x_t + W_t \tag{11}$$

2.3 Measurement Model

The measurement model is defined as

$$y_t = C_t x_t + e_t \tag{12}$$

$$y_t = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} x_t + e_t \tag{13}$$

We may need to estimate α or β for simplisity.

3 Estimator Design

A sentence

$$\begin{aligned}
z_{t+1} =& z_t + T_s v_t^z + \frac{1}{2} T_s^2 (\alpha_t \tau_t + \beta_t) + w_t^z (\text{notInUse}) && (14) \\
z_{t+1} =& z_t + T_s v_t + w_t^z && (15) \\
v_{t+1}^z =& v_t^z + T_s(\alpha_t \tau_t + \beta_t) + w_t^v && (16) \\
\alpha_{t+1} =& \alpha_t + w_t^\alpha, && (17) \\
\beta_{t+1} =& \beta_t + w_t^\beta && (18) \\
& && (19)
\end{aligned}$$

Consider using quation 14 for grater accuracy

where,

$$a_t = \alpha_t \tau_t + \beta_t \tag{20}$$

We define w_t^z and w_t^v as the process noise elements. w_t^z is distributed normally with zero mean and variance σ_z^2, expressed as $w_t^z \sim \mathcal{N}(0, \sigma_z^2)$, and similarly, w_t^v follows a normal distribution with $w_t^v \sim \mathcal{N}(0, \sigma_v^2)$. Additionally, w_t^α and w_t^β represent white noise processes with distributions $w_t^\alpha \sim \mathcal{N}(0, T_s \sigma_\alpha^2)$ and $w_t^\beta \sim \mathcal{N}(0, T_s \sigma_\beta^2)$ respectively. The system's state needing estimation is denoted by $x_t = (z_t, v_t^z, \alpha_t, \beta_t)$, with the system's dynamic model being describable as follows,

$$x_{t+1} = A_t x_t + w_t^z, \tag{21}$$

3.1 State transition matrix A_t

The state transition matrix A describes how the state at time t evolves to the state at time $t+1$. For the given system, A is defined as:

Given the state vector $x_t = \begin{bmatrix} z_t & v_t^z & \alpha_t & \beta_t \end{bmatrix}$, the state transition matrix A_t from the system's dynamic model is defined as:

$$A_t = \begin{bmatrix} 1 & T_s & T_s \tau_t & T_s \\ 0 & 1 & T_s \tau_t & T_s \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{22}$$

where T_s is the sampling time, and τ_t represents the throttle signal at time t.

3.2 Process Noise Covariance Matrix Q

The process noise covariance matrix Q represents the covariance of the process noise, accounting for the uncertainty in the model dynamics:

$$Q = \begin{bmatrix} \sigma_z^2 & 0 & 0 & 0 \\ 0 & \sigma_v^2 & 0 & 0 \\ 0 & 0 & T_s\sigma_\alpha^2 & 0 \\ 0 & 0 & 0 & T_s\sigma_\beta^2 \end{bmatrix} \tag{23}$$

Here, σ_z^2, σ_v^2, σ_α^2, and σ_β^2 represent the variances of the altitude, velocity, and the coefficients α and β, which relate the throttle signal to the lift.

3.3 Measurement Matrix C

The measurement matrix C links the state vector to the measurement vector:

$$C_t = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{24}$$

This matrix considers the direct measurement of altitude by all sensors and accounts for biases in the ToF and barometer sensors.

3.4 Measurement Noise Covariance Matrix R

The measurement noise covariance matrix R accounts for the uncertainty in sensor measurements:

$$R = \begin{bmatrix} \sigma_{barom}^2 & 0 & 0 \\ 0 & \sigma_{gps}^2 & 0 \\ 0 & 0 & \sigma_{ToF}^2 \end{bmatrix} \tag{25}$$

where σ_{barom}^2, σ_{gps}^2, and σ_{ToF}^2 are the variances of the measurement noises for the barometer, GPS, and Time-of-Flight sensors, respectively.

4 Relationship between throttle signal and lift

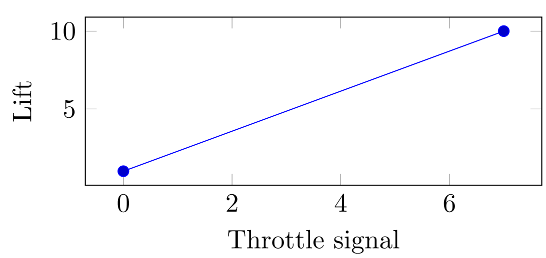

Figure 1:

This graph represents the relationship between throttle signal From our radio, and how this effects the lifing force on the quad. (please note that this graph is not to scale) This relationship is catagorised by the equation,

$$\alpha_t \tau_t + \beta_t$$

/sectionControler In order to simulate if our kalman filter will work we will have to design a controler which will graph the real position and the estimated on. To do this we will first need to define the values for the Bias of Tof and Baramoter.

$$d^{Tof} = \tag{26}$$

$$d^{bar} = \tag{27}$$

Next we may need to set a value for α or β as we may have introduced too many variables. This can be tested using the controller in order to find an optimal value for either.