The Greatest Game in the World: 24

Quincy Bayer
Mrs. Bayer's Son

ISLAND OF MATHEMATICIANS
WHO ENJOY TRICKING CHILDREN

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What is 24 on the island of mathematicians?

Given a multiset of four numbers:

$$S = \{a_1, a_2, a_3, a_4\}, \text{ s.t. } a_i \in \mathbb{N}, \ 1 \le a_i \le 9.$$

Define the set of binary operators

$$\mathcal{O} = \{+, -, \times, \div\}.$$

Define $\mathcal{E}(S)$ to be the set of all arithmetic expressions that can be formed by:
- Using each element of S exactly once,
- Combining them with binary operators from $\mathcal{O}$,
- Using parentheses to specify order of operations.

Formally, $\mathcal{E}(S)$ is the closure of S under binary operations in $\mathcal{O}$, with the restriction that all a_i are used exactly once.

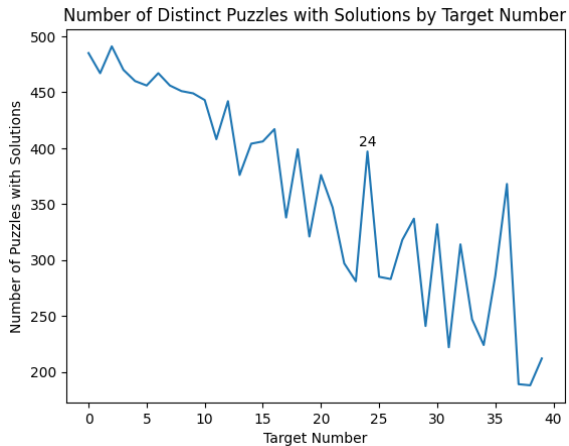The game is won if $\exists E \in \mathcal{E}(S)$ s.t. $E = 24$.

Why is it called 24?

The target number is 24. Why not 23 or 25 or 24,000,000?

Ideas I came up With

1. Most possible puzzles?
2. Average sum of numbers?
3. Has a lot of multiplication solutions?
4. Has a lot of difficult puzzles?

Number of Distinct Puzzles with Solutions by Target Number
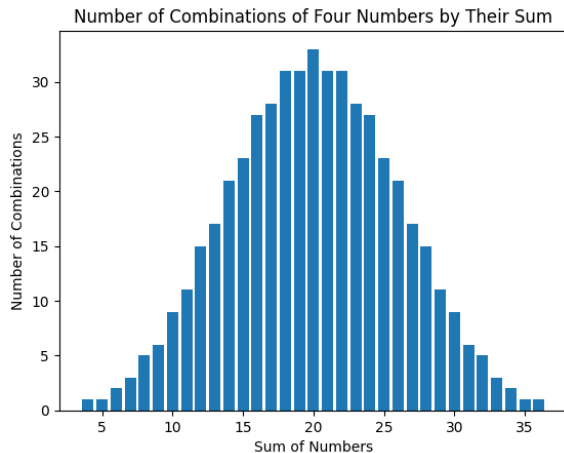
Number of Combinations of Four Numbers by Their Sum

Figure 1: If the goal was to sum often, the target would be 20.

A lot of multiplications solutions?

1	1									21	1	3	7	21					
2	1	2								22	1	2	11	22					
3	1	3								23	1	23							
4	1	2	4							24	1	2	3	4	6	8	12	24	
5	1	5								25	1	5	25						
6	1	2	3	6						26	1	2	13	26					
7	1	7								27	1	3	9	27					
8	1	2	4	8						28	1	2	4	7	14	28			
9	1	3	9							29	1	29							
10	1	2	5	10						30	1	2	3	5	6	10	15	30	
11	1	11								31	1	31							
12	1	2	3	4	6	12				32	1	2	4	8	16	32			
13	1	13								33	1	3	11	33					
14	1	2	7	14						34	1	2	17	34					
15	1	3	5	15						35	1	5	7	35					
16	1	2	4	8	16					36	1	2	3	4	6	9	12	18	36
17	1	17								37	1	37							
18	1	2	3	6	9	18				38	1	2	19	38					
19	1	19								39	1	3	13	39					
20	1	2	4	5	10	20				40	1	2	4	5	8	10	20	40	

1	1						21	1	3	7	21					
2	1	2					22	1	2	11	22					
3	1	3					23	1	23							
4	1	2	4				24	1	2	3	4	6	8	12	24	
5	1	5					25	1	5	25						
6	1	2	3	6			26	1	2	13	26					
7	1	7					27	1	3	9	27					
8	1	2	4	8			28	1	2	4	7	14	28			
9	1	3	9				29	1	29							
10	1	2	5	10			30	1	2	3	5	6	10	15	30	
11	1	11					31	1	31							
12	1	2	3	4	6	12	32	1	2	4	8	16	32			
13	1	13					33	1	3	11	33					
14	1	2	7	14			34	1	2	17	34					
15	1	3	5	15			35	1	5	7	35					
16	1	2	4	8	16		36	1	2	3	4	6	9	12	18	36
17	1	17					37	1	37							
18	1	2	3	6	9	18	38	1	2	19	38					
19	1	19					39	1	3	13	39					
20	1	2	4	5	10	20	40	1	2	4	5	8	10	20	40	

Most possible puzzles?

Number of Distinct Puzzles with Solutions by Target Number
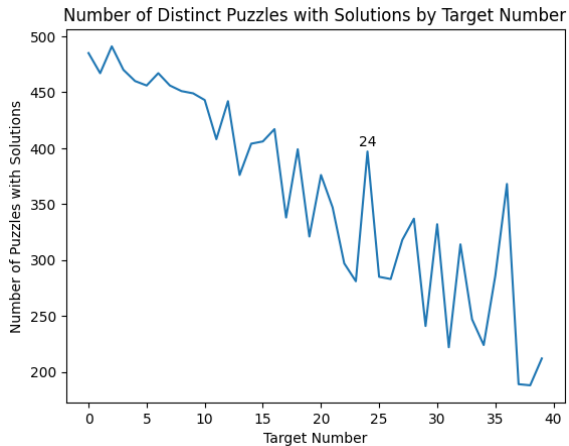
Most possible puzzles with highly divisible?
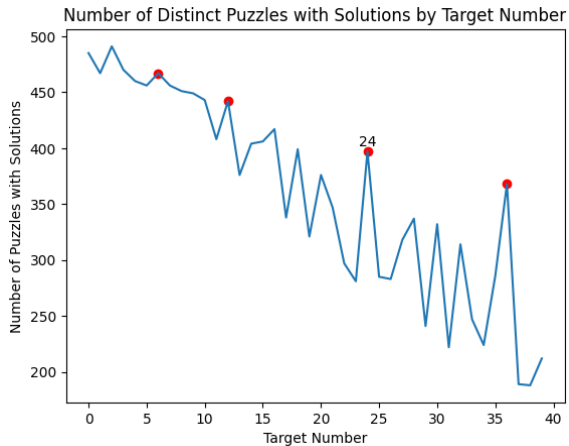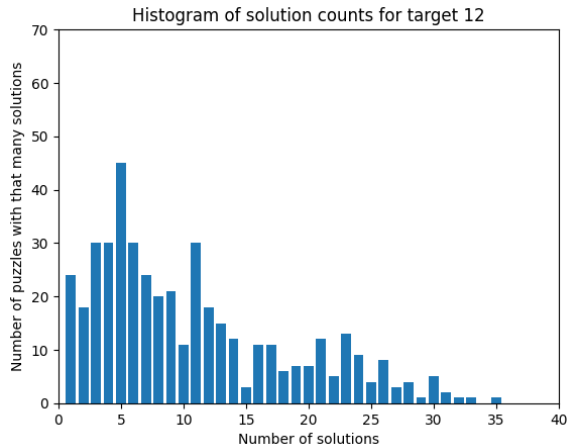
Number of Distinct Puzzles with Solutions by Target Number

How difficult is the game of 12?

Histogram of solution counts for target 12

How difficult is the game of 24?

Histogram of solution counts for target 24

Histogram of solution counts for target 36

Number of Distinct Puzzles with Solutions by Target Number
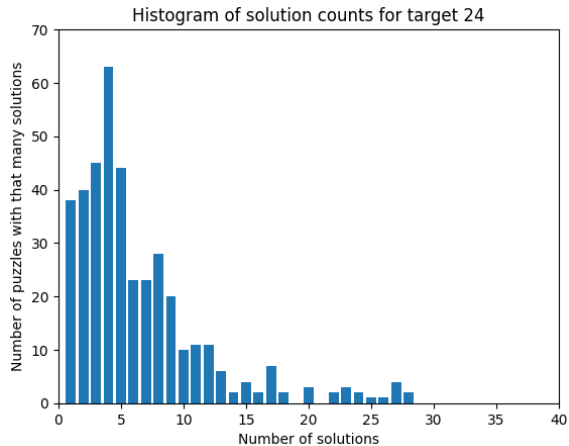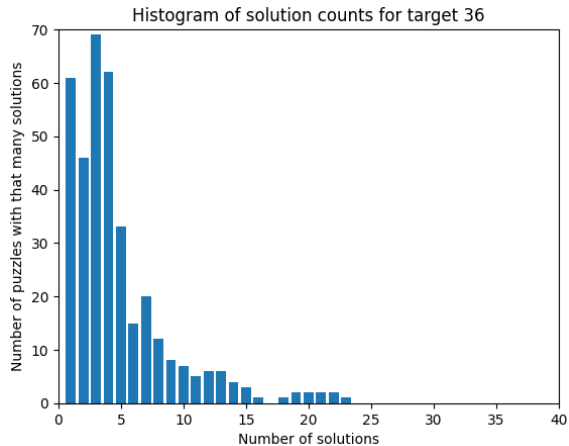
Histogram of solution counts for target 24
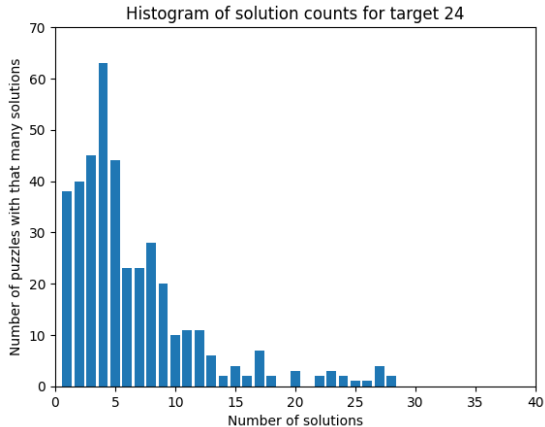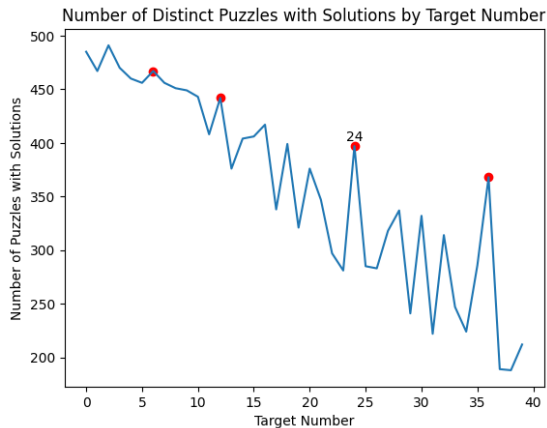
24 is highly divisible, it has a lot of possible puzzles, and its a good balance of difficulty

- There are 495 possible 24 puzzles
- Exactly 397 puzzles have a solution
- So just memorize all 397 solutions...easy

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- Exactly 397 puzzles have a solution
- So just memorize all 397 solutions...easy...or you can look for patterns.

- The final step is the most important step.
- In competitive 24, the last step needs to be spoken first.
- In all possible puzzles and solutions, there are only 45 final steps.

Equation	Puzzles	Equation	Puzzles	Equation	Puzzles	Equation	Puzzles	Equation	Puzzles
4*6	201	25-1	36	10+14	24	11+13	12	48-24	2
3*8	200	3+21	35	2+22	22	31-7	11	168/7	2
2*12	116	7+17	33	12+12	21	96/4	9	192/8	2
1*24	79	72/3	31	1+23	19	144/6	7	45-21	1
6+18	66	30-6	29	0+24	18	35-11	4	56-32	1
8+16	61	27-3	28	26-2	15	40-16	4	49-25	1
9+15	57	28-4	27	33-9	15	120/5	4	54-30	1
4+20	40	5+19	26	29-5	12	42-18	3	64-40	1
48/2	37	32-8	26	36-12	12	216/9	3	72-48	1

And some final steps have easier solutions

- While all 45 final steps are in possible solutions, many of them have easier solutions.

Equation	Puzzles	Equation	Puzzles	Equation	Puzzles
4*6	201	9+15	6	30-6	2
3*8	95	10+14	4	32-8	2
6+18	24	1+24	3	2+22	1
8+16	18	27-3	3	144/6	1
25-1	12	48/2	2	54-30	1
2*12	9	26-2	2		
28-4	9	4+20	2		

How do you win 24?

My approach to win 24 almost every time

1. Look for a way to make 6
   1. Look for a way to make 4
   2. Look for a way to make 18
2. Look for a way to make 8
   1. Look for a way to make 3
   2. Look for a way to make 16
3. See if the numbers add up to 24 or a number close to 24.
4. Good luck. This one is pretty hard.