THE UNIVERSITY OF BUEA

FACULTY OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF COMPUTER ENGINEERING

CEF 401: OPERATIONAL RESEARCH

TITLE: AN OPERATION RESEARCH ON HOW TO SOLVE LINEAR PROGRAMMING PROBLEMS BY GRAPHICAL METHOD.

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Introduction to Linear Programming

It is an **optimization method** for a linear **objective function** and a system of **linear inequalities or equations**. The linear inequalities or equations are known as **constraints**. The quantity which needs to be maximized or minimized (optimized) is reflected by the objective function. The fundamental objective of the linear programming model is to look for the values of the variables that optimize (maximize or minimize) the objective function.

We know that in linear programming, we subject linear functions to multiple constraints. These constraints can be written in the form of linear inequality or linear equations. This method plays a fundamental role in finding optimal resource utilization. The word "linear" in linear programming depicts the relationship between different variables. It means that the variables have a **linear relationship** between them. The word "programming" in linear programming shows that the **optimal solution** is selected from different alternatives.

The following were assumed while solving the linear programming problems:

- The constraints are expressed in the **quantitative values**
- There is a **linear relationship** between the objective function and the constraints
- The objective function which is also a linear function needs optimization

Linear Programming Problems

This report addresses problems in various fields using linear programming techniques: education, production and health.

Problem Description 1: Optimization of Toy Car and Video Surveillance Drone Production

The Faculty of Engineering and Technology of the University of Buea produces two Products, toy cars and video surveillance drones. For the toy car the materials needed (g/unit) is 5 and that for video surveillance drone is 3 (g/unit). Their production rates are 60 and 30 units/hour respectively. The selling price of each unit is 13 and 11 respectively.
Also space is needed to store the products of which $4cm^2$ is needed for 1 unit of the toy car and $5cm^2$ for 1 unit of the video surveillance drone. The total amount of equipment available per day for both products is 1575 grams. The total storage space for all products is 1500 cm^2 and a maximum of 7 hours per day can be used for production.

Solution:

Step 1 - Identify the decision variables

The first step is to discern the decision variables which control the behavior of the objective function. Objective function is a function that requires optimization.

The school has decided that it wants to maximize its sale income, which depends on the number of units of toy cars and video surveillance drones that it produces.

• Therefore, the decision variables, x1 and x2 can be the number of units of toy cars and video surveillance drones, respectively, produced per day.

Step 2 - Write the objective function

The decision variables that you have just selected should be employed to jot down an algebraic expression that shows the quantity we are trying to optimize. In other words, we can say that the objective function is a linear equation that is comprised of decision variables.

$Z = 13x_1 + 11x_2$
Subject to the constraints on storage space, materials, and production time.

Step 3 - Identify Set of Constraints

Constraints are the limitations in the form of equations or inequalities on the decision variables. Remember that all the decision variables are non-negative; i.e. they are either positive or zero.

• Each unit of toy cars requires $4cm^2$ of storage space and each unit of video surveillance drone requires $5cm^2$.

Thus a total of $4x_1 + 5x_2$ cm^2 of storage space is needed each day. This space must be less than or equal to the available storage space, which is $1500cm^2$

Therefore, $4x_1 + 5x_2 <= 1500$

• Similarly, each unit of toy cars and video surveillance drone produced requires 5 and 3 grams, respectively, of materials. Hence a total of $5x_1 + 3x_2$ grams of materials is used. This must be less than or equal to the total amount of materials available, which is 1575 grams.

Therefore, $5x_1 + 3x_2 <= 1575$

• Toy cars can be produced at the rate of 60 units per hour. Therefore, it must take 1 minute or 1/60 of an hour to produce 1 unit. Similarly, it requires 1/30 of an hour to produce 1 unit of video surveillance drone. Hence a total of $x_1/60 + x_2/30$ hours are required for the daily production. This quantity must be less than or equal to the total production time available each day.

Therefore, we get $x_1 + 2x_2 <= 420$

• Finally, the school cannot produce a negative quantity of any product, therefore x1 and x2 must each be greater than or equal to zero. The linear programming model for this example can be summarized as:

Maximize $Z = 13x_1 + 11x_2$

Subject to $4x_1 + 5x_2 <= 1500$

$5x_1 + 3x_2 <= 1575$

$x_1 + 2x_2 <= 420$

$x_1 , x_2 >= 0$

Step 4 - Choose the method for solving the linear programming problem

Multiple techniques can be used to solve a linear programming problem. These techniques include:

- Simplex method
- Solving the problem using R
- Solving the problem by employing the graphical method
- Solving the problem using an open solver

Here, it is specifically discussed on how to solve linear programming problems using the **graphical method**.

Step 5 - Construct the graph

After you have selected the graphical method for solving the linear programming problem, you should construct the graph and plot the constraints lines.

Graph goes here

Step 6 - Identify the feasible region

This is the region of the graph that satisfies all the constraints in the problem. Selecting any point in the feasible region yields a valid solution for the objective function.

Therefore, the region below and including the line $4x_1 + 5x_2 = 1500$ in the figure above represents the region defined by $4x_1 + 5x_2 <= 1500$. The same thing applies to other equations as well.

The shaded area of the figure comprises the area common to all the regions defined by the constraints and contains all pairs of x_1 and x_2 that are feasible solutions to the problem.

This area is known as the feasible region or feasible solution space. The optimal solution must lie within this region.

Step 7 - Find the optimum point

Any point in the feasible region that gives the maximum or minimum value of the objective function represents the optimal solution. Next, we need to find the corner points of the feasible region and evaluate the objective function at each corner point.

• There are various pairs of x_1 and x_2 that satisfy the constraints such as:

$$X=[x_1, x_2]=[0,0]$$

Trying different solutions, the optimal solution will be [270, 75]

Conclusion (Optimal Solution)

- This indicates that maximum income of $4335 is obtained by producing 270 units of toy cars and 75 units of video surveillance drones.
- In this solution, all the materials and available time are used, because the optimal point lies on the two constraint lines for these resources.
- However, 1500- [4(270) + 5(75)], or $45cm^2$
- If storage space, is not used. Thus the storage space is not a constraint on the optimal solution that is, more products could be produced before the school runs out of storage space. Thus this constraint is said to be slack.

Problem Description 2: Balancing School and Work for Maximum Productivity

A student needs to balance school and work. The student is required to spend a minimum of 14 hours studying per week and work a maximum of 12 hours. The total combined time spent on studying and work should not exceed 40 hours weekly. The student aims to schedule their study and work time to maximize productivity at the end of the week. The productivity coefficient for studying is 2, and for work is 1.

Solution:

Step 1 - Identify the decision variables

Let x1 represent the number of hours spent on studying, and x2 represent the number of hours spent on work.

Step 2 - Write the objective function

Maximize the student's productivity at the end of the week, given by the objective function: $P = 2 * x1 + 1 * x2$.

Step 3 - Identify Set of Constraints

- Minimum Study Time Constraint: The student must spend a minimum of 14 hours studying per week: $x1 \geq 14$.
- Maximum Work Time Constraint: The student should work a maximum of 12 hours per week: $x2 \leq 12$.
- Total Time Constraint: The combined time spent on studying and work should not exceed 40 hours per week: $x1 + x2 \leq 40$.
- Non-negativity Constraint: The variables representing study time and work time should be non-negative: $x1 \geq 0$, $x2 \geq 0$

Step 4 - Choose the method for solving the linear programming problem

Here, we use the graphical method.

Step 5 - Construct the graph

We plot a graph which satisfies all the constraints above.

Graph Goes Here

Step 6 - Identify the feasible region

This region of the graph satisfies all the constraints in the problem. Selecting any point in the feasible region yields a valid solution for the objective function.

Step 7 - Find the optimum point

Next, we need to find the corner points of the feasible region and evaluate the objective function at each corner point.

The corner points of the feasible region are:
A: (14, 0), B: (14, 12), C: (26, 14) and D: (40, 0).

Now, we can calculate the value of the objective function P = 2 * x1 + 1 * x2 at each corner point:

P(A) = 2 * 14 + 1 * 0 = 28
P(B) = 2 * 14 + 1 * 12 = 40
P(C) = 2 * 26 + 1 * 14 = 66
P(D) = 2 * 40 + 1 * 0 = 80

The maximum productivity is achieved at point D, where P(D) = 80.

Conclusion (Optimal Solution)

Therefore, the optimal solution is to spend 40 hours on studying (x1 = 40) and no time on work (x2 = 0), resulting in a productivity of 80