THE UNIVERSITY OF BUEA

FACULTY OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF COMPUTER ENGINEERING

CEF 401: OPERATIONAL RESEARCH

TITLE: AN OPERATION RESEARCH ON HOW TO SOLVE LINEAR PROGRAMMING PROBLEMS BY SIMPLEX METHOD AND SENSITIVITY ANALYSIS.

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SOLVING LINEAR PROGRAMMING PROBLEMS

The two methods described here in solving a linear programming problem are

- Simplex method
- Sensitivity analysis

Simplex method

This is a standard technique in linear programming for solving an optimization problem, typically one involving a function and several constraints expressed as inequalities. When decision variables are more than 2, it is always advisable to use Simplex Method to avoid lengthy graphical procedure.

- The simplex method is not used to examine all the feasible solutions.
- It deals only with a small and unique set of feasible solutions, the set of vertex points (i.e., extreme points) of the convex feasible space that contains the optimal solution.

Steps involved

Step 1: Formulate the LPP

- Clearly define the objective function to be maximized or minimized.
- Identify the decision variables and their constraints.

Step 2: Convert the LPP to standard form

- Convert any inequality constraints to equality constraints by introducing slack, surplus, or artificial variables.
- Express all variables as non-negative.

Step 3: Set up the initial Simplex tableau

- Create a table called the Simplex tableau that represents the LPP.
- Label the columns with the decision variables, slack, surplus, artificial variables, and the right-hand side (RHS) values.
- Write the coefficients of the objective function in the bottom row of the tableau.

Step 4: Select a pivot column

- Choose the most negative coefficient in the bottom row of the tableau as the pivot column.
- If there are multiple negative coefficients, select the one corresponding to the variable with the highest coefficient in the objective function.

Step 5: Select a pivot row

- Divide the RHS values by the values in the pivot column.
- Choose the smallest positive ratio as the pivot row.
- If all ratios are negative or zero, the problem is unbounded, and the solution is not feasible.

Step 6: Perform row operations

- Use row operations to make all the other values in the pivot column zero.
- Perform elementary row operations to achieve this, such as multiplying and adding rows.

Step 7: Update the tableau

- Update the tableau with the new values obtained from the row operations.
- The pivot element becomes 1, and all other values in the pivot column become 0.

Step 8: Repeat steps 4-7

- Repeat steps 4-7 until there are no negative coefficients in the bottom row of the tableau.
- This indicates that the optimal solution has been reached.

Step 9: Read the solution

- The optimal solution is found in the rightmost column of the tableau.
- The values of the decision variables are given by the corresponding entries in the rightmost column.

Step 10: Perform sensitivity analysis (optional)

- Perform sensitivity analysis to determine how changes in the coefficients or RHS values affect the optimal solution.

Below is a brief solution of the linear programming problem using simplex method

Problem Description:

The Faculty of Engineering and Technology of the University of Buea produces two products; toy cars and video surveillance drones. For the toy car the materials needed (g/unit) is 5 and that for video surveillance drone is 3. Their production rates are 60 and 30 units/hour respectively. The selling price of each unit is 13 and 11 respectively. Also space is needed to store the products of which 4cm2 is needed for 1 unit of the toy car and 5cm2 for 1 unit of the video surveillance drone. The total amount of equipment's available per day for both products is 1575 grams. The total storage space for all products is 1500 cm2 and a maximum of 7hours per day can be used for production. All products manufactured are shipped out of the storage area at the end of the day. Therefore, the two products must share the total raw material, storage space, and production time. The company wants to determine how many units of each product to produce per day to maximize its total income.

Solution

- **Step 1**: Convert all the inequality constraints into equalities by the use of slack variables. Let:

S_1 = unused storage space

S_2 = unused materials

S_3= unused production time

As already developed, the LP model is:

Maximize $Z = 13X_1 + 11X_2$

Subject to
$4 X_1 + 5 X_2 <= 1500$

$5 X_1 + 3 X_2 <= 1575$ ...Eq(4)

$X_1 + 2 X_2 <= 420$

$X_1 , X_2 >= 0$

Introducing these slack variables into the inequality constraints and rewriting the objective function such that all variables are on the left-hand side of the equation. Equation 4 can be expressed as:

$Z - 12 X_1 - 11 X_2 = 0$ (A1)

$4 X_1 + 5 X_2 + S_1 = 1500$ (B1)

$5 X_1 + 3 X_2 + S_2 = 1575$ (C1) ....Eq(5)

$X_1 + 2 X_2 + S_3 = 420$ (D1)

$X_i >= 0 \quad i = 1, 2$

From the above equations above, it is obvious that one feasible solution that satisfies all the constraints is:

$X_1 = 0$, $X_2 = 0$, S_1 = 15OO , $S_2 = 1575$, $S_3 = 420$, and $Z = 0$

Since the coefficients of X_1 and X_2 in Eq. (A1) are both negative, the value of Z can be increased by giving either X_1 or X_2 some positive value in the solution.

- In Eq. (B1), if $X_1 = S_1 = 0$, then $X_1 = 1500/4 = 375$. That is, there is only sufficient storage space to produce 375 units of toy cars.
- From Eq. (C1), there is only sufficient materials to produce 1575/5 = 315 units of toy cars.
- From Eq. (D1), there is only sufficient time to produce 420/1 = 420 units of toy cars.
- Therefore, considering all three constraints, there is sufficient resource to produce only 315 units of X_1. Thus the maximum value of X_1 is limited by Eq. (C1).

- **Step 2**: From Equation C(1), which limits the maximum value of X_1.

  $X_1 = -3/5\ X_1 - 1/5S_2 + 315$ ...Eq(6)

Substituting this equation into Eq. (5) yields the following new formulation of the model.

$Z – (16/5)X_2 + (13/5)S_2 = 4095$ (A2)

$(13/5)X_2 + S_1 – (4/5)S_2 = 240$ (B2) ...Eq(7)

$X_1 + (3/5)X_2 + (1/5)S_2 = 315$ (C2)

$(7/5)X_2 - 1/5S_2 + S_3 = 105$ (D2)

It is now obvious from these equations that the new feasible solution is:

$X_1 = 315$, $X_2 = 0$, $S_1 = 240$, $S_2 = 0$, $S_3 = 105$, and $Z = 4095$

- It is also obvious from Eq.(A2) that it is also not the optimum solution. The coefficient of X_1 in the objective function represented by A2 is negative (-16/5), which means that the value of Z can be further increased by giving X_2 some positive value.

Following the same analysis procedure used in step 1, it is clear that:

- In Eq. (B2), if $S_1 = S_2 = 0$, then $X_2 = (5/13)(240) = 92.3$
- From Eq. (C2), X_2 can take on the value $(5/3\ )(315) = 525$ if $X_1 = S2 = 0$
- From Eq. (D2), X_2 can take on the value $(5/7)(105) = 75$ if $S_2 = S_3 = 0$
- Therefore, constraint D2 limits the maximum value of X_2 to 75. Thus a new feasible solution includes $X_2 = 75$, $S_2 = S3 = 0$

- **Step 3**: From Equation D2:

  $X_2 = (1/7)S_2 – (5/7)S_3 + 75$ ...Eq(8)

  Substituting this equation into Eq. (7) yield:

  $Z + (15/7)S_2 + (16/7)S_3 = 4335$ (A3)

$S_1 - (3/7)S_2 - (13/7)S_3 = 45$ (B3)

$X_1 + (3/7)S_2 - (3/7)S_3 = 270$ (C3) ...Eq(9)

$X_2 - (1/7)S_2 + (5/7)S_3 = 75$ (D3)

From these equations, the new feasible solution is readily found to be:

$X_1 = 270, X_2 = 75, S_1 = 45, S_2 = 0, S_3 = 0, Z = 4335$

Because the coefficients in the objective function represented by Eq. (A3) are all positive, this new solution is also the optimum solution.

Simplex Tableau for maximization

- **Step 1**: Set up the initial tableau using Eq. (5).

$Z - 12X_1 - 11X_2 = 0$ (A1)

$4X_1 + 5X_2 + S_1 = 1500$ (B1)

$5X_1 + 3X_2 + S_2 = 1575$ (C1) ....Eq(5)

$X_1 + 2X_2 + S_3 = 420$ (D1)

$X_i >= 0 \quad i = 1, 2$

In any iteration, a variable that has a nonzero value in the solution is called a ***basic variable***.

Row Number	Basic Variable	Coefficients of: Z	x_1	x_2	S_1	S_2	S_3	Right-Hand Side	Upper Bound on Entering Variable
Initial tableau									
A1	Z	1	−13	−11	0	0	0	0	
B1	S_1	0	4	5	1	0	0	1500	375
C1	S_2	0	5	3	0	1	0	1575	315
D1	S_3	0	1	2	0	0	1	420	420

- **Step 2**: Identify the variable that will be assigned a nonzero value in the next iteration so as to increase the value of the objective function. This variable is called the entering variable.
- ➢ It is that non basic variable which is associated with the smallest negative coefficient in the objective function.

- ➢ If two or more non basic variables are tied with the smallest coefficients, select one of these arbitrarily and continue.
- **Step 3**: Identify the variable, called the leaving variable, which will be changed from a nonzero to a zero value in the next solution.

- **Step 4**: Enter the basic variables for the second tableau. The row sequence of the previous tableau should be maintained, with the leaving variable being replaced by the entering variable.

Row Number	Basic Variable	Coefficients of: Z	x_1	x_2	S_1	S_2	S_3	Right-Hand Side	Upper Bound on Entering Variable
Initial tableau									
A1	Z	1	-13	-11	0	0	0	0	
B1	S_1	0	4	5	1	0	0	1500	375
C1	S_2	0	5	3	0	1	0	1575	315
D1	S_3	0	1	2	0	0	1	420	420
	Cj-zj		**13**	**11**	**0**	**0**	**0**		

For maximization, Cj – Zj >= 0

Here Cj – Zj are not all >= 0, so we do the second table.

Second table at end of iteration

A2	Z	1	0	$-\frac{16}{5}$	0	$+\frac{13}{5}$	0	4095	
B2	S_1	0	0	$\frac{13}{5}$	1	$-\frac{4}{5}$	0	240	92.3
C2	x_1	0	1	$\frac{3}{5}$	0	$\frac{1}{5}$	0	315	525
D2	S_3	0	0	$\frac{7}{5}$	0	$-\frac{1}{5}$	1	105	75
	Cj-zj		**0**	**3.2**	**0**	**-13/5**	**0**		

Since a value in Cj – Zj is greater than zero, do another table.

- **Step 5**: Compute the coefficients for the second tableau. A sequence of operations will be performed so that at the end the X_1 column in the second tableau will have the following coefficients:

	X_1
Z	0
S_1	0
X_1	1
S_3	0

The second tableau yields the following feasible solution:

$X_1 = 315$, $X_2 = 0$, $S_1 = 240$, $S_2 = 0$, $S_3 = 105$, and $Z = 4095$

The row operations proceed as follows:

- The coefficients in row C2 are obtained by dividing the corresponding coefficients in row C1 by 5.
- The coefficients in row A2 are obtained by multiplying the coefficients of row C2 by 13 and adding the products to the corresponding coefficients in row A1.
- The coefficients in row B2 are obtained by multiplying the coefficients of row C2 by -4 and adding the products to the corresponding coefficients in row B1.
- The coefficients in row D2 are obtained by multiplying the coefficients of row C2 by -1 and adding the products to the corresponding coefficients in row D1.

- **Step 6**: Check for optimality. The second feasible solution is also not optimal, because the objective function (row A2) contains a negative coefficient. Another iteration beginning with step 2 is necessary.
- In the third tableau (BELOW), all the coefficients in the objective function (row A3) are positive. Thus an optimal solution has been reached and it is as follows:

$X_1 = 270$, $X_2 = 75$, $S_1 = 45$, $S_2 = 0$, $S_3 = 0$, and $Z = 4335$

Row Number	Basic Variable	Coefficients of: Z	x_1	x_2	S_1	S_2	S_3	Right-Hand Side	Upper Bound on Entering Variable
Initial tableau									
A1	Z	1	-13	-11	0	0	0	0	
B1	S_1	0	4	5	1	0	0	1500	375
C1	S_2	0	5	3	0	1	0	1575	315
D1	S_3	0	1	2	0	0	1	420	420

Second tableau at end of first iteration

A2	Z	1	0	$-\frac{16}{5}$	0	$+\frac{13}{5}$	0	4095	
B2	S_1	0	0	$\frac{13}{5}$	1	$-\frac{4}{5}$	0	240	92.3
C2	x_1	0	1	$\frac{3}{5}$	0	$\frac{1}{5}$	0	315	525
D2	S_3	0	0	$\frac{7}{5}$	0	$-\frac{1}{5}$	1	105	75
	Zj		**13**	**23.2**	**0**	**2/5**	**0**		
	Cj-zj		**0**	**-12.2**	**0**	**-2/5**	**0**		

Since, Cj – Zj <= 0, therefore our optimal value has been obtained.

Third tableau at end of first iteration

A3	Z	1	0	0	0	$+\frac{15}{7}$	$+\frac{16}{7}$	4335
B3	S_1	0	0	0	1	$-\frac{3}{7}$	$-\frac{13}{7}$	45
C3	x_1	0	1	0	0	$\frac{2}{7}$	$-\frac{3}{7}$	270
D3	x_2	0	0	1	0	$-\frac{1}{7}$	$\frac{5}{7}$	75

Conclusion

We can thus say that an optimal solution has been reached and it is as follows:

$X_1 = 270$, $X_1 = 75$, $S_1 = 45$, $S_2 = 0$, $S_3 = 0$, and $Z = 4335$

Sensitivity Analysis

- Sensitivity analysis helps to test the sensitivity of the optimum solution with respect to changes of the coefficients in the objective function, coefficients in the constraints inequalities, or the constant terms in the constraints.
- For Example, in the case study discussed:
- The actual selling prices (or market values) of the two products may vary from time to time. Over what ranges can these prices change without affecting the optimality of the present solution?
- Will the present solution remain the optimum solution if the amount of materials, production time, or storage space is suddenly changed because of shortages, machine failures, or other events?
- The amount of each type of resources needed to produce one unit of each type of product can be either increased or decreased slightly. Will such changes affect the optimal solution?