THE UNIVERSITY OF BUEA
P.O BOX 63
Buea, Southwest Cameroon
Tel: (237) 674354327
Fax: (237) 3332 22 72

REPUBLIC OF CAMEROON
Peace -Work-FatherLand

FACULTY OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF COMPUTER ENGINEERING

COURSE TITLE: FEEDBACK SYSTEMS LABORATORY

COURSE CODE: EEF 460

LAB 4

QUINUEL TABOT NDIP-AGBOR

FE21A300

Course Supervisor:

Dr. WINKAR BASIL, PhD

MAY 2024

Question 1 :

a) $q(s) = s^5 + s^4 + 3s^3 + 4s^2 + s + 2$

Row	s^5	s^4	s^3	s^2	s
1	1	1	3	4	1
2	1	(3 - 4) = -1	4 + 3	(4^1 – 3^1) = 1	
3	1	-1	7		

Analysis Of Sign Changes:

There is one sign change in the first column (from 1 to -1).

Interpretation: Since there's one sign change, the system has **one root** in the right-half plane (positive real part). Therefore, the system is **unstable**

b) $q(s) = s^5 + s^4 + 4s^3 + 4s^2 + 2s + 1$

Row	s^5	s^4	s^3	s^2	s
1	1	1	4	4	2
2	1	(4 - 4) = 0	(4^2 – 4^1) = 4	(4^1 – 2^2) = 0	
3	1	0	4		

Analysis Of Sign Changes:

There are zero sign changes in the first column.

Interpretation: Since there are no sign changes, the system has **all roots** in the left-half plane (negative real parts). Therefore, the system is **stable**.

c) $q(s) = s^8 + 3s^7 + 5s^6 + 6s^5 + s^4 + 4s^3 + 4s^2 + 2s + 1$

1	5	1	4
3	6	4	0
0	4	1	0
6	2	0	0
4	1	0	0

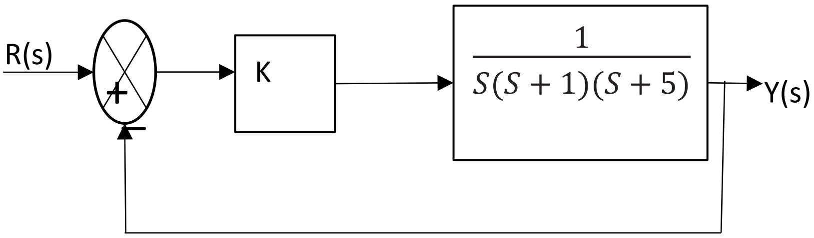
1	0	0	0

Analysis Of Sign Changes:

The first column has three sign changes (from 1 to 3 to 0 to 4), indicating that there are three poles in the right-half plane.

Interpretation: Therefore, the system represented by q(s) = s^8 + 3s^7 + 5s^6 + 6s^5 + s^4 + 4s^3 + 4s^2 + 2s + 1 is **unstable**.

Question 2 :



The feedback system consists of a block with transfer function G(s)= s(s+1)(s+5)K and a unity feedback path (H(s) = 1).

Closed-Loop Transfer Function:

The closed-loop transfer function (W(s)) of the system can be derived using the following formula:

W(s) = 1 + G(s) / H(s)G(s)

In this case:

W(s) = s(s+1)(s+5)+K / K

Stability Analysis:

The stability of the closed-loop system is determined by the roots of the characteristic equation, which is obtained by setting the denominator of the closed-loop transfer function to zero:

s(s+1)(s+5) + K = 0

This expands to a third-order polynomial:

s^3 + 6s^2 + (5 + K)s + K = 0

For the system to be stable, all the roots of this polynomial must lie in the left-half of the complex plane (i.e have negative real parts).

Applying Routh-Hurwitz Criterion:

For the given system, the characteristic equation is:

s^3 + 6s^2 + (5 + K)s + K = 0

Conclusion:

This implies that the gain K can be any positive value. The system remains stable for any positive value of K.

q(s) =k + s^3 + 6s^2 + 5s = 0

S^3	1	5	0
S^2	6	k	0
S^1	(30 – k)/ 6	0	0
S^0	k		

For stability , 0 < k < 30

Question 3 :

$$L(s) = \frac{K}{s(s+1)(0.1s+1)}$$

a) For the Nyquist Plot:

$$L(s) = \frac{K}{s(s+1)(0.1s+1)}$$

$$\Rightarrow \quad L(\omega) = \frac{K}{\omega\,(\omega+1)(0.1\,\omega+1)}$$

⇨ $L(j\omega) = \dfrac{K}{j\omega\,(j\omega + 1)(0.1j\,\omega + 1)}$

⇨ $/L(j\omega)/ = \dfrac{K}{\omega\,(\omega\text{^}2 + 1)(0.01\omega\text{^}2 + 1)}$

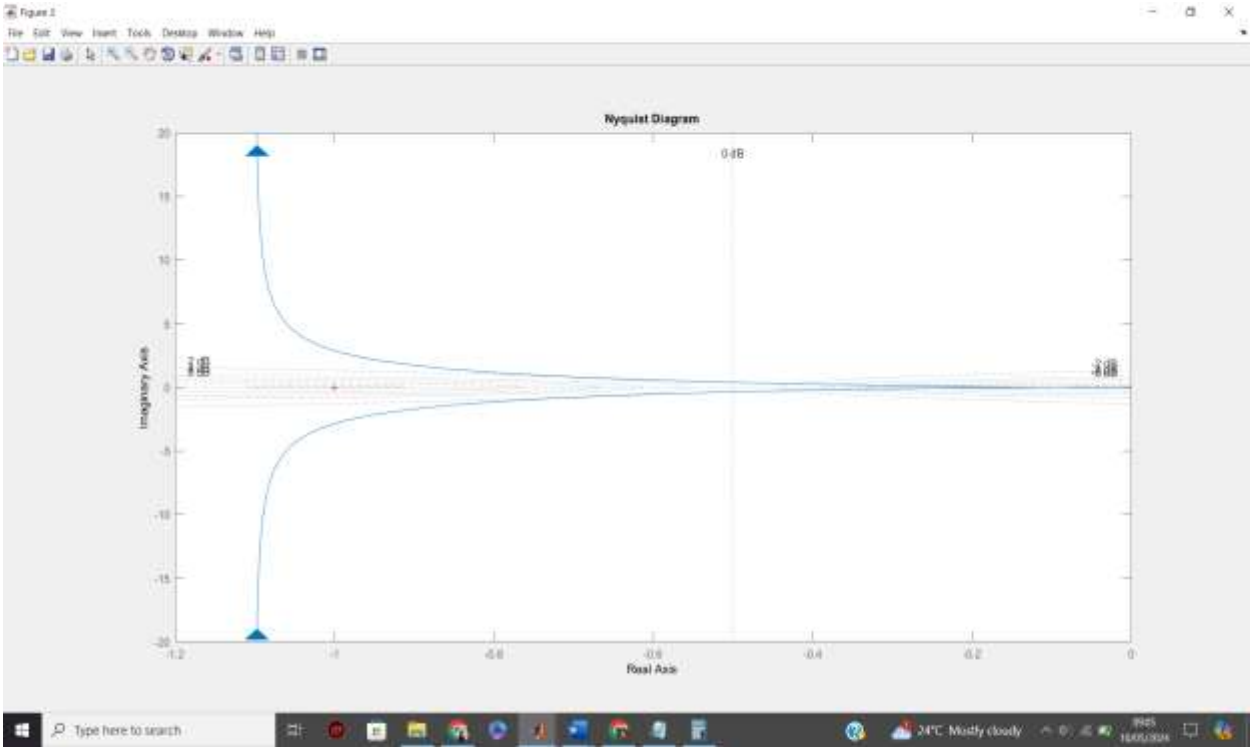
⇨ $\Theta(\omega) = -\pi/2 - \tan^{-1}(\omega) - \tan^{-1}(0.1\,\omega)$

For $\omega = 0$; $/L(j\omega)/= \infty$; $\Theta(\omega) = -\pi/2$

For $\omega = \infty$; $/L(j\omega)/= \mathbf{0}$; $\Theta(\omega) = -3\pi/2$

Matlab Code:

```
% System parameters (marginal stability case)
k = 1;
num = [k];
den = [0.1 1.1 1 0];
sys = tf(num, den);

% Plot Nyquist diagram (approximate)
figure;
nyquist(sys);
hold on;  % Plot imaginary part vs magnitude for approximation
title('Nyquist Diagram');
grid on;
```



b) The maximum value of K for which the system is marginally stable

The polar diagram crosses the negative real axis at (-1+0j) , when $\Theta(\omega) = -\pi$, if the system is marginally stable.

We have that: $\Theta(\omega) = -\pi/2 - \tan^{-1}(\omega) - \tan^{-1}(0.1/10) = -\pi$

⇨ $\tan^{-1}(11wc/10.wc\text{^}2) = \pi/2$

From above; $10 - wc\text{^}2 = 0 \Rightarrow wc = \sqrt{10}$

At (-1,0j) => $\mathbf{k = 11}$

c) For the Bode Plot:

For k = 10,

$$L(s) = \frac{K}{s(s+1)(0.1s+1)}$$

⇨ $$/L(j\omega)/ = \frac{10}{\omega\,(\omega\text{^}2+1)(0.01\omega\text{^}2+1)}$$

GdB = 20log10 – 20log w - 20log$\sqrt{1 + w\text{^}2}$ – 20 log$\sqrt{1 + 0.001w\text{^}2}$

w	**0.01**	**0.1**	**1**	**10**
GdB	60	39.96	16.95	-23.05

Gain Margin = 1 / /L(jw)/ = 11 / 10 = 1.1

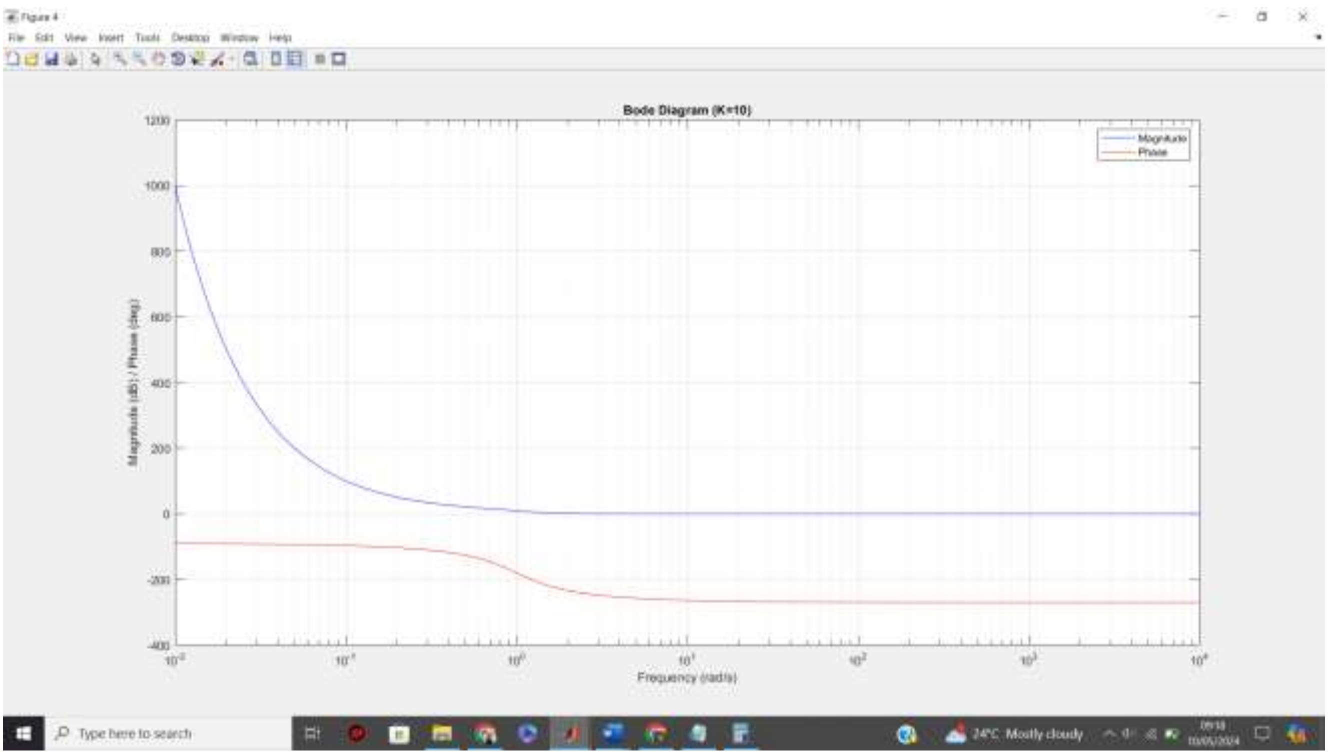
Phase Margin = $180^\circ + (-10 - \tan^{-1}(0.79) - \tan^{-1}(0.79/10))$

$= 47.17^\circ$

This signifies the relative stability of the system.

MatLab Code:

```
Compute Bode response
[mag, phase] = bode(num, den, w);

% Plot Bode diagram
figure;
semilogx(w, mag, 'b');
hold on;
semilogx(w, phase, 'r');
title('Bode Diagram (K=10)');
xlabel('Frequency (rad/s)');
ylabel('Magnitude (dB) / Phase (deg)');
legend('Magnitude', 'Phase');
grid on;
```



Question 4 :

$$L(S) = \frac{K}{S(S+1)(S+2)(S+4)}$$

a)

Number of Loci : m = 0, n = 4

Origin of Loci : s = 0, -1, -2, -4

Centre of asymptotes =(∑ poles - ∑ zeroes)/ n-m

= - 7 / 4 = -1.75

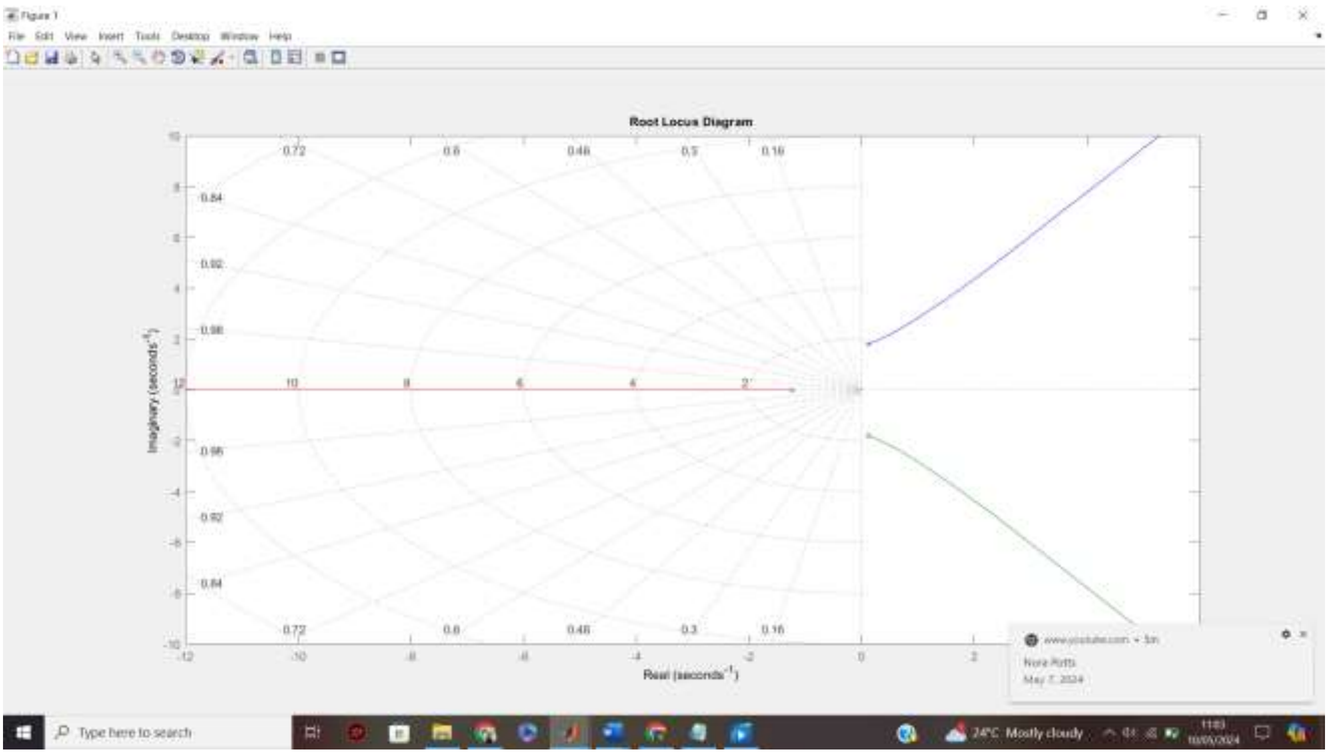
⇨ Angle of asymptotes : Θ(ω) = (180 + 360j) / 3

Θ = { 60, 180, 300}

Matlab Code:

```
% System parameters
K = 1; % Gain (adjust as needed)
num = K;
den = [1 1 3 4]; % Denominator with coefficients

% Create the root locus object
sys = tf(num, den);
rlocus(sys);

% Title and labels
title('Root Locus Diagram');
xlabel('Real');
ylabel('Imaginary');
grid on;

% Breakaway points calculation
% Use rlocus function with additional argument 'Breakaway'
[r, k_breakaway] = rlocus(sys, 'Breakaway');

% Display breakaway points (if any)
if ~isempty(k_breakaway)
  hold on;
  scatter(real(r), imag(r), 'o', 'MarkerSize', 10,
'MarkerEdgeColor', 'r');
  for i = 1:length(k_breakaway)
    text(real(r(i)), imag(r(i)), strcat('K = ',
num2str(k_breakaway(i))), ...
```

```
        'HorizontalAlignment', 'center', 'VerticalAlignment',
'middle');
  end
  legend('Root Locus', 'Breakaway Points');
else
  disp('No breakaway points found.');
end
```



b) q(s) = k + s(s+1)(s+2)(s+4) = 0

= k + 2s^3 + 20s^2 +8 = 0

⇨ k = - 2s^3 - 20s^2 – 8

⇨ dk/ds = -6s^2 – 40s – 8 = 0, Using the quadratic formular s = - 0.21 or s = -6.460

Therefore the break-away point is **s = - 0.21**

c)

$$L(S) = \frac{K}{S(S+1)(S+2)(S+4)}$$

k +s^3+6s^2+s^3+6s^2+8s = 0

⇨ 2s^3 +20s^2+8s+k = 0

S^3	2	8	0
S^2	20	k	0
S^1	(160 – 2k)/ 20	0	0
S^0	k		

For stability, $k > 0 \Rightarrow 160 > 2k$

Therefore **$0 < k < 80$**

Question 5 :

$$L(S) = \frac{K(S+0.5)(S+1.5)}{S(S+1)(S+2)(S+4)}$$

a) L(s)= K(s+0.5)(s+1.5) / s(s+1)(s+2)(s+4)

Number of loci ;m=2, n=4

Origin of loci: s=0,-1,-2,-4

Destination of loci: s=-0.5,-1.5

Angle of asymptote=(∑ poles - ∑ zeroes)/ n-m

= (-7+7)/2 = -5/2=-2.5

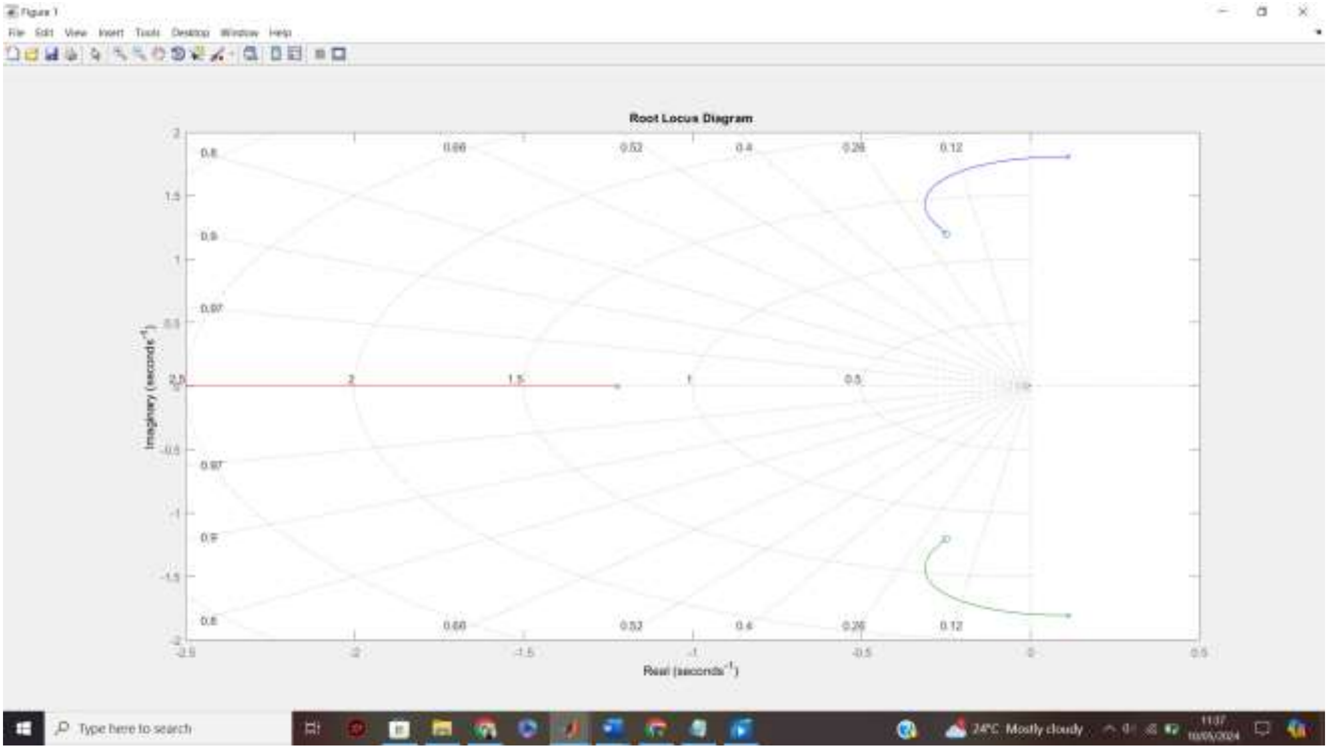
Θ(ω) = (180+360j)/2, j=0,...,2

Θ(ω) = { 90, 270}

Matlab Code:

```
% System parameters
K = 0.1;  % Initial gain value (adjust as needed)
num = K * [1 0.5 1.5];  % Numerator with gain and zeros
den = [1 1 3 4];        % Denominator with coefficients

% Create the root locus object
sys = tf(num, den);

% a) Root Locus diagram and Stability Analysis
figure(1);
rlocus(sys);
title('Root Locus Diagram');
xlabel('Real');
ylabel('Imaginary');
```

```
grid on;

% Analyze stability from the root locus plot
all_stable = all(real(roots(sys)) < 0);
if all_stable
  disp('System is stable for all positive K values.');
else
  disp('System is not stable for all positive K values.');
end

% b) Breakaway Points calculation
[r, k_breakaway] = rlocus(sys, 'Breakaway');

% Display breakaway points (if any)
if ~isempty(k_breakaway)
  hold on;
  scatter(real(r), imag(r), 'o', 'MarkerSize', 10,
'MarkerEdgeColor', 'r');
  for i = 1:length(k_breakaway)
    text(real(r(i)), imag(r(i)), strcat('K = ',
num2str(k_breakaway(i))), ...
         'HorizontalAlignment', 'center', 'VerticalAlignment',
'middle');
  end
  legend('Root Locus', 'Breakaway Points');
else
  disp('No breakaway points found.');
end
```



b) $k(s+0.5)(s+1.5)+2s^3+20s^2+8s=0$

$k(s+2s+0.75)+2\ s^3+20\ s^2+8s$

$2\ s^3+\ (20+k)\ s^2+(8+2k)s+0-75=0$

$K=(2\ s^3+20\ s^2+8s)/\ s^2+2s+0.75$

$dk/ds=(\ s^2+2s+0.75)(6\ s^2+40s+8)-25$

$=[(\ s^2+2s+0.75)(6\ s^2+40s+8)-(2\ s^3+20\ s^2+8s)(2s+2)]/(\ s^2+2s+0.75)\ ^2$

$S=\ -2.2778$

$S_2=-0.5966$

$S_3=0.6601+1.1632$

$S_4=0.6601-1.1632$

Break away, s_2

$2s+(20+k)s+(8+2k)s+0.75=0$

S^3	2	8+2k
S^2	20+k	0.75

S^1	[(20+k)(8+2k)]/20+k	0
S^0	0.75	

For stability,

$160+40k+2k^2-1.5$

K=18.43 or k= -22.43

K<-22.43 or k>18.43.

Question 6 :

$$L(S) = \frac{K(S+2)}{S^2+2S+3}$$

a)

L(s)= ks(s+2) / (s+2)(s+1)

Num of loci, m=2, n=2

Origin of loci s=0,-1,-2,

Center of asymptode ∑poles-∑zeros/n-m

-7-0/4

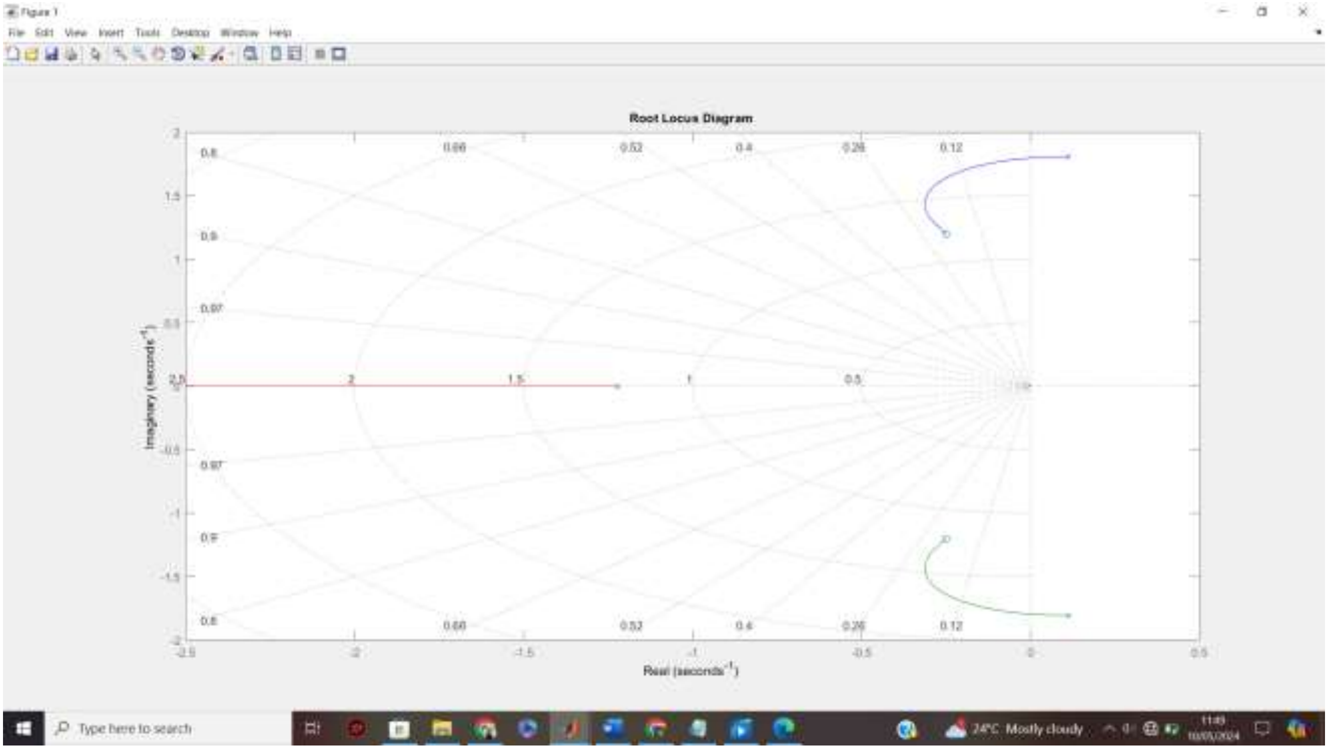
-7/4= -1.75

Angle of asymptode

Φ=180+360j, J= 0,……4

Φ=60, 180, 300

Diagram.



b)$q(1)=k+s(s+1)(s+2)(s+4)=0$

$k+(s+s^2)(s+6s^2+8)=0$

$k+s^3+6s^2+8s^2+s+6s^2+8s=0$

$k=-2s^3-20s^2-8s=0$

dk/ds=, $3s^3+20s^2+4=0$

$s= \frac{-20\pm\sqrt{20^2-4(3)(4)}}{2(3)}$

s=-0.21 or -6.460

break away, s= -0.21

c)$k/s(s+1)(s+2)(s+4)$

$k+(s^2+s)(s^2+6s+8)=0$

$k+s^3+6s^2+8s^2+s^3+6s^2+8s=0$

$2s^3+20s^2+8s+k=0$

s^3	2	8	0
S^2	20	k	0

S^1	(160-2k)/20	0	
S^1	k		

For stability, k>0

(160-2k)/20>0

Therefore 160>2k

K<80

0<k<80