JEE MAIN (2023-24) Mock Test Series

Paper - 02

DURATION : 180 Minutes

M. MARKS : 300

ANSWER KEY

PHYSICS

1.	(1)
2.	(1)
3.	(3)
4.	(3)
5.	(3)
6.	(3)
7.	(2)
8.	(2)
9.	(2)
10.	(3)
11.	(2)
12.	(4)
13.	(3)
14.	(1)
15.	(3)
16.	(2)
17.	(2)
18.	(3)
19.	(2)
20.	(2)
21.	(3)
22.	(494)
23.	(12)
24.	(4)
25.	(5)
26.	(8)
27.	(16)
28.	(17)
29.	(10)
30.	(3)

CHEMISTRY

31.	(3)
32.	(3)
33.	(3)
34.	(1)
35.	(4)
36.	(1)
37.	(2)
38.	(2)
39.	(3)
40.	(3)
41.	(4)
42.	(2)
43.	(3)
44.	(2)
45.	(2)
46.	(3)
47.	(4)
48.	(1)
49.	(3)
50.	(2)
51.	(166)
52.	(35)
53.	(11)
54.	(6)
55.	(2)
56.	(50)
57.	(30)
58.	(1)
59.	(10)
60.	(9)

MATHEMATICS

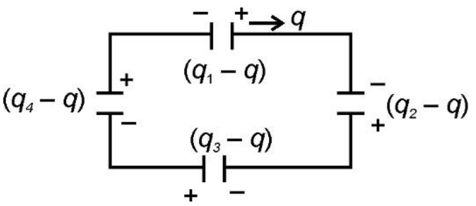
61.	(4)
62.	(4)
63.	(2)
64.	(2)
65.	(4)
66.	(2)
67.	(3)
68.	(3)
69.	(2)
70.	(4)
71.	(3)
72.	(3)
73.	(2)
74.	(1)
75.	(3)
76.	(1)
77.	(1)
78.	(4)
79.	(4)
80.	(1)
81.	(2)
82.	(2)
83.	(75)
84.	(80)
85.	(3)
86.	(3)
87.	(32)
88.	(3)
89.	(13)
90.	(10)

SECTION-I (PHYSICS)

1. **(1)**

Use KVL

Let a charge q flow in circuit in clockwise direction



By loop law,

$$\frac{q_1 - q}{C} + \frac{q_2 - q}{2C} + \frac{q_3 - q}{3C} + \frac{q_4 - q}{4C} = 0$$

$\Rightarrow$ q_1 = CV, q_2 = 4CV, q_3 = 9CV, q_4 = 16CV

$$q = \frac{24}{5}\text{CV}$$

$$V_1 = \left|\frac{q_1 - q}{c}\right| = \frac{19V}{5}$$

$$V_2 = \left|\frac{q_2 - q}{2c}\right| = \frac{2V}{5}$$

$$V_3 = \left|\frac{q_3 - q}{3c}\right| = \frac{7V}{5}$$

$$V_4 = \left|\frac{q_4 - q}{4c}\right| = \frac{14V}{5}$$

2. **(1)**

$$\because\ Mg \times R = \left(\frac{5MR^2}{2}\right) \times \left(\frac{a_c}{R}\right)$$

$$\Rightarrow a_c = \frac{2g}{5}$$

$$\therefore\ T = M\left(g - \frac{2g}{5}\right) = \frac{3Mg}{5}$$

3. **(3)**

For outside point sphere behaves as a point mass.

Gravitational field at centre of sphere due to disc is

$$E = \frac{G\,ma}{[a^2 + [\sqrt{3}a]^{3/2}} = \frac{G\,ma\sqrt{3}}{8a^3}$$

$$E = \frac{G\,m\sqrt{3}}{8a^2}$$

Force on sphere due to this field is

$$F = ME = \frac{\sqrt{3}G\,Mm}{8a^2}$$

4. **(3)**

$$\vec{B} = \frac{-B_0 y\hat{i}}{R} - \frac{B_0 x}{R}\hat{j} = -\frac{B_0}{R}\left(y\hat{i} + x\hat{j}\right)$$

$$\left|\vec{B}\right| = \frac{B_0}{R}\sqrt{x^2 + y^2} = \frac{\mu_0 I}{2\pi R}$$

$$\Rightarrow I = \frac{2\pi B_0 R}{\mu_0}$$

5. **(3)**

$\frac{d^2\theta}{dt^2} = -\omega^2\theta$ for angular SHM

$$\tau = k\frac{L}{6}\theta \cdot \frac{L}{6} + k\frac{L}{3}\theta \cdot \frac{L}{3}$$

$$\Rightarrow \left(\frac{2mL^2}{3} + mL^2\right) \cdot \frac{d^2\theta}{dt^2} = -kL^2\theta\left(\frac{1}{36} + \frac{1}{9}\right)$$

$$\Rightarrow \frac{5mL^2}{3} \cdot \frac{d^2\theta}{dt^2} = -\frac{5kL^2}{36} \cdot \theta$$

$$\Rightarrow \frac{d^2\theta}{dt^2} = -\frac{k}{12m} \cdot \theta$$

$$\Rightarrow \omega = \sqrt{\frac{k}{12m}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{12m}}$$

$$= \frac{1}{4\pi}\sqrt{\frac{k}{3m}}$$

6. **(3)**

$F = T$

Also, $3T = 40$

$$\Rightarrow T = \frac{40}{3}\text{ N}$$

7. **(2)**

$$\gamma = \frac{1}{V}\frac{dV}{dT}$$

$\gamma = \frac{1}{V}\frac{dV}{dT}$, PT^2 = Constant

$\Rightarrow T^3 = kV$

$$\Rightarrow\ 3T^2 = k\frac{dV}{dT} \quad \Rightarrow \frac{3T^2}{k} = \frac{dV}{dT}$$

$$\Rightarrow\ \frac{3T^2}{kT^3}k = \frac{1}{V}\frac{dV}{dT} = \frac{3}{T}$$

8. **(2)**

$$i = \frac{v}{R_1+R_2}e^{-\frac{t}{(R_1+R_2)c}}$$

$$H = \int_0^\infty \left(\frac{v}{R_1+R_2}\right)^2 e^{-\frac{2t}{(R_1+R_2)c}} \times R_1 dt$$

$$= \frac{v^2 c R_1}{2(R_1+R_2)}$$

9. **(2)**

Angular fringe width = 3°

$$3\times\frac{\pi}{180} = \frac{\lambda}{d}$$

10. **(3)**

Long wire does not contribute to and rectangular loop produces magnetic field which are cancelling in pair.

11. **(2)**

Power is zero when $\vec{F}.\vec{v} = 0$

Now $\vec{F} = \frac{q\sigma}{\varepsilon_0}(-\hat{j})$

$$\vec{v} = v\cos\alpha\hat{i} + \left(v\sin\alpha - \frac{q\sigma t}{m\varepsilon_0}\right)\hat{j}$$

$$\vec{F}.\vec{v} = \left(v\sin\alpha - \frac{q\sigma t}{m\varepsilon_0}\right).\frac{q\sigma}{\varepsilon_0} = 0$$

$$t = \frac{mv\sin\alpha\,\varepsilon_0}{q\sigma}$$

12. **(4)**

$$I_P = I_{\text{cm}} + M\left(\frac{R}{2}\right)^2$$

$$I_{CD} = I_{\text{cm}} + M\left(\frac{R}{2}\right)^2$$

Hence, $I_{AB} = I_{CD}$

13. **(3)**

$$\oint \vec{B}\cdot\vec{dl} = \mu_0\Sigma i = \mu_0(2+5-3) = 4\mu_0$$

14. **(1)**

$W_{\text{total}} = \Delta KE$

$(3mg)X + mgX = \frac{1}{2}kX^2$

$$\Rightarrow\ X = \frac{8mg}{k}$$

15. **(3)**

Momentum of photon = Momentum of ion

$$\Delta E_0 Z\left[\frac{1}{9} - \frac{1}{49}\right] = \frac{hc}{\lambda}$$

$$\Rightarrow\ \frac{40}{9\times49}\times Rch\times9 = \frac{hc}{\lambda}$$

$$\Rightarrow\qquad v = \frac{40\,Rh}{49\text{ m}}$$

16. **(2)**

$$X_C = \frac{1}{WC};\quad I_{\text{rms}} = \frac{E_0}{\sqrt{2}\,X_C}$$

$$X_C = \frac{1}{WC} = \frac{1}{100\times10^{-6}} = 10^4\ \Omega$$

$$i_{\max} = \frac{200\sqrt{2}}{10^4} = 20\sqrt{2}\text{ mA}$$

$\Rightarrow$ Reading of AC ammeter

$$= \ l_{\text{rms}} = \frac{20\sqrt{2}}{\sqrt{2}}\text{ mA}$$

= 20 mA

17. **(2)**

$$V = \frac{\rho}{6\varepsilon_0}\left(3R^2 - r^2\right)$$

$$V_0 = \frac{\rho 3R^2}{6\varepsilon_0} - \frac{\rho}{6\varepsilon_0}\left[3\left[\left(\frac{R}{1}\right)^2\right] - \left(\frac{R}{2}\right)^2\right]$$

$$= \frac{\rho R^2}{2\varepsilon_0} - \frac{\rho}{6\varepsilon_0}\left[2.\frac{R^2}{4}\right]$$

$$= \frac{\rho R^2}{2\varepsilon_0} - \frac{\rho R^2}{12\varepsilon_0} = \frac{5\rho R^2}{12\varepsilon_0}$$

18. **(3)**

Use the formula

$$r = \frac{P}{qB}$$

19. **(2)**

Terminal velocity is attained when magnetic force is equal to $mg\sin\theta$.

$F_m = mg\sin\theta$

$iBl = mg\sin\theta$

$\frac{BV_T l}{R} Bl = mg\sin\theta$

$\Rightarrow V_T = \frac{mgR\sin\theta}{B^2 l^2}$

20. **(2)**

$mv_0 = 3m\, v_x$

$v_x = \frac{v_0}{3}$

Using energy conservation

$\frac{1}{2}mv_0{}^2 + 0 = \frac{1}{2}mv_1^2 + \frac{1}{2}(2m)\times\left(\frac{v_0}{3}\right)^2 + mgR$

$\Rightarrow v_1^2 = \frac{8gR}{3}$

$v_x^2 + v_y^2 = v_1^2$

$\left(\frac{v_0}{3}\right)^2 + v_y^2 = \frac{8gR}{3}$

$v_y^2 = 2gR$

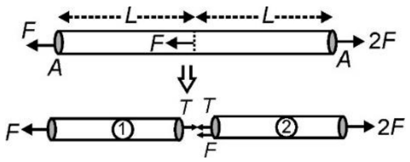
h max = $\frac{v_y^2}{2g}$

$\Rightarrow h$ max = R

$\Rightarrow$ maximum this maximum height attained by the block with respect to its initial level is 2 R

21. **(3)**

$\Delta l = \frac{\sum F_i L_i}{AY}$



Stress in part (1) is $\frac{F}{A}$ and stress in part (2) is $\frac{2F}{A}$

$\Delta l = \frac{2FL}{AY} + \frac{FL}{AY}$

$= \frac{3FL}{AY}$

22. **(494)**

Energy of incident photons

= 13.6 Z^2

= 13.6 × 2^2

= 54.4 eV

Max K.E. of photoelectrons

= 54.4 – 5

= 49.4 eV

Stopping potential = 49.4 volt

23. **(12)**

$F_B = i\, dl \times B$

$F_{\text{net}} = \frac{\mu_0 I_0 \times 2}{2\pi a} Ia - \frac{\mu_0 I_0 \times 2}{2\pi 3a} Ia$

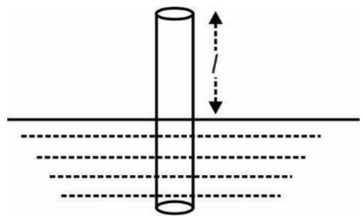
$\therefore \quad F_{\text{net}} = \frac{\mu_0 I_0}{2\pi a} 2Ia\,(1 - 1/3)$

$\Rightarrow \quad F_{\text{net}} = \frac{\mu_0 I_0 I}{\pi} \times \frac{2}{3} = 4\times10^{-7} \times \frac{2}{3} \times 5 \times \frac{9}{10}$

= 1.2 μN

24. **(4)**

Fundament frequency, $F = \frac{c}{4x}$



Fundamental frequency, $f = \frac{c}{4l}$

$f = \frac{c}{4} l^{-1}$

$\frac{df}{dt} = \frac{d}{dt}\left[\frac{c}{4} l^{-1}\right] = \frac{c}{4}(-1)\, l^{-2} \frac{dl}{dt}$

$= -\frac{c}{4l^2}\frac{dl}{dt} = \frac{-c}{4l^2}(-v)$

$\frac{df}{dt} = \frac{cv}{4l^2} = \frac{cv}{x\, l^2}$ where, $x = 4$

25. **(5)**

If t be the tension in the string, then

$F = 2T = 20t$

$\Rightarrow \quad T = 10t$ newton

Let the block A lose its contact with the floor at time $t = t_1$ (say). This happens when the tension in string becomes equal to the weight of block A.

So,

$T = mg$

$\Rightarrow \quad 10t_1 = 1\times10$

$\Rightarrow \quad t_1 = 1\ s$

Similarly, for block B, we have

$10t_2 = 2\times10$

$\Rightarrow \quad t_2 = 2\ s$ …..(2)

i.e., the block B loses contact with the floor after $t_2 = 2\ s$.

For block A, at time t such that $t \geq t_1$ let a be its acceleration in upward direction. Then

$$10t-(1)(10)=(1)(a)=\left(\frac{dv}{dt}\right)$$

$$\Rightarrow dv=10(t-1)dt \qquad \text{…..(3)}$$

Integrating, we get

$$\int_0^v dv=10\int_1^t (t-1)dt$$

$$\Rightarrow v=5t^2-10t+5 \qquad \text{…..(4)}$$

Substituting $t = t_2 = 2\ s$, we get

$v = 20 - 20 + 5 = 5\ \text{ms}^{-1}$ ….(5)

26. **(8)**

V_m (just before collision) = $2\sqrt{2gh_0}$

V_m (just before collision) = $2\sqrt{2gh_0}$

$\Rightarrow V_{3m}$ (just after collision) = $\left(\frac{V_m}{5}\right)$

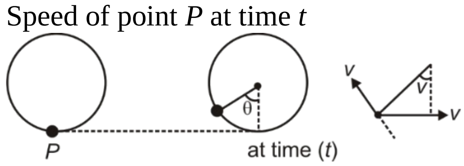
$a_{3m}=\frac{g}{5}$ (Downward)

$$\Rightarrow \quad H_{\max}=\frac{(V_{3m})^2}{a_{3m}}=\frac{4h_0}{5}=8\,\text{cm}$$

27. **(16)**

$s=\int vdt$

Speed of point P at time t



$$v_0=\sqrt{v^2+v^2+2v^2\cos(180-\theta)}$$

$$=2v\sin\left(\frac{\theta}{2}\right)$$

$\therefore$ Distance covered in one revolution

$$s=\int_0^T v_0dt$$

where $\theta=\omega t$

$$=\frac{v}{R}t$$

$s = 8R = 16$ m

28. **(17)**

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$(n_A-1)\frac{2}{R_A}=(n_B-1)\frac{2}{R_B}$$

$$n_B=1.7$$

29. **(10)**

For solenoid $B=\mu_0 nI$

Resistance $R_0=(2\pi r)\times400\frac{1}{100}$

n = (no. of turns per unit length) = $\frac{400}{20}\times100$

$\Rightarrow \ n = 2000$

$$\therefore \quad B=\frac{\mu_0\times2000\times E_0\times100}{2\pi r\times400}$$

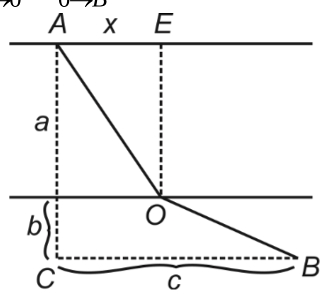
$$\Rightarrow \quad E_0=\frac{1}{10}\times2\pi\times\frac{1}{100}\times\frac{400}{4\pi\times10^{-7}\times2000\times100}$$

$\Rightarrow \quad E_0 = 10$ volts

30. **(3)**

$T=t_{A\to0}+t_{0\to B}$

$T=t_{A\to0}+t_{0\to B}$



$$T=\frac{\sqrt{a^2+x^2}}{v_1}+\frac{\sqrt{b^2+(c-x)^2}}{v_2}$$

for minimum T, $\frac{dT}{dx}=0$

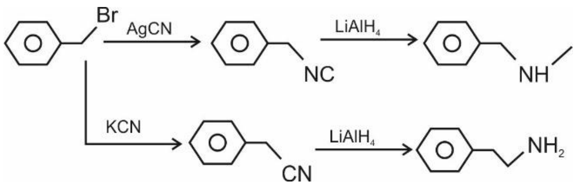
$$\Rightarrow \quad \frac{\frac{x}{\sqrt{a^2+x^2}}}{\frac{(c-x)}{\sqrt{b^2+(c-x)^2}}}=\frac{v_1}{v_2}\Rightarrow\frac{\sin\alpha_1}{\sin\alpha_2}=\frac{6}{2}=3$$

SECTION-II (CHEMISTRY)

31. **(3)**

Diazonium salts of aromatic amines are more stable

32. **(3)**



33. **(3)**

Stability Order :

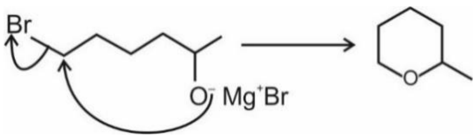
Conjugated > Isolated > Cumulative > Anti aromatic.

$$HOH \propto \frac{1}{\text{Stability of Alkene}}$$

34. **(1)**

Ease of Carbocation formation

35. **(4)**



36. **(1)**

Inversion + Inversion = Retention

37. **(2)**

$$3O_2(g) \rightleftharpoons 2O_3(g), \quad \Delta H \longrightarrow \text{positive}$$

High temperature favours forward path and hence

$T_1 > T_2 > T_3$

38. **(2)**

Potential energy $= (2)\left(\frac{-13.6}{4}\right) eV$

$$= \frac{-13.6}{2} = -6.8 \text{ eV}$$

39. **(3)**

$[Ag(NH_3)_2]^+$ is linear

$[Cu(NH_3)_4]^{2+}$ is square planar

40. **(3)**

$t_{2g}^3 e_g^2$ is for sp^3d^2 complex

41. **(4)**

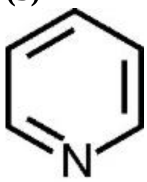
For CO, x = 2

For I_2O_5, x = 10

42. **(2)**

Primary valencies may or may not ionisable

43. **(3)**



sp^2 hybridised N-atom (more electronegative than sp^3 hybridised N-atom) is less basic.

Due to aromaticity, Pyrrole is least basic

44. **(2)**

$C_3H_8 + 5O_2 \rightarrow 3CO_2 + 4H_2O$

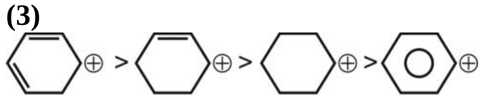
$\Delta H = -\{6(C{=}O) + 8(OH)\} + 2(C-C) + 8(C-H) + 5(O{=}O)$

$= 8a_1 + 5a_2 + 2a_5 - 6a_3 - 8a_4$

45. **(2)**

Correct order : $Na^+ > Li^+ \simeq Mg^{2+} > Be^{2+}$

46. **(3)**



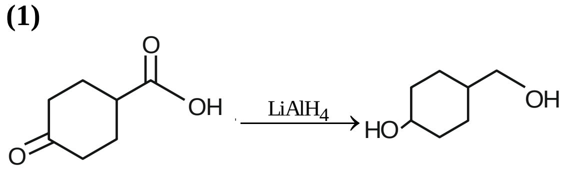
Ease of S_N1 and hence the correct order is $\propto$ stability of carbocation intermediate

I > IV > III > II

47. **(4)**

In Reimer-Tiemann reaction intermediate is carbene

48. **(1)**



49. **(3)**

pH = 6

$[H^+] = 10^{-6}$

Upon dilution 100 times

$[H^+] = 10^{-8}$

New pH = 6.97 due to contribution of H^+ from H_2O.

50. **(2)**

n factor of $A^{n-} = 3$

hence, final O.S. = 3 – n

51. **(166)**

Rate constant (K) = $Ae^{-E_a/RT}$

$\therefore \quad E_a = 0$

$\therefore \quad K = A$

52. **(35)**

$BaCl_2 \rightarrow Ba^{+2} + 2Cl^-$

0.1 m 2(0.1) = 0.20 m

$BaSO_4 \rightleftharpoons Ba^{+2} + SO_4^{-2}$

(0.1 + S) S

$(0.1)(S) = 1.5 \times 10^{-9}$

$S = 1.5 \times 10^{-8}$ mol/L

$= 1.5 \times (137 + 32 + 64) \times 10^{-8}$ gram/L

$= 34.95 \times 10^{-7}$ gram/L

53. **(11)**

gm equivalent wt. of

$As_2S_3 = \dfrac{75\times2+32\times3}{22}$

≈ 11.18

54. **(6)**

Rate is dependent on concentration of both reactants HCHO and NaOH. It is second order with respect to HCHO and first order with respect to NaOH. Hence, x = 3.

Similarly, the second reaction is second order with respect to benzaldehyde and first order with respect to CN^- ion. Hence y = 3.

So $\boxed{x+y=6}$

55. **(2)**

Heat gain by Neon = Heat loss by Helium

$\Rightarrow \quad n_1 C_v (500 - 400) = 0.1 \times C_v (700 - 500)$

$\Rightarrow \quad n_1 \times 100 = 0.1 \times 200$

$n_1 = 0.2$

56. **(50)**

$CaCO_3 \longrightarrow CaO + CO_2$

$CO_2 + H_2O \longrightarrow H_2CO_3$

$H_2CO_3 + 2NaOH \longrightarrow Na_2CO_3 + 2H_2O$

m mole of NaOH = 0.2 × 500 = 100

$\therefore$ m mole of $H_2CO_3 = \left(\dfrac{100}{2}\right) = 50$

m mole of $CaCO_3 = 50$

Mole of $CaCO_3 = 50 \times 10^{-3}$

Mass of $CaCO_3 = 50 \times 10^{-3} \times 100 = 5$ g

% purity of $CaCO_3 = \left(\dfrac{5}{10}\times100\right) = 50\%$

57. **(30)**

$Zn(s) + CuSO_4(aq) \rightarrow Cu(s) + ZnSO_4(aq)$

$E° = E_i$

$E_f = E° - \dfrac{0.06}{2}\log\dfrac{1}{10}$

$(E_f - E_i) = \dfrac{0.06}{2}\log(10)$

= 0.03 Volt

58. **(1)**

	A	+ B	⇌ C	+ D
t = 0	x	x	0	0
t = t eq.	$x-\frac{x}{2}$	$x-\frac{x}{2}$	$\frac{x}{2}$	$\frac{x}{2}$

$\therefore \quad k_{eq} = 1$

59. **(10)**

$HA + OH^- \rightleftharpoons A^- + H_2O$

$K_{eq} = \dfrac{[A^-]}{[OH^-][HA]}$

$HA + H_2O \rightleftharpoons A^- + H_3O^+$

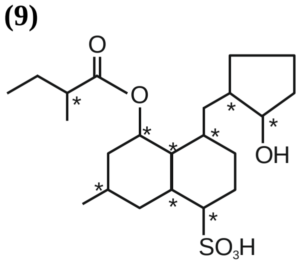
$K_a = \dfrac{[A^-][H_3O^+]}{[HA]}$

$K_{eq} = \dfrac{[A^-][H_3O^+]}{[OH^-][HA][H_3O^+]} = \dfrac{K_a}{K_\omega} = \dfrac{4\times10^{-8}}{10^{-14}} = 4\times10^6$

A = 4, B = 6

A + B = 10

60. **(9)**



SECTION-III (MATHEMATICS)

61. **(4)**

Mean $\bar{x}=\dfrac{x_1+x_2+\ldots x_{100}}{100}=55$

If each data is increased by 2, then

$\bar{x}_{\text{new}}=\dfrac{(x_1+x_2+\ldots x_{100})+2\times100}{100}=\bar{x}+2$

$\Rightarrow \bar{x}_{\text{new}}=57$

Variance $=\dfrac{\Sigma(x-\bar{x})^2}{n}=16$

Variance will not change if each data is increased by 2.

$\therefore$ Variance new = 16

$\Rightarrow$ Standard deviation

$=\sqrt{\text{variance}}=4$

$\therefore$ Sum of mean and standard deviation

= 57 + 4 = 61

62. **(4)**

Reflexivity:

We have $\sin^2 p+\cos^2 p=1$

$\forall\ a\in R$

$\Rightarrow pR'p\ \forall\ p\in R \Rightarrow R'$ is reflexive

Symmetry: Let $pR'q$

$\Rightarrow \sin^2 p+\cos^2 q=1$

$\Rightarrow 1-\cos^2 p+1-\sin^2 q=1$

$\Rightarrow \sin^2 q+\cos^2 p=1$

$\Rightarrow qR'p\ \Rightarrow R'$ is symmetric

Transitivity:

Let $pR'q$ and $qR'r$

$\Rightarrow \sin^2 p+\cos^2 q=1$ ...(i)

and $\sin^2 q+\cos^2 r=1$ ...(ii)

By [(i) + (ii)]

$\sin^2 p+(\cos^2 q+\sin^2 q)+\cos^2 r=2$

$\Rightarrow \sin^2 p+\cos^2 r=1$

$\Rightarrow pR'r\ \Rightarrow R'$ is transitive also.

Hence R is equivalence relation.

63. **(2)**

$$f(x)=\begin{cases}k_1(x-\pi)^2-1 & ;\ x\le\pi\\ k_2\cos x & ;\ x>\pi\end{cases}$$

$$f'(x)=\begin{cases}2k_1(x-\pi) & ;\ x\le\pi\\ -k_2\sin x & ;\ x>\pi\end{cases}$$

and $$f''(x)=\begin{cases}2k_1 & ;\ x\le\pi\\ -k_2\cos x & ;\ x>\pi\end{cases}$$

Θ $f(x)$ is twice differentiable at x = π, then (i) $\lim\limits_{x\to\pi^+}f(x)=\lim\limits_{x\to\pi^-}f(x)\ \Rightarrow -1=-k_2\Rightarrow k_2=1$ (ii)

$\lim\limits_{x\to\pi^+}f''(x)=\lim\limits_{x\to\pi^-}f''(x)\ \Rightarrow k_2=2k_1\Rightarrow k_1=\dfrac{1}{2}$

64. **(2)**

IF = $e^{-\int\frac{x}{1+x}dx}=e^{-\int\frac{x+1-1}{x+1}dx}=e^{-x+\ln|(x+1)|}$

$=e^{-x}\cdot|(x+1)|$

$y\cdot|(x+1)|e^{-x}=\displaystyle\int\frac{|x+1|}{x+1}e^{-x}dx+c$

$y\cdot|(x+1)|e^{-x}=-\dfrac{|x+1|}{x+1}e^{-x}+c$

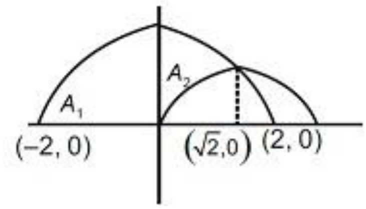
$\Rightarrow\ y=-\dfrac{1}{x+1}+\dfrac{ce^x}{|x+1|}$

$x=0, y=-1\ \Rightarrow\ -1=-1+c\ \Rightarrow\ c=0$

$\Rightarrow y=-\dfrac{1}{1+x}$

$y(2)=-\dfrac{1}{3}$

65. **(4)**



$A_2=\displaystyle\int_0^{\sqrt2}\sqrt{4-x^2}-\int_0^{\sqrt2}\sqrt2\sin\left(\frac{\pi}{2}\frac{x}{\sqrt2}\right)dx$

$A_2=1+\dfrac{\pi}{2}-\dfrac{4}{\pi}.$

$A_1=\pi$

$\therefore\ \dfrac{A_1}{A_2}=\dfrac{2\pi^2}{2\pi+\pi^2-8}$

66. **(2)**

Clearly $x=0, 1, 2, 3\ldots, 10$.

Let $x=k$; then $0\le k\le10$

When $x=k$, $y+z=21-2k$

The number of non-negative integral solutions

= the number of ways to distribute $(21-2k)$ identical things (each thing is the number 1) among 2 persons

$={}^{21-2k+2-1}C_{2-1}={}^{22-2k}C_1=22-2K$

$\therefore$ The required number of solutions

$$\sum_{k=0}^{10}(22-2k)=22+20+18+\ldots+2$$

$$=2(1+2+3+\ldots+11)=2\times\frac{11\times 12}{2}=132$$

67. **(3)**

$f(x) = \sin^2 x - 3(1 - \sin^2 x) + 2ax - 4$

$= 4\sin^2 x + 2ax - 7$

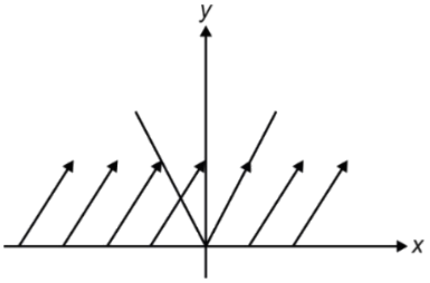
$\therefore\ f'(x) = 8\sin x\cos x + 2a \geq 0$ for all x

$\therefore\ -4 + 2a \geq 0$ i.e., $a \geq 2$.

68. **(3)**

$$f(x)=\sqrt{|x|-\{x\}}$$

$$|x|\geq\{x\}$$



$$X\in\left(-\infty,-\frac{1}{2}\right]\cup[0,\infty)\ \text{and}\ Y\in[0,\infty)$$

$Y \in [0, \infty)$ and $f(x)$ is many one

69. **(2)**

$$\left(A'-\frac{1}{2}I\right)\left(A-\frac{1}{2}I\right)=I\ \text{and}$$

$$\left(A'+\frac{1}{2}I\right)\left(A+\frac{1}{2}I\right)=I$$

$\Rightarrow\ A + A' = 0$ (subtracting the two results)

$$\Rightarrow A'=-A\Rightarrow A^2=-\frac{3}{4}I$$

$$\Rightarrow\left(\frac{-3}{4}\right)^n=(\det(A))^2\Rightarrow n\text{ is even}$$

70. **(4)**

$$\int\left(x^4+2x^2+2\right)dx=\frac{x^5}{5}+\frac{2x^3}{3}+2x+c$$

71. **(3)**

Here we use $\lim\limits_{\theta\to 0}\dfrac{\tan^{-1}(\theta)}{\theta}=1$

$$\lim_{x\to\infty}x\left[\tan^{-1}\left(\frac{x+1}{x+2}\right)-\tan^{-1}\left(\frac{x}{x+2}\right)\right]$$

$$=\lim_{x\to\infty}x\cdot\tan^{-1}\left\{\frac{x+2}{2x^2+5x+4}\right\}$$

$$=\lim_{x\to\infty}\frac{\tan^{-1}\left\{\dfrac{x+2}{2x^2+5x+4}\right\}}{\dfrac{x+2}{(2x^2+5x+4)}}\cdot\lim_{x\to\infty}$$

$$x\left[\frac{x+2}{2x^2+5x+4}\right]=\frac{1}{2}$$

72. **(3)**

$$x\in[-1,1],\ 0\leq\frac{x^2}{2}+\sqrt{1-x^2}\sqrt{1-\frac{x^2}{4}}\leq 1$$

$$\text{and }\cos^{-1}\frac{x}{2}\geq\cos^{-1}x$$

$$\Rightarrow\ x\in[0,1]$$

73. **(2)**

By graph,

$x_2 > x_4 > x_3 > x_1$

74. **(1)**

$A + C = \pi$

$\Rightarrow C = \pi - A$

and $B + D = \pi$

$\Rightarrow D = \pi - B$

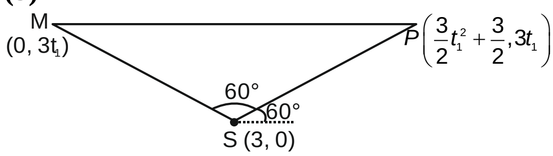
$\therefore\ \cos A + \cos B + \cos(\pi - A) + \cos(\pi - B) = 0$

Since, $\tan 3\theta=\dfrac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}$

Put $\theta=\dfrac{\pi}{12}$, so $3\tan\dfrac{\pi}{12}-\tan^3\dfrac{\pi}{12}=1-3\tan^2\dfrac{\pi}{12}$

Put $\tan\dfrac{\pi}{12}=x$; $x^3 - 3x^2 - 3x + 1 = 0$

75. **(3)**



Slope of $SM = \tan 120° = -\sqrt{3}$

$\Rightarrow t_1=\sqrt{3}$

Side $=\sqrt{9+27}=6$

76. **(1)**

$$\text{Here, } f(x)=\begin{cases}e^{\{e^{|x|}\}} & ,\ \text{if } x>0\\ 1 & ,\ \text{if } x=0\\ e^{\{-e^{|x|}\}} & ,\ \text{if } x<0\end{cases}$$

and $g(x)=\begin{cases} e^{\left[e^{|x|}\right]} & , \text{ if } x>0 \\ 1 & , \text{ if } x=0 \\ e^{\left[-e^{|x|}\right]} & , \text{ if } x<0 \end{cases}$

$$h(x)=\ln(f(x))+\ln(g(x))=\begin{cases} e^{|x|} & , \text{ if } x>0 \\ 0 & , \text{ if } x=0 \\ -e^{|x|} & , \text{ if } x<0 \end{cases}$$

77. (1)

If $x < 0$, then $f(x) = 10^x - 10^{-x}$

$f'(x) = 10^x \log_e 10 + 10^{-x} \log_e 10$

$= (10^x + 10^{-x}) \log_e 10 > 0$, for $x < 0$

$\therefore\ f(x)$ is one-one onto.

78. (4)

$$\frac{|\text{adj } B|}{|C|}=\frac{|\text{adj(adj } A)|}{|3A|}=\frac{|A|^{2^2}}{3^3|A|}=\left(\frac{|A|}{3}\right)^3$$

$$|A|=\begin{vmatrix} 1 & 1 & 2 \\ 1 & 3 & 4 \\ 1 & -1 & 3 \end{vmatrix}=1(13)-1(-1)+2(-4)=6$$

Hence, $\dfrac{|\text{adj } B|}{|C|}=\left(\dfrac{6}{3}\right)^3=8$

79. (4)

$2018 + \lambda = 0 \Rightarrow \lambda = -2018$

80. (1)

$$\tan^{-1}\left(\frac{2x}{1-x^2}\right)+\cot^{-1}\left(\frac{1-x^2}{2x}\right)=\frac{\pi}{3}$$

When $x \in (0, 1)$

then $\dfrac{2x}{1-x^2} > 0$

then $\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)+\tan^{-1}\left(\dfrac{2x}{1-x^2}\right)=\dfrac{\pi}{3}$

$\Rightarrow \tan^{-1}\left(\dfrac{2x}{1-x^2}\right)=\dfrac{\pi}{6}$

$\Rightarrow \dfrac{2x}{1-x^2}=\dfrac{1}{\sqrt{3}}$

$\Rightarrow \dfrac{1-x^2}{2x}=\sqrt{3}$

$\Rightarrow \dfrac{1}{x}-x=2\sqrt{3}$

$\Rightarrow \dfrac{1}{x^2}+x^2-2=12$

$\dfrac{1}{x^2}+x^2=14$

$\Rightarrow \dfrac{1}{x^4}+x^4+2=196$

$\Rightarrow x^4+\dfrac{1}{x^4}=194$

81. (2)

$$f(x)=\tan^{-1}\left(\frac{\sin x-\cos x}{\sin x+\cos x}\right)=x-\frac{\pi}{4}$$

$\therefore$ let $y=\dfrac{x}{2}$

$\dfrac{d}{dy}\left(f(x)\right)=2$

82. (2)

$$\lim_{x\to 1}\left(\frac{1+x}{2+x}\right)^{\frac{1}{1+\sqrt{x}}}=\sqrt{\frac{2}{3}}$$

83. (75)

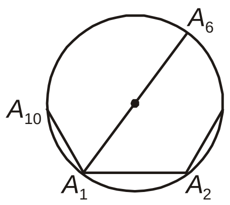
$\bar{w}=a\bar{x}+b \Rightarrow 55=48a+b$ …(i)

$\sigma_w^2=a^2\sigma_x^2$

$15^2 = a^2 \cdot 12^2 \Rightarrow a=\dfrac{5}{4},\ b=-5$

84. (80)

Diameter of circle = $2r = A_1A_6$



$A_1A_2=2r\sin\dfrac{\pi}{10}=A_1A_{10}$

$A_1A_3=2r\sin\dfrac{2\pi}{10}=A_1A_9$

Similarly

$A_1A_5=2r\sin\dfrac{4\pi}{10}=A_1A_7$

$A_1A_6 = 2r$ where $r = 2$

Required result

$$=(4\times r^2)\sum_{k=1}^{4}2\left(\sin^2\frac{k\pi}{10}\right)+4r^2$$

$= 16\sum_{k=1}^{4}\left(1-\cos\frac{2k\pi}{10}\right)+16$

$= 16 \times 4 + 16 = 80$

85. **(3)**

Let $f'(x) = \lambda(x-1)(x+1) = \lambda(x^2-1)$

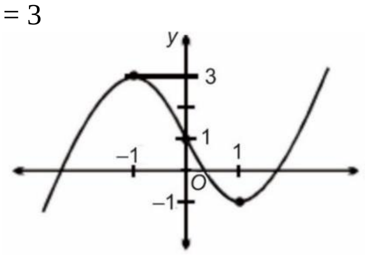
Integrating on both sides, we get

$$f(x)=\lambda\left(\frac{x^3}{3}-x\right)+c$$

Now, $f(1)=-1 \Rightarrow -1=\frac{-2\lambda}{3}+c$ ….(1)

Similarly, $f(-1)=3 \Rightarrow 3=\frac{2\lambda}{3}+c$ …(2)

$\therefore$ From (1) and (2), we get $2c = 2 \Rightarrow c = 1$ and l = 3

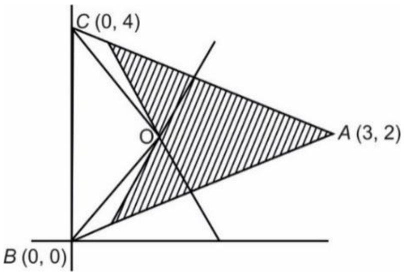


$$\therefore \quad f(x)=3\left(\frac{x^3}{3}-x\right)+1=x^3-3x+1$$

So, $f(2) = 8 - 6 + 1 = 3$

86. **(3)**

In the figure O is circumcentre and shaded region shows location of P.



Distance of P from side BC is maximum if P is at A.

87. **(32)**

$$\text{Area} = 2\int_0^2\left(x^2+4-2x^2\right)dx=\frac{32}{3}$$

88. **(3)**

For range $[0, \infty)$, $D = 0$

$$4(\sin^{-1}\beta)^2-4\left(\frac{\pi}{\sqrt{2}}+\sin^{-1}\alpha\right)\left(\frac{\pi}{\sqrt{2}}-\sin^{-1}\alpha\right)=0$$

$$\therefore\ (\sin^{-1}\alpha)^2+(\sin^{-1}\beta)^2=\frac{\pi^2}{2}$$

$$\Rightarrow \left(\sin^{-1}\alpha\right)^2+\left(\sin^{-1}\beta\right)^2=\frac{\pi^2}{2}$$

$$\Rightarrow \sin^{-1}\alpha=\pm\frac{\pi}{2}$$

$$\sin^{-1}\beta=\pm\frac{\pi}{2}$$

$\Rightarrow \alpha = \pm 1$

$\beta = \pm 1$

89. **(13)**

$$\left(\sqrt[3]{\frac{a}{b}}+\sqrt[3]{\frac{b}{\sqrt{a}}}\right)^{21}$$

$$t_{r+1}={}^{21}C_r\left(\frac{a}{b}\right)^{\frac{21-r}{3}}\frac{b^{r/3}}{a^{r/6}}={}^{21}C_r\ a^{\frac{42-3r}{6}}\ b^{\frac{2r-21}{3}}$$

$\therefore\ 42 - 3r = 4r - 42$

i.e. $r = 12$

$\therefore$ 13th term contains same power of a and b

90. **(10)**

$$\text{Adj } A=\begin{bmatrix}3 & -2 & 7\\ 2 & -6 & -4\\ 1 & 3 & 2\end{bmatrix}$$

$|\text{Adj } A| = \qquad |A|^2$

$|\text{Adj } A| = 100 \Rightarrow |A| = \pm 10$