JEE MAIN (2023-24) Mock Test Series

Paper - 05

DURATION : 180 Minutes

M. MARKS : 300

ANSWER KEY

PHYSICS

1. (2)
2. (1)
3. (2)
4. (4)
5. (3)
6. (2)
7. (2)
8. (2)
9. (3)
10. (3)
11. (2)
12. (1)
13. (2)
14. (2)
15. (2)
16. (2)
17. (2)
18. (3)
19. (1)
20. (4)
21. (6)
22. (20)
23. (24)
24. (8)
25. (15)
26. (70)
27. (12)
28. (4)
29. (40)
30. (5)

CHEMISTRY

31. (4)
32. (1)
33. (1)
34. (3)
35. (4)
36. (3)
37. (3)
38. (4)
39. (2)
40. (1)
41. (1)
42. (4)
43. (1)
44. (4)
45. (1)
46. (1)
47. (3)
48. (2)
49. (4)
50. (4)
51. (5)
52. (2)
53. (6)
54. (1)
55. (2)
56. (7)
57. (4)
58. (3)
59. (7)
60. (6)

MATHEMATICS

61. (3)
62. (2)
63. (4)
64. (4)
65. (3)
66. (1)
67. (1)
68. (2)
69. (3)
70. (1)
71. (3)
72. (3)
73. (3)
74. (1)
75. (2)
76. (1)
77. (4)
78. (1)
79. (2)
80. (4)
81. (2)
82. (43)
83. (9)
84. (5)
85. (6)
86. (9)
87. (208)
88. (49)
89. (3)
90. (1)

SECTION-I (PHYSICS)

1. **(2)**

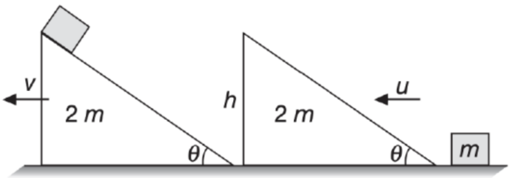
When the block just reaches the top of the wedge then the velocity of block with respect to wedge at the top of the wedge is zero. Let v be the horizontal velocity of both at this instant. By Law of conservation of Linear Momentum, we have

$(2m + m)\, v = mu$

$$\Rightarrow v = \frac{u}{3}$$

By law of conservation of of Mechanical energy, we get

$$\frac{1}{2}mu^2 = \frac{1}{2}(3m)v^2 + mgh$$



$$\Rightarrow u^2 = 3\left(\frac{u^2}{9}\right) + 2gh$$

$$\Rightarrow \frac{2}{3}u^2 = 2gh$$

$$\Rightarrow u = \sqrt{3gh}$$

2. **(1)**

$$f = \frac{1}{2\ell}\sqrt{\frac{\mathrm{T}}{m}};$$

In air: $T = mg = \rho V g$

$$\therefore\ f = \frac{1}{2\ell}\sqrt{\frac{\rho V g}{m}} \qquad \ldots(1)$$

In water : $T = mg$ – upthrust

$$= V\rho g - \frac{V}{2}\rho_\omega g = \frac{Vg}{2}(2\rho - \rho_\omega)$$

$$\therefore\ f' = \frac{1}{2\ell}\sqrt{\frac{\frac{Vg}{2}(2\rho - \rho_\omega)}{m}}$$

$$= \frac{1}{2\ell}\sqrt{\frac{Vg\rho}{m}}\sqrt{\frac{(2\rho - \rho_\omega)}{2\rho}}$$

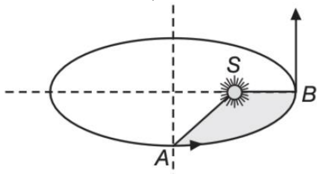
$$\frac{f'}{f} = \sqrt{\frac{2\rho - \rho_\omega}{2\rho}}$$

$$f' = f\left(\frac{2\rho - \rho_\omega}{2\rho}\right)^{1/2}$$

$$300\left(\frac{2\rho - 1}{2\rho}\right)^{1/2} \text{ Hz}$$

3. **(2)**

Since, we know that areal velocity of planet is constant. So, we have



$$\frac{\text{Area of Ellipse}}{\text{Period of Revolution}} = \frac{\text{Area } SAB}{t_{AB}}$$

$$\Rightarrow t_{AB} = \frac{T\left(\frac{\pi ab}{4} - \frac{1}{2}(b)(ea)\right)}{\pi ab}$$

$$\Rightarrow t_{AB} = T\left(\frac{1}{4} - \frac{e}{2\pi}\right)$$

4. **(4)**

In electromagnetic wave, the electric field vector is given as

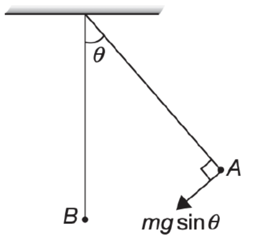
$$E = (E_1\hat{i} + E_2\hat{j})\cos(kz - \omega t)$$

In electromagnetic wave, the associated magnetic field vector,

$$B = \frac{E}{c} = \frac{(E_1\hat{i} + E_2\hat{j})}{c}\cos(kz - \omega t)$$

As, E and B are perpendicular to each other and the propagation of electromagnetic wave is perpendicular to E as well as B, so the given electromagnetic wave is plane polarised.

5. **(3)**



At extreme position A, since the ball is at rest so, net acceleration is equal to the tangential acceleration.

$$\Rightarrow a_A = g\sin\theta$$

At lowermost position B, net acceleration is centripetal acceleration, i.e.,

$$a_B = \frac{v^2}{L} \text{ where } v = \sqrt{2gL(1 - \cos\theta)}$$

$$\Rightarrow a_B = 2g(1 - \cos\theta)$$

Since, $a_A = a_B$

$$\Rightarrow g\sin\theta = 2g(1 - \cos\theta)$$

$$\Rightarrow 2g\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)=2g\left(1-\left(1-2\sin^2\frac{\theta}{2}\right)\right)$$

Since $\sin\theta=2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)$ and

$$1-\cos\theta=2\sin^2\left(\frac{\theta}{2}\right)$$

$$\Rightarrow 2g\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)=2g\times2\sin^2\left(\frac{\theta}{2}\right)$$

$$\Rightarrow \tan\frac{\theta}{2}=\frac{1}{2}$$

$$\Rightarrow \frac{\theta}{2}=26.5^\circ$$

$$\Rightarrow \theta=53^\circ$$

6. **(2)**

$$X_C=\frac{1}{\omega C};\quad I_{\text{rms}}=\frac{E_0}{\sqrt{2}\,X_C}$$

$$X_C=\frac{1}{\omega C}=\frac{1}{100\times10^{-6}}=10^4\ \Omega$$

$$i_{\max}=\frac{200\sqrt{2}}{10^4}=20\sqrt{2}\text{ mA}$$

$\Rightarrow$ Reading of AC ammeter

$$=l_{\text{rms}}=\frac{20\sqrt{2}}{\sqrt{2}}\text{ mA}$$

= 20 mA

7. **(2)**

$$V=\frac{\rho}{6\varepsilon_0}\left(3R^2-r^2\right)$$

$$V=\frac{\rho 3R^2}{6\varepsilon_0}-\frac{\rho}{6\varepsilon_0}\left[3\left[\left(\frac{R}{2}\right)^2\right]-\left(\frac{R}{2}\right)^2\right]$$

$$=\frac{\rho R^2}{2\varepsilon_0}-\frac{\rho}{6\varepsilon_0}\left[2.\frac{R^2}{4}\right]$$

$$=\frac{\rho R^2}{2\varepsilon_0}-\frac{\rho R^2}{12\varepsilon_0}=\frac{5\rho R^2}{12\varepsilon_0}$$

8. **(2)**

$$R=\sqrt{\frac{L}{C}}\Rightarrow RC=\frac{L}{R}$$

$\Rightarrow$ Time constant of both circuits are equal

$$l_L=i_C$$

$$\frac{V}{R}\left(1-e^{-\frac{t}{\tau}}\right)=\frac{V}{R}e^{-\frac{t}{\tau}}$$

$$\Rightarrow t=\tau\ln(2)=RC\ln(2)$$

9. **(3)**

Length of air column on both side is 45 cm when one side at 0°C and the other is at 273°C. The pressure must be same on both sides. Hence

$$\frac{l_1}{T_1}=\frac{l_2}{T_2}\Rightarrow\frac{l_1}{273}=\frac{l_2}{(273+273)}\Rightarrow l_1=\frac{l_2}{2}$$

Applying gas equation to the side at 0°C, we get

$$\frac{P_1 l_2}{T_1}=\frac{Pl}{T}\Rightarrow\frac{P_1\times30}{273}=\frac{76\times45}{(273+81)}\Rightarrow P_1=88.16$$

10. **(3)**

$$\frac{1}{2}K\left(A'\right)^2=\frac{1}{2}KA^2+\frac{1}{2}m\omega^2A^2$$

$$\text{K.E.}=\frac{1}{2}m\omega^2(A^2-x^2)$$

$$=\frac{1}{2}m\omega^2\left(A^2-\frac{3}{4}A^2\right)$$

$$=\frac{1}{8}m\omega^2A^2$$

If kinetic energy increased by $\frac{1}{2}m\omega^2A^2$ at position $\frac{\sqrt{3}A}{2}$, then new kinetic energy at that instant

$$\text{K.E.}=\frac{1}{8}m\omega^2A^2+\frac{1}{2}m\omega^2A^2$$

$$=\frac{1}{2}m\omega^2\left(A'^2-\left(\frac{\sqrt{3}A}{2}\right)^2\right)$$

$$\Rightarrow \boxed{A'=\sqrt{2}A}$$

11. **(2)**

Current in R_1 is $I_1=\frac{5}{500}=10\times10^{-3}A=10\text{mA}$

Current in R_2 is $I_2=\frac{10}{1500}\text{A}=\frac{20}{3}\text{mA}$

Current through Zener diode is

$$I_z=I_1-I_2=\left(10-\frac{20}{3}\right)\text{mA}=\frac{10}{3}\text{mA}\approx3.3\text{mA}$$

12. **(1)**

$$\frac{dt}{dx}=(2\alpha x+\beta)$$

$$\therefore \frac{dx}{dt} = v = \left(\frac{1}{2\alpha x + \beta}\right)$$

$$a = \frac{dv}{dt} = -2\alpha\left(\frac{1}{2\alpha x + \beta}\right)^2 \cdot \frac{dx}{dt}$$

$$= -2\alpha\left(v^2\right)(v) = -2\alpha v^3$$

13. **(2)**

For floating, $W = U$

$$\Rightarrow W_{\text{sphere}} + W_{\text{chain}} = U$$

$$W = mg + (\lambda h) g$$

Now $U = V_{\text{sphere}}\rho_{\text{water}} g + V_{\text{chain}}\rho_{\text{water}} g$

$$\Rightarrow U = V_{\text{sphere}}\left(3\rho_{\text{sphere}}\right) g + V_{\text{chain}}\left(\frac{\rho_{\text{chain}}}{7}\right) g$$

$$\Rightarrow U = 3mg + \frac{1}{7}\left(m_{\text{chain}}\, g\right)$$

$$\Rightarrow U = 3mg + \frac{\lambda hg}{7}$$

Since $W = U$

$$\Rightarrow mg + \lambda hg = 3mg + \frac{\lambda hg}{7}$$

$$\Rightarrow 2mg = \frac{6\lambda hg}{7}$$

$$\Rightarrow h = \frac{7m}{3\lambda}$$

14. **(2)**

For equilibrium of sphere net torque on it due to its weight and that due to magnetic forces must balance about bottom point of contact which is given as

$$\tau_{mg} = \tau_B$$

$$\Rightarrow mgR\sin\theta = \pi R^2 iB\sin\theta$$

$$\Rightarrow B = \frac{mg}{\pi i R}$$

15. **(2)**

Path difference between the waves reaching O will be $\Delta x = 2d \sin\alpha$

For dark fringe at O, we have

$$2d\sin\alpha = \frac{\lambda}{2}$$

$$\Rightarrow \sin\alpha = \frac{\lambda}{4d}$$

$$\Rightarrow \alpha = \sin^{-1}\left(\frac{\lambda}{4d}\right)$$

16. **(2)**

For perfectly absorbing, $F_n = \dfrac{P}{c}$

For perfectly reflecting, $F_n = \dfrac{2P}{c}$

For the given situation, $F_n = \dfrac{P}{c} + \dfrac{2}{5}\dfrac{P}{c}$

$$\Rightarrow F_n = \frac{7}{5}\frac{P}{c} = 1.4\frac{P}{c}$$

17. **(2)**

$$\vec{\tau}_{\text{Hinge}} = I\alpha$$

$$\frac{\sigma QL}{2\varepsilon_0} = \frac{ML^2}{3}\alpha$$

$$\alpha = \frac{3\sigma Q}{2ML\varepsilon_0}$$

18. **(3)**

$F = T$

Also, $3T = 40$

$$\Rightarrow T = \frac{40}{3}\text{ N}$$

19. **(1)**

About point O ring is in pure rotation.

20. **(4)**

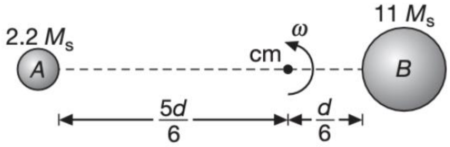
$$F = F_R + F_M$$

$$F_R = \frac{Mg}{2}$$

$$F_M = \frac{\Delta P}{\Delta t} = Mg$$

$$F = \frac{3}{2}Mg$$

21. **(6)**



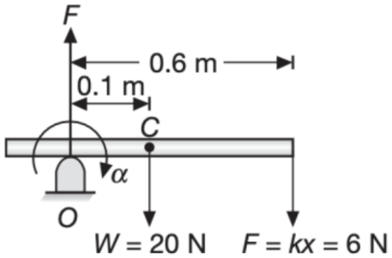
$$\frac{\text{Total angular momentum about cm}}{\text{Angular momentum of B about cm}} = \frac{L}{L_B}$$

$$\Rightarrow \frac{L}{L_B} = \frac{(2.2M_S)\left(\frac{5\omega d}{6}\right)\left(\frac{5d}{6}\right) + (11M_S)\left(\frac{\omega d}{6}\right)\left(\frac{d}{6}\right)}{(11M_S)\left(\frac{\omega d}{6}\right)\left(\frac{d}{6}\right)}$$

$$\Rightarrow \frac{L}{L_B} = 6$$

22. **(20)**

Just after the thread is burnt, the forces acting on the rod are as shown in Figure.



Torque due to forces about O is

$\tau = (20)(0.1) + (6)(0.6) = 5.6$ N

Angular acceleration about O is given by

$$\alpha = \frac{\tau}{I} = \frac{5.6}{\left(\frac{(2)(1)^2}{12} + (2)(0.1)^2\right)}$$

$\Rightarrow \alpha \cong 30 \text{rads}^{-2}$

Now, $a_c = r\alpha = (0.1)\alpha$

$\Rightarrow a_c = 3 \text{ ms}^{-2}$ {downwards}

Since $W + kx - F = ma_C$

$\Rightarrow F = W + kx - ma_C$

$\Rightarrow F = 20 + 6 - (2)(3)$

$\Rightarrow F = 20$ N

23. **(24)**

$$C = \frac{K \in_0 A}{d} = \text{a constant}$$

For A to be minimum, d must be minimum. The separation between the plates is limited by the breakdown strength of the dielectric.

For air capacitor

$$\frac{V}{d_{\text{min}}} = E_{\text{air}} \quad [E_{\text{air}} = \text{Breakdown field for air}]$$

$$\therefore d_{\text{min}} = \frac{V}{E_{\text{air}}}$$

Now $\frac{\in_0 A_{\text{min}}}{d_{\text{min}}} = C$

$$\Rightarrow A_{\text{min}} = \frac{C}{\in_0} \frac{V}{E_{\text{air}}}$$

$$\therefore A_1 = \frac{CV}{\in_0 E_{\text{air}}}$$

With dielectric, similar calculation gives

$$A_2 = \frac{CV}{K \in_0 E_{\text{dielec}}}$$

$$\therefore \frac{A_1}{A_2} = \frac{KE_{\text{dielec}}}{K_{\text{air}}} = 3 \times 8 = 24$$

24. **(8)**

$$a = \frac{v^2}{r} = \frac{Z^2}{n^2} \times \frac{Z}{n^2}$$

$\Rightarrow a \propto Z^3$

$$\Rightarrow \frac{a_{\text{He}^+}}{a_{\text{H}}} = \frac{Z_{\text{He}}^3}{Z_{\text{H}}^3} = \frac{2^3}{1^3} = \frac{8}{1}$$

25. **(15)**

Restoring torque, $\tau = -kl^2\theta - k\frac{l^2}{4}\theta = -\frac{5}{4}kl^2\theta$

$$\Rightarrow \left(\frac{ml^2}{3}\right)\alpha = -\frac{5}{4}kl^2\theta$$

$$\Rightarrow f = \frac{1}{2\pi}\sqrt{\left|\frac{\alpha}{\theta}\right|} = \frac{1}{2\pi}\sqrt{\frac{15k}{4m}}$$

26. **(70)**

$qE = 30$ N, vertical component of electric force $= 30\sin 30^\circ = 15$ N and horizontal component of electric force $= 30\cos 30^\circ = 15\sqrt{3}$ N

$$a_y = \frac{mg - 15}{m} = \frac{30 - 15}{3} = 5 \text{ m/s}^2 \text{ (downwards)}$$

$$a_x = \frac{15\sqrt{3}}{3} = 5\sqrt{3} \text{ m/s}^2$$

$$T_1 = \frac{2u_y}{a_y} = \frac{2 \times 20\sin 30^\circ}{5} = 4 \text{ s}$$

$T_2 = eT_1 = 2$ s

Horizontal velocity after first drop

$= (20\cos 30^\circ) + a_x T_1$

$= (10\sqrt{3}) + (5\sqrt{3})4$

$= 30\sqrt{3}$ m/s

$\therefore$ Horizontal distance travelled between first drop and second drop

$$= (30\sqrt{3})T_2 + \frac{1}{2}a_x T_2^2$$

$$= (30\sqrt{3})(2) + \frac{1}{2}(5\sqrt{3})(2)^2$$

$= 70\sqrt{3}$ m

27. **(12)**

$$F = mg\sin\theta + (\eta)(a^2)\,\frac{v_0}{t}$$

$$3mg = (mg) \times \frac{3}{5} + \eta a^2 \frac{v_0}{t}$$

$\Rightarrow\ \eta = \dfrac{12\,mgt}{5v_0 a^2}$

28. **(4)**

Wavelength of the emitted photon is

$$\frac{1}{\lambda} = Z^2 R\left[\frac{1}{n_i^2} - \frac{1}{n_f^2}\right]$$

$\therefore\ \nu = \dfrac{5}{36}Rc \quad \therefore \quad n = 4$

29. **(40)**

$W_T = \Delta KE$

$\Rightarrow\ W_F + W_{2F} = k_f$

$\Rightarrow\ -20 + (20)4 = k_f \Rightarrow k_f = 60\text{ J}$

$\Rightarrow\ k_T + k_R = 60 \quad \text{also} \quad \dfrac{k_T}{k_R} = 2$

$\Rightarrow\ k_T = \dfrac{2}{3} \times 60 = 40\text{ J}$

30. **(5)**

$(T)_{\text{mid point}} = \dfrac{F}{2}$

$E = \dfrac{F}{2AY}$

SECTION-II (CHEMISTRY)

31. **(4)**

Energy of 1 mole photon

$$E = \frac{6.626\times10^{-34}\times3\times10^{8}}{4\times10^{-7}}\times6.022\times10^{23}$$

$\approx 297\text{ kJ / mole}$

Percentage of energy converted to KE

$$= \frac{297 - 246.5}{297}\times100 = 17\%$$

32. **(1)**

Let the equivalents of Na_2CO_3 is X

Equivalents of $NaHCO_3$ is Y

Phenolphthalein indicator

$\dfrac{X}{2} = 2.5\times0.1\times2\times10^{-3}$

$X = 1\times10^{-3}$ in 10 mL

$\therefore$ In one litre $= 1\times10^{-1}$

Mass of Na_2CO_3 = 5.3gm

Methyl orange indicator

$\dfrac{X}{2} + Y = 2.5\times0.2\times2\times10^{-3}$

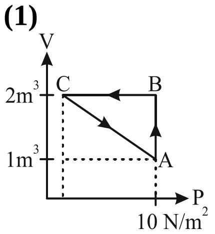
$Y = 1\times10^{-3} - 0.5\times10^{-3} = 0.5\times10^{-3}$ in 10 mL

$\therefore$ Equivalents of $NaHCO_3$ in 1 litre = 0.05

Mass of $NaHCO_3$ = 0.05 × 84

= 4.2 gm

33. **(1)**



$\Delta U = q + w$

or $-q = w$ ($\Delta U = 0$)

$-5 = (w_{AB} + w_{BC} + w_{CA})$

$-5 = (-10 + 0 + w_{CA})$

$\Rightarrow\ w_{CA} = +5J$

34. **(3)**

Its equilibrium constant

$$K_{eq} = \frac{K_a\times K_b}{K_w} = \frac{3.24\times10^{-10}}{10^{-14}}$$

$= 1.8\times1.8\times10^{4}$

35. **(4)**

$Mg(OH)_2(s) \rightleftharpoons Mg^{2+}(aq) + 2OH^-(aq.)$

$[Mg^{2+}][OH^-]^2 = 1.2\times10^{-11}$

$[OH^-]^2 = 1.2\times10^{-10}$

$[OH^-] = 1.1\times10^{-5}$

pOH $= -\log 1.1\times10^{-5} = 5 - \log 1.1 = 5 - 0.04 = 4.96$

pH = 14 – 4.96 = 9.04

36. **(3)**

As in (III) the correct order of increasing first ionization enthalpy is B < C < O < N.

37. **(3)**

Number of ions increases considerably only for weak electrolytes.

38. **(4)**

(I) < (II) = (IV) < (III)

O_2	14 e^-	B.O. = 2.0
NO	15 e^-	B.O. = 2.5
NO^+	14 e^-	B.O. = 3.0
O_2^+	15 e^-	B.O. = 2.5

39. (2)

Basic strength: III > I > IV > II

Acidic strength of conjugate acids:

(II) > (IV) > (I) > (III)

40. (1)

– NO_2 is a meta-directing group. As it is also a deactivating group so no chance of introduction of second-Br atom

41. (1)

CO, NO and N_2O are neutral oxides.

42. (4)

XeF_2–sp^3d– linear (3 lone pairs)

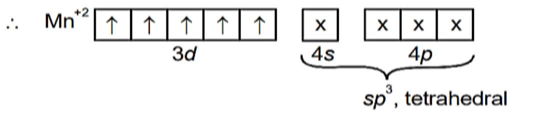
XeF_4–sp^3d^2– square planar (2 lone pairs)

XeF_6–sp^3d^3– distorted octahedral (one lone pair)

43. (1)

$\sqrt{n(n+2)} = 5.92$

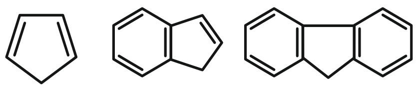
n = 5



44. (4)

Theoretical

45. (1)



Pka = 15 Pka = 20 Pka = 23

46. (1)

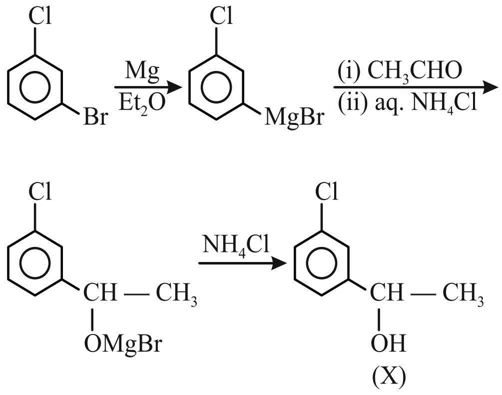
Equal volumes of both will consume and hence,

$[CH_3COONa] = \frac{0.01}{2} = 0.005\ M$

Now, $P^H = 7 + \frac{1}{2}(P^{K_a} + \log C)$

$= 7 + \frac{1}{2}(4.7 + \log(0.005)) = 8.2$

47. (3)



48. (2)

Chromyl chloride test → Confirmatory test for Cl^-

Brown ring test → Confirmatory test for NO_3^-

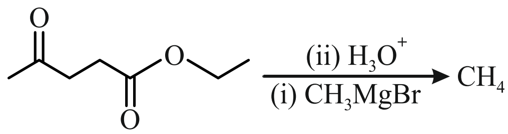
Smell of vinegar → indicatory test for CH_3COO^-

Smell of rotten eggs → indicates S^{2-}

49. (4)

SF_6 is thermodynamically very stable due to high S – F bond strength.

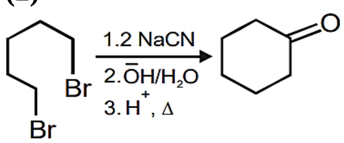
50. (4)



51. (5)

All of the reagents can be used for this purpose.

52. (2)



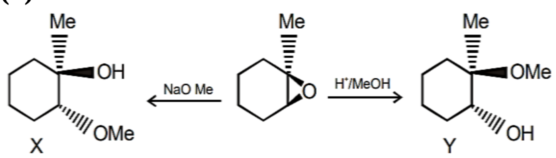
Degrees of unsaturation = 2.

53. (6)

Possible number of tripeptide

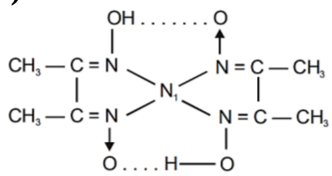
= !3 = 3 × 2 × 1 = 6

54. (1)



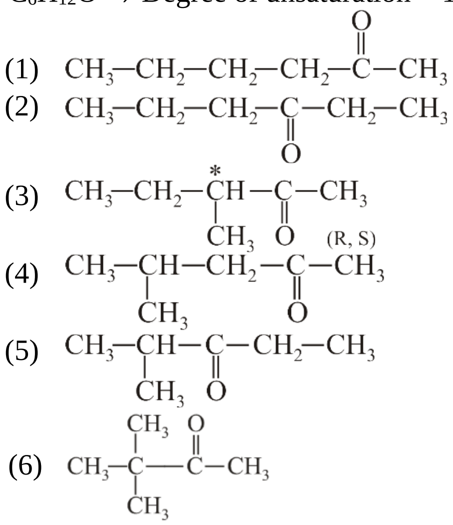
Mol. wt. of X and Y are identical.

55. (2)



56. (7)

$C_6H_{12}O \Rightarrow$ Degree of unsaturation = 1



57. (4)

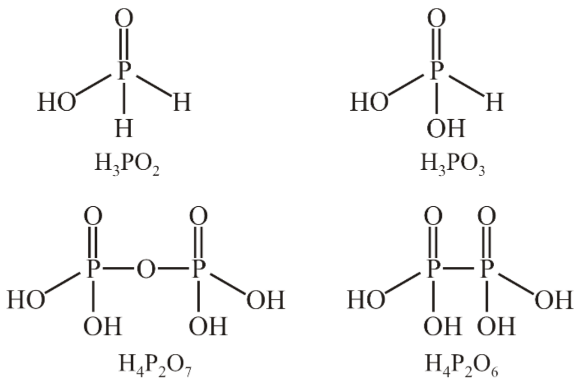
$$\Delta T_f = K_f \cdot m \Rightarrow 0.29 = 1.86 \times \frac{1.04/267}{100/1000} \times n$$

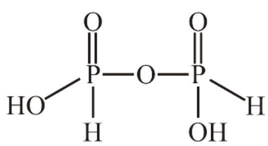
$\Rightarrow n \approx 4$

58. (3)

Only primary amines undergo carbylamine reaction.

59. (7)





$H_4P_2O_5$

X = 5
Y = 2
X + Y = 7

60. (6)

	$3A_2(g)$	$\rightleftharpoons$	$A_6(g)$,	$K_{P1} = 1.6\ atm^{-2}$
Initial partial pressure	$2P_0$		0	
Equilibrium Partial pressure	$2P_0 - 3a - b$		a	

	$A_2(g)$	+	$C(g)$	$\rightleftharpoons$	$A_2C(g)$	$K_{P2} = x\ atm^{-1}$
Initial partial pressure	$2P_0$		P_0		0	
Equilibrium Partial pressure	$2P_0 - 3a - b$		$P_0 - b$		b	

From question, a = 0.2,

$$\frac{P_{A_6}}{P_{A_2}^3} = 1.6 \Rightarrow \frac{0.2}{P_{A_2}^3} = 1.6$$

$\Rightarrow P_{A_2} = 0.5 = 2P_0 - 3a - b$

and $(2P_0 - 3a - b) + a + (P_0 - b) + b = 1.4$

$\Rightarrow P_0 = 0.7$ and $b = 0.3$

Now,

$$K_{P_2} = \frac{b}{(2P_0 - 3a - b)(P_0 - b)} = \frac{0.3}{0.5 \times 0.4} = 1.5\ atm^{-1}$$

SECTION-III (MATHEMATICS)

61. **(3)**

$PQ = 3(AB) = 7\sqrt{2}$

$C(2,1);\ r = \sqrt{5+C}$

$M\left(\frac{3}{2},\frac{3}{2}\right);\ CM = \frac{1}{\sqrt{2}}$

$\therefore \quad 7\sqrt{2} = 2\sqrt{5+c-\frac{1}{2}} \Rightarrow C = 20$

$\therefore \quad r = 5$

62. **(2)**

$A^2 = A \cdot A$

$= (AB)(AB) = A(BA)B = (AB)B = AB = A$

Similarly $B^2 = B$

$\therefore \quad A = A^2 = A^3 = \ldots B = B^2 = B^3 = \ldots$

$(A^{2021} + B^{2021})^{2022} = (A+B)^{2022}$

$(A+B)^2 = (A+B)(A+B) = A^2 + AB + BA + B^2$

$= A^2 + A + B + B^2 = 2(A+B)$

$(A+B)^3 = 2^2(A+B)$

$\therefore \quad (A+B)^{2022} = 2^{2021}(A+B)$

63. **(4)**

$y = \frac{x}{x-c_1} + \frac{c_2 x}{(x-c_1)(x-c_2)} + \frac{c_3 x^2}{(x-c_1)(x-c_2)(x-c_3)}$

$= \frac{x^2}{(x-c_1)(x-c_2)} + \frac{c_3 x^2}{(x-c_1)(x-c_2)(x-c_3)}$

$= \frac{x^3}{(x-c_1)(x-c_2)(x-c_3)}$

$\therefore \quad \ln y = \ln x^3 - \ln(x-c_1) - \ln(x-c_2) - \ln(x-c_3)$

$\therefore \quad \ln y = 3\ln x - (x-c_1) - \ln(x-c_2) - \ln(x-c_3)$

$\frac{y'}{y} = \frac{3}{x} - \frac{1}{x-c_1} - \frac{1}{x-c_2} - \frac{1}{x-c_3}$

$y' = \frac{y}{x}\left[3 - \frac{x}{x-c_1} - \frac{x}{x-c_2} - \frac{x}{x-c_3}\right]$

$= \frac{y}{x}\left[\frac{c_1}{c_1-x} + \frac{c_2}{c_2-x} + \frac{c_3}{c_3-x}\right]$

64. **(4)**

$I_2 = \int_0^1 \left(\frac{x}{5+x}\right)^{\frac{7}{2}} \cdot \left(\frac{1-x}{5+x}\right)^{\frac{9}{2}} \cdot \frac{dx}{(5+x)^2}$

Put $\frac{x}{5+x} = t \ \frac{5}{(5+x)^2}dx = dt$

$\Rightarrow \quad \frac{dx}{(5+x)^2} = \frac{1}{5}dt$

$\therefore \quad I_2 = \int_0^{\frac{1}{6}} (t)^{\frac{7}{2}} (1-6t)^{\frac{9}{2}} \frac{dt}{(5)^{11/2}}$; Now Put $6t = \mu$

and simplify we get $I_2 = \frac{1}{5^{9/2} \times 6^{7/2}} I_1$ we conclude $a = 30$

65. **(3)**

$\frac{m}{s} = \sin^2\theta;\ \frac{n}{t} = \cos^2\theta$

$\Rightarrow \quad s = m\operatorname{cosec}^2\theta \qquad t = n\sec^2\theta$

$s + t = m\operatorname{cosec}^2\theta + n\sec^2\theta$

$= m + n + n\cot^2\theta + n\tan^2\theta$

$= 3 + \underbrace{m\cot^2\theta + n + n + n\tan^2\theta}_{Use\ Am \geq Gm}$

$\geq 3 + 2\sqrt{mn}$

$\therefore \quad mn = 2 \qquad m + n = 3$

$\therefore \quad m = 1$

$n = 2$ ($\therefore\ m < n$) Point is (1, 2)

$T = S_7$

66. **(1)**

$\int_0^x \sqrt[2023]{\cos x}\,dx = 0$

67. **(1)**

$x^2 - 3y^2 = 3$

Fouse of ellipse = $(\pm ae, 0) = (\pm\sqrt{3}, 0)$

So, the hyperbola passes through $(\pm\sqrt{3}, 0)$

$\Rightarrow \quad \frac{3}{a^2} = 1$

Or $a = \sqrt{3}$

So, by equation (1), we get $b = 1$

So, the equation of hyperbola is

$\frac{x^2}{3} - \frac{y^2}{1} = 1$

Or, $x^2 - 3y^2 = 3$

Focus of hyperbola $= (\pm ae, 0) = (\pm 2, 0)$

68. **(2)**

$\Rightarrow \quad \sum x_i = 2500$

Correct $\sum x_i = 2500 - 99 = 2401$

Correct $\bar{x} = \frac{2401}{49} = 49$

$\sigma^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$

Correct $\Rightarrow \quad 100 = \frac{\sum x_i^2}{50} - 2500$

$\Rightarrow \quad \sum x_i^2 = 130000$

Correct $\sum x_i^2 = 130000 - 99^2 = 120199$

Correct $\sigma^2 = \frac{\sum x_i^2}{n} - \bar{x}^2$

$= \frac{120199}{49} - (49)^2$

$= 2453 - 2401$

$= 52.04$

69. **(3)**

$e^x f(x) + c$ (where c is integration constant) so $f(0)$ is.

$\int e^x \left(f + f' - f' - f''\right)dx = e^x\left(f - f'\right) + c$

$\int e^x \left(f + f' - f' - f''\right)dx = e^x\left(f - f'\right) + c$

$f = \frac{1}{\sqrt{1+x^2}}$ and $\frac{x^4 + 2x^2 + 1 - 2x^2 + 1}{\left(x^2+1\right)^{5/2}}$

$f(x) = \frac{1}{\sqrt{1+x^2}} + \frac{1-2x^2}{\left(x^2+1\right)^{5/2}} - \frac{x}{\left(1+x^2\right)^{\frac{3}{2}}}$

70. **(1)**

00108.00

$\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow |\vec{a} + \vec{b}| = |\vec{c}|$

$\Rightarrow 36 + |\vec{b}|^2 + 60 = 196 \Rightarrow |\vec{b}| = 10$

$|\vec{a}\times\vec{b} + \vec{b}\times\vec{c}| = |\vec{b}\times(\vec{c} - \vec{a})| = ^*(\vec{a} + \vec{c})\times$

$(\vec{c} - \vec{a})| = 2|\vec{a}\times\vec{c}^*$

$|\vec{a} + \vec{c}| = |\vec{b}| \Rightarrow 36 + 196 + 2(\vec{a}\cdot\vec{c}) = 100 \Rightarrow$

$\vec{a}\cdot\vec{c} = -66$

$\therefore \quad |\vec{a}||\vec{c}|\cos\theta = -66 \Rightarrow \cos\theta = -\frac{11}{14}$

$|\vec{a}\times\vec{b} + \vec{b}\times\vec{c}|^2 = 4|\vec{a}\times\vec{c}|^2 = 4\times 36\times 196\times$

$\frac{75}{196} = 10800$

71. **(3)**

$a_r = \left(cis\frac{2\pi}{9}\right)^r$ or $e^{i\frac{2\pi r}{9}}$ $\quad r = 1, 2, 3\ldots..$

$\therefore \quad a_1, a_2, \ldots..$are in GP

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix} = \begin{vmatrix} a_1 & a_1^2 & a_1^3 \\ a_1^4 & a_1^5 & a_1^6 \\ a_1^7 & a_1^8 & a_1^9 \end{vmatrix}$$

$$= a_1 \cdot a_1^4 \cdot a_1^7 \begin{vmatrix} 1 & a_1 & a_1^2 \\ 1 & a_1 & a_1^2 \\ 1 & a_1 & a_1^2 \end{vmatrix}$$

Now $a_1a_9 = a_3a_7 = a_1^{10} - a_1^{10} = 0$

72. **(3)**

Now $a_1a_9 - a_3a_7 = a_1 - a_1 = 0$

$$f(x) = \begin{cases} (x-3)(x+1) & e^{(3x-2)^2} & x \in (3, \infty) \\ -(x-3)(x+1) & e^{(3x-2)^2} & x \in [-1, 3] \\ (x-3)(x+1) & e^{(3x-2)^2} & x \in (-\infty, -1) \end{cases}$$

Clearly non differentiable of $x = -1$ & 3

73. **(3)**

$\left|2\bar{a} + 3\bar{b}\right| = \left|3\bar{a} + \bar{b}\right|$

S.O.B.S

$4\bar{a}^2 + 9\bar{b}^2 + 12\bar{a}\cdot\bar{b} = 9\bar{a}^2 + \bar{b}^2 + 6\bar{a}\cdot\bar{b}$

$5\bar{a}^2 - 6\bar{a}\cdot\bar{b} = 8\left|\bar{b}\right|^2$

$5\bar{a}^2 - 6\cdot 8\cdot\left|\bar{b}\right|\cos 60° = 8\left|\bar{b}\right|^2 \because \frac{1}{8}|\bar{a}| = 1$

$40 - 3\left|\bar{b}\right| = \left|\bar{b}\right|^2$

$\left|\bar{b}\right|^2 + 3\left|\bar{b}\right| - 40 = 0$

$\left(\left|\bar{b}\right| + 8\right)\left(\left|\bar{b}\right| - 5\right) = 0$

$\therefore \quad \left|\bar{b}\right| = 5$

74. **(1)**

Note that

$\sin\left(\frac{\pi}{6} - \frac{\pi}{18}\right) = \sin\frac{\pi}{9},$

Therefore $\dfrac{\cos\dfrac{\pi}{18}}{4\cos\dfrac{\pi}{18}+\sqrt{3}}=\sin\dfrac{\pi}{18}.$

Thus

$$\cos^2\frac{\pi}{18}+\frac{\cos^2\dfrac{\pi}{18}}{\left(4\cos\dfrac{\pi}{18}+\sqrt{3}\right)}=\cos^2\frac{\pi}{18}+\sin^2\frac{\pi}{18}=1.$$

75. **(2)**

$|z+5|\le 4$

$(x+5)^2+y^2\le 16$

So, points (x, y) lie inside or on the circle whose center is $(-5, 0)$ and radius is 4.

Comparing thes with $\alpha+\beta\sqrt{2}$, we get

$\alpha = 32$, $\beta = 16$

$\therefore\quad \alpha+\beta = 48$

76. **(1)**

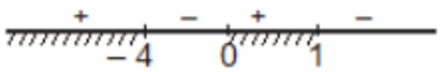
Here, $\dfrac{(16)^{1/x}}{\left(2^{x+3}\right)}>1$

$\Rightarrow\quad \dfrac{2^{4/x}}{2^{x+3}}>1$ or $2^{\frac{4}{x}-x-3}>1$ i.e. $2^{\frac{4}{x}-x-3}>2^{\circ}$

$\Rightarrow\quad \dfrac{4}{x}-x-3>0 \quad\Rightarrow\quad \dfrac{\left(x^2+3x-4\right)}{x}>0$ or

$\dfrac{-(x+4)(x-1)}{x}>0$

Using number line rule,



$\Rightarrow\quad x\in(-\infty,-4)\cup(0,1)$

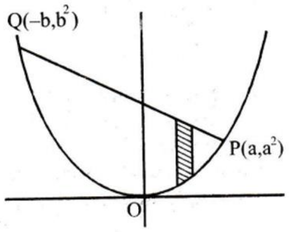
Hence, Options (1) is correct.

77. **(4)**

$||x+2|-3| = 1$

But rejected all x values

78. **(1)**



$m_{PQ}=\dfrac{a^2-b^2}{a+b}=a-b$

Equation of PQ

$y-a^2=\dfrac{a^2-b^2}{a+b}(x-a)$

$y-a^2=\dfrac{a^2-b^2}{a+b}\left(x-a\right)$

or $y-a^2=(a-b)(x-a)$

$y=a^2+x(a-b)-a^2+ab$

$y=(a-b)x+ab$

$S_1=\int_{-b}^{a}(a-b)x+ab+x^2dx$

Which simplifies to $\dfrac{(a+b)^3}{6}$

Also,

$$S_2=\frac{1}{2}\begin{vmatrix} a & a^2 & 1\\ -b & b^2 & 1\\ 0 & 0 & 1\end{vmatrix}=\frac{1}{2}\left[ab^2+a^2b\right]=\frac{1}{2}ab(a+b)$$

$$\therefore\quad \frac{S_1}{S_2}=\frac{(a+b)^3}{6}\cdot\frac{2}{ab(a+b)}=\frac{(a+b)^2}{3ab}=\frac{1}{3}\left[\frac{a}{b}+\frac{b}{a}+2\right]$$

$$\therefore\quad \left.\frac{S_1}{S_2}\right|_{\min.}=\frac{4}{3}$$

79. **(2)**

The given expression is the coefficient of x^4 in

${}^4C_0(1+x)^{404}-{}^4C_1(1+x)^{303}+{}^4C_2(1+x)^{202}$

$-{}^4C_3(1+x)^{101}+{}^4C_4$

= Coefficient of x^4 in $\left[(1+x)^{101}-1\right]^4$

= Coefficient of x^4 in $\left({}^{101}C_1x+{}^{101}C_2x^2+\cdots\right)^4$

$=(101)^4$

80. **(4)**

$y\sin 2x-\cos x+\left(1+\sin^2 x\right)\dfrac{dy}{dx}=0$ where

$y=f(x)$

$\dfrac{dy}{dx}+\left(\dfrac{\sin 2x}{1+\sin^2 x}\right)y=\dfrac{\cos x}{1+\sin^2 x}$

I.F. $= e^{\int \frac{\sin 2x}{1+\sin^2 x}dx} = e^{\int \frac{dt}{t}} = e^{\ln\left(1+\sin^2 x\right)} = 1+\sin^2 x$

(by putting $1 + \sin^2 x = t$)

$y\left(1+\sin^2 x\right) = \int \cos x dx$

$y\left(1+\sin^2 x\right) = \sin x + C; \{y(0) = 0)\} \Rightarrow C = 0$

Hence, $\quad y = \dfrac{\sin x}{1+\sin^2 x}$

$y\left(\dfrac{\pi}{6}\right) = \dfrac{2}{5}$

81. (2)

Let $x_i - 5 = y_i$

$\sum_{i=1}^{9}(x_i - 5) = 9 \text{ and } \sum_{i=1}^{9}(x_i - 5)^2 = 45$

So, required standard deviation is

$$\sigma = \sqrt{\sum_{i=1}^{9} y_i^2 - \left(\frac{\sum_{i=1}^{9} y_i}{9}\right)} = \sqrt{\frac{45}{9} - \left(\frac{9}{9}\right)^2} = 2$$

82. (43)

$A_1.A_3.A_5.A_7 = \dfrac{1}{1296}$

$\Rightarrow \quad (A_4)^4 = \dfrac{1}{1296} \Rightarrow A_4 = \dfrac{1}{6}$

$A_2 + A_4 = \dfrac{7}{36} \Rightarrow A_2 = \dfrac{1}{36}$

$\Rightarrow \quad A_6 = 1, A_8 = 6$ and $A_{10} = 36$

$\Rightarrow \quad A_6 + A_8 + A_{10} = 43$

83. (9)

General term in the expansion

$= \dfrac{10!}{\alpha!\beta!\gamma!} \alpha^{\alpha} (2b)^{\beta} \cdot (4ab)^{\gamma}$

For term containing $a^7 b^8$, we have

$\alpha + \gamma = 7$

$\beta + \gamma = 8$

$\alpha + \beta + \gamma = 10$

Solving we get and $\gamma = 5$, $\alpha = 2$ and $\beta = 3$

So, coefficient $= \dfrac{10!}{2!3!5!} 2^3.2^{10}$

$= 315 \times 2^{16} \Rightarrow K = 315$

84. (5)

$$= \int_{-1}^{1}\left(\sqrt{5-x^2} - (1-x)\right)dx + \int_{1}^{2}\left(\sqrt{5-x^2} - (1-x)\right)dx$$

$$= 2\left[1 + \frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}}\right] - 2 + \left[1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}} - 1 - \frac{5}{2}\sin^{-1}\frac{1}{\sqrt{5}}\right]$$

$$= \frac{5}{2}\left[\tan^{-1}\frac{1}{2} + \tan^{-1} 2\right] - \frac{1}{2}$$

$$= \frac{5\pi}{4} - \frac{1}{2}$$

85. (6)

$\alpha = \max_{x \in R}\left\{8^{2\sin 3x}.4^{4\cos 3x}\right\}$

$= \max \{2^{6\sin 3x + 8\cos 3x}\} = 2^{10}$

$\beta = \max(2^{6\sin 3x + 8\cos 3x}) = 2^{-10}$

$\alpha^{1/5} = 2^2$, $\beta^{1/5} = 2^{-2}$

$\therefore \quad b = -34$ and $c = 8$

So, $c - b = 42$

86. (9)

Required Probability $= \dfrac{{}^3C_1 \cdot {}^3C_2}{{}^7C_4} = \dfrac{9}{35}$

87. (208)

Let $I = \int_0^1 {}^{207}C_7 \cdot \underset{II}{x^{200}} \cdot \underbrace{(1-x)^7}_{I} dx$

$$I = {}^{207}C_7[\underbrace{(1-x)^7 \cdot \frac{x^{201}}{201}\Bigg|_0^1}_{zero} + \frac{7}{201}\int_0^1 (1-x)^6 \cdot x^{201} dx]$$

$$= {}^{207}C_7 \cdot \frac{7}{201}\int_0^1 (1-x)^6 \cdot x^{201} dx$$

$$= \frac{(207)!}{7!(200)!} \cdot \frac{7!}{201 \cdot 202 \ldots 207} \cdot \frac{1}{208}$$

$$= \frac{(207)!}{(207)!7!} \cdot \frac{7!}{208} = \frac{1}{208} = \frac{1}{k}$$

88. **(49)**

D ≤ 0

$4(p + q - 7)^2 - 8pq \leq 0$

$\Rightarrow$ $(p - 7)^2 + (q - 7)^2 \leq 72$ interior & circumference of this represents an circle with centre (7, 7) & radius 7

$\therefore$ Area = 49π.

89. **(3)**

Let $f(x)$ = sin(sin(sin(sin(sin(x)))))

$f'(0)=1>\frac{1}{3}$. Therefore, $f(x)>\frac{x}{3}$ is some neighbourhood of 0

So, there are 3 solutions

90. **(1)**

$$\sum_{r=1}^{3}\cos(2\theta_r)=1$$

$$\Rightarrow \sum_{r=1}^{3}\frac{1-t_1^2}{1+t_1^2}=1 \text{ where } t_1^2=\tan^2\theta_1$$

$$\Rightarrow \frac{1-t_1^2}{1+t_1^2}+\frac{1-t_2^2}{1+t_2^2}+\frac{1-t_3^2}{1+t_3^2}=1$$

$$\Rightarrow \sum_{r=1}^{3}t_1^2\cdot t_2^2+2\prod_{r=1}^{3}t_1^2=1$$