JEE MAIN (2023-24) Mock Test Series

Paper - 06

DURATION : 180 Minutes | M. MARKS : 300

ANSWER KEY

PHYSICS

1.	(3)
2.	(2)
3.	(3)
4.	(2)
5.	(4)
6.	(4)
7.	(3)
8.	(1)
9.	(2)
10.	(3)
11.	(1)
12.	(1)
13.	(2)
14.	(3)
15.	(4)
16.	(1)
17.	(3)
18.	(4)
19.	(2)
20.	(2)
21.	(8)
22.	(5)
23.	(8)
24.	(6)
25.	(2)
26.	(4)
27.	(10)
28.	(5)
29.	(494)
30.	(27)

CHEMISTRY

31.	(2)
32.	(2)
33.	(1)
34.	(1)
35.	(4)
36.	(2)
37.	(4)
38.	(2)
39.	(3)
40.	(4)
41.	(1)
42.	(1)
43.	(2)
44.	(3)
45.	(4)
46.	(2)
47.	(3)
48.	(4)
49.	(1)
50.	(3)
51.	(5)
52.	(8)
53.	(2)
54.	(5)
55.	(1)
56.	(5)
57.	(5)
58.	(5)
59.	(8)
60.	(59)

MATHEMATICS

61.	(4)
62.	(2)
63.	(3)
64.	(4)
65.	(2)
66.	(1)
67.	(3)
68.	(3)
69.	(2)
70.	(4)
71.	(1)
72.	(1)
73.	(2)
74.	(4)
75.	(4)
76.	(2)
77.	(3)
78.	(3)
79.	(4)
80.	(1)
81.	(8)
82.	(2)
83.	(3)
84.	(3)
85.	(2)
86.	(12)
87.	(48)
88.	(900)
89.	(30)
90.	(191)

SECTION-I (PHYSICS)

1. **(3)**

Two surfaces are formed in the capillary

Since force due to surface tension becomes twice

$\therefore h' = 2h$

2. **(2)**

$\Delta KE = \Delta PE$ and $\vec{p_i} = \vec{p_f}$

$$\frac{J^2}{2m} = \frac{1}{2}kx^2 + \frac{J^2}{2(5m)}$$

$$\Rightarrow \quad x = \frac{2J}{\sqrt{5mk}} = \frac{2\times 10}{\sqrt{5\times 1\times 1000}}$$

$$= \frac{2}{\sqrt{50}} = \frac{\sqrt{2}}{5}\text{ m}$$

3. **(3)**

As the detector moves from Q along x-axis, maxima are observed when $\Delta x = 4\lambda, 3\lambda, 2\lambda, \lambda$

At first minima from Q, $\Delta x = \frac{7\lambda}{2}$

$$\sqrt{x^2 + (4\lambda)^2} - x = \frac{7\lambda}{2}$$

$$\Rightarrow \quad x = \frac{15\lambda}{28}$$

4. **(2)**

$$V_c = \int_0^R -4\pi r^2 dr \times \frac{1}{r} \times \frac{G}{r}$$

$$M = \int_0^R 4\pi r^2 dr \times \frac{1}{r} = 4\pi \times \frac{R^2}{2}$$

$$V_c = \int_0^R -(4\pi r^2 dr) \times \frac{1}{r} \times \frac{G}{r} = -\frac{2GM}{R}$$

5. **(4)**

Both voltmeters are parallel.

Both the voltmeters A and B are effectively in parallel and hence give the same reading every time.

6. **(4)**

Work done by a gas in a cyclic process is negative if P-V graph is in anticlockwise sequence.

$$W_{\text{by gas}} = -\frac{1}{2}\times 1\times 40 = -20\text{ J}$$

7. **(3)**

$u = u_s + u_i$

Total energy of the system = $u_1 + u_2 + u_{12}$

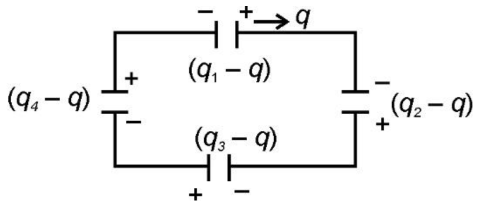
$$= \frac{q_1^2}{8\pi\varepsilon_0 a} + \frac{q_2^2}{8\pi\varepsilon_0 b} + q_1 V_2$$

$$= \frac{q_1^2}{8\pi\varepsilon_0 a} + \frac{q_2^2}{8\pi\varepsilon_0 b} + \frac{q_1 \cdot q_2}{4\pi\varepsilon_0 b}$$

8. **(1)**

Use KVL

Let a charge q flow in circuit in clockwise direction



By loop law,

$$\frac{q_1 - q}{C} + \frac{q_2 - q}{2C} + \frac{q_3 - q}{3C} + \frac{q_4 - q}{4C} = 0$$

$\Rightarrow q_1 = CV, q_2 = 4CV, q_3 = 9CV, q_4 = 16CV$

$$q = \frac{24}{5}CV$$

$$V_1 = \left|\frac{q_1 - q}{c}\right| = \frac{19V}{5}$$

$$V_2 = \left|\frac{q_2 - q}{2c}\right| = \frac{2V}{5}$$

$$V_3 = \left|\frac{q_3 - q}{3c}\right| = \frac{7V}{5}$$

$$V_4 = \left|\frac{q_4 - q}{4c}\right| = \frac{14V}{5}$$

9. **(2)**

$$\gamma = \frac{1}{V}\frac{dV}{dT}$$

$$\gamma = \frac{1}{V}\frac{dV}{dT}, \; PT^2 = \text{Constant}$$

$\Rightarrow T^3 = kV$

$$\Rightarrow \quad 3T^2 = k\frac{dV}{dT} \quad \Rightarrow \frac{3T^2}{k} = \frac{dV}{dT}$$

$$\Rightarrow \frac{3T^2}{kT^3}k = \frac{1}{V}\frac{dV}{dT} = \frac{3}{T}$$

10. (3)

$$\frac{dr}{dt} = R\sqrt{\frac{2g}{r}}$$

$$u = \sqrt{2gR}$$

$$\frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{1}{2}mu^2 - \frac{GMm}{R}$$

$$\Rightarrow v = R\sqrt{\frac{2g}{r}}$$

$$\Rightarrow \frac{dr}{dt} = R\sqrt{\frac{2g}{r}}$$

$$\Rightarrow \int_R^{4R} \sqrt{r}\, dr = R\sqrt{2g}\int_0^t dt$$

$$\Rightarrow t = \frac{7}{3}\sqrt{\frac{2R}{g}}$$

11. (1)

$$q = \int i\, dt$$

$$q = \int_0^{\tau} idt = \int_0^{\tau} l_{\max}\left(1 - e^{-\frac{t}{\tau}}\right)dt$$

$$q = \frac{l_{\max}\tau}{e}$$

12. (1)

$x^2 = 4y$

$2xv_x = 4v_y$

$v_x^2 + xa_x = 2a_y$

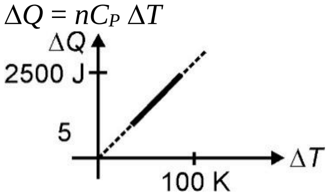
At (0, 0)

$ax = 0|\vec{v}| = 0$

$$a_y = \frac{v_x^2}{2}$$

13. (2)

$\Delta Q = nC_P\, \Delta T$



$\Delta Q = nC_P\, \Delta T$

$$C_P = \left(\frac{f}{2} + 1\right)R$$

14. (3)

$kx = 3mg\sin\theta$

$kx = 3mg\sin\theta$

$$x = \frac{3mg\sin\theta}{k}$$

15. (4)

$$R = \frac{mv}{qB}$$

$$\therefore \text{ Area}(A) = \pi R^2 = \frac{\pi(mv)^2}{(qB)^2}$$

16. (1)

At equilibrium

$$\Delta x_1 = \frac{mg}{k}$$

$$\Delta x_2 = \frac{mg}{2k}$$

$$\Rightarrow \max.\Delta x_2 = 2\Delta x_2 = \left(\frac{mg}{k}\right)$$

17. (3)

Use dipole

$dq = \lambda_0 \sin\phi\, a\, d\phi$

$|\vec{dp}| = dq \times 2a - 2a^2\lambda_0 \sin\phi d\phi$

$|\vec{dp}| = 2a^2\lambda_0 \sin\phi d\phi\left[\cos\phi\hat{i} + \sin\phi\,\hat{j}\right]$

$d\vec{\tau} = \vec{dp} \times \vec{E}$

$d\vec{\tau} = 2a^2\lambda_0 d\phi\left[\sin\phi\cos\phi\hat{i} + \sin^2\phi\hat{j}\right] \times$

$\left[E_0\hat{i} + E_0\hat{j}\right]$

$$\int d\vec{\tau} = \int_0^{\pi} 2a^2\lambda_0 E_0 d\phi\left[\sin\phi\cos\phi - \sin^2\phi\right]\hat{k}$$

$$= 2a^2\lambda_0 E_0\left[\int_0^{\pi}\sin\phi\cos\phi d\phi - \int_0^{\pi}\sin^2\phi d\phi\right]\hat{k}$$

$$= 2a^2\lambda_0 E_0\left[\int_0^{\pi}\frac{\sin^2\phi}{2}d\phi - \int_0^{\pi}\left(\frac{1-\cos 2\phi}{2}\right)d\phi\right]\hat{k}$$

$$= 2a^2\lambda_0 E_0\left[0 - \frac{\pi}{2}\right]\hat{k} = a^2\lambda_0\pi E_0\,\hat{k}$$

18. (4)

$V_y^A = V_y^B$

$10 \times \sin 37° = (V)\sin 53°$

$$\Rightarrow 10 \times \frac{3}{5} = (V) \times \frac{4}{5}$$

$$\Rightarrow V = \frac{15}{2}\text{ m/s}$$

19. **(2)**

Power is zero when $\vec{F}.\vec{v} = 0$

Now $\vec{F} = \frac{q\sigma}{\varepsilon_0}(-\hat{j})$

$$\vec{v} = v\cos\alpha\hat{i} + \left(v\sin\alpha - \frac{q\sigma t}{m\varepsilon_0}\right)\hat{j}$$

$$\vec{F}.\vec{v} = \left(v\sin\alpha - \frac{q\sigma t}{m\varepsilon_0}\right).\frac{q\sigma}{\varepsilon_0} = 0$$

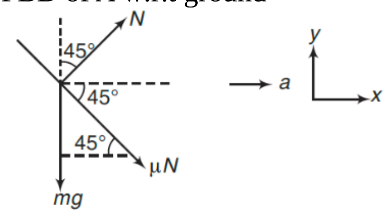
$$t = \frac{mv\sin\alpha\,\varepsilon_0}{q\sigma}$$

20. **(2)**

Let a = maximum acceleration of A.

Under no slip condition acceleration of B is also a

FBD of A w.r.t ground



$$\Sigma F_y = 0$$

$$\therefore \frac{N}{\sqrt{2}} = mg + \frac{\mu N}{\sqrt{2}}$$

$$\Sigma \frac{NF_x}{\sqrt{2}} + \frac{\mu N}{\sqrt{2}} = ma$$

Solving these two equations, we get

$$a = g\left(\frac{1+\mu}{1-\mu}\right)$$

21. **(8)**

$qv = \Delta$ K.E. and $\lambda \propto \frac{1}{V}$

$$\lambda = \frac{h}{\sqrt{2\,emV_1}}$$

$$\frac{\lambda}{3} = \frac{h}{\sqrt{2\,em(V_1+V_2)}}$$

$$\therefore \quad 3 = \sqrt{\frac{V_1+V_2}{V_1}}$$

$$\Rightarrow \frac{V_1}{V_2} = \frac{1}{8}$$

22. **(5)**

ω = 10 rad/s

From Eqn. $a = -100x' = -\omega^2 x'$

$\Rightarrow \omega = 10$

Time period of motion is, $T = \frac{2\pi}{\omega} = \frac{2\pi}{10} = \frac{\pi}{5}$

Then, $t = \frac{T}{4} = \frac{\pi}{20} = \frac{\pi}{\alpha}$

$$\therefore \quad \frac{\alpha}{4} = \frac{20}{4} = 5$$

23. **(8)**

v_m (just before collision) = $2\sqrt{2gh_0}$

v_m (just before collision) = $2\sqrt{2gh_0}$

$\Rightarrow v_{3m}$ (just after collision) = $\left(\frac{v_m}{5}\right)$

$a_{3m} = \frac{g}{5}$ (Downward)

$$\Rightarrow \quad H_{\max} = \frac{(v_{3m})^2}{a_{3m}} = \frac{4h_0}{5} = 8\,\text{cm}$$

24. **(6)**

$$i = \frac{Bvl}{R_{eq}}; \; F = iBl$$

$$i = \frac{Bvl}{R_{eq}} = \frac{3\times2\times2}{12} = 1\,\text{A}$$

$$F = iBl = 1\times3\times2 = 6\,\text{N}$$

25. **(2)**

$$V = \frac{kq}{R}$$

$$\frac{kq_A}{R} + \frac{kq_B}{2R} = 2V \qquad \ldots (1)$$

$$\frac{kq_A}{2R} + \frac{kq_B}{2R} = \frac{3}{2}V \qquad \ldots (2)$$

From equation (1) and (2), $\frac{q_A}{q_B} = \frac{1}{2}$

After B is earthed $V_B = 0$

$\therefore \quad q_B = -q_A$

After earthing

$$V_A - V_B = kq_A\left[\frac{1}{R} - \frac{1}{2R}\right] = \frac{kq_A}{2R}$$

Putting $q_B = 2q_A$ in equation (1)

$$\frac{kq_A}{2R} = \frac{V}{2}, V_A - V_B = \frac{V}{2}$$

$$\because \quad V_B = 0 \Rightarrow V_A = \frac{V}{2}$$

26. **(4)**

$\overrightarrow{L_i} = \overrightarrow{L_f}$ (About point on horizontal surface)

$$\Rightarrow \frac{mR^2}{2}\omega_0 = 2\left(\frac{3}{2}mR^2\right)\omega$$

$$\Rightarrow \omega = \frac{\omega_0}{6}$$

$$\Rightarrow V_{cm} = \frac{R\omega_0}{6} \Rightarrow J = \frac{mR\omega_0}{6}$$

$$\Rightarrow J = \frac{(2)(1)12}{6} = 4\ \text{kg}\cdot\text{m/s}$$

27. **(10)**

For solenoid $B = \mu_0 nI$

Resistance $R_0 = (2\pi r)\times 400\frac{1}{100}$

n = (no. of turns per unit length) = $\frac{400}{20}\times 100$

$\Rightarrow$ $n = 2000$

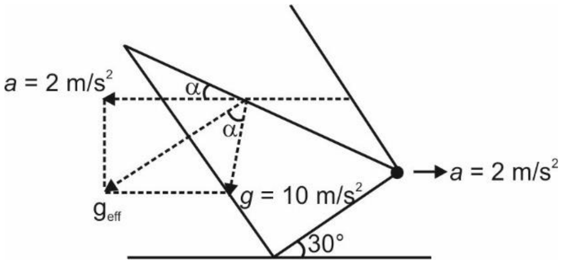
$$\therefore \quad B = \frac{\mu_0 \times 2000 \times E_0 \times 100}{2\pi r \times 400}$$

$$\Rightarrow \quad E_0 = \frac{1}{10}\times 2\pi \times \frac{1}{100}\times \frac{400}{4\pi \times 10^{-7}\times 2000\times 100}$$

$\Rightarrow$ E_0 = 10 volts

28. **(5)**

$$\tan\theta = \frac{a}{g}$$



$$\tan\alpha = \frac{a}{g} = \frac{2}{10} = \frac{1}{5}$$

$$\alpha = \tan^{-1}\left(\frac{1}{5}\right) = \tan^{-1}\left(\frac{1}{x}\right)$$

i.e. $x = 5$

29. **(494)**

Energy of incident photons

= $13.6\ Z^2$

= 13.6×2^2

= 54.4 eV

Max K.E. of photoelectrons

= 54.4 – 5

= 49.4 eV

Stopping potential = 49.4 volt

30. **(27)**

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

SECTION-II (CHEMISTRY)

31. **(2)**

In compound 'Y', 4 g of A will combine with $\frac{7}{3}\times 12 =$ 28 g of B, and hence, 8 g of A will combine with 2 × 28 = 56 g of B.

32. **(2)**

$$V_{n,z} = 2.188\times 10^6 \frac{z}{n}\ \text{m/s}$$

Or, $1094 \times 10^3 = 2.188 \times 10^6 \times \frac{z}{4} \Rightarrow z = 2$

$\Rightarrow$ He^{+}ion

33. **(1)**

For minimum work in compression, the process must be reversible.

$$w = -nRT.\ln\frac{P_1}{P_2} = -\frac{16}{32}\times 8.3\times 300\times \ln\frac{1.01325\times 10^3}{1.01325\times 10^5}$$

= + 2727 J

34. **(1)**

$$\frac{P^\circ - P}{P} = \frac{n_1}{n_2} \Rightarrow \frac{1}{99} = \frac{w/180}{500/18} \Rightarrow w = 50.5\text{gm}$$

35. **(1)**

$$\frac{1}{2}Hg_2Cl_2(s) + e^- \rightarrow Hg(l) + Cl^-(aq)$$

36. **(2)**

$4C_3H_6 + 6NO \rightarrow 4C_3H_3N + 6H_2O + N_2$

$$n_{C_3H_6} = \frac{420}{42} = 10\ \text{moles}$$

4 moles C_3H_6 = 4moles C_3H_3N

$\therefore$ 10 moles C_3H_6 = 10 moles C_3H_6N

= 10 × 53 = 530 kg.

37. (4)

$n_{eq}H_2O_2 = n_{eq}KMnO_4$

$$\text{or } \frac{1\times\frac{x}{100}}{34}\times 2 = \frac{x\times N}{1000} \Rightarrow N = 0.588$$

38. (2)

Apply Fajans' rule.

Covalent character:

$$\overset{4+}{C}Cl_4 > \overset{3+}{B}Cl_3 > \overset{2+}{Be}Cl_2 > \overset{\oplus}{Li}Cl$$

39. (3)

In $N_2^{\oplus}$, there is one unpaired electron in bonding MO (i.e., s $2p_z^1$)

40. (4)

Lesser is the positive charge, higher is the radius. Moreover the size increases down the group. Therefore,

$$Na^{\oplus} > Li^{\oplus} > Mg^{2+} > Be^{2+}$$

Na^{+}= 102 pm

Li^{+} = 76 pm

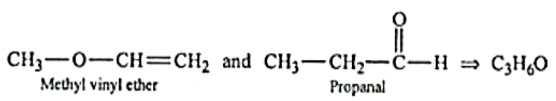
Mg^{2+} = 72 pm

Be^{2+} = 31pm

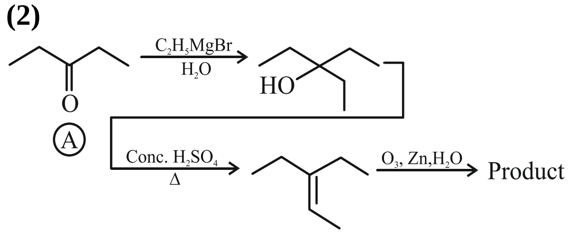
41. (1)

E_2 reaction

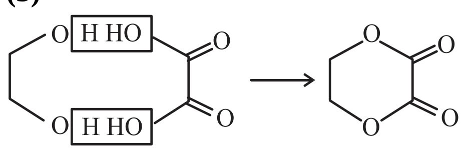
42. (1)



43. (2)



44. (3)



45. (4)

Molecule having almost negligible tendency to form hydrogen bonds is HI as hydrogen bonding depends on two factors:

(i) Higher electronegativity of X in HX

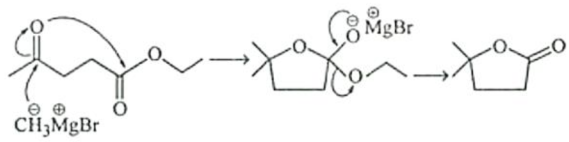
(ii) Small size of X

Electronegativity of I is low and its size is also large. Therefore, both the factors fail here. Hence, no hydrogen bonding is present in HII.

46. (2)

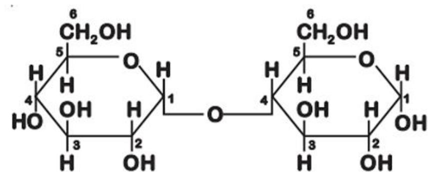
Both are enantiomers.

47. (3)

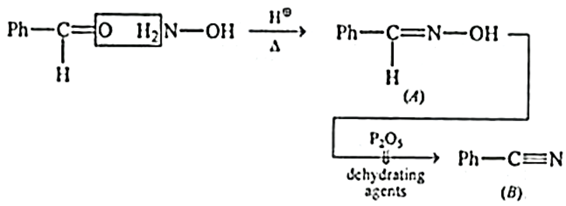


48. (4)

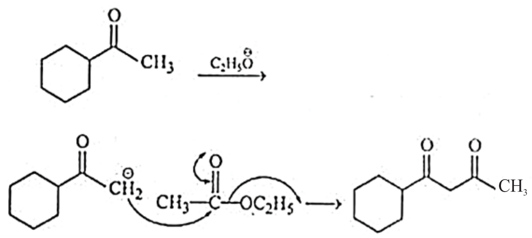
Maltose is composed of two units of a-D glucose which are joined through C_1 – C_4 glycosidic linkage.



49. (1)



50. (3)



51. (5)

$$\frac{160}{96} = \frac{2\times n}{2+n} \Rightarrow n = 10 \Rightarrow$$ Oxidation state of Br in unknown product $= 5$

52. (8)

Heat released by $\frac{6\cdot 3}{64000}$ mole haemoglobin

$= 25\times 4.2\times 0.03 = 3.15$ J

$\therefore$ Heat released per mole haemoglobin

$$= \frac{3\cdot 15\times 64000}{6\cdot 3} = 32000 \text{ J}$$

$\therefore$ Heat released per mole

$$O_2 = \frac{32000}{4} = 8000 \text{ J}$$

53. (2)

$$\Delta T_f = K_f \cdot m$$

$$Hg(CN)_2 + mCN^- \rightleftharpoons Hg(CN)_{m+2}^{m-}$$

0.1 mole 0.2 mole 0

Final 0 (0.2–0.1 m) mole 0.1 mole

For KCN solution: $0.80 = K_f \times 0.2 \times 2$

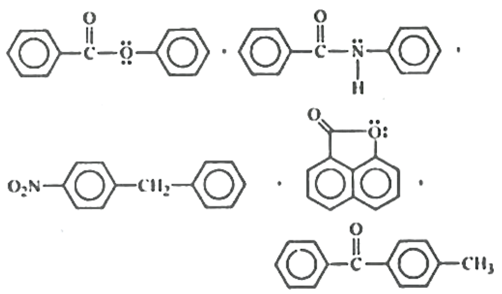
Final effective molality = (0.2–0.1 m) + 0.1 + 0.2

= 0.5–0.1 m

Now, $0.60 = K_f \times (0.5–0.1\text{ m})$

From (1) and (2): m =2

54. (5)



55. (1)

Carbon 1 is anomeric carbon.

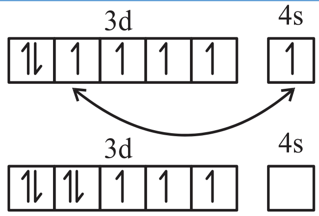
56. (5)

$$\left[\overset{\oplus}{Fe}\left(\overset{0}{H_2O}\right)_5\overset{+1}{NO}\right]^{2+}$$

$$Fe^{+1} = 3d^6 4s^1$$

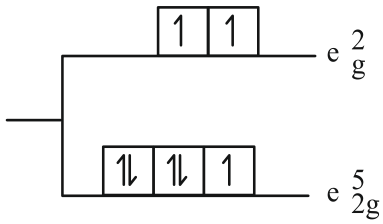
Since (NO) is strong ligand, so one pairing occurs.

Fe^{+1}



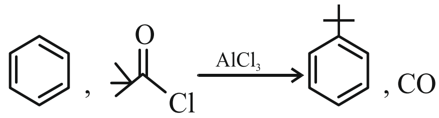
Fe^{+1} in complex

It has sp^3d^2 hybridization with octahedral geometry according to CFT, e^-distribution is as follow:

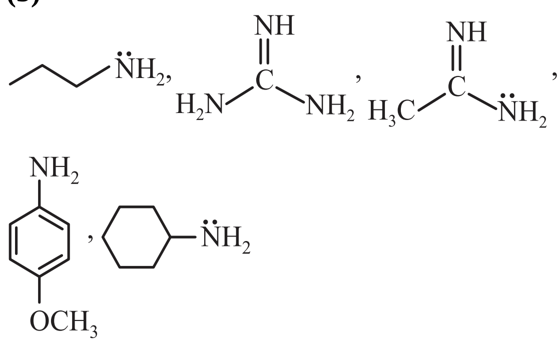


57. (5)

(i), (ii), (iii), (iv) and (v)



58. (5)



59. (8)

$$H_3C-CH\overset{\bullet}{=}CH-CH\overset{\bullet}{=}CH-CH\overset{\bullet}{=}CH-Ph$$

Total stereoisomer $=2^3 = 8$

60. (59)

$K = Ae^{-Ea/RT}$

$$\ln\left(\frac{2K_1}{K_1}\right) = \frac{Ea}{R}\left(\frac{1}{300}-\frac{1}{309}\right)$$

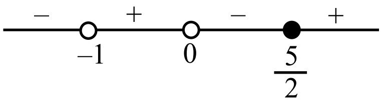
$$Ea = \frac{0.3\times 8.3\times 2.3\times 300\times 309}{9} \approx 59 \text{ kJ mol}^{-1}$$

SECTION-III (MATHEMATICS)

61. **(4)**

$$\frac{(2x-5)}{x(x+1)} \ge 0;$$

$\because \quad a^x > 0$ and $\cos x - 2 < 0$



62. **(2)**

Put $\log_3(x-1) = t$

$\Rightarrow \; t = 2 + \frac{1}{t^2} - 2t$

$\Rightarrow \; 3t^3 - 2t^2 - 1 = 0$

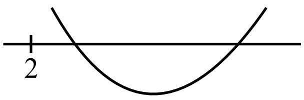
$\therefore \; (t-1)(3t^2 + t + 1) = 0$

$\Rightarrow \; t = 1 \; \therefore \; x = 4$

63. **(3)**

$$\alpha + \frac{1}{\alpha} \ge 2, \beta + \frac{1}{\beta} \ge 2$$

$\Rightarrow$ both the roots are real and greater than 2.



$$\begin{cases} f(2) \ge 0 \\ D \ge 0 \\ -\frac{b}{2a} > 2 \end{cases}$$

$$\Rightarrow \begin{cases} 4 - 10 - a \ge 0 \\ 25 + 4a \ge 0 \;\; \Rightarrow a \in \left[\frac{-25}{4}, -6\right] \\ \frac{5}{2} > 2 \end{cases}$$

64. **(4)**

$$t_r = \frac{r(r+1)(2r+1)4}{6r^2(r+1)^2} = \frac{2(2r+1)}{3r(r+1)} = \frac{2}{3}\left[\frac{1}{r} + \frac{1}{r+1}\right]$$

$$S_n = \frac{2}{3}\sum_{r=1}^{n}(-1)^r\left(\frac{1}{r} + \frac{1}{r+1}\right)$$

$\Rightarrow$

$$S_\infty = \frac{2}{3}\left[\left(-1 + \frac{1}{2} - \frac{1}{3} + \ldots\right) + \left(-\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\right)\right]$$

$$= \frac{2}{3}[-1]$$

65. **(2)**

The given equation is $(z^2 + z + 1)(z^2 + 1) = 0$.

$z = \pm I, w, w^2$, w being an imaginary cube root of unity. Thus, $|z| = 1$

66. **(1)**

$$\left(y + 3x^2y^2e^{x^3}\right)dx = xdy$$

$$\Rightarrow \; \frac{dy}{dx} = \frac{y + 3x^2y^2e^{x^3}}{x}$$

$$\Rightarrow \; \frac{dy}{dx} - \frac{y}{x} = y^2\left(3x^{e^{x^3}}\right)$$

$$\Rightarrow \; \frac{1}{y^2}\frac{dy}{dx} - \frac{1}{xy} = 3x^{e^{x^3}}$$

Put $-\frac{1}{y} = t \Rightarrow \frac{dt}{dx} + \frac{t}{x} = 3x^{e^{x^3}}$

I.F. $= e^{\int \frac{dx}{x}} = x$

$$-\frac{1}{y} = t \;\; \frac{dt}{dx} + \frac{t}{x} = 3x^{e^{x^3}}$$

$$(I.F.) = e^{\int \frac{dx}{x}} = x$$

$$\frac{d}{dx}(tx) = 3x_2e^{x^3}$$

$$\Rightarrow \; tx = \int 3x^2e^{x^3}dx$$

$$\Rightarrow \; \frac{-x}{y} = e^{x^3} + c$$

Alternate solution

$$\frac{ydx - xdy}{y^2} + e^{x^3} \cdot d\left(x^3\right) = 0$$

$$\frac{x}{y} + e^{x^3} = c$$

67. **(3)**

$$I = \int\sqrt{\frac{x-1}{x+1}} \times \frac{1}{x^2}dx$$

Put $\frac{1}{x} = \cos 2\theta \Rightarrow -\frac{dx}{x^2} = -2\sin 2\theta d\theta$

$$I = \int\sqrt{\frac{1-\cos 2\theta}{1+\cos 2\theta}}2\sin 2\theta \; d\theta$$

$$= \int 4\sin^2\theta\ d\theta = 2\int (1-\cos 2\theta)d\theta$$

$$= 2\theta - \sin 2\theta + C = \cos^{-1}\left(\frac{1}{x}\right) - \sqrt{1-\frac{1}{x^2}} + C$$

68. **(3)**

$$\lim_{x\to\frac{\pi}{2}} \tan^2 x\left(\sqrt{2\sin^2 x+3\sin x+4}-\sqrt{\sin^2 x+6\sin x+2}\right)$$

$$= \lim_{x\to\frac{\pi}{2}} \tan^2 x\left(\frac{2\sin^2 x+3\sin x+4-(\sin^2 x+6\sin x+2)}{\sqrt{2\sin^2 x+3\sin x+4}+\sqrt{\sin^2 x+6\sin x+2}}\right)$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{1}{\cos^2 x}\frac{(\sin^2 x-3\sin x+2)}{\sqrt{2+3+4}+\sqrt{1+6+2}}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{1}{6}\left[\frac{\sin^2 x-3\sin x+2}{\cos^2 x}\right]\left(\frac{0}{0}\text{form}\right)$$

(Use L' Hospital rule)

$$= \frac{1}{6}\lim_{x\to\frac{\pi}{2}} \frac{2\sin x\cos x-3\cos x}{2\cos x(-\sin x)}$$

$$= \frac{1}{6}\lim_{x\to\frac{\pi}{2}} \frac{2\sin x-3}{-2\sin x} = \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) = \frac{1}{12}$$

69. **(2)**

$$\Delta = \begin{vmatrix} S_0 & S_1 & S_2 \\ S_1 & S_2 & S_3 \\ S_2 & S_3 & S_4 \end{vmatrix}$$

$$= \begin{vmatrix} 3 & \alpha+\beta+\gamma & \alpha^2+\beta^2+\gamma^2 \\ \alpha+\beta+\gamma & \alpha^2+\beta^2+\gamma^2 & \alpha^3+\beta^3+\gamma^3 \\ \alpha^2+\beta^2+\gamma^2 & \alpha^3+\beta^3+\gamma^3 & \alpha^4+\beta^4+\gamma^4 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 1 & 1 \\ \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \end{vmatrix}^2 = (\alpha-\beta)^2(\beta-\gamma)^2(\gamma-\alpha)^2$$

70. **(4)**

Total n-digit numbers using 1, 2 or 3 = 3^n

total n-digit numbers using any two digits out of 1, 2 or 3 = ${}^3C_2 \times 2^n - 6 = 3 \times 2^n - 6$

total n-digit numbers using only one digit of 1, 2 or 3 = 3

$\therefore$ the numbers containing all three of the digits 1, 2 and 3 at least once = $3^n - (3 \times 2^n - 6) - 3$ $= 3^n - 3 \cdot 2^n + 3$

71. **(1)**

$$\begin{vmatrix} x^3+1 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & yz^2 & z^3+1 \end{vmatrix} = \begin{vmatrix} x^3 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & yz^2 & z^3+1 \end{vmatrix}$$

$$+\begin{vmatrix} 1 & x^2y & x^2z \\ 0 & y^3+1 & y^2z \\ 0 & yz^2 & z^3+1 \end{vmatrix}$$

$$= x\begin{vmatrix} x^2 & x^2y & x^2z \\ y^2 & y^3+1 & y^2z \\ z^2 & yz^2 & z^3+1 \end{vmatrix} + (y^3+1)(z^3+1) - y^3z^3$$

$C_2 \to C_2 - yC_1$ and $C_3 \to C_3 - zC_1$

$$= x\begin{vmatrix} x^2 & 0 & 0 \\ y^2 & 1 & 0 \\ z^2 & 0 & 1 \end{vmatrix} + y^2z^3 + z^3 + y^3 + 1 - y^3z^3$$

$= x^3 + y^3 + z^3 + 1$

Given, $x^3 + y^3 + z^3 + 1 = 30 \Rightarrow x^3 + y^3 + z^3 = 29$

As, $29 = 3^3 + 1^3 + 1^3$, then solutions are (3, 1, 1), (1, 3, 1), (1, 1, 3)

Sum of all the possible values of x is = 3 + 1 + 1 = 5

72. **(1)**

$|A| = 2$; $A^{2011} - 5A^{2010} = A^{2010}(A - 5I)$

$\therefore$ $|A^{2011} - 5A^{2010}| = 2^{2010}\begin{vmatrix} -2 & 11 \\ 2 & 3 \end{vmatrix}$

$$= 2^{2010}(-28) = -2^{2012}7$$

73. **(2)**

$f(x) = \text{sgn}(x - x^4 + x^7 - x^8 - 1)$

For $x \in (0, 1)$; $x - 1 < 0$, $x^7 - x^4 < 0$

$\therefore$ $x - x^4 + x^7 - x^8 - 1 < 0$

For $x \in (1, \infty)$; $x < x^4$, $x^7 < x^8$

$\therefore$ $x - x^4 + x^7 - x^8 - 1 < 0]$

Also for $x = 1$; $x - x^4 + x^7 - x^8 - 1 = -1$

Thus $x - x^4 + x^7 - x^8 - 1 < 0$ for all $x \in R^+$

Hence $\text{sgn}(x - x^4 + x^7 - x^8 - 1) = -1\ \forall\ x \in R^+$

Therefore $f(x)$ is many-one and into

74. **(4)**

$$\lim_{x\to\frac{\pi}{2}} \frac{(1-\sin x)(8x^3-\pi^3)\cos x}{(\pi-2x)^4}$$

$$= \lim_{x\to\frac{\pi}{2}} \frac{\left(1-\cos\left(\frac{\pi}{2}-x\right)\right)(2x-\pi)\left(4x^2+\pi^2+2\pi x\right)\sin\left(\frac{\pi}{2}-x\right)}{16\left(\frac{\pi}{2}-x\right)^4}$$

$$= \lim_{h\to 0} \frac{(1-\cos h)(-2h)\sin h}{16h^4} \lim_{x\to\frac{\pi}{2}} \left(4x^2+\pi^2+2\pi x\right)$$

$$= \lim_{h\to 0} \frac{(1-\cos h)(-2h)\sin h}{16h^4} \lim_{x\to\frac{\pi}{2}} \left(4x^2+\pi^2+2\pi x\right)$$

$$= \lim_{h\to 0} \frac{(1-\cos h)}{8h^2}\left(4\left(\frac{\pi}{2}\right)^2+\pi^2+2\pi\left(\frac{\pi}{2}\right)\right)$$

$$= -\left(3\pi^2\right)\lim_{h\to 0} \frac{2\sin^2\frac{h}{2}}{32\frac{h^2}{4}}$$

$$= -\frac{3\pi^2}{16}$$

75. (4)

$$f(x) = \begin{cases} 2\sqrt{1-x^2}, & x \le 0 \\ 0, & x > 0 \end{cases}$$

Clearly, $f\ (x)$ is discontinuous, hence non-differentiable at $x = 0$.

76. (2)

$$\cos(A-B) = \frac{3}{5} \ \& \ \tan A \tan B = 2$$

$$\cos A \cos B + \sin A \sin B = \frac{3}{5}$$

$$\Rightarrow \ (1 + \tan A \tan B) \cos A \cos B = \frac{3}{5}$$

$$\Rightarrow \ (1+2)\times \cos A \cos B = \frac{3}{5}$$

$$\Rightarrow \ \cos A \cos B = \frac{1}{5}$$

$$\therefore \ \sin A \sin B = \frac{3}{5} - \frac{1}{5} = \frac{2}{5}$$

$$\cos(A + B) = \cos A \cos B - \sin A \sin B = \frac{1}{5} - \frac{2}{5} = \frac{-1}{5}$$

77. (3)

Let position vector of A be origin

Position vector of $M \equiv \frac{\vec{c}}{3}$

Position vector of $N \equiv \left(-\vec{c} + 2\vec{b}\right)$

$\therefore$ equation of line BC is $\vec{r} = \vec{b} + \lambda\left(\vec{b} - \vec{c}\right)$

$\therefore$ equation of line AB is $\vec{r} = \vec{0} + \mu\vec{b}$

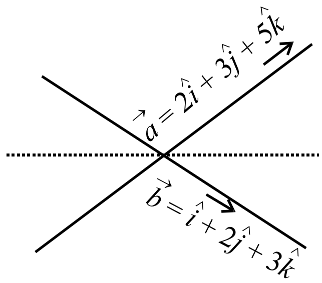
$\therefore$ equation of line MN is $\vec{r} = \frac{\vec{c}}{3} + t\left(\frac{4\vec{c}}{3} - 2\vec{b}\right)$

$$\Rightarrow \ \mu = -2t,\ 0 = \frac{1}{3} + \frac{4}{3}t$$

Which gives $\mu = \frac{1}{2}$

$\Rightarrow$ Position vector of X is $\frac{\vec{b}}{2}$.

78. (3)



$$\frac{x}{2} = \frac{y}{3} = \frac{z}{5} \qquad \text{.....(i)}$$

$$\frac{x}{1} = \frac{y}{2} = \frac{z}{3} \qquad \text{....(ii)}$$

$$\hat{a} + \hat{b} = \frac{2\hat{i} + 3\hat{j} + 5\hat{k}}{\sqrt{38}} + \frac{\hat{i} + 2\hat{j} + 3\hat{k}}{\sqrt{14}}$$

$\Rightarrow$ (A) and (B) will be incorrect

Let the dr's of line $\perp$ to (1) and (2) be a, b, c

$\Rightarrow$ $2a + 3b + 5c = 0$ ....(iii) and

$a + 2b + 3c = 0$ .....(iv)

$$\therefore \ \frac{a}{9-10} = \frac{b}{5-6} = \frac{c}{4-3}$$

$$\Rightarrow \ \frac{a}{-1} = \frac{b}{-1} = \frac{c}{1}$$

$$\Rightarrow \ \frac{a}{1} = \frac{b}{1} = \frac{c}{-1}$$

$\therefore$ equation of line passing through (0, 0, 0) and is $\perp_r$ to the lines (1) and (2) is

$$\frac{x}{1} = \frac{y}{1} = \frac{z}{-1}$$

79. **(4)**

$$I=\int\sqrt{\frac{1-\cos x}{\cos\alpha-\cos x}}\,dx$$

$0<\alpha<x<\pi$

$$=\int\frac{\sqrt{2}\sin\frac{x}{2}dx}{\sqrt{2\cos^2\frac{\alpha}{2}-1-2\cos^2\frac{x}{2}+1}}$$

$$=\int\frac{\sin\frac{x}{2}dx}{\sqrt{\cos^2\frac{\alpha}{2}-\cos^2\frac{x}{2}}}$$

Put $\cos\frac{x}{2}=t\Rightarrow-\frac{1}{2}\sin\frac{x}{2}dx=dt$

$$\Rightarrow\quad I=\int\frac{-2dt}{\sqrt{\cos^2\frac{\alpha}{2}-t^2}}=-2\sin^{-1}\left(\frac{\cos\frac{x}{2}}{\cos\frac{\alpha}{2}}\right)+C$$

80. **(1)**

The equation of required hyperbola is

$$\frac{\left(\frac{2x-y+4}{\sqrt{5}}\right)^2}{\left(\frac{\sqrt{2}}{2}\right)^2}-\frac{\left(\frac{x+2y-3}{\sqrt{5}}\right)^2}{\left(\frac{1}{\sqrt{3}}\right)^2}=1$$

$\Rightarrow\quad x^2-4xy-2y^2+10x+4y=0$

Since centre of hyperbola is intersection of axis C (–1, 2) and foci lies on T.A. at a distance ae from centre

$\therefore$ focus $((-1\pm ae\cos\theta), (2\pm ae\sin\theta))$

transverse axis slope = $-\frac{1}{2}$

We get $\cos\theta=\frac{-2}{\sqrt{5}},\sin\theta=\frac{1}{\sqrt{5}}$

$\Rightarrow\quad\left(-1\mp\frac{2}{\sqrt{6}},2\pm\frac{1}{\sqrt{6}}\right)$

81. **(8)**

Solving given curves

$x^2+2=2|x|-\cos\pi x$

$\Rightarrow\ x^2-2|x|+2=-\cos\pi x$

$\Rightarrow\ (|x|-1)^2+1=-\cos\pi x$

$\Rightarrow\ x=\pm1$

$\therefore$ Required Area =

$$\int_{-1}^{1}\left(x^2+2-2|x|+\cos\pi x\right)dx=\frac{8}{3}$$

82. **(2)**

$$\frac{dx}{dy}+\frac{x^2}{y^2}-\frac{x}{y}+1=0$$

Put $x=vy$

$\Rightarrow\quad\frac{dx}{dy}=v+y\frac{dv}{dy}$

$\therefore\quad v+y\frac{dv}{dy}+v^2-v+1=0$

$\Rightarrow\quad -y\frac{dv}{dy}=\left(1+v^2\right)$

$\Rightarrow\quad\frac{dv}{v^2+1}=-\frac{dy}{y}$

Integrating

$\tan^{-1}v+c=-\ln y$

$\Rightarrow\quad\tan^{-1}\left(\frac{x}{y}\right)+\ln y+c=0$

Where c is arbitrary constant

83. **(3)**

Let $\vec{b}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$, $\vec{a}\times\vec{b}=\hat{j}-\hat{k}$

$$\Rightarrow\quad\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\1&1&1\\\alpha&\beta&\gamma\end{vmatrix}=\hat{j}-\hat{k}$$

$\Rightarrow\ \beta-\gamma=0,\ \alpha-\gamma=1,\ \alpha-\beta=1$

$\Rightarrow\ \beta=\gamma,\ \alpha=1+\gamma,\ \alpha=1+\beta,\ \vec{a}\cdot\vec{b}=1$

$\Rightarrow\ \alpha+\beta+\gamma=1,\Rightarrow\beta+1+\beta+\beta=1$

$\Rightarrow\ \beta=0$

$\therefore\ \alpha=1,\ \gamma=0,$

$\therefore\ \vec{b}=\hat{i}$

84. **(3)**

Eliminating n we have $(l^2+m^2)-(l+m)^2=0$

$\therefore\ 2lm=0$

When $l=0$ then $m+n=0$

$\therefore\quad\frac{l}{0}=\frac{m}{1}=\frac{m}{-1}\qquad\ldots(1)$

When $m=0$ then $l+n=0$

$\therefore\quad\frac{l}{1}=\frac{m}{0}=\frac{n}{-1}\qquad\ldots..(2)$

$\therefore$ D.R.'s are 0, 1, –1 and 1, 0, –1

$\therefore\quad\cos\theta=\frac{0.1+1.0+1}{\sqrt{1+1}\sqrt{1+1}}=\frac{1}{2}$

$\therefore\quad\theta=\frac{\pi}{3}$

85. (2)

Equation of AU is

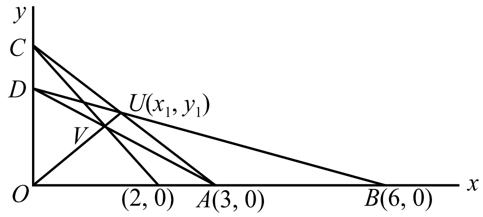
$$y-y_1=\frac{0-y_1}{3-x_1}(x-x_1)$$

So that the coordinates of C are $\left(0, \frac{3y_1}{(3-x_1)}\right)$

Similarly, the coordinates of D are $\left(0, \frac{6y_1}{(6-x_1)}\right)$

Now, equation of AD is $\frac{x}{3}+\frac{y(6-x_1)}{6y_1}=1$ …(i)

and equation of OU is $y_1x = x_1y$ …(ii)



Solving (i) and (ii), we get

$$\frac{x_1y}{3y_1}+\frac{y(6-x_1)}{6y_1}=1$$

$\Rightarrow$ $y(2x_1+6-x_1)=6y_1$

$\Rightarrow$ $y=\frac{6y_1}{6+x_1} \Rightarrow x=\frac{6x_1}{6+x_1}$

Hence, the coordinates of V are $\left(\frac{6x_1}{6+x_1}, \frac{6y_1}{6+x_1}\right)$

Therefore, equation of CV is

$$y-\frac{3y_1}{3-x_1}=\frac{\frac{6y_1}{6+x_1}-\frac{3y_1}{3-x_1}}{\frac{6x_1}{6+x_1}-0}(x-0)$$

$\Rightarrow$ $y=\frac{3y_1}{3-x_1}-\frac{9x_1y_1}{6x_1(3-x_1)}x$

$\Rightarrow$ $y=\frac{3y_1}{3-x_1}\left(1-\frac{x}{2}\right)$

Which passes through the point (2, 0) for all values of (x_1, y_1)

86. (12)

$x_3+5x_2+px+q=0$ — α, β, x_1

$\Rightarrow$ $\alpha+\beta+x_1=-5$, $\alpha\beta+\beta x_1+\alpha x_1=p$ …..(1)

$x_3+7x_2+px+r=0$ — α, β, x_2

$\Rightarrow$ $\alpha+\beta+x_2=-7$, $\alpha\beta+\beta x_2+\alpha x_2=p$ ….(2)

$\alpha\beta+\beta x_1+\alpha x_1=p$

$$\Rightarrow \frac{\alpha\beta+\beta x_2+\alpha x_2=p}{\alpha(x_1-x_2)+\beta(x_1-x_2)=0}$$

$\Rightarrow$ $(x_1-x_2)(\alpha-\beta)=0$ $[x_1\neq x_2]$

$\therefore$ $\alpha+\beta=0$

$\Rightarrow$ $x_1=-5$

$\Rightarrow$ $x_2=-7$

87. (48)

$$f(x)=\sqrt{-\log_{\frac{x+4}{2}}\left(\log_2\frac{2x-1}{3+x}\right)}$$

For domain : $\log_{\frac{x+4}{2}}\left(\log_2\frac{2x-1}{3+x}\right)\le 0$

Case I $0<\frac{x+4}{2}<1$

$\Rightarrow$ $-4<x<-2$ ……A

then $\log_{\frac{x+4}{2}}\left(\log_2\frac{2x-1}{3+x}\right)\le 0$

$\Rightarrow$ $\log_2\frac{2x-1}{3+x}\ge 1$

$\Rightarrow$ $\frac{2x-1}{3+x}\ge 2$

$\Rightarrow$ $x<-3$ …..B

$\Rightarrow$ on $A\cap B$ $x\in(-4,-3)$ …..(i)

Case-II

$\frac{x+4}{2}>1$ or $x>-2$ …..A

$\log_{\frac{x+4}{2}}\left(\log_2\frac{2x-1}{3+x}\right)\le 0$

$\Rightarrow$ $0<\log_2\frac{2x-1}{3+x}\le 1$

$\Rightarrow$ $1<\frac{2x-1}{3+x}\le 2$

$\Rightarrow$ $x\in(4,\infty)$ ….(ii)

$\therefore$ (i) $\cup$ (ii)

Domain $x\in(-4,-3)\cup(4,\infty)$

88. **(900)**

$a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$

$\Rightarrow$ $3(a_1 + a_{24}) = 225$

(sum of terms equidistant from beginning and end are equal) $a_1 + a_{24} = 75$

Now $a_1 + a_2 + \ldots\ldots + a_{23} + a_{24} = \frac{24}{2}[a_1 + a_{24}]$

$= 12 \times 75 = 900$

89. **(30)**

$$\int_1^2 x \cdot x^{x^2}(1+2\ln x)dx$$

$x^{x^2} = t$

$x^2 \ln x = \ln t$

$$\left(x^2 \cdot \frac{1}{x} + \ln x \cdot 2x\right)dx = \frac{1}{t}dt$$

$$(x + 2\ln x)dx = \frac{1}{t}dt$$

$$= \int t \cdot \frac{1}{t}dt$$

$$= t = \left(x^{x^2}\right)_1^2 = 2^4 - 1 = 15$$

90. **(191)**

Let No. of children of John & Angelina = y

$\therefore$ $x + (x+1) + y = 24$

$y = 23 - 2x$

Number of fights

$F = x(x+1) + x(23-2x) + (x+1)(23-2x)$

$F = -3x^2 + 45x + 23$

$\frac{dF}{dx} = 0 \Rightarrow -6x + 45 = 0$

$\Rightarrow$ $x = 7.5$

But 'x' will be integral.

check $x = 8$ or $x = 7$

$F = 191$