JEE MAIN (2023-24) Mock Test Series

Paper - 03

DURATION : 180 Minutes | **M. MARKS : 300**

ANSWER KEY

PHYSICS

1. (2)
2. (1)
3. (3)
4. (2)
5. (2)
6. (2)
7. (3)
8. (3)
9. (4)
10. (3)
11. (4)
12. (4)
13. (2)
14. (1)
15. (2)
16. (2)
17. (2)
18. (1)
19. (3)
20. (3)
21. (5)
22. (12)
23. (1)
24. (6)
25. (18)
26. (2)
27. (17)
28. (15)
29. (2)
30. (50)

CHEMISTRY

31. (1)
32. (2)
33. (2)
34. (4)
35. (4)
36. (3)
37. (1)
38. (3)
39. (1)
40. (3)
41. (4)
42. (1)
43. (3)
44. (4)
45. (4)
46. (3)
47. (2)
48. (2)
49. (1)
50. (1)
51. (5)
52. (4)
53. (36)
54. (6)
55. (12)
56. (2)
57. (5)
58. (0)
59. (5)
60. (5)

MATHEMATICS

61. (2)
62. (4)
63. (3)
64. (3)
65. (3)
66. (2)
67. (1)
68. (1)
69. (2)
70. (3)
71. (4)
72. (2)
73. (3)
74. (3)
75. (3)
76. (3)
77. (4)
78. (3)
79. (1)
80. (3)
81. (4)
82. (2)
83. (1)
84. (2)
85. (15)
86. (6)
87. (1)
88. (1)
89. (1)
90. (5)

SECTION-I (PHYSICS)

1. **(2)**

$f = \mu_s N$

$T\cos 37 = \frac{4}{7}N$ ...(1)

$N = 20 - T\sin 37$ ...(2)

from (1) and (2)

$m_1 = 2$ kg

2. **(1)**

$U = 2x^2 - 3x^3$

$F = \frac{-dU}{dx} = -\left[4x - 9x^2\right]$

At equilibrium

$F = 0$

$x = 0, \frac{4}{9}$ are equilibrium points

$\frac{d^2U}{dx^2} > 0$ for $x = 0$ therefore $x = 0$ is stable equilibrium

$\frac{d^2U}{dx^2} < 0$ for $x = \frac{4}{9}$ therefore $x = \frac{4}{9}$ is unstable equilibrium

3. **(3)**

$k' = \frac{k}{l}x + k$

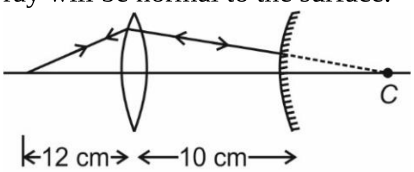
$dC = \frac{ldx \in_0 k'}{d}$

$\Rightarrow \quad c = \frac{l}{d}\in_0 \int_0^l \left(\frac{k}{l}x + k\right)dx$

$= \frac{3kl^2 \in_0}{2d}$

4. **(2)**

For lens $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$. And on mirror the incident ray will be normal to the surface.



Using lens formula

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$u = -12$ cm, $f = 10$ cm

$\Rightarrow v = 60$ cm

$\Rightarrow$ Distance of C from mirror is 50 cm

$\Rightarrow R = 50$ cm

$\Rightarrow f = 25$ cm

5. **(2)**

$I_1 = I_2$

$\frac{V_1 - V_C}{R_1} = \frac{V_C - V_2}{R_2}$

6. **(2)**

In one quarter time electric field energy will completely change into magnetic field energy.

$t = \frac{T}{4} = \frac{2\pi\sqrt{LC}}{4}$

$t = 1.57$ ms

7. **(3)**

$I = Q\omega/2\pi$

8. **(3)**

$\frac{dr}{dt} = R\sqrt{\frac{2g}{r}}$

$u = \sqrt{2gR}$

$\frac{1}{2}mv^2 - \frac{GMm}{r} = \frac{1}{2}mu^2 - \frac{GMm}{R}$

$\Rightarrow v = R\sqrt{\frac{2g}{r}}$

$\Rightarrow \frac{dr}{dt} = R\sqrt{\frac{2g}{r}}$

$\Rightarrow \int_R^{4R} \sqrt{r}\,dr = R\sqrt{2g}\int_0^t dt$

$\Rightarrow t = \frac{7}{3}\sqrt{\frac{2R}{g}}$

9. **(4)**

Individual magnetic field gets cancelled out due to symmetry

10. **(3)**

$\frac{1}{2}K(A')^2 = \frac{1}{2}KA^2 + \frac{1}{2}m\omega^2 A^2$

$\text{K.E.} = \frac{1}{2}m\omega^2(A^2 - x^2)$

$= \frac{1}{2}m\omega^2\left(A^2 - \frac{3}{4}A^2\right)$

$= \frac{1}{8}m\omega^2 A^2$

If kinetic energy increased by $\frac{1}{2}m\omega^2 A^2$ at position $\frac{\sqrt{3}A}{2}$, then new kinetic energy at that instant

K.E. = $\frac{1}{8}m\omega^2 A^2 + \frac{1}{2}m\omega^2 A^2$

= $\frac{1}{2}m\omega^2\left(A^2 - \left(\frac{\sqrt{3}A}{2}\right)^2\right)$

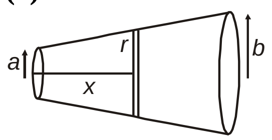
$\Rightarrow \boxed{x = \sqrt{2}A}$

11. **(4)**

Work done by a gas in a cyclic process is negative if *P-V* graph is in anticlockwise sequence.

$W_{\text{by gas}} = -\frac{1}{2}\times 1\times 40 = -20 \text{ J}$

12. **(4)**



$dR = \frac{\rho dx}{\pi r^2}$ ...(i)

Also, $r = \frac{b-a}{l}x + a \Rightarrow dx = \frac{ldr}{(b-a)}$

$R = \frac{\rho l}{\pi(b-a)}\int_a^b \frac{dr}{r^2} = \frac{\rho l}{\pi ab}$

13. **(2)**

$g(x, t) = f((x - v(t - t_0)), t)$

14. **(1)**

$mg + N = \frac{mv^2}{r}$

$N = \frac{mv^2}{r} - mg = 70\left(\frac{120\times 120}{500} - 10\right)$

$= 70\left(\frac{144}{5} - 10\right)$

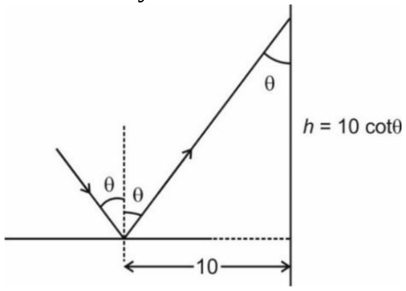
= 70 (28.8 – 10) = 1316 N

15. **(2)**

Since charge on the outer part of outer sphere is 0 therefore no electric filed can be present out side the outer sphere.

16. **(2)**

When mirror is rotated with angular speed 'ω' the reflected ray will rotate with 2ω.



When mirror is rotated with angular speed 'ω' the reflected ray will rotate with 2ω = 36 rad/s

Speed of the spot

$= \left|\frac{dh}{dt}\right| = \left|\frac{d}{dt}(10\cot\theta)\right|$

$= \left|-10\,\text{cosec}^2\theta\frac{d\theta}{dt}\right| = 1000 \text{ m/s}$

17. **(2)**

$RC = \frac{L}{R}$

$R = \sqrt{\frac{L}{C}} \Rightarrow RC = \frac{L}{R}$

$\Rightarrow$ Time constant of both circuits are equal

$l_L = i_C$

$\frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right) = \frac{V}{R}e^{-\frac{t}{\tau}}$

$\Rightarrow t = \tau\ln(2) = RC\ln(2)$

18. **(1)**

$\Delta U = mg\Delta H$

$|\Delta U| = \Delta U_A + \Delta U_B$

$= \frac{mgl}{2}$

$= 1\times\frac{10}{2}\times\frac{10}{100}$

$= \frac{1}{2}$ J

19. **(3)**

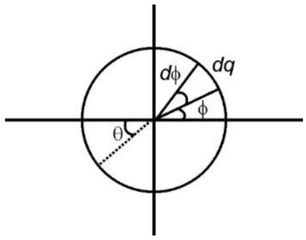
$mv_0 \times \frac{l}{4} = \left(\frac{7}{48}ml^2 + \frac{ml^2}{16}\right)\times\omega$

$\Rightarrow \omega = \frac{6v_0}{5l}$

$\therefore t = \frac{\pi\times 5l}{2\times 6v_0} = \frac{5\pi l}{12v_0}$

20. **(3)**

Use dipole



$dq = \lambda_0 \sin\phi \, a \, d\phi$

$|\overrightarrow{dp}| = dq \times 2a - 2a^2\lambda_0 \sin\phi d\phi$

$|\overrightarrow{dp}| = 2a^2\lambda_0 \sin\phi d\phi\left[\cos\phi\hat{i} + \sin\phi\,\hat{j}\right]$

$d\vec{\tau} = \overrightarrow{dp} \times \vec{E}$

$d\vec{\tau} = 2a^2\lambda_0 d\phi\left[\sin\phi\cos\phi\hat{i} + \sin^2\phi\hat{j}\right] \times \left[E_0\hat{i} + E_0\hat{j}\right]$

$$\int d\vec{\tau} = \int_0^{\pi} 2a^2\lambda_0 E_0 d\phi\left[\sin\phi\cos\phi - \sin^2\phi\right]\hat{k}$$

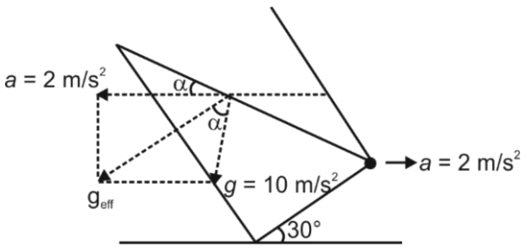
$$= 2a^2\lambda_0 E_0\left[\int_0^{\pi}\sin\phi\cos\phi d\phi - \int_0^{\pi}\sin^2\phi d\phi\right]\hat{k}$$

$$= 2a^2\lambda_0 E_0\left[\int_0^{\pi}\frac{\sin^2\phi}{2}d\phi - \int_0^{\pi}\left(\frac{1-\cos 2\phi}{2}\right)d\phi\right]\hat{k}$$

$$= 2a^2\lambda_0 E_0\left[0 - \frac{\pi}{2}\right]\hat{k} = a^2\lambda_0\pi E_0\,\hat{k}$$

21. (5)

$\tan\theta = \dfrac{a}{g}$



$\tan\alpha = \dfrac{a}{g} = \dfrac{2}{10} = \dfrac{1}{5}$

$\alpha = \tan^{-1}\left(\dfrac{1}{5}\right) = \tan^{-1}\left(\dfrac{1}{x}\right)$

i.e. $x = 5$

22. (12)

$Y \quad = \text{LCM of } \left(\dfrac{\lambda_1 D}{d} \text{ and } \dfrac{\lambda_2 D}{d}\right)$

= LCM (400×10^{-6} and 600×10^{-6})
= 1200×10^{-6} m = 1.2 mm

23. (1)

We know that $v = \sqrt{\dfrac{T}{\mu}}$

$\Rightarrow \quad T = \mu v^2 = (2.5 \times 10^{-3}) \times (20)^2 = 1$ N

24. (6)

For angular momentum to be conserved, $\tau = 0$

$$\because \quad \vec{\tau} = \begin{vmatrix} \hat{i} & j & k \\ 2 & 1 & 3 \\ 4 & 2 & \lambda \end{vmatrix} = 0$$

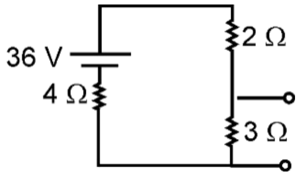
$= \hat{i}(\lambda - 6) + j(12 - 2\lambda) + k(4 - 4) = 0$

$\Rightarrow \lambda = 6$

25. (18)

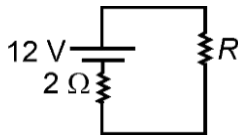
If load resistance matches with source resistance, then maximum power transfer happens at load resistance.

If we open the circuit across 'R' then



$V_{Th} = \left(\dfrac{36}{9}\right) \times 3 = 12$ volts

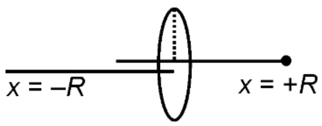
And $\quad r_0 = \dfrac{6 \times 3}{9} = 2\,\Omega$



So value of R should be 2 Ω and $I = 3$ A

$\therefore \quad F_{(\max)} = I^2R = 18$ watt

26. (2)

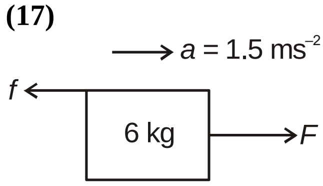


Alternately, we shall calculate the magnetic field by straight wire with current I and then find its line integral on circle.

$\therefore \quad \displaystyle\int B \cdot dl = \frac{\mu_0 I}{4\pi R} \cdot \frac{2.1}{\sqrt{2}} \cdot 2\pi R$

$\Rightarrow \quad \displaystyle\int B.dl = \frac{\mu_0 I}{\sqrt{2}}$

27. (17)



$F = 0.4 \times 2 \times 10 = 8$ N

$F - 8 = 6 \times 1.5$

$F = 17$ N

28. **(15)**

$$I = \frac{Bvl}{R};\ P = I^2 R.$$

$$l = \frac{Bvl}{R} = \frac{0.5 \times 2 \times 2}{6} = \frac{1}{3}\ \text{A}$$

$$P = l^2 R = \frac{1}{9} \times 6 = \frac{2}{3}\ \text{W}$$

29. **(2)**

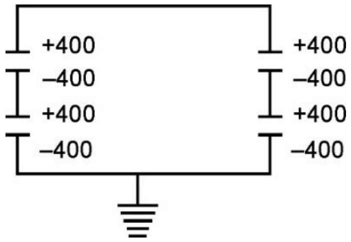
$p_x - p_y = p_1$

$$\frac{h}{\lambda_0} - \frac{h}{2\lambda_0} = \frac{h}{\lambda}$$

$\lambda = 2\lambda_0$

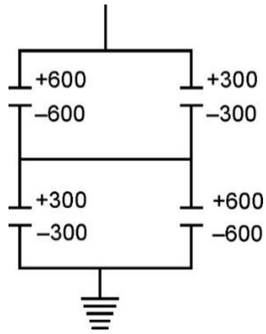
30. **(50)**

Before closing the switch, the charges on capacitor are as shown



$$q = 200 \times \left[\frac{6 \times 3}{6+3}\right] = \frac{200 \times 6 \times 3}{8} = +400\,\mu\text{C}$$

After closing the switch,



Hence, charge flown is 300 μC.

SECTION-II (CHEMISTRY)

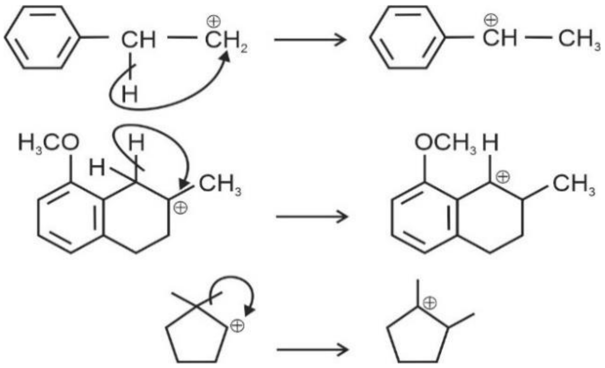
31. **(1)**

More the stable carbocation higher will be its reactivity towards S_N1. Thus, the order is b < d < a < c, where “c” and “a” are resonance stabilized. "d” would form 1º C^+ and least would be “b” since C^+ is not formed on carbon with bridge head.

32. **(2)**

Except glycine every amino acid is optically active. Thus among the given options we check which is not an essential amino acid *i.e.* Asparagine.

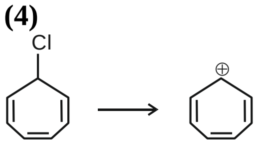
33. **(2)**



34. **(4)**

$CH_3 - \overset{\overset{\large O}{\|}}{C} - Cl$ is most reactive since it has Cl which is a good leaving group whereas $CH_3 - \overset{\overset{\large O}{\|}}{C} - NHCH_3$ is least reactive since it has –NH – CH_3 which is weakest leaving group among all.

35. **(4)**



On removal of Cl it attains aromaticity.

36. **(3)**

The correct order for –ve electron gain enthalpy for oxygen family is S > Se > Te > O.

37. **(1)**

Both statements are true as maleic acid is cis dicarboxylic acid which on heating can form anhydride whereas fumaric acid is trans isomer which restricts its anhydride formation.

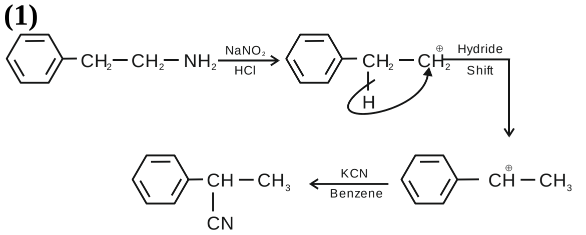
$$\underset{\text{Malonic acid}}{HOOC - CH_2 - COOH} \xrightarrow{\Delta} \underset{\text{Acetic acid}}{CH_3 - COOH} + CO_2$$

38. **(3)**

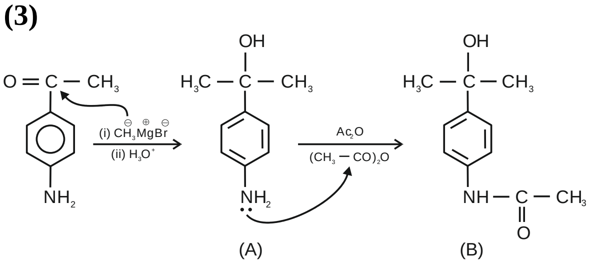
Hoffmann Bromamide Degradation reaction results in decrease of number of carbons in carbon chain.

$$R-\overset{\overset{\displaystyle O}{\|}}{C}-NH_2 + Br_2 + NaOH \longrightarrow R-NH_2 + Na_2CO_3$$

39. **(1)**



40. **(3)**



41. **(4)**

Gradual addition of ethylene diamine (en) affects the colour observed

$$[Ni(H_2O)_6]^{2+} + en \longrightarrow \underset{\text{Pale Blue}}{[Ni(H_2O)_4(en)]^{2+}} \xrightarrow{+en(aq)}$$

$$\underset{\text{Blue/purple}}{[Ni(H_2O)_2(en)_2]^{2+}} \xrightarrow{+en(aq)} \underset{\text{Violet}}{[Ni(en)_3]^{2+}}$$

42. **(1)**

K_2MnO_4 is paramagnetic whereas $KMnO_4$ is diamagnetic. K_2MnO_4 (manganate) has one unpaired electron but permanganate $KMnO_4$ is diamagnetic.

43. **(3)**

Lactose is a combination of β-D-Galactose and β-D-Glucose instead of α-D-Galactose.

44. **(4)**

$$\underset{(A)}{SO_3^{2-}} + H_2SO_4 \rightarrow SO_4^{2-} + H_2O + \underset{(B)}{SO_2}$$

$K_2Cr_2O_7 + H_2SO_4 + SO_2 \rightarrow K_2SO_4 + Cr_2(SO_4)_3 + H_2O$

45. **(4)**

$[Co(NH_3)_6]Cl_3$, d^6 configuration

NH_3 causes pairing in Co^{+3}.

46. **(3)**

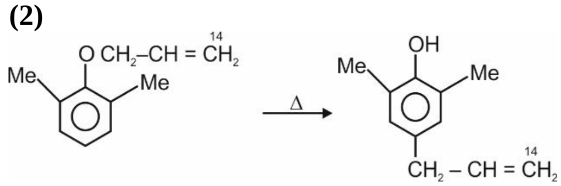
$[PtCl_6]^{2-}$ is formed which is octahedral.

$$Pt + H^+ + NO_3^- + Cl^- \longrightarrow \underset{(P)}{PtCl_6^{2-}} + NO + H_2O$$

47. **(2)**

$[CoF_6]^{3-}$ is paramagnetic.

48. **(2)**



(Claisen Rearrangement)

49. **(1)**

Gabriel phthalimide synthesis is best method to prepare aliphatic primary amine.

50. **(1)**

2,4,6-trinitrophenol is picric acid.

Picric acid give CO_2 with $NaHCO_3$.

51. **(5)**

Heat Capacity, Enthalpy, Entropy, Internal Energy, Volume are extensive properties.

52. **(4)**

$R = K[A]^m$

$$\frac{-dA}{dt} = K[A]^m$$

$1 = K[1]^m$

$9 = K[3]^m$

$25 = K[5]^m$

$m = 2$

$m^2 = 4$

53. **(36)**

$$E_n = -13.6 \times \frac{Z^2}{n^2} \qquad Z = 2$$

$$\frac{-13.6(2)^2}{(3)^2} : \frac{-13.6(2)^2}{(2)^2} = \frac{1}{9} \times \frac{4}{1} = \frac{4}{9}$$

$a \times b = 4 \times 9 = 36$

54. **(6)**

$$\begin{array}{lll} Ca(NO_3)_2 & \rightleftharpoons Ca^{2+} & + 2NO_3^- \\ 1 & 0 & 0 \\ 1-\alpha & \alpha & 2\alpha \end{array}$$

$i = (1 + 2\alpha)$

$$\begin{array}{lll} K_4[Fe(CN)_6] & \rightleftharpoons 4K^+ & + [Fe(CN)_6]^{4-} \\ 1 & 0 & 0 \\ 1-\alpha & 4\alpha & \alpha \end{array}$$

$i = (1 + 4\alpha)$

Total $i = (1 + 2\alpha) + (1 + 4\alpha)$

$= (1 + 2 \times 0.6) + (1 + 4 \times 0.7)$

$= 2.2 + 3.8$

$= 6.0$

55. **(12)**

800 ml – 0.5 M H_2SO_4

$0.5 \times 2 = 1$ N

200 ml – 2 M HCl

$2 \times 1 = 2$ N HCl

$(1 \times 800) + (2 \times 200) = N_3(800 + 200)$

$800 + 400 = N_3 \times 1000$

$1.2 = N_3$

x = 1.2

10x = 1.2 × 10 = 12

56. **(2)**

Before electrolysis,

$$Z = \frac{\theta}{F} = \frac{i \times t}{96500}$$

$$= \frac{2.681 \times 2 \times 60 \times 60}{96500}$$

= 0.20003 equivalents

= 200.03 milliequivalents

N = 2.1 × 2 = 4.2 N

= 4200 milliequivalents in 1 L

After electrolysis,

= 4200 – 200

= 4000 milliequivalents

= 4 equivalents

$$N = \frac{4}{1\,(L)} = 4\text{ N}$$

M × 2 = 4

M = 2 M

57. **(5)**

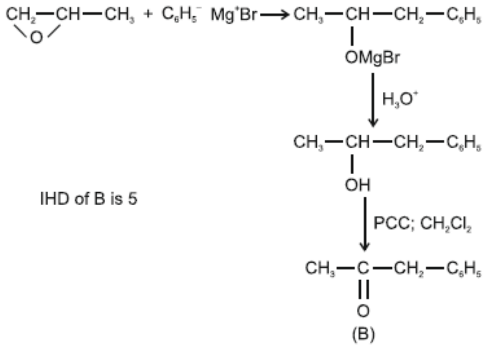
Fe^{3+} has d^5 configuration and oxalate ion is not a very strong ligand and hence cannot cause pairing.

58. **(0)**

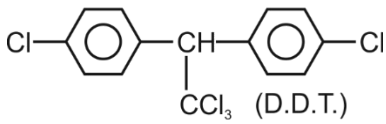
$Co_2(CO)_8$ – Oxidation number of Co is zero.

59. **(5)**

Reaction of Grignard reagent with cyclic ether



60. **(5)**



(D.D.T.)

SECTION-III (MATHEMATICS)

61. **(2)**

$|A^{2005} - 6A^{2004}| \quad = |A|^{2004}\,|A - 6I|$

$$= 2^{2004}\begin{vmatrix} 0 & 11 \\ 2 & -2 \end{vmatrix}$$

$= (-22)2^{2004}$

$= -2 \times 11 \times 2^{2004}$

$= (-11)(2)^{2005}$

62. **(4)**

$$\sin^{-1}\left(\cos\left(\cos^{-1}(\cos x) + \sin^{-1}(\sin x)\right)\right)$$

$$x \in \left(\frac{\pi}{2}, \pi\right)$$

$$= \sin^{-1}\left[\cos(x + \pi - x)\right]$$

$$= \sin^{-1}(-1) = -\frac{\pi}{2}$$

63. **(3)**

The value of $\{x\} = x - 2$, as $x \to 2 + h$

$$\lim_{x \to 2^+} \frac{(x-2)\sin(x-2)}{(x-2)^2} = \lim_{h \to 0} \frac{h \sin h}{h^2} = 1$$

64. **(3)**

Player *A* can win if *A* throws (1, 6) or (6, 1) and *B* throws ((1, 1,), (2, 2), (3, 3), (4, 4), (5, 5) or (6, 6)). Thus the number of ways is 12.

Similarly the number of ways in which *B* can win is 12.

Total number of ways in which either *A* wins or *B* wins = 24.

Thus the number of ways in which none of the two wins = $6^4 - 24$.

$\therefore$ The required probability = $\dfrac{6^4 - 24}{6^4} = \dfrac{53}{54}$.

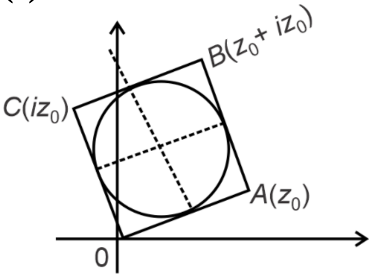
65. **(3)**

Put $\sqrt{x}=t \Rightarrow \frac{1}{2\sqrt{x}}\ dx=dt$

$\Rightarrow \int e^t\left(t^2+t\right)2dt=2e^t\left(t^2-t+1\right)+c$

$=2e^{\sqrt{x}}\left(x-\sqrt{x}+1\right)+c$

66. **(2)**



Clearly mid-point of OB is centre of the circle and radius is equal $\frac{|z_0|}{2}$

$\Rightarrow$ Required equation is $\left|z-\frac{z_0}{2}(1+i)\right|=\frac{|z_0|}{2}$

67. **(1)**

We have,

$\tan\left(\theta+\frac{\pi}{4}\right)=3\tan 3\theta$

$\Rightarrow \frac{1+\tan\theta}{1-\tan\theta}=3\left(\frac{3\tan\theta-\tan^3\theta}{1-3\tan^2\theta}\right)$

$\Rightarrow (1+\tan\theta)(1-3\tan^2\theta)=(1-\tan\theta)$
$(9\tan\theta-3\tan^3\theta)$

$\Rightarrow 1+\tan\theta-3\tan^2\theta-3\tan^3\theta$
$=9\tan\theta-9\tan^2\theta-3\tan^3\theta+3\tan^4\theta$

$\Rightarrow 3\tan^4\theta-6\tan^2\theta+8\tan\theta-1=0$

So, $\tan\alpha+\tan\beta+\tan\gamma+\tan\delta=0$

68. **(1)**

We observe that product of roots = $2b$ = even number. Since its given equation has prime roots only and 2 is only even prime number, hence 2 must be one root of the equation and consequently $4+2a+2b=0$

$\Rightarrow a+b=-2$

69. **(2)**

$(1+y^2)\frac{dx}{dy}+x=2e^{\tan^{-1}y}$

$\Rightarrow \frac{dx}{dy}+\frac{1}{1+y^2}x=\frac{2e^{\tan^{-1}y}}{1+y^2}$

I.F. $= e^{\int\frac{dy}{1+y^2}}=e^{\tan^{-1}y}$

$\Rightarrow xe^{\tan^{-1}y}=2\int e^{\tan^{-1}y}\cdot\frac{e^{\tan^{-1}y}}{1+y^2}dy$

$\Rightarrow xe^{\tan^{-1}y}=e^{2\tan^{-1}y}+k$

70. **(3)**

Area $=\int_0^a(\sqrt{a}-\sqrt{x})^2dx$

$=\int_{x=0}^{a}(a+x-2\sqrt{a}\sqrt{x})\,dx$

$=a^2+\frac{1}{2}a^2-\frac{2\sqrt{a}\,a^{3/2}}{\frac{3}{2}}=\frac{a^2}{6}$

71. **(4)**

Rearranging gives : $x^{2020}+(y-1)^2=1$

Clearly x can take values 0,1, –1 only otherwise $(y-1)^2$ becomes negative putting these values of x we get.

$R=\{(0,0),(1,1),(-1,1),(0,2)\}$

72. **(2)**

$x\cos\left(\frac{\alpha+\beta}{2}\right)+\frac{y}{b}\sin\left(\frac{\alpha+\beta}{2}\right)=\cos\left(\frac{\alpha-\beta}{2}\right)\left(\frac{\sqrt{1-b^2}}{b}>0\right)$

$\Rightarrow\sqrt{1-b^2}\cos\left(\frac{7\pi}{6}\right)=\cos\left(\frac{3\pi}{4}\right)$

$\Rightarrow 1-b^2=\frac{2}{3}$

$\Rightarrow b=\frac{1}{\sqrt{3}}$

73. **(3)**

$g(x)$ is discontinuous at $x=0$ and 1

74. **(3)**

Put $x=\frac{1}{t}$

$\therefore I=\frac{\pi}{4}\int_0^\infty\frac{t^2+at+1}{1+t^4}dt$

$I=\frac{\pi^2}{16}\left(a+2\sqrt{2}\right)$

75. **(3)**

$$f'(x)=2-\frac{1}{1+x^2}-\frac{1}{\sqrt{1+x^2}}$$

Here $f'(x)>0,\ \forall x\in R$

76. **(3)**

$$e_H=\sqrt{1+\frac{9}{16}}=\frac{5}{4}$$

$$\because \frac{1}{e_C^2}+\frac{1}{e_H^2}=1\Rightarrow e_C=\frac{5}{3}$$

$F_1(5, 0)$, $F_2(-5, 0)$, $F_3(0, 5)$ and $F_4(0, -5)$

Area of quadrilateral = 50 square units

77. **(4)**

$f(x)$ is continuous at $x = 0$ so

$f(0) = f(0^+) = f(0^-)$

$$\Rightarrow \lim_{x\to 0}\frac{(\sin x)\left(e^x-1\right)}{x\left(\log(1+x)\right)}=k$$

$$\Rightarrow \lim_{x\to 0}\frac{\sin x}{x}\times\frac{e^x-1}{x}\times\frac{x}{\ln(1+x)}=k$$

$k = 1$

78. **(3)**

$$y(x)=\begin{cases} x, & \text{if } x\ge 0\\ \dfrac{x}{3}, & \text{if } x<0\end{cases}$$ is non-differentiable at

$x = 0$.

79. **(1)**

Coefficient of x in $\Delta(x) = \Delta'(0) = 0$

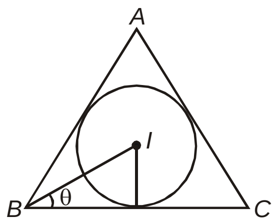
$\Rightarrow k = 0$

$\Rightarrow \{k\} = 0$

80. **(3)**

Slope of IB, $m_1 = \dfrac{3-2}{2-1} = 1$

Slope of BC, $m_2 = 2+\sqrt{3}$



$\therefore\ \angle IBC = \theta$

$$=\tan^{-1}\left|\frac{m_2-m_1}{1+m_2m_1}\right|$$

$$=\tan^{-1}\left|\frac{2+\sqrt{3}-1}{1+\left(2+\sqrt{3}\right)}\right|$$

$$=\tan^{-1}\left|\frac{1+\sqrt{3}}{3+\sqrt{3}}\right|$$

$$=\tan^{-1}\left|\frac{1+\sqrt{3}}{\sqrt{3}(1+\sqrt{3})}\right|$$

$$=\tan^{-1}\left|\frac{1}{\sqrt{3}}\right|$$

$= 30°$

$\therefore\ \angle ABC = 2\theta = 60°$

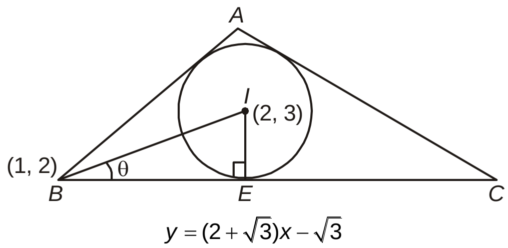
Second method:

$$\sin\theta=\frac{IE}{BI}$$

$$IE=\frac{\sqrt{3}+1}{2\times\sqrt{\sqrt{3}+2}}$$

$$BI=\sqrt{2}$$

$$\sin\theta=\frac{\sqrt{3}+1}{2\sqrt{2}\times\sqrt{\sqrt{3}+2}}=\frac{\sqrt{3}+1}{2\sqrt{4+2\sqrt{3}}}=\frac{1}{2}$$



$\theta = 30°$, $2\theta = 60°$

81. **(4)**

$A^TA = I$

As $A^T = A$

$\Rightarrow A^2 = I$

$\Rightarrow a^2 + b^2 + c^2 = 1$ and $ab + bc + ca = 0$

As $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$

$\Rightarrow (a + b + c) = \pm 1$

$\Rightarrow a + b + c = 1$

Now, $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

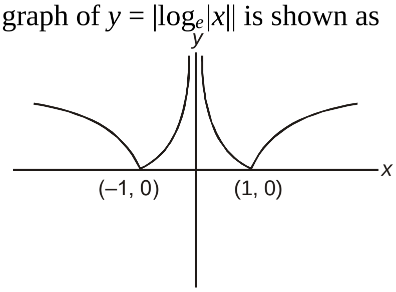
$\Rightarrow a^3 + b^3 + c^3 - 3 = (a + b + c)$

$\Rightarrow a^3 + b^3 + c^3 = 3 + 1$

$\Rightarrow a^3 + b^3 + c^3 = 4$

82. **(2)**

The graph of $y = |\log_e|x||$ is shown as



Clearly the function is continuous at $x = \pm 1$, but non-differentiable at $x = \pm 1$.

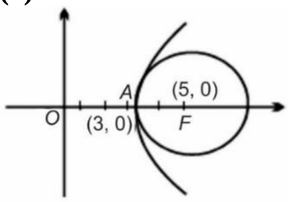
83. **(1)**

Series is

$1 + 1, (1 + n) + n, (1 + 2n) + n^2, (1 + 3n) + n^3, ...,$

$1 + 2n + n^2 = (n + 1)^2$ is a perfect square for all $n \in N$

84. **(2)**



$$e^2 = 1 + \frac{16}{9} = \frac{25}{9} \Rightarrow e = \frac{5}{3}$$

$\therefore$ focus = (5, 0)

Use reflection property to conclude that circle cannot touch at two points.

It can only be tangent at the vertex $r = 5 - 3 = 2$

85. **(15)**

$$\lim_{n\to\infty}\left(\frac{1}{a_1}+\frac{1}{a_2}+...+\frac{1}{a_n}\right)$$

$$=\left(1+\frac{1}{3}+\frac{1}{3^2}+....\right)\left(1+\frac{1}{5}+...\right)$$

$$=\frac{15}{8}$$

86. **(6)**

$x(x^2 + 3y^2)dx + y(y^2 + 3x^2)\,dy = 0$

$$\Rightarrow \frac{4x^3dx+4y^3dy+12xy^2dx+12x^2ydy}{x^4+y^4+6x^2y^2}=0$$

$\Rightarrow \ln(x^4 + y^4 + 6x^2y^2) = \ln c$

$\Rightarrow x^4 + y^4 + 6x^2y^2 = c$

87. **(1)**

For 301, 302, ..., 400

$\bar{x} = 350.5$

$$\therefore V_B = \frac{\sum\left(x_i-\bar{x}\right)^2}{N} = \frac{49.5^2+50.5^2+...}{100}$$

Also for 201, 202, ..., 300

$\bar{x} = 250.5$

$$V_A = \frac{49.5^2+50.5^2+...}{100}$$

$\therefore V_A = V_B$

88. **(1)**

From given relation

$$\frac{\pi}{2} - \cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \tan^{-1} a$$

$$\Rightarrow \frac{\pi}{2} - \cos^{-1}\left(\frac{1-\frac{y^2}{x^2}}{1+\frac{y^2}{x^2}}\right) = \tan^{-1} a$$

$$\Rightarrow \frac{\pi}{2} - \left[2\tan^{-1}\left(\frac{y}{x}\right)\right] = \tan^{-1} a$$

$$\Rightarrow \frac{\pi}{2} - \tan^{-1} a = 2\tan^{-1}\left(\frac{y}{x}\right)$$

$$\Rightarrow 2\tan^{-1}\left(\frac{y}{x}\right) = \cot^{-1} a$$

Differentiate both sides with respect to x, and we get

$$\Rightarrow \frac{2}{1+\frac{y^2}{x^2}}\left(\frac{xdy-ydx}{x^2}\right)=0$$

$$\Rightarrow \frac{dy}{dx}=\frac{y}{x}]$$

89. **(1)**

$$\lim_{x\to 1}\frac{f(1)g(x)-f(1)-g(1)f(x)+g(1)}{f(1)g(x)-f(x)g(1)},$$

form : $\frac{0}{0}$

$$\lim_{x\to 1}\frac{f(1)g'(x)-g(1)f'(x)}{f(1)g'(x)-f'(x)g(1)}=1$$

90. **(5)**

$Y = x^3 - 2x^2 + x + 5$

$\Rightarrow f(0) = 5$