JEE MAIN (2023-24) Mock Test Series

Paper - 07

DURATION : 180 Minutes

M. MARKS : 300

ANSWER KEY

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(4)	31.	(2)	61.	(1)
2.	(2)	32.	(2)	62.	(2)
3.	(3)	33.	(3)	63.	(2)
4.	(2)	34.	(1)	64.	(1)
5.	(2)	35.	(1)	65.	(1)
6.	(3)	36.	(2)	66.	(4)
7.	(1)	37.	(2)	67.	(3)
8.	(2)	38.	(4)	68.	(4)
9.	(2)	39.	(3)	69.	(3)
10.	(3)	40.	(4)	70.	(1)
11.	(2)	41.	(2)	71.	(2)
12.	(2)	42.	(4)	72.	(1)
13.	(2)	43.	(2)	73.	(1)
14.	(2)	44.	(2)	74.	(4)
15.	(3)	45.	(4)	75.	(2)
16.	(3)	46.	(3)	76.	(3)
17.	(3)	47.	(3)	77.	(3)
18.	(1)	48.	(1)	78.	(2)
19.	(2)	49.	(2)	79.	(2)
20.	(4)	50.	(2)	80.	(2)
21.	(12)	51.	(366)	81.	(15)
22.	(3)	52.	(3)	82.	(330)
23.	(18)	53.	(2)	83.	(0)
24.	(7)	54.	(6)	84.	(18)
25.	(2)	55.	(9)	85.	(87)
26.	(3)	56.	(4)	86.	(1)
27.	(17)	57.	(3)	87.	(6)
28.	(5)	58.	(1)	88.	(5)
29.	(4)	59.	(6)	89.	(5)
30.	(8)	60.	(4)	90.	(1)

SECTION-I (PHYSICS)

1. **(4)**

$$\frac{\mu_0 I_1}{2\pi x}=\frac{\mu_0 I_2}{2\pi y}$$

$$\Rightarrow y=\left(\frac{I_2}{I_1}\right)x$$

2. **(2)**

Magnetic force should balance the centrifugal force.

$$\frac{q_0}{2\pi R}dl\,\omega RB>\frac{m}{2\pi R}dl\,\omega^2 R$$

$$\Rightarrow\ q_0\,\omega RB>m\omega^2 R$$

$$\Rightarrow\ \frac{q_0 B}{m}>\omega$$

3. **(3)**

$$\frac{d^2\theta}{dt^2}=-\omega^2\theta \text{ for angular SHM}$$

$$\tau=k\frac{L}{6}\theta\cdot\frac{L}{6}+k\frac{L}{3}\theta\cdot\frac{L}{3}$$

$$\Rightarrow\left(\frac{2mL^2}{3}+mL^2\right)\cdot\frac{d^2\theta}{dt^2}=-\,kL^2\theta\left(\frac{1}{36}+\frac{1}{9}\right)$$

$$\Rightarrow\frac{5mL^2}{3}\cdot\frac{d^2\theta}{dt^2}=-\frac{5kL^2}{36}\cdot\theta$$

$$\Rightarrow\frac{d^2\theta}{dt^2}=-\frac{k}{12m}\cdot\theta$$

$$\Rightarrow\omega=\sqrt{\frac{k}{12m}}$$

$$f=\frac{1}{2\pi}\sqrt{\frac{k}{12m}}$$

$$=\frac{1}{4\pi}\sqrt{\frac{k}{3m}}$$

4. **(2)**

$$\frac{E_S}{E_P}=\frac{N_S}{N_P}$$

$$\frac{E_S}{E_P}=\frac{N_S}{N_P}$$

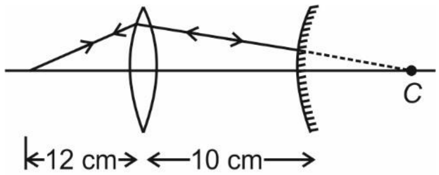
$$\frac{E_S}{20}=\frac{5000}{500}$$

$$E_S=200\text{ V}$$

Frequency remains same = 50 Hz

5. **(2)**

For lens $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$. And on mirror the incident ray will be normal to the surface.



Using lens formula

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$u=-12$ cm, $f=10$ cm

$\Rightarrow v=60$ cm

$\Rightarrow$ Distance of C from mirror is 50 cm

$\Rightarrow R=50$ cm

$\Rightarrow f=25$ cm

6. **(3)**

$$\sum\vec{F}=0$$

$$(M+m)g=(50+25)g=\left(T+\frac{T}{2}\right)$$

$$\Rightarrow\ T=500\text{ N}$$

7. **(1)**

Pressure $=\dfrac{\sigma^2}{2\varepsilon_0}$

Considering any point on hemispherical shell

Pressure $=\dfrac{\sigma^2}{2\varepsilon_0}$

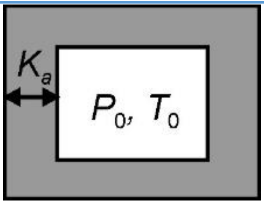
Hence force required $=P\propto A$

$=\dfrac{\sigma^2}{2\varepsilon_0}\times\pi R^2$ (cross-sectional area)

$=\dfrac{\sigma^2\pi R^2}{2\varepsilon_0}$

8. **(2)**

$$\frac{dQ}{dt}=c\frac{dT}{dt}$$



$$\frac{dQ}{dt} = c\frac{dT}{dt} \qquad \text{...(i)}$$

$$\frac{dQ}{dt} = \frac{kA}{t_0}\left(T - \frac{T_0}{2}\right) \qquad \text{...(ii)}$$

From equation (i) and equation (ii)

$$\frac{dQ}{dt} = C\frac{dT}{dt} = -\frac{kA}{t_0}\left(T - \frac{T_0}{2}\right)$$

$$\Rightarrow \int_{T_0}^{T} \frac{C}{T - \frac{T_0}{2}} dT = \int_0^t -\frac{kA}{t_0} dt$$

$$\Rightarrow \quad T = \frac{T_0}{2}\left(1 + e^{\frac{-kA}{Ct_0}t}\right)$$

9. **(2)**

$$P = \frac{V_0 I_0}{2}\cos\phi$$

$$P = \frac{V_0 I_0}{2}\cos\phi$$

$$\frac{V_0 I_0}{4} = \frac{V_0 I_0}{2}\cos\phi$$

$$\cos\phi = \frac{1}{2}$$

$$\phi = \frac{\pi}{3}$$

10. **(3)**

$$T_{\max} = mg + \frac{mv_{\max}^2}{l}$$

$$V_{\max}^2 = 4gl + 2gl = 6gl$$

$$T_{\max} = mg + 6mg = 7mg$$

11. **(2)**

Angular fringe width = 3°

$$3 \times \frac{\pi}{180} = \frac{\lambda}{d}$$

12. **(2)**

$$f = \mu_s N$$

$$\Rightarrow T\cos 37 = \frac{4}{7}N \qquad \ldots(1)$$

$$N = 20 - T\sin 37 \qquad \ldots(2)$$

From (1) and (2)

m_1 = 2 kg

13. **(2)**

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Point $A(-2f, 2f)$

$$\Rightarrow 2f = 40$$

$$\Rightarrow f = 20\,\text{cm}$$

14. **(2)**

$g(x, t) = f((x - v(t - t_0)), t)$

15. **(3)**

The ray SM after reflection undergoes a phase change of π, for maxima at P, path difference between S & S' = $\frac{\lambda}{2}$

(S' is virtual source producing back PM, symmetric to S)

Comparing with YDSE, $d = 4x$, $D = 600x$

Path difference = $\frac{x.d}{D}$

$$\frac{\lambda}{2} = \frac{x\,(4x)}{600\,x} \quad \Rightarrow \quad x = 75\,\lambda$$

16. **(3)**

Power, P = Area under E_λ versus λ

$$\frac{P_1}{P_2} = 4 = \left(\frac{T_1}{T_2}\right)^4 \Rightarrow T_1 = T_2\,4^{1/4} = T_2\sqrt{2}$$

17. **(3)**

If mirror rotates then angle rotated by reflected ray is twice of mirror rotation

Total angle rotated by reflected ray

= 20° + 10° = 30° in anticlockwise

18. **(1)**

$\Delta Q = \Delta U + \Delta W$

Heat energy exchange,

$$\Delta Q = \int_{T_0}^{\eta T_0} nc\, dT = \int_{T_0}^{\eta T_0} n\frac{\alpha}{T}\, dT$$

$= n\,\alpha\,\ln\eta$

Change in internal energy, $\Delta U = nC_v \Delta T$

$$= n\frac{R}{r-1}(\eta T_0 - T_0)$$

$$= \frac{nRT_0}{r-1}(\eta - 1)$$

From first law of thermodynamics

$$W = \Delta Q - \Delta U = n\,\alpha\,\ln\eta - \frac{nRT_0}{r-1}(\eta-1)$$

For one mole, $n = 1$

$$W = \alpha\ln\eta - \frac{RT_0}{r-1}(\eta - 1)$$

19. **(2)**

$Q = [(7.835 \times 231) + (7.07 \times 4) - (7.8 \times 235)]$

$= 5.165$ MeV

20. **(4)**

For maximum intensity, path difference = $n\lambda$

Path difference = 3 × 8.5 cm = 25.5 cm

21. **(12)**

$$F_B = i\,dl \times B$$

$$F_{\text{net}} = \frac{\mu_0 I_0 \times 2}{2\pi a} Ia - \frac{\mu_0 I_0 \times 2}{2\pi 3a} Ia$$

$$\therefore \quad F_{\text{net}} = \frac{\mu_0 I_0}{2\pi a} 2Ia\,(1 - 1/3)$$

$$\Rightarrow \quad F_{\text{net}} = \frac{\mu_0 I_0 I}{\pi} \times \frac{2}{3} = 4 \times 10^{-7} \times \frac{2}{3} \times 5 \times \frac{9}{10}$$

$= 1.2$ μN

22. **(3)**

$$T = t_{A\to 0} + t_{0\to B}$$

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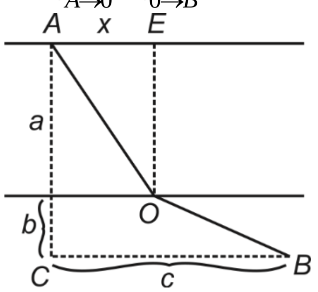


$$T = \frac{\sqrt{a^2 + x^2}}{v_1} + \frac{\sqrt{b^2 + (c-x)^2}}{v_2}$$

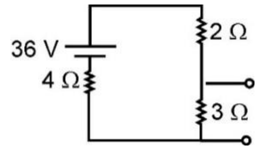
for minimum T, $\frac{dT}{dx} = 0$

$$\Rightarrow \quad \frac{\dfrac{x}{\sqrt{a^2+x^2}}}{\dfrac{(c-x)}{\sqrt{b^2+(c-x)^2}}} = \frac{v_1}{v_2} \Rightarrow \frac{\sin\alpha_1}{\sin\alpha_2} = \frac{6}{2} = 3$$

23. **(18)**

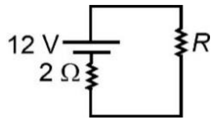
If load resistance matches with source resistance, then maximum power transfer happens at load resistance.

If we open the circuit across 'R' then



$$V_{Th} = \left(\frac{36}{9}\right) \times 3 = 12 \text{ volts}$$

$$\text{And} \quad r_0 = \frac{6 \times 3}{9} = 2\,\Omega$$



So value of R should be 2 Ω and I = 3 A

$\therefore \quad F_{(\max)} = I^2R = 18$ watt

24. **(7)**

$W_f = |k_f - k_i|$

$L_i = L_f$

$$\Rightarrow v_f = \frac{5}{7}v_i$$

$$\Rightarrow k_f = \frac{5}{7}k$$

$$\Delta KE = k - \frac{5}{7}k = \frac{2}{7}k$$

$$\Rightarrow \quad |W_f| = \frac{20}{7}\text{ J}$$

25. **(2)**

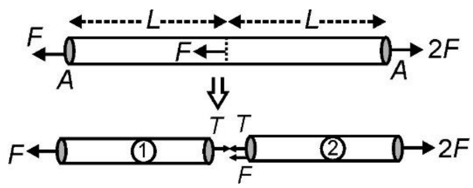
$$I = \frac{Bvl}{R};\; P = I^2R.$$

$$l = \frac{Bvl}{R} = \frac{0.5 \times 2 \times 2}{6} = \frac{1}{3}\text{ A}$$

$$P = l^2R = \frac{1}{9} \times 6 = \frac{2}{3}\text{ W}$$

26. **(3)**

$$\Delta l = \frac{\sum F_i L_i}{AY}$$



Stress in part (1) is $\frac{F}{A}$ and stress in part (2) is $\frac{2F}{A}$

$$\Delta l = \frac{2FL}{AY} + \frac{FL}{AY}$$

$$= \frac{3FL}{AY}$$

27. (17)

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$(n_A - 1)\frac{2}{R_A} = (n_B - 1)\frac{2}{R_B}$$

$$n_B = 1.7$$

28. (5)

$$mg - N_A = ma_{\text{cm}}$$

$$mg - N_A = ma_{\text{cm}}$$

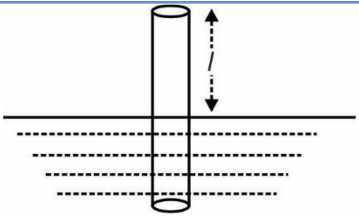
$$a_{\text{cm}} = \frac{\alpha l}{2}$$

Also $\tau_A = I_A \alpha \;\Rightarrow\; mg\frac{l}{2} = I\alpha$

$$\Rightarrow \alpha = \frac{3g}{2l} \Rightarrow N_A = \frac{mg}{4} = 5\text{ N}$$

29. (4)

Fundamental frequency, $F = \frac{c}{4x}$



Fundamental frequency, $f = \frac{c}{4l}$

$$f = \frac{c}{4} l^{-1}$$

$$\frac{df}{dt} = \frac{d}{dt}\left[\frac{c}{4} l^{-1}\right] = \frac{c}{4}(-1)\, l^{-2} \frac{dl}{dt}$$

$$= -\frac{c}{4l^2}\frac{dl}{dt} = \frac{-c}{4l^2}(-v)$$

$$\frac{df}{dt} = \frac{cv}{4l^2} = \frac{cv}{x\,l^2} \quad \text{where, } x = 4$$

30. (8)

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

For 1st reflection

$$u_1 = -15,\; f = -10$$

$$\Rightarrow v_1 = -30\text{ cm}$$

For 2nd reflection

$$u_2 = -(40-30) = -10\text{ cm}$$

$$\Rightarrow v_2 = +10\text{ cm}$$

For 3rd reflection

$$u_3 = -(40+10) = -50\text{ cm}, \quad f = -10\text{ cm}$$

$$\Rightarrow v_3 = -12.5\text{ cm}$$

SECTION-II (CHEMISTRY)

31. (2)

	A $\longrightarrow$	2B	+ C
t = 0	P_0	0	0
t = t	$P_0 - x$	2x	x
t = ∞	≈ 0	$2P_0$	P_0

From question: $P_\infty = 2P_0 + P_0$

$$\Rightarrow P_0 = \frac{P_\infty}{3}$$

and $P = (P_0 - x) + 2x + x$

$$\Rightarrow x = \frac{P - P_0}{2} = \frac{3P - P_\infty}{6}$$

Now,

$$K = \frac{1}{t}\cdot \ln\frac{P_A^\circ}{P_A} = \frac{1}{t}\cdot \ln\frac{P_0}{P_0 - x}$$

$$= \frac{1}{t}\cdot \ln\frac{\frac{P_\infty}{3}}{\frac{P_\infty}{3} - \frac{3P - P_\infty}{6}} = \frac{1}{t}\ln\frac{\frac{P_\infty}{3}}{\frac{P_\infty}{3} - \frac{3P}{6} + \frac{P_\infty}{6}}$$

$$= \frac{1}{t}\ln\frac{2P_\infty}{3(P_\infty - 3P)}$$

32. (2)

q =ΔU-w and from the question, q = | w | = w or 2q = ΔU

$$2\cdot n\cdot C_m\cdot \Delta T = n\cdot C_{v,m}\cdot \Delta T \Rightarrow C_m = \frac{C_{v,m}}{2}$$

Or $= \frac{1}{2}\times\frac{R}{1.5-1} = R$

33. (3)

$$N_2O_4 \Rightarrow 2NO_2$$

Equilibrium $1-\alpha$ $\quad 2\alpha$

$$K_p = \frac{P^2_{NO_2}}{P_{N_2O_4}} = \frac{\left(\frac{2\alpha}{1+\alpha}\cdot P\right)^2}{\left(\frac{1-\alpha}{1+\alpha}\cdot P\right)} = \frac{4\alpha^2\cdot P}{1-\alpha^2}, \alpha = \left(\frac{Kp}{Kp+4P}\right)^{\frac{1}{2}}$$

$$\therefore \alpha = \sqrt{\frac{Kp/4p}{1+Kp/4p}}$$

34. (1)

$$\frac{\overline{v_2}}{\overline{v_1}} = \frac{z_2^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)_1}{z_1^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)_2}$$

$$\Rightarrow \frac{\overline{v_2}}{2.5\times10^5} = \frac{3^2\left(\frac{1}{3^2}-\frac{1}{5^2}\right)}{4^2\left(\frac{1}{2^2}-\frac{1}{3^2}\right)}$$

$$\therefore \overline{v_2} = 7.2\times10^4\ cm^{-1}$$

35. (1)

$$\Delta T_{f(\text{theo})} = K_f\cdot m = 1.72\times\frac{20/172}{50/1000} = 4\ K$$

$$\text{Now, } i = \frac{\Delta T_f\ (\exp)}{\Delta T_f\ (\text{theo})} = \frac{2}{4} = 0.5$$

36. (2)

$$\Delta G° = \Delta H° - T.\Delta S° = -RT.\ln K_P^o$$

$$\text{Or} \quad 18-300\times\frac{30}{1000} = -\frac{2}{1000}\times300\times\ln K_P^o$$

$$\text{Or,} \quad \ln K_P^o = -15 \Rightarrow K_P^o = e^{-15}$$

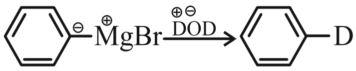
37. (2)

$$\frac{r_{H_2}}{3} = \frac{r_{NH_3}}{2}$$

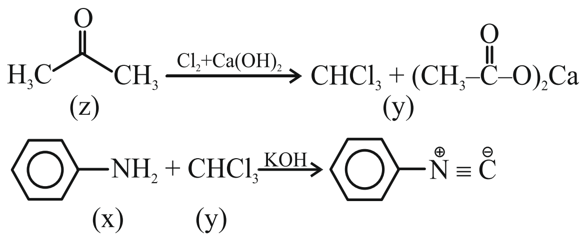
$$\Rightarrow \quad r_{H_2} = \frac{3}{2}\times\frac{10^{-3}}{17}\times10^3\ mol\ hr^{-1}$$

$$= \frac{3}{34}\times\frac{2}{10^3}Kg\ hr^{-1} = 1.76\times10^{-4}\ Kg\ hr^{-1}$$

38. (4)



39. (3)



40. (4)

–I power of – F is maximum in

41. (2)

C > Si > Ge > Pb > Sn (correct order of M.P. and $\Delta_i H_1$)

42. (4)

$\overset{\oplus}{Ga} < \overset{\oplus}{In} < \overset{\oplus}{Tl}$ (correct order of stablity)

$SiF_4 > SiCl_4 > SiBr_4 > SiI_4$

(correct order of stablity)

43. (2)

Bohr's theory is applicable for unelectronic species only.

Li+ has two electrons.

Bohr's theory could not explain the splitting of spectral lines in the presence of external magnetic field (Zeeman effect)

Statement I - false

Statement II - true

44. (2)

$$XeF_4 + SbF_5 \rightarrow \underset{\left(\substack{sp^3d,\text{bent}\\ -T-\text{Shape}}\right)}{XeF_3^{\oplus}}\ \underset{\left(\substack{sp^3d^2\\ \text{octahederal}}\right)}{SbF_6^{-}}$$

45. (4)

(III) is largest as it is complete single bond.

(IV) is shortest as it is complete triple bond.

Bond length of (I) increases because of hyperconjugation.

46. (3)

The correct order of bond angles (smallest first) in H_2S, NH_3, BF_3 and SiI_4 is

Species	Lp	Bp	VSEPR	Bond angle

H_2S	2	2	lp-lp lp-bp	92°
NH_3	1	3	Bp-bp lp-bp bp-bp	107°
BF_3	0	3	Bp-bp	120°
SiH_4	0	4	bp-bp	109°28°

Hence, bond angle $H_2S < NH_3 < SiH_4 < BF_3$

47. **(3)**

Chlorine being the group 17 clement has maximum electronegativity. 'N' has zero electron affinity because extra stability is associated with exactly half-filled orbitals. Sulphur has more electron affinity than 'O' because the effect of small size of O atom is more than offset by the repulsion of electrons already present in 2p-orbirlas of O atom.

48. **(1)**

Element X can lose its first two outermost electrons easily. It is most likely bivalent. By similar reasoning element Y is tetravalent. Therefore, the compound may be X_2Y.

49. **(2)**

The correct order of boiling point

$PH_3 < AsH_3 < NH_3 < SbH_3$

50. **(2)**

$$S = \frac{K \times 1000}{\lambda^{o}_{AgCl}} = \frac{2.3 \times 10^{-6} \times 1000}{(61.9 + 76.3)}$$

$$= 1.66 \times 10^{-5}\ M$$

51. **(366)**

In first case

$\Delta Tb = Kb \times m$

$$0.17 = 1.7 \times \frac{1.22}{x \times 100 \times 10^{-3}}$$

x = 122

In 2nd case

$$0.13 = 2.6 \times \frac{1.22}{y \times 100 \times 10^{-3}}$$

y = 244

In 2nd case it exist as dimer.

52. **(3)**

Final excited state = 5th orbit

As only two wavelengths are longer than absorbed radiation initial excited state = 3rd orbit

53. **(2)**

$$A^{n+} + \overset{+7}{Mn}O_4^- \rightarrow \overset{+5}{A}O_3^- + Mn^{2+}$$

Number of lost electrons = Number of Gained electrons

$5 \times 1.5 \times 10^{-3} = 2.5 \times 10^{-3} \times (5–n)$

$\therefore\ (5–n) = \dfrac{7.5}{2.5} = 3$

n = 2

54. **(6)**

$MnO_4^{\ominus}, Cr_2O_7^{2-}, CrO_4^{2-}$

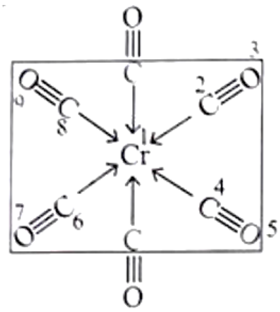
(They have $3d^0$ configuration yet they are coloured due to charge transfer theory)

$Cu^{2+}(3d^9), Fe^{2+}(3d^6), Fe^{3+}(3d^5)$, all are coloured dur to the presence of unpaired electrons.

Colourless ions are:

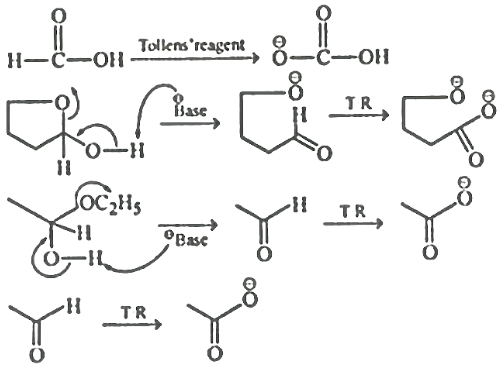
$Sc^{3+}\left(3d^0\right), Ti^{4+}\left(3d^0\right), Zn^{2+}\left(3d^{10}\right)$

55. **(9)**



9 atoms (as marked) are present in same plane.

56. **(4)**



57. (3)

$\lambda = \lambda_1 + \lambda_2$

$\lambda = \dfrac{0.693}{20} = \lambda_1 + \lambda_2$

$\dfrac{\lambda_1}{\lambda_2} = \dfrac{4}{96}$

$\Rightarrow \lambda_2 = 24\,\lambda_1$

$25\,\lambda_1 = \dfrac{0.693}{20}$

$\lambda_1 = \dfrac{0.693}{20 \times 25}$

$\lambda_1 = 1.386 \times 10^{-3}$

58. (1)

$$2KIO_3 + 10KI + 12Na_2S_2O_3 + 12HCl \rightarrow 12KCl + 12NaI + 6Na_2S_4O_6 + 6H_2O$$

$Na_2S_2O_3$ required $= \left(\dfrac{2.14}{214}\right) \times \dfrac{12}{2} = 6 \times 10^{-2}$ mole

$10^{-3} \times\ M \times 60\ = 6 \times 10^{-2}$

$M = \dfrac{6 \times 10^{-2}}{60 \times 10^{-3}} = 1$

59. (6)

CrO_5 have butterfly structure

60. (4)

N = 4

Metal ion is Hg^{2+}.

$Hg^{2+} + Co^{2+} + 4SCN^- \rightarrow Co[Hg(SCN)_4]\downarrow$ deep blue

SECTION-III (MATHEMATICS)

61. (1)

(i) Let P be perimeter

$P = 2x + 2y$; $A = xy$

$\dfrac{dP}{dt} = 2\dfrac{dx}{dt} + 2\dfrac{dy}{dt}$

$\dfrac{dA}{dt} = \dfrac{dx}{dt}y + x\dfrac{dy}{dt} = -6 + 4 = -2$

(ii) Let A be area

$A = xy$

$\dfrac{dA}{dt} = \dfrac{dx}{dt}y + x\dfrac{dy}{dt}$

$= -18 + 20 = 2$

62. (2)

$T_{r+1} = {}^{21}C_r \left(\dfrac{a}{b}\right)^{\frac{21-r}{3}} \left(\dfrac{b}{\sqrt{a}}\right)^{\frac{r}{3}}$

$= {}^{21}C_r \cdot a^{\frac{21-r}{3} - \frac{r}{6}}\, b^{\frac{r}{3} - \frac{21-r}{3}}$

$= {}^{21}C_r \cdot a^{\frac{42-3r}{2}} \cdot b^{\frac{2r-21}{3}}$

$= {}^{21}C_r \cdot a^{\frac{14-r}{2}} \cdot b^{\frac{2r-21}{3}}$

$\dfrac{14-r}{2} = \dfrac{2r-21}{3}$

$42 - 3r = 4r - 42$

$7r = 84$

$r = 12$

$\Rightarrow$ 13th term

63. (2)

$x^2 + y^2 - 10x + \lambda\,(2x - y) = 0$....(i)

$x^2 + y^2 + 2x\,(\lambda - 5) - \lambda y = 0$

Centre $\left(-(\lambda - 5), \dfrac{\lambda}{2}\right)$

Using on $y = 2x$

$\Rightarrow\ \dfrac{\lambda}{2} = -2(\lambda - 5) \Rightarrow \dfrac{5\lambda}{2} = 10$

Putting $\lambda = 4$

$\Rightarrow\ x^2 + y^2 - 2x - 4y = 0$

64. (1)

$\log_{\frac{1}{2}}\left(\dfrac{|z-1|+4}{3|z-1|-2}\right) > 1$

$0 < \dfrac{|z-1|+4}{3|z-1|-2} < \dfrac{1}{2}$

$|z - 1| = t$

$0 < \dfrac{t+4}{3t-2} < \dfrac{1}{2}$

$0 < \dfrac{t+4}{3t-2} < \dfrac{1}{2}$

$\Rightarrow\ t > 10$ So true

65. (1)

$\sqrt{(x-3)^2 + (y-4)^2} = \dfrac{|x + 3y - 3|}{\sqrt{1+9}}$

$\Rightarrow$ 10 {(x^2 + 9 – 6x) + [y^2 + 16 – 8y]} = (x + 3y – 3)^2
= $x^2 + 9y^2 + 9 + 6xy - 6xy - 6x - 18y$
$\Rightarrow$ $9x^2 + y^2 - 6xy - 54x - 62y + 241 = 0$

66. **(4)**
$(z - i)(z^2 + 2iz - 2) = 0$
$\Rightarrow$ $z = i, 1 - i, -1 - i$
Area of ΔABC = $\frac{1}{2}\begin{vmatrix} -1 & 1 \\ 2 & 2 \end{vmatrix} = 2$ square units.

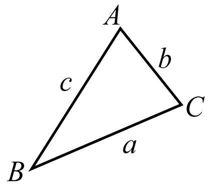
67. **(3)**
a_{124} = 111….1 (124 times)
= $1 + 10 + 10^2 + 10^{123}$
The remainder when 1, 10, 10^2, 10^3, 10^4 are divided by 271
Are respectively 1, 10, 100, 187, 244
These are only possible remainders when
10^k ($k \in W$) is divided by 271.
Required remainder = Remainder of {24 (1 +10 + 100 + 187 + 244) + (1 + 10 + 100 + 187)} = 27

68. **(4)**
Choose any three elements from set A such that $f(x) = y_1$ by ${}^7C_3$ ways, and rest 4 elements of set A and 3 elements of set B can arranged by $3^4 - 3(2^4) + 3$ ways. So total number of functions are ${}^7C_3\,(3^4 - 3.2^4 + 3) = 1260$.

69. **(3)**
$$\frac{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}}{\frac{4}{5}\times\frac{3}{4}\times\frac{3}{4}+\frac{4}{5}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{5}\times\frac{1}{4}\times\frac{3}{4}}$$
$$=\frac{36+4}{36+4+6}=\frac{20}{23}$$

70. **(1)**
According to the given condition,
$$6.80=\frac{\left[(6-a)^2+(6-b)^2+(6-8)^2+(6-5)^2+(6-10)^2\right]}{5}$$
$\Rightarrow$ $34 = (6 - a)^2 + (6 - b)^2 + 4 + 1 + 16$
$\Rightarrow$ $(6 - a)^2 + (6 - b)^2 = 13 = 9 + 4 = 3^2 + 2^2$
$\Rightarrow$ $a = 3, b = 4$

71. **(2)**
$$P(A)=\frac{3}{11};P(B)=\frac{2}{7};P(C)=?$$
$$P(A)+P(B)+P(C)=1$$
$$\Rightarrow\ P(C)=\frac{34}{77}$$

72. **(1)**
We, know that $a + b > c$, $b + c > a$ and $c + a > b$
$\Rightarrow$ $c - a < b$, $a - b < c$, $b - c < a$
squaring on both sides and adding $(c - a)^2 + (a - b)^2 + (b - c)^2 < a^2 + b^2 + c^2$
$a^2 + b^2 + c^2 - 2(ab + bc + ca) < 0$
$\Rightarrow$ $(a + b + c)^2 - 4(ab + bc + ca) < 0$
$$\Rightarrow\ \frac{(a+b+c)^2}{ab+bc+ca}<4 \quad ….(i)$$
Now roots of equation
$x^2 + 2(a + b + c)\,x + 3\lambda\,(ab + bc + ca) = 0$ are real, then $D \geq 0$
$\Rightarrow$ $4\,(a + b + c)^2 - 4 \cdot 3\lambda\,(ab + bc + ca) \geq 0$
$$\Rightarrow\ \frac{(a+b+c)^2}{ab+bc+ca}\geq 3\lambda$$
$$\text{So } 3\lambda \leq \frac{(a+b+c)^2}{ab+bc+ca}<4$$
$$\Rightarrow\ \lambda<\frac{4}{3}$$


73. **(1)**
$$\text{Given } f(x)=\begin{cases} -\frac{x^2}{2} & \text{for } x\leq 0 \\ x^n\sin\frac{1}{x} & \text{for } x>0 \end{cases}$$
and f (x) is continuous at x = 0 clearly f (0) = 0
$$\text{Now L.H.L.} = \lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}\left(\frac{-x^2}{2}\right)=0$$
$$\text{and R.H.L.} = \lim_{x\to 0^+} f(x)=\lim_{x\to 0^+} x^n\sin\frac{1}{x}$$
$\therefore$ For continuity at $x = 0$
$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0^-} f(x)=f(0)$$

$\Rightarrow \lim\limits_{x\to 0^+} x^n \sin\left(\frac{1}{x}\right) = 0$

$\Rightarrow$ limit is defined only when $n > 0$ ….(i)

since $f(x)$ is non-differentiable at $x = 0$,

L.H.D. $\neq$ R.H.D

Now

L.H.D = $f'\left(0^-\right) = \lim\limits_{h\to 0^+} \frac{f(0-h) - f(0)}{-h}$

$= \lim\limits_{h\to 0^+} \frac{-\frac{h^2}{2} - 0}{-h} = 0$

and R.H.D =

$$f'\left(0^+\right) = \lim\limits_{h\to 0^+} \frac{f(0+h) - f(0)}{h}$$

$= \lim\limits_{h\to 0^+} \frac{h^n \sin\frac{1}{h}}{h} = \lim\limits_{h\to 0^+} h^{n-1} \sin\frac{1}{h}$

Now L.H.D. $\neq$ R.H.D

$\Rightarrow = \lim\limits_{h\to 0^+} h^{n-1} \sin\left(\frac{1}{h}\right) \neq 0$, which is possible only when $n - 1 \leq 0$

$\Rightarrow n \leq 1$ …(ii)

$\therefore$ from equation (i) and (ii) $n \in (0, 1]$

74. (4)

Given, $x_1 + x_2 + \ldots\ldots + x_{10} = 12$

And $x_1^2 + x_2^2 + \ldots\ldots + x_{10}^2 = 18$

$\therefore \sigma^2 = \frac{1}{n}\sum x^2 - \left(\frac{1}{n}\sum x\right)^2 = \frac{18}{10} - \left(\frac{12}{10}\right)^2$

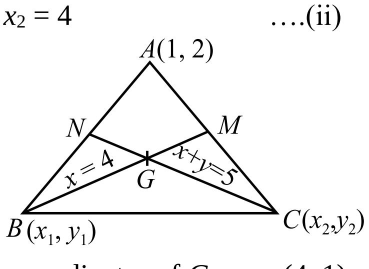
$= \frac{9}{5} - \frac{36}{25} = \frac{9}{25}$

$\Rightarrow SD = \frac{3}{5}$

75. (2)

$x_1 + y_1 = 5$ ….(i)

$x_2 = 4$ ….(ii)



co-ordinates of G are $\equiv (4, 1)$

$\Rightarrow \frac{1 + x_1 + x_2}{3} = 4$ ….(iii)

and $\frac{y_1 + y_2 + 2}{3} = 1$ ….(iv)

Solving above equations, we get B & C.

76. (3)

$f(x + 1) = x + f(x)$

$\Rightarrow e^{f(x+1)} = e^{x + f(x)} = e^x \times e^{f(x)}$

$\Rightarrow g(x + 1) = e^x \times g(x)$

$\Rightarrow \ln(g(x + 1)) = x + \ln(g(x))$

$\Rightarrow \frac{g'(x+1)}{g(x+1)} - \frac{g'(x)}{g(x)} = 1$

$\Rightarrow \frac{g'\left(\frac{1}{2}+1\right)}{g\left(\frac{1}{2}+1\right)} - \frac{g'\left(\frac{1}{2}\right)}{g\left(\frac{1}{2}\right)} = 1$

$\frac{g'\left(\frac{1}{2}+2\right)}{g\left(\frac{1}{2}+2\right)} - \frac{g'\left(1+\frac{1}{2}\right)}{g\left(1+\frac{1}{2}\right)} = 1$

$\vdots \qquad \vdots$

$\frac{g'\left(n+\frac{1}{2}\right)}{g\left(n+\frac{1}{2}\right)} - \frac{g'\left(n-\frac{1}{2}\right)}{g\left(n-\frac{1}{2}\right)} = 1$

Adding $\frac{g'\left(n+\frac{1}{2}\right)}{g\left(n+\frac{1}{2}\right)} - \frac{g'\left(\frac{1}{2}\right)}{g\left(\frac{1}{2}\right)} = n$

77. (3)

$u = \sqrt{x^2 + 16}$

$\therefore \frac{du}{dx} = \frac{2x}{2\sqrt{x^2+16}} = \frac{x}{\sqrt{x^2+16}}$

$v = \frac{x}{x-1}$

$\Rightarrow \frac{dv}{dx} = \frac{-1}{(x-1)^2}$

$\therefore \frac{du}{dv} = \frac{du/dx}{dv/dx} = \frac{-12}{5}$

78. **(2)**

We have $(x-41)^{49} + (x-49)^{41} + (x-2009)^{2009} = 0$

Let $f(x) = (x-41)^{49} + (x-49)^{41} + (x-2009)^{2009}$

$\therefore \; f'(x) = 49(x-41)^{48} + 41(x-49)^{40} + 2009(x-2009)^{48} > 0$

Hence, $f(x)$ will cut x-axis only once $\Rightarrow$ 1 real root

79. **(2)**

$$I = \int \frac{\left(e^x + \cos x + 1\right) - \left(e^x + \sin x + x\right)}{e^x + \sin x + x}\, dx$$

$= \log_e(e^x + \sin x + x) - x + c$

$\therefore \; f(x) = e^x + \sin x + x$ and $g(x) = -x$

$f(x) + g(x) = e^x + \sin x$

80. **(2)**

$$I = \int_0^a \ln\left(\cot a + \tan x\right) dx$$

$$= \int_0^a \ln\left(\frac{\cos(a-x)}{\sin a \cos x}\right) dx$$

$$\therefore \; I = \int_0^a \ln\left(\frac{\cos x}{\sin a \cos(a-x)}\right) dx$$

Adding (1) and (2) we get $2I = \int_0^a \ln\left(\frac{1}{\sin^2 a}\right) dx$

$$= -2\int_0^a \ln(\sin a)\, dx$$

$= -2a \ln(\sin a)$

81. **(15)**

If $x = \alpha + i\beta$ is a root then

$$\frac{A_1^2}{\alpha - a_1 + i\beta} + \frac{A_2^2}{\alpha - a_2 + i\beta} + \ldots\ldots + \frac{A_n^2}{\alpha - a_n + i\beta} = K$$

& taking conjugate

$$\frac{A_1^2}{\alpha - a_1 - i\beta} + \frac{A_2^2}{\alpha - a_2 - i\beta} + \ldots\ldots + \frac{A_n^2}{\alpha - a_n - i\beta} = K$$

Substracting

$$\frac{2\beta A_1^2}{(\alpha - a_1)^2 + \beta^2} + \frac{2\beta A_2^2}{(\alpha - a_2)^2 + \beta^2} + \ldots\ldots + \frac{2\beta A_n^2}{(\alpha - a_n)^2 + \beta^2} = 0$$

$\Rightarrow \; \beta = 0$

$\Rightarrow \; x = \alpha + i\,0$

Which is purely real.

82. **(330)**

$n(A) = 40\%$ of $10{,}000 = 4{,}000$

$n(B) = 20\%$ of $10{,}000 = 2{,}000$

$n(C) = 10\%$ of $10{,}000 = 1{,}000$

$n(A \cap B) = 5\%$ of $10{,}000 = 500$

$n(B \cap C) = 3\%$ of $10{,}000 = 300$

$n(C \cap A) = 4\%$ of $10{,}000 = 400$

$n(A \cap B \cap C) = 2\%$ of $10{,}000 = 200$

$n(A \cap B^c \cap C^c) = n[A \cap (B \cup C)^c]$

$= n(A) - n[A \cap (B \cup C)] = n(A) - n[(A \cap B) \cup (A \cap C)]$

$= n(A) - [n(A \cap B) + n(A \cap C) - n(A \cap B \cap C)]$

$= 4000 - [500 + 400 - 200] = 4000 - 700 = 3300$

83. **(0)**

$$\begin{vmatrix} a & a^3 & a^4 - 1 \\ b & b^3 & b^4 - 1 \\ c & c^3 & c^4 - 1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a & a^3 & a^4 \\ b & b^3 & b^4 \\ c & c^3 & c^4 \end{vmatrix} = \begin{vmatrix} a & a^3 & 1 \\ b & b^3 & 1 \\ c & c^3 & 1 \end{vmatrix}$$

$abc(a-b)(b-c)(c-a)(ab+bc+ca) = (a-b)(b-c)(c-a)(a+b+c)$

$\therefore \; abc(ab+bc+ca) = (a+b+c)$

84. **(18)**

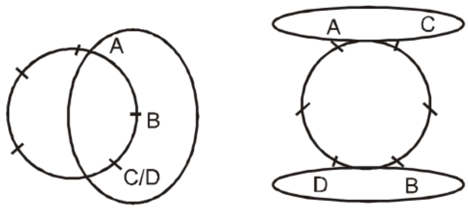
Case-I If B is right on A

Subcase -I C is right on B

then no. of ways = $(4-1)! = 6$ Subcase- II If D is right on B then no. of ways = $(4-1)! = 6$

Case-II If C is right on $A \Rightarrow D$ must be right on $B = (4-1)! = 3! = 6$

Hence total no. of ways is $6 + 6 + 6 = 18$



85. **(87)**

Let $A_1 \rightarrow$ Ball drawn from urn A is red and ball returned is also red, $P(A_1) = \frac{6}{10} \times \frac{5}{11}$

$B_1 \rightarrow$ Ball drawn from urn A is red but ball returned to it is black, $P(B_1) = \frac{6}{10}\times\frac{6}{11}$

$C_1 \rightarrow$ Ball drawn from urn A is black and ball of same colour is returned, $P(C_1) = \frac{4}{10}\times\frac{7}{11}$

$D_1 \rightarrow$ Ball drawn from urn A is black and ball returned is red, $P(D_1) = \frac{4}{10}\times\frac{4}{11}$

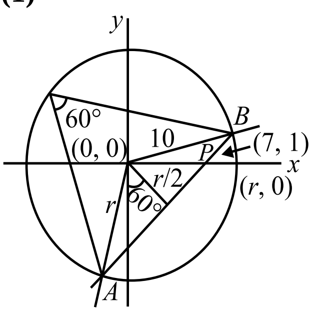
Required probability $P(R) = P(A_1)\times P\left(\frac{R}{A_1}\right)+$

$P(B_1)\times P\left(\frac{R}{B_1}\right)+P(C_1)\times P\left(\frac{R}{C_1}\right)+P(D_1)\times P\left(\frac{R}{D_1}\right)$

$=\frac{6}{10}\times\frac{5}{11}\times\frac{6}{10}+\frac{6}{10}\times\frac{6}{11}\times\frac{5}{10}+\frac{4}{10}\times\frac{7}{11}\times$

$\frac{6}{10}+\frac{4}{10}\times\frac{4}{11}\times\frac{7}{10}=\frac{32}{55}$

86. **(1)**



Let the slope of the chord through point (7, 1) be m,

Thus, equation of line is

$y - 1 = m\,(x - 7)$

or $mx - y + 1 - 7m = 0$

Perpendicular distance from $(0,0)=\frac{r}{2}$

$\Rightarrow \frac{|7m-1|}{\sqrt{1+m^2}}=5$

$\Rightarrow (7m-1)^2 = 25\,(1+m^2)$

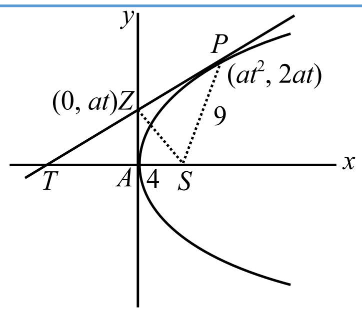
$\Rightarrow 49m^2 - 14m + 1 = 25 + 25m^2$

$\Rightarrow 24m^2 - 14m - 24 = 0$

$\Rightarrow m_1m_2 = -1$

87. **(6)**

Let $P(at^2, 2at)$ be any point on the parabola $y^2 = 4ax$, then the equation of the tangent at P is $yt = x + at^2$. It cuts y-axis at $(0, at)$.



Clearly, SZ is perpendicular to PT.

Now, $SZ=\sqrt{a^2+a^2t^2}=a\sqrt{1+t^2}$

$SP = a + at^2$ and $AS = a$

$\therefore\ SZ = a^2\,(1 + t^2)$ and $AS \cdot SP = a^2\,(t^2 + 1)$

Clearly, $SZ^2 = AS \cdot SP$

So, from the figure, we have

$\Rightarrow SZ^2 = (4)(9) \Rightarrow SZ = 6$

88. **(5)**

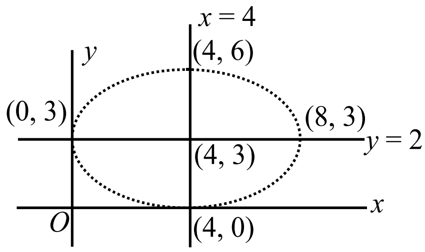
Let $(4\cos\theta+4, 3\sin\theta+3)$ be any point on the ellipse $\frac{(x-4)^2}{16}+\frac{(y-3)^2}{9}=1$

Image of $(4\cos\theta + 4, 3\sin\theta + 3)$ about the line $x - y - 2 = 0$ is (h, k), then

$\frac{h-4\cos\theta-4}{1}=\frac{k-3\sin\theta-3}{-1}$

$=-\frac{2(4\cos\theta-3\sin\theta-1)}{2}$

$\Rightarrow h = 3\sin\theta + 5$ and $k = 4\cos\theta + 2$



Locus of (h, k) is $\left(\frac{x-5}{3}\right)^2+\left(\frac{y-2}{4}\right)^2=1$

$\Rightarrow 16x^2 + 9y^2 - 160x - 36y + 292 = 0$

$\Rightarrow k_1 + k_2 = 25$

89. **(5)**

$\frac{\sin^2 x-2\cos^2 x+1}{\sin^2 x+2\cos^2 x-1}=4$

$\Rightarrow \sin^2x - 2\cos^2x + 1$

$= 4\sin^2x + 8\cos^2x - 4$

$\Rightarrow 10\cos^2 x + 3\sin^2x - 5 = 0$

$\Rightarrow$ $10 + 3\tan^2 x - 5(1 + \tan^2 x) = 0$

$\Rightarrow$ $2\tan^2 x = 5$

$\therefore$ $\tan^2 x = \frac{5}{2}$

90. **(1)**

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma$

$$= \frac{\tan^2\alpha}{1+\tan^2\alpha} + \frac{\tan^2\beta}{1+\tan^2\beta} + \frac{\tan^2\gamma}{1+\tan^2\gamma}$$

$$= \frac{x}{1+x} + \frac{y}{1+y} + \frac{z}{1+z}$$

[where $x = \tan^2\alpha$, $y = \tan^2\beta$, $z = \tan^2\gamma$]

$$= \frac{(x+y+z)+(xy+yz+zx+2xyz)+xy+yz+zx+xyz}{(1+x)(1+y)(1+z)}$$

$$= \frac{1+x+y+z+xy+yz+zx+xyz}{(1+x)(1+y)(1+z)} = 1$$

$[\because xy+yz+zx+2xyz = 1]$