JEE MAIN (2023-24) Mock Test Series

Paper - 09

DURATION : 180 Minutes **M. MARKS : 300**

ANSWER KEY

PHYSICS

1.	(2)
2.	(4)
3.	(4)
4.	(1)
5.	(1)
6.	(2)
7.	(2)
8.	(4)
9.	(4)
10.	(3)
11.	(3)
12.	(1)
13.	(4)
14.	(4)
15.	(3)
16.	(1)
17.	(1)
18.	(1)
19.	(3)
20.	(2)
21.	(15)
22.	(10)
23.	(16)
24.	(12)
25.	(50)
26.	(2)
27.	(75)
28.	(2)
29.	(175)
30.	(13)

CHEMISTRY

31.	(4)
32.	(4)
33.	(4)
34.	(1)
35.	(4)
36.	(3)
37.	(4)
38.	(1)
39.	(4)
40.	(2)
41.	(2)
42.	(4)
43.	(3)
44.	(2)
45.	(2)
46.	(2)
47.	(2)
48.	(1)
49.	(1)
50.	(3)
51.	(10)
52.	(7)
53.	(9)
54.	(3)
55.	(7)
56.	(8)
57.	(2)
58.	(8)
59.	(2)
60.	(4)

MATHEMATICS

61.	(3)
62.	(3)
63.	(3)
64.	(2)
65.	(3)
66.	(2)
67.	(2)
68.	(1)
69.	(1)
70.	(1)
71.	(3)
72.	(1)
73.	(3)
74.	(1)
75.	(2)
76.	(2)
77.	(3)
78.	(3)
79.	(1)
80.	(3)
81.	(2)
82.	(0)
83.	(4)
84.	(0)
85.	(3)
86.	(3)
87.	(3)
88.	(6)
89.	(6)
90.	(2)

SECTION-I (PHYSICS)

1. **(2)**

$$RC = \frac{L}{R}$$

$$R = \sqrt{\frac{L}{C}} \Rightarrow RC = \frac{L}{R}$$

$\Rightarrow$ Time constant of both circuits are equal

$$l_L = i_C$$

$$\frac{V}{R}\left(1 - e^{-\frac{t}{\tau}}\right) = \frac{V}{R}e^{-\frac{t}{\tau}}$$

$$\Rightarrow t = \tau \ln(2) = RC\ln(2)$$

2. **(4)**

$$I_P = I_{cm} + M\left(\frac{R}{2}\right)^2$$

$$I_{CD} = I_{cm} + M\left(\frac{R}{2}\right)^2$$

Hence, $I_{AB} = I_{CD}$

3. **(4)**

Fundamental frequency, $f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$

T = force on dielectric slab.

To calculate force on slab, consider capacitor as combination of two capacitors one with slab and one with air.

$$F = -\frac{dU}{dx}; U = \frac{1}{2}\left[\frac{k \in_0 bx}{d} + \frac{\in_0 b(b-x)}{d}\right]V^2$$

$$\frac{dU}{dx} = \frac{1}{2}\left[\frac{k \in_0 b}{d} - \frac{\in_0 b}{d}\right]V^2 = \frac{1}{2}\frac{\in_0 bV^2}{d}(k-1)$$

Thus, $f = \frac{1}{2L}\sqrt{\frac{\in_0 bV^2(k-1)}{2d\mu}}$

4. **(1)**

$\Delta U = mg\Delta H$

$|\Delta U| = \Delta U_A + \Delta U_B$

$= \frac{mgl}{2}$

$= 1 \times \frac{10}{2} \times \frac{10}{100}$

$= \frac{1}{2}$ J

5. **(1)**

Let refractive index of glass be μ.

Let after first refraction, image distance b v then

$$\frac{\mu}{v} - \frac{1}{\infty} = \frac{\mu - 1}{R} \Rightarrow v = \frac{\mu R}{\mu - 1}$$

Now second refraction will take place.

So, distance of first image from O is

$$u_1 = \frac{\mu R}{\mu - 1} - R = \frac{R}{\mu - 1}$$

and image is formed at R

$$\therefore \frac{1}{R} - \frac{\mu(\mu-1)}{R} = \frac{2(1-\mu)}{R}$$

$$\Rightarrow \mu^2 - 3\mu + 1 = 0,\ \mu = \frac{3+\sqrt{5}}{2}$$

6. **(2)**

The frictional force μmg is the only horizontal force acting on the two bodies. So each body has an acceleration $\frac{\mu mg}{m} = \mu g$ in opposite direction.

So, relative acceleration is $2\mu g$

7. **(2)**

Initially field due to both is along positive x-axis. Due to the ring, field will first increase and then decrease to zero at centre. While field due to the solid sphere, will continuously increase in positive x-direction. On the other side of the ring field is now towards negative x-axis.

Hence, the correct answer is (2).

8. **(4)**

Since, $h = \frac{2T\cos\theta}{r\rho g}$

$$\Rightarrow r = \frac{2T\cos\theta}{h\rho g}$$

$$\Rightarrow \frac{r_{Hg}}{r_{water}} = \frac{r_1}{r_2} = \left(\frac{T_{Hg}}{T_W}\right)\left(\frac{\rho_W}{\rho_{Hg}}\right)\left(\frac{\cos\theta_{Hg}}{\cos\theta_W}\right)$$

$$\Rightarrow \frac{r_1}{r_2} = 7.5 \times \frac{1}{13.6} \times \frac{1}{\sqrt{2}} = 0.4 = \frac{2}{5}$$

9. **(4)**

Let x be the displacement of bead. Displacement of particle with respect to bead is $L(1 - \cos\theta)$, i.e., displacement of particle with respect to ground will be $L(1-\cos\theta)-x$. Since net force in horizontal direction on the system is zero. Therefore, the centre of mass will not move in horizontal direction.

$\Rightarrow$ $2mx = m\,[L(1-\cos\theta) - x]$

$\Rightarrow$ $3mx = mL\,(1-\cos\theta)$

$\Rightarrow$ $x = \frac{L}{3}(1-\cos\theta)$

10. **(3)**

The direction of light is given by the normal vector

$\vec{n} = \hat{i} + 2\hat{j} + 3\hat{k}$. So, angle made by the $\vec{n}$ with y-axis is given by $\cos\beta = \frac{2}{\sqrt{1^2+2^2+3^2}} = \frac{2}{\sqrt{14}}$

11. **(3)**

For outside point sphere behaves as a point mass. Gravitational field at centre of sphere due to ring is

$$E = \frac{G\,ma\sqrt{3}}{[a^2+[\sqrt{3}a]^{3/2}} = \frac{G\,ma\sqrt{3}}{8a^3}$$

$$E = \frac{G\,m\sqrt{3}}{8a^2}$$

Force on sphere due to this field is

$$F = M \times E = \frac{\sqrt{3}G\,Mm}{8a^2}$$

12. **(1)**

$$r = \frac{mv}{qB}$$

$$\sin\theta = \frac{x}{r}$$

$$= \frac{\frac{mv}{\sqrt{2}qB}}{\frac{mv}{qB}} = \frac{1}{\sqrt{2}}$$

Or $\theta = \frac{\pi}{4}$

Time to complete the circle (π), $T = \frac{2\pi m}{qB}$

$\therefore$ Time taken to transverse $\frac{\pi}{4}$, $t = \frac{\pi m}{4qB}$

$$t_1 = \frac{\frac{mv}{\sqrt{2}qB}}{\frac{v}{\sqrt{2}}} = \frac{m}{qB}$$

Total time taken = $2t + 2t_1$

$= \frac{m}{2qB}(\pi+4)$

13. **(4)**

Using $v = \sqrt{\frac{T}{\mu}}$, μ = mass per unit length of the rope If v_t is velocity at top and v_b is velocity at bottom then

$$\frac{\lambda_t}{\lambda_b} = \frac{v_t}{v_b} = \sqrt{\frac{T_t}{T_b}} = \sqrt{\frac{(M+m)g}{mg}} = \sqrt{\frac{M+m}{m}}$$

$$\lambda_t = \lambda\sqrt{\frac{M+m}{m}}$$

14. **(4)**

$W_{1-2} + W_{2-3} + W_{3-1} = -300$ ($\Delta U = 0$ in a cyclic process).

$W_{3-1} = 0$ $W_{1-2} = P(V_2 - V_1) = nR(T_2 - T_1) = 600R = 4980$ J

$= -4980 - 300 = -5280$ J

15. **(3)**

$$\frac{1}{2}k(A')^2 = \frac{1}{2}kA^2 + \frac{1}{2}m\omega^2A^2$$

$$\Rightarrow \boxed{A' = \sqrt{2}A}$$

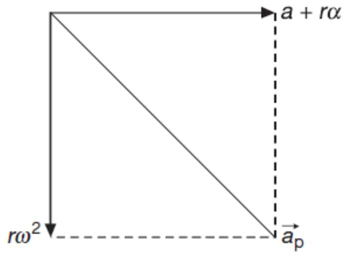
16. **(1)**

$\vec{a}_p = \vec{a}_{p_0+\vec{a}_0}$

Here, $\vec{a}_{p_0}$ = acceleration of P with respect to O, so

$\vec{a}_{p_0} = \vec{a}_{p_0t} + \vec{a}_{p_0n}$

$\Rightarrow$ $\vec{a}_p = \left(\vec{a}_{p_0t} + \vec{a}_{p_0n}\right) + \vec{a}_0$



Where, $\vec{a}_{p_0t}$ = tangential component of $\vec{a}_{p_0}$

and $\vec{a}_{p_0}$ = normal component of $\vec{a}_{p_0}$

So, $\left|\vec{a}_0 + \vec{a}_{p_0t}\right| = a + r\alpha$ and

$\left|\vec{a}_{p_0n}\right| = r\omega^2$

$$\Rightarrow \left|\vec{a}_p\right| = \sqrt{\left(a + r\alpha\right)^2 + \left(r\omega^2\right)^2}$$

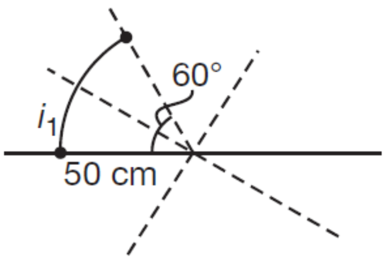
17. (1)

In interference, the energy is redistributed from dark fringes to bright fringes.

18. (1)

Image formed by the lens is at (75, 0)

This acts as a virtual image for the mirror



Assuming that the mirror is not tilted, then for mirror

$u = +25, v = ?\ f = +50$

$$\Rightarrow \frac{1}{v} + \frac{1}{25} = \frac{1}{50}$$

$\Rightarrow v = -50$ cm ...(1)

This image is formed on x-axis

Now, when the mirror is rotated clockwise by 30°, image rotates by 60° clockwise. Since the ray strikes the pole of the mirror, so this ray rotates by 60° and image lies on this ray. Hence image rotates by 60°. New co-ordinates of image will be

$x = 50 - 50\cos(60°) = 25$ cm

$y = 50\sin(60°) = \frac{50\sqrt{3}}{2}\text{cm} = 25\sqrt{3}$ cm

19. (3)

Work done by the electric force on the particle

$$W = \int_A^B q\vec{E}.\vec{d}t = qE.\pi R$$

$\therefore\ K_B - K_A = qE\pi R$

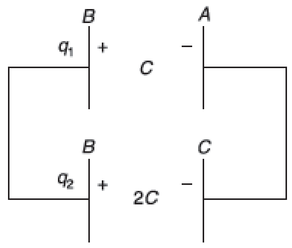
$\therefore\ K_B = \pi qER$ [$\because K_A = 0$]

20. (2)

Capacitance between A and B is $C = \frac{\epsilon_0 S}{2d}$.

Capacitance between B and C is $\frac{\epsilon_0 S}{d} = 2C$.

Initial energy stored $U_i = \frac{Q^2}{2(2C)} = \frac{Q^2}{4C}$



After the switch is closed, we have two capacitors in parallel, as shown in Figure

$q_1 + q_2 = Q$ ...(i)

and $\frac{q_1}{C} = \frac{q_2}{2C}$

$\Rightarrow q_1 = \frac{q_2}{2}$ ...(ii)

Solving (i) and (ii) we get $q_1 = \frac{Q}{3}; q_2 = \frac{2Q}{3}$

Final energy stored in the system

$$U_f = \frac{1}{2}\frac{q_1^2}{C} + \frac{1}{2}\frac{q_2^2}{2C} = \frac{Q^2}{6C}$$

$\therefore$ Loss in energy $\Delta U = U_i - U_f$

$$= \frac{Q^2}{4C} - \frac{Q^2}{6C} = \frac{Q^2}{12C}$$

$$= \frac{Q^2}{12\frac{\epsilon_0 S}{2d}} = \frac{Q^2.d}{6\epsilon_0 .S}$$

21. (15)

This silvered concavo-convex lens behaves like mirror whose focal length can be calculated by the formula

$$\frac{1}{f} = \frac{2}{f_1} + \frac{1}{f_2}$$

f_1 = focal length of concave surface.

f_2 = focal length of concave mirror.

$$\therefore \frac{1}{f} = \frac{2}{-60} + \frac{1}{-10} = \frac{4}{30}$$

$\therefore$ f = –7.5 cm

Using mirror formula

$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u} \Rightarrow \frac{1}{-7.5} = \frac{1}{-x} + \frac{1}{-x} \Rightarrow x = 15 \text{ cm}$$

22. **(10)**

$V = \frac{kQ}{R}$

Let charge given to $A = q$

$\frac{kq}{a} = 20$

$a = 1$ m

$kq = 20$ …(1)

Now, when they are connected, charge will go on sphere B.

$V_B = \frac{kq}{b} = \frac{kq}{2} = 10\,\text{V}$

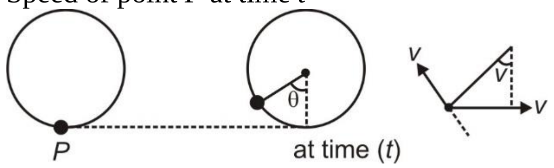
Now, $V_A = V_B$ ($\because$ connected)

$V_A = 10$ V

23. **(16)**

$s = \int v\,dt$

Speed of point P at time t



$v_0 = \sqrt{v^2 + v^2 + 2v^2\cos(180 - \theta)}$

$= 2v\sin\left(\frac{\theta}{2}\right)$

$\therefore$ Distance covered in one revolution

$s = \int_0^T v_0\,dt$

where $\theta = \omega t$

$= \frac{v}{R}t$

$s = 8R = 16$ m

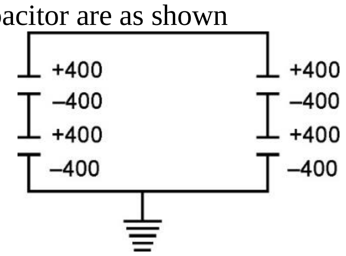
24. **(12)**

For maximum, path difference = $n\lambda$

In a quadrant path difference varies continuously from 3λ to $0 \Rightarrow 4$.

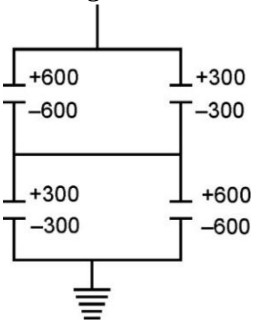
25. **(50)**

Before closing the switch, the charges on capacitor are as shown



$q = 200 \times \left[\frac{6\times 3}{6+3}\right] = \frac{200\times 6\times 3}{8} = +400\,\mu\text{C}$

After closing the switch,



Hence, charge flown is 300 μC.

26. **(2)**

In equilibrium

$\frac{KQq}{x_0^2} = mg\sin\theta$ …(1)

If the charge is displaced by x ($\ll x_0$)

$ma = \frac{KQq}{(x_0 + x)^2} - mg\sin\theta$

$= \frac{KQq}{x_0^2\left(1 + \frac{x}{x_0}\right)^2} - mg\sin\theta$

$= \frac{KQq}{x_0^2}\left(1 + \frac{x}{x_0}\right)^2 - mg\sin\theta$

Using binomial expansion and neglecting higher order terms-

$ma = \frac{KQq}{x_0^2}\left[1 - \frac{2x}{x_0}\right] - mg\sin\theta$

$ma = -\frac{2KQq}{x_0^2}.x$ [using 1]

$ma = -\left(2\frac{mg\sin\theta}{x_0}\right).x$ [again using 1]

$\therefore$ $a = -\left(\frac{2g\sin\theta}{x_0}\right)x$ $\therefore \omega = \sqrt{\frac{2g\sin\theta}{x_0}}$

$f = \frac{1}{2\pi}\sqrt{\frac{2g\sin\theta}{x_0}}$

27. **(75)**

Just before the switch is opened

$V_R = IR = 10$ Volt.

$\therefore$ p.d. across C_1 at this instant

$V_0 = V_R = 10$ Volt.

Energy stored in C_1 at this instant

$$U_1 = \frac{1}{2}C_1V_0^2 = \frac{1}{2}\times 3\times 10^{-6}\times 10^2 = 150\ \mu J.$$

Charge on C_1 at this instant $Q_0 = 3 \times 10^{-6} \times 10$

$= 30\ \mu C$

Now this charge gets shared between C_1 and C_2 so that p.d across both of them becomes equal.

Final common pd.

$$V = \frac{Q_0}{C_1 + C_2} = \frac{30\mu C}{4\mu F} = 7.5\ \text{Volt.}$$

$\therefore$ Finaly energy stored in the capacitor system

$$U_2 = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}\times 4\times 10^{-6}\times (7.5)^2$$

$\therefore$ Heat liberated = Energy lost

$37.5\mu J$.

28. **(2)**

Current (and hence current density) in the entire loop must be same.

From microscopic form of Ohm's law we can write

$\sigma_1 E_1 = \sigma_2 E_2$

$$\Rightarrow\ R_2E_1 = R_1E_2 \Rightarrow \frac{E_1}{E_2} = \frac{R_1}{R_2}$$

29. **(175)**

$$\text{Shift} = \frac{(\mu - 1)tD}{d}$$

$$x = \frac{(1.5 - 1)tD}{d} \qquad \ldots(1)$$

$$\text{and } \frac{3}{2}x = \frac{(\mu - 1)tD}{d} \qquad \ldots(2)$$

Dividing equation (1) by (2)

$$\frac{2}{3} = \frac{0.5}{\mu - 1}$$

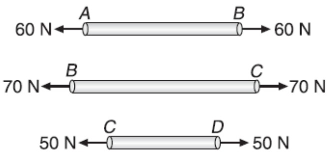
$2\mu - 2 = 1.5$

$2\mu = 3.5$

$\mu = 1.75$

30. **(13)**

The action of forces on each part of rod is shown in Figure.



We know that the extension due to external force F is given by

$$\Delta L = \frac{FL}{AY}$$

$$\Delta L_{AB} = \frac{(60\times 10^3)\times 1.5}{1\times 2\times 10^{11}} = 4.5\times 10^{-7}\ \text{m}$$

$$\Delta L_{BC} = \frac{(70\times 10^3)X_1}{1\times 2\times 10^{11}} = 3.5\times 10^{-7}\ \text{m and}$$

$$\Delta L_{CD} = \frac{(50\times 10^3)X_2}{1\times 2\times 10^{11}} = 5.0\times 10^{-7}\ \text{m}$$

The total extension is given by

$\Delta L = \Delta L_{AB} + \Delta L_{BC} + \Delta L_{CD}$

$\Rightarrow\ \Delta L = 4.5 \times 10^{-7} + 3.5 \times 10^{-7} + 5.0 \times 10^{-7}$

$\Rightarrow\ \Delta L = 13 \times 10^{-7}$ m = 1.3 μm

SECTION-II (CHEMISTRY)

31. **(4)**

72 g Mg is present in

1 mole $Mg_3(PO_4)_2$

$\therefore$ 8 moles of O atoms

4 moles of O_2 molecules

4 gm molecules of O_2 are present

32. **(4)**

(I)
$$\begin{array}{lcccccccccc} \text{Pd} & 1s^2 & 2s^2 & 2p^6 & 3s^2 & 3p^6 & 4s^2 & 3d^{10} & 4p^6 & 5s^0 & 4d^{10} \\ l=0 & \downarrow & \downarrow & & \downarrow & & \downarrow & & & \downarrow & \\ & 2 & 2 & & 2 & & 2 & & & 0 & = 8\ \text{electrons} \end{array}$$

(II) Z is different, therefor, Z_{eff} will be different.

(III) Configuration of ${}_{26}Fe \rightarrow [Ar]\ 4s^2\ 3d^6$

Configuration of $Ni^{+2} \rightarrow [Ar]\ 3d^8\ 4s^0$

Number of unpaired electrons are different.

(IV) 57 electron (La) $5d^1$

n = 5

$l = 2 \qquad \left(\frac{l}{n}\right) = 0.4$

33. **(4)**

$K_P = P_{CO_2}$ and active mass of solid is constant

34. **(1)**

$\Delta T_b = K_b\ .\ m$

$$\Rightarrow 0.1 = K_b \times \frac{1.8/180}{100/1000} \Rightarrow K_b = 1.0\ K/m$$

35. **(4)**

$$r = K_3\,[CHCl_3]\,[\dot{Cl}] \text{ and } \frac{K_1}{K_2} = \frac{[\dot{Cl}]^2}{[Cl_2]}$$

$$\therefore r = \sqrt{\frac{K_1}{K_2}}.K_3[CHCl_3][Cl_2]^{1/2}$$

36. **(3)**

Eq. of SO_2 formed = Eq. of $KMnO_4$ used

$= 0.2 \times 5 = 1.0$ ($Mn^{+7} + 5e \rightarrow Mn^{+2}$)

Moles of SO_2 formed $= \frac{1}{2} = 0.5$ ($S^{+4} \rightarrow S^{+6} + 2e$)

$$2SO_3 \rightleftharpoons 2SO_2 + O_2$$

	$2SO_3$	$2SO_2$	O_2
t = 0	1	0	0
t = t,	0.5	0.5	0.25

$$K_1 = \frac{(0.5)^2 \times 0.25}{(0.5)^2} = 0.25$$

$$K_2 = \sqrt{K_1} = \sqrt{0.25} = 0.5.$$

37. **(4)**

$R - OH + CH_3MgI \rightarrow CH_4 + ROMgI$

1 mol. 22400 ml

11.2 ml CH_4 evolved from 0.037 gm R – OH

$$22400 \text{ ml evolved from} = \frac{0.037 \times 22400}{11.2} = 74\text{gm}$$

$\therefore$ 12n + 2n + 1 + 17 = 74

$$\therefore \quad n = \frac{56}{14} = 4$$

38. **(1)**

meq. $FeSO_4$ = meq. $KMnO_4$

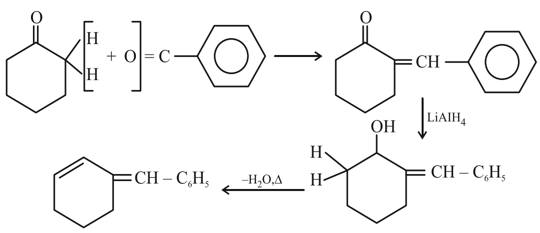
$$\frac{W}{152} \times 1000 = 200 \times 1$$

$\therefore$ W = 30.4 gm

39. **(4)**

$NH_2^- > OH^- > NH_3$ is correct order of basic strength.

40. **(2)**



41. **(2)**

F_2 due to greater inter electronic repulsions.

42. **(4)**

Diamond has high refractive index. The value of μ = 2, only some synthetic compound having such a high value of refractive index.

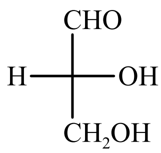
43. **(3)**

$K_3[Cr(CN)_6]$ has d^3 configuration so paramagnetic.

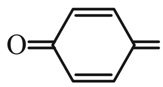
44. **(2)**

Al^{3+} cannot form an amine complex ion with excess of NH_3.

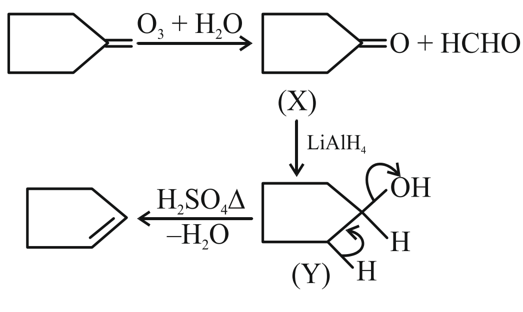
45. **(2)**



46. **(2)**

, it has no α-H-atom

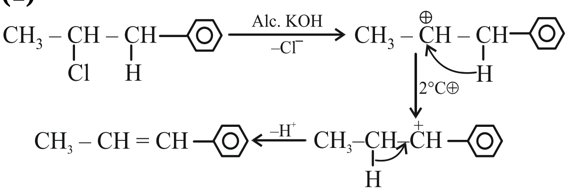
47. **(2)**



48. **(1)**

$$Ph{-}C \equiv C - CH_3 \xrightarrow{H^+OH^-} \underset{\text{(Enol)}}{Ph - \underset{OH}{\underset{|}{C}} = CH - CH_3} \rightleftharpoons \underset{\text{(Keto)}}{Ph - \underset{O}{\underset{||}{C}} - CH_2 - CH_3}$$

49. **(1)**



50. **(3)**

Allylic $C^{\oplus}$ > $2°C^{\oplus}$ > $1C^{\oplus}$

$\longrightarrow$

Reactivity Decreases

51. **(10)**

$$Ka = C\alpha^2 \Rightarrow \frac{\alpha_2}{\alpha_1} = \sqrt{\frac{C_1}{C_2}} = \sqrt{\frac{1}{1/100}} = 10$$

52. **(7)**

$$\frac{q}{w} = \frac{nC_{P,m}.\Delta T}{-nR.\Delta T}$$

$$\Rightarrow \frac{q}{-2} = \frac{\frac{7}{2}R}{-R}$$

$\Rightarrow q = 7$ J

53. **(9)**

Cathode: $2H_2O(l) + 2e^- \longrightarrow H_2(g) + 2OH^-(aq)$

Anode: $2Cl^-(aq) \longrightarrow Cl_2(g) + 2e^-$

n_{eq} OH^- produced = $\frac{Q}{F}$

$$\Rightarrow n_{OH^{\ominus}} \times 1 = \frac{9.65 \times 10}{96500} = 10^{-3}$$

$$\therefore [OH^-]_{final} = \frac{10^{-3}}{100} = 10^{-5} M \Rightarrow P^H = 9.0$$

54. **(3)**

In S_8, SO_4^{2-} and H_2S, S is sp^3 hybridised.

55. **(7)**

Let oxidation state of I in H_5IO_6 is 'x'

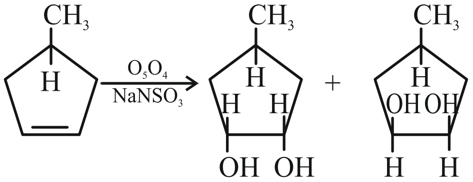
$(+1) \times 5 + x + (-2) \times 6 = 0$, $x = +7$

56. **(8)**

It has 8α-H–atoms that can participate in Hyperconjugation.

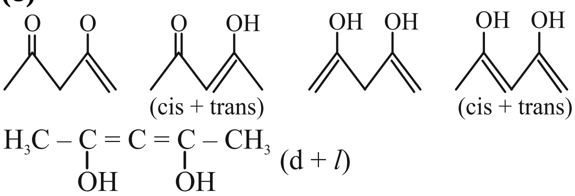
57. **(2)**

OsO_4–$NaHSO_3$(aq) brings about cis-hydroxylation of alkenes.



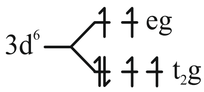
Both are optically inactive (Meso) i.e. only two Isomers.

58. **(8)**



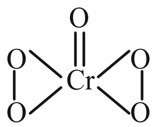
59. **(2)**

CoF_6^{-3}; O.N. of Co=+3, Co + 3 $\rightarrow$ [Ar] $3d^6$



60. **(4)**

$$K_2CrO_4 + H_2O_2 \xrightarrow[{[H^+]}]{\text{Amyl alcohol}} \underset{(X)}{CrO_5} \text{ Blue liquid}$$



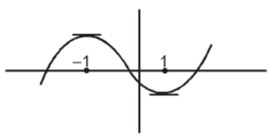
SECTION-III (MATHEMATICS)

61. **(3)**

$f(x) = x^3 - 3x + k$, $k = [a]$

$f'(x) = 3(x-1)(x+1)$

–1 is maxima and 1 is minima



Figure

for three roots $f(-1)\, f(1) < 0$

$\Rightarrow \quad (k+2)(k-2) < 0$

$k \in (-2, 2) \qquad \Rightarrow -2 < [a] < 2$

$\Rightarrow -1 \le a < 2$

62. **(3)**

Expression = $x.\, x^2.\, x^3 \ldots\ldots\ldots\ldots x^{20}$

$$\left(1-\frac{1}{x}\right)\left(1-\frac{2}{x^2}\right)\left(1-\frac{3}{x^3}\right).......\left(1-\frac{20}{x^{20}}\right)$$

Let $E = \left(1-\frac{1}{x}\right)\left(1-\frac{2}{x^2}\right)\left(1-\frac{3}{x^3}\right)......\left(1-\frac{20}{x^{20}}\right)$

Now Co-efficient of x^{203} in original expression

$\Rightarrow$ Co-efficient of x^{-7} in E.

But

$$E=1-\left(\frac{1}{x}+\frac{2}{x^2}+\frac{3}{x^3}+.......\right)+\left(\frac{1}{x}\cdot\frac{6}{x^6}+\frac{2}{x^2}\cdot\frac{5}{x^5}+\frac{3}{x^3}\cdot\frac{4}{x^4}+......\right)$$
$$\left(\frac{1}{x}\cdot\frac{2}{x^2}\cdot\frac{4}{x^4}+.....\right)$$

= Co-efficient of $x^{-7} = -7 + 6 + 10 + 12 - 8 = 13$

63. **(3)**

$2x - 3y = 1$, $x^2 + y^2 \le 6$

$$S \equiv \left\{\left(2,\frac{3}{4}\right),\left(\frac{5}{2},\frac{3}{4}\right),\left(\frac{1}{4},-\frac{1}{4}\right),\left(\frac{1}{8},\frac{1}{4}\right)\right\}$$

(I) (II) (III) (IV)

Plot the two curves

I, III, IV will lie inside the circle and point (I, III, IV) will lie on the P region if (0, 0) and the given point will lie opposite to the line $2x - 3y - 1 = 0$

$P(0, 0)$ = negative, $P\left(2,\frac{3}{4}\right)$ = positive,

$P\left(\frac{1}{4},-\frac{1}{4}\right)$ = positive $P\left(\frac{1}{8},\frac{1}{4}\right)$ = negative

$P\left(\frac{5}{2},\frac{3}{4}\right)$ = positive, but it will not lie in the given circle

so point $\left(2,\frac{3}{4}\right)$ and $\left(\frac{1}{4},-\frac{1}{4}\right)$ will lie on the opp side of the line so two points $\left(2,\frac{3}{4}\right)$ and $\left(\frac{1}{4},-\frac{1}{4}\right)$

Further $\left(2,\frac{3}{4}\right)$ and satisfy $S_1\left(\frac{1}{4},-\frac{1}{4}\right) < 0$

64. **(2)**

$$z^2+\bar{z}=0 \Rightarrow (x+iy)^2+x-iy=0$$

$$x^2-y^2+x+i(2xy-y)=0$$

$$y(2x-1)=0 \text{ and } x^2-y^2+x=0$$

$$y=0 \text{ (or) } 2x-1=0$$

of $y=0, x=0,-1$

of $x=\frac{1}{2}, y=\pm\frac{\sqrt{3}}{2}$

$$\sum_{z\in S}(\text{Re}\,z+\text{Im}\,z)$$

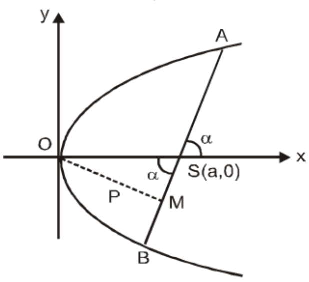
$$=(0+0)+(-1+0)+\left(\frac{1}{2}+\frac{\sqrt{3}}{2}\right)+\left(\frac{1}{2}-\frac{\sqrt{3}}{2}\right)=0$$

65. **(3)**

Distance of focal chord from (0, 0) is p

equation of chord; $2x - (t_1 + t_2)y + 2a\, t_1 t_2 = 0$

$2x - (t_1 + t_2)\, y - 2a = 0$ ..... (i)



so perpendicular length from (0, 0)

$$\left|\frac{2a}{\sqrt{4+\left(t_1-\frac{1}{t_1}\right)^2}}\right|=p \Rightarrow \left(t_1+\frac{1}{t_1}\right)=\frac{2a}{p}$$

Now length of focal chord is = $a\left(t_1+\frac{1}{t_1}\right)^2=\frac{2a}{p}$

$$a\frac{4a^2}{p^2}=\frac{4a^3}{p^2}$$

66. **(2)**

$$\sum_{r=0}^{n}\frac{{}^nC_{r-1}}{{}^{n+1}C_r}=\sum_{r=0}^{n}\frac{r}{n+1}$$

$$=\frac{1}{n+1}\cdot\frac{n(n+1)}{2}=\frac{n}{2}$$

67. **(2)**

$(2k_1 + 1) + (2k_2 + 1) + (2k_3 + 1) + (2k_4 + 1) = 20$

$\rightarrow k_1 + k_2 + k_3 + k_4 = 8$

$k_i \ge 0$

Required number = ${}^{4+8-1}C_8 = {}^{11}C_3 = 165$

68. **(1)**

Coefficient x^{20} in $(1 + x + x^2 + \ldots. + x^{10})^4$

Coefficient x^{20} in $(1 - x^{11})^4 (1 - x)^{-4}$

= Coefficient x^{20} in $(1 - 4x^{11} + \ldots) (1 - x)^{-4}$

$= {}^{4+20-1}C_{20} - {}^{4+9-1}C_9$

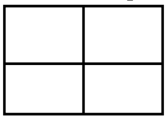
$= {}^{23}C_3 - 4\,({}^{12}C_3) = 891$

69. **(1)**

$n(s) = {}^{64}C_3$

Let 'E' be the event of selecting three squares which form the letter 'L'

The number of ways selecting squares consisting of 4 unit squares is $7 \times 7 = 49$.



Each square with 4 unit squares form 4L shapes consisting of 3 unit squares

$n(E) = 7 \times 7 \times 4 = 196$ $\dfrac{n(E)}{n(S)} = \dfrac{196}{{}^{64}C_3}$

70. **(1)**

P (get a number bigger than 3) $= \dfrac{1}{2}$

P (get 5 in throw) = $\dfrac{1}{6}$

$E \rightarrow$ get 5 in last throw when he gets a number bigger than 3

$$P(E) = \frac{1}{6} + \frac{1}{2}\cdot\frac{1}{6} + \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{6} + \ldots\ldots\infty$$

$$= \frac{1}{6} \times \frac{1}{1-\frac{1}{2}} = \frac{1}{3}$$

71. **(3)**

$$A_1^2 - A_2^2 + A_3^2 - A_4^2 + A_5^2 - A_6^2$$

$= -d\,(A_1 + A_2 + \ldots\ldots. + A_6)$

$$= -\left(\frac{b-a}{7}\right)\left(3(b+a)\right) = 3\left(\frac{a^2-b^2}{7}\right) = \text{Prime}$$

$\Rightarrow a = 4, b = 3$

72. **(1)**

$f(x) = [x]\,(\sin kx)^p$

$(\sin kx)^p$ is continuous and differentiable function $\forall\ x \in R$, $k \in R$ and $p > 0$.

$[x]$ is discontinuous at $x \in I$

For $k = n\,\pi$, $n \in I$

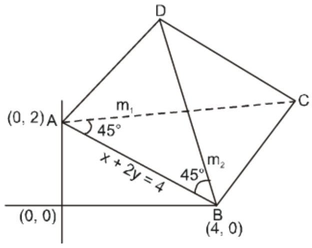
$f(x) = [x]\,(\sin (n\pi\, x))^p$

$$\lim_{x\to a} f(x) = 0, a \in I$$

and $f(a) = 0$

So, $f(x)$ becomes continuous for all $x \in R$

73. **(3)**



$$\tan 45° = \left|\frac{m+\frac{1}{2}}{1-\frac{m}{2}}\right| \Rightarrow \pm 1 = \frac{2m+1}{2-m} \Rightarrow m = \frac{1}{3}, -3$$

$\therefore$ Equation of AC

$y - 2 = \dfrac{1}{3}(x)$ $\quad\Rightarrow\quad$ $x - 3y + 6 = 0$ …..(i)

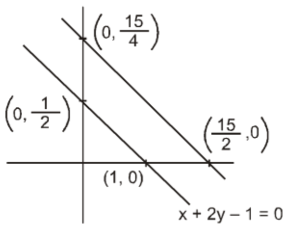
Equation of BD

$y = -3(x - 4)$ $\quad\Rightarrow\quad$ $3x + y - 12 = 0$.... (ii)

Solving (i) & (ii) $\Rightarrow$ $x = 3$ & $y = 3$

74. **(1)**

Point $P\left(1+\dfrac{t}{\sqrt{2}}, 2+\dfrac{t}{\sqrt{2}}\right)$ lies between given lines



Hence $L_1(P)$ $= \left(1+\dfrac{t}{\sqrt{2}}\right) + 2\left(2+\dfrac{t}{\sqrt{2}}\right) - 1 = 0$

$$5 + \frac{3t}{\sqrt{2}} - 1 = 0 \Rightarrow t = -\frac{4\sqrt{2}}{3}$$

Now, $L_2(P)$ $2\left(1+\dfrac{t}{\sqrt{2}}\right) + 4\left(2+\dfrac{t}{\sqrt{2}}\right) - 15 = 0$

$$\Rightarrow 10 + \frac{6t}{\sqrt{2}} - 15 = 0 \Rightarrow t = \frac{5\sqrt{2}}{6}$$

and $L_1(P) \times L_2(P) < 0$

Hence $t \in -\dfrac{4\sqrt{2}}{3} < t < \dfrac{5\sqrt{2}}{6}$

75. **(2)**

$\because$ $\tan A < 0$ and $A + B + C = 180°$

$\Rightarrow A > 90° \Rightarrow B + C < 90° \Rightarrow \tan(B + C) > 0$

$\Rightarrow \dfrac{\tan B + \tan C}{1 - \tan B \tan C} > 0$

$\Rightarrow 1 - \tan B \tan C > 0 \Rightarrow \tan B \tan C < 1$

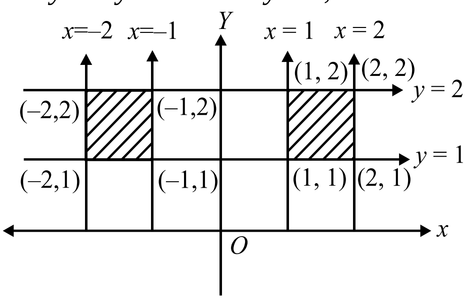
76. **(2)**

$x^2 - 3|x| + 2 = 0$

$\Rightarrow |x| = 1, 2$

$\Rightarrow x = \pm 1, \pm 2$

and $y^2 - 3y + 2 = 0 \Rightarrow y = 1, 2$



So the vertices can be (–2, 1), (–1, 1), (–1, 2), (–2, 2) or (1, 1) (2, 1), (2, 2), (1, 2)

Both are of unit area.

Hence there are 2 such squares.

77. **(3)**

Since the triangle is right angled formed by the line $x = 0$, $y = 0$ and $x + y = 1$ the orthocentre lies at the vertex (0, 0), the point of intersection of the perpendicular lines $x = 0$ and $v = 0$

78. **(3)**

Let O be the centre of the circle $x^2 + y^2 = 4$, and let AB be a chord of this circle, so that $\angle AOB = \dfrac{\pi}{2}$. Let $M(h, k)$ be the mid-point of AB. Then OM is perpendicular to AB

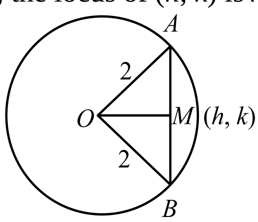
$\therefore$ $(AB)^2 = (OA)^2 + (OB)^2 = 4 + 4 = 8$

$\Rightarrow AM = \left(\dfrac{1}{2}\right) AB = \sqrt{2}$

$\Rightarrow (OM)^2 = (OA)^2 - (AM)^2 = 4 - 2 = 2$

$\Rightarrow h^2 + k^2 = 2$

Therefore, the locus of (h, k) is $x^2 + y^2 = 2$.



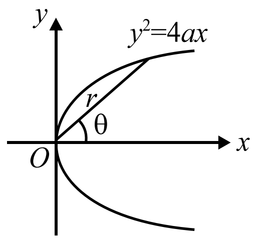
79. **(1)**

Putting $x = r\cos\theta$, $y = r\sin\theta$ in $y^2 = 4ax$, we get

$r^2 \sin^2\theta = 4ar\cos\theta$

$\Rightarrow r = \dfrac{4a\cos\theta}{\sin^2\theta} = 4a\cot\theta \operatorname{cosec}\theta$

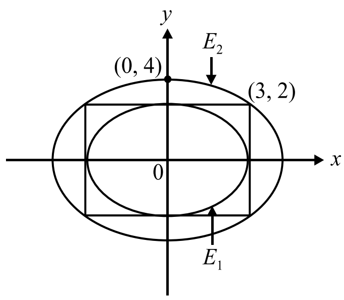
$\therefore$ Length of the required chord = $4a\,|\cot\theta|\operatorname{cosec}\theta$



80. **(3)**

Sides of R are given by

$x = \pm 3$, $y = \pm 2$



Let equation of E_2 be

$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

As it passes through (0, 4) and (3, 2) we get

$\dfrac{16}{b^2} = 1 \Rightarrow b^2 = 16$

and $\dfrac{9}{a^2} + \dfrac{4}{b^2} = 1 = a^2 = 12$

Eccentricity e of E_2 is given by

$a^2 = b^2(1 - e^2)$

$\Rightarrow 12 = 16(1 - e^2) \Rightarrow e = 1/2$

81. **(2)**

$PS = e\left(\dfrac{a}{e} - a\cos\theta\right) = a - ae\cos\theta$

$PS' = e\left(\dfrac{a}{e} + a\cos\theta\right) = a + ae\cos\theta$ and

$SS' = 2ae$

Let incentre be (h, k)

$$h=\frac{-ae\left(a-ae\cos\theta\right)+ae\left(a+ae\cos\theta\right)+2a^2e\cos\theta}{2a\left(1+e\right)}$$

$$\Rightarrow\ h=\frac{2a^2e^2\cos\theta+2a^2e\cos\theta}{2a\left(1+e\right)}$$

$$=\frac{ae^2\cos\theta+ae\cos\theta}{\left(1+e\right)}=ae\cos\theta$$

$k=\dfrac{b\sin\theta\times 2ae}{2a\left(1+e\right)}=\dfrac{be\sin\theta}{1+e}$ equation of ellipse

$$\left(\frac{h}{ae}\right)^2+\left(\frac{k\left(1+e\right)}{be}\right)^2=1\Rightarrow\frac{x^2}{a^2e^2}+\frac{y^2\left(1+e\right)^2}{b^2e^2}$$

$$e_2=\sqrt{1-\frac{b^2e^2}{\left(1+e\right)^2a^2e^2}}$$

$$=\sqrt{1-\frac{1-e}{1+e}}=\sqrt{\frac{2e}{1+e}}\Rightarrow\left(1+\frac{1}{e}\right)e_2^2=2$$

82. (0)

$\tan\alpha=2\alpha\ \ \&\ \ \tan\beta=2\beta$

Now $I=\dfrac{1}{2}\displaystyle\int_0^1\left\{\cos\left(\alpha-\beta\right)x-\cos\left(\alpha+\beta\right)x\right\}dx$

$$=\frac{1}{2}\left[\frac{\sin\left(\alpha-\beta\right)x}{\alpha-\beta}-\frac{\sin\left(\alpha+\beta\right)x}{\alpha+\beta}\right]_0^1$$

$$I=\frac{1}{2}\left\{\frac{\sin\left(\alpha-\beta\right)}{\alpha-\beta}-\frac{\sin\left(\alpha+\beta\right)}{\alpha+\beta}\right\}$$

since $\tan\alpha = 2\alpha$ & $\tan\beta = 2\beta$

adding them we get $\dfrac{\sin\left(\alpha+\beta\right)}{\alpha+\beta}=2\cos\alpha\cos\beta$

subtracting them we get $\dfrac{\sin\left(\alpha-\beta\right)}{\alpha-\beta}=2\cos\alpha\cos\beta$

Hence $I = 0$

83. (4)

$$y\left(\frac{dy}{dx}\right)^2+x\frac{dy}{dx}-y\frac{dy}{dx}-x=0$$

$$y\frac{dy}{dx}\left(\frac{dy}{dx}-1\right)+x\left(\frac{dy}{dx}-1\right)=0$$

$$\left(y\frac{dy}{dx}+x\right)\left(\frac{dy}{dx}-1\right)=0$$

$\therefore$ either $ydy + xdx = 0$ or $dy – dx = 0$

Since the curves pass through the point (3, 4)

$\therefore\ x^2 + y^2 = 25$ or $x – y + 1 = 0$

$\Rightarrow 2x – 2y + 2 = 0 \quad \Rightarrow \quad A = 2\ \&\ B = –2$

$\Rightarrow A – B = 4$

84. (0)

$C = ABA^T$ where $B^T = –B$

$\Rightarrow C^T = (A^T)^T B^TA^T = –ABA^T = –C$

$\Rightarrow C$ is skew matrix $\Rightarrow C_3, C_5,$ .... $,C^{99}$ are also skew matrix

$\Rightarrow$ trace of $C + C_3 + C_5 + ...+ C_{99}$ is zero

85. (3)

Let p and q denote the probability of things going to man and woman respectively.

Therefore $p=\dfrac{1}{1+\mu}$ and $q=\dfrac{\mu}{1+\mu}$

Probability of men receiving r things is given by

$P_r = {}^nC_r\cdot q^{n–r}\cdot p^r$

So required probability is given by

$P_1 + P_3 + P_5 +$ ....

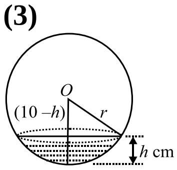
$$=\frac{1}{2}\left[\left(q+p\right)^n-\left(q-p\right)^n\right]=\frac{1}{2}\left[1-\left(\frac{\mu-1}{\mu+1}\right)^n\right]$$

$$=\frac{1}{2}-\frac{1}{2}\left(\frac{\mu-1}{\mu+1}\right)^n$$

By comparison, we have $\left(\dfrac{\mu-1}{\mu+1}\right)=\dfrac{1}{2}$

$\Rightarrow\quad 2\mu - 2 = \mu + 1$. Thus $\mu = 3$

86. (3)



$\dfrac{dh}{dt}=-2, r=10$ cm

We have to find $\dfrac{dx}{dt}$ when $h = 4$, where x is the radius of the top surface.

From the figure $r^2 = x^2 + (10 – h)^2$

$$\therefore\quad 2x\frac{dx}{dt}=2\left(10-h\right)\frac{dh}{dt}$$

$$\Rightarrow\quad \frac{dx}{dt}=\frac{\left(10-h\right)}{x}\left(-2\right)$$

$$\Rightarrow\quad \frac{dx}{dt}=\frac{-2\left(10-4\right)}{x}=-\frac{12}{x}\qquad ...(i)$$

When $h = 4$, then $x^2 = 10^2 – 6^2 = 64$ or $x = 8$.

$$\therefore\quad -\frac{dx}{dt}=\frac{-12}{8}=-\frac{3}{2}$$

87. (3)

$$f\left(x\right)=\frac{\tan\left(x+\dfrac{\pi}{6}\right)}{\tan x}=\cot x\tan\left(x+\frac{\pi}{6}\right)$$

$$f'(x)=\cot x\sec^2\left(x+\frac{\pi}{6}\right)-\operatorname{cosec}^2 x\tan\left(x+\frac{\pi}{6}\right)$$

$$\therefore \quad f''(x)=2\cot x\sec^2\left(x+\frac{\pi}{6}\right)\tan\left(x+\frac{\pi}{6}\right)$$

$$-\operatorname{cosec}^2 x\sec^2\left(x+\frac{\pi}{6}\right)-\operatorname{cosec}^2 x\sec^2\left(x+\frac{\pi}{6}\right)$$

$$+2\operatorname{cosec}^2 x\cot x\tan\left(x+\frac{\pi}{6}\right)$$

Now $f'(x)=0$

$$\Rightarrow \frac{1}{2}\sin 2x=\frac{1}{2}\sin\left(2x+\frac{\pi}{3}\right)$$

$$\Rightarrow 2x=\pi-2x-\frac{\pi}{3}$$

$$\Rightarrow x=\frac{\pi}{6}$$

There $f''\left(\frac{\pi}{6}\right)>0$

$\therefore$ At $x=\frac{\pi}{6}, f(x)$ is minimum and there is no other minimum in $\left(0,\frac{\pi}{2}\right)$.

$\therefore$ The minimum value of

$$f(x)=f\left(\frac{\pi}{6}\right)=\frac{\tan\left(\frac{\pi}{6}+\frac{\pi}{6}\right)}{\tan\frac{\pi}{6}}=\frac{\sqrt{3}}{\frac{1}{\sqrt{3}}}=3$$

88. (6)

$$I=\int\frac{3(\tan x-1)\sec^2 x}{(\tan x+1)\sqrt{\tan^3 x+\tan^2 x+\tan x}}dx$$

Put $\tan x=t$

$$\Rightarrow I=3\int\frac{1-\frac{1}{t^2}}{\left(t+2+\frac{1}{t}\right)\sqrt{t+\frac{1}{t}+1}}dt$$

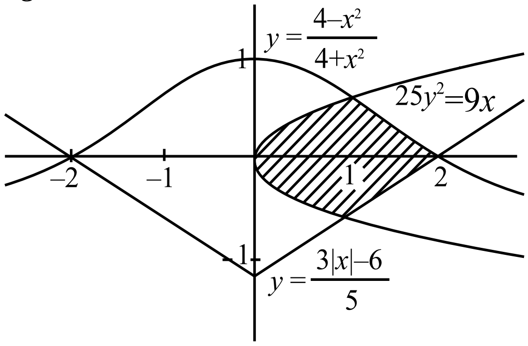
Put $t+\frac{1}{t}+1=z^2$

$$\Rightarrow I=3.2\tan^{-1}\left(\sqrt{\cot x+\tan x+1}\right)+C$$

$\Rightarrow K=6$

89. (6)

Graph of the functions is shown in the following figure



Required area =

$$2\int_0^1\frac{3}{5}\sqrt{x}dx+\int_1^2\left\{\frac{4-x^2}{4+x^2}-\frac{3x-6}{5}\right\}dx$$

$$=\frac{4}{5}+\int_1^2\left\{\frac{8}{4+x^2}-\frac{3x-1}{5}\right\}dx$$

$$=\frac{4}{5}+\left[4\tan^{-1}\frac{x}{2}\right]_1^2-\frac{1}{5}\left[\frac{3x^2}{2}-x\right]_1^2$$

$$=\frac{4}{5}+\pi-4\tan^{-1}\frac{1}{2}-\frac{7}{10}$$

$$=\left\{\pi-4\tan^{-1}\frac{1}{2}+\frac{1}{10}\right\}\text{sq.units}$$

90. (2)

$x^2dx=ydx-xdy$

$$dx=-\frac{(xdy-ydx)}{x^2}$$

$$dx=-d\left(\frac{y}{x}\right)$$

$$x=-\frac{y}{x}+c$$

$1=-1+c$

$c=2$

$$x=-\frac{y}{x}+2$$

$y=2x-x^2$

$x=0, x=2$