JEE MAIN (2023-24) Mock Test Series

Paper - 04

DURATION : 180 Minutes

M. MARKS : 300

ANSWER KEY

PHYSICS

1.	(2)
2.	(2)
3.	(3)
4.	(1)
5.	(1)
6.	(1)
7.	(2)
8.	(1)
9.	(4)
10.	(2)
11.	(2)
12.	(2)
13.	(4)
14.	(2)
15.	(3)
16.	(1)
17.	(4)
18.	(4)
19.	(2)
20.	(2)
21.	(10)
22.	(70)
23.	(1)
24.	(95)
25.	(50)
26.	(4)
27.	(12)
28.	(1)
29.	(7)
30.	(100)

CHEMISTRY

31.	(4)
32.	(1)
33.	(3)
34.	(1)
35.	(1)
36.	(1)
37.	(4)
38.	(2)
39.	(2)
40.	(1)
41.	(4)
42.	(4)
43.	(3)
44.	(2)
45.	(2)
46.	(3)
47.	(4)
48.	(3)
49.	(2)
50.	(1)
51.	(8)
52.	(6)
53.	(3)
54.	(3)
55.	(5)
56.	(2)
57.	(4)
58.	(6)
59.	(2)
60.	(5)

MATHEMATICS

61.	(3)
62.	(1)
63.	(4)
64.	(4)
65.	(1)
66.	(4)
67.	(3)
68.	(2)
69.	(4)
70.	(3)
71.	(1)
72.	(2)
73.	(4)
74.	(2)
75.	(1)
76.	(3)
77.	(1)
78.	(1)
79.	(1)
80.	(2)
81.	(50)
82.	(10)
83.	(28)
84.	(2)
85.	(0)
86.	(9)
87.	(9)
88.	(5)
89.	(3)
90.	(1)

SECTION-I (PHYSICS)

1. **(2)**

$$mv_0\frac{\ell}{2}=4m\left(\frac{\ell}{\sqrt{2}}\right)^2\omega$$

$$mv_0\frac{\ell}{2}=4m\frac{\ell^2}{2}\omega$$

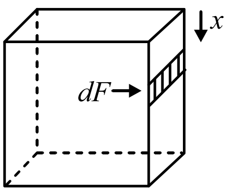
$$\omega=\frac{v_0}{4\ell}$$

2. **(2)**

$$P=\int_0^x \rho_0 g\left(1+\frac{x}{h}\right)dx$$

$$P=\rho_0 g\left(x+\frac{x^2}{2h}\right)$$

Force on the face *ABCD*



$$F=\int_0^h \rho_0 g\left(x+\frac{x^2}{2h}\right)bdx$$

$$F=\frac{2}{3}\rho_0 bgh^2$$

3. **(3)**

$$\lambda_{\text{rod}}=\frac{M}{4R}$$

now, $dF=\dfrac{GM\frac{M}{4R}dx}{x^2+4R^2}$

Force acting on the rod due to sphere is

$$F=\int_{-2R}^{2R} dF\cos(\theta)$$

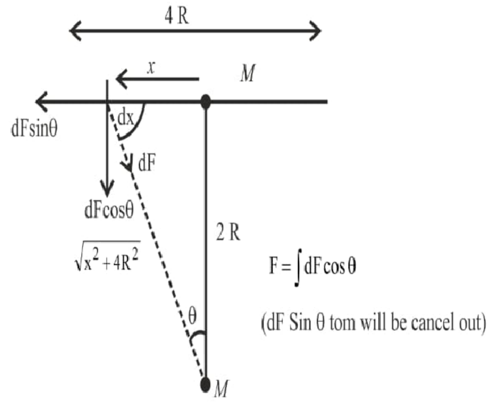
$$=\int_{-2R}^{2R}\frac{GM\frac{M}{4R}dx}{x^2+4R^2}\left(\frac{2R}{\sqrt{x^2+4R^2}}\right)$$

$$=\int_{-2R}^{2R}\frac{GM^2}{2\left(x^2+4R^2\right)^{\frac{3}{2}}}dx$$

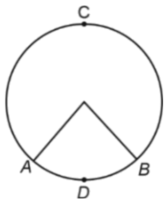
after solving this integration we get

$$F=\frac{GM^2}{4\sqrt{2}R^2}\text{ N}$$

Direction of force will be along $(-\hat{j})$.



4. **(1)**



Let $R_{ADB}=R=\dfrac{l}{KA}$

$\therefore\ R_{ACB}=\dfrac{3l}{KA}=3R$

$\therefore\ H=\dfrac{T_1-T_2}{\frac{3}{4}R}$

Now new $R_{ADB}=R_1$

$\therefore\ R_{eq}=\dfrac{3RR_1}{R+R_1}$

and $2H=\dfrac{T_1-T_2}{\mathrm{R_{eq}}}$

$R_{eq}=\dfrac{l}{k_{\text{new}}A}$

Solving $k_{\text{new}}=\dfrac{7}{3}k$

5. **(1)**

$Q = VC = (C + c)\,V_1 \quad V_1=\dfrac{VC}{C+c}$

$$V_n=\left(\frac{C}{C+c}\right)^n V$$

$$Q_n=V_nC=\left(\frac{C}{C+c}\right)^n VC=\left(\frac{C}{C+c}\right)^n\cdot Q$$

6. **(1)**

A line can leave $+q_1$ in a cone of apex angle α and then enter $-q_2$ in a cone of apex angle β.

So, flux due to the charge $+q_1$ is $\phi_1 = \frac{q_1}{2\varepsilon_0}(1-\cos\alpha)$ and that due to the charge $-q_2$ is $\phi_2 = \frac{q_2}{2\varepsilon_0}(1-\cos\beta)$.

Since, we know that only one line is leaving q_1 to enter $-q_2$. So, we can say

$$\frac{\phi_1}{\phi_2} = \frac{N_1}{N_2} = 1$$

$$\Rightarrow \frac{q_1}{2\varepsilon_0}(1-\cos\alpha) = \frac{q_2}{2\varepsilon_0}(1-\cos\beta)$$

$$\Rightarrow q_1\left[2\sin^2\left(\frac{\alpha}{2}\right)\right] = q_2\left[2\sin^2\left(\frac{\beta}{2}\right)\right]$$

$$\Rightarrow \sin\left(\frac{\beta}{2}\right) = \sqrt{\frac{q_1}{q_2}}\sin\left(\frac{\alpha}{2}\right)$$

$$\Rightarrow \beta = 2\sin^{-1}\left[\sqrt{\frac{q_1}{q_2}}\sin\left(\frac{\alpha}{2}\right)\right]$$

7. **(2)**

$Q = W + \Delta U$

= area under the graph + ΔU

$$= -p_0V_0 - \frac{3p_0V_0}{2}$$

$$= -\frac{5}{2}p_0V_0$$

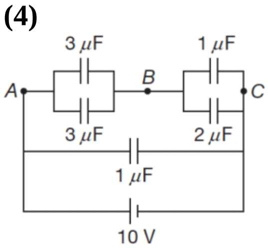
8. **(1)**

$Y \propto v^{\alpha}A^{\beta}F^{\gamma}$

$[ML^{-1}T^{-2}] = [LT^{-1}]^{\alpha}[LT^{-2}]^{\beta}[M^1L^1T^{-2}]^{\gamma}$

Solve α, β, γ

9. **(4)**

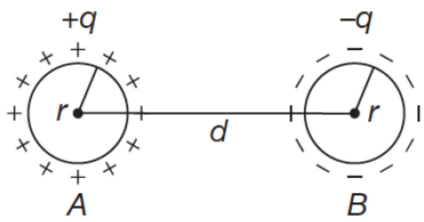


$$\frac{V_{AB}}{V_{BC}} = \frac{(C)_{BC}}{(C)_{AB}} = \frac{3}{6} = \frac{1}{2}$$

$$\Rightarrow V_{AB} = \left(\frac{1}{1+2}\right)(10)\text{ V}$$

$$\Rightarrow V_{AB} = \frac{10}{3}\text{ V}$$

10. **(2)**



Since $V_A - V_B = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{r} - \left(-\frac{Kq}{r}\right)\right) = \frac{2q}{4\pi\varepsilon_0 r}$

$$\Rightarrow C = \frac{q}{V_A - V_B} = 2\pi\varepsilon_0 r$$

11. **(2)**

$$\frac{dx}{dt} = \sqrt{\frac{\mu xg}{\mu}}$$

$$\int_0^{\ell} x^{-1/2}dx = \sqrt{g}\int_0^t dt$$

$$t = 2\sqrt{\frac{\ell}{g}} = 2\sqrt{\frac{2.45}{9.8}} \Rightarrow t = 1\text{ sec}$$

12. **(2)**

PT^{-5} = constant

$PV^{5/4}$ = constant

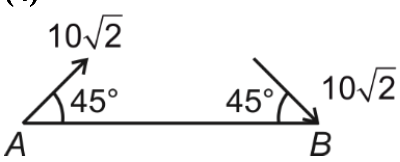
$$\alpha = \frac{5}{4}$$

$$C = C_V + \frac{R}{(1-\alpha)}$$

$$C = \frac{3R}{2} + \frac{R}{1-\frac{5}{4}} = \frac{3R}{2} + \frac{4R}{(-1)}$$

$$C = \frac{3R}{2} - 4R = \frac{3R - 8R}{2} = \frac{-5R}{2}$$

13. **(4)**



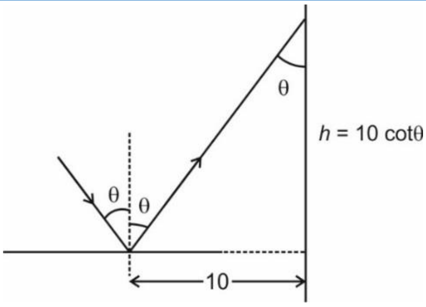
$t = T$

$$= \frac{(2)\times 10}{10}$$

= 2 s

14. **(2)**

When mirror is rotated with angular speed ‘ω’ the reflected ray will rotate with 2ω.



When mirror is rotated with angular speed 'ω' the reflected ray will rotate with 2ω = 36 rad/s

Speed of the spot

$$=\left|\frac{dh}{dt}\right|=\left|\frac{d}{dt}(10\cot\theta)\right|$$

$$=\left|-10\,\mathrm{cosec}^2\theta\frac{d\theta}{dt}\right|=1000\text{ m/s}$$

15. **(3)**

$$\overline{\overline{\overline{A}+\overline{B}}}=\overline{A.B}$$

16. **(1)**

Time constant of the left branch of circuit is given as

$$\tau_L=\frac{L}{R}=\frac{0.01}{10}=10^{-3}\text{ s}$$

Time constant of the right branch of circuit is given as

$$\tau_C=CR=\left(0.1\times10^{-3}\right)(10)=10^{-3}\text{ s}$$

Steady state currents in left and right branch of circuit are given as

$$\left(i_0\right)_L=\frac{20}{10}=2\text{ A}$$

$$\left(i_0\right)_C=\frac{20}{10}=2\text{ A}$$

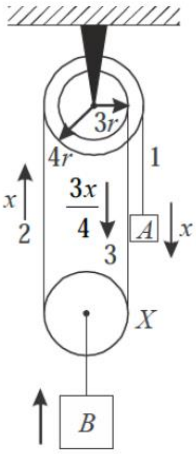
The given time is the half-life time of both left and right branches of the circuit so current in branches at this instant are given as

$$i_L=i_C=\frac{2}{2}=1\text{ A}$$

Thus total current through the battery at this instant is 2A.

17. **(4)**

If block A goes down by a distance x, string 2 will go up by same distance x and due to this, string 3 will go down by $\frac{3x}{4}$



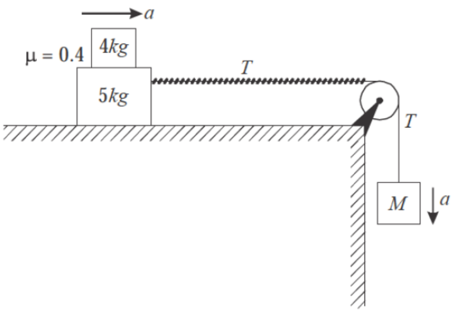
So, pulley X has to go up by distance

$$\frac{x/4}{2}=\frac{x}{8}$$

The same constrained relation exists for velocities and acceleration of blocks A and B so we use

$$a_B=\frac{a_A}{8}=\frac{2}{8}=\frac{1}{4}=0.25\text{m/s}^2$$

18. **(4)**



Common acceleration of system is

$$a=\frac{Mg}{M+9}$$

for 4kg block we use $f=4a=\frac{4Mg}{M+9}$

to slide $=0.4\times4\times10=16$ N

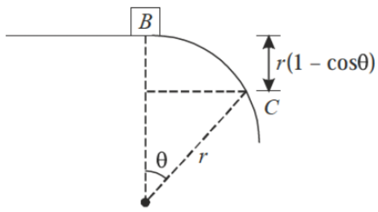
$$\Rightarrow\frac{4Mg}{M+9}=16$$

$$\Rightarrow40M=16M+144$$

$$\Rightarrow M=6\text{ kg}$$

19. **(2)**

From conservation of energy,



$$\frac{1}{2}mv_0^2 + mgr(1-\cos\theta) = \frac{1}{2}mv^2$$

$$v = \sqrt{v_0^2 + 2gr(1-\cos\theta)}$$

when block leaves the surface, $N = 0$

$$mg\cos\theta = \frac{mv^2}{r}$$

$$rg\cos\theta = v_0^2 + 2gr(1-\cos\theta)$$

$$3rg\cos\theta = \frac{rg}{4} + 2rg$$

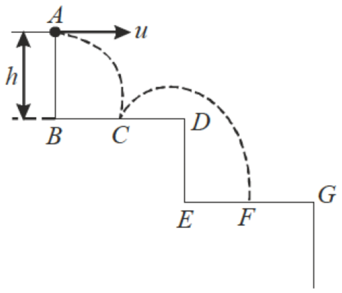
$$3\cos\theta = \frac{9}{4}$$

$$\cos\theta = \frac{3}{4}$$

$$\theta = \cos^{-1}\left(\frac{3}{4}\right)$$

20. **(2)**

The horizontal velocity of the ball during the motion remains constant



Thus, the journey from C to F takes twice the time as taken from A to C

Time of flight from A to C, $t = \sqrt{\frac{2h}{g}}$

and velocity with which ball strikes at C ,

$$v^2 = 0^2 + 2gh$$

$$v = \sqrt{2gh}$$

The velocity with which ball rebounds,

$$v_1 = ev$$

$$-h = ev(2t) - \frac{1}{2}g(2t)^2$$

$$-h = 2e(2h) - 4h$$

$$3h = 4eh$$

$$e = \frac{3}{4}$$

21. **(10)**

For coherent source $I_{\max} = 4I$

$\therefore$ For incoherent source $I = I_1 + I_2 = 2I$

$\therefore$ Difference = $2I_0$ = 10 W/m^2

22. **(70)**

In a cyclic process,

$\Delta U = 0$

$\Rightarrow Q_{\text{Cycle}} = W_{\text{Cycle}}$

$\Rightarrow W_{CA} = 0$ (Isochoric process)

$\Rightarrow W_{A\to B} = P\Delta V = nR\Delta T$

$= 1 \times R \times (370 - 350)$

$= 20R$

Now, $-50R = 20R + W_{BC}$

$\Rightarrow W_{BC} = -70R$

23. **(1)**

$$\overrightarrow{M_1} = I \cdot L^2 \cdot \hat{k}$$

$$\overrightarrow{M_1} = -I.L^2 - \left[\cos 60°\hat{k} + \sin 60\hat{i}\right]$$

$$\vec{M} = \overrightarrow{M_1} + \overrightarrow{M_2}$$

$$\vec{M} = -IL^2\left[\left(1-\frac{1}{2}\right)\hat{k} + \frac{\sqrt{3}}{2}\hat{i}\right]$$

$$|\vec{M}| = IL^2\left[\sqrt{\frac{1}{4}+\frac{3}{4}}\right]$$

$$|\vec{M}| = IL^2$$

24. **(95)**

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

Let the frequency of tuning fork be 'n', then in the first case the fundamental frequency of the wire will be (n + 5), which is given

$$n+5 = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \quad \text{....(1)}$$

Here, T = 100 N, l = 50 cm = 0.5 cm, then

$$n+5 = \frac{1}{2\times.5}\times\sqrt{\frac{100}{\mu}} = \frac{10}{\sqrt{\mu}} \quad \text{....(2)}$$

In the second case, T = 81 N, in this case the frequency of wire will be (n – 5)

$$n-5 = \frac{1}{2\times.5}\times\sqrt{\frac{81}{\mu}} = \frac{9}{\sqrt{\mu}} \quad \text{....(3)}$$

From Equation (2) and Equation (3),

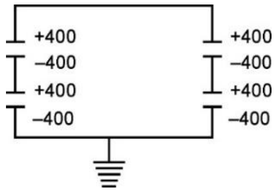
$$\frac{n+5}{n-5} = \frac{10/\sqrt{\mu}}{9/\sqrt{\mu}} = \frac{10}{9}$$

$\Rightarrow$ $9n + 45 = 10n - 50$

$n = 95$

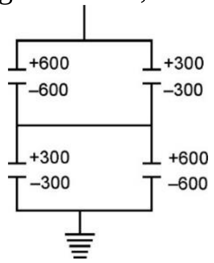
25. **(50)**

Before closing the switch, the charges on capacitor are as shown



$$q = 200\times\left[\frac{6\times3}{6+3}\right] = \frac{200\times6\times3}{8} = +400\,\mu\text{C}$$

After closing the switch,

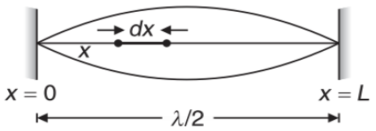


Charge flown through switch

= 0 – (–600+300) = 300 μC

26. **(4)**

Mass of string is $m = \mu\,\ell$



Since, $\dfrac{\lambda}{2} = \ell$

$\Rightarrow\ \lambda = 2\ell$

$\Rightarrow\ k = \dfrac{2\pi}{\lambda} = \dfrac{\pi}{\ell}$

Now, $A(x) = A\sin(kx)$

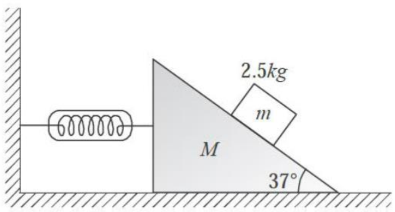
$$\Rightarrow\ dE = \frac{1}{2}(\mu dx)\omega^2A^2\sin^2 kx$$

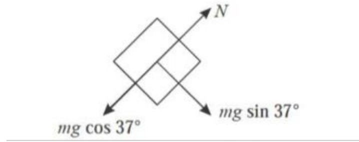
$$\Rightarrow\ dE = \frac{1}{4}\left(\mu\omega^2A^2\right)\left(1-\cos(2kx)\right)dx$$

$$\Rightarrow\ E = \int_{x=0}^{x=\ell} dE = \frac{\mu\omega^2A^2}{4}\left(x - \frac{\sin 2kx}{2}\right)\Bigg|_0^{\ell}$$

$$\Rightarrow\ E = \frac{\mu\ell\omega^2A^2}{4} = \frac{m\omega^2A^2}{4}$$

27. **(12)**

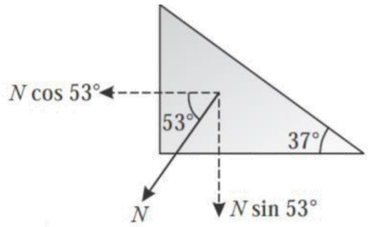




$N = mg\cos 37°$

$N = 2.5\times10\times\dfrac{4}{5}$

$N = 20\text{N}$



Reading of spring balance, $R = N\cos 53°$

$R = 20\times\dfrac{3}{5}$

$R = 12\text{N}$

28. **(1)**

In hydrogen atom the magnetic moment due to the motion of electron in n$^{\text{th}}$ orbit is given as

$$M_n = \frac{enh}{4\pi m}$$

29. **(7)**

$BE = (\Delta m)c^2$

$\Rightarrow BE = 0.0302\times930$

$\Rightarrow BE = 28.086$

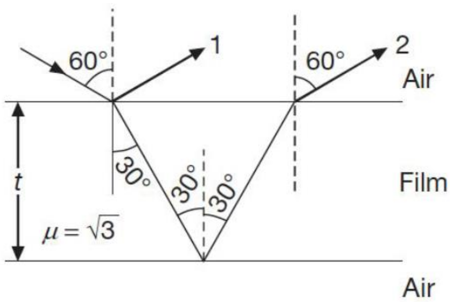
$\Rightarrow \dfrac{BE}{4} = \dfrac{28.086}{4} \approx 7\,\text{MeV}$

30. **(100)**

According to Snell's Law, we have

$1\sin(30°) = \sqrt{3}\sin r$

$\Rightarrow r = 30°$



The Optical path difference is given by

$\Delta x = 2\mu t\sec r - 2t\tan r\sin i$

$\Rightarrow \Delta x = 2\sqrt{3}\left(t\sec 30^\circ\right) - 2t\tan\left(30^\circ\right)\sin\left(60^\circ\right)$

$$\Delta x = 4t - 2t\left(\frac{1}{\sqrt{3}}\right)\left(\frac{\sqrt{3}}{2}\right) = 3t$$

Since the Ray 1 is reflected at the surface of the denser medium so it suffers an additional phase change of π or a path change of $\frac{\lambda}{2}$. So, for constructive interference, we have

$$3t = \frac{\lambda}{2}$$

$$\Rightarrow t = \frac{\lambda}{6} = 1000Ð = 100 \text{ nm}$$

SECTION-II (CHEMISTRY)

31. **(4)**

$10 \to 4, 9 \to 4, 8 \to 4, 7 \to 4, 6 \to 4, 5 \to 4$

32. **(1)**

Both NO_3^- and CO_3^{2-} have same number electrons (32 electrons) and sp^2 hybridized central atom (isostructural).

33. **(3)**

Molecular mass of the compound

$$= \frac{1}{6.06 \times 10^{-3}} \approx 165$$

Empirical formula mass

$= 3 \times 12 + 3 \times 1 + 1 \times 16 = 55$

Now, $n = \frac{165}{55} = 3$ and hence, the molecular formula = $(C_3H_3O)_3 = C_9H_9O_3$.

34. **(1)**

Given pair represents constitutional isomers.

35. **(1)**

$[Fe(CN)_6]^{4-}$ is a inner orbital complex, i.e., has d^2sp^3 hybridization with no unpaired electron.

$[MnCl_4]^2$ is a tetrahedral complex (sp^3) with 5 unpaired electrons.

$[CoCl_4]^{2-}$ is a tetrahedral complex (sp^3) with three unpaired electrons.

36. **(1)**

$$E^\circ_{cell} = E^\circ_{RP\ cathode} - E^\circ_{RP\ anode}$$

$$= 0.13 - (-0.34) = 0.47$$

$2TI + Sn^{4+} \rightarrow Sn^{2+} + 2TI^+$

$$E_{cell} = 0.47 - \frac{0.059}{2}\log\frac{[TI^+]^2[Sn^{2+}]}{[Sn^{4+}]}$$

$$= 0.47 - \frac{0.059}{2}\log[10]^2$$

$= 0.47 - 0.059 = 0.411$ V

37. **(4)**

For ‘B’

$$M = \frac{1000 \times K_b \times 1}{1 \times 100}\left(M = \text{ molar mass of B}\right)$$

$$K_b = \frac{M}{10}$$

For ‘A’

$$M_1 = \frac{1000 \times M \times 2}{1 \times 10 \times 100}\ \left(M_1 = \text{ molar mass of A}\right)$$

$M_1 = 2M$

38. **(2)**

Take V=1 L

$$-\frac{d[O_2]}{dt} = \frac{6.4}{32} \times 10^3 \text{ mol / L / min} = 0.2 \times 10^3$$

$$\therefore \frac{d[SO_3]}{dt} = 2\left[\frac{-d[O_2]}{dt}\right]$$

$= 2 \times 0.2 \times 10^3$ mole/L/min

$$\Rightarrow \frac{2 \times 0.2}{60} \times 80 \text{ kg / s.} \simeq 0.53 \text{ kg / s.}$$

39. **(2)**

Electronic configuration of X (atomic number =34) is $1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^4$

Group number =10 + number of valence electrons =10 + 6 = 16

40. **(1)**

The geometry of IF_7 is pentagonal bipyramidal.

41. **(4)**

$\Delta U = \Delta H - \Delta n_g \cdot RT$

$$= (-72.3) - (-1) \times \frac{8.314}{1000} \times 298 = -69.8 \text{ kJ}$$

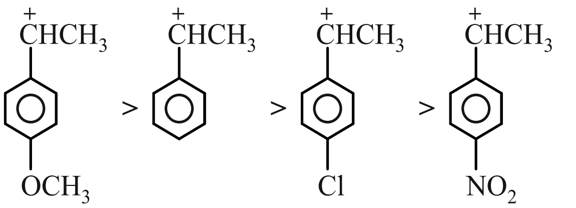
As HCl is limiting reagent, for the given amount,

$\Delta U = 2 \times (-69.8) = -139.6$ kJ

42. **(4)**
The composition of distillate and solution remains same in azeotrope.

43. **(3)**
$CH_3CH = CHCH = CH_2 \rightarrow$
$CH_3CHO + CHO - CHO + HCHO$

44. **(2)**
The reaction involves the formation of carbocation as intermediate. Hence more the stability of the carbocation, more will be the rate of reaction.

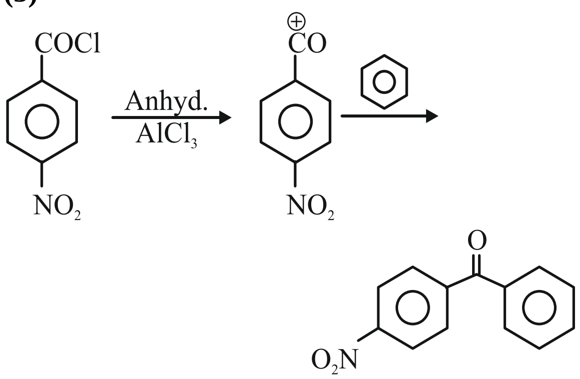


45. **(2)**
Nitrogen has no d-orbitals in its valence shell.

46. **(3)**
Negative charge will delocalize in vacant orbital of Cl atom.

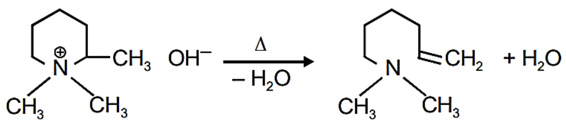
47. **(4)**
H_3O^+ gives (A), i.e., Markownikov's addition, through carbocation formation. (B) formed by hydroboration oxidation that gives anti-Markownikov's product.

48. **(3)**

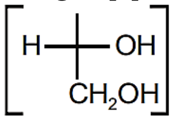


Given product.

49. **(2)**
Less substituted alkene is more stable



50. **(1)**
OH group present on right side



Single digit

51. **(8)**
$n_{eq}KHC_2O_4 \cdot H_2C_2O_4 \cdot 2H_2O = n_{eq}NaOH$
or $n \times 3 = 30 \times 1 \Rightarrow n = 10$
Now, $n_{eq}KHC_2O_4 \cdot H_2C_2O_4 \cdot 2H_2O = n_{eq}KMnO_4$
or $10 \times 4 = n \times 5 \Rightarrow n = 8$

52. **(6)**

$$\frac{q}{\Delta U} = \frac{n \cdot C_m \cdot \Delta T}{n \cdot C_{V,m} \cdot \Delta T} = \frac{Q}{Q - \frac{Q}{2}}$$

$$\Rightarrow \frac{C_m}{\frac{3}{2}R} = 2$$

$\Rightarrow C_m = 6$ cal/K mol

53. **(3)**
ii, iv, v $\rightarrow$ order = molecularity

54. **(3)**
$CO_3^{2-}, CH_3COO^{\ominus}, SO_4^{2-} \rightarrow$ flexidentate

55. **(5)**

$$E^o_{Fe^{3+}|Fe^{2+}} = \frac{3 \times E^o_{Fe^{3+}|Fe} - 2 \times E^o_{Fe^{2+}|Fe}}{3-2}$$

$$= \frac{3 \times (-0.04) - 2 \times (-0.44)}{1} = 0.76 \text{ V}$$

Now, $E_{Fe^{3+}|Fe^{2+}} = E^o_{Fe^{3+}|Fe^{2+}} - \frac{0.06}{1} \log \frac{[Fe^{2+}]}{[Fe^{3+}]}$

or $0.718 = 0.76 - 0.06 \log \frac{[Fe^{2+}]}{[Fe^{3+}]} \Rightarrow \frac{[Fe^{2+}]}{[Fe^{3+}]} = 5$

56. **(2)**
$Al_4C_3 \rightarrow 4Al^{3+} + 3C^{-4}$

57. **(4)**
There are total four isomers are possible for the given complex

58. **(6)**
Chromic acid = CrO_3

59. **(2)**

Concept of limiting reagent should be applied first

$H^+ + OH^- \longrightarrow H_2O$

(2.5 + 1) 2.5

m mole m mole

L.R.

$$\left[H^+\right]_{final} = \frac{(3.5-2.5)}{25+50+25} = 10^{-2}\,M$$

$pH = 2$

60. **(5)**

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$\left(\frac{V_2}{1.25}\right)^{\gamma} = 10$$

$$\frac{V_2}{1.25} = (10)^{3/5} \text{ as } \gamma = \frac{5}{3}$$

$V_2 = 4.975$

SECTION-III (MATHEMATICS)

61. **(3)**

$$|A(t)| = \frac{\sin 4t}{\sin t}$$

$$|A(4t)| = \frac{\sin 16t}{\sin 4t}.$$

62. **(1)**

Let $f(x) = x^3 + bx^2 + cx + 1.$

$f(0) = 1 > 0,\ f(-1) = b - c < 0$

So, $\alpha \in (-1,0). So, 2\tan^{-1}(\operatorname{cosec}\alpha) + \tan^{-1}\left(2\sin\alpha\sec^2\alpha\right)$

$$= 2\tan^{-1}\left(\frac{1}{\sin\alpha}\right) + \tan^{-1}\left(\frac{2\sin\alpha}{1-\sin^2\alpha}\right)$$

$$= 2\left[\tan^{-1}\left(\frac{1}{\sin\alpha}\right) + \tan^{-1}(\sin\alpha)\right]$$

$$= 2\left(-\frac{\pi}{2}\right) = -\pi(\ as\ \sin\alpha < 0)$$

63. **(4)**

(a) $AM \ge GM$

Function $\frac{e^x + e^{-x}}{2} \ge 1$ so not onto function

(b) $f'(x) = 4x^3 - 9x^2 = x^2(4x - 9)$ so, not one-one function

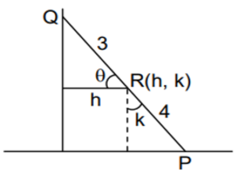
(c) $f'(x) = 54x^2 - 42x^2 + 8$

$= 2\left(27x^2 - 21x + 4\right)$ So, not one-one function

64. **(4)**

(5, 12) $\in R$ but (12, 5) $\notin R$.

65. **(1)**



$$\cos\theta = \frac{h}{3}, \cos(90-\theta) = \frac{k}{4}$$

$$\sin\theta = \frac{k}{4}$$

$$\frac{x^2}{3^2} + \frac{y^2}{4^2} = 1$$

66. **(4)**

$$\sum_{n=1}^{\infty} \sin^{-1} \frac{(2n+1)\left[\sqrt{n^2+2n} - \sqrt{n^2-1}\right]}{n(n+1)\left[\left(n^2+2n\right) - \left(n^2-1\right)\right]}$$

$$\sum_{n=1}^{\infty} \sin^{-1}\left[\frac{\sqrt{n^2+2n} - \sqrt{n^2-1}}{n(n+1)}\right]$$

$$\sum_{n=1}^{\infty} \sin^{-1}\left[\frac{\sqrt{(n+1)^2-1} - \sqrt{n^2-1}}{n(n+1)}\right]$$

$$\sum_{n=1}^{\infty} \sin^{-1}\left[\frac{1}{n}\sqrt{1-\frac{1}{(n+1)^2}} - \frac{1}{n+1}\sqrt{1-\frac{1}{n^2}}\right]$$

$$\sum_{n=1}^{\infty}\left(\sin^{-1}\frac{1}{n} - \sin^{-1}\frac{1}{n+1}\right)$$

$$\therefore \quad S_n = \left(\sin^{-1}1 - \sin^{-1}\frac{1}{2}\right) +$$

$$\left(\sin^{-1}\frac{1}{2} - \sin^{-1}\frac{1}{3}\right) + \left(\sin^{-1}\frac{1}{3} - \sin^{-1}\frac{1}{4}\right) + ..$$

$$..... + \left(\sin^{-1}\frac{1}{n} - \sin^{-1}\frac{1}{n+1}\right)$$

$$S_n = \frac{\pi}{2} - \sin^{-1}\frac{1}{n+1}$$

$$\lim_{n\to\infty} S_n = \frac{\pi}{2}.$$

67. (3)

$$\frac{\alpha^2+\alpha+1}{\alpha^2-\alpha+1}$$

$$=\frac{\alpha^3-1}{(\alpha-1)\left(\alpha^2-\alpha+1\right)}$$

$$=\frac{2\alpha^2-6\alpha}{\alpha^3-2\alpha^2+2\alpha-1}=\frac{2\alpha(\alpha-3)}{-4\alpha}=\frac{3-\alpha}{2}$$

Similarly remaining

$$\frac{\alpha(3-\alpha)+\beta(3-\beta)+\gamma(3-\gamma)}{2}$$

$$\frac{3(\alpha+\beta+\gamma)+\left(\alpha^2+\beta^2+\gamma^2\right)}{2}=7$$

68. (2)

$$\lim_{n\to\infty}\sum_{r=1}^{n}\ln\left(\sqrt[n]{\frac{4r^2}{n^2}}\right)=\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}\ln\left(\frac{4r^2}{n^2}\right)$$

$$=\int_0^1\ln\left(4x^2\right)dx$$

$$=\int_0^1(2\ln 2+2\ln x)\,dx$$

69. (4)

The sum of the first n integers is given by $\frac{n(n+1)}{2}$, so $\frac{37(37+1)}{2}=703.$

Therefore, $703 - x - y = xy$

Rearranging, $xy + x + y = 703$. We can factor this equation by SFFT to get $(x + 1)(y + 1) = 704$

Looking at the possible divisors of $704 = 2^6 . 11$, 22 and 32 are within the constraints of $0 < x \le y \le 37$ so try those:

$(x + 1)(y + 1) = 22 \cdot 32$

$x + 1 = 22, y + 1 = 32$

$x = 21, y = 31$

Therefore, the difference $y - x = 31 - 21 = 10$

70. (3)

$$x\cdot f'(x)+f(x)=1+f(x)\Rightarrow f'(x)=\frac{1}{x}$$

$f(x) = \ln x + k$

But $f(1) = 1 \therefore k = 1$

$\therefore f(x) = \ln x + 1$

$$\sum_{k=1}^{10}f\left(e^k\right)=\sum_{k=1}^{10}\ln e^k+1=\sum_{k=1}^{10}(k+1)=65$$

71. (1)

at $x=\frac{\pi}{4}$

$$\lim_{x\to\frac{\pi}{4}^-}f(x)=\lim_{x\to\frac{\pi}{4}^+}f(x)=f\left(\frac{\pi}{4}\right)$$

$$\Rightarrow\frac{\pi}{4}+a=\frac{\pi}{2}+b\Rightarrow a-b=\frac{\pi}{4}\qquad\text{.....(i)}$$

at $x=\frac{\pi}{2}$ we observe that

$$\lim_{x\to\frac{\pi}{2}^-}f(x)=b,\ \lim_{x\to\frac{\pi}{2}^+}f(x)=-a-b\ and\ f\left(\frac{\pi}{2}\right)=$$

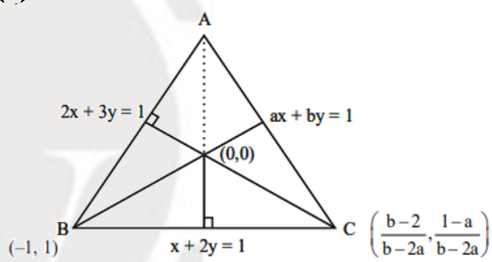
So, $2b+a=0$ ....(ii)

On solving (i) and (ii), we get

$\Rightarrow b=-\frac{\pi}{12}$ and $a=\frac{\pi}{6}$

Hence, $5\left(\frac{a}{b}\right)^2=20$

72. (2)



73. (4)

Image of vertex of P_1 in $3x - y + 10 = 0$ will be vertex of P_2

$$\frac{x_2-1}{3}=\frac{y_2-3}{-3}=\frac{-2(3-8+1)}{10}$$

$\therefore$ Vertex of $P_2 = (-2, 9)$

Similarly focus of P2 is $(-9, 3)$ $\therefore$ $\alpha=-9$ and $\beta=3$

Slop of directrix $=-\frac{7}{6}$

Equation of directrix $y-15=-\frac{7}{6}(x-5)$

$7x + 6y = 125 \therefore \gamma = 125$

74. (2)

Let $\vec{r}_1=a\hat{i}+b\hat{j}+c\hat{k}$

$\vec{r}_2=3\hat{i}+4\hat{j}+5\hat{k}$

$$|\vec{r}_1\times\vec{r}_2|^2\le|\vec{r}_1|^2|\vec{r}_2|^2\qquad\text{.....(1)}$$

$$\Rightarrow \vec{r}_1 \times \vec{r}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & c \\ 3 & 4 & 5 \end{vmatrix}$$

$$= \hat{i}(5b-4c)+\hat{j}(3c-5a)+\hat{k}(4a-3b)$$

So, from (1)

$$(5b-4c)^2+(3c-5a)^2+(4a-3b)^2 \le 50.$$

75. **(1)**

Case I :

$$x^2-11|x|+30=0$$
$$(|x|-5)(|x|-6)=0$$
$$|x|=5,6 \Rightarrow x=\pm 5,\pm 6$$

Case II:

$$x^2-5|x|+5=1$$
$$x^2-5|x|+4=0$$
$$(|x|-1)(|x|-4)=0$$
$$|x|=1,4$$
$$x=\pm 1,\pm 4$$

Case III:

$x^2-5|x|+5=1$ and power is even

$$x^2-5|x|+6=0$$
$$(|x|-2)(|x|-3)=0$$
$$|x|=2,3$$
$$x=\pm 2,\pm 3$$

Power $x^2-11|x|+30$ is even in both cases

So total number of solution is 12

76. **(3)**

$$L=\lim_{x\to\beta}\frac{e^{2\left(x^2+bx+c\right)}-1-2\left(x^2+bx+c\right)}{(x-\beta)^2}$$

$$L=\lim_{x\to\beta}\frac{\left[e^{2\left(x^2+bx+c\right)}-1-2\left(x^2+bx+c\right)\right](x-\alpha)^2}{(x-\alpha)^2(x-\beta)^2}$$

$= 2\,(\beta-\alpha)^2$

$= 2\,[(\beta+\alpha)^2 - 4\alpha\beta] = 2\,(b^2-4c)$

77. **(1)**

$L=\lim_{x\to 1}\dfrac{f(x)}{x-1}$ (finite value)

$\therefore \quad f(1)=0$ and $L=\lim_{x\to 1} f'(1)$

Now $f(x)+f'(x)+f''(x)=x^5+64$ ....(1)

So, clearly $f(x)$ is polynomial of degree 5.

Differentiating $f'(x)+f''(x)+f'''(x)=5x^4$

$\Rightarrow \quad f''(x)+f'''(x)+f^{\text{iv}}(x)=20x^3$ ....(2)

$\Rightarrow \quad f'''(x)+f^{\text{iv}}(x)+f^{\text{v}}(x)=60x^2$ ....(3)

From (2)-(3), we get

$f^{\text{v}}(x)-f''(x)=60x^2-20x^3$ ....(3)

Also, from (3), differentiating two more times we get

$f^{\text{v}}(x)=120$

78. **(1)**

Given differential equation

$$\frac{dy}{dx}+\left(8+4\cot 2x\right)y=\frac{2e^{-4x}}{\sin^2 2x}\left(2\sin 2x+\cos 2x\right)$$

I.F. $= \int\left(8+4\cot 2x\right)dy = e^{8x+2\log_e(\sin 2x)}$

$$=e^{8x}\cdot\sin^2 2x$$

$\therefore \quad$ Solution is

$$y\left(e^{8x}\cdot\sin^2 2x\right)=\int 2e^{4x}\left(2\sin 2x+\cos 2x\right)dx+C$$

$$\therefore \quad y(x)=\frac{e^{-4x}}{\sin 2x}$$

$$\therefore \quad y\left(\frac{\pi}{6}\right)=\frac{e^{-4\frac{\pi}{6}}}{\sin\left(2\cdot\frac{\pi}{6}\right)}=\frac{2}{\sqrt{3}}e^{-\frac{2\pi}{3}}$$

79. **(1)**

Period of both $\sin^2 x$ and $\left\{\dfrac{x}{\pi}\right\}$ is π.

So, $I=\displaystyle\int_0^{100\pi}\frac{\sin^2 x}{e^{\left\{\frac{x}{\pi}\right\}}}dy=100\int_0^{\pi}\frac{\sin^2 x}{e^{\frac{x}{\pi}}}dx$

$$=50\left\{\int_0^{\pi}e^{-x/\pi}dx-\int_0^{\pi}e^{-x/\pi}\cos 2x\,dx\right\}$$

$$=\pi\left(1-e^{-1}\right)-2\pi\int_0^{\pi}e^{-x/\pi}\sin 2x\,dx$$

$$\therefore \quad I=50\left\{\pi\left(1-e^{-1}\right)-\frac{\pi\left(1-e^{-1}\right)}{1+4\pi^2}\right\}$$

$$=\frac{200\left(1-e^{-1}\right)}{1+4\pi^2}$$

80. **(2)**

Let C, S, B and T be the events of Ramesh using car, scooter, bus or train. Let L be the event of Ramesh reaching offices late. By hypothesis

$P(C)=\frac{1}{7}, P(S)=\frac{3}{7}, P(B)=\frac{2}{7}$ and $P(T)=\frac{1}{7}$

$P\left(\frac{\bar{L}}{C}\right)=1-\frac{2}{9}=\frac{7}{9}, P\left(\frac{\bar{L}}{T}\right)=1-\frac{1}{9}=\frac{8}{9}$

By baye's theorem,

$$P\left(\frac{C}{\bar{L}}\right)=\frac{P(C)P\left(\frac{\bar{L}}{C}\right)}{P(C)P\left(\frac{\bar{L}}{C}\right)+P(S)P\left(\frac{\bar{L}}{S}\right)+P(B)P\left(\frac{\bar{L}}{B}\right)+P(T)P\left(\frac{\bar{L}}{T}\right)}$$

81. **(50)**

$$\begin{vmatrix} \frac{20}{1} & \frac{20}{2} & \frac{20}{3} \\ \frac{20}{4} & \frac{20}{5} & \frac{20}{6} \\ \frac{20}{7} & \frac{20}{8} & \frac{20}{9} \end{vmatrix}$$

$= \frac{50}{21}$

82. **(10)**

$=\tan^{-1}\left|\frac{1-\cos\pi/5}{\sin\pi/5}\right| \quad \left\{\frac{1-\cos\alpha}{\sin\alpha}=\tan\frac{\alpha}{2}\right\}$

$=\tan^{-1}\left|\tan\frac{\pi}{10}\right|$

$=\tan^{-1}\tan\frac{\pi}{10}=\frac{\pi}{10}$

$\tan^{-1}\tan t=t \quad$ if $t\in\left(\frac{-\pi}{2},\frac{\pi}{2}\right)$

$1-\cos\frac{\pi}{5}>0,$

$\sin\frac{\pi}{5}>0$

83. **(28)**

If a_1 is mapped to 2, we have ${}^7C_5$ ways of mapping rest of the elements.

If a_1 is mapped to 3, we have ${}^6C_5$ ways of mapping rest of the elements.

If a_1 is mapped to 4, we have ${}^5C_5$ ways of mapping rest of the elements.

If a_1 is mapped to 4, we have ${}^5C_5$ ways of mapping rest of the elements.

Hence total number of increasing function = ${}^7C_5$ + ${}^6C_5$ + ${}^5C_5$ = 28.

84. **(2)**

$f'(x)=-2-3x^2<0\Rightarrow f(x)$ is decreasing

$\therefore f(f(x))<f(-x)\Rightarrow f(x)>-x$

$\Rightarrow 30-x-x^3>0\Rightarrow x^3+x-30<0$

$\Rightarrow (x-3)\left(x^2+3x+10\right)<0$

$\Rightarrow x<3$

No. of values = 2

85. **(0)**

$\because \quad x\to 0^-\Rightarrow\{x\}=1+x$

$\therefore \quad \lim_{x\to 0^-} f(x)=\lim_{x\to 0}\frac{\sin^{-1}(-x)\cos^{-1}(-x)}{\sqrt{2(1+x)}\cdot(-x)}=\frac{\pi}{2\sqrt{2}}$

& for $x\to 0^+\Rightarrow\{x\}=x$

$\lim_{x\to 0^+} f(x)=\lim_{x\to 0}\frac{\sin^{-1}(1-x)\cos^{-1}(1-x)}{\sqrt{2x}(1-x)}=\frac{\pi}{2}$

86. **(9)**

Circle $x^2 + y^2 - 2x - 4y + 4 = 0$

Center of the circle is C (1, 2) and radius, $r = 1$

Line $3x + 4y - k = 0$ intersects the circle at two distinct points. So, distance of the center of the circle from the line must be less than 'r'.

$\therefore \quad \left|\frac{3\times 1+4\times 2-k}{\sqrt{3^2+4^2}}\right|<1$

$\Rightarrow \quad |k-11| < 5$

$\Rightarrow \quad k \in \{7, 8, 9, \ldots..15\}$ since $k \in Z$

87. **(9)**

$36 = 2^2 \times 3^2$

So, the digits of 5-digit numbers can be

(1, 1, 1, 9, 4), (1, 1, 1, 6, 6), (1, 1, 2, 2, 9), (1, 1, 3, 3, 4), (1, 1, 6, 2, 3), (1, 2, 2, 3, 3)

So, total number of numbers

$=\frac{5!}{3!}+\frac{5!}{3!\times 2!}+3\times\frac{5!}{2!\times 2!}+\frac{5!}{2!}=180$

88. **(5)**

We must have $0 \le p(E_i) \le 1$ for $i = 1, 2, 3$

$0\le\frac{2+3p}{6}\le 1\Rightarrow -\frac{2}{3}\le p\le\frac{4}{3}$ ….(1)

$0\le\frac{2-p}{8}\le 1\Rightarrow -6\le p\le 2$ …..(2)

$0\le\frac{1-p}{2}\le 1\Rightarrow -1\le p\le 1$ …...(3)

From (1), (2) and (3), we get

$$-\frac{2}{3} \le p \le 1 \quad .....(4)$$

$$\Rightarrow \quad p \ge \frac{2}{3} \quad ......(5)$$

From (4) and (5), we get

$$\frac{2}{3} \le p \le 1$$

$\therefore \quad P_1 = 1$ and $P_2 = \frac{2}{3}$,

So, $P_1 + P_2 = \frac{5}{3}$

89. **(3)**

From the given information,

$p'(x) = a(x - 1)(x - 2)$

$$\therefore \quad p(x) = a\left(\frac{x^3}{3} - 3\frac{x^2}{2} + 2x\right) + C$$

Now, $p(1) = 8 \Rightarrow \frac{5}{6}a + C = 8 \quad ....(1)$

$p(2) = 4 \Rightarrow \frac{2}{3}a + C = 4 \quad ....(2)$

Solving (1) and (2), we get $a = 24$, $C = 12$

$\therefore \quad P(0) = C = -12$

90. **(1)**

$$\frac{h - \lambda^2}{1} = \frac{k - 2\lambda}{-1} = \frac{-2(\lambda^2 - 2\lambda + 1)}{2}$$

$h_1 = 2\lambda - 1$; $k = \lambda^2 + 1$

Eliminating $\lambda, k = \left(\frac{h+1}{2}\right)^2 + 1$

$4K = h^2 + 2h + 3$

Locus is $(x + 1)^2 = 4(y - 1)$