JEE MAIN (2023-24) Mock Test Series

Paper - 01

DURATION : 180 Minutes **M. MARKS : 300**

ANSWER KEY

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(2)	31.	(4)	61.	(3)
2.	(2)	32.	(4)	62.	(2)
3.	(4)	33.	(2)	63.	(3)
4.	(2)	34.	(1)	64.	(1)
5.	(3)	35.	(4)	65.	(3)
6.	(2)	36.	(2)	66.	(1)
7.	(4)	37.	(1)	67.	(3)
8.	(4)	38.	(4)	68.	(1)
9.	(4)	39.	(3)	69.	(2)
10.	(2)	40.	(1)	70.	(2)
11.	(4)	41.	(4)	71.	(2)
12.	(3)	42.	(2)	72.	(4)
13.	(3)	43.	(4)	73.	(3)
14.	(2)	44.	(3)	74.	(2)
15.	(2)	45.	(4)	75.	(4)
16.	(2)	46.	(2)	76.	(2)
17.	(4)	47.	(4)	77.	(1)
18.	(1)	48.	(2)	78.	(3)
19.	(3)	49.	(4)	79.	(2)
20.	(1)	50.	(3)	80.	(4)
21.	(40)	51.	(10)	81.	(36)
22.	(6)	52.	(20)	82.	(70)
23.	(27)	53.	(12)	83.	(0)
24.	(1)	54.	(40)	84.	(3)
25.	(4)	55.	(6)	85.	(1)
26.	(2)	56.	(25)	86.	(81)
27.	(2)	57.	(73)	87.	(1)
28.	(125)	58.	(2)	88.	(15)
29.	(95)	59.	(25)	89.	(4)
30.	(7)	60.	(14)	90.	(5)

SECTION-I (PHYSICS)

1. **(2)**

$Q = [(7.835 \times 231) + (7.07 \times 4) - (7.8 \times 235)]$

$= 5.18$ MeV

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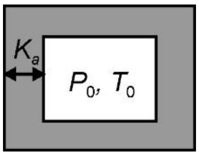
$= 5.18$ MeV

2. **(2)**

$$\frac{dQ}{dt} = c\frac{dT}{dt}$$



$$\frac{dQ}{dt} = c\frac{dT}{dt} \qquad \text{...(i)}$$

$$\frac{dQ}{dt} = \frac{kA}{t_0}\left(T - \frac{T_0}{2}\right) \qquad \text{...(ii)}$$

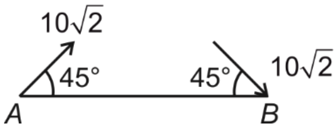
From equation (i) and equation (ii)

$$\frac{dQ}{dt} = C\frac{dT}{dt} = -\frac{kA}{t_0}\left(T - \frac{T_0}{2}\right)$$

$$\Rightarrow \quad \int_{T_0}^{T} \frac{C}{T - \frac{T_0}{2}} dt = \int_0^t -\frac{kA}{t_0} dt$$

$$\Rightarrow \quad T = \frac{T_0}{2}\left(1 + e^{\frac{-kA}{Ct_0}t}\right)$$

3. **(4)**



$t = T$

$= \frac{(2)10}{10}$

$= 2$ s

4. **(2)**

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

Point $A(-2f, 2f)$

$\Rightarrow 2f = 40$

$\Rightarrow f = 20$ cm

5. **(3)**

$u = u_s + u_i$

Total energy of the system $= u_1 + u_2 + u_{12}$

$$= \frac{q_1^2}{8\pi\varepsilon_0 a} + \frac{q_2^2}{8\pi\varepsilon_0 b} + q_1 v_2$$

$$= \frac{q_1^2}{8\pi\varepsilon_0 a} + \frac{q_2^2}{8\pi\varepsilon_0 b} + \frac{q_1 . q_2}{4\pi\varepsilon_0 b}$$

6. **(2)**

$$E = \frac{KQ}{a^2}\sqrt{3 + 3\times 1} = \frac{\sqrt{6}KQ}{a^2}$$

7. **(4)**

$F = F_R + F_M$

$F_R = \frac{Mg}{2}$

$F_M = \frac{\Delta P}{\Delta t} = Mg$

$F = \frac{3}{2}Mg$

8. **(4)**

For maximum intensity, path difference $= n\lambda$

Path difference $= 2 \times 8.5$ cm $= 17$ cm

9. **(4)**

Assume upper hemisphere,

$B = \frac{2}{3}\pi R^3 \times \rho g$

$F_2 = \pi R^2 \times \rho g(2R)$

$\therefore \quad F_1 = F_2 - B = \frac{4}{3}\pi R^3 \rho g$

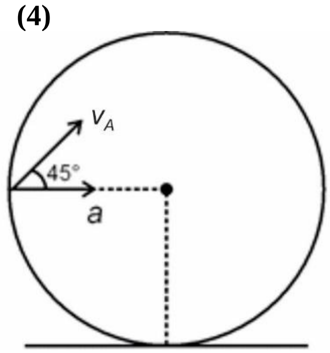
10. **(2)**

$$V_c = \int_0^R -4\pi r^2 dr \times \frac{1}{r} \times \frac{G}{r}$$

$$M = \int_0^R 4\pi r^2 dr \times \frac{1}{r} = 4\pi \times \frac{R^2}{2}$$

$$V_c = \int_0^R -(4\pi r^2 dr) \times \frac{1}{r} \times \frac{G}{r} = -\frac{2GM}{R}$$

11. **(4)**



$$v_A = \left(\frac{v_0}{R}\right) \times \left(\sqrt{2}\,R\right)$$

$= \sqrt{2}\,v_0$

$$a_A = \omega^2 \times R = \frac{v_0^2}{R}(\text{right})$$

$$\therefore \quad a_\perp = \frac{1}{\sqrt{2}} \times \left(\frac{v_0^2}{R}\right)$$

$$\therefore \text{ rad of curve } = \frac{\left(\sqrt{2}\,v_0\right)^2}{\left(\dfrac{v_0^2}{\sqrt{2}\,R}\right)} = 2\sqrt{2}\,R$$

12. **(3)**

The ray SM after reflection undergoes a phase change of π, for maxima at P & minima at X, path difference between S & S' = $\frac{\lambda}{2}$

(S' is virtual source producing back PM, symmetric
o S)

Comparing with YDSE, $d = 4x$, $D = 600x$

Path difference = $\frac{x.d}{D}$

$$\frac{\lambda}{2} = \frac{x\,(4x)}{600\,x} \quad \Rightarrow \quad x = 75\,\lambda$$

13. **(3)**

$$T_{\max} = mg + \frac{mv_{\max}^2}{l}$$

$$V_{\max}^2 = 4gl + 2gl \;= 6gl$$

$$T_{\max} = mg + 6mg \;= 7mg$$

14. **(2)**

For perfectly absorbing,

$$F_n = \frac{P}{C}$$

For perfectly reflecting,

$$F_n = \frac{2P}{C}$$

For the given situation,

$$F_n = \frac{P}{C} + \frac{2}{5}\frac{P}{C}$$

$$\Rightarrow \; F_n = \frac{7}{5}\frac{P}{C} = 1.4\frac{P}{C}$$

15. **(2)**

$$\frac{E_S}{E_P} = \frac{N_S}{N_P}$$

$$\frac{E_S}{E_P} = \frac{N_S}{N_P}$$

$$\frac{E_S}{20} = \frac{5000}{500}$$

$$E_S = 200\text{ V}$$

Frequency remains same = 50 Hz

16. **(2)**

$$\vec{\tau}_{\text{Hinge}} = I\,\alpha$$

$$\frac{\sigma QL}{2\varepsilon_0} = \frac{ML^2}{3}\alpha$$

$$\alpha = \frac{3\sigma Q}{2ML\varepsilon_0}$$

17. **(4)**

$$\frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi y}$$

$$\frac{\mu_0 I_1}{2\pi x} = \frac{\mu_0 I_2}{2\pi y}$$

18. **(1)**

$$q = \int i\,dt$$

$$q = \int_0^\tau i dt = \int_0^\tau l_{\max}\left(1 - e^{-\frac{t}{\tau}}\right)dt$$

$$q = \frac{l_{\max}\tau}{e}$$

19. **(3)**

$$\sum \vec{F} = 0$$

$(M + m)g = (50 + 25)g = \left(T + \frac{T}{2}\right)$

$$\Rightarrow \; T = 500\text{ N}$$

20. **(1)**

Output will be high when both diodes do not conduct.

If $A = 0$, $B = 5$ V, then D_1 conducts and $y = 0$

If $A = 5$ V, $B = 0$, then D_1 and D_2 conducts and $y = 0$

If $A = 0$, $B = 0$, then D_1 and D_2 conducts and $y = 0$

If $A = 5$ V, $B = 5$ V, both do not conduct and
$y = 9$ V

$\Rightarrow$ AND gate

21. **(40)**

$W_T = \Delta KE$

$\Rightarrow \quad W_F + W_{2F} = k_f$

$\Rightarrow \quad -20 + (20)4 = k_f \Rightarrow k_f = 60$ J

$\Rightarrow \quad k_T + k_R = 60$ also $\dfrac{k_T}{k_R} = 2$

$\Rightarrow \quad k_T = \dfrac{2}{3} \times 60 = 40$ J

22. **(6)**

$i = \dfrac{Bvl}{R_{eq}}; \; F = iBl$

$i = \dfrac{Bvl}{R_{eq}} = \dfrac{3 \times 2 \times 2}{12} = 1$ A

$F = iBl = 1 \times 3 \times 2 = 6$ N

23. **(27)**

$$\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

24. **(1)**

$$T = \frac{2\mu \sin\alpha}{g\cos\theta} = T_1 = T_2 \, ; \frac{T_1}{T_2} = 1$$

25. **(4)**

$\vec{L_i} = \vec{L_f}$ (About point on horizontal surface)

$\Rightarrow \dfrac{mR^2}{2}\omega_0 = 2\left(\dfrac{3}{2}mR^2\right)\omega$

$\Rightarrow \quad \omega = \dfrac{\omega_0}{6}$

$\Rightarrow V_{\text{cm}} = \dfrac{R\omega_0}{6} \Rightarrow J = \dfrac{mR\omega_0}{6}$

$\Rightarrow \text{J} = \dfrac{(2)(1)12}{6} = 4 \text{ kg} \cdot \text{m/s}$

26. **(2)**

$V = \dfrac{kq}{R}$

$\dfrac{kq_A}{R} + \dfrac{kq_B}{2R} = 2V \qquad \ldots (1)$

$\dfrac{kq_A}{2R} + \dfrac{kq_B}{2R} = \dfrac{3}{2}V \qquad \ldots (2)$

From equation (1) and (2), $\dfrac{q_A}{q_B} = \dfrac{1}{2}$

After B is earthed $V_B = 0$

$\therefore \quad q_B = -q_A$

After earthing

$V_A - V_B = kq_A\left[\dfrac{1}{R} - \dfrac{1}{2R}\right] = \dfrac{kq_A}{2R}$

Putting $q_B = 2q_A$ in equation (1)

$\dfrac{kq_A}{2R} = \dfrac{V}{2}, V_A - V_B = \dfrac{V}{2}$

$\because \quad V_B = 0 \Rightarrow V_A = \dfrac{V}{2}$

27. **(2)**

$\because \quad c = \dfrac{\varepsilon_0 A}{d}$

On increasing temperature,

$c' = \dfrac{\varepsilon_0 A'}{d'} = \dfrac{\varepsilon_0 A(1+2\alpha_1 T)}{d(1+\alpha_2 T)}$

$c' = \dfrac{\varepsilon_0 A(1+2\alpha_1 T)(1-\alpha_2 T)}{d}$

$= \dfrac{\varepsilon_0 A}{d}(1 + (2\alpha_1 - \alpha_2)T - 2\alpha_1\alpha_2 T^2)$

c' = constant with temperature

$\therefore \quad 2\alpha_1 - \alpha_2 = 0$

$2\alpha_1 = \alpha_2$

28. **(125)**

$\dfrac{1}{v} + \dfrac{1}{u} = \dfrac{1}{f}$

For 1st reflection

$u_1 = -15, \; f = -10$

$\Rightarrow v_1 = -30$ cm

For 2nd reflection

$u_2 = -(40 - 30) = -10$ cm

$\Rightarrow v_2 = +10$ cm

For 3rd reflection

$u_3 = -(40 + 10) = -50$ cm, $\quad f = -10$ cm

$\Rightarrow v_3 = -12.5$ cm

29. **(95)**

$$f = \frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

Let the frequency of tuning fork be 'n', then in he first case the fundamental frequency of the wire will be (n + 5), which is given

$$n+5 = \frac{1}{2l}\sqrt{\frac{T}{\mu}} \qquad ...(i)$$

Here, T = 100 N, l = 50 cm = 0.5 cm, then

$$n+5 = \frac{1}{2\times.5}\times\sqrt{\frac{100}{\mu}} = \frac{10}{\sqrt{\mu}} \qquad ...(ii)$$

In the second case, T = 81 N, in this case the frequency of wire will be (n – 5)

$$n-5 = \frac{1}{2\times.5}\times\sqrt{\frac{81}{\mu}} = \frac{9}{\sqrt{\mu}} \qquad ...(iii)$$

From Equation (ii) and Equation (iii),

$$\frac{n+5}{n-5} = \frac{10/\sqrt{\mu}}{9/\sqrt{\mu}} = \frac{10}{9}$$

$\Rightarrow$ $9n + 45 = 10n - 50$

$n = 95$

30. **(7)**

$W_f = |k_f - k_i|$

$L_i = L_f$

$$\Rightarrow v_f = \frac{5}{7}v_i$$

$$\Rightarrow k_f = \frac{5}{7}k$$

$$\Delta KE = k - \frac{5}{7}k = \frac{2}{7}k$$

$$\Rightarrow \left|W_f\right| = \frac{20}{7}\text{ J}$$

SECTION-II (CHEMISTRY)

31. **(4)**

Only in n = 2 $\rightarrow$ n = 1 (one single type of photon is emitted)

32. **(4)**

Meq of $K_2Cr_2O_7$ must be greater than the other given species.

33. **(2)**

N_3^-, I_3^-, NO_2^+ are linear.

34. **(1)**

$\Delta G = -T\Delta S_{universe}$

35. **(4)**

$\because\ \Delta x \cdot \Delta p \geq \frac{h}{4\pi}$

According to question,

$\Delta x \cdot m \cdot \Delta v = 0.527 \times 10^{-34}$

$$\Delta x = \frac{0.527\times10^{-34}}{9.1\times10^{-31}\times7.98\times10^{6}}$$

$$\frac{\Delta p}{\Delta x} = 10^{-12}$$

36. **(2)**

$Na_2S + AgNO_3 \rightarrow Ag_2S$

$NaCN + AgNO_3 \rightarrow AgCN$

$Na_2CrO_4 + AgNO_3 \rightarrow Ag_2CrO_4$

$Na_3PO_4 + AgNO_3 \rightarrow Ag_3PO_4$

37. **(1)**

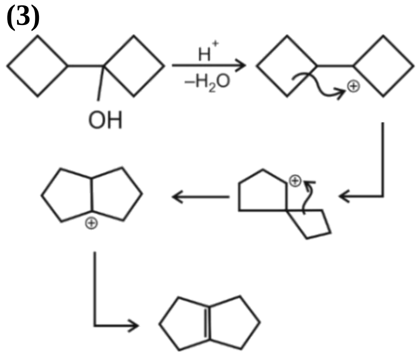
Reaction with greater E_a is more temperature sensitive.

38. **(4)**

$[Co(C_2O_4)_3]^{3-}$ - d^2sp^3

- inner orbital complex

39. **(3)**



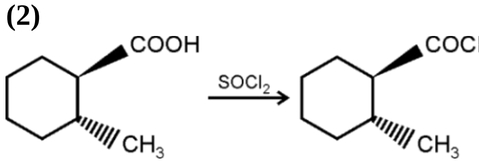
40. **(1)**

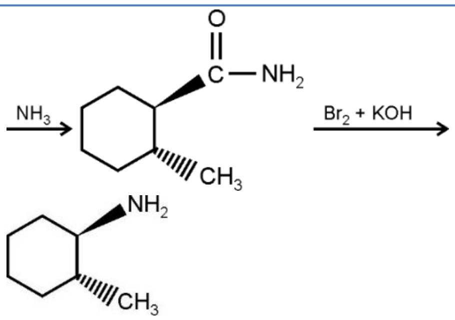
$[M(AA)_3]^{x+}$ complexes are chiral

41. **(4)**

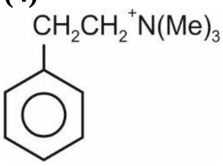
Formic acid is most acidic among all.

42. **(2)**





43. **(4)**

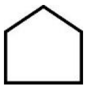


Ring is most activated in this case due to maximum number of hyperconjugable hydrogens among all.

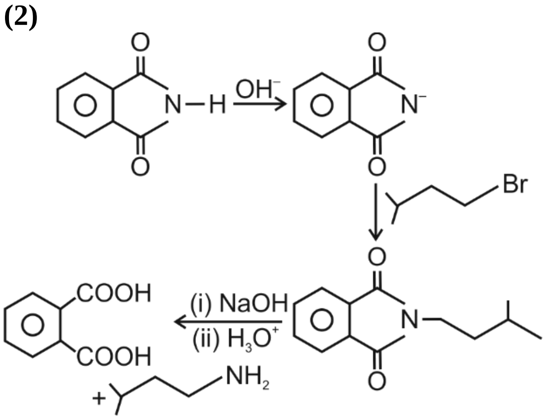
44. **(3)**

To give haloform reaction, carbonyl group must have methyl group.

45. **(4)**

Given C_5H_{10} molecule is 

46. **(2)**



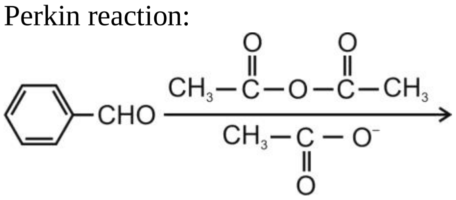
47. **(4)**

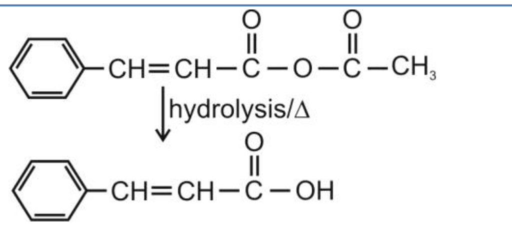
X is NH_3.

With mercurous nitrate it forms $\underbrace{Hg + HgNH_2Cl}_{\text{Black}}$

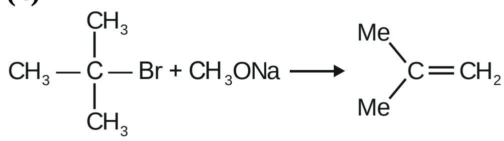
48. **(2)**

Perkin reaction:

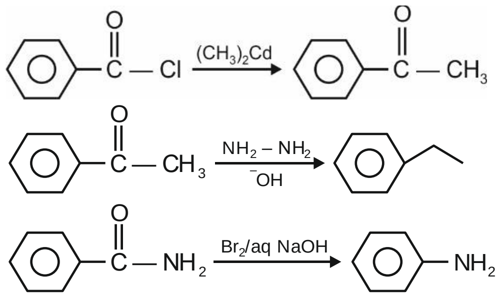




49. **(4)**



50. **(3)**



51. **(10)**

$$\text{ppm of } O_2 = \frac{\text{wt. of } O_2}{\text{wt. of } H_2O} \times 10^6$$

$$= \frac{10.3\text{ mg}}{1.03 \times 10^6\text{ mg}} \times 10^6$$

= 10 ppm

52. **(20)**

$C_3H_8 + 5O_2 \rightarrow 3CO_2 + 4H_2O$

$\Delta H = -\{6(C{=}O) + 8(OH)\} + 2(C-C) + 8(C-H) + 5(O{=}O)$

$= 8 \times 9 + 5 \times 12 + 2 \times 6 - 6 \times 10 - 8 \times 8$

= 20 kJ/mole

53. **(12)**

$K_{sp} = 4s^3 = 4 \times 10^{-12}$

$s = 10^{-4}$ molar

$= 10^{-4} \times 120$ g/L

$= 1.2 \times 10^{-2}$ g/L

54. **(40)**

$$\Delta T_f = 1.8 \times \frac{\left(\frac{1}{3} + \frac{1}{2} \times 2\right)}{60} \times 1000 = 40\text{ K}$$

55. **(6)**

$[Ni(NH_3)_6]^{2+}$ is paramagnetic.

56. **(25)**

$BCl \rightarrow B^+ + Cl^-$

$$\begin{array}{llllll} B^+ & + \ H_2O & \rightleftharpoons & BOH & + \ H^+, & K_h = \dfrac{K_w}{K_b} \\ 0.25 & - & & - & & \\ (0.25-x) & - & & x & x & \end{array}$$

Given, pH = 2.7 $\Rightarrow$ $[H^+] = 2 \times 10^{-3}$

$\therefore \quad \dfrac{x^2}{0.25} = \dfrac{10^{-14}}{K_b}$

$\Rightarrow \quad 4 \times 10^{-6} \times 4 \times K_b = 10^{-14}$

$\Rightarrow \quad K_b = \dfrac{1}{16} \times 10^{-8} = 6.25 \times 10^{-10}$

57. **(73)**

Milli moles of NaOH in sol = 10

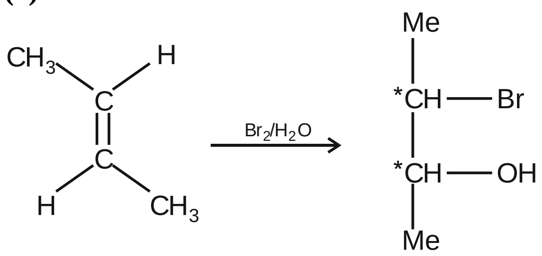
Milli moles of HCl in sol = 10

At any instant millimoles of NaCl

formed = $\dfrac{0.5265}{58.5} \times 10^3 = 9$

$\therefore$ Amount of HCl left = 36.5 mg

58. **(2)**



Number of optical centre = 2

59. **(25)**

Volume of HCl required $= \dfrac{0.5 \times 1000 \times 2}{100 \times 0.8}$

= 12.5 ml

60. **(14)**

$kt = 2.303 \log \dfrac{v_\infty}{v_\infty - v_t}$

$= \dfrac{2.303}{20} \log \dfrac{40}{40-10} = \dfrac{2.303}{20} \log \dfrac{4}{3}$

$= \dfrac{0.28}{20} = 0.014 \ \text{min}^{-1}$

SECTION-III (MATHEMATICS)

61. **(3)**

$\dfrac{n(n-1)}{2} - n = 44$

$\Rightarrow n = 11$,

Number of required triangle = $n(n-4) = 11 \times 7$

= 77

62. **(2)**

$\sum_{r=0}^{2020} {}^{2020}C_r (x-2018)^{2020-r} (2017)^r = (x-1)^{2020}$

Hence coefficient of $x^{2017} = -({}^{2020}C_{2017})$

63. **(3)**

Consider the graph of $h(x) = \max(x, x^2)$ at

$x = 0$ and $x = 1$

for D: $h(x) = \max.(x^2, -x^2)$

64. **(1)**

From graph of $3\sin^{-1}x$, it is one-one and onto function

65. **(3)**

$\dfrac{1}{2}\int \ln\left(\dfrac{x-1}{x+1}\right)\left(\dfrac{1}{x-1} - \dfrac{1}{x+1}\right)dx$

let $\ln\left(\dfrac{x-1}{x+1}\right) = t$

$\dfrac{1}{2}\int t\,dt$

$\dfrac{t^2}{4} + c = \dfrac{1}{4}\left(\ln\left(\dfrac{x-1}{x+1}\right)\right)^2 + c$

66. **(1)**

It is clear that a should be greater than one.

Hence $b^2 - 10b + 25 > 1$

$(b-4)(b-6) > 0$

$b > 6 \cup b < 4$

67. **(3)**

$xRy \Leftrightarrow x < y$ is not reflexive on the set of integers

68. **(1)**

Since origin and the point $(a^2, a+1)$ lie on the same side of both lines.

$\Rightarrow 3a^2 - (a+1) + 1 > 0$, *i.e.*, $a(3a-1) > 0$ gives

$a \in (-\infty, 0) \cup \left(\dfrac{1}{3}, \infty\right)$

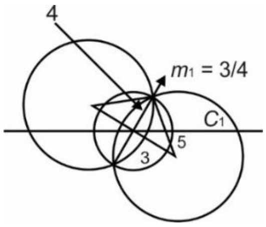
and $a^2 + 2(a + 1) - 5 < 0$, *i.e.*, $a^2 + 2a - 3 < 0$, *i.e.*, $(a - 1)(a + 3) < 0$ gives $a \in (-3, 1)$

Intersection of the above inequalities gives

$$a \in (-3, 0) \cup \left(\frac{1}{3}, 1\right)$$

69. **(2)**

$$m_2 = -\frac{4}{3} = \tan\theta$$



$$\sin\theta = \frac{4}{5} : \cos\theta = -\frac{3}{5}$$

Required centre

$$= \left(0 \pm 3\left(-\frac{3}{5}\right), 0 \pm 3\left(\frac{4}{5}\right)\right) = \left(\mp\frac{9}{5}, \pm\frac{12}{5}\right)$$

70. **(2)**

$$V\left(-\frac{3}{4a}, -\frac{35}{16}a\right)$$

$\therefore\ xy = \dfrac{105}{64}$ is the required locus

71. **(2)**

$k^2 + 2k + 5 < k + 11$

$\Rightarrow k \in (-3, 2)$

72. **(4)**

Equation AB is $x + 2y = 2$

So intersection point of P is $\left(-\dfrac{24}{5}, \dfrac{17}{5}\right)$

73. **(3)**

$A^2 - 4A + 3I = 0$

$\Rightarrow A(A + 3I) - 7A + 3I = 0$

$\Rightarrow (A + 3I)(A - 7I) = -24I$

$$\Rightarrow (A + 3I)\left(\frac{7}{24}I - \frac{A}{24}\right) = I$$

$$\therefore\ (A + 3I)^{-1} = \frac{7}{24}I - \frac{A}{24}$$

74. **(2)**

$\because \dfrac{|x| - 2}{4} \in [-1, 1]$ and $3 - x > 0$ and $3 - x \neq 1$

$\Rightarrow x \in [-6, 3) - \{2\}$

75. **(4)**

$$\frac{1}{2^6}\left[{}^7C_1 + {}^7C_3(4x+1) + {}^7C_5(4x+1)^2 + {}^7C_7(4x+1)^3\right]$$

76. **(2)**

We have $b^2 = ac$ and $2(\log 2b - \log 3c)$

$= \log a - \log 2b + \log 3c - \log a$

$\Rightarrow b^2 = ac$ and $2b = 3c$

$\Rightarrow b = \dfrac{2a}{3}$ and $c = \dfrac{4a}{9}$

Since $a + b = \dfrac{5a}{3} > c,\ b + c = \dfrac{10a}{9} > a$ and $c + a = \dfrac{13a}{9} > b$, therefore a, b, c are the sides of a triangle. As a is the greatest side, the greatest angle A is given by

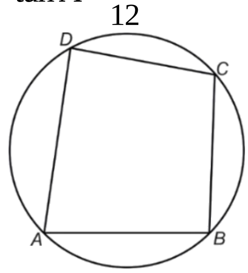
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = -\frac{29}{48} < 0$$

Hence, ΔABC is an obtuse-angled triangle.

Hence (2) is the correct answer.

77. **(1)**

$$\tan A = \frac{5}{12}$$



$$\cos B = -\frac{3}{5}$$

$$\therefore \cos C = -\cos A = -\frac{12}{13},\ \tan D = -\tan B = \frac{4}{3}$$

$$\therefore \cos C + \tan D = \frac{16}{39} \text{ and } \cos C \cdot \tan D = -\frac{48}{39}$$

$\therefore$ The required equation is $x^2 - \dfrac{16}{39}x - \dfrac{48}{39} = 0$

i.e. $39x^2 - 16x - 48 = 0$

78. **(3)**

A denote the event that a sum of 4 occurs

$$P(A) = \frac{1}{12}$$

B denote the event that a sum of 6 occurs

$$P(B) = \frac{5}{36}$$

C denote that neither a sum of 4 nor a sum of 6 occurs.

$P(C) = \frac{28}{36} = \frac{14}{18} = \frac{7}{9}$

P (A occurs before B)

$= P(A) + P(C). P(A) + P(C)^2. P(A) + \ldots \infty$

$= \frac{1}{12} + \frac{7}{9} \times \frac{1}{12} + \left(\frac{7}{9}\right)^2 \times \frac{1}{12} + \ldots.\infty$

$= \frac{3}{8}$

79. **(2)**

$$2A = \begin{bmatrix} 3 & 5 & 8 \\ -3 & 2 & -1 \\ -1 & 6 & 5 \end{bmatrix}$$

$$2B = \begin{bmatrix} -1 & -1 & -6 \\ 5 & 0 & -1 \\ -5 & -2 & 3 \end{bmatrix}$$

$$\Rightarrow 2A + 4B = \begin{bmatrix} 1 & 3 & -4 \\ 7 & 2 & -3 \\ -11 & 2 & 11 \end{bmatrix}$$

80. **(4)**

$[x + 1] \neq 0$

$[x] \neq -1$

$x \notin [-1, 0)$

i.e. $x \quad R - [-1, 0)$

81. **(36)**

The difference between the focal distances is a constant for a hyperbola. For a rectangular hyperbola latusrectum = transverse axis.

S(2, 0) S$^|$(h, k) P(0, 0) $\left|S^| p - Sp\right| = 4$

$\left|\sqrt{h^2 + k^2} - 2\right| = 4$

$\Rightarrow \quad \sqrt{h^2 + x^2} = 6$

$\Rightarrow \quad h^2 + k^2 = 36$

Locus of (h, k) is $x^2 + y^2 = 36$

82. **(70)**

$1^2.\left({}^5C_1\right)^2 + 2^2.\left({}^5C_2\right)^2 + \ldots. + 5^2.\left({}^5C_5\right)^2 = 1750$

83. **(0)**

$x^2 - 14x + 40 \leq 0$

$x \in [4, 10]$

$x^2 - 6ax + 5a^2 \leq 0$

$x \in [a, 5a]$

$4 < a$, $5a < 10$, $a < 2$

So there is no possible value of 'a'

84. **(3)**

$A = \{1, 2, 3, 4, 5\}$

$B = \{3, 5, 7\}$ U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

$C = \{5, 10\}$

$n(A^C \cap B^C \cap C^C) = n(\text{U}) - n(A \cup B \cup C)$

$= 10 - 7 = 3$

85. **(1)**

$$I_1 = \int_0^3 \frac{\sin x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx = \int_0^3 2\sin x \, dx$$

$$I_2 = \int_{-3}^0 \frac{\sin x}{\left[\frac{x}{\pi}\right] + \frac{1}{2}} dx = \int_{-3}^0 \frac{\sin x}{-1 + \frac{1}{2}} dx$$

$$= -\int_{-3}^0 2\sin x \, dx$$

$$= -\int_3^0 2\sin(-t)(-dt) = \int_0^3 2\sin x \, dx = I_2$$

$\therefore I_2 = I_1$

86. **(81)**

$\det(3A) = 3^3 \bullet 3 = 81$

87. **(1)**

$|z + \bar{z}| + |z - \bar{z}| = 2 \quad \ldots$(i)

$|z + i| + |z - i| = 2 \quad \ldots$(ii)

From equation (i)

$\Rightarrow |2x| + |2iy| = 2$

$\Rightarrow |x| + |y| = 1$

So from (i) & (ii)

$z = \pm i$

88. **(15)**

$\sum_{i=1}^{n} P_i = 3$ and $M = 12$

$P + M = 15$

89. **(4)**

$$(a+\sqrt{2}b\cos x)(a-\sqrt{2}b\cos y)=a^2-b^2$$

Differentiating both sides

$$(-\sqrt{2}b\sin x)(a-\sqrt{2}\,b\cos y)+(a+\sqrt{2}b\cos x)$$

$$(\sqrt{2}b\sin y)y'=0$$

at $\left(\frac{\pi}{4},\frac{\pi}{4}\right)$ $-b(a-b)+(a+b)by' = 0$

$$\frac{dy}{dx}=\frac{a-b}{a+b}$$

$$\Rightarrow \frac{dx}{dy}=\frac{a+b}{a-b}=\frac{5+3}{5-3}=\frac{8}{2}=4$$

90. **(5)**

If $g(x)=x^5\sin\left(\frac{1}{x}\right)$ and $h(x)=x^5\cos\left(\frac{1}{x}\right)$

then $g''(0) = 0$ and $h''(0) = 0$

So, $f''(0^+) = g''(0^+) + 10 = 10$

and $f''(0^-) = h''(0^-) + 2\lambda = f''(0^+)$

$\Rightarrow 2\lambda = 10$

$\lambda = 5$