JEE MAIN (2023-24) Mock Test Series

Paper - 08

DURATION : 180 Minutes

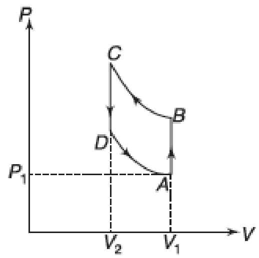
M. MARKS : 300

ANSWER KEY

PHYSICS		CHEMISTRY		MATHEMATICS	
1.	(2)	31.	(1)	61.	(4)
2.	(3)	32.	(3)	62.	(4)
3.	(1)	33.	(3)	63.	(1)
4.	(1)	34.	(2)	64.	(1)
5.	(3)	35.	(1)	65.	(2)
6.	(1)	36.	(2)	66.	(3)
7.	(4)	37.	(2)	67.	(1)
8.	(2)	38.	(1)	68.	(2)
9.	(1)	39.	(4)	69.	(1)
10.	(2)	40.	(3)	70.	(1)
11.	(1)	41.	(1)	71.	(4)
12.	(2)	42.	(4)	72.	(3)
13.	(1)	43.	(3)	73.	(3)
14.	(3)	44.	(3)	74.	(1)
15.	(4)	45.	(3)	75.	(4)
16.	(2)	46.	(1)	76.	(4)
17.	(3)	47.	(2)	77.	(3)
18.	(2)	48.	(4)	78.	(2)
19.	(1)	49.	(2)	79.	(2)
20.	(3)	50.	(4)	80.	(1)
21.	(3)	51.	(3)	81.	(7)
22.	(5)	52.	(3)	82.	(101)
23.	(83)	53.	(2)	83.	(36)
24.	(18)	54.	(5)	84.	(6)
25.	(4)	55.	(5)	85.	(9)
26.	(95)	56.	(2)	86.	(3)
27.	(2)	57.	(4)	87.	(6)
28.	(2)	58.	(3)	88.	(1)
29.	(10)	59.	(2)	89.	(4)
30.	(2)	60.	(4)	90.	(60)

SECTION-I (PHYSICS)

1. **(2)**



$V_1 = \frac{nM}{\rho_1}$ and $V_2 = \frac{nM}{\rho_2}$

$W_{AB} = W_{CD} = 0$

$W_{BC} = P_B V_B \quad \text{In} \quad \left(\frac{V_C}{V_B}\right) = P_2 V_1 \quad \text{In}$

$\left(\frac{V_2}{V_1}\right) = P_2 \frac{nM}{\rho_1} \quad \text{In} \left(\frac{\rho_1}{\rho_2}\right)$

$W_{DA} = P_A V_A \text{ In} \left(\frac{V_A}{V_D}\right) = P_1 V_1 \text{ In} \left(\frac{V_1}{V_2}\right) = P_1 \frac{nM}{\rho_1}$

$\text{In} \left(\frac{\rho_1}{\rho_2}\right)$

$\therefore \quad W = \frac{nM}{\rho_1}\left[-P_2 \text{In}\left(\frac{\rho_2}{\rho_1}\right) + P_1 \text{In}\left(\frac{\rho_2}{\rho_1}\right)\right]$

$= -\frac{nM}{\rho_1} \text{In}\left(\frac{\rho_2}{\rho_1}\right)(P_2 - P_1)$

2. **(3)**

When A and B are mixed

Heat gained by A = Heat lost by B

$m_A s_A$ (16 – 12) = $m_B s_B$ (19 – 16)

$\therefore \quad m_B s_B = \frac{4}{3} m_A s_A \qquad \ldots\text{(i)}$

Similarly, when B and C are mixed –

$m\text{B}s_B(23 - 19) = m_C s_C(28 - 13)$

$\Rightarrow \quad m_C s_C = \frac{4}{5} m_B s_B \qquad \ldots\text{(ii)}$

Usign (i) and (ii)

When A and C are mixed, let the final temperature be θ.

$m_A s_A (\theta - 12) = m_C s_C (28 - \theta)$

$\Rightarrow \quad \theta - 12 = \frac{16}{15}(28 - \theta) \Rightarrow \theta = \frac{628}{31} = 20.26°\text{C}$

3. **(1)**

Net electric field inside the conductor is zero.

4. **(1)**

Time of travel for the bullet from one disc to the other

$t = \frac{H}{V} \therefore \theta = \omega t$

$\theta = \omega \frac{H}{V} \Rightarrow V = \frac{\omega H}{\theta}$

5. **(3)**

Wet ball takes time to reach ground 2.5 seconds so water drops detach from the ball is 2.

6. **(1)**

Output will be high when both diodes do not conduct.

If $A = 0$, $B = 5$ V, then D_1 conducts and $y = 0$

If $A = 5$ V, $B = 0$, then D_1 and D_2 conducts and $y = 0$

If $A = 0$, $B = 0$, then D_1 and D_2 conducts and $y = 0$

If $A = 5$ V, $B = 5$ V, both do not conduct and $y = 5$ V

$\Rightarrow$ AND gate

7. **(4)**

Assume upper hemisphere,

$B = \frac{2}{3}\pi R^3 \times \rho g$

$F_2 = \pi R^2 \times \rho g(2R)$

$\therefore \quad F_1 = F_2 - B = \frac{4}{3}\pi R^3 \rho g$

8. **(2)**

In one quarter time electric field energy will completely change into magnetic field energy.

$t = \frac{T}{4} = \frac{2\pi\sqrt{LC}}{4}$

$t = 1.57$ ms

9. **(1)**

$W_{\text{total}} = \Delta KE$

$(3mg)x + mgx = \frac{1}{2}kx^2$

$\Rightarrow \quad X = \frac{8mg}{k}$

10. **(2)**

For perfectly absorbing,

$$F_n = a\frac{P}{c}$$

For perfectly reflecting,

$$F_n = \frac{2rP}{c}$$

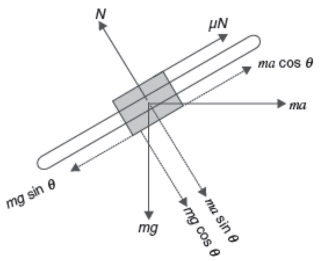
For the given situation,

$$F_n = \frac{P}{c}(a+2r)$$

$$\Rightarrow \quad F_n = \frac{P}{c}(1+r) = 1.4\frac{P}{c}$$

11. **(1)**

Figure shows the free body diagram of the sleeve in a reference frame attached to the rod when a is small, the rod has a tendency to slid down, hence friction is up the rod.



For the minimum value of a for which the sleeve does not slide, friction will take its maximum possible value, i.e., μN

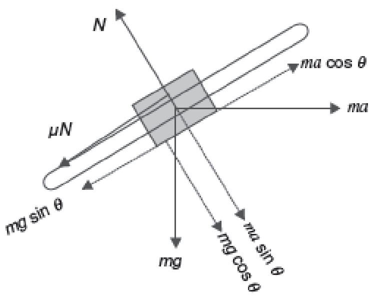
$N = m\,(a \sin\theta + g\cos\theta)$

and $\mu N + ma\cos\theta = mg\sin\theta$

$\mu m\,(a\sin\theta + g\cos\theta) + ma\cos\theta = mg\sin\theta$

$$\Rightarrow \quad a = g\frac{(\sin\theta - \mu\cos\theta)}{(\cos\theta + \mu\sin\theta)}$$

This is the minimum value of a for which the sleeve does not slide.



When a increase the sleeve has a tendency to move up.

Thus friction is directed down the rod.

a is maximum when friction is μN

$N = m\,(a\sin\theta + g\cos\theta)$

and $mg\sin\theta + \mu N = ma\cos\theta$

or

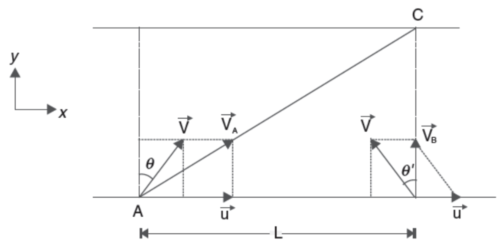
$mg\sin\theta + \mu m\,(a\sin\theta + g\cos\theta) = ma\cos\theta$

or

$$a = \frac{g(\sin\theta + \mu\cos\theta)}{\cos\theta - \mu\sin\theta}$$

$$\therefore \quad g = \frac{(\sin\theta - \mu\cos\theta)}{(\cos\theta - \mu\sin\theta)} \le a \le g\frac{(\sin\theta + \mu\cos\theta)}{\cos\theta - \mu\sin\theta}$$

12. **(2)**



V = velocity of boat relative to water

V_A and V_B = actual velocity to two boats.

From the condition given in the problem it follows that

$V_{Ay} = V_{By}$

$\Rightarrow \quad V\cos\theta = V\cos\theta'$

$\Rightarrow \quad \theta = \theta'$

Also, $V\sin\theta' = u \qquad [\because V_{Bx} = 0]$

$5\sin\theta' = 3$

$\sin\theta' = 3/5$

$$\theta = \theta' = \sin^{-1}\left(\frac{3}{5}\right)$$

$$\therefore \quad V_{Ay} = V_{By} = V\cos\theta = 5\times\frac{4}{5} = 4 \text{ km/hr}$$

time to cross the river $t = \dfrac{3.0\text{km}}{4.0\text{km/hr}}$

$$= \frac{3}{4}\text{ hr.}$$

For A, $V_{ax} = V\sin\theta + u\ 5\times\frac{3}{5} + 3 = 6$ km/hr

$$\therefore \quad L = V_{Ax}t = 6\times\frac{3}{4} = 4.5 \text{ km}$$

13. **(1)**

Net electric field will be zero.

14. **(3)**

The capacitance depends upon the geometrical parameters only. And if Q is increased then V increase.

Hence, the correct answer is (3).

15. (4)

A→ (p, r, s)
B → (p, r)
C → (p, r, s)
D → (p, r)

16. (2)

Time of flight $= \dfrac{2u\sin\theta}{g} = \dfrac{2\times 20\times\frac{1}{2}}{10} = 2s$

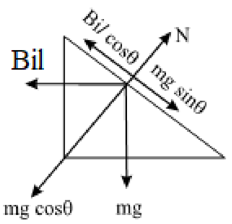
After 1 sec, the projectile is at maximum height.
Maximum height

$$(H) = \frac{u^2\sin^2\theta}{2g} = \frac{20^2\times\left(\frac{1}{2}\right)^2}{2\times 10} = 5m$$

$$\text{Range } (R) = \frac{u^2\sin 2\theta}{g} = \frac{20^2\times\frac{\sqrt{3}}{2}}{10} = 20\sqrt{3}\,m$$

$$\overline{AB} = \sqrt{H^2+\left(\frac{R}{2}\right)^2} = \sqrt{5^2+\left(10\sqrt{3}\right)^2} = \sqrt{325} = 5\sqrt{13}\,m$$

17. (3)



$F\cos\theta = Mg\sin\theta$

$$B = \frac{mg\tan\theta}{i\ell}$$

18. (2)

$$v_e = \sqrt{\frac{2GM}{R}} = 11.2 \text{ km/s}$$

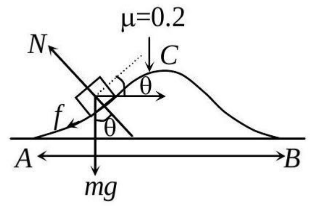
$$v_e = \sqrt{\frac{2GM}{R/4}} = 2\sqrt{\frac{2GM}{R}} = 2\times 11.2 = 22.4\,\text{km/s}$$

19. (1)

Electrostatics force on $q = \dfrac{\lambda q}{2\pi\varepsilon_0 r}$ away from line charge Magnetic force $= \dfrac{\mu_0\lambda v}{2\pi r}\times q\times v$ away from line charge

$$\therefore \text{ total force } = \frac{\lambda q}{2\pi r}\left[\frac{1}{\varepsilon_0}+\mu_0 v^2\right]$$

20. (3)



Work done by friction $= \int \vec{F}\cdot d\vec{s}$

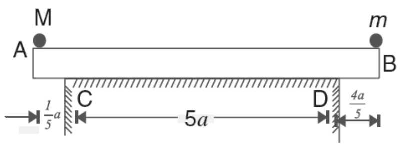
$$= \int_0^x \mu mg\cos\theta\frac{dx}{\cos\theta} = \mu mgx = 20 \text{ J}$$

21. (3)

When the second insect with large mass sits at end A, the bar has a tendency to topple about C (see figure). If M increases COM of the system shifts to left. M is maximum (for not toppling) when COM is at C

Distance of COM from A =

$$\frac{M\times 0+4m\times 3a+m\times 6a}{M+5m}$$



$\Rightarrow \dfrac{a}{5} = \dfrac{18ma}{M+5m}$

$\Rightarrow$ M + 5m = 90 m

$\Rightarrow$ m = 85 m

22. (5)

If $\vec{C} = a\hat{i}+b\hat{j}$ them

$\vec{A}.\vec{C} = \vec{A}.\vec{B}$

$\Rightarrow$ a+ b = 1 …(1)

$\vec{B}.\vec{C} = \vec{A}.\vec{B}$

$\Rightarrow$ $2a - b = 1$ …(2)

Solving equations (1) and (2), we get

$$a = \frac{1}{3},\quad b = \frac{2}{3}$$

$$\Rightarrow \left|\vec{C}\right| = \sqrt{\frac{1}{9}+\frac{4}{9}} = \sqrt{\frac{5}{9}}$$

23. (83)

$n_1\lambda_1 = n_2\lambda_2$

$\Rightarrow$ 31 × 5893 = n_2 × 4358

$\Rightarrow$ n_2 = 41

Number of fringes = 2 (41) + 1 = 83

24. **(18)**

Distance of object from mirror is

$$15+\frac{33.25}{1.33}=40\,\text{cm}$$

Distance of image from mirror is

$$15+\frac{25}{1.33}=33.8\,\text{cm}$$

Applying mirror formula, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$, we get

$$\Rightarrow \quad \frac{1}{-33.8}+\frac{1}{40}=\frac{1}{f}$$

$\Rightarrow \quad f = -18.3$ cm

25. **(4)**

The escape velocity is

$$v_e=\sqrt{\frac{2GM}{R}}$$

So, $v_A=\sqrt{\frac{2GM}{R}}$ and $v_B=\sqrt{\frac{2G(M/2)}{R/2}}=\sqrt{\frac{2GM}{R}}$

$$\Rightarrow \quad \frac{v_A}{v_B}=1=\frac{m}{4}$$

$\Rightarrow \quad n = 4$

26. **(95)**

$$f=\frac{1}{2l}\sqrt{\frac{T}{\mu}}$$

Let the frequency of tuning fork be 'n', then in the first case the fundamental frequency of the wire will be (n + 5), which is given

$$n+5=\frac{1}{2l}\sqrt{\frac{T}{\mu}} \qquad \text{...(i)}$$

Here, T = 100 N, l = 50 cm = 0.5 cm, then

$$n+5=\frac{1}{2\times .5}\times\sqrt{\frac{100}{\mu}}=\frac{10}{\sqrt{\mu}} \qquad \text{...(ii)}$$

In the second case, T = 81 N, in this case the frequency of wire will be (n – 5)

$$n-5=\frac{1}{2\times .5}\times\sqrt{\frac{81}{\mu}}=\frac{9}{\sqrt{\mu}} \qquad \text{...(iii)}$$

From Equation (ii) and Equation (iii),

$$\frac{n+5}{n-5}=\frac{10/\sqrt{\mu}}{9/\sqrt{\mu}}=\frac{10}{9}$$

$\Rightarrow \quad 9n + 45 = 10n - 50$

$n = 95$

27. **(2)**

$$\because \quad c=\frac{\varepsilon_0 A}{d}$$

On increasing temperature,

$$c'=\frac{\varepsilon_0 A'}{d'}=\frac{\varepsilon_0 A(1+2\alpha_1 T)}{d(1+\alpha_2 T)}$$

$$c'=\frac{\varepsilon_0 A(1+2\alpha_1 T)\,(1-\alpha_2 T)}{d}$$

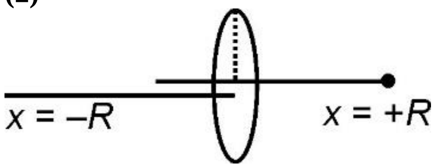
$$=\frac{\varepsilon_0 A}{d}(1+(2\alpha_1-\alpha_2)T-2\alpha_1\alpha_2 T^2)$$

c' = constant with temperature

$\therefore \quad 2\alpha_1 - \alpha_2 = 0$

$2\alpha_1 = \alpha_2$

28. **(2)**

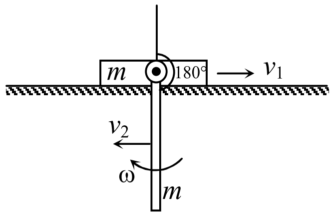


Alternately, we shall calculate the magnetic field by straight wire with current I and then find its line integral on circle.

$$\therefore \quad \int B\cdot dl=\frac{\mu_0 I}{4\pi R}\cdot\frac{2.1}{\sqrt{2}}\cdot 2\pi R$$

$$\Rightarrow \quad \int B.dl=\frac{\mu_0 I}{\sqrt{2}}$$

29. **(10)**



There is no horizontal force, momentum is conserved

$mv_1 - mv_2 = 0$

For hinged point

$$v_1=\frac{L}{2}\omega-v_2$$

Energy conservation,

$$mgL=\frac{1}{2}mv_1^2+\frac{1}{2}mv_2^2+\frac{1}{2}\frac{mL^2\omega^2}{12}$$

Solving, $v_1=\sqrt{\frac{3}{5}gL}$, v_1 = 10 m/s.

30. **(2)**

$$\left[\frac{e^2}{2h\varepsilon_0 c}\right]=M^0L^0T^0$$

$\Rightarrow \quad x = 2$

SECTION-II (CHEMISTRY)

31. **(1)**

$C_xH_{2y}O_y + xO_2 \rightarrow xCO_2 + yH_2O$

Amount of O_2 is twice the needed amount i.e., 2x.

The hot gases when cooled to 0ºC and 1 atm pressure = 2.24 litres = 2x

$\therefore$ x = 1.12 litres CO_2

$n_{CO_2} = \frac{1.12}{22.4} = 0.5$ moles CO_2

$n_{H_2O} = \frac{0.9}{18} = 0.5$ moles H_2O

x : y = 1 : 1

EF = CH_2O

$\frac{p^o - p}{p^o} = \frac{W_2 \times M_1}{M_2 \times W_1}$

Or, $\frac{0.104}{17.5} = \frac{50 \times 18}{M_2 \times 1000}$

$M_2 = 151.1$ g ≈ 151 g

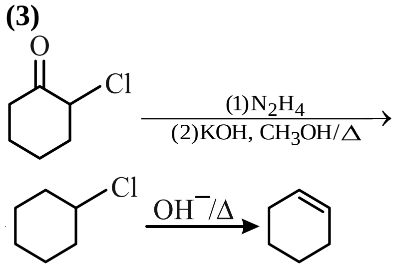
$\therefore n = \frac{151}{30} \approx 5$

Molecular formula = $5(CH_2O) = C_5H_{10}O_5$

32. **(3)**

For the given question only (3) & (4) compounds are possible. In (3) H bonded to the carbon adjacent to the benzoic acid is less acidic.

33. **(3)**



34. **(2)**

Both are true statements.

35. **(1)**

$NaCl + H_2SO_4 \rightarrow HCl + NaHSO_4$

$MnO_2 + 4HCl \rightarrow MnCl_2 + \underset{\text{(Green)}}{Cl_2} + 2H_2O$

36. **(2)**

Hydroquinol undergoes removal of hydrogen, i.e., oxidation and hence, it acts as a reducing agent.

37. **(2)**

$C(s) + O_2(g) \rightarrow CO_2(g)$; $\Delta H = -393$ kJ

Now, (–393) = [718 + 498] –2 × 539 – $|R.E.|_{CO_2}$

$\therefore |R.E.|_{CO_2} = 531$ kJ/mol

38. **(1)**

Hybridization

$I_3^+ = sp^3$

$I_3^- = sp^3d$

39. **(4)**

I. CH_3^+; 6 + 3 – 1 = 8 (electrons)

II. H_3O^+; 8 + 3 – 1 = 10

III. NH_3; 7 + 3 = 10

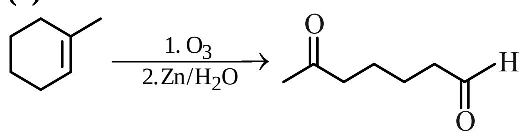
IV. CH_3^-; 6 + 3 + 1 = 10

Thus, II, III and IV are isoelectronic structures.

40. **(3)**

The difference in atomic radii is maximum in Na and K.

41. **(1)**



Aromatic aldehydes and ketones do not give positive Fehling's test.

42. **(4)**

Presence of unpaired electrons.

43. **(3)**

Chromyl chloride test.

44. **(3)**

$E^o_{Zn^{+2}/Zn} = -0.76\,V$

45. **(3)**

A $\rightarrow$ R; B $\rightarrow$ S; C $\rightarrow$ P; D $\rightarrow$ Q

$\rightarrow PhSO_2Cl \rightarrow$ Hinsberg reagent

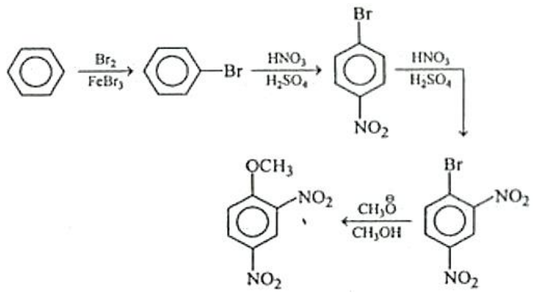
$\rightarrow Ph-\underset{\underset{O}{\|}}{C}H-NH_2 \rightarrow Ph-N=C=O$

Hoffmann bromamide reaction

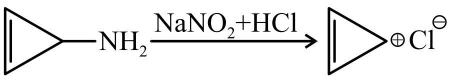
$\rightarrow$ $\underset{1^\circ \text{ amine}}{R-NH_2}$ $\rightarrow$ carbylamine reaction

$\rightarrow$ Saytzeff product vs Hoffman product

46. **(1)**

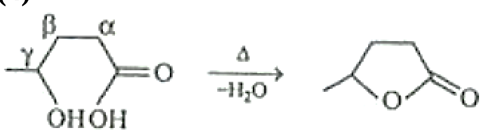


47. **(2)**



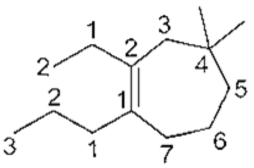
Aromatic Salt

48. **(4)**



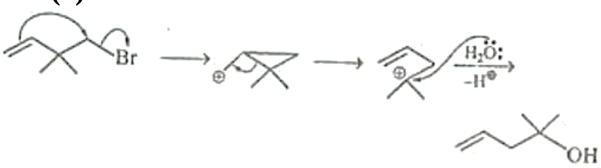
γ-hydroxy acid

49. **(2)**



2-Ethyl-4,4-dimethyl-1propylcyclohept-1-ene

50. **(4)**



51. **(3)**

$$\Delta T_f = \frac{1000 \times K_f \times w}{m \times w}$$

For the solution in benzene using the date given

$$1.28 = \frac{1000 \times 5.12 \times w}{m_w \times 100} \qquad \ldots(i)$$

For the solution in water in which solute dissociates

$$1.40 = \frac{1000 \times 1.86 \times w}{m_{exp} \times 100} \qquad \ldots(ii)$$

Dividing eq. (ii) by (i)

$$i = \frac{m_N}{m_{exp}} = \frac{1.40}{1.28} \times \frac{5.12}{1.86} = 3.01 = 3.0$$

Now, suppose that formula of solute is

A_xB_y	$\rightleftharpoons$	xA^+ +	yB^-
1		0	0
$(1-\alpha)$		$x\alpha$	$y\alpha$

i = 1 – α + xα + yα

i = 3 and α = 1 (Given than α = 1)

No. of ions given (x + y) = 3

52. **(3)**

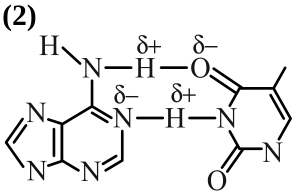
$$K = \frac{2.303}{t_2 - t_1} \log \frac{R_1}{R_2}$$

$$= \frac{2.303}{60} \log \frac{1.24 \times 10^{-2}}{0.2 \times 10^{-2}}$$

$$= \frac{2.303}{60} \log 6.2$$

= 0.0304 = 3 × 10^{-2}

53. **(2)**



Adenine Thymine

H–bond

54. **(5)**

$$\lambda = \frac{h}{mv} = \frac{6.62 \times 10^{-34}}{1.675 \times 10^{-27} \times 800 \times 10^{-12}} = 4.94 \times 10^2 \text{ ms}^{-1}$$

55. **(5)**

$Ba(OH)_2$ + 2HCl $\rightarrow$ $BaCl_2 + 2H_2O$

$H^+ = 10^{-12}$ $H^+ = 10^{-2}$ V = 2 litres

$OH^- = 10^{-2}$ $OH^- = 10^{-12}$

0.005 moles of Barium chloride in 2L.

56. **(2)**

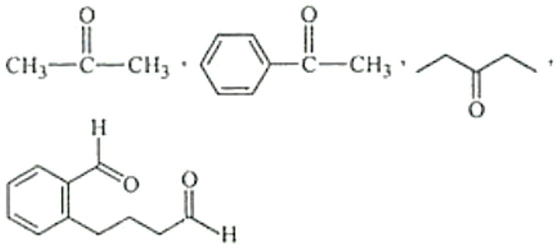
$2NaOH + NaH_2PO_4 \longrightarrow Na_3PO_4 + 2H_2O$

$$\frac{12}{120} = 0.1 \text{ Mole}$$

V × 1= 0.1 × 2

V = 0.2 litre = 200 ml.

57. (4)



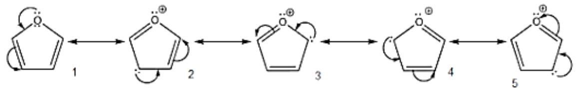
58. (3)

$$\begin{array}{l} CH_2-OH \\ | \\ C=O \\ | \\ CH-OH \\ | \\ CH-OH \\ | \\ CH-OH \\ | \\ CH_2-OH \end{array} \xrightarrow{5HIO_4} HCHO + \text{'3'}\ HCOOH + CO_2$$

59. (2)

Ring 2 get reduced to release strain.

60. (4)



SECTION-III (MATHEMATICS)

61. (4)

$|z_2 + iz_1| = |z_1| + |z_2| \Rightarrow z_2, iz_1, 0$ are collinear

$\arg(iz_1) = \arg z_2$

$\Rightarrow \arg z_2 - \arg z_1 = \dfrac{\pi}{2}$

$\Rightarrow z_3 = \dfrac{z_2 - iz_1}{1-i} \Rightarrow z_3 - z_2 = i(z_3 - z_1)$

$\Rightarrow \text{Arg}\left(\dfrac{z_3 - z_2}{z_3 - z_1}\right) = \dfrac{\pi}{2}$ and $|z_3 - z_2| = |z_3 - z_1|$

$\Rightarrow BC = AC$ and $AB^2 = AC^2 + BC^2$

$\Rightarrow AB^2 = 25 \Rightarrow 2AC^2 = 25$

Required area $= \dfrac{1}{2} AC(BC) = \dfrac{25}{4}$ Sq.units

62. (4)

$$\begin{vmatrix} 1+\sin^2 x & \text{Cos}^2 x & \text{Sin}\,2x \\ \text{Sin}^2 x & 1+\text{Cos}^2 x & \text{Sin}\,2x \\ \text{Sin}^2 x & \text{Cos}^2 x & 1+\text{Sin}\,2x \end{vmatrix} = \begin{vmatrix} 1+\text{Sin}^2 x & \text{Cos}^2 x & \text{Sin}\,2x \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix}$$

$= (1+\text{Sin}^2 x)(1+0) - \text{Cos}^2 x(-1-0) + \text{Sin}\,2x(1-0)$

$= \ = 2 + \text{Sin}\,2x$

A triangle can be constructed its sides as α = 3, β = 2 is false

63. (1)

$3x - y + 4z = 3 \rightarrow$

$x + 2y - 3z = -2 \rightarrow$

$6x + 5y + kz = -3 \rightarrow 3$

$1 \times 2 + 2 \Rightarrow 7x + 5z = 4$

$1 \times 5 + 3 \Rightarrow 21x + (20 + k)\, z = 12$

$\Rightarrow \dfrac{21}{7} = \dfrac{20+k}{5} = \dfrac{12}{4} \Rightarrow 20 + k = 15 \Rightarrow k = -5$

64. (1)

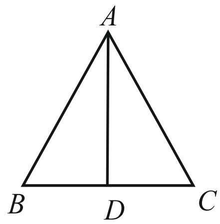
${}^{12}C_2 \times {}^{10}C_3 \times 2^3 = 63360$

65. (2)

$A(4, 7, 8)$, $B(2,3,4)$, $C(2,5,7)$

D Divides BC in $AB : AC = 6 : 3 = 2 : 1$

$D\left(\dfrac{2(2)+1(2)}{2+1}, \dfrac{2(5)+1(3)}{2+1}, \dfrac{2(7)+1(4)}{2+1}\right)$



$\Rightarrow \overline{OD} = \dfrac{1}{3}\left(6\bar{i} + 13\bar{j} + 18\bar{k}\right)$

66. **(3)**

$$\int \frac{1}{\cos^6 x+\sin^6 x}dx$$

$$\int \frac{1}{\left(\cos^2 x+\sin^2 x\right)\left(\cos^4 x+\sin^4 x-\cos^2 x\sin^2 x\right)}dx$$

$$=\int \frac{dx}{1-3\sin^2 x\cos^2 x}=\int \frac{\sec^4 x}{\sec^4 x-3\tan^2 x}dx$$

$$=\int \frac{\left(1+\tan^2 x\right)\sec^2 x}{\left(1+\tan^2 x\right)^2-3\tan^2 x}dx$$

$$=\int \frac{\sec^2 x\left(1+\tan^2 x\right)}{\tan^4 x-\tan^2 x+1}dx$$

$$=\int \frac{\left(1+\frac{1}{\tan^2 x}\right)\sec^2 x}{\left(\tan^2 x+\frac{1}{\tan^2 x}-2\right)+1}dx$$

Put $\tan x = t$

$\Rightarrow \sec^2 x\, dx = dt$

$$\Rightarrow I=\int \frac{\left(1+\frac{1}{t^2}\right)}{\left(t^2+\frac{1}{t^2}-2\right)+1}dt$$

$$=\tan^{-1}\left(t-\frac{1}{t}\right)+c=\tan^{-1}\left(\tan x-\cot x\right)+c.$$

67. **(1)**

$$\text{R.H.L} = \lim_{x\to 0^+}\left[\left(1-e^x\right)\frac{\sin x}{x}\right]$$

When $x \in (0, h)$ and $h \to 0$

then $(1-e^x) \in (-1, 0)$ and $\frac{\sin x}{x}<1$

So $-1 < (1-e^x)\frac{\sin x}{x}<0$; $\lim_{x\to 0^+}\left[\left(1-e^x\right)\frac{\sin x}{x}\right]=-1$

L.H.L

$$=\lim_{x\to 0^-}\left[\left(1-e^x\right)\frac{\sin x}{-x}\right]=\lim_{x\to 0^-}\left[\left(e^x-1\right)\frac{\sin x}{x}\right]$$

When $x \in (-h, 0)$ and $h \to 0$,

then $e^x - 1 \in (-1, 0)$ and $\frac{\sin x}{x}<1$

So $-1 < (e^x - 1)\frac{\sin x}{x}<0$

so $\lim_{x\to 0^-}\left[\left(e^x-1\right)\frac{\sin x}{x}\right]=-1$

L.H.L = R.H.L = –1

68. **(2)**

$$\begin{vmatrix}\cos(\theta+\phi) & -\sin(\theta+\phi) & \cos 2\phi\\ \sin\theta & \cos\theta & \sin\phi\\ -\cos\theta & \sin\theta & \cos\phi\end{vmatrix}=\frac{1}{\sin\phi\cos\phi}$$

$$\begin{vmatrix}\cos(\theta+\phi) & -\sin(\theta+\phi) & \cos 2\phi\\ \sin\theta\sin\phi & \sin\phi\cos\theta & \sin^2\phi\\ -\cos\theta\cos\phi & \sin\theta\cos\phi & \cos^2\phi\end{vmatrix}$$

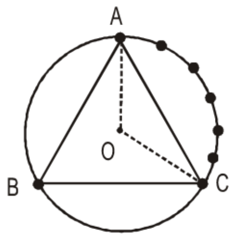
Applying $R_1 \to R_1 + R_2 + R_3$

$$=\frac{1}{\sin\phi\cos\phi}\begin{vmatrix}0 & 0 & 2\cos^2\phi\\ \sin\theta\sin\phi & \sin\phi\cos\theta & \sin^2\phi\\ -\cos\theta\cos\phi & \sin\theta\cos\phi & \cos^2\phi\end{vmatrix}$$

$$=\begin{vmatrix}0 & 0 & 2\cos^2\phi\\ \sin\theta & \cos\theta & \sin\phi\\ -\cos\theta & \sin\theta & \cos\phi\end{vmatrix}$$

$= 2\cos^2\phi\,(\sin^2\theta + \cos^2\theta) = 2\cos^2\phi$

69. **(1)**



If between A and C, there are 'r' vertices, then AC will subtends $\frac{2\pi}{2n+1}(r+1)$ at the centre.

According to condition

$$\frac{2\pi}{2n+1}(r+1)<\pi \Rightarrow r<n-1$$

So required number of triangles will be number of solutions of $a_1 + a_2 + a_3 = 2n - 2$

$a_1 \le n - 1$, $a_2 \le n - 1$, $a_3 \le n - 1$

Which is ${}^{2n}C_2 - 3.\,{}^{n}C_2$

70. **(1)**

Let the correct equation be $ax^2 + bx + c = 0$

now Sachin's equation $\Rightarrow ax^2 + bx + c' = 0$ $\left\langle\begin{matrix}4\\3\end{matrix}\right.$

Krishna's equation $\Rightarrow ax^2 + b'x + c = 0$ $\left\langle\begin{matrix}3\\2\end{matrix}\right.$

$$-\frac{b}{a}=7 \qquad \text{.......(i)}$$

$$\frac{c}{a}=6 \qquad \text{......(ii)}$$

from (i) and(ii)

correct equation is $x^2 - 7x + 6 = 0$; and roots are 6 and 1.

71. **(4)**

Let

$$I=\int_{\pi/6}^{\pi/3}\tan^3 x.\sin^2 3x\left(2\sec^2 x.\sin^2 3x+3\tan x.\sin 6x\right)dx$$

$$=\int_{\pi/6}^{\pi/3}\left[\tan^3 x.\sin^4 3x.2\sec^2 x+3\tan^4 x.\sin^2 3x.\sin 6x\right]dx$$

$$=\int_{\pi/6}^{\pi/3}\left[\frac{\frac{d}{dx}\left(\tan^4 x\right)\cdot\sin^4 3x}{2}+\tan^4 x.\frac{\frac{d}{dx}\left(\sin^4 3x\right)}{2}\right]dx$$

$$=\frac{1}{2}\int_{\pi/6}^{\pi/3}\frac{d}{dx}\left(\tan^4 x.\sin^4 3x\right)dx$$

$$=\frac{1}{2}\left[\tan^4 x.\sin^4 3x\right]_{\pi/6}^{\pi/3}$$

$$=\frac{1}{2}\left[\left(\sqrt{3}\right)^4\times 0-\left(\frac{1}{\sqrt{3}}\right)^4\times 1\right]=\frac{1}{2}\left[-\frac{1}{9}\right]=-\frac{1}{18}$$

72. **(3)**

$$(100)^2+...+(100-99)(100+99)$$

$$=(100)^2+\left(100^2-1^2\right)+\left(100^2-2^2\right)+.....$$

$$......+\left(100^2-99^2\right)$$

$$=(100)^2+99(100)^2-\left(1^2+2^2....+99^2\right)$$

73. **(3)**

We have, det

$$A=\begin{bmatrix}-2 & 4+d & (\sin\theta)-2\\ 1 & (\sin\theta)+2 & d\\ 5 & (2\sin\theta)-d & (-\sin\theta)+2+2d\end{bmatrix}$$

Applying $R_1\to R_1+R_3-2R_2$, we get

$$\det A=\begin{bmatrix}1 & 0 & 0\\ 1 & (\sin\theta)+2 & d\\ 5 & (2\sin\theta)-d & 2+2d-\sin\theta\end{bmatrix}$$

$$=(2+\sin\theta)(2+2d-\sin\theta)-d(2\sin\theta-d)$$

$$=4+4d-2\sin\theta+2\sin\theta+2d\sin\theta-\sin^2\theta-2d\sin\theta+d^2$$

$$=d^2+4d+4-\sin^2\theta$$

For a given d, minimum value of det

$(A)=(d+2)^2-1=8\Rightarrow d=1 or -5$

74. **(1)**

When have, $z_0=\omega$ or ω^2

(where ω is a non real cube root of unity)

Now, $z=3+6iz_0^{81}-3iz_0^{93}$

$$=3+6i(\omega)^{81}-3i(\omega)^{93}$$

$$=3+6i-3i=3+3i$$

$$\therefore\ \arg(z)=\tan^{-1}\left(\frac{3}{3}\right)=\tan^{-1}(1)=\frac{\pi}{4}$$

75. **(4)**

Clearly, the integers from 8 through 14 must be in different pairs, and 7 must pair with 14. Note that 6 can pair with either 12 or 13. From here, we consider casework:

If 6 pairs with 12, then 5 can pair with one of 10, 11, 13. After that, each of 1, 2, 3, 4 does not have any restrictions. This case produces 3.4! = 72 ways.

If 6 pairs with 13, then 5 can pair with one of 10, 11, 12. After that, each of 1, 2, 3, 4 does not have any restrictions. This case produces 3.4! = 72 ways.

Together, the answer is 72+72= 144.

76. **(4)**

$\vec{a}.\left(\vec{b}+\vec{c}\right)=0$, $\vec{b}.\left(\vec{c}+\vec{a}\right)=0$, $\vec{c}.\left(\vec{a}+\vec{b}\right)=0$

$\Rightarrow\quad \vec{a}.\vec{b}+\vec{a}.\vec{c}=0,\quad \vec{b}.\vec{c}+\vec{b}.\vec{a}=0, \vec{c}.\vec{a}+\vec{c}.\vec{b}=0$,

$\vec{a}.\vec{b}=\vec{b}.\vec{c}=\vec{c}.\vec{a}=0$

$$\left|\vec{a}+\vec{b}+\vec{c}\right|=\sqrt{\vec{a}^2+\vec{b}^2+\vec{c}^2+2\left(\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}\right)}$$

$$=\sqrt{9+16+25}=\sqrt{50}$$

77. **(3)**

$f(10-x)=f(x)=f(4-x)$

$\Rightarrow f(10-x)=f(4-x)$

Let $4 - x = t$
$\Rightarrow f(6 + t) = t$
$\Rightarrow f(x)$ is periodic with period 6.
$\Rightarrow f(x) = 101$ at $x = 0, 6, 12, 18, 24, 30$
Since $f(2 + x) = f(2 - x)$
$\Rightarrow f(x)$ is symmetric about $x = 2$
$\Rightarrow f(0) = f(4)$
$\Rightarrow$ using periodic nature
$f(x) = 101$ at $x = 4, 10, 16, 22, 28$ $\Rightarrow$ $f(5 + x) = f(5 - x)$
x is symmetric about $x = 5$ $f(0) = f(10)$
$\Rightarrow x = 4, 10, 16, 22$
$f(6) = f(4)$
$\Rightarrow x = 0, 6, 12, 18,$
Total different values of x are 0, 4, 6, 10, 12, 16, 18, 22, 24, 28, 30

78. **(2)**
Let $x_1, x_2, \ldots\ldots, x_n$ be the variates corresponding to n sets of data, each having the same number of observations say K and x be their product.
Then, $x = x_1 \cdot x_2 \ldots\ldots x_n$
i.e. $\log x = \log x_1 + \log x_2 + \ldots\ldots + \log x_n$
or $\frac{\sum \log x}{K} = \frac{\sum \log x_1}{K} + \frac{\sum \log x_2}{K} + \ldots\ldots + \frac{\sum \log x_n}{K}$
or $\log G = \log G_1 + \log G_2 + \ldots\ldots + \log G_n$
$\Rightarrow G = G_1 \cdot G_2 \ldots\ldots G_n$

79. **(2)**
Since, 1 rad $= \frac{7\pi}{22}$
$\therefore$ 12 $rad = \frac{7\pi}{22} \times 12 = \frac{42\pi}{11} = 4\pi - \frac{2\pi}{11}$ ....(ii)
and 14 rad $= \frac{7\pi}{22} \times 14 = \frac{49\pi}{11} = 4\pi + \frac{5\pi}{11}$ ......(ii)
$\therefore$ $\cos^{-1}(\cos 12) - \sin^{-1}(\sin 14)$
$$= \cos^{-1}\left[\cos\left(4\pi - \frac{2\pi}{11}\right)\right] - \sin^{-1}\left[\sin\left(4\pi + \frac{5\pi}{11}\right)\right]$$
$$= \cos^{-1}\left[\cos\left(\frac{2\pi}{11}\right)\right] - \sin^{-1}\left(\sin\frac{5\pi}{11}\right) = \frac{2\pi}{11} - \frac{5\pi}{11}$$
$= 4\pi - 12 - (14 - 4\pi) = 8\pi - 26$
[using Eqs. (i) and (ii)]

80. **(1)**
(i) $z_1 = r_1 e^{i\theta_1}$
$z_2 = r_2 e^{i\theta_2}$
$$|z_1 + z_2| = \sqrt{r_1^2 + r_2^2 + 2r_1 r_2 \cos(\theta_1 - \theta_2)}$$
$$\left|\frac{z_1}{|z_1|}|z_2| + \frac{z_2}{|z_2|}|z_1|\right| = \left|r_2 e^{i\theta_1} + r_1 e^{i\theta_2}\right|$$
$$\sqrt{r_1^2 + r_2^2 + 2r_1 r_2 \cos(\theta_1 - \theta_2)}$$
LHS = RHS

81. **(7)**
$a - 3 > 0, \Delta < 0$
$\Rightarrow 12^2 - 4(a-3)(a+6) < 0$
$\Rightarrow 36 - \left(a^2 + 6a - 3a - 18\right) < 0$
$\Rightarrow a^2 + 3a - 54 > 0 \Rightarrow (a+9)(a-6) > 0$

$\Rightarrow a < -9$ or $a > 6$ but $a > 3 \Rightarrow$ least value of a is 7

82. **(101)**
The coefficient of x^4 in
$4c_o(1+x)^{404} - 4c_1(1+x)^{303} + 4c_2(1+x)^{202}$
$-4c_3(1+x)^{101} + 4c_4$
$\left[(1+x)^{101} - 1\right]^4 = \left[101c_1 x + 101c_2 x^2 + \ldots. + 101c_{101} x^{101}\right]^4$
is $(101)^4$

83. **(36)**
E_1, E_2, E_3 be the events that two headed coin, biased coin, unbiased coin
$\Rightarrow p(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$
E be the event that head shows
$$p\left(\frac{E}{E_1}\right) = \frac{\frac{1}{3}(1)}{\frac{1}{3}(1) + \frac{1}{3}\left(\frac{3}{4}\right) + \frac{1}{3}\left(\frac{1}{2}\right)} = \frac{1}{1 + \frac{3}{4} + \frac{1}{2}}$$
$$= \frac{4}{4+3+2} = \frac{4}{9} = p$$
$\Rightarrow 81p = 36$

84. **(6)**
$|\vec{v}_2| = |\vec{v}_1|$
$\Rightarrow 2p^2 - 2p - 4 = 0 \Rightarrow (p-2)(p+1) = 0 \Rightarrow p = 2 (p > 0)$
$$\text{Cos}\,\theta = \frac{2\sqrt{3}p + (p+1)}{\sqrt{(4 + (p+1)^2)(3p^2 + 1)}}$$
$$= \frac{4\sqrt{3} + 3}{\sqrt{13(13)}}$$

$$\sec\theta = \frac{13}{4\sqrt{3}+3}$$

$$\Rightarrow \quad \tan^2\theta = \frac{112-24\sqrt{3}}{57+24\sqrt{3}}$$

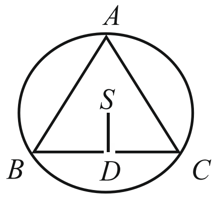
$$= \left(\frac{6\sqrt{3}-2}{4\sqrt{3}+3}\right)^2 \Rightarrow \alpha = 6$$

85. (9)

Slope of $BC = \frac{-1}{2}, D\left(2, \frac{17}{2}\right)$,

Equation of SD is $2x - y = -\frac{9}{2}$

$x = 0 \Rightarrow \frac{9}{2} \Rightarrow \left(0, \frac{9}{2}\right) = \left(0, \frac{\alpha}{2}\right) \qquad \Rightarrow \alpha = 9$



86. (3)

For rational roots, D must be perfect square

$D = 121 - 24\alpha = k^2$

for $121 - 24\alpha$ to be perfect square α must be equal to 3, 4, 5 (observation) so number of possible values of α is 3.

87. (6)

$S_n = (1 + 2T_n)(1 - T_n)$

$\Rightarrow S_1 = (1 + 2T_1)(1 - T_1)$

$T_1 = 1 - T_1 + 2T_1 - 2T_1^2$

$\Rightarrow 2T_1^2 = 1$

$\Rightarrow T_1 = \frac{1}{\sqrt{2}}$

$S_2 = T_1 + T_2 = (1 + 2T_2)(1 - T_2)$

$\Rightarrow T_1 + T_2 = 1 - T_2 + 2T_2 - 2T_2^2$

$T_1 = 1 - 2T_2^2 \Rightarrow 2T_2^2 = 1 - \frac{1}{\sqrt{2}}$

$\Rightarrow T_2^2 = \frac{\sqrt{2}-1}{2\sqrt{2}}$

$\Rightarrow \quad T_2^2 = \frac{2-\sqrt{2}}{4} \quad \Rightarrow a = 4, b = 2 \Rightarrow a + b = 6$

88. (1)

$$\frac{2\sin 2^\circ + 4\sin 4^\circ + \dots\dots\dots + 178\sin 178^\circ}{90}$$

$$\frac{2\sin 2^\circ \times \sin 1^\circ + 2(2\sin 4^\circ \times \sin 1^\circ) + \dots. \dots. + 89(2\sin 178^\circ \times \sin 1^\circ)}{90\sin 1^\circ}$$

We know that

$2\sin 2k^\circ \sin 1^\circ = \cos(2k-1)^\circ - \cos(2k+1)^\circ$

We have

$$\frac{2\sin 2^\circ \times \sin 1^\circ + 2(2\sin 4^\circ \times \sin 1^\circ) + 3(2\sin 6^\circ \sin 1^\circ) + 4(2\sin 8^\circ \sin 1^\circ) + \dots\dots + 89(2\sin 178^\circ \times \sin 1^\circ)}{90\sin 1^\circ}$$

$$= \frac{(\cos 1^\circ - \cos 3^\circ) + 2(\cos 3^\circ - \cos 5^\circ) + \dots. \dots. + 89(\cos 177^\circ - \cos 179^\circ)}{90\sin 1^\circ}$$

$$= \frac{\cos 1^\circ + \cos 3^\circ + \cos 5^\circ + \cos 177^\circ + 89\cos 1^\circ}{90\sin 1^\circ}$$

$$= \frac{\cos 1^\circ + 89\cos 1^\circ + (\cos 3^\circ + \cos 5^\circ + \dots\dots + \cos 177^\circ)}{90\sin 1^\circ}$$

$$= \frac{90\cos 1^\circ + 0}{90\sin 1^\circ}$$

$$= \frac{90\cos 1^\circ}{90\sin 1^\circ} = \cot 1^\circ$$

89. (4)

$L_1 : \frac{x}{0} = \frac{y}{b} = \frac{z-c}{-c} = r;\ L_2 : \frac{x}{a} = \frac{y}{0} = \frac{z+c}{c} = \ell$

$A(0, br, -cr + c)\ B(a\ell, 0, c\ell - c)$

Dr's of AB are $-a\ell, br, -cr - c\ell + 2c$

$\Rightarrow AB$ is perpendicular to both the lines

$0(-a\ell) + b \cdot br + (-c)(-cr - c\ell + 2c) = 0$

$\Rightarrow (b^2 + c^2)r + c^2\ell = 2c^2 \qquad \dots\dots.(1)$

and $a(-a\ell) + 0(br) + c(-cr - c\ell + 2c) = 0$

$\Rightarrow (a^2 + c^2)\ell - c^2r + 2c^2 = 0$

$(a^2 + c^2)\ell + c^2r = 2c^2 \qquad \dots\dots(2)$

from (1) and (2)

$$\ell = \frac{2b^2c^2}{a^2b^2 + b^2c^2 + c^2a^2}, r = \frac{2a^2c^2}{a^2b^2 + b^2c^2 + c^2a^2}$$

$$A\left(0, \frac{2a^2bc^2}{a^2b^2 + b^2c^2 + c^2a^2}, c\left(\frac{a^2b^2 + b^2c^2 - c^2a^2}{a^2b^2 + b^2c^2 + c^2a^2}\right)\right)$$

$$B\left(\frac{2ab^2c^2}{a^2b^2+b^2c^2+c^2a^2}, 0, c\left(\frac{b^2c^2-a^2b^2-c^2a^2}{a^2b^2+b^2c^2+c^2a^2}\right)\right)$$

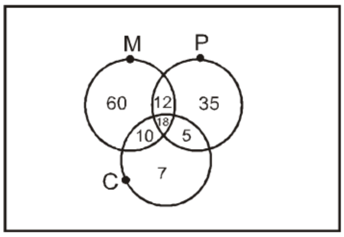
$$d^2=\frac{4a^2b^4c^4}{\left(a^2b^2+b^2c^2+c^2a^2\right)^2}+\frac{4a^4b^2c^4}{\left(a^2b^2+b^2c^2+c^2a^2\right)^2}$$

$$+\frac{4c^2\left(a^4b^4\right)}{\left(a^2b^2+b^2c^2+c^2a^2\right)^2}$$

$$\frac{4}{d^2}=\frac{\left(a^2b^2+b^2c^2+c^2a^2\right)^2}{a^2b^4c^4+a^4b^2c^4+a^4b^4c^2}$$

$$=\frac{a^2b^2+b^2c^2+c^2a^2}{a^2b^2c^2}\Rightarrow\frac{4}{d^2}=\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}$$

90. **(60)**



Number of students offered maths alone = 60

$n(M) = 100$

$n(P) = 70$

$n(C) = 40$

$n(M \cap P) = 30$

$n(M \cap P) = 28$

$n(P \cap C) = 23$

$n(M \cap P \cap C) = 18$