Finding Basic Antiderivative

Polaris

2025/04/16

1 What is Indefinite integral

Recall definite integral and Fundamental Theorem of Calculus:

$$\int_a^b f(x)dx = F(b) - F(a)$$

Where $F'(x) = f(x)$, note definite integral only returns a number, what if we define a new operation that will return a function that is kind of like integrals?

This operation is called indefinite integral, given a function $f(x)$, it returns a function which has a derivative of $f(x)$, we denote this operation as this:

$$\int f(x)dx = F(x)$$

Where $F'(x) = f(x)$.

However this is not the complete result, recall that the derivative of a constant is 0, which means $\frac{d}{dx}F(x) + C = f(x)$, thus for an indefinite integral, the correct answer would be

$$\int f(x)dx = F(x) + C$$

Where C is any constant and $F'(x) = f(x)$

Indefinite follows two important rules:

1. $$\int (f(x) + g(x))dx = \int f(x)dx + \int g(x)dx$$

2. $$\int cf(x)dx = c\int f(x)dx$$

2 How to calculate Indefinite Integral

Let's take a look at an example question:

$$\int (\cos x + 7e^x)dx$$

We can split this integral into 2 and evaluate them separately:

$$\int (\cos x + 7e^x)dx = \int \cos x dx + \int 7e^x dx$$

Recall the defintion of indefinite integral, if we want to find the indefintite integral of $\cos x$, we are simply finding a function that has a derivative of $\cos x$, and this function is $\sin x$, thus we have:

$$\int \cos x dx = \sin x + C_1$$

Same logic apply for $7e^x$, we have

$$\int 7e^x dx = 7e^x + C_2$$

Sum everything up, we have

$$\int (\cos x + 7e^x)dx = \sin x + C_1 + 7e^x + C_2$$

Here C_1 and C_2 are simply constant, we denote their sum as a new constant C, thus we have

$$\int (\cos x + 7e^x)dx = \sin x + 7e^x + C$$

3 List of Basic Integral

We know how to calculate basic integrals now, but it is ineffcient to go back and use the definition everytime, here we start from derivatives and created a list for basic integrals, these are the building blocks of more complex integral and must be remember:

1. $\int dx = x + C$
2. $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ (if $n \neq -1$)
3. $\int \frac{1}{x} dx = \ln|x| + C$ (remember the absolute value)
4. $\int e^x dx = e^x + C$
5. $\int b^x dx = \frac{1}{\ln b} b^x + C$
6. $\int \sin x dx = -\cos x + C$
7. $\int \cos x dx = \sin x + C$
8. $\int \sec^2 x dx = \tan x + C$
9. $\int \csc^2 x dx - \cot x + C$
10. $\int \sec x \tan x dx = \sec x + C$
11. $\int \csc x \cot x = -\csc x + C$
12. $\int \frac{1}{\sqrt{1-x^2}} dx = \arcsin x$ (if $|x| < 1$)
13. $\int \frac{1}{1+x^2} dx = \arctan x + C$
14. $\int \frac{1}{x\sqrt{x^2-1}dx} = \operatorname{arcsec} x + C$

If any of those feel rusty, consult back to the derivative chapter.

With this chart, we can solve for basic integrals, here are some example question:

$$\begin{aligned}
\int \left(\frac{1}{\sqrt[3]{x^2}} - \frac{4}{5x} + \frac{8}{1+x^2} dx \right) &= \int \frac{1}{\sqrt[3]{x^2}} dx - \int \frac{4}{5x} dx + \int \frac{8}{1+x^2} dx \\
&= \int x^{-\frac{2}{3}} dx - \frac{4}{5} \int \frac{1}{x} dx + 8 \int \frac{1}{1+x^2} dx \\
&= 3x^{\frac{1}{3}} - \frac{4}{5} \ln|x| + 8 \arctan\big(1+x^2\big) + C
\end{aligned}$$

Here we used equation 2, 3 and 13.

Let's take a look at another example:

$$\begin{aligned}\int \tan^2 x dx &= \int (\sec^2 x - 1) dx \\ &= \int \sec^2 x dx - \int dx \\ &= \tan x - x + C\end{aligned}$$

Here we used an trig equation of $\sec^2 x = \tan^2 x + 1$ and equation 8.