Odd and Even Functions

Polaris

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1 Definition

1.1 Odd Functions

A function is said to be odd if:

$$f(-x) = -f(x) \tag{1.1}$$

The graph of an odd function is symmetrical with respect to $(0,0)$

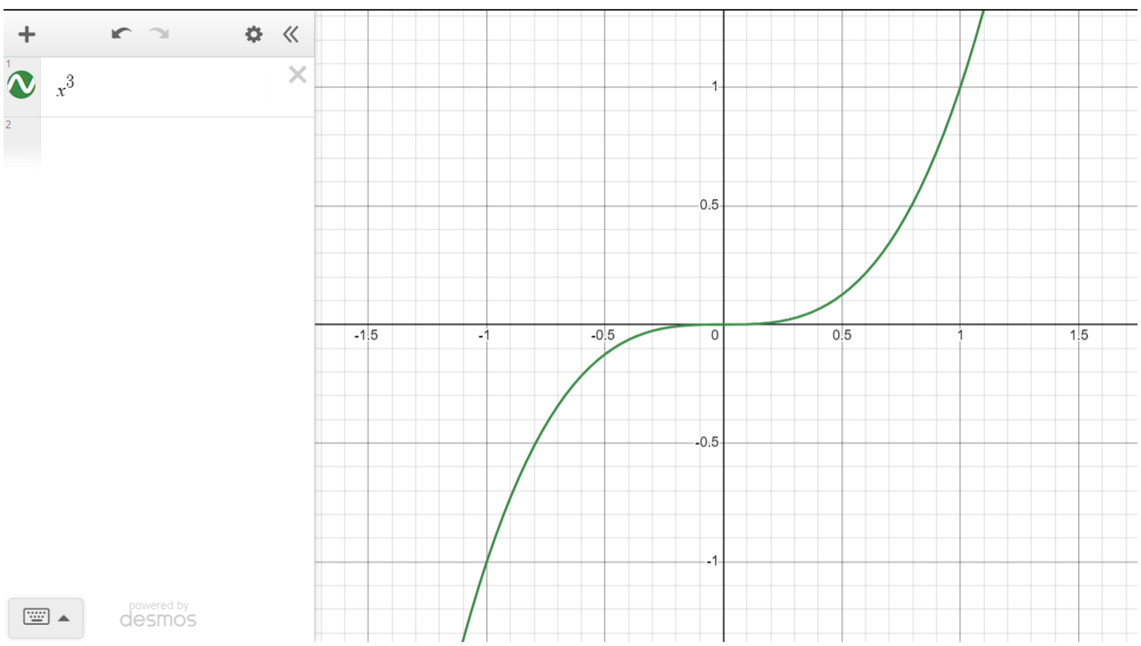

Figure 1: The graph of $y = x^3$, an odd function

1.2 Even function

A function is said to be even if:

$$f(x) = f(-x) \tag{1.2}$$

The graph of an even function is symmetrical with respect to the y axis.

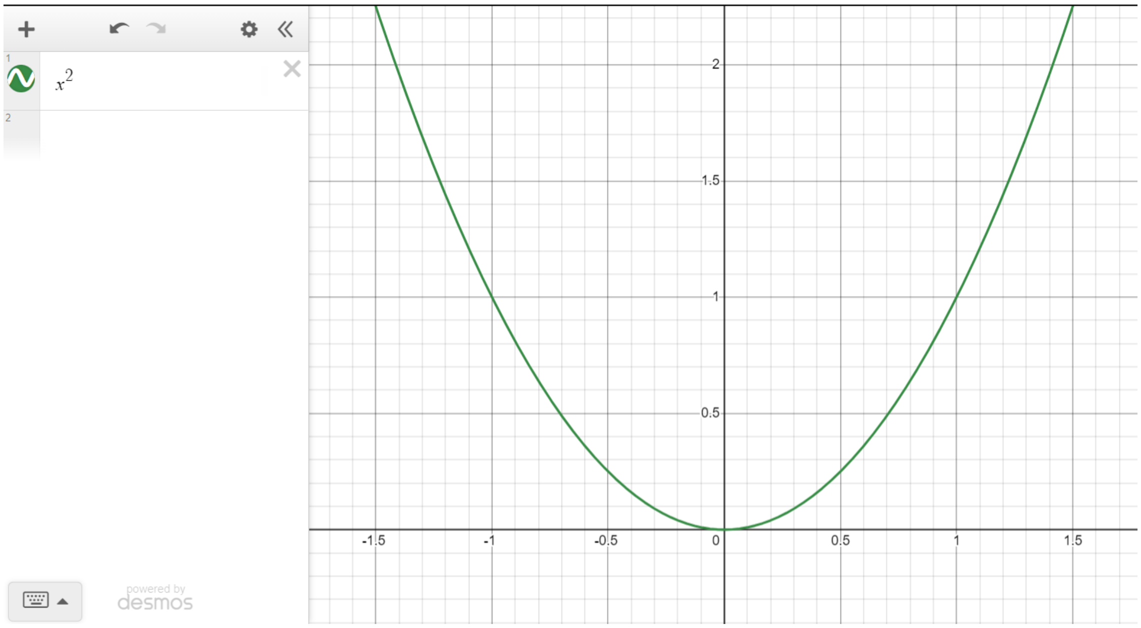

Figure 2: The graph of $y = x^2$, an even function

2 Properties

Let $f(x), g(x)$ be odd function, $a(x), b(x)$ be even function

We will first start with the basic properties:

1. If a function is both even and odd, it is equal to 0 everywhere
2. If a function is odd, the absolute value of that function is even

2.1 Odd Function

The properties of odd functions include:

1. The sum/difference of two odd functions is odd
2. The product/quotient of two odd functions is even
3. The composition of two odd functions is odd: $h(x) = f(g(x))$ is odd

2.2 Even Function

1. The sum/difference of two even functions is even
2. The product/quotient of two even functions is even
3. The composition of two even functions is even: $j(x) = a(b(x))$ is even

2.3 Odd Function and Even Function

When calculations are done between an odd function and an even function, the result holds the following properties:

1. The sum/difference of an odd function and an even function is not odd or even, unless one of the function equals to 0
2. The product/quotient of an odd function and an even function is an odd function
3. The composition of any function with an even function is even (not vice versa): $k(x) = a(f(x))$ is even

3 Examples

3.1 Odd Function

1. $f(x) = x^n$, where n is an odd number
2. $f(x) = \sin x, f(x) = \tan x f(x) = \cot x, f(x) = \csc x$
3. $f(x) = \arcsin x, f(x) = \arctan x, f(x) = \operatorname{arccsc} x$

3.2 Even Function

1. $f(x) = x^n$, where n is an even number
2. $f(x) = \cos x, f(x) = \sec x$

4 In Calculus

For an odd function $f(x)$, its integral over a symmetrical interval $(-a, a)$ is 0:

$$\int_{-a}^{a} f(x)dx = 0 \tag{4.1}$$

For an even function $g(x)$, its integral over a symmetrical interval $(-a, a)$ is 2 times its integral from $(0, a)$:

$$\int_{-a}^{a} g(x)dx = 2\int_{0}^{a} g(x)dx \tag{4.2}$$