Integration using subsitution

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1 U-subsitution

1.1 Indefinite integral

U-subsutition is the first techniques we will learn, let's first look at an example question:

Evaluate

$$\int 2xe^{-x^2}\mathrm{d}x$$

To solve this integral: Let $u = -x^2$, then $\mathrm{d}u = -2x\mathrm{d}x$, let's first substitute u back to the integral:

$$\int 2xe^{u}\mathrm{d}x$$

At first glance it seems that we make this integral more complicated as we introduce 2 variables in, but if we examine the integral carefully, we notice that we already have $2x\mathrm{d}x$ present in the integrand, we just need a minus sign. So if we add the minus sign like this, we can substitute $\mathrm{d}u$ in and evaluate the integral:

$$\int -e^{u}(-2x\mathrm{d}x) = \int -e^{u}\mathrm{d}u = -e^{u} + C$$

All that is left is to do is substitute $u = -x^2$ back and we get the final result:

$$\int 2xe^{-x^2}\mathrm{d}x = -e^{-x^2} + C$$

If we differentiate our results, we will arrive at the integrand, meaning that our process is correct.

This is essentially U-substitution, our thought process can be summarized as follow:

1. Let something equals to u
2. Calculate $\mathrm{d}u = \text{some expression} \cdot \mathrm{d}x$
3. Manipulate the integral so we found the some expression $\cdot\, \mathrm{d}x$
4. Substitute $\mathrm{d}u$ and evaluate the integral
5. Replace u with x and finish the integral

Let's look at another example:

$$\int 2x \sin\left(x^2\right) \mathrm{d}x$$