AP MCQs

Polaris

2025/01/13

1 AP Calculus BC

Evaluate this integral:

$$\int e^{2x} \sin x \mathrm{d}x$$

A. $\frac{1}{2}e^{2x}\sin x - \frac{1}{2}e^{2x}\cos x + C$

B. $\frac{2}{3}e^{2x}\sin x - \frac{1}{3}e^{2x}\cos x + C$

C. $\frac{2}{5}e^{2x}\sin x - \frac{1}{5}e^{2x}\cos x + C$

D. $\frac{1}{3}e^{2x}\sin x - \frac{1}{3}e^{2x}\cos x + C$

Explanation:

By integration by parts, the correct way to do the integral is as follow, let $I = \int e^{2x}\sin x \mathrm{d}x$:

$$\begin{aligned}
\int e^{2x}\sin x \mathrm{d}x &= \frac{1}{2}\int \sin x \mathrm{d}(e^{2x}) (1)\\
&= \frac{1}{2}e^{2x}\sin x - \frac{1}{2}\int e^{2x}\cos x \mathrm{d}x (2)\\
&= \frac{1}{2}e^{2x}\sin x - \frac{1}{4}\int \cos x \mathrm{d}(e^{2x}) (3)\\
&= \frac{1}{2}e^{2x}\sin x - (\frac{1}{4}e^{2x}\cos x - \frac{1}{4}\int e^{2x}\mathrm{d}(\cos x)) (4)\\
&= \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x - \frac{1}{4}\int e^{2x}\sin x \mathrm{d}x (5)\\
I &= \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x - \frac{1}{4}I (6)\\
\frac{5}{4}I &= \frac{1}{2}e^{2x}\sin x - \frac{1}{4}e^{2x}\cos x (7)\\
I &= \frac{2}{5}e^{2x}\sin x - \frac{1}{5}e^{2x}\cos x + C (8)
\end{aligned}$$

A common mistake is when taking the antiderivative of e^{2x}, $\frac{1}{2}$ gets dropped (mistake on line 1), resulting in a missing coefficient of the fraction and leading to choice A.

Another mistake is forgetting to change the sign in line 4, which results in choice B.

Another mistake that could be made is forgetting to multiply $\frac{1}{2}$ on line 3, because there is already a $\frac{1}{2}$, this will results in choice D.

Evaluate this integral:

$$\int \frac{1}{x\sqrt{x^2-1}} dx$$

A. $-\arcsin\frac{1}{x} + C$

B. $-\arcsin x + C$

C. $\arctan\frac{1}{x} + C$

D. $\arctan x + C$

Evaluate this integral:

$$\int_2^3 \frac{2x-3}{\sqrt{4x-x^2}} \mathrm{d}x$$

A. $\frac{\pi}{12} + 4 - 2\sqrt{3}$

B. $\frac{\pi}{6} + 4 - 2\sqrt{3}$

C. $\frac{\pi}{6} - 4 + 2\sqrt{3}$

D. $\frac{\pi}{3} - 4 + 2\sqrt{3}$

$$\int \frac{ax}{\sqrt{x^2+bx}} \mathrm{d}x = a\sqrt{x^2+bx} - \frac{ab}{2}\ln\left|x + \sqrt{x^2+bx} + \frac{b}{2}\right| + C$$

$$W = \tau\Delta\theta = 100\mathrm{Nm} \cdot 3000 \cdot 2\pi = 1.885 \cdot 10^6\mathrm{J}$$

$$\begin{aligned} W &= \Delta E_k = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2 \\ W &= \frac{1}{2}I\omega_f^2 - 0 \\ \omega_f &= \sqrt{\frac{2W}{I}} = 5284.436/s \end{aligned}$$

$$\bar{x}$$

$$E_{tot} = \frac{1}{2}mv^2 + \frac{1}{5}mv^2$$

$$t = \sqrt{\frac{2h}{g}}$$

$$g = \frac{GM}{r^2}$$

$$mr^2\sqrt{\frac{\frac{2E}{m} + \frac{2GM}{r} - \frac{L^2}{m^2r^2}}{L^2}}$$