

Defining Convergent and Divergent Infinite Series

Polaris

2025/04/17

We are going to introduce to some terms about this unit:

1. Sequence: a list of number in a particular order, ex. 1, 4, 7, 10...
2. n-th term formula: the formula for the n th term of the sequence, in the sequence given above,
 $a_n = 3n - 2$
3. Series: the sum of all terms in the sequence.
4. Partial Sum: the first n term of the sequence.

With this we can define the **convergence** and **divergence** of infinite series.

Let S_n be the partial sum of a sequence for the first n th term, if

$$\lim_{n \rightarrow \infty} S_n = L$$

Then we say this infinite series **converge to** L , if the limit does not exist, we say this infinite series **diverge**.

We can also understand the convergence series as sequence with a finite sum, while divergent series as sequence with infinite sum.

It is also important to look at how to denote series, for example:

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots + \frac{1}{n^2}$$

is the sum of all the reciprocal of natural number squared. (This series is also called the Basel Problem, later on, we will prove that it converges, you can look up how to calculate the value the series converge to on the internet)

In general, we use Greek letter Σ (pronounced as "sigma") to represent sum, it is used like this:

$$\sum_{n=\text{first term}}^{\text{last term}} \text{nth term formula}$$

For example

$$\sum_{n=1}^5 n^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2$$

means adding 1^2 to 5^2 , while

$$\sum_{n=1}^{\infty} n^2 = 1^2 + 2^2 + 3^2 + 4^2 + 5^2 + \dots + n^2$$

means adding 1^2 all the way to infinity.

1 Practice Problems

1. Find the value of $\sum_{n=0}^4 n$

A. 10

B. 6

C. 14

D. 15

2. What is an equivalent form of $\sum_{n=1}^5 \frac{1}{n}$?

A. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$

B. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

C. $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$

D. ∞

3. What is an equivalent form of $\sum_{n=1}^4 2^n$

A. $2 + 2^2 + 2^3$

B. $2 + 2^2 + 2^3 + 2^4$

C. $2^2 + 2^3 + 2^4 + 2^5$

D. $1 + 2^2 + 2^3 + 2^4$

2 Solution

1. By definition, this means adding 1 to 4, the sum is therefore 10, the answer is A.

2. This means adding 1 to $\frac{1}{5}$, the answer is C.

3. This means adding 2 to 2^4 , the answer is B.