Book recommendations

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Many people, especially college students, ask what books I recommend for learning math, but it's impossible to give a universal answer to such a question. Aside from the obvious fact that it depends on what exactly you want to learn, it's entirely possible for two students with similar backgrounds and goals to approach the same book, with one having a fantastic experience while the other is left perpetually banging their head against the page in confusion. Likewise, a book might work for you at one time in your life but not another.

I've put together a small list of textbooks I've thoroughly enjoyed at some time in the past, with the added condition that each one stands out in some way from the surrounding body of math texts. Take each recommendation with a grain of salt, precisely because the pairing of a book to a learner can be so personal, and who I was when I read each of these may be fairly different from who you are now.

Books for college students and beyond

Vector Calculus, Linear Algebra, and Differential Forms: A Unified Approach

by John Hubbard and Barbara Burke Hubbard

This is what I wish I learned from in my freshman year of college. The part of it about linear algebra is one of the best linear algebra resources out there, and remainder shows you how those tools apply to non-linear mathematics.

It doesn't pull any punches for the pure math students, but unlike many books with that target audience, it stays grounded in frequent numerical examples and scientific applications. The culmination of the generalized Stokes' theorem, as well as the way to frame Maxwell's equations with differential forms, are particularly satisfying.

An Introduction to Thermal Physics

by Daniel Schroeder

Despite my aim to keep this list purely about math books, I cannot help but include this one in the list. Simply put, it's one of the most enjoyable textbooks I've ever read, giving a very satisfying first-principles understanding of what thermodynamic concepts like temperature and entropy really are, and how we can to understand them.

Statistical mechanics is all about taking surprisingly minimal assumptions, and mathematically analyzing the life out of them to explain where the laws of thermodynamics come from. I feel all math students should at least expose themselves to this topic, if for no other reason than to see what powerful results can come from simple combinatorics. There's also value in practicing what it feels like to take a complicated expression, write an approximation that capture the core part that really matters, and use it to make experimental predictions about insanely complicated systems.

Visual Complex Analysis

As the title suggests, this offers a highly pictorial introduction to complex analysis. I would not use this as the *only* text to learn about complex analysis, but Needham does a marvelous job of conveying the beauty of the subject, with numerous helpful intuitions for otherwise famously tricky ideas.

Visual Group Theory

by Nathan Carter

Another text with a titular promise to focus on pictures, this one tackles a subject which is more commonly taught purely symbolically. One of the most important things for any student learning about groups to understand is just how many different ways there are to think about a group, and how different views might be helpful in different contexts. Again, I wouldn't use this as the only book to understand the topic, but it offers a refreshingly different perspective on the topic from most others out there to help arm your arsenal of intuitions.

Proofs from the book

by Martin Aigner and Günter Ziegler

If you like clever proofs, this is the book for you. It's filled with some of the most elegant arguments in math, taken from combinatorics, geometry, graph theory, analysis, set theory, and more. I can't think of any other books with a higher density of "aha" moments.

Elegant proofs can often be difficult to understand due to their density, but Aigner and Ziegler skillfully set the stage for each argument to make it more understandable. There are also numerous helpful illustrations in the margin which help elucidate the charm in all the proofs they've collected.

Nonlinear Dynamics and Chaos

by Steven Strogatz

Steven Strogatz is one of the most well-known math communicators, and for good reason. He has a way of making you love a problem before diving into telling you how it's solved. Chaos is an incredibly fascinating field, which has only emerged relatively recently in the history of math. This is the well-deserved gold standard for understanding what it's all about.

Ordinary Differential Equations

by Vladimir Arnold

If you want to understand differential equations, but more from the pure math side of things, Arnold gives a wonderful picture of what the field is all about, with a mixture of high-level intuitions and low-level details, with a satisfying focus on visual and physical intuition along the way.

The Art Of Probability

by Richard Hamming

Hamming was a practitioner, spending much of his career in Bell Labs, so unlike many books on probability, this stays grounded in the kind of intuitions that come up a lot in solving practical problems. It has countless helpful puzzles and problem-solving tricks which are organized in such a way as to give a sense of the deeper theory underlying probability and statistics.

Information Theory, Inference and Learning Algorithms

by David MacKay

Information theory has a way of shaping the way you view many things in math, computer science, physics, and beyond. It's one of those fields that takes ideas that you wouldn't think of as being quantifiable or rigorous and makes them so.

My *first* recommendation for anyone looking to learn about information theory is to read Shannon's original paper on the topic. After that, for a fuller view of what it's become, MacKay's book is fantastic.

The Cauchy-Schwarz Master Class

by J. Michael Steele

The strength of this book is the central role it places on problems. Its aim is to teach you about inequalities, which are the backbone of analysis. But it's all too easy to teach the statement of an inequality without conveying the intuition for when and how you'd use it. By focusing on well-chosen puzzles, this book does an admirable job circumventing that risk and leaving you with the feeling of having trained a skill, rather than having learned a list of facts.

Fourier Analysis: An Introduction

by Elias Stein and Rami Shakarchi

Not only is this a wonderful foundation for Fourier analysis, with a surprising and satisfying application to prime numbers given in the last chapter, but it's also a good starting point for analysis as a whole. Stein and Shakarchi have a series of four excellent books on the foundations of analysis, based on a now-famous series of lectures given at Princeton.

Analysis, in all its forms, can be very tricky to wrap your mind around, and very fiddly to work with in trying to form airtight proofs and definitions. This set of books, though, is a good place to start for anyone feeling ambitious.

Primes of the Form x^2 + n y^2

by David A. Cox

This might seem like a bizarrely specific question to write a whole book about, but that's actually the charm. It covers deep and general ideas, such as class field theory and elliptic functions, which are usually only taught to graduate students, but Cox approaches it all from an extremely concrete question which Fermat thought about.

The typical culture in modern math texts, especially graduate texts, is to first build up an abstract theory, with indications of what you can do with these constructions appearing as offhanded as exercises or examples later on. In any other book on class field theory, the question of which primes can be expressed as $x^2 + ny^2$x2+ny2 for a particular nn would be some exercise sitting in the back of a later chapter, easily ignored. The fact that Cox puts one little example so front-and-center as to be the title reflects his general adherence to strong motivation before any abstraction.

The Princeton Companion to Mathematics

by Timothy Gowers (and many, many others)

Books can never replace the intuition available if there's a professor down the hall whose door you can knock on to start asking questions. However, this book gets pretty close.

Where it shines is in section IV, which includes many expository introductions to various fields of modern math. If there's a field of research you've heard of, say something like analytic number theory, and you're curious to get a feel for what it's all about, the corresponding essay in that section is likely to do a fantastic job.

For any textbooks that you read, try to avoid being passive. Read with a pencil and paper in hand to jot down notes and work on exercises (yes, you should actually do the exercises!). Try to predict what proofs will look like before reading them. Be willing to meditate on what the right way to think about a given object is, and ask what would happen if definitions were tweaked.

Ask yourself if each new construct feels motivated, or if it's out of the blue. If it is out of the blue, it's okay to move forward anyway, just keep note of the fact that there is a lurking question mark.

Also, although it’s not quite a book, you may also enjoy the expository papers written by Keith Conrad, especially if you’re looking to learn group theory or number theory.

Pre-college

The recommendations above are targeted at those in college or beyond. There are undoubtedly many great books for those in high school and below, but I feel less well-positioned to give recommendations in that direction. It was really only once I got to college that I began learning meaningfully from math books, with more of my learning before then coming from interactions with teachers, poking around online, and getting lost in my own head.

That's not to say I didn't learn from books at all before then, there are a handful of texts such as the original Art of Problem Solving books which were very influential. And when I was younger I remember a certain fascination with many of the wooden books my dad gave me. But part of me feels like the best thing for any young learner will be a set of good problems to chew on, more so than a set of good books.

Popular books

There are countless wonderful works out there about math aimed at the general public. Here's just a very small handful which I’d recommend taking a look at.

Chaos: Making a New Science

by James Gleick

This is one of my all-time favorite books, closely followed by everything else Gleick has written. It gives an inspiring look at the birth of a new field, and unlike many popular science books, it manages to talk substantively about the math that was being discovered.

Change Is the Only Constant: The Wisdom of Calculus in a Madcap World

by Ben Orlin

If you haven't already come across math with bad drawings, you're severely missing out.

I feel like Ben Orlin solved what was previously an unsolved problem with this book: Teach calculus in a way that is as enjoyable to those unfamiliar with the topic as it is to experts who use it every day. I, for one, was laughing from page 1. One of the more delightful aspects of this book is how Orlin doesn't just draw from the usual suspects like physics and economics to get practical insight about calculus, he also looks to history, poetry, literature, and swimming corgis.

Infinite Powers: How Calculus Reveals the Secrets of the Universe

by Steven Strogatz

This is another wonderful book aiming to popularize calculus (2019 was a good year for that goal). What I enjoyed about this one is how broad a view Strogatz takes of the origins of calculus, ranging from the clever proofs Archimedes was developing up to Fourier's study of heat.

Humble Pi: A Comedy of Maths Errors

by Matt Parker

Leave it to Matt Parker to write an entire book about making mistakes in math.