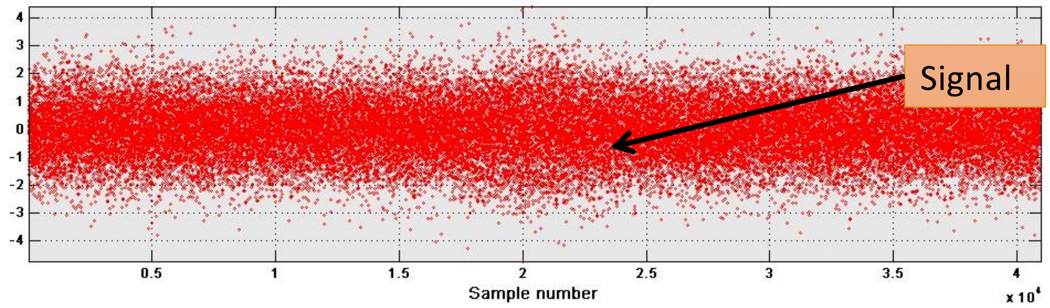
Beyond MLE and GLRT

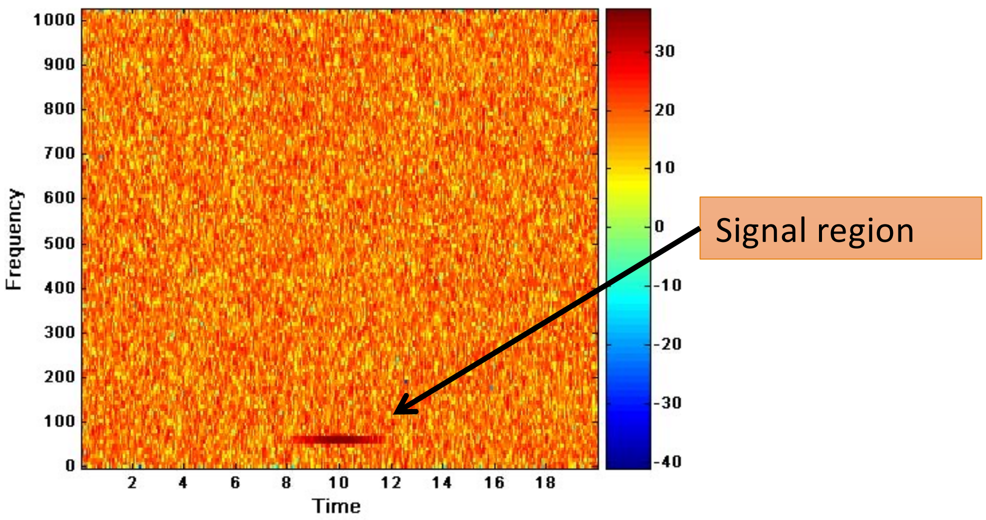
SIGNAL MODELS

- MLE and GLRT both require reliable signal models
- Many anticipated GW sources do not have reliable signal models, for example:
  - Core-collapse supernova signals for ground-based detectors: turbulent dynamics → fundamentally unpredictable waveforms
  - Extreme mass ratio inspiral signals for LISA: two-body problem in GR but very challenging to compute accurately
- It is possible to extend MLE and GLRT to the case of where signal models are not known: Requires regularization techniques
- Various types of time-frequency analysis methods may also be used
- MLE and GLRT also require reliable noise models
- Data analysis techniques (e.g., vetoes) are needed to bring the performance of MLE and GLRT closer to the ideal one

TIME-FREQUENCY ANALYSIS

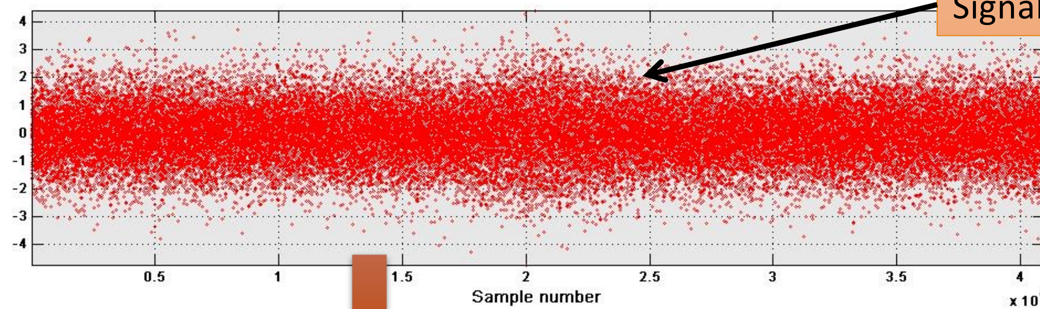
TIME FREQUENCY ANALYSIS OF NOISY DATA



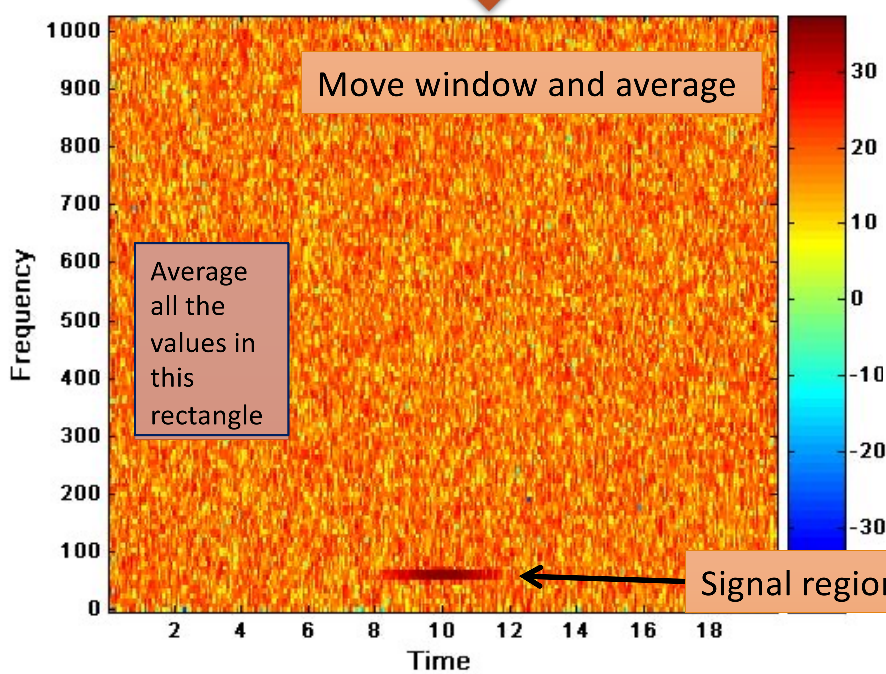


EXCESS POWER

Signal



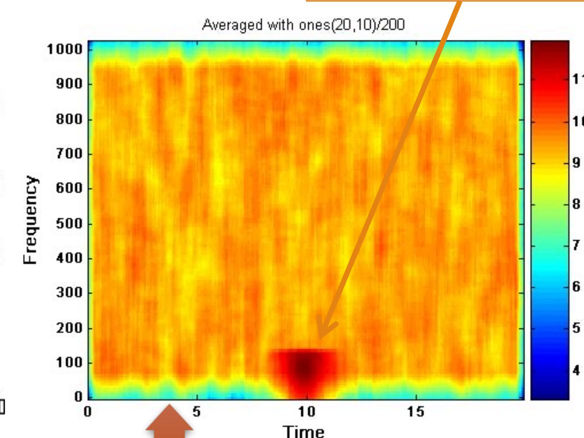
Threshold on the maximum value



Move window and average

Average all the values in this rectangle

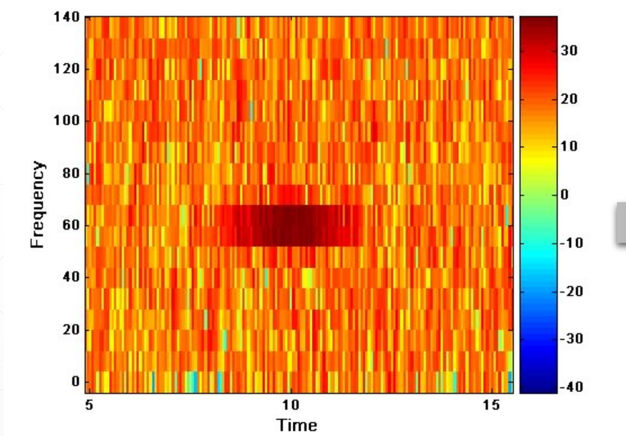
Signal region



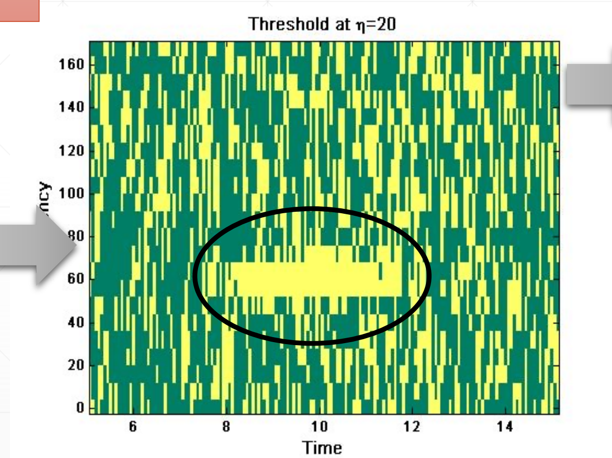
Use a range of window dimensions to catch signals of varying time-frequency area

TF clustering

Assume that a signal occupies a simply connected region of the time-frequency plane



Apply a threshold: set pixels below threshold to 0 and above to 1



Cluster
Find groups of pixels that are "connected" to each other

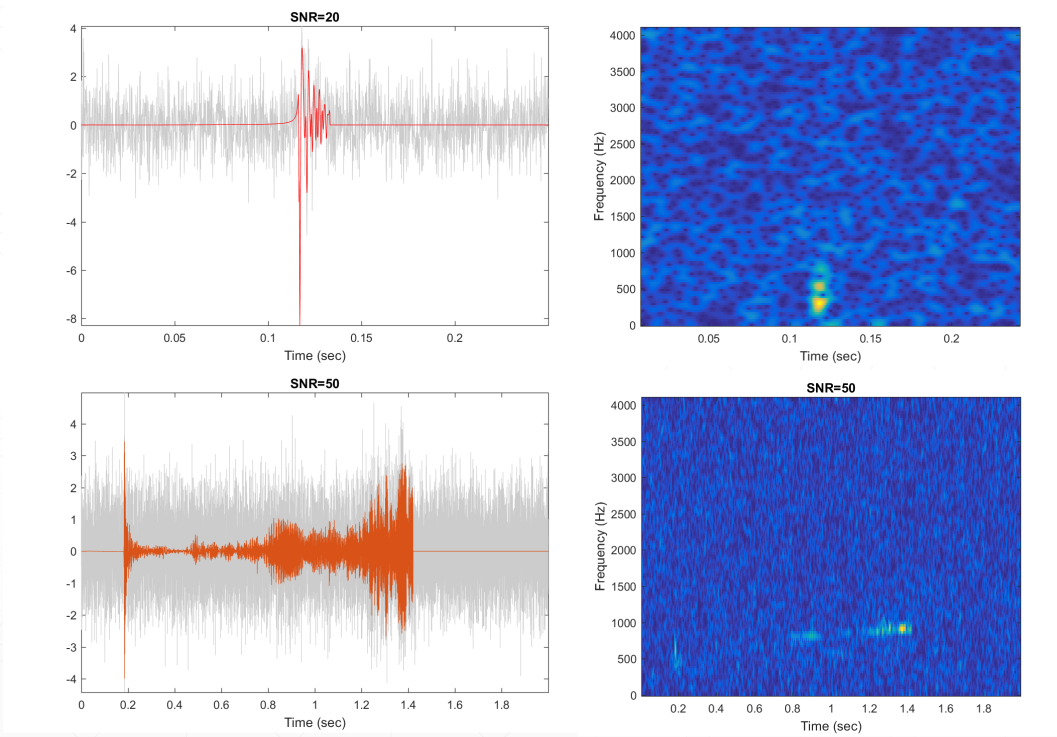
Retain only clusters above a certain threshold size

Further threshold on average value of pixels in the cluster

TF methods in GW data analysis

- Time-frequency methods plays an important role in searches for unmodeled gravitational wave signals
  - GW150914 discovered with Coherent WaveBurst (CWb): TF analysis with Wavelets (*Klimenko, Yakushin, Rakhmanov, Mitselmakher, CQG, 2004)* combined with regularized network analysis (*Klimenko, Mohanty, Rakhmanov, Mitselmakher, PRD, 2005)*
  - *Klimenko et al, PRD, 2016*
- Q-transform (used in Omicron)
  - *Brown, JASA, 1991; Chatterji, Blackburn, Martin, Katsavounidis, CQG, 2004*

TF clustering and CCSN signals



Core bounce

- Single Cluster
- Signal power well-localized $\Rightarrow$ cluster easily detectable

Post-Shock

- Fragmented Cluster
- Individual clusters can be too weak or too small
- Aggregation of multiple clusters $\Rightarrow$ additional ad hoc algorithm parameters

Note: Multi-resolution TF analysis is a must (not shown here)

Analyzing real data

- The concepts we have learnt so far are adequate to do a preliminary analysis of real GW data
- Matlab live scripts provided in DATASCIENCE_COURSE / GWDATA

Homework

Readings

- GWTC-3: Compact Binary Coalescences Observed by LIGO and Virgo During the Second Part of the Third Observing Run, arXiv:2111.03606
- Papers related to GW150914 and GW170817

Windowing

- Application of convolution theorem in the reverse direction
- Example: Windowed sinusoid

$$s(t) = \begin{cases} \cos(2\pi f_0 t)\,; t \in [-T/2, T/2] \\ \quad 0; \quad \text{otherwise} \end{cases}$$

- **Windowing:** Box-car window

$$s(t) = \Pi(t/T)\cos(2\pi f_0 t)$$

$$\Pi(t) = \begin{cases} 1, & t \in [-1,1] \\ 0, & \text{otherwise} \end{cases}$$

- Fourier transform of Box car window:

$$\tilde{\Pi}(\mathrm{f}) = T\mathrm{sinc}(fT) = T\sin(\pi f T)/(\pi f T)$$

- Fourier transform of sinusoid (Lecture 3):

$$\frac{1}{2}(\delta(f - f_0) + \delta(f + f_0))$$

- Convolution theorem:

$$\tilde{s}(f) = \int_{-\infty}^{\infty} df'\, T\mathrm{sinc}((f - f')T)\left[\frac{1}{2}(\delta(f' - f_0) + \delta(f' + f_0))\right]$$

$$= \frac{T}{2}\left[\mathrm{sinc}\big((f - f_0)T\big) + \mathrm{sinc}((f + f_0)T)\right]$$

- Using a different window (e.g., Hann) results in a different trade-off between the main lobe width and side lobe height