KIAA Intern

Matrix Calculate

> Reference
> J.H.Wilkinson: Rounding errors in algebraic process
> 矩阵分析与应用 张贤达

$\frac{\partial |C|}{\partial C_{ij}} = det(C)Tr[C^{-1}\frac{\partial C}{\partial C_{ij}}]$

$Tr[C^{-1}\frac{\partial C}{\partial C_{ij}}] = C^{-1}_{ii}$

$\frac{\partial C^{-1}}{\partial C_{ij}} = -C^{-1}\frac{\partial C}{\partial C_{ij}}C^{-1}$

$\Lambda = -\frac{1}{2}\ln|C| - \frac{1}{2}X^TC^{-1}X$

$\delta\Lambda_{ij} = \frac{\partial\Lambda}{\partial C_{ij}}\delta C_{ij} = (-\frac{1}{2}C^{-1}_{ji} + \frac{1}{2}X^TC^{-1}V_{ij}C^{-1}X)\delta C_{ij}$

> $V_{ij} = \frac{\partial C}{\partial C_{ij}}$

Rounding error analysis of matrix inversion

Use $CX = I$ to calculate inversion of C, X is inversion of C.
if C is not singular then we have

如果矩阵 A 非奇异，可以用矩阵 A 的逆来表示误差：

$$x - x_* = A^{-1}(b - Ax_*)$$

因此

$$\| x - x_* \| \leqslant \| A^{-1} \| \; \| E \| \; \| x_* \|$$

根据它可方便地比较误差的范数和数值解的范数，因此相对误差满足

$$\frac{\| x - x_* \|}{\| x_* \|} \leqslant \rho \| A \| \; \| A^{-1} \| \varepsilon$$

即

$$\frac{\| x - x_* \|}{\| x_* \|} \leqslant \rho \kappa(A) \varepsilon$$

so the relative error for matrix C is less than $\rho\kappa(C)\epsilon$, where ρ is usually less than 10, ϵ is the machine precision.

矩阵奇异上述方法用不了，转战SVD

$$\Lambda = -\frac{1}{2}\ln |C| - \frac{1}{2}X^T C^{-1} X$$

$$C = USV^T$$

$$C^{-1} = VS^{-1}U^T$$

$$\Lambda = -\frac{1}{2}\sum \ln(S_i) - \frac{1}{2}Y^T S^{-1} Y$$

where $Y = U^T X$ and because the matrix C is symmetric so $U = V$
So we have

$$\Lambda = -\frac{1}{2}\sum \ln(S_i) - \frac{1}{2}\sum \frac{y_i^2}{S_i}$$

$$\delta\Lambda = -\frac{1}{2}\sum \frac{\delta S_i}{S_i} + \frac{1}{2}\sum \frac{y_i^2}{S_i^2}\delta S_i$$

where we let $\Delta_i = \frac{\delta S_i}{S_i}$, so

$$\delta\Lambda = -\frac{1}{2}\sum \Delta_i + \frac{1}{2}\sum \frac{y_i^2}{S_i}\Delta_i$$

For $Y = U^T X$ and X satisfies $< X^T C^{-1} X >= 1$, so we have the expectation of $\delta\Lambda$

$$< \delta\Lambda >= 0$$