MITx: 8.421.1x Atomic and Optical Physics I part 1: Resonance

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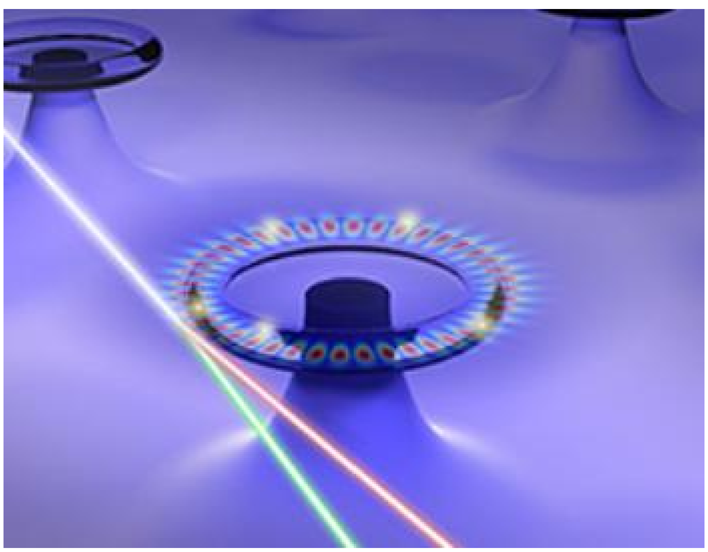
1 Resonances - an overview

1.1 Introduction to resonance

- Classical resonance - any periodic variation of some variable. When you drive a system with a variable frequency, you observe a peak at resonant frequency.
- Atomic Physics is interested with every single possible aspect of resonance. Resonance is the language physicists talk to atoms with.
- Oscillators are characterized by the sharpness of the resonance (Q - quality factor) which is a ratio of the frequency width of the resonance and its resonant frequency. Q is the number of oscillations that can be observed before the oscillation decays away.
- In Atomic Physics oscillators are characterized by extremely high Q factors
  - optical oscillators: 10^{15} Hz (light frequency) $\Rightarrow Q = 10^6$ (with doppler broadening), $Q = 10^{15}$ (without doppler broadening, eg. atom in optical lattice in metastable levels)
  - mechanical oscillators: eg. quartz $Q = 10^4$ - 10^6
  - micromechanical oscillators: $Q = 10^5$ (eg. whispering gallery mode can have $Q = 10^9$, where light circulates around the circle of the mushroom-like structure)

  - astronomical oscillators: earth rotation: $Q = 10^7$, neutron star: $Q = 10^{10}$
- "Useful" resonances are reproducible and connected by theory to fundamental constants.
- Rydberg constant ($R = 1.097... * 10^7 m^{-1}$) is the most accurately known constant in physics because it can be directly measured by performing spectroscopy experiments with hydrogen. Measuring fundamental constants more accurately is very important in understanding the world.
- Typical resonance lineshape is lorentzian which is proportional to $\mathrm{Im}\left(\frac{1}{\omega_0-\omega+i*\frac{\gamma}{2}}\right)$. For lorentzian: $\gamma =$FWHM and $Q = \frac{\omega_0}{\gamma}$.

1.2 Angular frequency units

- All parameters should be measured in angular frequency units which are technically $\frac{rad}{s}$, sometimes we use s^{-1}. Frequencies (not angular) are measured in Hz.
- Good manner is to write angular frequency as $\omega_0 = 2\pi * 1\text{MHz} = 6.28 * 10^6 s^{-1}$, *NEVER* $6.28 * 10^6$ Hz.
- Units for γ (temporal decay rate, inverse of dumping time) are s^{-1} (*NOT* Hz).

2 Resonance widths and uncertainty relations

2.1 How precisely can you measure frequencies?

- Some of the most accurate experiments in physics are done by measuring frequency.
- For oscillator oscillating for time Δt with finite width of frequency spectrum $\Delta\omega$: $\Delta\omega\Delta t \geq \frac{1}{2}$

2.2 Heisenberg limits on quantum and classical systems

- Heisenberg uncertainty relation is about single measurement on a single quantum system. To increase accuracy of the measurement it can be repeated many times or can be done on a system with many photons.
- Using nonlinear process it can be achieved for a single quantum system and a single photon utilizing many energy levels at the same time and jumping many levels at the same time. Precision of such an experiment is better by $\frac{1}{n}$ times, where n is the number of energy levels.

2.3 Frequency measurement example: atomic clocks

- Cesium atom fountain clock, $\omega = 2\pi * 10\text{GHz}$, interrogation time $\Delta t = 1\text{s}$ has fractional linewidth $\frac{\Delta\omega}{\omega} = 10^{-11}$. Accuracy of the best cesium fountains is now 10^{-16}.
- Strontium optical clock - extremely narrow transition. Accuracy $6 * 10^{-18}$!

3 Harmonic oscillators and two-level systems

3.1 Harmonic oscillators vs. two-level systems

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