MITx: Atomic and Optical Physics I

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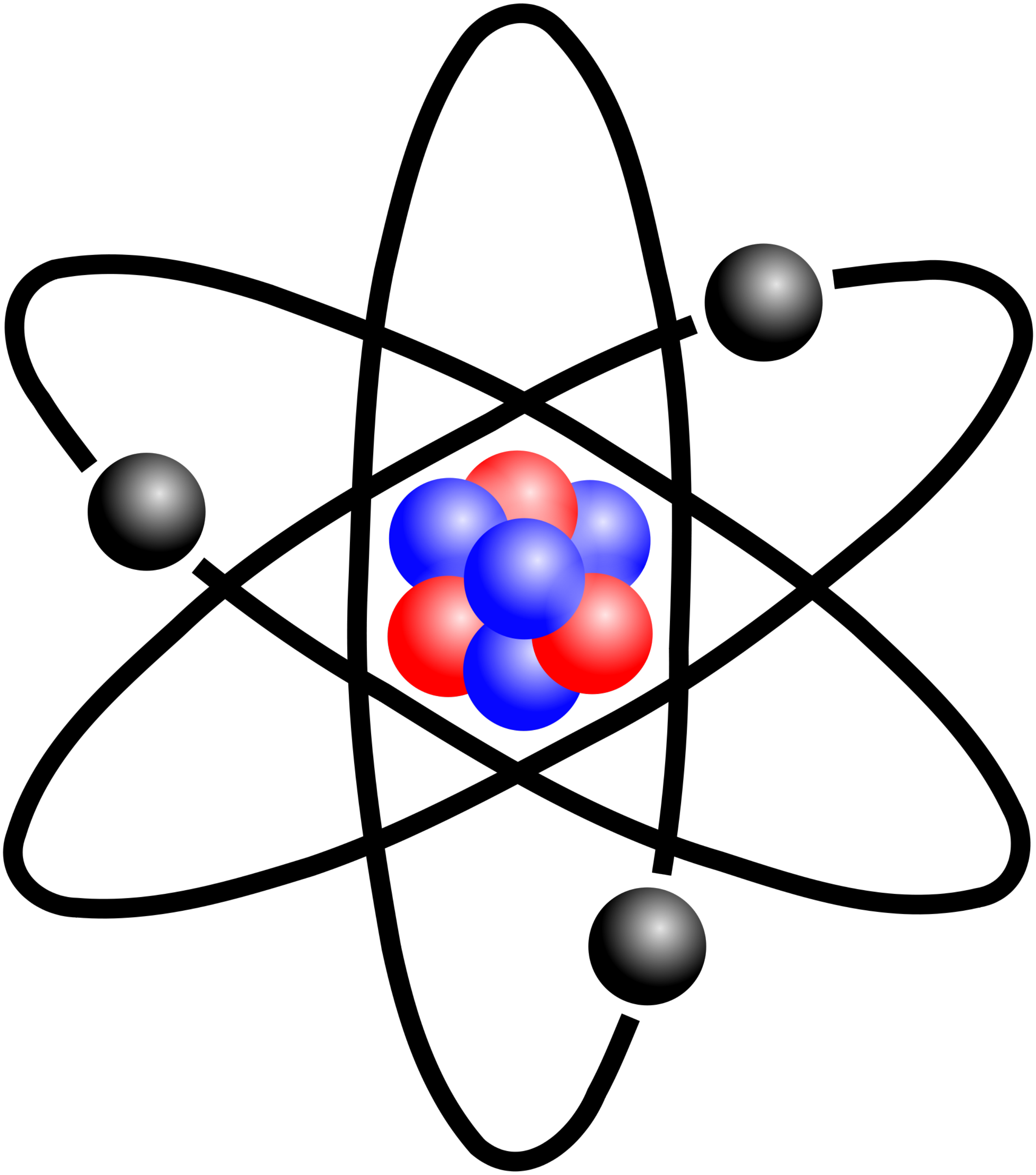


LaTeX notes for the MITx: 8.421 Atomic and Optical Physics I MOOC

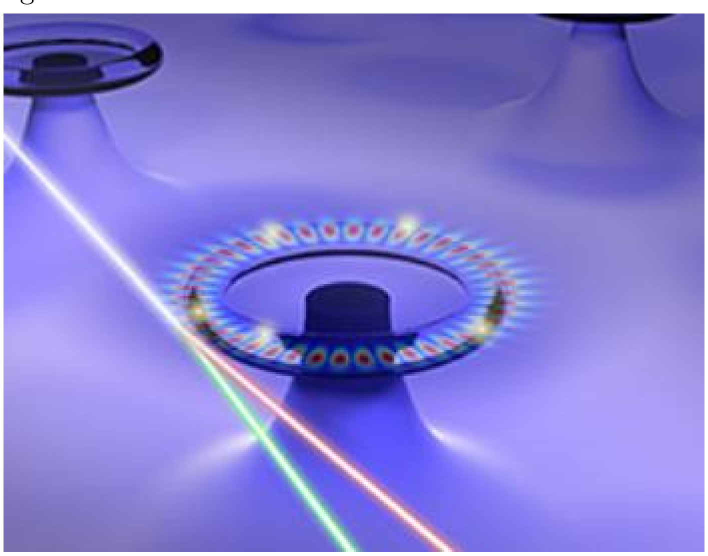
Part 1: Resonance

Week 1: Classical Resonances

1 Resonances - an overview

1.1 Introduction to resonance

- Resonance - any periodic variation of some variable. When you drive a system with a variable frequency, you observe a peak at resonant frequency.
- Atomic Physics is interested with every single possible aspect of resonance. Resonance is the language physicists talk to atoms with.
- Oscillators are characterized by the sharpness of the resonance (Q - quality factor) which is a ratio of the frequency width of the resonance and its resonant frequency. Q is the number of oscillations that can be observed before the oscillation decays away.
- In Atomic Physics oscillators are characterized by extremely high Q factors
  - optical oscillators: 10^{15} Hz (light frequency) $\Rightarrow Q = 10^6$ (with doppler broadening), $Q = 10^{15}$ (without doppler broadening, eg. atom in optical lattice in metastable levels)
  - mechanical oscillators: eg. quartz $Q = 10^4$ - 10^6
  - micromechanical oscillators: $Q = 10^5$ (eg. whispering gallery mode can have $Q = 10^9$, where light circulates around the circle of the mushroom-like structure)



  - astronomical oscillators: earth rotation: $Q = 10^7$, neutron star: $Q = 10^{10}$
- "Useful" resonances are reproducible and connected by theory to fundamental constants.
- Rydberg constant ($R = 1.097... * 10^7 m^{-1}$) is the most accurately known constant in physics because it can be directly measured by performing spectroscopy experiments with hydrogen. Measuring fundamental constants more accurately is very important in understanding the world.
- Typical resonance lineshape is lorentzian which is proportional to $\mathrm{Im}\left(\frac{1}{\omega_0 - \omega + i * \frac{\gamma}{2}}\right)$. For lorentzian: $\gamma =$FWHM and $Q = \frac{\omega_0}{\gamma}$.

1.2 Angular frequency units

- All parameters should be measured in angular frequency units which are technically $\frac{rad}{s}$, sometimes we use s^{-1}. Frequencies (not angular) are measured in Hz.
- Good manner is to write angular frequency as $\omega_0 = 2\pi * 1\text{MHz} = 6.28 * 10^6 s^{-1}$, *NEVER* $6.28 * 10^6$ Hz.
- Units for γ (temporal decay rate, inverse of dumping time) are s^{-1} (*NOT* Hz).

2 Resonance widths and uncertainty relations

2.1 How precisely can you measure frequencies?

- Some of the most accurate experiments in physics are done by measuring frequency.
- For oscillator oscillating for time Δt with finite width of frequency spectrum $\Delta\omega$: $\Delta\omega\Delta t \geq \frac{1}{2}$

2.2 Heisenberg limits on quantum and classical systems

- Heisenberg uncertainty relation is about single measurement on a single quantum system. To increase accuracy of the measurement it can be repeated many times or can be done on a system with many photons.
- Using nonlinear process it can be achieved for a single quantum system and a single photon utilizing many energy levels at the same time and jumping many levels at the same time. Precision of such an experiment is better by $\frac{1}{n}$ times, where n is the number of energy levels.

2.3 Frequency measurement example: atomic clocks

- Cesium atom fountain clock, $\omega = 2\pi * 10\text{GHz}$, interrogation time $\Delta t = 1\text{s}$ has fractional linewidth $\frac{\Delta\omega}{\omega} = 10^{-11}$. Accuracy of the best cesium fountains is now 10^{-16}.
- Strontium optical clock - extremely narrow transition. Accuracy $6 * 10^{-18}$!

3 Harmonic oscillators and two-level systems

3.1 Harmonic oscillators vs. two-level systems

- Two-level system is a system with 2 levels. Harmonic oscillator is a system with infinite number of equidistant levels.
- When some energy is put into the second state of HO, some small energy also goes into every higher level. In two-level system there are no higher levels than second one, so nothing can go higher.
- Quantum system can be described as a harmonic oscillator for weak excitation (when there is small energy in excited state).
- Two-level system can be saturated (harmonic oscillator can never be saturated). When the second energy level is full adding more energy can't go higher. If a two-level system is not saturated, it behaves like a harmonic oscillator so it behaves completely classical.

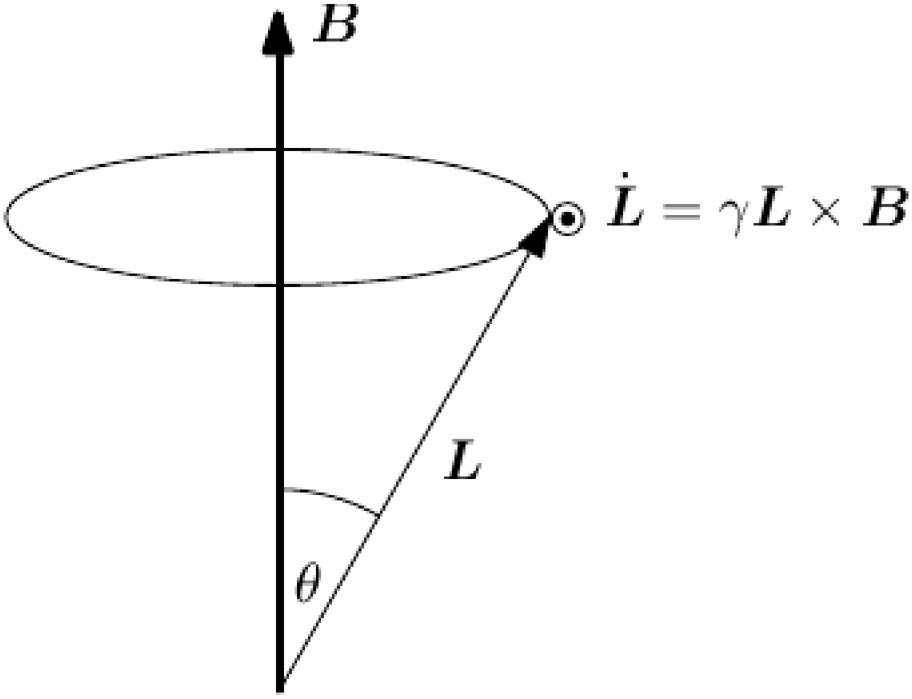
3.2 Rotating systems vs. 2-level systems

- Precessing gyroscope has a bound on amplitude. Motion of classical magnetic moments provides a model which captures almost all features of the quantum mechanical two-level system (except for projection, quantum measurement).

4 Classical magnetic moment in a uniform field

4.1 Magnetic resonance

- Gyromagnetic ratio (γ) is a ratio between magnetic moment and angular momentum ($\boldsymbol{\mu} = \gamma \mathbf{L}$). From that we can find that derivative of angular momentum is given by: $\dot{\mathbf{L}} = \gamma \mathbf{L} \times \mathbf{B}$. Solution of that equation is a pure precession of $\mathbf{L}$ around $\mathbf{B}$. Precession around the $\mathbf{B}$ is happening at the constant tipping angle with an angular frequency called Larmor frequency $\Omega_L = -\gamma B$.



- For an electron $\gamma_e = 2\pi * 2.8\text{MHz/G}$. For classical charge distribution (of particles with the same charge to mass ratio as electrons) gyromagnetic ratio is a half of that: $\gamma = 2\pi * \mu_B$ (where $\mu_B = 1.4\text{MHz/G}$ and is called Bohr magneton).
- Proton is heavier than the electron ($\frac{m_p}{m_e} = 1836.153$) and therefore $\gamma_P = 2\pi * 4.2\text{kHz/G}$.
- The magnetic moment of the electron is 1 Bohr magneton.
- Precession frequency of a system depends on energy difference of two neighbouring energy levels.

4.2 Rotating coordinate transform for equations of motion of a classical magnetic moment

- Going into rotating frame is useful to solve certain problems.
- Rotating vector $\mathbf{A}$ which rotates with a constant angular frequency $\boldsymbol{\Omega}$ is described by: $\dot{\mathbf{A}} = \boldsymbol{\Omega} \times \mathbf{A}$.
- When we view the system in a coordinate system rotating at $\boldsymbol{\Omega}$ then derivatives of the vector $\mathbf{A}$ in rotating and inertial frames are related by: $\dot{\mathbf{A}}_{\mathbf{in}} = \dot{\mathbf{A}}_{\mathbf{rot}} + \boldsymbol{\Omega} \times \mathbf{A}_{\mathbf{in}}$. From that follow two special cases:
  - If $\mathbf{A}$ is constant in the rotating system then: $\dot{\mathbf{A}}_{\mathbf{in}} = \boldsymbol{\Omega} \times \mathbf{A}_{\mathbf{in}}$.
  - If rotating frame is not rotating ($\boldsymbol{\Omega} = 0$) then: $\dot{\mathbf{A}}_{\mathbf{in}} = \dot{\mathbf{A}}_{\mathbf{rot}}$.
- From above there can be derived an operator equation for transforming a vector from inertial to rotating frame: $\left(\frac{d}{dt}\right)_{\text{rot}} = \left(\frac{d}{dt}\right)_{\text{in}} - \boldsymbol{\Omega} \times \left(\right)_{\text{in}}$.

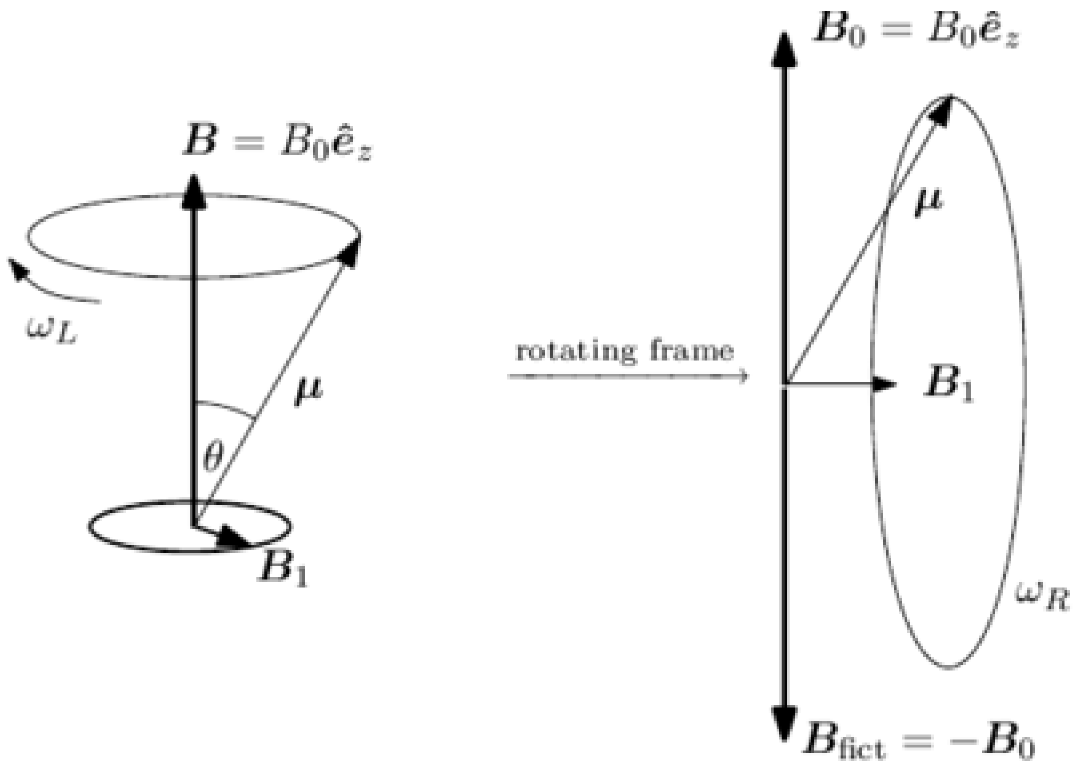
- We can apply this operator equation to angular momentum vector $\mathbf{L}$ and we get: $\dot{\mathbf{L}}_{\mathbf{rot}} = \gamma \mathbf{L}_{\mathbf{in}} \times (\mathbf{B} + \mathbf{B}_{\mathbf{fict}})$, where $\mathbf{B}_{\mathbf{fict}} = \frac{\mathbf{\Omega}}{\gamma}$.

- If we choose the rotating frequency to be Larmor frequency $\mathbf{\Omega} = \mathbf{\Omega_L} = -\gamma \mathbf{B}$ then our effective magnetic field $\mathbf{B}_{\mathbf{eff}} = \mathbf{B} + \mathbf{B}_{\mathbf{fict}} = 0$ and because in that situation there is no magnetic field, we know that angular momentum $\mathbf{L}$ is constant in the rotating frame. To get the knowledge what happens in the real frame the answer has to be 'rotated back'.

- For classical charge distribution (of electrons) forming magnetic moment, Larmor frequency at which magnetic moment of the distribution is precessing is: $\Omega_L = \frac{e}{2m} B$. Frequency of the cyclotron motion for free electron in a magnetic field is twice that: $\Omega_{cyclotron} = \frac{e}{m} B$.

4.3 Rotating magnetic field on resonance

- We put a magnetic moment in a time-dependent field: $\mathbf{B}(t) = B_1(\hat{e_x} cos\Omega_L t - \hat{e_y} sin\Omega_L t) + B_0 \hat{e_z}$



- In the frame rotating with Larmor frequency we have: $\mathbf{B}_{\mathbf{eff}}(t) = \mathbf{B}(t) - \frac{\Omega_L}{\gamma}\hat{e_z} = B_1 \hat{e_{x'}}$, where $\hat{e_{x'}} = \hat{e_x} cos(\Omega_L t) - \hat{e_y} sin(\Omega_L t)$. Therefore in the rotating frame we have a static field of value B_1, and we know that in the static magnetic field magnetic moment is just precessing around the field vector with the Rabi frequency $\omega_R = \gamma B_1$.

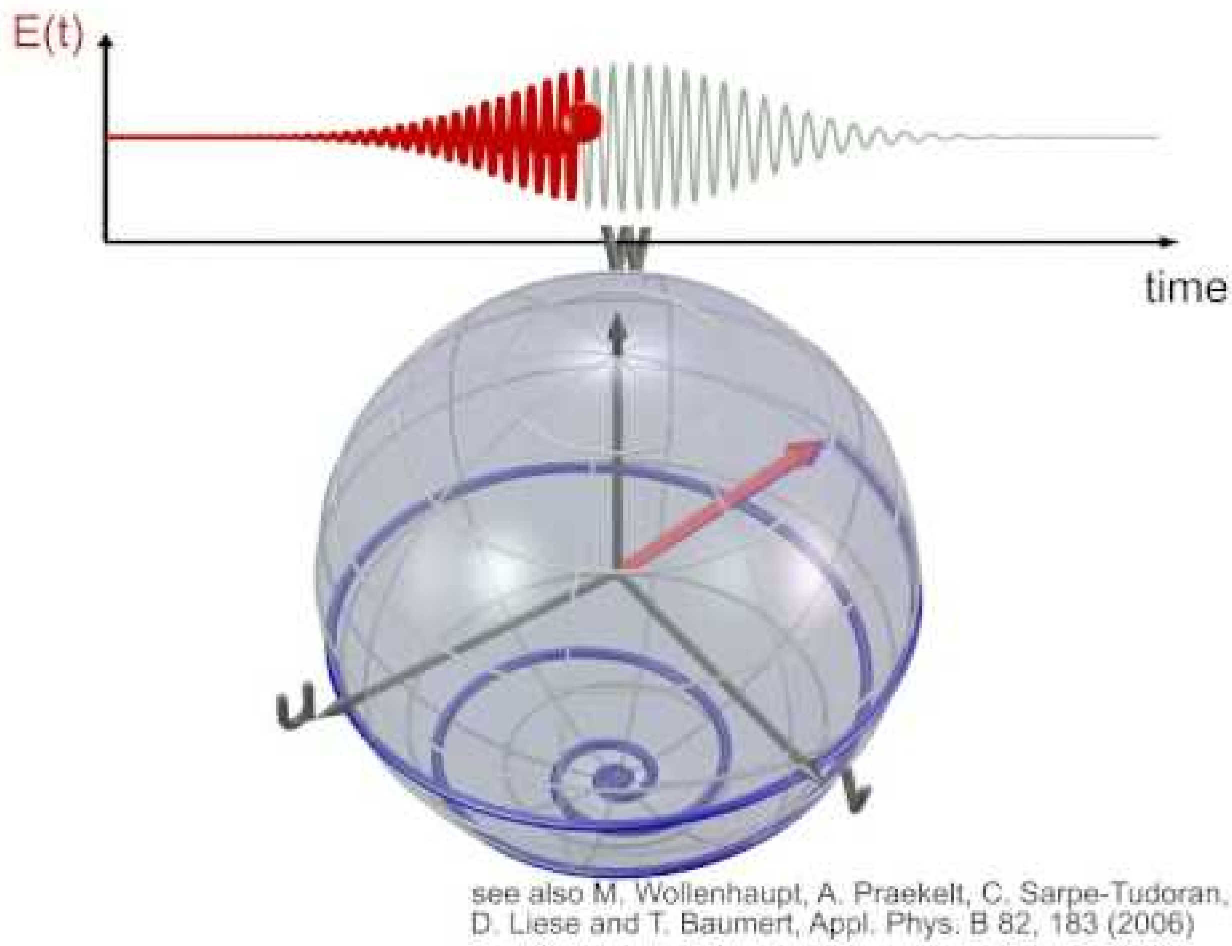
- If we start with the magnetic moment aligned with the z-axis (at $t = 0$ $\mu = \mu \hat{e_z}$), after half a Rabi cycle the magnetic moment will be inverted (at $t = \frac{\pi}{\omega_R}$ $\mu = -\mu \hat{e_z}$). This is called π pulse, because it rotates a spin by π or we can call it a 'spin flip'.

- For the off-resonant case, the field in the z direction will not be canceled and the resulting field $\mathbf{B}_{\mathbf{eff}}$ is a sum of the field in the z direction and the rotating field. The magnetic moment precesses at generalized Rabi frequency: $\Omega_R = \gamma B_{eff} = \sqrt{\omega_R^2 + (\omega_L - \omega)^2} = \sqrt{\omega_R^2}$. Therefore the generalized Rabi frequency is the resonant Rabi frequency added in quadrature with the detuning from resonance. When the rotating field is at frequency lower or higher than the Larmor frequency, oscillation frequency of the magnetic moment will be larger than the resonant Rabi frequency. Driving the system off-resonance, the spin will never fully invert.

- Classical magnetic moment time dependence while in a magnetic field rotating at the generalized Rabi frequency: $\mu_z(t) = \mu\left(1 - 2\frac{\omega_R^2}{\Omega_R^2}\sin^2\frac{\Omega_R t}{2}\right)$. This result is correct in quantum-mechanical treatment and the spin flip probability is: $P = \frac{\omega_R^2}{\Omega_R^2}\sin^2\frac{\Omega_R t}{2}$.

4.4 Rapid adiabatic passage

- Rapid (compared to decoherence and relaxation processes) adiabatic passage is a technique for inverting spins by slowly (compared to the Larmor frequency) sweeping the frequency of the drive field across the resonance. As the frequency of the rotating field is increased, the effective field changes its position from pointing up to pointing in the x-direction on resonance to pointing down after the process. As the precession around the effective magnetic field is at small angle and is happening fast compared to the frequency change, the magnetic moment is moved down together with the field and after the process is complete, the magnetic moment points downwards.



Visualization of a spin flip with chirped femtosecond laser pulses

- Adiabaticity condition is: $|\dot{\omega}| \ll \omega_R^2$.

- It doesn't matter if we start with low frequency and sweep to high or start with high and sweep to low, the effect is the same.

- This technique is easier to use than π pulse because in π pulse you have to be exactly at resonance, and with RAP you just sweep across it.

- Similar process is used in magnetic trap experiments (quadrupole traps).

Week 2: Quantized Spin in a Magnetic Field

Week 3: Resonance and Decoherence Processes