THE STATE UNIVERSITY OF NEW JERSEY
RUTGERS

School of Engineering
Department of Electrical and Computer Engineering

332:224 Principles of Electrical Engineering II Laboratory

Experiment 1

The R-C series circuit

1 Introduction

Objectives

- To study the behavior of the R-C Series Circuit under different conditions
- To use different methods for the determination of the RC time constant from experimental results

Overview

The aim of this experiment is to study the R-C Series Circuit under different conditions by observing input and output waveforms and studying their interrelation.
In particular the following are explored:
(a) *Natural response of an R-C Circuit*: The capacitor is charged to a certain value and its decay is observed as a function of time. Such a decay is known as the *natural response* of the R-C Circuit as there are no forcing inputs applied to the circuit.
(b) *Response of an R-C Circuit to a periodic square wave input*: When the input to a circuit is a periodic signal (wave), the output voltage is a periodic wave as well but not necessarily of the same waveform as that at the input. The output voltage across the capacitance is studied for a square wave input.
(c) *Differentiation And Integration Properties*: Under certain appropriate conditions, an R-C Circuit can function approximately as an integrating circuit or as a differentiating circuit. These properties are also observed.

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2 Theory

2.1 The Natural Response of an R-C Circuit [1]

Consider the R-C circuit of fig. 1 where the source voltage V_s is a DC voltage source. Assuming that the switch has been closed for a long period of time, the circuit has reached a steady state condition. In steady state, the capacitor has been charged to

$$V_g = V_s R/(R+R_i)$$

Therefore, when the switch opens, at t = 0, the initial voltage on the capacitor is V_g volts. With the capacitor so charged, it would be desirable to compute the natural response of the R-C Series Circuit shown in fig. 2.

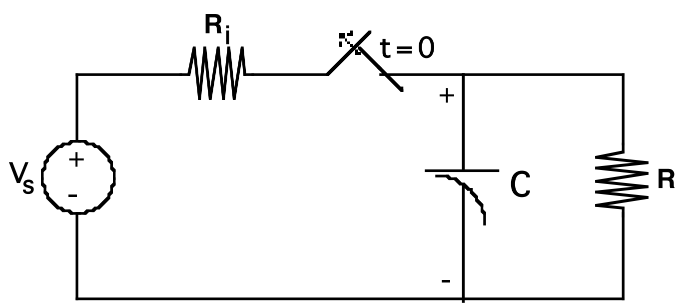


Fig. 1

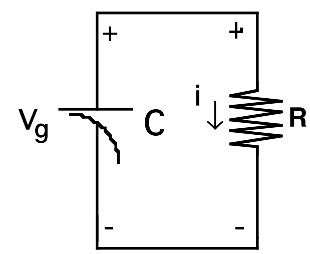


Fig. 2

Analysis of the circuit in fig. 2 yields the general solution

$$v_c(t) = v_c(0)\, e^{-t/RC}, \quad t \geq 0$$

With initial condition: $v_c(0^-) = v_c(0^+) = V_g = V_s R/(R+R_i)$ from which

$$v_c(t) = V_g\; e^{-t/\tau}, \qquad (1)$$

where $\tau = RC$ is the time constant. This means that the natural response of the R-C circuit is an exponential decay of the initial voltage. The rate of this decay is governed by the time constant RC. The graphical plot of Eq. 1 is given in fig. 3 where the graphical interpretation of the time constant is also shown.

[1] A more detailed description can be found in section 7.2 of the text.

The time constant $\tau = RC$ can be measured in several ways. Assuming that R and C are not known, τ can be measured from the discharge data of the capacitor as will be seen in section 5 below.

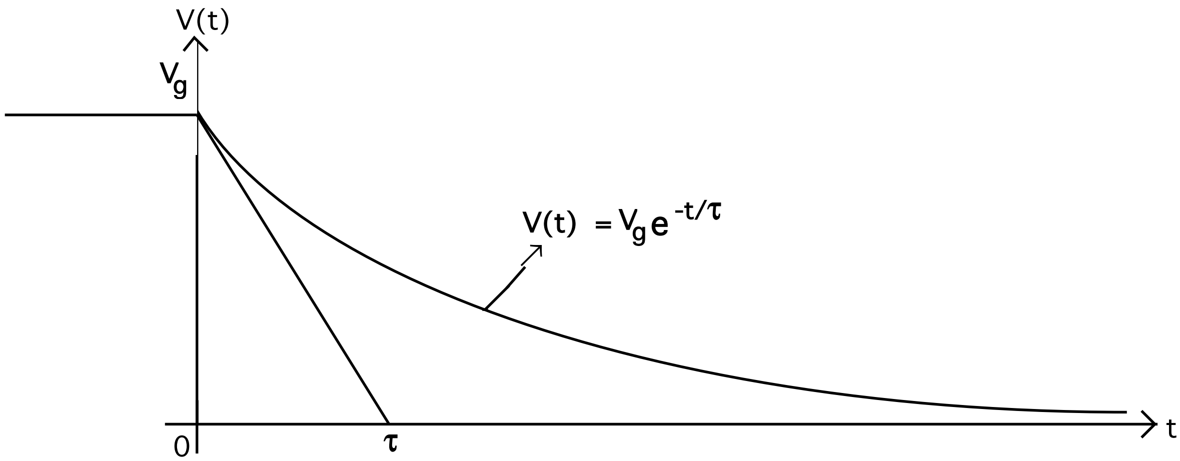
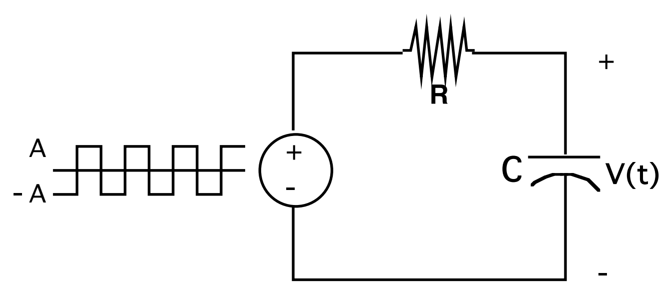


Fig. 3 Exponential Decay Of An R-C Circuit

2.2 Square Wave Response[2]

Let the source voltage V_s be a square wave of frequency f and amplitude A, applied to the R-C circuit as shown in fig. 4.

Fig. 4 A square wave input applied across a series R-C Circuit



Since the input is a periodic wave, the output voltage across the capacitor is also a periodic wave, albeit not a square wave. In each period, the output voltage across the capacitor consists of two parts:

- During the half period when the input is a positive constant, the capacitor gets charged exponentially. Hence the output voltage v(t) during this half period is an *exponentially increasing* signal. At the end of this half period, v(t) has attained a certain positive peak value.

[2] A detailed description of the step response can be found in section 7.3 of the text.

- During the half period when the input is a negative constant, the capacitor gets discharged exponentially. Hence the output voltage v(t) during this half period is an *exponentially decreasing* signal. By the end of this half period, v(t) has attained a certain negative peak value.

The difference between the positive and negative peaks is called the *peak-to-peak voltage of the capacitor*. It can be shown that

$$V_{CPP} = V_{PP}\,(1-e^{-K})/(1+e^{-K}) \qquad (2)$$

where V_{CPP} is the peak-to-peak voltage of the capacitor,
V_{PP} is the peak-to-peak voltage of the input square wave, and
K is a number such that $K\tau$ is the input square wave half-period when $\tau = RC$.

2.3 Differentiation and Integration Properties

Consider the loop equation of the R-C series circuit,

$$v_R(t) + v_C(t) = v(t),$$

where v(t) is the source voltage v_s which could be any time varying signal.

If $v_C(t)$ is kept small with respect to $v_R(t)$, then $v(t) \approx v_R(t) = R\,i(t)$, and

$$v_c(t) = \frac{1}{C}\int i(t)dt = \frac{1}{RC}\int v_R(t)dt \approx \frac{1}{\tau}\int v(t)dt$$

i.e. the capacitor voltage is very closely proportional to the integral of the source voltage. If v(t) is a periodic function, $v_C(t)$ can be kept small by making the period $T \ll \tau$. In this way v_C never gets time to grow large.

On the other hand, when the period $T \gg \tau$, v_C tends to follow v(t) almost exactly. In such a case,

$$v_R(t) = Ri(t) = RC\frac{dv_C(t)}{dt} \approx \tau\frac{dv(t)}{dt}$$

i.e. the resistor voltage is very closely proportional to the derivative of the source voltage.

3 Prelab Exercises

3.1 Following the discussion in section 7.2 of the textbook, write down the differential equation of the series R-C circuit in the absence of any forcing input. Then, explain or derive equation (1) in your own way.

3.2 For $R = 10\text{M}\Omega$ and $C = 15\mu\text{F}$, determine the expected time constant $\tau = RC$.

3.3 Equation (2) for V_{CPP} is rather difficult to prove at this time. Take it as a challenge to derive it as you learn increasingly more on the topic of differential equations.

3.4 Explain in your own words why an R-C series circuit can act approximately as an integrator as well as a differentiator and under what conditions.

4 Experiments

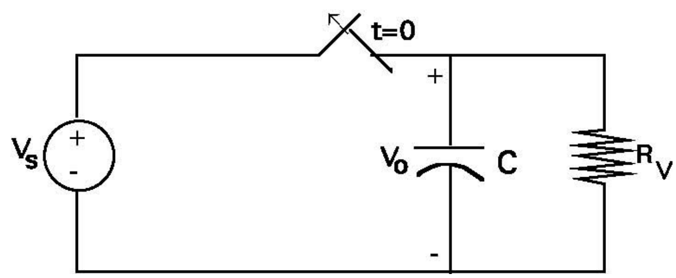
Suggested Equipment:

Function Generator: Tektronix AFG1022
Oscilloscope: Tektronix DPO 3012 or MDO3024
Digital Multimeter: HP/Agilent 34405A, 34401A, or 34461A
Power Supply: Keithley 2231-30-3
Stopwatch
Resistors
Capacitors
Your own USB memory or cell phone camera to record waveforms

4.1 Time Constant of an R-C Circuit

Construct the R-C circuit shown in fig. 5.

Fig. 5 Natural Response of an R-C Circuit



Let the source voltage VS be a DC voltage of 10 V, C = 15 µF, and R_V = 10 MΩ which is the internal resistance of the DMM. Neglect the internal resistance R_i of the source. Then, when the switch is closed, the capacitor charges quickly to the voltage source V_S.

Notes:

1. Observe polarity of the electrolytic capacitor. The negative terminal is marked on the part. Reversing polarity will lead to excessive internal leakage causing the capacitor to discharge too quickly.
2. On the 34461 DMM, the main menu at the bottom of the display should display internal resistance 10M (in yellow). This confirms 10M resistance.
3. We won't use a switch in the experiment, simply make the connection with a wire.

At t=0, the switch opens (break the connection) and the voltage source is disconnected from the R-C circuit. The capacitor will now discharge through the internal resistance of the DMM. Using a timer or stopwatch, record the DMM DC voltage readings for an interval of 5 minutes taking data every 15 seconds. Repeat the same procedure until you are assured that you have a representative set of data. Fill Table 1 in with the data using the set of data that, in your opinion, corresponds to the run that is most representative of the capacitor discharge.

T (min)	v(t) V	v(t) V	v(t) V	v(t) V	v(t) V	f(t)=v(t)/v(0)	ln(f(t))
0.0							
0.15							
0.30							
1.00							
1.15							
1.30							
1.45							
2.00							
2.15							
2.30							
2.45							
3.00							
3.15							
3.30							
3.45							
4.00							
4.15							
4.30							
4.45							
5.00							

Table 1 Capacitor Discharge Data

4.2 Square Wave response

a- Take a C = 0.01 μF capacitor and use a resistor R so that one-half period of a 1.00 KHz square wave will be 5τ where the time constant $\tau = RC$. Arrange such an R-C circuit (fig. 4) with the function generator as the source. Observe the function generator output on Channel 1 and V_C on Channel 2.

b- Set the generator to square wave and set f = 500 Hz. Set the scope vertical sensitivities to 1 volt/div. Set horizontal to 0.5 msec/div. Adjust the generator amplitude for 6V peak-to-peak, 0 volt offset. Center both channels vertically on the scope.

c- Since f = 500Hz, a half period is 10τ. Notice that the capacitor has time to (practically) fully charge on each half cycle. (see eq. (2) and note that e^{-10} is negligible.) Change the sweep to 0.1 msec/div. Now a half period is 10 divisions wide. Let us write a theoretical expression for the capacitor voltage V_C. Taking the origin of time, t=0, when the square wave jumps from –A to A, we can determine the capacitor voltage V_C during the half cycle that follows t = 0 as

$$V_C(t) = A - 2Ae^{-t/\tau}$$

Verify that the trace on the scope follows the above equation. This can be done by checking whether four or so representative points of $V_C(t)$ on the scope are as predicted by the equation. Cursors on the scope can be used to make these measurements. Download the waveform for your report.

d- Increase R and decrease C by a factor of 10, and verify that the circuit still behaves as in part c. (same time constant!) Download the waveforms for your report. Note that the final capacitor amplitude is reduced slightly due to the 1 MΩ input impedance of the scope. This is expected.

e- From their *original* values, decrease R and increase C by a factor of 10 and again verify that the circuit behavior is substantially the same as part c. This time you will notice that the 50-ohm internal source resistance of the generator causes some distortion at the beginning of each half cycle, when the current is large. Again this is expected. Download the waveforms for your report.

f- Return to the original values of R and C and check the amplitude is still 6.0 V p-p. Change the frequency to 2.00 KHz. The half period is now $2.5\ \tau$. Measure peak to peak capacitor voltage V_{CPP} and keep this value for your report.

V_{CPP} = ___________V

Note: When measuring V_{CPP} use either cursors, or the automatic measurements provided by the scope. Choose the 'Amplitude' measurement, not the 'Peak' measurement. The 'Peak' measurement includes noise amplitude, so the readings will tend to be slightly high in this application.

4.3 Integration and Differentiation

4.3.1 Integration of a square wave:
Choose a frequency at which you are satisfied with the performance of the circuit as an integrator (adjust sensitivity and sweep rate as needed). Measure the peak to peak capacitor voltage V_{CPP} and record the frequency of the waveform from the fuction generator. Study the relation of the function and its integral and download or take a picture of the scope image for your report.

4.3.2 Integration of a triangular wave:
With the circuit functioning well as an integrator, switch to triangular input. Adjust the generator output to 6 V peak-to peak. Study the relation of the function and its integral and copy the scope image for your report.
The image of the integral on the scope may look like a sine wave but in fact it is parabolic. To prove this, make adjustments with the sweep, sensitivities, and variables until the image spans 8 divisions peak-to-peak and the half period is 4 divisions wide. Notice that the amplitude at 1/8 period is 3/4 of peak instead of 0.707 of peak as it would be in a sine wave. Download the waveform.

4.3.3 Integration of a sine wave:
Change to sine wave input. Set the sensitivity variables and the sweep variable to CAL. Study the image and download as before.

4.3.4 Differentiation of a triangular wave:
Return to triangular wave. Interchange the capacitor and the resistor in the circuit. Channel 2 should now be receiving v_R. Adjust the sensitivity of the scope so v_R becomes visible. Reduce frequency gradually, making changes in scope sensitivity and sweep rate as you go, until you are satisfied with the performance of the circuit as a differentiator.
Measure the peak to peak resistance voltage V_{RPP} and the frequency of the wave using the frequency counter. Study the images and download them for your report.

4.3.5 Differentiation of a square wave:
Change to a square wave and decrease Channel 2 sensitivity until the derivative becomes visible. Study and copy the images. With 1 volt/div on Channel 1 and 2 volts/div on Channel 2, notice that the peak V_R is twice the peak input.

4.3.6 Differentiation of a sine wave:
Change to sine wave. Adjust input to V_{PP} = 6.0 and make the usual positioning adjustments. Tweak the frequency to make the phase difference stand out. Study the images and download them.

5 Report

5.1 Determine the time constant τ in the following two ways:

(a). Plot f(t) data from table 1 on a graph paper with rectangular coordinates. The value of τ is the time at which

$$f(t) = f(0)e^{-1} = 0.368 f(0)$$

(b). $f(t) = A\, e^{-t/\tau}\, u(t)$

- $\Rightarrow \ln(f(t)) = (-1/\tau)t + \ln(A)$ for $t \geq 0$ which is in the form: $Y = A_1\, t + A_0$ and $\ln(f(t))$ vs t should plot as a straight line.
- Plot $\ln(f(t))$ vs t on a graph paper with rectangular coordinates and find the best straight line fit to the data.
- The slope of the straight line must be $(-1/\tau)$, hence the value of τ can be computed from the slope.

5.2 Determine the time constant by integration using the following method:

(a) From the equation $f(t) = Ae^{-t/\tau}$, it is easy to show that the area under the complete f(t) curve is equal to $A\tau$.

(b) The trapezoidal rule is used to find the area. For a set of observations $f_1 \ldots f_N$ spaced at a common interval Δt, the area τ in the interval $t_1 \leq t_i \leq t_N$ is given approximately by:

$$Area = \tau = \Delta t \left[\frac{f_1 + f_N}{2} + \sum_{i=2}^{N-1} f_i \right]$$

5.3 Determine the time constant by differentiation using the following method:

(a) At any point on the graph of f(t) vs t, the slope line (tangent) will always reach zero in a time $t = \tau$. Given

$$f(t) = f(t_o)e^{-\frac{t-t_o}{\tau}}$$

it can be shown that (do not prove)

$$\tau = f(t_o)\frac{\Delta t}{f(t_o) - f(t_o + \Delta t)}$$

(b) Apply the above equation for τ to each of the data points to the first 3 minutes and average the values of τ to obtain the time constant.

5.4 From the value of V_{CPP} measured in Section 4.2f, determine V_{PP}/V_{CPP}. Compare with the theoretical value obtained from Eq. 2.

5.5 Submit all images copied from the scope, using the appropriate scales and label with the appropriate descriptive labels.

5.6 From Section 4.3.1, determine the minimum τ/T for good integration of a square wave. Determine the minimum V_{PP}/V_{CPP}. Compare with the theoretical value obtained from Eq. 2.

5.7 From Section 4.3.2, Show that the integration of the triangular wave is a parabolic wave, i.e, show that the amplitude of the wave at 1/8 period is 3/4 of peak.

5.8 From Section 4.3.4, determine the minimum T/τ for good differentiation of the triangular wave. Determine the minimum V_{PP}/V_{RPP}. What does the output waveform look like?

5.9 Simulate in Spice all parts of section 4.3. Use the frequencies obtained in the Lab. Plot the output waveforms. Compare with the experimental ones.

5.10 Design a first order RC circuit that produces the following response:

$$v_c(t) = 10 - 5\, e^{-3000t} \text{ V for } t \geq 0.$$

5.11 Prepare a summary.