THE STATE UNIVERSITY OF NEW JERSEY
RUTGERS

School of Engineering
Department of Electrical and Computer Engineering

332:224 Principles of Electrical Engineering II Laboratory

Experiment II

Step response of an RLC series circuit

1 Introduction

Objectives

- To study the behavior of an underdamped RLC Series Circuit for different damping coefficients

Overview

This experiment is a study of the step response of an underdamped RLC series circuit. The natural frequency is chosen and that determines the values of L and C. Then the damping coefficient is varied by changing the value of R.

A short theory is presented. The prelab exercise is rudimentary but crucial for the performance of the experiment which includes comparison of experimentally determined and theoretically derived waveform maxima and minima.

Authored by P. Sannuti
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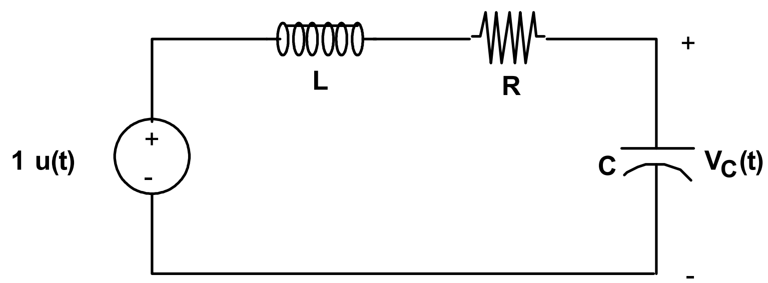
2 Theory[1]

Consider an RLC series circuit subject to a unit step voltage as shown in Fig. 1.

Fig. 1
A Series RLC Circuit
with a Unit Step Input



For a second order linear differential equation with step function input

$$a_2 \frac{d^2 y(t)}{dt^2} + a_1 \frac{dy(t)}{dt} + a_0 y = Au(t)$$

the step response is the general solution for $t \geq 0$. This step response can be partitioned into forced and natural components. Natural response is the general solution of the homogeneous equation (with u(t) = 0), while the forced response is a particular response of the above equation.

Following the notation of the textbook, the following notation is used for a second order circuit:

- α = The damping factor
- ζ = The damping coefficient
- ω_d = The damped frequency
- ω_o = The natural frequency

For the RLC series circuit of Fig. 1, the expression for the capacitance voltage in the oscillatory (*underdamped*) case is

$$v_C(t) = 1 + Ke^{-at}\cos(\omega_d t + \phi) \quad (1)$$

where $\alpha = R/2L = \zeta\omega_o$ and

$$\omega_d = \sqrt{\omega_o^2 - \alpha^2} = (\frac{1}{LC} - \frac{R^2}{4L^2})^{1/2} = \omega_o(1-\zeta^2)^{1/2}$$

[1] A more detailed treatment can be found in the text starting with section 8.4.

Also, $\omega_o = \dfrac{1}{\sqrt{LC}}$ and $\zeta = \dfrac{R}{2}\sqrt{\dfrac{C}{L}}$

In equation (1) for **v_C(t),** the term $Ke^{-at}\cos(\omega_d t + \phi)$ represents the natural response and the unity represents the particular solution.

Consider the initially quiescent state (both the initial inductance current and the initial capacitor voltage are zero). In this case,

$$\phi = -\tan^{-1}\left(\frac{\alpha}{\omega_d}\right) = -\tan^{-1}\left(\frac{\zeta}{\sqrt{1+\zeta^2}}\right) \qquad (2)$$

$$K = -\frac{1}{\cos\phi} = -\frac{1}{\sqrt{1-\zeta^2}} \qquad (3)$$

The maxima and minima occur alternately when

$\tan(\omega_d + \phi) = -\alpha/\omega_d$, i.e. $\omega_d\, t = k\,\pi$; $k = 0, 1, 2, ...$ (4)

From the above, the maximum and the minimum values of **v_C** are given by

$$v_c\left(k\pi / \omega_d\right) = 1 - (-1)^k \exp\left[-\frac{k\pi\zeta}{\sqrt{1-\zeta^2}}\right] \quad k = 0, 1, 2, ... \qquad (5)$$

(odd values of **k** give the maxima)

3 Prelab Exercises

3.1 For L = 100mH, compute the value of C needed for the natural frequency $\omega_o = 2\pi f_o = 10^4\pi$. To vary the damping coefficient ζ, vary the value of R . Compute the values of R for each of the six different values of the damping coefficient $\zeta=0.1$, $\zeta=0.2$, $\zeta=0.4$, $\zeta=0.6$, $\zeta=0.8$, and $\zeta=1.0$, (see Table 1 under section 4.4).

3.2 From Eq. 5, calculate and fill the theoretical values of the maxima and minima in Table 1 under section 4.4. Use k = 1, 2, and 3.

4 Experiments

Suggested Equipment:
Function Generator: Tektronix AFG1022
Oscilloscope: Tektronix DPO3012 or MDO3024
Digital Multimeter (DMM) HP/Agilent 34405A, 34401A, or 34461A
Decade Resistance/Capacitance Box
Decade Inductance Box

Sources, like function generators, are designed with internal source impedance. The source impedance is there to match a transmission line (coaxial cable) and thus eliminate high frequency reflections. In this experiment the internal source impedance will need to be considered as it contributes to the overall series resistance of your circuit.

The inductor also has internal resistance due to it's construction. The resistance is the resistance of the wire used to wind the coil, and will also need to be considered.

4.1 Function Generator Resistance

The internal impedance (resistance) of the function generator will affect the damping of an RLC circuit to which it is connected. Measure the resistance in the following way:

a- Connect the function generator to scope channel 1 using the BNC to banana breakout box. Generate a 1 Vpp, 1 kHz sine wave. Set the scope to MEASURE – AMPLITUDE and fine tune the generator output to 1.0 Vpp.

Note: On the scope you can use ACQUIRE – MODE – AVERAGE. Set average to, say 16. Averaging 'cleans up' your displayed waveform by averaging out random noise.

b- Connect a variable resistance load (Decade box R=200 ohms) to the generator output, thus forming a voltage divider. The divider is formed by the first resistance inside the generator, and the second resistance being the decade box. The scope is connected to the center node of the divider.
Adjust R until the measured voltage falls to one-half the open circuit value. At this point the two resistances of the voltage divider must be equal. Therefore, the resistance of the decade box should now be equal to the internal resistance of the function generator. Note this resistance.

4.2 Inductor Internal Resistance

Use the digital ohmmeter (DMM in 2 wire ohms mode) to measure the internal resistance of the inductor used. Again note the resistance. The values of R that should be used in the series RLC circuit are those computed in Section 3 (Pre-lab exercise) *minus the internal resistance of the function generator* and *minus the internal resistance of the inductor*.

4.3 Step Response of RLC Branch

Arrange a series RLC series circuit (as shown in the 'Theory' section) using the values calculated for $\zeta = 0.1$ in Section 3 (Pre-lab exercise). Remember that R includes the generator and inductor resistances. Connect the + side of C to the scope channel 2 scope input. Keep channel 1 connected to the generator as it will provide a stable trigger for the scope.

Set the scope sweep rate to 400 usec/div, and sensitivity to 500mV/div. Set the square wave output from 0 Volts (low level) to 1 Volt (high level). With the square wave input to the circuit, set f = 200Hz and observe the typical oscillatory response to a step input. (Instead of a step input, a square wave of low enough frequency is used so that the repetitive wave form of the capacitor voltage can be easily plotted on the scope). Check the generator voltage so that the steady state value (high level) of the capacitor voltage is 1V corresponding to the particular solution for a unit step input. Download (using a USB flash drive) or photograph the waveform.

Note: Horizontal position may be used to reposition the trigger to the left or right to display just the ½ cycle of interest.

4.4 Measurement of Peaks

Use Cursors on the scope to measure the values of the maxima and the minima based on a unit step input. Cursor FINE mode should be on. Each cursor displays a small crosshair on the waveform that reads time (relative to the trigger or the other cursor) and voltage. Repeat 4.3 for different values of ζ , fill all the values in Table 1 and compare with the theoretical values that were calculated in the pre-lab exercise 3.2. In each case download (using a USB flash drive) or photograph the waveform.

ζ_τ	1st Max	1st Min	2nd Max	
0.1				Measured
				Theoretical
0.2				Measured
				Theoretical
0.4				Measured
				Theoretical
0.6				Measured
				Theoretical
0.8				Measured
				Theoretical
1.0				Measured
	Critically Damped			Theoretical

Table 1 Unit step response of 2nd order system

4.5 Natural frequency

Change the sweep to 100 usec/div and reduce damping as much as possible by reducing the value of R. Notice that the period of the oscillation is close to 200 usec which corresponds to the natural frequency ω_o used to determine the L and C values.

5 Report

5.1 Derive Equation 1 in Section 2 for the underdamped case of an series RLC circuit.

5.2 Derive and prove Equation 4 in Section 2.

5.3 In pre-lab exercise 3.2 you have already filled in the theoretical values of the maxima and minima into Table 1. Compare them now with the experimental values. Print out the stored or photographed waveforms and label them appropriately.

5.4 Prepare a summary.