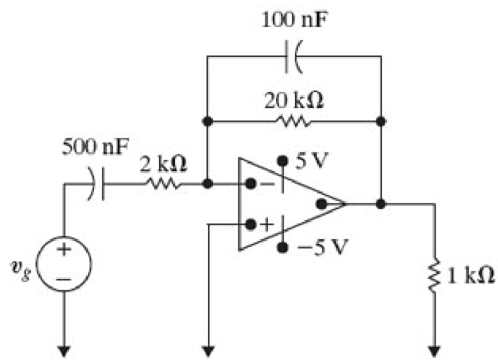
Problem 10.8

The op amp in the circuit shown is ideal.



Calculate the average power delivered to the $1\ k\Omega$ **resistor when** $v_g = 1.1\cos(1000t)\ V$**.** First convert all the elements to the frequency domain and from there the equations of ideal op amps can be used to determine the output voltage and then the power delivered to the output resistor.

$$100\ nF = \frac{-j}{(1000)(100 * 10^{-9})} = -j10000$$

$$500\ nF = \frac{-j}{(1000)(500 * 10^{-9})} = -j2000$$

$$v_g = 1.1\underline{/0^\circ} = 1.1\ V$$

Since this is an inverting amplifier, the equation for the output voltage is given in this case as,

$$v_o = -\frac{Z_f}{Z_s}v_s.$$

Here, R_f will be equal to the impedance of the 100 nF capacitor and the 20 $k\Omega$ resistor in parallel, and R_s will be the impedance of the 500 nF capacitor with the 2 $k\Omega$ resistor in series.

$$Z_f = \frac{(-j10000)(20 * 10^3)}{(-j10000 + 20 * 10^3)} = 4000 - j8000\ \Omega$$

$$Z_s = 2000 - j2000\ \Omega$$

$$v_o = -\frac{4000 - j8000}{2000 - j2000}(1.1) = -3.3 + j1.1\ V$$

The value attained is the value of the output voltage however, the rms value of the voltage is need in order to calculate power.

$$v_{o_{rms}} = \frac{-3.3 + j1.1}{\sqrt{2}}$$

With this, the power will be equal to,

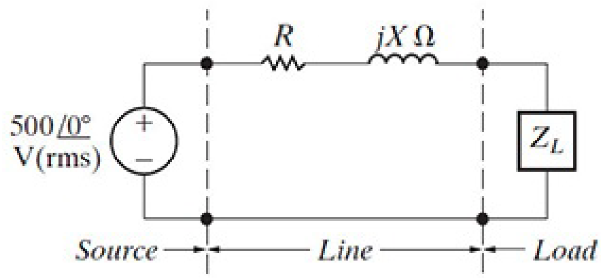
$$\frac{v_{o_{rms}}^2}{R} = \frac{(-3.3 + j1.1)^2}{2}\frac{1}{1000} = 4.84 - j3.63 \; mW.$$

Finally,

$$P = \sqrt{4.84^2 + 3.63^2} = \boxed{6.05 \; mW}$$

Problem 10.10

The load impedance in the circuit absorbs 2.5 kW and generates 5 $kVAR$. The sinusoidal voltage source develops 7.5 kW. Suppose that $R = 20.5\ \Omega$.



Find the minimum and maximum values of inductive line reactance that will satisfy these constraints.

Here, the line loss will be equal to the power absorbed by the resistor subtracted from that which is generated by the source,

$$\text{line loss} = 7.5 - 2.5 = 5 \; kW$$

The line loss is also equivalent to the current in the branch squared times the resistor in the branch, I^2R. With this, the current in the branch can be found and from there the values of the impedance of the load, R_L and X_L, can be determined.

$$I^2(20.5) = 5000$$

$$I^2 = \frac{5000}{20.5} \; A$$

Now, this current squared multiplied by the individual elements within the load can give possible values for the resistor and the inductor in said load.

$$I^2R_L = \frac{5000}{20.5}R_L = 2500$$

$$I^2X_L = \frac{5000}{20.5}X_L = -5000$$

$$R_L = 10.25\ \Omega$$

$$X_L = -20.5\ \Omega$$

With this, the equivalent impedance of the circuit can be written as the sum of the real and imaginary parts,

$$Z_{eq} = (20.5 + 10.25) + j(X - 20.5).$$

Since $V_s = 500\ V$, the current in the circuit can be expressed as the voltage divided by the magnitude of the equivalent impedance.

$$I = \frac{500}{\sqrt{(30.75)^2 + (X - 20.5)^2}}$$

The value of the current squared is known so this can be solved easily with some manipulation.

$$I^2 = \frac{500^2}{(30.75)^2 + (X - 20.5)^2}$$

$$\frac{5000}{20.5} = \frac{500^2}{(30.75)^2 + (X - 20.5)^2}$$

$$\frac{(30.75)^2(5000)}{20.5} + \frac{5000}{20.5}(X - 20.5)^2 = 500^2$$

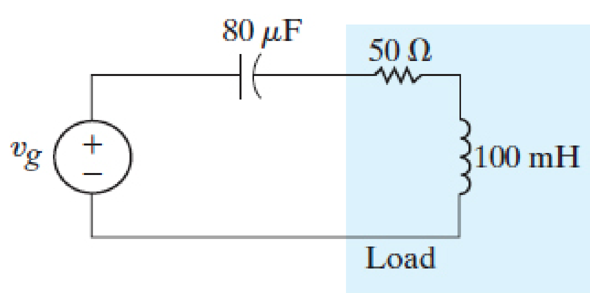
$$X = \pm\sqrt{\frac{20.5}{5000}\left(500^2 - \frac{(30.75)^2(5000)}{20.5}\right)} + 20.5$$

$$X = \pm\sqrt{\frac{(20.5)(500^2)}{5000} - 30.75^2} + 20.5$$

$$\boxed{X_{min} = 11.6\ \Omega \quad X_{max} = 29.4\ \Omega}$$

Problem 10.20

Consider the circuit shown. Suppose that $v_g = 180\cos(250t)$.



Find the average, reactive, and apparent power absorbed by the load in the circuit. Use a positive value if the power is absorbed and a negative value if the power is delivered.

Converting to the frequency domain,

$$v_g = 180\underline{/0^\circ}$$

$$80\ \mu F = \frac{-j}{(250)(80 * 10^{-6})} = -j50$$

$$100\ mH = j(250)(100 * 10^{-3}) = j25$$

The equivalent impedance can be expressed as,

$$Z_{eq} = 50 - j25\ \Omega,$$

and the current as,

$$I = \frac{V}{Z} = \frac{180\underline{/0^\circ}}{50 - j25} = 2.88 + j1.44\ A.$$

Using the current, the apparent power can be defined as,

$$S = \frac{1}{2}VI^* = \frac{1}{2}(180)(2.88 - j1.44)$$

$$S = 259.2 - j129.6\ VA$$

$$|S| = \sqrt{259.2^2 + 129.6^2} = \boxed{289.79\ VA.}$$

The values of average and reactive power can be taken from the apparent power as well,

$$\boxed{P = 259.2\ W \quad Q = 129.6\ VAR}.$$

Problem 10.24

Three loads are connected in parallel across a $V_o = 350\ V$ (rms) line. Load 1 absorbs 16 kW and 18 kVAR; Load 2 absorbs 10 kVA at 0.6 leading; Load 3 absorbs 8 kW at unity power factor.

Find the impedance that is equivalent to the three parallel loads.

$$S_1 = 16 + j18\ kVA$$

$$S_2 = 6 - j8\ kVA$$

$$S_3 = 8\ kVA$$

$$S_{eq} = (16 + j18) + (6 - j8) + 8 = 30 + j10\ kVA$$

Since,

$$S = VI^*,$$

$$30 + j10 = 180I^*,$$

$$I = 85.7 - j28.57\ A.$$

Now, using the current and the voltage, the impedance can be found.

$$Z = \frac{V}{I} = \frac{350}{85.7 - j28.57}$$

$$\boxed{Z = 3.675 + j1.225\ \Omega}$$

Find the power factor of the equivalent load as seen from the line's input terminals.

$$Z = 3.675 + j1.225 = 3.87\underline{/18.435^\circ}\ \Omega$$

$$\text{pf} = \cos(18.435^\circ) = \boxed{0.9487}$$

The power factor is lagging since the angle $\theta_v - \theta_i$ is positive.

Problem 10.42

The phasor voltage $\mathbf{V}_{ab}$ in a circuit is $240\underline{/0^\circ}$ V (rms) when no external load is connected to the terminals a and b. When a load having an impedance of $80 - j60\ \Omega$ is connected across a and b, the value of $\mathbf{V}_{ab}$ is $115.2 + j33.6$ V (rms).

Find the impedance that should be connected across a and b for maximum average power transfer.

Using the concept of the Thevenin's equivalent voltage and resistance, the circuit can be interpreted as a source along with one resistor. This means that there are only two impedances connected to the source and since we know the voltage when the load is connected, voltage division can be applied to determine the value of the Thevenin's resistance.

$$\mathbf{V}_{ab} = \frac{Z_L}{Z_{TH} + Z_L}\mathbf{V}_s$$

$$(115.2 + j33.6) = \frac{(80 - j60)}{Z_{TH} + (80 - j60)}(240\underline{/0^\circ})$$

Using this, the value of Z_{TH} can be determined to be,

$$\boxed{Z_{TH} = 40 - j100\ \Omega}$$

Find the maximum average power transferred to the load.
For maximum power,

$$Z_L = Z_{TH}^*$$

$$Z_L = 40 + j100\ \Omega$$

The combined impedance will simply be a resistance of 80 Ω, meaning the current in the circuit is 3 A. Since the power will be equal to I^2R, the maximum power will equal $3^2(40)$.

$$\boxed{P = 360\ W}$$