Problem 9.3

At $t = -2\ ms$, a sinusoidal voltage is known to be zero and going positive. The voltage is next zero at 12 ms. It is also known tat the voltage is 80.9 V at $t = 0$.

What is the frequency of $v(t)$ in hertz?

$$\frac{T}{2} = 12\ ms + 2\ ms = 14\ ms$$

$$f = \frac{1}{T} = \frac{1}{28 * 10^{-3}\ s} = \boxed{35.714\ Hz}$$

What is the expression for $v(t)$?

$$v(t) = v_m \cos(\omega t + \theta)$$

$$\omega = 2\pi f = 2\pi(35.714) = 71.429\pi$$

$$v(t) = 80.9 \cos(71.429\pi t + \theta)$$

since $v(t) = 0$ at $t = 2\ ms$,

$$80.9 \cos(71.429\pi(-2 * 10^{-3}) + \theta) = 0$$

$$71.429\pi(-2 * 10^{-3}) + \theta = \frac{\pi}{2}$$

$$\theta = \frac{\pi}{2} - 71.429\pi(-2 * 10^{-3})$$

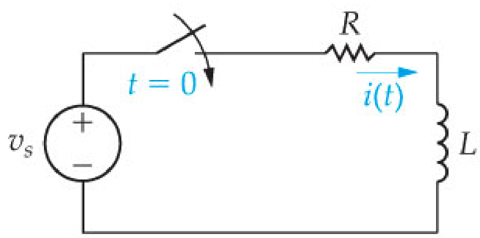
$$\theta = 64.2857°$$

with that the expression for $v(t)$ is,

$$\boxed{v(t) = 80.9 \cos(71.429\pi t + 64.2857°}$$

Problem 9.10

The voltage applied to the circuit shown in figure 1 at $t = 0$ is $20 \cos(800t + 25°)\ V$, where t is in seconds. The circuit resistance is 80 Ω and the initial current in the 75 mH inductor is zero.



Select the correct expression for $i(t)$ for $t \geq 0$.
convert the circuit to the frequency domain,

$$\mathbf{V} = 20\angle 25^\circ$$

$$\begin{aligned}\mathbf{Z} &= R + j\omega L\ \Omega \\ &= 80 + j(800)(75 * 10^{-3})\ \Omega \\ &= 80 + 60\ \Omega \\ &= 100\angle 36.87^\circ\end{aligned}$$

from Ohm's Law

$$I = \frac{V}{R},$$

and in the frequency domain,

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}}.$$

With this the current can be found to be,

$$\mathbf{I} = \frac{20\angle 25^\circ}{100\angle 36.87^\circ} = 0.2\angle -11.8699^\circ\ A$$

$$\boxed{i(t) = 200\cos(800t - 11.9^\circ\ mA}$$

Problem 9.13

A 350 Hz sinusoidal voltage with a maximum amplitude of 90 V at $t = 0$ is applied across the terminals of an inductor. The maximum amplitude of the steady-state current in the inductor is 20 A.
If the phase angle of the voltage is zero, what is the phase angle of the current?

Since the current of the voltage is zero and the circuit is purely inductive, the angle of the current will be lagging by 90°

$$\boxed{\theta_i = -90^\circ}.$$

What is the inductive reactance of the inductor?

$$\frac{90\ V}{\omega L} = 20\ A$$

$$\omega L = \frac{90}{20} = \boxed{4.5\ \Omega}$$

What is the inductance of the inductor?

$$\omega L = 4.5\ \Omega$$

$$\omega = 2\pi(350)$$

$$L = \frac{4.5}{\omega} = 0.002046\ H = \boxed{2.05\ mH}$$

What is the impedance of the inductor?

$$j\omega L = j(4.5)\ \Omega$$

$$\boxed{Z_L = 0 + j4.5\ \Omega}$$

Problem 9.15

A 50 Ω resistor, a 5 mH inductor, and a 1.25 μF capacitor are connected in series. The series-connected elements are energized by a sinusoidal voltage source whose voltage is $600\cos(8000t + 20°)\ V$.
Determine the impedances of the elements in the frequency-domain equivalent circuit.

$$Z_R = R = \boxed{50\ \Omega}$$

$$Z_L = j\omega L = j(8000)(5 * 10^{-3}) = \boxed{j40\ \Omega}$$

$$Z_C = \frac{-j}{\omega C} = \frac{-j}{(8000)(1.25 * 10^{-6})} = \boxed{-j100\Omega}$$

Reference the current in the direction of the voltage rise across the source, and find the phasor current.

$$Z_{eq} = (50 + j40 + -j100)\ \Omega = 50 - j60\ \Omega = 78.1025\underline{/-50.19}\ \Omega$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{600\underline{/20°}}{78.1025\underline{/-50.19}} = \boxed{7.68\underline{/70.19}\ A}$$

steady state expression for the current:

$$\boxed{i(t) = 7.68\cos(8000t + 70.2°)\ A}$$

Problem 9.17

Three branches having impedances of $3 + j4\ \Omega$, $16 - j12\ \Omega$, and $-j4\ \Omega$, respectively, are connected in parallel.
What is the equivalent admittance of the parallel connection?

$$Y_{ab} = \frac{1}{Z_{ab}}$$

$$Y_{ab} = \frac{1}{3 + j4} + \frac{1}{16 - j12} + \frac{1}{-j4}$$

$$\mathbf{Y}_{ab} = 0.16 + j0.12 = \boxed{0.2\underline{/36.87°}\ S}$$

Conductance of the parallel connection: $\boxed{0.16\ S}$
Susceptance of the parallel connection: $\boxed{0.12\ S}$

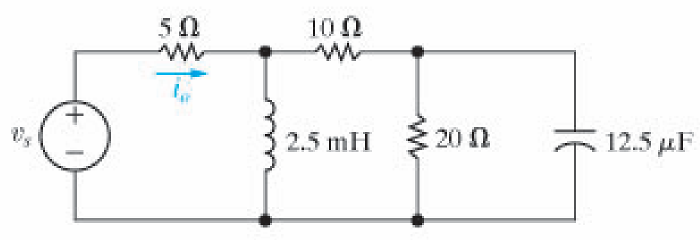
If the parallel branches are excited from a sinusoidal current source where $i(t) = 4\cos(\omega t)\ A$, what is the maximum amplitude of the current in the purely capacitive branch?

$$\mathbf{V} = \frac{\mathbf{I}}{\mathbf{Y}} = \frac{4\underline{/0^\circ}}{0.2\underline{/36.87^\circ}} = 20\underline{/-36.87^\circ}\ V$$

$$\mathbf{I}_C = \frac{\mathbf{V}}{\mathbf{Z}_C} = \frac{20\underline{/-36.87^\circ}}{4\underline{/-90^\circ}} = 5\underline{/53.12^\circ}$$

$$\boxed{\mathbf{I}_C = 5\ A}$$

Problem 9.30



Find $i_0(t)$ if $v_s = 45\sin(4000t)\ V$. Suppose that $i_0(t) = I_0\cos(\omega t + \phi)$, where $-360^\circ < \phi \leq 360^\circ$. Determine the values I_0, ω, and ϕ.
Converting to the frequency domain,

$$v_s = 45\sin(4000t) = 45\cos(4000t - 90^\circ) = 45\underline{/-90^\circ}$$

$$Z_C = \frac{-j}{(4000)(12.5 * 10^{-6})} = -j20\ \Omega$$

$$Z_L = j(4000)(2.5 * 10^{-3}) = j10\ \Omega$$

Now to compute the equivalent impedance seen by v_s:

$$20\ \Omega \,||\, -j20\Omega = 10 - j10\ \Omega$$

$$10\ \Omega + (10 - j10)\ \Omega = 20 - j10\ \Omega$$

$$(20 - j10)\ \Omega \,||\, j10\ \Omega = 5 + j10\ \Omega$$

$$5 + (5 + j10) = 10 + j10\ \Omega$$

$$Z_{eq} = 10 + j10\ \Omega$$

Now,

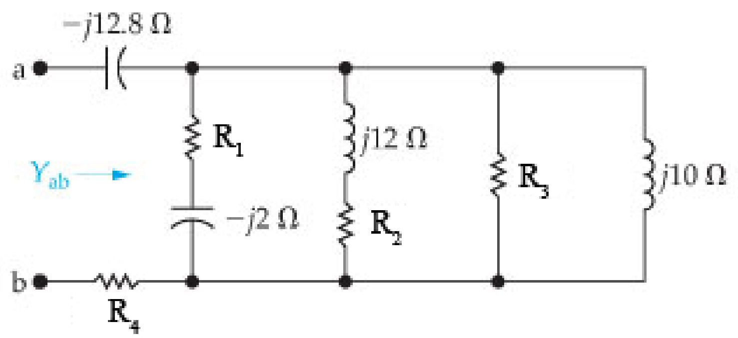
$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}_{eq}} = \frac{45\angle -90^\circ}{10 + j10} = \frac{45\angle -90^\circ}{14.14214\angle 45^\circ}$$

$$\mathbf{I} = 3.18198\angle -135^\circ$$

$$\boxed{I_0 = 3.18\ A,\ \omega = 4000\ rad/s,\ \phi = -135^\circ}$$

Problem 9.25

Find the admittance Y_{ab} in the circuit seen in figure 1. Take that $R_1 = 7\ \Omega$, $R_2 = 4\ \Omega$, $R_3 = 7\ \Omega$, and $R_4 = 12.2\ \Omega$. **Express Y_{ab} in rectangular form.**



$$Y_1 = \frac{1}{j10} + \frac{1}{7} + \frac{1}{4 + j12} + \frac{1}{7 - j2} = 0.2999 - j0.1373\ S$$

$$Z_1 = \frac{1}{Y_1} = \frac{1}{0.2999 - j0.1373} = 2.7567 + j1.2616\ \Omega$$

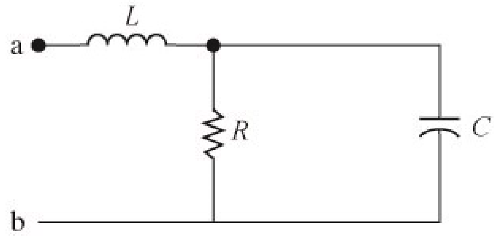
$$Z_{ab} = Z_1 + 4 - j12.8 = 6.767 - j11.5384$$

$$Y_{ab} = \frac{1}{Z_{ab}} = 0.0378 + j0.0645\ S$$

$$\boxed{Y_{ab} = 37.8 + j64.5\ mS}$$

Problem 9.27

Consider the circuit shown. Suppose $R = 110\ \Omega$, $L = 130\ \mu H$, and $C = 30\ nF$.



Find the frequency (in radians per second) at which the impedance Z_{ab} is purely resistive.

$$Z_C = \frac{-j}{\omega C} = \frac{-j}{\omega(30 * 10^{-9})}$$

$$Z_L = j\omega L = j\omega(130 * 10^{-6})$$

$$Z_{ab} = Z_L + (R||Z_C) = j\omega L + \frac{-jR/\omega C}{R - j/\omega C}$$

$$Z_{ab} = j\omega L + \frac{-jR}{\omega CR - j} = j\omega L + \frac{-jR(\omega CR + j)}{\omega^2 C^2 R^2 + 1}$$

Now,

$$\omega L - \frac{\omega CR^2}{\omega^2 C^2 R^2 + 1} = 0$$

$$L = \frac{CR^2}{\omega C^2 R^2 + 1}$$

$$\omega^2 C^2 R^2 + 1 = \frac{CR^2}{L}$$

$$\omega = \sqrt{\frac{CR^2/L - 1}{C^2 R^2}} = \boxed{4.06 * 10^5\ rad/s}$$

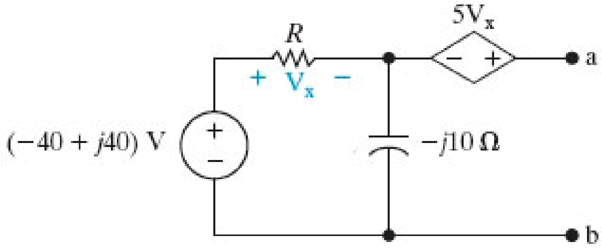
Find the value of Z_{ab} at the frequency of Part A.

$$Z_{ab} = j\omega L + \frac{-jR}{\omega CR - j}$$

$$Z_{ab} = \boxed{39.4\ \Omega}$$

Problem 9.48

Find the Norton equivalent with respect to terminals a and b in the circuit. Suppose that $R = 20\ \Omega$.



Find the value of $\mathbf{I}_N$.
Since the Norton's current is the short circuit current,

$$-v_s + Ri_1 + -j10(i_1 - I_N) = 0$$

$$-5V_x + -j10(I_N - i_1) = 0$$

$$V_x = Ri_1 = 20i_1$$

Solving for I_N:

$$\boxed{I_N = 5.5 + j4.5\ A}$$

Find the value of Z_N.

$$Z_N = \frac{V_{oc}}{I_N}$$

$$i_1 = \frac{v_s}{20 - j10} = -2.4 + j0.8\ A$$

$$V_x = -48 + j16\ V$$

$$V_{oc} = 5V_x - j10i_1 = -232 + j104\ V$$

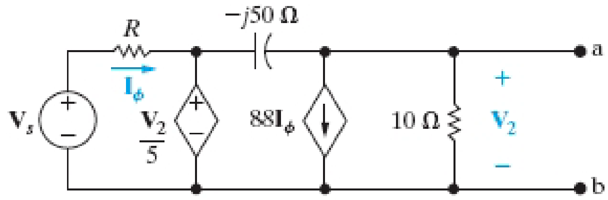
Finally,

$$Z_N = \frac{V_{oc}}{I_N} = \frac{-232 + j104}{5.5 + j4.5}$$

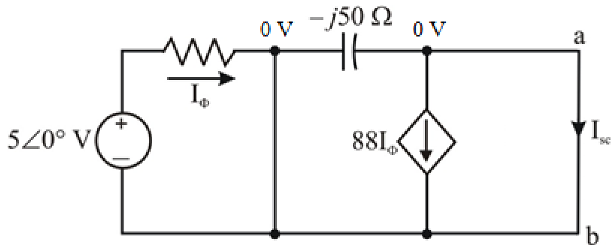
$$\boxed{Z_N = -16 + j32\ \Omega}$$

Problem 9.49

Find the Norton equivalent circuit with respect to the terminals a and b for the circuit where $\mathbf{V}_s = 5\underline{/0^\circ}\ V$ and $R = 100\ \Omega$.



Find the value of $\mathbf{I}_N$.
Since the Norton's current is the short circuit current, putting a wire along the terminals a and b will lead to V_2 being zero since there is no voltage along the 10 Ω resistor. This shuts off the dependant voltage source and shunts the resistor.



Both of the nodes, on the left and the right, are at zero volts since they are directly connected to ground.

$$-I_\phi = \frac{0-5}{R} = \frac{-5}{100} = -0.05\ A$$

$$I_\phi = 0.05\ A$$

At node 2,

$$\frac{0-0}{-j50} + 88I_\phi + I_N = 0$$

$$88(0.05) + I_N = 0$$

$$\boxed{I_N = -4.4\ A}$$

Find the value of $\mathbf{Z}_N$.
The value of $V_{oc} = V_{TH} = V_2$ must be found from which the equivalent impedance can be calculated.
The node above the current source and the resistor are the same node and its equation,

$$\frac{V_2 - V_2/5}{-j50} + 88I_\phi + \frac{V_2}{10} = 0$$

Since,

$$I_\phi = \frac{V_s - V_2/5}{100},$$

$$\frac{5V_2 - V_2}{-j250} + 88\left(\frac{V_s - V_2/5}{100}\right) + \frac{V_2}{10} = 0.$$

Solving for V_2,

$$\frac{4V_2}{-j250} + 88\left(\frac{25 - V_2}{500}\right) + \frac{V_2}{10} = 0$$

$$\left(\frac{4}{-j250} - \frac{88}{500} + \frac{1}{10}\right)V_2 = \frac{2200}{500}$$

$$V_2 = -55.4377 - j11.671\ V.$$

Now to use Ohm's Law to find Z_N,

$$Z_N = \frac{V_{oc}}{I_{sc}} = \frac{V_{TH}}{I_N}$$

$$Z_N = \frac{-55.4377 - j11.671}{-4.4}$$

$$Z_N = \boxed{12.599 + j2.653\ \Omega}$$