Problem 6.5

The current in a 20 mH inductor is known to be

$$i(t) = \begin{cases} 40\ mA & t \leq 0 \\ A_1 e^{-10000t} + A_2 e^{-40000t}\ A & t \geq 0 \end{cases}$$

i) Choose the correct expression for the voltage across the inductor for $t > 0$:

$$v(t) = L\frac{di}{dt}$$

$$v(t) = (20 * 10^{-3}) * \frac{d}{dt}(A_1 e^{-10000t} + A_2 e^{-40000t})$$

$$v(t) = (20 * 10^{-3}) * (-10000 A_1 e^{-10000t} - 40000 A_2 e^{-40000t})$$

$$v(t) = 200 A_1 e^{-10000t} - 800 A_2 e^{-40000t}$$

This is the voltage across the inductor at time $t > 0$. The initial voltage in the inductor is, $v(0) = 28$ V. Since $v(t) = 200A_1 e^{-10000t} - 800A_2 e^{-40000t}$, $v(0) = 200A_1 - 800A_2$.

$$28 = 200A_1 - 800A_2 \tag{1}$$

The equation for the current can then be used, $i(t) = A_1 e^{-10000t} + A_2 e^{-40000t}$, where $i(0) = A_1 + A_2$

$$40 * 10^{-3} = A_1 + A_2 \tag{2}$$

Solving these two equations:

$$A_1 = 0.1$$

$$A_2 = -0.06$$

Now to get the final answer for the voltage as a function of time, the values of A_1 and A_2 are substituted:

$$\boxed{v(t) = -20e^{-10000t} + 48e^{-40000t}}$$

ii) Find the time, greater than zero, when the power at the terminals of the inductor is zero.

$$power = iv = iL\frac{di}{dt}$$

$$power = (0.1e^{-10000t} - 0.06e^{-40000t})(-20e^{-10000t} + 48e^{-40000t})$$

$$power = -2e^{-20000t} + 4.8e^{-50000t} + 1.2e^{-50000t} - 2.88e^{-80000t}$$

$$power = -2e^{-20000t} + 6e^{-50000t} - 2.88e^{-80000t}$$

$$0 = -2e^{-20000t} + 6e^{-50000t} - 2.88e^{-80000t}$$

Setting $e^{-20000t} = x$,

$$0 = -2x + 6x^{2.5} + 2.88x^4$$

$$x = 0 \quad x = (\frac{5}{12})^{\frac{1}{1.5}}$$

$$e^{-20000t} = (\frac{5}{12})^{\frac{1}{1.5}}$$

$$\boxed{t = 29.18\ \mu s}$$

Problem 6.12

Initially there was no energy stored in the 5 H inductor in the circuit in the following figure when it was placed across the terminals of the voltmeter. At t = 0 the inductor was switched instantaneously to position b where it remained for 1.6 s before returning instantaneously to position a. The d'Arsonval voltmeter has a full-scale reading of 28 V and a sensitivity of 1000 Ω/V.

i) What will the reading of the voltmeter be at the instant the switch returns to position a if the inertia of the d'Arsonval movement is negligible?

$$i(t) = \frac{1}{L}\int_0^t v(t)\ dt$$

$$i(t) = \frac{1}{5}\int_0^t (3 * 10^{-3})\ dt$$

$$i(t) = \frac{3 * 10^{-3}}{5}t$$

$$i(1.6) = \frac{3 * 10^{-3}}{5}(1.6) = 0.00096\ A$$

The full scale reading of the meter is 28 V, and the sensitivity is 1000 Ω/V, therefore, the resistance of the meter is $28 * 1000\ \Omega\frac{V}{V} = 28000\ \Omega$.

$$V = iR$$

$$V = (0.00096)(28000) = \boxed{26.88\ V}$$

Problem 6.14

The voltage at the terminals of the capacitor in is known to be:

$$v(t) = \begin{cases} -10\ V & t \leq 0 \\ 40 - 1e^{-1000t}(50\cos(500t) + 20\sin(500t))\ V & t \geq 0 \end{cases}$$

where t is in seconds. Assume $C = 0.8\ \mu F$.

i) Find the current in the capacitor for $t > 0$.

$$i(t) = C\frac{dv}{dt}$$

$$i(t) = (0.8)\frac{d}{dt}(40 - 1e^{-1000t}(50\cos(500t) + 20\sin(500t)))$$

$$i(t) = 0.8e^{-1000t}(45000\sin(500t) + 40000\cos(500t))$$

$$i(t) = e^{-1000t}(36000\sin(500t) + 32000\cos(500t))\ A$$

$$\boxed{i(t) = e^{-1000t}(36\sin(500t) + 32\cos(500t))\ mA}$$

ii) How much energy (in millijoules) is stored in the capacitor at $t = \infty$?

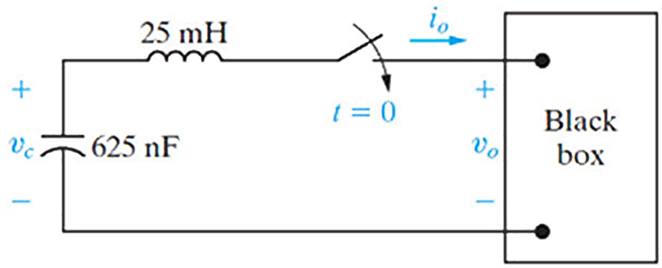
$$v(t) = 40, t = \infty$$

$$w = \frac{1}{2}Cv^2$$

$$w = \frac{1}{2}(0.8)(40)^2$$

$$\boxed{w = 0.64\ mJ}$$

Problem 6.34



At $t = 0$ a series-connected capacitor and inductor are placed across the terminals of a black box, as shown in Figure 1. For $t > 0$:

$$i_0 = 1.5e^{-16000t} - 0.5e^{-4000t}\ A$$

Voltage across the capacitor: $v(0) = -50\ V$

$$v(t) = -\frac{1}{C}\int_0^t i_0 dt + v(0)$$

$$v(t) = -\frac{1}{625 * 10^{-9}}\int_0^t (1.5e^{-16000t} - 0.5e^{-4000t})dt - 50$$

$$v(t) = -\frac{1}{625 * 10^{-9}} * \frac{1}{32000}(e^{-16000t}(4e^{12000t} - 3) - 1) - 50$$

$$v(t) = -50(e^{-16000t}(4e^{12000t} - 3) - 1) - 50$$

$$v(t) = -50e^{-16000t}(4e^{12000t} - 3)$$

$$v(t) = 150e^{-16000t} - 200e^{-4000t}\ V$$

Voltage across the inductor:

$$v(t) = L\frac{di}{dt}$$

$$v(t) = 25 * 10^{-3}\frac{d}{dt}(1.5e^{-16000t} - 0.5e^{-4000t})$$
$$v(t) = e^{-16000t}(50e^{12000t} - 600)$$

v_0 will equal $v_c - v_l$:

$$v_0 = (150e^{-16000t} - 200e^{-4000t}) - (e^{-16000t}(50e^{12000t} - 600))$$

$$\boxed{v_0 = 750e^{-16000t} - 250e^{-4000t}\ V}$$