Probability & Statistical Modelling

AQ077-3-2-PSMOD and Version VD1

A · P · U
ASIA PACIFIC UNIVERSITY
OF TECHNOLOGY & INNOVATION

Concept of Probability

Topic & Structure of The Lesson

- Introduction
- Terminologies
- Rules of Probability
- Tree Diagram
- Use of Venn diagram to solve probability
- Contingency tables
- Bayes' Theorem

Learning Outcomes

- At the end of this topic, You should be able to
  - Model /analyse simple business situations using probability.

Key Terms You Must Be Able To Use

- If you have mastered this topic, you should be able to use the following terms correctly in your assignments and exams:
  - Addition Rule
  - Mutually Exclusive
  - Not mutually exclusive
  - Multiplication Rule
  - With Replacement
  - Without Replacement
  - Independent events
  - Dependent events
  - Probability tree diagram
  - Conditional Probability

Introduction

- Probability is the likelihood or chance of something happening.
  - In an experiment in which all outcomes are equally likely, the probability of an event E is

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} = \frac{n(E)}{n(S)}$$

- **(e.g. we might say that there is a 80% chance or 0.8 chance that outcome A will happen; we would then implying that there is a 20% or 0.2 chance that A would not occur.)**
- **In Statistics , probabilities will be more commonly expressed as proportions than as percentages.**

- $0 \leq p(E) \leq 1$
- If P(E) = 0, E is called an impossible event
- If P(E) = 1, E is called a certain event.
- The sum of the probabilities of all the outcomes of an experiment must total 1.
- P(E does not occur) = 1 – P(E)
  - (Complementary probability)

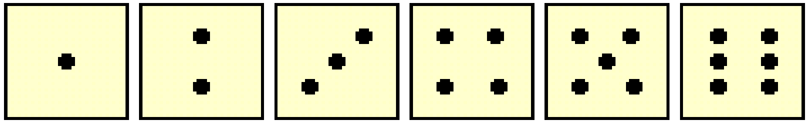
Terminologies

- **Statistical event**
  - it is defined as any subset of the given outcome set that is of interest
- **Statistical experiment**
  - it is described as any situation , specially set up or occurring naturally, which can be performed, enacted or otherwise considered in order to gain useful information

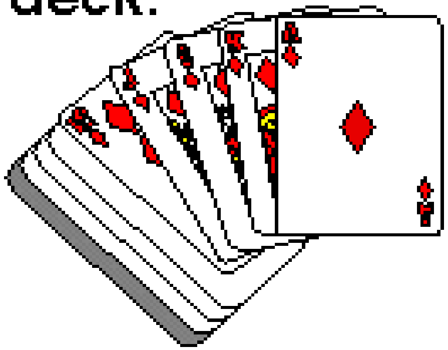
➢ Sample space

- The Sample Space is the collection of all possible outcomes

  e.g. All 6 faces of a die:



  e.g. All 52 cards of a bridge deck:

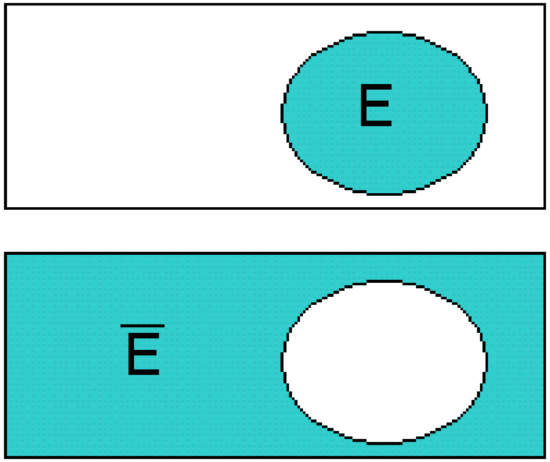


Complement Rule

- The complement of an event E is the collection of all possible elementary events **not** contained in event E. The complement of event E is represented by $\overline{E}$.

- Complement Rule:

$$P(\overline{E}) = 1 - P(E)$$



Or, $P(E) + P(\overline{E}) = 1$

Example 1.1

- A group of 20 university students contains eight who are in their first year of study. A student is picked at random. Find the probability that the student is not in the first year of study.

Probability Rule for Combined Events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Quick Review Question

Example 1.2

- **If a card is drawn from a deck of playing cards, what is the probability of getting a red or an ace?**

P(Red or Ace) = P(Red) +P(Ace) - P(Red and Ace)

$= 26/52 + 4/52 - \boxed{2/52} = 28/52$

Type	Color		Total
	Red	Black	
Ace	2	2	4
Non-Ace	24	24	48
Total	26	26	52

Don't count the two red aces twice!

Example 1.3

- In a class of 20 children, 4 of the boys and 3 of the eleven girls are in the athletics team. A child from the class is chosen at random. Find the probability that the child chosen is

  (a) in the athletics team

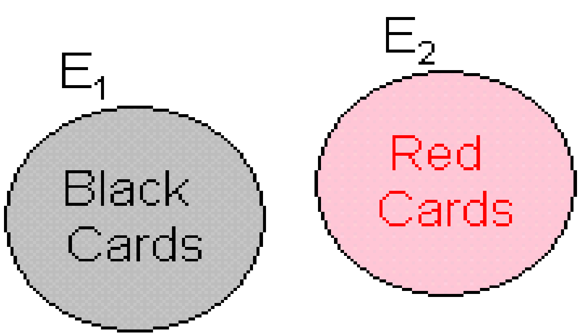
  (b) a girl

  (c) a girl member of the athletics team

  (d) a girl member or in the athletics team.

Mutually exclusive events

- **Two events of the same experiment are said to be mutually exclusive if their respective events do not overlap.**
  - If E_1 occurs, then E_2 cannot occur
  - E_1 and E_2 have no common elements



A card cannot be Black and Red at the same time.

- **Not mutually exclusive**
  - If two or more events occur at one time.
- **Independent events**
  - Two events are said to be independent if the occurrence (or not) of one of the events will in no way affect the occurrence (or not) of other.
  - Alternatively, two events that are defined on two physically different experiments are said to be independent.

- **Independent Events**

E_1 = heads on one flip of fair coin

E_2 = heads on second flip of same coin

Result of second flip does not depend on the result of the first flip.

- **Dependent Events**

E_1 = rain forecasted on the news

E_2 = take umbrella to work

Probability of the second event is affected by the occurrence of the first event

Example 1.4

In a race in which there are no dead heats, the probability that John wins is 0.3, the probability that Paul wins is 0.2 and the probability that Mark wins is 0.4. Find the probability that

(a) John or Mark wins.
(b) John or Paul or Mark wins.
(c) someone else wins.

Example 1.5

A card is drawn from a bag contains 5 red cards numbered 1 to 5 and 3 green cards numbered 1 to 3. Find the probability that the card is

(a) a green card or a red card,

(b) a green card or an even number.

Conditional event

- one of the outcomes of which is influenced by the outcomes of another event.
- If A and B are two events not necessarily from the same experiment, then the conditional probability that A occurs, given that B has already occurred, is written

$$\textbf{P(A, given B) = P(A|B)} = \frac{\text{P(A and B)}}{\text{P(B)}}$$

$$\textbf{P(B, given A) = P(B|A)} = \frac{\text{P(A and B)}}{\text{P(A)}}$$

Example 1.6

When a die was thrown, the score was an odd number. What is the probability that it was a prime number?

Example 1.7

In a certain college,

65% of the students are full time students

55% of the students are female

35% of the students are male full time students.

Find the probability that a student chosen at random

(a) from all the students in the college is a part time student.

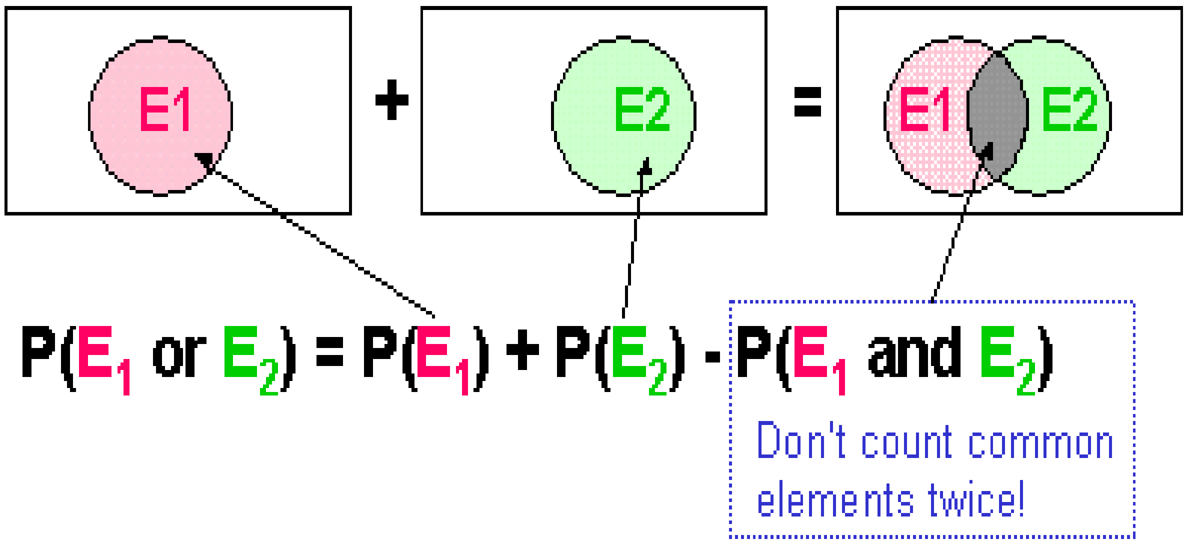
(b) from all the students in the college is female and a part time student.

(c) from all the female students in the college is a part time student.

Rules of Probability

- Addition Rule
- Multiplication Rule

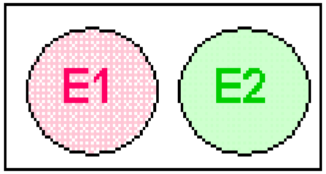
- **General Addition Principle**
  - **If E and F are not mutually exclusive events, then**
    $P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$



Rule of Addition

- If E1 and E2 are mutually exclusive, then

P(E1 and E2) = 0



So

$$P(E_1 \text{ or } E_2) = P(E_1) + P(E_2) - \underbrace{P(E_1 \text{ and } E_2)}_{= 0 \text{ if mutually exclusive}}$$

$$= P(E_1) + P(E_2)$$

(i.e.the probability that either E_1 or E_2 occurs is the probability that E_1 occurs, plus the probability that E_2 occurs)

- **Rule of multiplication**
  - **If A and B are two events, then the probability of P(A and B), i.e. probability that A and B occur can be calculated as below:**
    - **Probabilities under conditions of statistical independence**
    - **P(A and B) = P(A) x P(B)**
    - **Probabilities under conditions of statistical dependence**
      - **P(A and B) = P(A) x P(B\A)**
      - **Probability of event B given that A has occurred**

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$$

Example 1.8

(a) A fair die is thrown twice. Find the probability that two fives are thrown.

(b) Events A & B are independent and $P(A) = 0.3$, $P(A \cap B) = 0.12$. Find $P(B)$ and $P(A \cup B)$.

Example 1.9

The events A and B are independent and $P(A) = x$, $P(B) = x + 0.2$ and $P(A \cap B) = 0.15$.

(a) Find the value of x.

(b) $P(A \cup B)$

(c) $P(A' \mid B')$

Example 1.10

Three people in an office decide to enter a marathon race. The respective probabilities that they will complete the race are 0.9, 0.7 and 0.6. Assume that their performances are independent. Find the probability that

(a) they all complete the race.

(b) none complete the race.

(c) at least one completes the race.

Tree Diagrams

- **Tree diagram**
  - **It augments the fundamental principle of counting by exhibiting all possible outcomes of a sequence of events where each event can occur in a finite number of ways.**

Example 1.11

In a certain selection of flower seeds, 2/3 have been treated to improve germination. The seeds which have been treated have a probability of germination of 0.8, whereas the untreated seeds have a probability of germination of 0.5.

(a) Find the probability that a seed, selected at random, will germinate.

(b) Find the probability that a seed selected at random had been treated, given that it had germinated.

Example 1.12

Marie is getting married tomorrow, at an outdoor ceremony in the desert. In recent years, it has rained only 5 days each year. Unfortunately, the weatherman has predicted rain for tomorrow. When it actually rains, the weatherman correctly forecasts rain 90% of the time. When it doesn't rain, he incorrectly forecasts rain 10% of the time. What is the probability that it will rain on the day of Marie's wedding?

Venn Diagram

- When events are not mutually exclusive, Venn diagram is useful.
- Note that when completing a Venn diagram , it is essential to deal with the overlap area first.

Example 1.13

A group of 50 people was asked which of the three newspapers A, B or C they read. The results showed that 25 read A, 16 read B, 14 read C, 5 read both A and B, 4 read both B and C, 6 read C and A and 2 read all three.

(a) Represent these data on a Venn Diagram.

Find the probability that a person selected at random from this group reads

(b) only A.

(c) only one of the newspaper.

(d) at least one of the newspaper.

Contingency Tables

- A table used to classify sample observations according to two or more identifiable characteristics
- It is a cross tabulation that simultaneously summarizes two variables of interest and their relationship.
- Example:

  A survey of 150 adults classified each as to gender and the number of movies attended last month. Each respondent is classified according to two criteria –the number of movies attended and gender.

	Gender		
Movies Attended	Men	Women	Total
0	20	40	60
1	40	30	70
2 or more	10	10	20
Total	70	80	150

Posterior Probability

- **Bayes Theorem**
  - **It is a formula which can be thought of as 'reversing' conditional probability. That is , it finds a conditional probability (A\B) given, among other things, its inverse (B\A).**
  - **If A and B are two events of an experiment, then**

$$P(A\backslash B) = \frac{P(A) \times P(B \backslash A)}{P(B)}$$

Solution for example 1.12

P(R) = 5/365 P(R') = 360/365

P(F|R) = 0.9 P(F|R') = 0.1

Applying Bayes' Theorem:

$$P(R \mid F) = \frac{P(F \mid R)P(R)}{P(F)} = \frac{P(F \mid R)P(R)}{P(F \mid R)P(R) + P(F \mid R')P(R')}$$

$$= \frac{0.9(\frac{5}{365})}{0.9(\frac{5}{365}) + 0.1(\frac{360}{365})} = \frac{1}{9}$$

Summary of Main Teaching Points

Recall

- What is Probability ?
- Rules of Probability
- Using
  - Tree Diagram
  - Venn Diagram
  - Contingency table

  to solve the questions on Probability
- Conditional Probability & Posterior Probability

Question and Answer Session

Q & A

What we will cover next

- Summary Measures of Statistics