Probability & Statistical Modelling

AQ077-3-2-PSMOD and Version VD1

Decision Making Techniques

Topic & Structure of The Lesson

- Introduction
- Decision making Under Uncertainty
- Expected Value / Expected Opportunity Loss
- Expected Value of perfect information
- Decision Trees

Learning Outcomes

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- **At the end of this topic, You should be able to**
  - Understand how to calculate and use Expected Value
  - Know how to value Perfect and Imperfect information
  - Use various decision rules
  - Know what is meant by a Decision Tree
  - Be able to describe Decision Trees and Outcomes Nodes

- Understand how to draw Decision Trees using the Forward Pass
- Know that the outcome values are calculated using the backward pass
- Be able to incorporate Bayes' Theorem into a Decision Tree.

Key Terms You Must Be Able To Use

- If you have mastered this topic, you should be able to use the following terms correctly in your assignments and exams:

- State of Nature
- Decision Alternatives
- Payoff Table
- Maximax criterion
- Maximin criterion
- Minimax regret criterion
- Hurwicz criterion
- Expected Value
- Expected Opportunity Loss
- Expected value of perfect Information
- Decision Tree

Introduction

- Decision making involves the following steps:
  - Recognise and clearly define the problem
  - Collect the information needed to analyse possible alternatives
  - Choose and implement the most feasible alternative

Problem formulation

- A decision problem is characterized by decision alternatives, states of nature, and resulting payoffs.
- The decision alternatives are the different possible strategies the decision maker can employ.
- The states of nature refer to future events, not under the control of the decision maker, which may occur. States of nature should be defined so that they are mutually exclusive and collectively exhaustive.

Decision Theory

- a general approach to decision making when the outcomes associated with alternatives are often in doubt
  - List the feasible alternatives
  - List the events
  - Calculate the payoff
  - Estimate the likelihood of each event
  - Select a decision rule

- When a decision has to be made, there will be a range of possible action
- Each action will have a certain consequences or ***payoff***

Decision Making Under Certainty

- Manager knows which event will occur
- pick the alternative with the best payoff

	Possible Future Demand	
Alternative	**Low**	**High**
Small facility	200	270
Large facility	160	800
Do nothing	0	0

What is the best choice if future demand will be low?

Decision Making Under Uncertainty

- **Decision making without probability**
  (no probability of occurrence are assigned)

- **Decision making with probabilities**
  (probabilities can be assigned)

Decision Making Without Probabilities

- **Decision Criteria**
  - Maximax criterion
  - Maximin criterion
  - Minimax regret criterion
  - Hurwicz criterion

- **Maximax Criterion**
  - maximise the maximum profit
  - optimistic - looks at the best possible payoffs
  - ignores the possibility of a potential loss

N/B: maximax criterion deals with profit, if the payoff table consists of costs, then minimin criterion would be used (i.e. minimise the minimum costs)

Quick Review Question

- **Decision making without probability**
  (no probability of occurrence are assigned)

State of Nature

Decision (purchase)	Good Economic conditions	Poor Economic conditions
Apartment building	$50,000	$30,000
Office building	100,000	-40,000
Warehouse	30,000	10,000

Determine the maximax criterion for our investment.

Decision making without probability

(no probability of occurrence are assigned)

Decision (purchase)	
Apartment building	$50,000
Office building	100,000
Warehouse	30,000

The maximax criterion for our investment is to purchase office building.

- **Maximin Criterion**
  - looks at the worst possible payoffs
  - pessimistic and conservative
  - results in the maximum of the minimum profit

Its equivalent is minimax (i.e. minimise the maximum *cost*)

Decision making without probability

(no probability of occurrence are assigned)

	State of Nature	
Decision (purchase)	**Good Economic conditions**	**Poor Economic conditions**
Apartment building	$50,000	$30,000
Office building	100,000	-40,000
Warehouse	30,000	10,000

Determine the maximin criterion for our investment.

Quick Review Question

- **Decision making without probability**
  (no probability of occurrence are assigned)

Decision (purchase)	
Apartment building	30,000
Office building	-40,000
Warehouse	10,000

The maximin criterion for our investment is to purchase apartment building.

Minimax Regret Criterion

- Regret (opportunity loss) is the difference between the payoff from the best decision and all other decision payoffs in those circumstances

- In this criterion decision maker attempts to avoid regret by selecting the decision alternative that minimises the maximum regret

Decision making without probability

(no probability of occurrence are assigned)

Decision (purchase)	State of Nature: Good Economic conditions	State of Nature: Poor Economic conditions
Apartment building	$50,000	$30,000
Office building	100,000	-40,000
Warehouse	30,000	10,000

Decision making without probability

(no probability of occurrence are assigned)

Regret Table

Decision (purchase)	State of Nature: Good Economic conditions	State of Nature: Poor Economic conditions
Apartment building	$50,000	0
Office building	0	70,000
Warehouse	70,000	20,000

Decision making without probability

(no probability of occurrence are assigned)

Decision:

Decision (purchase)	
Apartment building	$50,000
Office building	70,000
Warehouse	70,000

According to minimax regret criterion, the decision should be to purchase apartment building because this decision will result, at most, $50,000 in regret.

Hurwicz Criterion

- a comprise between the maximax and maximin criteria
- introduce α, the coefficient of optimism
- $0 < \alpha < 1$; It is a measure of the decision maker's optimism
- Multiplies the best payoff by α and the worst by $(1 - \alpha)$, then select the decision which gives the maximum payoff

- When α = 0 it is effectively the maximin criterion
- when α = 1, it is the maximax criterion
- α = 0.5 means the states of nature are equally likely to occur, it is known as **Equally Likelihood Criterion**

Decision (purchase)	State of Nature: Good Economic conditions	State of Nature: Poor Economic conditions
Apartment building	$50,000	$30,000
Office building	100,000	-40,000
Warehouse	30,000	10,000

Determine the Hurwicz criterion decision of our investment example, let $\alpha = 0.4$

$\alpha = 0.4, (1 - \alpha) = 0.6$

Decision (purchase)	Values
Apartment building	
Office building	
Warehouse	

Decision (purchase)	Values
Apartment building	\$50,000(0.4) + \$30,000(0.6) = 38,000
Office building	100,000(0.4)+(-40,000)(0.6)= 16,000
Warehouse	30,000(0.4)+10,000(0.6)= 18,000

According to Hurwicz criterion (α =0.4), the maximum weighted value is \$38,000. Thus, the decision would be to purchase the apartment building.

Decision (purchase)	State of Nature: Good Economic conditions	State of Nature: Poor Economic conditions
Apartment building	$50,000	$30,000
Office building	100,000	-40,000
Warehouse	30,000	10,000

Determine the best decision using the equal likelihood criterion in our investment.

Decision (purchase)	Values
Apartment building	
Office building	
Warehouse	

Determine the best decision using the equal likelihood criterion in our investment.

Decision (purchase)	Values
Apartment building	$50,000(0.5) + $30,000(0.5) = 40,000
Office building	100,000(0.5)+(-40,000)(0.5)= 30,000
Warehouse	30,000(0.5)+10,000(0.5)= 20,000

In applying the equal likelihood criterion, we are assuming 50% chance, that either state of nature will occur. Since $40,000 is the highest weighted value, the decision would be to purchase the apartment building.

Summary of criteria results:

Criterion	Decision (purchase)
Maximax	Office building
Maximin	Apartment building
Minimax regret	Apartment building
Hurwicz	Apartment building
Equal likelihood	Apartment building

- **Conclusion**
  - **Which criterion is appropriate is dependent on the risk personality and philosophy of the decision maker.**

Example 1

Tutorial 2 Q2

A farmer in Georgia must decide which crop to plant next year on his land: corn, peanuts, or soyabeans. The return from each crop will be determined by whether a new trade bill with Russia passes the Senate. The profit the farmer will realize from each crop given the two possible results on the trade bill as shown in the following payoff table. Determine the best crop to plant using the following decision criteria.

	Trade Bill	
Crop	Pass ($)	Fail ($)
Corn	35000	$8000
Peanuts	18000	12000
Soyabeans	22000	20000

(a) Maximax
(b) Maximin
(c) Minimax regret
(d) Hurwicz ($\alpha = 0.3$)
(e) Equal likelihood

Decision Making with Probabilities

- **Expected Value Approach**
  - If probabilistic information regarding the states of nature is available, one may use the expected value (EV) approach.
  - Expected value is computed by multiplying each decision outcome under each state of nature by the probability of its occurrence
  - The decision yielding the best expected return is chosen.

Payoff Tables

- The consequence resulting from a specific combination of a decision alternative and a state of nature is a payoff.
- A table showing payoffs for all combinations of decision alternatives and states of nature is a payoff table.
- Payoffs can be expressed in terms of profit, cost, time, distance or any other appropriate measure.

Quick Review Question

- **Payoff Table for the real estate investments**

	State of Nature	
Decision (Purchase)	Good economic conditions (0.6)	Poor economic conditions (0.4)
Apartment building	$50,000	$30,000
Office building	100,000	-40,000
Warehouse	30,000	10,000

Calculate the expected value of each choice and make the best decision.

- **Expected Opportunity Loss**
  - Is the expected value of the regret for each decision.
    - Regret is the difference between the payoff from the best decision and all other decision payoffs.
  - Computed by multiplying each decision outcome (probability) by the regret i.e. opportunity loss

- **Payoff Table for the real estate investments**

	State of Nature	
Decision (Purchase)	Good economic conditions (0.6)	Poor economic conditions (0.4)
Apartment building	$50,000	$30,000
Office building	100,000	-40,000
Warehouse	30,000	10,000

Calculate the expected opportunity loss (EOL) that would be experienced by the decision maker ?

- **An investor must decide among an apartment building, an office building and a warehouse using expected opportunity loss.**

The regret values for each decision outcome were shown below with the probabilities of occurrence for each state of nature.

	State of Nature	
Decision (Purchase)	Good economic conditions (0.6)	Poor economic conditions (0.4)
Apartment building	$50,000	$0
Office building	0	70000
Warehouse	70000	20000

The expected opportunity loss for each decision is computed as follows:

EOL (apartment) = $50000(0.6) + 0(0.4) = $30000

EOL (Office) = $0(0.6) + 70000(0.4) = $28000

EOL (Warehouse) = $70000(0.6) + 20000(0.4) = $50000

The best decision results from minimizing the expected regret or opportunity loss. Since $28000 is the minimum expected regret, the decision is to purchase the office building (This decision will result, at most, $28,000 in regret)

- **Expected value of Perfect information(EVPI)**
  - **It is the maximum amount a decision maker would pay for addition information**
  - **Equals to the expected value given perfect information minus the expected value without perfect information.**

Payoff table with Decisions, given perfect information is as below:

	State of Nature	
Decision (Purchase)	Good economic conditions (0.6)	Poor economic conditions (0.4)
Apartment building	$50,000	$30,000
Office building	100,000	-40,000
Warehouse	30,000	10,000

Determine the expected value of perfect information.

Expected value given perfect information

= $100,000 (0.6) + $30,000(0.4) = $72,000

Expected value without perfect information

= $100000 (0.6) – $40000(0.4) =$44000

EVPI = $72000 - $44000 = $28000 (lowest EOL)

(This is the maximum amount that the investor would pay to purchase perfect information from other source, such as an economic forecaster)

Example 2

- The Loebuck Grocery must decide how many cases of milk to stock each week in order to meet demand. The probability distribution of demand during a week is shown in the table below:

Demand (cases)	Probability
15	0.20
16	0.25
17	0.40
18	0.15

Each case costs the grocer $10 and sells for $12. Unsold cases are sold to a local farmer (who mixes the milk with feed for livestock) for $2 per case. If there is a shortage, the grocer considers the cost of customer ill will and lost profit to be $4 per case. The grocer must decide how many cases of milk to order each week.

(a) Construct the payoff table.

(b) Compute the expected value of each alternative amount of milk that could be stocked, and select the best decision.

Decision Trees

- A decision tree is a chronological representation of the decision problem. It is a graphical device that forces the decision-maker to examine all possible outcomes, including unfavorable ones.
- makes easier the computation of the expected values
- easy to understand the process of making decision

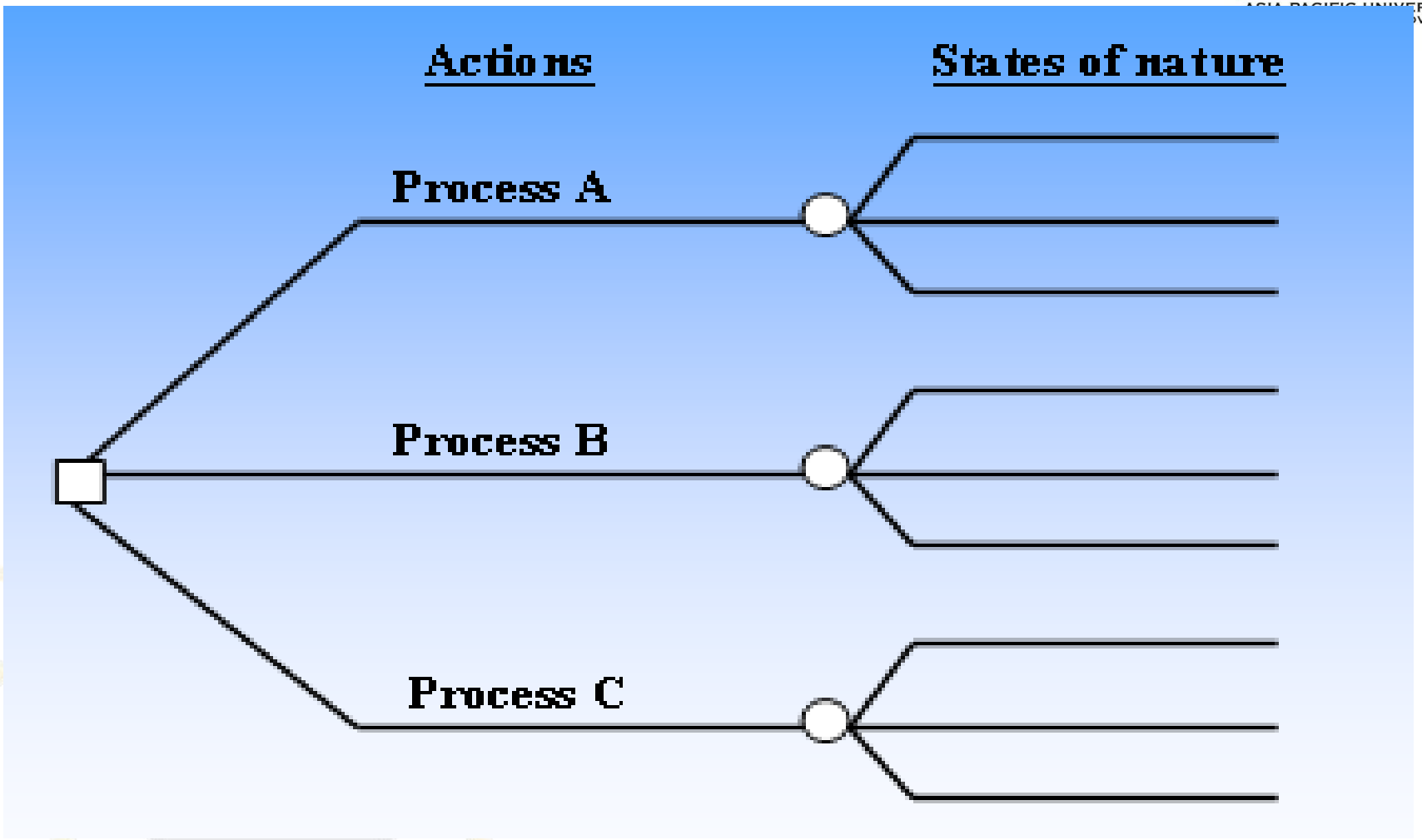
- **Each decision tree has two types of nodes; round nodes correspond to the states of nature while square nodes correspond to the decision alternatives.**
- **The branches leaving each round node represent the different states of nature while the branches leaving each square node represent the different decision alternatives.**

- **At the end of each limb of a tree are the payoffs attained from the series of branches making up that limb.**

- **How to draw a decision Tree ?**

All decision trees contain:

- Decision (or action) nodes
- Event (or state-of-nature) nodes
- Terminal nodes


Actions
States of nature
Process A
Process B
Process C

- Start a decision tree with a decision that needs to be made which is represented by a small square.
- From this square draw out lines towards the right for each possible solution, and write that solution along the line.
- Keep this lines as far apart as possible so that you can expand on your thoughts.

- At the end of each solution line, consider the results:
  - If the result of taking that decision is uncertain, draw a circle.
  - If the result is another decision that needs to be made, draw another square.
  - Write the decision or factor to be considered above the square or circle.
  - If you have completed the solution at the end of the line, just leave it blank.

- Starting from the new decision squares on your diagram, draw out lines representing the options that could be taken.
- From the circles draw out lines representing possible outcomes.
- Again mark a brief note on the line saying what it means.

- Keep on doing this until you have drawn down as many of the possible outcomes and decisions as you can see leading on from your original decision.
- Review your tree diagram. Challenge each square and circle to see if there are any solutions or outcomes you have not considered. If there are, draw them in.

- If necessary, redraft your tree if parts of it are too congested or untidy.

Evaluate Your decision Tree

- Start by assigning a cash or numeric value to each possible outcome – how much you think it is worth.
- Next look at each circle and estimate the probability of each outcome. (The total should be 100% or 1)

Calculating Tree Value

- Start on the right hand side of the decision tree, and work back towards the left.
- Record the result after completing the calculation on a node.
- All the calculations that lead to the result can be ignored – effectively that branch of the tree can be discarded. (Pruning the tree)

- **Calculating the value of uncertain outcome nodes**

  - Multiply the value of the outcomes by their probability, and noting the result.
  - The total value of that node of the tree is gained by adding the results together.

Calculating the value of decision nodes

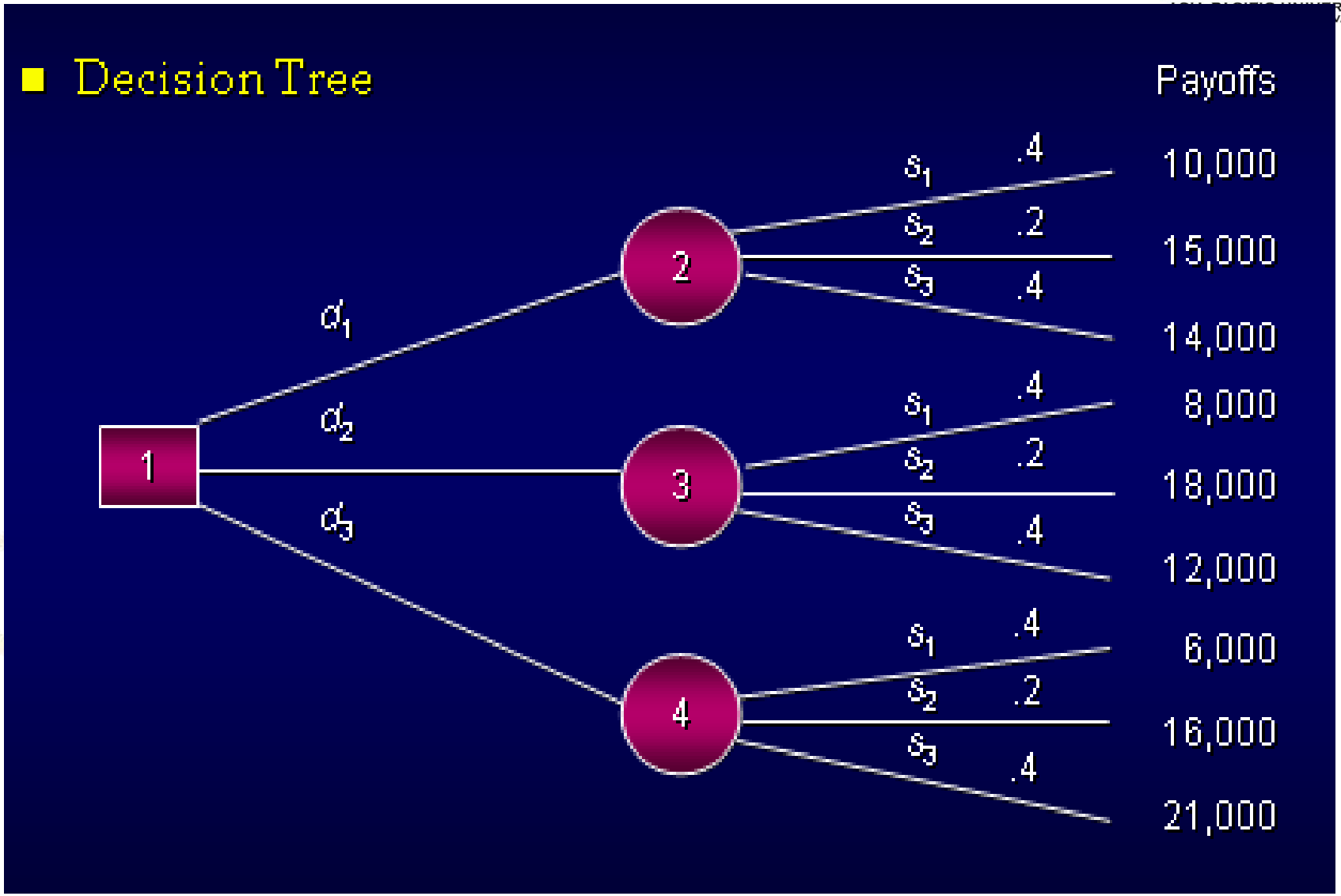
- Calculate the benefit of each decision (subtract the cost from the value of that outcome that you have already calculated)
- Select the decision which has the largest benefit, and take that as the decision made

Quick Review Question

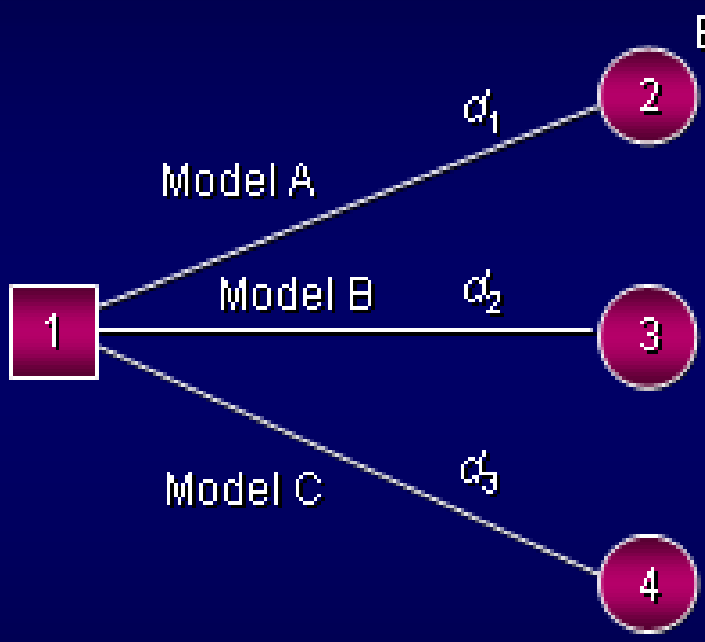
- **Example:**
  - Burger Prince Restaurant is contemplating opening a new restaurant on Main Street. It has three different models, each with a different seating capacity. Burger Prince estimates that the average number of customers per hour will be 80, 100, or 120. The payoff table for the three models is as follows:

	Average Number of Customers Per Hour		
	$S_1 = 80$	$S_2 = 100$	$S_3 = 120$
Model A	\$10000	\$15000	\$14000
Model B	\$8000	\$18000	\$12000
Model C	\$6000	\$16000	\$21000

- Calculate the expected value for each decision. The decision tree on the next slide can assist in this calculation. Here *d*1, *d*2, *d*3 represent the decision alternatives of models A, B, C, and *s*1, *s*2, *s*3 represent the states of nature of 80, 100, and 120.


Decision Tree
Payoffs
1
2
3
4
α_1
α_2
α_3
s_1 .4
s_2 .2
s_3 .4
10,000
15,000
14,000
8,000
18,000
12,000
6,000
16,000
21,000

Expected Value For Each Decision



EMV = .4(10,000) + .2(15,000) + .4(14,000)
= $12,600

EMV = .4(8,000) + .2(18,000) + .4(12,000)
= $11,600

EMV = .4(6,000) + .2(16,000) + .4(21,000)
= $14,000

Choose the model with largest EV, Model C.

Formula Spreadsheet for Expected Value Approach

	A	B	C	D	E	F
1	PAYOFF TABLE					
2						
3	Decision	State of Nature			Expected	Recommended
4	Alternative	s1 = 80	s2 = 100	s3 = 120	Value	Decision
5	Model A	10,000	15,000	14,000	=$B$8*B5+$C$8*C5+$D$8*D5	=IF(E5=$E$9,A5,"")
6	Model B	8,000	18,000	12,000	=$B$8*B6+$C$8*C6+$D$8*D6	=IF(E6=$E$9,A6,"")
7	Model C	6,000	16,000	21,000	=$B$8*B7+$C$8*C7+$D$8*D7	=IF(E7=$E$9,A7,"")
8	Probability	0.4	0.2	0.4		
9		Maximum Expected Value			=MAX(E5:E7)	

Solution Spreadsheet for Expected Value Approach

	A	B	C	D	E	F
1	PAYOFF TABLE					
2						
3	**Decision**	**State of Nature**			**Expected**	**Recommended**
4	**Alternative**	s1 = 80	s2 = 100	s3 = 120	**Value**	**Decision**
5	Model A	10,000	15,000	14,000	12600	
6	Model B	8,000	18,000	12,000	11600	
7	Model C	6,000	16,000	21,000	14000	**Model C**
8	**Probability**	0.4	0.2	0.4		
9		**Maximum Expected Value**			**14000**	

Example 3

The financial success of the Downhill Ski Resort in the Blue Ridge Mountains is dependent on the amount *of* snowfall during the winter months. If the snowfall averages more than 40 inches, the resort will be successful; if the snowfall is between 20 and 40 inches, the resort will receive a moderate financial return; and if snowfall averages less than 20 inches, the resort will suffer a financial loss. The financial return and probability given each level of snowfall follows.

Snowfall Level (in)	Prob	Financial Return ($)
>40,	0.4	120,000
20-40,	0.2	40,000
<20,	0.4	- 40,000

A large hotel chain has offered to lease the resort for the winter for $40,000. Draw a decision tree and use it to determine if the resort should operate or lease. Explain your answer.

Summary of Main Teaching Points

- **Elements common to decision theory problems:**
  - An objective the decision maker is trying to reach.
  - Several courses of action
  - A calculable measure of the benefit or worth of the various alternatives
  - Events beyond the control of the decision maker
  - Uncertainty concerning which outcome or state of nature will actually happen.

- Decision trees provide an effective method of decision making because they:

  - Clearly lay out the problem so that all choices can be viewed, discussed and challenged
  - Provides a framework to quantify of the values of outcomes and the probabilities of achieving them
  - Help us to make the best decisions on the basis of our existing information and best guesses.

Question and Answer Session

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Q & A

What we will cover next

- **END**