

SECTION A (50 marks)

Answer ALL questions in this section.

QUESTION A1 (5 marks)

The mean diameter of a sample of 200 microchips in computers produced by a machine is 5.02mm and the standard deviation is 0.05mm. The production of microchips allows a maximum tolerance in the diameter of 4.96mm to 5.08mm, otherwise the microchips are considered defective. Determine the percentage of non-defective microchips produced by the machine, assuming the diameters are normally distributed.

QUESTION A2 (5 marks)

The average number of radioactive particles passing through a counter during one millisecond in a laboratory experiment is 4. Find the probability at most one particle enters the counter in a given millisecond.

QUESTION A3 (5 marks)

Suppose that an experiment follows a binomial distribution with mean, $E(X) = 2$ and variance, $Var(X) = 1.6$. Find the probability of success, p .

QUESTION A4 (5 marks)

A survey of 35 individuals who passed the seven exams and obtained the rank of Fellow in the actuarial field finds the average salary to be \$150,000 per year. If the standard deviation for the population is \$15,000, construct a 95% confidence interval for the salary of all Fellows.

QUESTION A5 (5 marks)

Noise levels at various areas of urban hospitals were measured in decibels. The mean of the noise levels of a sample of 15 corridors was 61.2 decibels with the standard deviation of 7.9 decibels. Find the 95% confidence interval of the true mean of the noise levels.

QUESTION A6 (5 marks)

The proportion of students in a city that attended private schools is around 11%. A random sample of 450 students indicates that 55 attended private schools. Estimate the true proportion of students attending private schools with 95% confidence.

QUESTION A7 (5 marks)

A random sample of 36 drinks from a soft-drink machine has an average volume of 7.4 milliliters with a standard deviation of 0.48 milliliter. The null hypothesis is given by $\mu = 7.5$ milliliters against the alternative hypothesis $\mu < 7.5$ have been developed to be tested at 0.05 level of significance.

- (a) Find the critical value, Z_{α} .

(2 marks)

- (b) Compute the test statistic, Z .

(3 marks)

QUESTION A8 (5 marks)

Male students will spend, on average, \$8 for a Saturday evening fraternity party. The null hypothesis is $\mu = \$8$ against the alternative hypothesis $\mu \neq \$8$ if a random sample of 12 male students attending the fraternity party gives the test statistic value of $t_c = 1.8815$. State your decision on the rejection of null hypothesis:

- (a) at 0.10 level of significance.

(3 marks)

- (b) at 0.05 level of significance.

(2 marks)

QUESTION A9 (5 marks)

The following hypotheses are given.

$$H_0 : p = 0.45$$

$$H_1 : p > 0.45$$

A random sample of 96 observations gave sample proportion, $\hat{p} = 0.56$.

- (a) Find the value of test statistic, Z .

(3 marks)

- (b) Find the critical value from standard normal table given that $\alpha = 0.05$.

(2 marks)

QUESTION A10 (5 marks)

A company is considering the following marketing strategies and the corresponding payoff under different economic environments as shown in Table 1.

Table 1. Payoff

	State of nature	
	Good economic environment	Poor economic environment
Strategy P	\$200,000	-\$150,000
Strategy Q	\$100,000	-\$80,000
Strategy R	\$80,000	-\$20,000
Strategy S	\$50,000	\$30,000

Determine the best strategy using Hurwicz criterion, with the coefficient of optimism, $\alpha = 0.35$.

SECTION B (50 marks)

Answer ALL questions in this section.

QUESTION B1 (25 marks)

- (a) A survey was done to determine whether the age of voters was related to the political party they supported during the last election. The data had been taken and gathered in Table 2:

Table 2. Cross tabulation of frequency

Age of Voters	Political Party		
	A	B	C
Young adult	20	30	30
Middle age	20	10	10
Senior citizen	40	20	20

Determine if there exists any significant evidence to conclude that age of voters and the political party they supported are independent. Use $\alpha = 0.05$.

(15 marks)

- (b) Due to an unresolved national issue, the popularity of a certain politician is suspected to have decreased over the past year. His popularity vote percentage used to be 55%. To confirm the suspicion, a sample of 820 adult residents is surveyed. The survey reveals that 405 of the respondents still support him. Determine if there exists a significant decrease in his popularity vote percentage. Use significance level of 0.05 to conduct your hypothesis testing.

(10 marks)

QUESTION B2 (25 marks)

- (a) An investor is considering three investment sectors: construction, plantation, or health tourism. Profits from the construction or plantation will be affected by the availability of blue-collar workers. Whereas profits from the health tourism will be quite stable. The potential profits on the investments are estimated and tabulated in Table 3.

Table 3. Payoff table.

Investment	Blue-collar workers availability		
	Shortage	Stable supply	Surplus
<i>Construction</i>	-\$80000	\$250000	\$100000
<i>Plantation</i>	\$30000	\$150000	\$100000
<i>Health tourism</i>	\$90000	\$90000	\$80000

Determine the best investment using the following decision criteria.

- (i) Maximax criterion.

(2 marks)

- (ii) Maximin criterion.

(iii) Hurwicz criterion (coefficient of optimism = 0.6). (2 marks)

(iv) Equal likely criterion. (4 marks)

(b) An outdoor event organiser must decide whether to have the vendors sell sun visors or umbrella. There is a 25% chance of rain, 20% chance of overcast skies, and a 55% chance of sunshine, according to the weather forecast department. The organiser estimates that the following profits will result from each decision, given each state of weather condition as in Table 4. (4 marks)

Table 4. Weather conditions.

Decision	Weather conditions		
	Rain	Overcast	Sunshine
<i>Sun visors</i>	-\$15000	-\$6000	\$45000
<i>Umbrellas</i>	\$60000	0	\$27000

(i) Compute the expected value for each decision and select the best one. (3 marks)

(ii) Develop the opportunity loss table and compute the expected opportunity loss for each decision and determine which decision is the best. (6 marks)

(iii) Without further calculation, write down the expected value of perfection information (EVPI). (4 marks)

FORMULAE AND TABLES

(a) **Binomial Distribution**

$$P(X = x) = {}^n C_x \cdot p^x \cdot (1 - p)^{n-x}, x = 0, 1, \dots, n$$

Where

p is the probability of success in single trial
 n is the number of trials

Mean of a Binomial distribution, $E(X) = np$

Standard deviation of a Binomial distribution, $\sigma = \sqrt{np(1-p)}$

(b) **Poisson Distribution**

$$P(X = x) = \frac{e^{-m} \times m^x}{x!}, x = 0, 1, \dots$$

Where

m represents the mean

Mean of a Poisson distribution, $E(X) = m$

Standard deviation of a Poisson distribution, $\sigma = \sqrt{m}$

(c) **Normal Distribution**

$$Z = \frac{X - \mu}{\sigma}$$

Where

Z is the number of standard deviation from the mean
 X is the value of interest

Mean of a Normal distribution, $E(X) = \mu$

Standard deviation of a Normal distribution, $sd(X) = \sigma$

(d) Sampling Distribution

(i) Standard error of mean

$$se(\bar{X}) = \frac{\sigma}{\sqrt{n}}$$

Where

σ is the standard deviation for the population
 n represent the sample size

OR

$$se(\bar{X}) = \frac{s}{\sqrt{n}}$$

Where

s is the standard deviation for the sample
 n represent the sample size

(ii) Standard error of proportion

$$se(\hat{p}) = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

(e) Confidence Interval

(i) Confidence interval for μ :
$$\bar{X} \pm t_{\frac{\alpha}{2}, n-1} \left(\frac{s}{\sqrt{n}} \right)$$

Sample size:
$$n = \left(\frac{Z_{\alpha/2} \times \sigma}{e} \right)^2$$

(ii) Confidence interval for p :
$$\hat{p} \pm Z_{\frac{\alpha}{2}} \left(\sqrt{\frac{\hat{p}\hat{q}}{n}} \right)$$

Sample size,
$$n = \frac{(Z_{\alpha/2})^2 \times \hat{p}\hat{q}}{e^2}$$

(f) Hypothesis Testing:

(i) Hypothesis testing for μ :
$$t = \frac{\bar{X} - \mu}{s/\sqrt{n}}$$

(ii) Hypothesis testing for p :
$$Z = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

(iii) Non-parametric testing:
$$\chi^2 = \sum \frac{(O - E)^2}{E}$$

(g) Decision Making Techniques

(i) Expected Value:

$$EV(i) = \sum_{j=1}^N P_j X_{ij}$$

(ii) Expected Opportunity Loss:

$$EOL(i) = \sum_{j=1}^N P_j L_{ij}$$

(iii) Expected Value of Perfect Information:

$EVPI = \text{Expected value under certainty} - \text{expected value under uncertainty}$

STANDARD NORMAL CUMULATIVE PROBABILITY TABLE

Cumulative probabilities for NEGATIVE z-values are shown in the following table:



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

cum. prob	<i>t</i> _{.50}	<i>t</i> _{.75}	<i>t</i> _{.80}	<i>t</i> _{.85}	<i>t</i> _{.90}	<i>t</i> _{.95}	<i>t</i> _{.975}	<i>t</i> _{.99}	<i>t</i> _{.995}	<i>t</i> _{.999}	<i>t</i> _{.9995}
one-tail	0.50	0.25	0.20	0.15	0.10	0.05	0.025	0.01	0.005	0.001	0.0005
two-tails	1.00	0.50	0.40	0.30	0.20	0.10	0.05	0.02	0.01	0.002	0.001
df											
1	0.000	1.000	1.376	1.963	3.078	6.314	12.71	31.82	63.66	318.31	636.62
2	0.000	0.816	1.061	1.386	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.000	0.765	0.978	1.250	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.000	0.741	0.941	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.000	0.727	0.920	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.000	0.718	0.906	1.134	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.000	0.711	0.896	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.000	0.706	0.889	1.108	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.000	0.703	0.883	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.000	0.700	0.879	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.000	0.697	0.876	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.000	0.695	0.873	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.000	0.694	0.870	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.000	0.692	0.868	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.000	0.691	0.866	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.000	0.690	0.865	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.000	0.689	0.863	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.000	0.688	0.862	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.000	0.688	0.861	1.066	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.000	0.687	0.860	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.000	0.686	0.859	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.000	0.686	0.858	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.000	0.685	0.858	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.000	0.685	0.857	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.000	0.684	0.856	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.000	0.684	0.856	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.000	0.684	0.855	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.000	0.683	0.855	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.000	0.683	0.854	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.000	0.683	0.854	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3

Confidence Level

CHI-SQUARED(χ^2) DISTRIBUTION TABLE

df	Level of significance (α)									
	.995	0.99	0.975	0.95	0.90	0.10	0.05	0.025	0.01	0.005
1	-	-	0.001	0.004	0.016	2.706	3.841	5.024	6.635	7.879
2	0.010	0.020	0.051	0.103	0.211	4.605	5.991	7.378	9.210	10.597
3	0.072	0.115	0.216	0.352	0.584	6.251	7.815	9.348	11.345	12.838
4	0.207	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.412	0.554	0.831	1.145	1.610	9.236	11.070	12.833	15.086	16.750
6	0.676	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	0.989	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.344	1.646	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	1.735	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.156	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	2.603	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.074	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	3.565	4.107	5.009	5.892	7.042	19.812	22.362	24.736	27.688	29.819
14	4.075	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	4.601	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.142	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	5.697	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	6.265	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	6.844	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	7.434	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.034	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	8.643	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	9.260	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	9.886	10.856	12.401	13.848	15.659	33.196	36.145	39.364	42.980	45.559
25	10.520	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	11.160	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	11.808	12.879	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645

28	12.46 1	13.56 5	15.30 8	16.92 8	18.93 9	37.916	41.337	44.461	48.278	50.993
29	13.12 1	14.25 6	16.04 7	17.70 8	19.76 8	39.087	42.557	45.722	49.588	52.336
30	13.78 7	14.95 3	16.79 1	18.49 3	20.59 9	40.256	43.773	46.979	50.892	53.672
40	20.70 7	22.16 4	24.43 3	26.50 9	29.05 1	51.805	55.758	59.342	63.691	66.766
50	27.99 1	29.70 7	32.35 7	34.76 4	37.68 9	63.167	67.505	71.420	76.154	79.490
60	35.53 4	37.48 5	40.48 2	43.18 8	46.45 9	74.397	79.082	83.298	88.379	91.952

28	12.46 1	13.56 5	15.30 8	16.92 8	18.93 9	37.916	41.337	44.461	48.278	50.993
29	13.12 1	14.25 6	16.04 7	17.70 8	19.76 8	39.087	42.557	45.722	49.588	52.336
30	13.78 7	14.95 3	16.79 1	18.49 3	20.59 9	40.256	43.773	46.979	50.892	53.672
40	20.70 7	22.16 4	24.43 3	26.50 9	29.05 1	51.805	55.758	59.342	63.691	66.766
50	27.99 1	29.70 7	32.35 7	34.76 4	37.68 9	63.167	67.505	71.420	76.154	79.490
60	35.53 4	37.48 5	40.48 2	43.18 8	46.45 9	74.397	79.082	83.298	88.379	91.952
70	43.27 5	45.44 2	48.75 8	51.73 9	55.32 9	85.527	90.531	95.023	100.42 5	104.21 5
80	51.17 2	53.54 0	57.15 3	60.39 1	64.27 8	96.578	101.87 9	106.62 9	112.32 9	116.32 1
90	59.19 6	61.75 4	65.64 7	69.12 6	73.29 1	107.56 5	113.14 5	118.13 6	124.11 6	128.29 9

70	43.27 5	45.44 2	48.75 8	51.73 9	55.32 9	85.527	90.531	95.023	100.42 5	104.21 5
80	51.17 2	53.54 0	57.15 3	60.39 1	64.27 8	96.578	101.87 9	106.62 9	112.32 9	116.32 1
90	59.19 6	61.75 4	65.64 7	69.12 6	73.29 1	107.56 5	113.14 5	118.13 6	124.11 6	128.29 9

10 0	67.32 8	70.06 5	74.22 2	77.92 9	82.35 8	118.49 8	124.34 2	129.56 1	135.80 7	140.16 9
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