Probability and Statistical Modeling

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Chapter 1

Concept of Probability

1.1 Introduction

Probability is the likelihood or chance of something happening.
In an experiment in which all outcomes are equally likely, the probability of an event E is,

$$P(E) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}} = \frac{n(E)}{n(S)}$$

- $0 \leq P(E) \leq 1$
- $P(E) = 0$ is an impossible event
- $P(E) = 1$ is a certain event
- Sum of all probabilities of an experiment must total 1
- P(E does not occur) = 1 - P(E) (Complementary Probability)

1.2 Terminologies

- Statistical event

  It is defined as any subset of the given outcome set that is of interest

- Statistical experiment

  It is described as any situation, specially set up or occuring naturally, which can be performed, enacted or otherwise considered in order to gain useful information

➢ Sample Space

Sample space is the collection of all possible outcomes
Examples: 6 faces of a die, 52 cards of a bridge deck

➢ Complement Rule

The complement of an event E is the collection of all possible elementary events not contained in event E. The complement of event E is represented by $\overline{E}$.

$$P(\overline{E}) = 1 - P(E)$$
$$or, P(E) + P(\overline{E}) = 1$$

1.3 Events

➢ Combined Events

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

➢ Mutually Exclusive Events

Two events of the same experiment are said to be mutually exclusive if their respective events do not overlap.

♦ If E_1 occurs, then E_2 cannot occur

♦ If E_1 and E_2 have no common elements

➢ Not mulually exclusive

If two or more events occur at one time.

➢ Independent events

Two events are said to be independent if the occurance (or not) of one of the events will in no way affect the occurence (or not) of other.

Alternatively, two events that are definde on two physically different experiments are said to be independent.

$$E_1 = \text{heads on one flip of fair coin}$$
$$E_2 = \text{heads on second flip of same coin}$$

Result of second flip does not depend on the result of the first flip.

➢ Dependent events

$$E_1 = \text{rain forecasted on the news}$$
$$E_2 = \text{take umbrella to work}$$

Probability of the second event is affected by the occurence of the first event.

➢ Conditional event

One of the outcomes of which is influenced by the outcomes of another event.

If A and B are two events not necessarily from the same experiment, then the conditional probability that A occurs, given that B has already occured, is written,

$$P(\text{A, given B}) = P(A \mid B) = \frac{P(\text{A and B})}{P(B)}$$
$$P(\text{B, given A}) = P(B \mid A) = \frac{P(\text{A and B})}{P(A)}$$

1.4 Rules of Probability

➢ Addition Rule

If A and B are not mutually exclusive events, then

$$\begin{aligned} P(A \cup B) &= P(A) + P(A) - P(A \cap B) \\ P(A \cup B \cup C) &= P(A) + P(B) + P(C) - P(A \cap B) \\ &\quad - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) \end{aligned}$$

If A and B are mutually exclusive events, then

$$P(A \cup B) = P(A) + P(B)$$
$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$

➢ Multiplication Rule

If A and B are two events, then the probability of P(A and B), i.e, probability that A and B occur can be calculated as below:

Probabilities under conditions of statistical independence

$$P(A \cap B) = P(A)\dot{P}(B)$$

Probabilities under conditions of statistical dependence

$$P(A \cap B) = P(A) \times P(B \setminus A)$$

Probability of event B given that A has occured

$$P(B \mid A) = \frac{P(\text{A and B})}{P(A)}$$

1.5 Tree Diagram

It augments the fundamental principle of counting by exhibiting all possible outcomes of a sequence of events where each event can occur in a finite number of ways.

1.6 Contingency Tables

- ➢ A table used to classify sample observations according to two or more identifiable characteristics
- ➢ It is a cross tabulation that simultaneously summarizes two variables of interest and their relationship
- ➢ Example:

  A survey of 150 adults classified each as to gender and the number of movies attended last month. Each respondent is classified according to two criteria - the number of movies attended and gender.

Gender			
Movies Attended	Men	Women	Total
0	20	40	60
1	40	30	70
2 or more	10	10	20
Total	70	80	150

1.7 Posterior Probability

➢ Bayes Theorem

➢ It is a formula which can be thought of as ’reversing’ conditional probability. That is, it finds a conditional probability (A|B) given, among other things, its inverse (B|A).

➢ If A and B are two events of an experiment, then

$$P(A|B) = \frac{P(A)\dot{P}(B|A)}{P(B)}$$