

# Package ‘IsoplotR’

March 20, 2019

**Title** Statistical Toolbox for Radiometric Geochronology

**Version** 2.5

**Date** 2019-03-07

**Description** Plots U-Pb data on Wetherill and Tera-Wasserburg concordia diagrams. Calculates concordia and discordia ages. Performs linear regression of measurements with correlated errors using 'York', 'Titterton' and 'Ludwig' approaches. Generates Kernel Density Estimates (KDEs) and Cumulative Age Distributions (CADs). Produces Multidimensional Scaling (MDS) configurations and Shepard plots of multi-sample detrital datasets using the Kolmogorov-Smirnov distance as a dissimilarity measure. Calculates  $^{40}\text{Ar}/^{39}\text{Ar}$  ages, isochrons, and age spectra. Computes weighted means accounting for overdispersion. Calculates U-Th-He (single grain and central) ages, logratio plots and ternary diagrams. Processes fission track data using the external detector method and LA-ICP-MS, calculates central ages and plots fission track and other data on radial (a.k.a. 'Galbraith') plots. Constructs total Pb-U, Pb-Pb, K-Ca, Re-Os, Sm-Nd, Lu-Hf, Rb-Sr and  $^{230}\text{Th}$ -U isochrons as well as  $^{230}\text{Th}$ -U evolution plots.

**Author** Pieter Vermeesch [aut, cre]

**Maintainer** Pieter Vermeesch <p.vermeesch@ucl.ac.uk>

**Depends** R (>= 3.0.0)

**Imports** MASS, grDevices, graphics, stats, utils

**License** GPL-3

**URL** <http://isoplotr.london-geochron.com>

**LazyData** true

**RoxygenNote** 6.1.0

**Encoding** UTF-8

## R topics documented:

age . . . . .	2
agespectrum . . . . .	6
cad . . . . .	8
central . . . . .	11

concordant . . . . .	14
concordia . . . . .	15
data2york . . . . .	18
ellipse . . . . .	20
evolution . . . . .	21
examples . . . . .	23
helioplot . . . . .	25
isochron . . . . .	27
IsoplotR . . . . .	34
kde . . . . .	34
ludwig . . . . .	39
mds . . . . .	41
peakfit . . . . .	43
radialplot . . . . .	46
read.data . . . . .	51
scatterplot . . . . .	55
set.zeta . . . . .	56
settings . . . . .	58
titterington . . . . .	61
weightedmean . . . . .	62
york . . . . .	67

<b>Index</b>	<b>70</b>
--------------	-----------

---

age	<i>Calculate isotopic ages</i>
-----	--------------------------------

---

## Description

Calculates U-Pb, Pb-Pb, Ar-Ar, K-Ca, Re-Os, Sm-Nd, Rb-Sr, Lu-Hf, U-Th-He, Th-U and fission track ages and propagates their analytical uncertainties. Includes options for single grain, isochron and concordia ages.

## Usage

```
age(x, ...)
```

## Default S3 method:

```
age(x, method = "U238-Pb206", exterr = TRUE,
    J = c(NA, NA), zeta = c(NA, NA), rhoD = c(NA, NA), U48 = 1,
    Th0U8 = 1, Ra6U8 = 1, Pa1U5 = 1, ...)
```

## S3 method for class 'UPb'

```
age(x, type = 1, wetherill = TRUE, exterr = TRUE,
    i = NA, sigdig = NA, common.Pb = 0, show.p = FALSE, ...)
```

## S3 method for class 'PbPb'

```
age(x, isochron = TRUE, common.Pb = 1, exterr = TRUE,
```

```

    i = NA, sigdig = NA, ...)

## S3 method for class 'ArAr'
age(x, isochron = FALSE, i2i = TRUE, exterr = TRUE,
    i = NA, sigdig = NA, ...)

## S3 method for class 'KCa'
age(x, isochron = FALSE, i2i = TRUE, exterr = TRUE,
    i = NA, sigdig = NA, ...)

## S3 method for class 'UThHe'
age(x, isochron = FALSE, central = FALSE, i = NA,
    sigdig = NA, ...)

## S3 method for class 'fissiontracks'
age(x, central = FALSE, i = NA, sigdig = NA,
    exterr = TRUE, ...)

## S3 method for class 'ThU'
age(x, isochron = FALSE, i2i = TRUE, exterr = TRUE,
    i = NA, sigdig = NA, detritus = 0, ...)

## S3 method for class 'ReOs'
age(x, isochron = TRUE, i2i = TRUE, exterr = TRUE,
    i = NA, sigdig = NA, ...)

## S3 method for class 'SmNd'
age(x, isochron = TRUE, i2i = TRUE, exterr = TRUE,
    i = NA, sigdig = NA, ...)

## S3 method for class 'RbSr'
age(x, isochron = TRUE, i2i = TRUE, exterr = TRUE,
    i = NA, sigdig = NA, ...)

## S3 method for class 'LuHf'
age(x, isochron = TRUE, i2i = TRUE, exterr = TRUE,
    i = NA, sigdig = NA, ...)

```

## Arguments

- x can be:
- a scalar containing an isotopic ratio,
  - a two element vector containing an isotopic ratio and its standard error, or the spontaneous and induced track densities  $N_s$  and  $N_i$  (if method='fissiontracks'),
  - a four element vector containing Ar40Ar39, s[Ar40Ar39], J, s[J],
  - a two element vector containing K40Ca40 and s[K40Ca40],
  - a six element vector containing U, s[U], Th, s[Th], He and s[He],

- an eight element vector containing U, s[U], Th, s[Th], He, s[He], Sm and s[Sm]
- a six element vector containing Rb, s[Rb], Sr, s[Sr], Sr87Sr86, and s[Sr87Sr86]
- a six element vector containing Re, s[Re], Os, s[Os], Os187Os188, and s[Os187Os188]
- a six element vector containing Sm, s[Sm], Nd, s[Nd], Nd143Nd144, and s[Nd143Nd144]
- a six element vector containing Lu, s[Lu], Hf, s[Hf], Hf176Hf177, and s[Hf176Hf177]
- a five element vector containing  $\theta/8$ , s[ $\theta/8$ ], 4/8, s[4/8] and cov[ $\theta/8$ , 4/8]

OR

- an object of class UPb, PbPb, ArAr, KCa, ThU, RbSr, SmNd, ReOs, LuHf, UThHe or fissiontracks.

...	additional arguments
method	one of either 'U238-Pb206', 'U235-Pb207', 'Pb207-Pb206', 'Ar-Ar', 'K-Ca', 'Th-U', 'Re-Os', 'Sm-Nd', 'Rb-Sr', 'Lu-Hf', 'U-Th-He' or 'fissiontracks'
exterr	propagate the external (decay constant and calibration factor) uncertainties?
J	two-element vector with the J-factor and its standard error.
zeta	two-element vector with the zeta-factor and its standard error.
rhoD	two-element vector with the track density of the dosimeter glass and its standard error.
U48	the initial $^{234}\text{U}/^{238}\text{U}$ -activity ratio (only used if method= 'U238-Pb206', 'U238-Pb206' or 'Pb207-Pb206').
Th0U8	the initial $^{230}\text{Th}/^{238}\text{U}$ -activity ratio (only used if method= 'U238-Pb206', 'U238-Pb206' or 'Pb207-Pb206').
Ra6U8	the initial $^{226}\text{Ra}/^{238}\text{U}$ -activity ratio (only used if method= 'U238-Pb206', 'U238-Pb206' or 'Pb207-Pb206').
Pa1U5	the initial $^{231}\text{Pa}/^{235}\text{U}$ -activity ratio (only used if method= 'U238-Pb206', 'U238-Pb206' or 'Pb207-Pb206').
type	scalar flag indicating whether <ol style="list-style-type: none"> <li>1: each U-Pb analysis should be considered separately,</li> <li>2: all the measurements should be combined to calculate a concordia age,</li> <li>3: a discordia line should be fitted through all the U-Pb analyses using the maximum likelihood algorithm of Ludwig (1998), which assumes that the scatter of the data is solely due to the analytical uncertainties.</li> <li>4: a discordia line should be fitted ignoring the analytical uncertainties.</li> <li>5: a discordia line should be fitted using a modified maximum likelihood algorithm that accounts for overdispersion by adding a geological (co)variance term.</li> </ol>
wetherill	logical flag to indicate whether the data should be evaluated in Wetherill (TRUE) or Tera-Wasserburg (FALSE) space. This option is only used when type=2
i	(optional) index of a particular aliquot

sigdig	number of significant digits for the uncertainty estimate (only used if type=1, isochron=FALSE and central=FALSE).
common.Pb	apply a common lead correction using one of three methods: 1: use the Stacey-Kramer two-stage model to infer the initial Pb-composition 2: use the isochron intercept as the initial Pb-composition 3: use the Pb-composition stored in settings('iratio', 'Pb206Pb204') and settings('iratio', 'Pb207Pb204')
show.p	Show the p-value for concordance for each aliquot to the output table. Note: it would be unwise to use the p-value value as a concordance filter. Doing so would 'punish' high precision measurements, which are more likely to fail the Chi-square test than low precision measurements. The latter would therefore be 'rewarded' by such a criterion.
isochron	logical flag indicating whether each Ar-Ar analysis should be considered separately (isochron=FALSE) or an isochron age should be calculated from all Ar-Ar analyses together (isochron=TRUE).
i2i	'isochron to intercept': calculates the initial (aka 'inherited', 'excess', or 'common') $^{40}\text{Ar}/^{36}\text{Ar}$ , $^{40}\text{Ca}/^{44}\text{Ca}$ , $^{207}\text{Pb}/^{204}\text{Pb}$ , $^{87}\text{Sr}/^{86}\text{Sr}$ , $^{143}\text{Nd}/^{144}\text{Nd}$ , $^{187}\text{Os}/^{188}\text{Os}$ or $^{176}\text{Hf}/^{177}\text{Hf}$ ratio from an isochron fit. Setting i2i to FALSE uses the default values stored in settings('iratio', ...). When applied to data of class ThU, setting i2i to TRUE applies a detrital Th-correction.
central	logical flag indicating whether each analysis should be considered separately (central=FALSE) or a central age should be calculated from all analyses together (central=TRUE).
detritus	detrital $^{230}\text{Th}$ correction (only applicable when x\$format = 1 or 2). 0: no correction 1: project the data along an isochron fit 2: correct the data using an assumed initial $^{230}\text{Th}/^{232}\text{Th}$ -ratio for the detritus. 3: correct the data using the measured present day $^{230}\text{Th}/^{238}\text{U}$ , $^{232}\text{Th}/^{238}\text{U}$ and $^{234}\text{U}/^{238}\text{U}$ -ratios in the detritus.

## Value

1. if x is a scalar or a vector, returns the age using the geochronometer given by method and its standard error.
2. if x has class UPb and type=1, returns a table with the following columns: t.75, err[t.75], t.68, err[t.68], t.76, err[t.76], t.conc, err[t.conc], err[p.conc], containing the  $^{207}\text{Pb}/^{235}\text{U}$ -age and standard error, the  $^{206}\text{Pb}/^{238}\text{U}$ -age and standard error, the  $^{207}\text{Pb}/^{206}\text{Pb}$ -age and standard error, the single grain concordia age and standard error, and the p-value for concordance, respectively.
3. if x has class UPb and type=2, 3, 4 or 5, returns the output of the [concordia](#) function.
4. if x has class PbPb, ArAr, KCa, RbSr, SmNd, ReOs, LuHf, ThU or UThHe and isochron=FALSE, returns a table of Pb-Pb, Ar-Ar, K-Ca, Rb-Sr, Sm-Nd, Re-Os, Lu-Hf, Th-U or U-Th-He ages and their standard errors.
5. if x has class ThU and isochron=FALSE, returns a 5-column table with the Th-U ages, their standard errors, the initial  $^{234}\text{U}/^{238}\text{U}$ -ratios, their standard errors, and the correlation coefficient between the ages and the initial ratios.

6. if x has class PbPb, ArAr, KCa, RbSr, SmNd, ReOs, LuHf, UThHe or ThU and isochron=TRUE, returns the output of the [isochron](#) function.
7. if x has class fissiontracks and central=FALSE, returns a table of fission track ages and standard errors.
8. if x has class fissiontracks or UThHe and central=TRUE, returns the output of the [central](#) function.

### See Also

[concordia](#), [isochron](#), [central](#)

### Examples

```
data(examples)
tUPb <- age(examples$UPb,type=1)
tconc <- age(examples$UPb,type=2)
tdisc <- age(examples$UPb,type=3)
tArAr <- age(examples$ArAr)
tiso <- age(examples$ArAr,isochron=TRUE,i2i=TRUE)
tcentral <- age(examples$FT1,central=TRUE)
```

---

agespectrum

*Plot a (40Ar/39Ar) release spectrum*

---

### Description

Produces a plot of boxes whose widths correspond to the cumulative amount of  $^{39}\text{Ar}$  (or any other variable), and whose heights express the analytical uncertainties. Only propagates the analytical uncertainty associated with decay constants and J-factors *after* computing the plateau composition.

### Usage

```
agespectrum(x, ...)
```

```
## Default S3 method:
```

```
agespectrum(x, alpha = 0.05, plateau = TRUE,
  random.effects = TRUE, levels = NA, clabel = "",
  plateau.col = c("#00FF0080", "#FF000080"),
  non.plateau.col = "#00FFFF80", sigdig = 2, line.col = "red",
  lwd = 2, xlab = "cumulative fraction", ylab = "age [Ma]",
  hide = NULL, ...)
```

```
## S3 method for class 'ArAr'
```

```
agespectrum(x, alpha = 0.05, plateau = TRUE,
  random.effects = TRUE, levels = NA, clabel = "",
  plateau.col = c("#00FF0080", "#FF000080"),
  non.plateau.col = "#00FFFF80", sigdig = 2, exterr = TRUE,
  line.col = "red", lwd = 2, i2i = FALSE, hide = NULL, ...)
```

**Arguments**

<code>x</code>	a three-column matrix whose first column gives the amount of $^{39}\text{Ar}$ in each aliquot, and whose second and third columns give the age and its uncertainty. OR an object of class <code>ArAr</code>
<code>...</code>	optional parameters to the generic plot function
<code>alpha</code>	the confidence level of the error bars/boxes and confidence intervals.
<code>plateau</code>	logical flag indicating whether a plateau age should be calculated. If <code>plateau=TRUE</code> , the function will compute the weighted mean of the largest succession of steps that pass the Chi-square test for age homogeneity. If <code>TRUE</code> , returns a list with plateau parameters.
<code>random.effects</code>	if <code>TRUE</code> , computes the weighted mean using a random effects model with two parameters: the mean and the dispersion. This is akin to a ‘model-3’ isochron regression. if <code>FALSE</code> , attributes any excess dispersion to an underestimation of the analytical uncertainties. This akin to a ‘model-1’ isochron regression.
<code>levels</code>	a vector with additional values to be displayed as different background colours of the plot symbols.
<code>clabel</code>	label of the colour legend
<code>plateau.col</code>	a vector of two fill colours of the rectangles used to mark the steps belonging to the age plateau. If <code>levels=NA</code> , then only the first colour is used. If <code>levels</code> is a vector of numbers, then <code>bg</code> is used to construct a colour ramp.
<code>non.plateau.col</code>	if <code>plateau=TRUE</code> , the steps that do NOT belong to the plateau are given a different colour.
<code>sigdig</code>	the number of significant digits of the numerical values reported in the title of the graphical output.
<code>line.col</code>	colour of the average age line
<code>lwd</code>	width of the average age line
<code>xlab</code>	x-axis label
<code>ylab</code>	y-axis label
<code>hide</code>	vector with indices of aliquots that should be removed from the plot.
<code>exterr</code>	propagate the external (decay constant and calibration factor) uncertainties?
<code>i2i</code>	‘isochron to intercept’: calculates the initial (aka ‘inherited’, ‘excess’, or ‘common’) $^{40}\text{Ar}/^{36}\text{Ar}$ ratio from an isochron fit. Setting <code>i2i</code> to <code>FALSE</code> uses the default values stored in <code>settings('iratio', ...)</code>

**Details**

IsoplotR defines the ‘plateau age’ as the weighted mean age of the longest sequence (in terms of cumulative  $^{39}\text{Ar}$  content) of consecutive heating steps that pass the modified Chauvenet criterion (see [weightedmean](#)). Note that this definition is different (and simpler) than the one used by Isoplot (Ludwig, 2003). However, it is important to mention that all definitions of an age plateau are heuristic by nature and should not be used for quantitative inference.

**Value**

If plateau=TRUE, returns a list with the following items:

**mean** a 3-element vector with:

x: the plateau mean

s[x]: the standard error of x

ci[x]: the width of a  $100(1-\alpha)\%$  confidence interval of t

**disp** a 3-element vector with:

w: the overdispersion, i.e. the standard deviation of the Normal distribution that is assumed to describe the true ages.

l1: the width of the lower half of a  $100(1-\alpha)\%$  confidence interval for the overdispersion

u1: the width of the upper half of a  $100(1-\alpha)\%$  confidence interval for the overdispersion

**df** the degrees of freedom for the weighted mean plateau fit

**mswd** the mean square of the weighted deviates of the plateau

**p.value** the p-value of a Chi-square test with  $df = n - 2$  degrees of freedom, where  $n$  is the number of steps in the plateau and 2 degrees of freedom have been removed to estimate the mean and the dispersion.

**fract** the fraction of  $^{39}\text{Ar}$  contained in the plateau

**plotpar** plot parameters for the weighted mean (see [weightedmean](#))

**i** indices of the steps that are retained for the plateau age calculation

**See Also**

[weightedmean](#)

**Examples**

```
data(examples)
agespectrum(examples$ArAr, ylim=c(0, 80))
```

---

cad

---

*Plot continuous data as cumulative age distributions*


---

**Description**

Plot a dataset as a Cumulative Age Distribution (CAD), also known as a ‘empirical cumulative distribution function’.



**Usage**

```

cad(x, ...)

## Default S3 method:
cad(x, pch = NA, verticals = TRUE,
     xlab = "age [Ma]", colmap = "heat.colors", col = "black",
     hide = NULL, ...)

## S3 method for class 'detritals'
cad(x, pch = NA, verticals = TRUE,
     xlab = "age [Ma]", colmap = "heat.colors", hide = NULL, ...)

## S3 method for class 'UPb'
cad(x, pch = NA, verticals = TRUE, xlab = "age [Ma]",
     col = "black", type = 4, cutoff.76 = 1100, cutoff.disc = c(-15,
     5), common.Pb = 0, hide = NULL, ...)

## S3 method for class 'PbPb'
cad(x, pch = NA, verticals = TRUE, xlab = "age [Ma]",
     col = "black", common.Pb = 1, hide = NULL, ...)

## S3 method for class 'ArAr'
cad(x, pch = NA, verticals = TRUE, xlab = "age [Ma]",
     col = "black", i2i = FALSE, hide = NULL, ...)

## S3 method for class 'KCa'
cad(x, pch = NA, verticals = TRUE, xlab = "age [Ma]",
     col = "black", i2i = FALSE, hide = NULL, ...)

## S3 method for class 'ThU'
cad(x, pch = NA, verticals = TRUE, xlab = "age [ka]",
     col = "black", i2i = FALSE, detritus = 0, hide = NULL, ...)

## S3 method for class 'ReOs'
cad(x, pch = NA, verticals = TRUE, xlab = "age [Ma]",
     col = "black", i2i = TRUE, hide = NULL, ...)

## S3 method for class 'SmNd'
cad(x, pch = NA, verticals = TRUE, xlab = "age [Ma]",
     col = "black", i2i = TRUE, hide = NULL, ...)

## S3 method for class 'RbSr'
cad(x, pch = NA, verticals = TRUE, xlab = "age [Ma]",
     col = "black", i2i = TRUE, hide = NULL, ...)

## S3 method for class 'LuHf'
cad(x, pch = NA, verticals = TRUE, xlab = "age [Ma]",
     col = "black", i2i = TRUE, hide = NULL, ...)

```

```
## S3 method for class 'UThHe'
cad(x, pch = NA, verticals = TRUE, xlab = "age [Ma]",
    col = "black", hide = NULL, ...)

## S3 method for class 'fissiontracks'
cad(x, pch = NA, verticals = TRUE,
    xlab = "age [Ma]", col = "black", hide = NULL, ...)
```

## Arguments

x	a numerical vector OR an object of class UPb, PbPb, ArAr, KCa, UThHe, fissiontracks, ReOs, RbSr, SmNd, LuHf, ThU or detritals
...	optional arguments to the generic plot function
pch	plot character to mark the beginning of each CAD step
verticals	logical flag indicating if the horizontal lines of the CAD should be connected by vertical lines
xlab	x-axis label
colmap	an optional string with the name of one of R's built-in colour palettes (e.g., heat.colors, terrain.colors, topo.colors, cm.colors), which are to be used for plotting data of class detritals.
col	colour to give to single sample datasets (not applicable if x has class detritals)
hide	vector with indices of aliquots that should be removed from the plot.
type	scalar indicating whether to plot the $^{207}\text{Pb}/^{235}\text{U}$ age (type=1), the $^{206}\text{Pb}/^{238}\text{U}$ age (type=2), the $^{207}\text{Pb}/^{206}\text{Pb}$ age (type=3), the $^{207}\text{Pb}/^{206}\text{Pb}$ - $^{206}\text{Pb}/^{238}\text{U}$ age (type=4), or the (Wetherill) concordia age (type=5)
cutoff.76	the age (in Ma) below which the $^{206}\text{Pb}/^{238}\text{U}$ -age and above which the $^{207}\text{Pb}/^{206}\text{Pb}$ -age is used. This parameter is only used if type=4.
cutoff.disc	two element vector with the maximum and minimum percentage discordance allowed between the $^{207}\text{Pb}/^{235}\text{U}$ and $^{206}\text{Pb}/^{238}\text{U}$ age (if $^{206}\text{Pb}/^{238}\text{U} < \text{cutoff.76}$ ) or between the $^{206}\text{Pb}/^{238}\text{U}$ and $^{207}\text{Pb}/^{206}\text{Pb}$ age (if $^{206}\text{Pb}/^{238}\text{U} > \text{cutoff.76}$ ). Set cutoff.disc=NA if you do not want to use this filter.
common.Pb	apply a common lead correction using one of three methods: 1: use the isochron intercept as the initial Pb-composition 2: use the Stacey-Kramer two-stage model to infer the initial Pb-composition 3: use the Pb-composition stored in settings('iratio', 'Pb206Pb204') and settings('iratio', 'Pb207Pb204')
i2i	'isochron to intercept': calculates the initial (aka 'inherited', 'excess', or 'common') $^{40}\text{Ar}/^{36}\text{Ar}$ , $^{40}\text{Ca}/^{44}\text{Ca}$ , $^{207}\text{Pb}/^{204}\text{Pb}$ , $^{87}\text{Sr}/^{86}\text{Sr}$ , $^{143}\text{Nd}/^{144}\text{Nd}$ , $^{187}\text{Os}/^{188}\text{Os}$ , $^{230}\text{Th}/^{232}\text{Th}$ or $^{176}\text{Hf}/^{177}\text{Hf}$ ratio from an isochron fit. Setting i2i to FALSE uses the default values stored in settings('iratio', ...) or zero (for the Pb-Pb method). When applied to data of class ThU, setting i2i to TRUE applies a detrital Th-correction.

detritus      detrital  $^{230}\text{Th}$  correction (only applicable when `x$format == 1` or `2`).  
                  0: no correction  
                  1: project the data along an isochron fit  
                  2: correct the data using an assumed initial  $^{230}\text{Th}/^{232}\text{Th}$ -ratio for the detritus.  
                  3: correct the data using the measured present day  $^{230}\text{Th}/^{238}\text{U}$ ,  $^{232}\text{Th}/^{238}\text{U}$  and  $^{234}\text{U}/^{238}\text{U}$ -ratios in the detritus.

## Details

Empirical cumulative distribution functions or cumulative age distributions CADs are the most straightforward way to visualise the probability distribution of multiple dates. Suppose that we have a set of  $n$  dates  $t_i$ . The CAD is a step function that sets out the rank order of the dates against their numerical value:

$$CAD(t) = \sum_i 1(t < t_i)/n$$

where  $1(*) = 1$  if  $*$  is true and  $1(*) = 0$  if  $*$  is false. CADs have two desirable properties (Vermeesch, 2007). First, they do not require any pre-treatment or smoothing of the data. This is not the case for histograms or kernel density estimates. Second, it is easy to superimpose several CADs on the same plot. This facilitates the intercomparison of multiple samples. The interpretation of CADs is straightforward but not very intuitive. The prominence of individual age components is proportional to the steepness of the CAD. This is different from probability density estimates such as histograms, in which such components stand out as peaks.

## References

Vermeesch, P., 2007. Quantitative geomorphology of the White Mountains (California) using detrital apatite fission track thermochronology. *Journal of Geophysical Research: Earth Surface*, 112(F3).

## See Also

[kde](#), [radialplot](#)

## Examples

```
data(examples)
cad(examples$DZ, verticals=FALSE, pch=20)
```

---

central

*Calculate U-Th-He and fission track central ages and compositions*

---

## Description

Computes the geometric mean composition of a continuous mixture of fission track or U-Th-He data and returns the corresponding age and fitting parameters.

**Usage**

```
central(x, ...)

## Default S3 method:
central(x, alpha = 0.05, ...)

## S3 method for class 'UThHe'
central(x, alpha = 0.05, model = 1, ...)

## S3 method for class 'fissiontracks'
central(x, mineral = NA, alpha = 0.05, ...)
```

**Arguments**

x	an object of class UThHe or fissiontracks, OR a 2-column matrix with (strictly positive) values and uncertainties
...	optional arguments
alpha	cutoff value for confidence intervals
model	choose one of the following statistical models: 1: weighted mean. This model assumes that the scatter between the data points is solely caused by the analytical uncertainty. If the assumption is correct, then the MSWD value should be approximately equal to one. There are three strategies to deal with the case where $MSWD > 1$ . The first of these is to assume that the analytical uncertainties have been underestimated by a factor $\sqrt{MSWD}$ . 2: unweighted mean. A second way to deal with over- or underdispersed datasets is to simply ignore the analytical uncertainties. 3: weighted mean with overdispersion: instead of attributing any overdispersion ( $MSWD > 1$ ) to underestimated analytical uncertainties (model 1), one could also attribute it to the presence of geological uncertainty, which manifests itself as an added (co)variance term.
mineral	setting this parameter to either apatite or zircon changes the default efficiency factor, initial fission track length and density to preset values (only affects results if <code>x\$format=2</code> )

**Details**

The central age assumes that the observed age distribution is the combination of two sources of scatter: analytical uncertainty and true geological dispersion.

1. For fission track data, the analytical uncertainty is assumed to obey Poisson counting statistics and the geological dispersion is assumed to follow a lognormal distribution.
2. For U-Th-He data, the U-Th-(Sm)-He compositions and uncertainties are assumed to follow a logistic normal distribution.
3. For all other data types, both the analytical uncertainties and the true ages are assumed to follow lognormal distributions.

The difference between the central age and the weighted mean age is usually small unless the data are imprecise and/or strongly overdispersed.

**Value**

If `x` has class `UThHe`, returns a list containing the following items:

**uvw** (if the input data table contains `Sm`) or **uv** (if it does not): the mean  $\log[\text{U/He}]$ ,  $\log[\text{Th/He}]$  (, and  $\log[\text{Sm/He}]$ ) composition.

**covmat** the covariance matrix of `uvw` or `uv`.

**mswd** the reduced Chi-square statistic of data concordance, i.e.  $mswd = SS/df$ , where  $SS$  is the sum of squares of the  $\log[\text{U/He}]$ - $\log[\text{Th/He}]$  compositions.

**model** the fitting model.

**df** the degrees of freedom ( $2n - 2$ ) of the fit (only reported if `model=1`).

**p.value** the p-value of a Chi-square test with `df` degrees of freedom (only reported if `model=1`.)

**age** a three- or four-element vector with:  
     **t**: the central age.  
     **s[t]**: the standard error of **t**.  
     **ci[t]**: the width of a  $100(1 - \alpha)\%$  confidence interval for **t**.  
     **disp[t]**: the studentised  $100(1 - \alpha)\%$  confidence interval enhanced by a factor of  $\sqrt{mswd}$  (only reported if `model=1`).

**w** the geological overdispersion term. If `model=3`, this is a three-element vector with the standard deviation of the (assumedly) Normal dispersion and the lower and upper half-widths of its  $100(1 - \alpha)\%$  confidence interval. `w=0` if `code-model<3`.

OR, otherwise:

**age** a three-element vector with:

**t**: the central age.

**s[t]**: the standard error of **t**.

**ci[t]**: the width of a  $100(1 - \alpha)\%$  confidence interval for **t**.

**disp** a three-element vector with the overdispersion (standard deviation) of the excess scatter, and the upper and lower half-widths of its  $100(1 - \alpha)\%$  confidence interval.

**mswd** the reduced Chi-square statistic of data concordance, i.e.  $mswd = X^2/df$ , where  $X^2$  is a Chi-square statistic of the EDM data or ages

**df** the degrees of freedom ( $n - 2$ )

**p.value** the p-value of a Chi-square test with `df` degrees of freedom

**References**

Galbraith, R.F. and Laslett, G.M., 1993. Statistical models for mixed fission track ages. *Nuclear Tracks and Radiation Measurements*, 21(4), pp.459-470.

Vermeesch, P., 2008. Three new ways to calculate average (U-Th)/He ages. *Chemical Geology*, 249(3), pp.339-347.

**See Also**

[weightedmean](#), [radialplot](#), [helioplot](#)

## Examples

```
data(examples)
print(central(examples$UThHe)$age)
```

---

concordant

*Test whether the U-Pb compositions are concordant*

---

## Description

Assess whether U-Pb  $^{206}\text{Pb}/^{238}\text{U}$ - and  $^{207}\text{Pb}/^{206}\text{Pb}$ -ages, or the  $^{206}\text{Pb}/^{238}\text{U}$ - and  $^{207}\text{Pb}/^{235}\text{U}$  ages are concordant.

## Usage

```
concordant(x, cutoff.76 = 1100, cutoff.disc = c(-15, 5))
```

## Arguments

x	an object of class UPb
cutoff.76	the age (in Ma) below which the concordance between the $^{206}\text{Pb}/^{238}\text{U}$ and $^{207}\text{Pb}/^{238}\text{U}$ ages are tested, and above which the concordance between the $^{206}\text{Pb}/^{238}\text{U}$ and $^{207}\text{Pb}/^{206}\text{Pb}$ ages is tested.
cutoff.disc	two element vector with the minimum (negative) and maximum (positive) percentage discordance allowed between the $^{207}\text{Pb}/^{235}\text{U}$ and $^{206}\text{Pb}/^{238}\text{U}$ age (if $^{206}\text{Pb}/^{238}\text{U} < \text{cutoff.76}$ ) or between the $^{206}\text{Pb}/^{238}\text{U}$ and $^{207}\text{Pb}/^{206}\text{Pb}$ age (if $^{206}\text{Pb}/^{238}\text{U} > \text{cutoff.76}$ ).

## Details

Returns TRUE if the  $^{206}\text{Pb}/^{238}\text{U}$  and  $^{207}\text{Pb}/^{235}\text{U}$ -ages agree within the limits specified in cutoff.disc (if  $t[^{206}\text{Pb}/^{238}\text{U}] < \text{cutoff.76}$ , OR if the  $^{206}\text{Pb}/^{238}\text{U}$  and  $^{207}\text{Pb}/^{206}\text{Pb}$ -ages agree within the limits specified in cutoff.disc (if  $t[^{206}\text{Pb}/^{238}\text{U}] > \text{cutoff.76}$ ). Returns FALSE otherwise.

## Value

A vector of boolean flags marking the concordant aliquots.

## Examples

```
data(examples)
conc <- concordant(examples$UPb, cutoff.disc=c(-1,1))
radialplot(examples$UPb, omit=which(!conc))
```

concordia

*Concordia diagram***Description**

Plots U-Pb data on Wetherill and Tera-Wasserburg concordia diagrams, calculate concordia ages and compositions, evaluates the equivalence of multiple ( $^{206}\text{Pb}/^{238}\text{U}$ - $^{207}\text{Pb}/^{235}\text{U}$  or  $^{207}\text{Pb}/^{206}\text{Pb}$ - $^{206}\text{Pb}/^{238}\text{U}$ ) compositions, computes the weighted mean isotopic composition and the corresponding concordia age using the method of maximum likelihood, computes the MSWD of equivalence and concordance and their respective Chi-squared p-values. Performs linear regression and computes the upper and lower intercept ages (for Wetherill) or the lower intercept age and the  $^{207}\text{Pb}/^{206}\text{Pb}$  intercept (for Tera-Wasserburg), taking into account error correlations and decay constant uncertainties.

**Usage**

```
concordia(x = NULL, tlim = NULL, alpha = 0.05, wetherill = TRUE,
  show.numbers = FALSE, levels = NA, clabel = "",
  ellipse.col = c("#00FF0080", "#FF000080"),
  concordia.col = "darksalmon", exterr = FALSE, show.age = 0,
  sigdig = 2, common.Pb = 0, ticks = 5, anchor = list(FALSE, NA),
  hide = NULL, omit = NULL, omit.col = NA, ...)
```

**Arguments**

x	an object of class UPb
tlim	age limits of the concordia line
alpha	probability cutoff for the error ellipses and confidence intervals
wetherill	logical flag (FALSE for Tera-Wasserburg)
show.numbers	logical flag (TRUE to show grain numbers)
levels	a vector with length(x) values to be displayed as different background colours within the error ellipses.
clabel	label for the colour legend (only used if levels is not NA.
ellipse.col	a vector of two background colours for the error ellipses. If levels=NA, then only the first colour is used. If levels is a vector of numbers, then ellipse.col is used to construct a colour ramp.
concordia.col	colour of the concordia line
exterr	show decay constant uncertainty?
show.age	one of either: 0: plot the data without calculating an age 1: fit a concordia composition and age 2: fit a discordia line through the data using the maximum likelihood algorithm of Ludwig (1998), which assumes that the scatter of the data is solely due to the

	<p>analytical uncertainties. In this case, IsoplotR will either calculate an upper and lower intercept age (for Wetherill concordia), or a lower intercept age and common (<math>^{207}\text{Pb}/^{206}\text{Pb}</math>)-ratio intercept (for Tera-Wasserburg). If <math>\text{mswd} &gt; 0</math>, then the analytical uncertainties are augmented by a factor <math>\sqrt{\text{mswd}}</math>.</p> <p>3: fit a discordia line ignoring the analytical uncertainties</p> <p>4: fit a discordia line using a modified maximum likelihood algorithm that includes accounts for any overdispersion by adding a geological (co)variance term.</p>
sigdig	number of significant digits for the concordia/discordia age
common.Pb	<p>apply a common lead correction using one of three methods:</p> <p>1: use the Stacey-Kramers two-stage model to infer the initial Pb-composition</p> <p>2: use the isochron intercept as the initial Pb-composition</p> <p>3: use the Pb-composition stored in <code>settings('iratio', 'Pb207Pb206')</code> (if <code>x\$format &lt; 4</code>) or <code>settings('iratio', 'Pb206Pb204')</code> and <code>settings('iratio', 'Pb207Pb204')</code> (if <code>x\$format &gt; 3</code>)</p>
ticks	either a scalar indicating the desired number of age ticks to be placed along the concordia line, OR a vector of tick ages.
anchor	<p>control parameters to fix the intercept age or common Pb composition of the discordia fit. This is a two-element list.</p> <ul style="list-style-type: none"> <li>• The first element is a boolean flag indicating whether the discordia line should be anchored. If this is FALSE, then the second item is ignored and both the common Pb composition and age are estimated.</li> <li>• If the first element is TRUE and the second element is NA, then the common Pb composition is fixed at the values stored in <code>settings('iratio', ...)</code>. item If the first element is TRUE and the second element is a number, then the discordia line is forced to intersect the concordia line at an age equal to that number.</li> </ul>
hide	vector with indices of aliquots that should be removed from the concordia diagram
omit	vector with indices of aliquots that should be plotted but omitted from concordia or discordia age calculation
omit.col	colour that should be used for the omitted aliquots.
...	optional arguments to the generic plot function

## Details

The concordia diagram is a graphical means of assessing the internal consistency of U-Pb data. It sets out the measured  $^{206}\text{Pb}/^{238}\text{U}$ - and  $^{207}\text{Pb}/^{235}\text{U}$ -ratios against each other ('Wetherill' diagram) or, equivalently, the  $^{207}\text{Pb}/^{206}\text{Pb}$ - and  $^{206}\text{Pb}/^{238}\text{U}$ -ratios ('Tera-Wasserburg' diagram). The space of concordant isotopic compositions is marked by a curve, the 'concordia line'. Isotopic ratio measurements are shown as  $100(1-\alpha)\%$  confidence ellipses. Concordant samples plot near to, or overlap with, the concordia line. They represent the pinnacle of geochronological robustness. Samples that plot away from the concordia line but are aligned along a linear trend form an isochron (or 'discordia' line) that can be used to infer the composition of the non-radiogenic ('common') lead or to constrain the timing of prior lead loss.



**Value**

if `show.age=1`, returns a list with the following items:

**x** a named vector with the (weighted mean) U-Pb composition

**cov** the covariance matrix of the (weighted mean) U-Pb composition

**mswd** a vector with three items (equivalence, concordance and combined) containing the MSWD (Mean of the Squared Weighted Deviates, a.k.a the reduced Chi-squared statistic) of isotopic equivalence, age concordance and combined goodness of fit, respectively.

**p.value** a vector with three items (equivalence, concordance and combined) containing the p-value of the Chi-square test for isotopic equivalence, age concordance and combined goodness of fit, respectively.

**df** a three-element vector with the number of degrees of freedom used for the `mswd` calculation. These values are useful when expanding the analytical uncertainties if `mswd`>1.

**age** a 4-element vector with:

`t`: the concordia age (in Ma)

`s[t]`: the estimated uncertainty of `t`

`ci[t]`: the studentised  $100(1 - \alpha)\%$  confidence interval of `t` for the appropriate degrees of freedom

`disp[t]`: the studentised  $100(1 - \alpha)\%$  confidence interval for `t` augmented by  $\sqrt{mswd}$  to account for overdispersed datasets.

if `show.age=2, 3 or 4`, returns a list with the following items:

**model** the fitting model (=show.age-1).

**x** a two-element vector with the upper and lower intercept ages (if `wetherill=TRUE`) or the lower intercept age and  $^{207}\text{Pb}/^{206}\text{Pb}$  intercept (if `wetherill=FALSE`).

**cov** the covariance matrix of the elements in `x`.

**err** a [2 x 2] or [3 x 2] matrix with the following rows:

`s`: the estimated standard deviation for `x`

`ci`: the studentised  $100(1 - \alpha)\%$  confidence interval of `x` for the appropriate degrees of freedom

`disp[t]`: the studentised  $100(1 - \alpha)\%$  confidence interval for `x` augmented by  $\sqrt{mswd}$  to account for overdispersed datasets (only reported if `show.age=2`).

**df** the degrees of freedom of the concordia fit (concordance + equivalence)

**p.value** p-value of a Chi-square test for age homogeneity (only reported if `type=3`).

**mswd** mean square of the weighted deviates – a goodness-of-fit measure. `mswd > 1` indicates overdispersion w.r.t the analytical uncertainties (not reported if `show.age=3`).

**w** three-element vector with the standard deviation of the (assumedly) Normal overdispersion term and the lower and upper half-widths of its  $100(1 - \alpha)\%$  confidence interval (only important if `show.age=4`).

**n** the number of aliquots in the dataset

## References

Ludwig, K.R., 1998. On the treatment of concordant uranium-lead ages. *Geochimica et Cosmochimica Acta*, 62(4), pp.665-676.

## Examples

```
data(examples)
concordia(examples$UPb, show.age=2)

dev.new()
concordia(examples$UPb, wetherill=FALSE,
          xlim=c(24.9, 25.4), ylim=c(0.0508, 0.0518),
          ticks=249:254, exterr=TRUE)

dev.new()
concordia(examples$UPb, wetherill=FALSE, show.age=2, anchor=list(TRUE, 0))
```

---

data2york

*Prepare geochronological data for York regression*

---

## Description

Takes geochronology data as input and produces a five-column table as output, which can be used for York regression.

## Usage

```
data2york(x, ...)
```

## Default S3 method:

```
data2york(x, format = 1, ...)
```

## S3 method for class 'UPb'

```
data2york(x, option = 1, ...)
```

## S3 method for class 'ArAr'

```
data2york(x, inverse = TRUE, ...)
```

## S3 method for class 'KCa'

```
data2york(x, inverse = FALSE, ...)
```

## S3 method for class 'PbPb'

```
data2york(x, inverse = TRUE, ...)
```

## S3 method for class 'PD'

```
data2york(x, exterr = FALSE, inverse = FALSE, ...)
```

```
## S3 method for class 'UThHe'
data2york(x, ...)

## S3 method for class 'ThU'
data2york(x, type = 2, generic = TRUE, ...)
```

### Arguments

<code>x</code>	a five or six column matrix OR an object of class UPb, PbPb, ArAr, ThU, UThHe, or PD (which includes objects of class RbSr, SmNd, LuHf and ReOs), generated by the <code>read.data(...)</code> function
<code>...</code>	optional arguments
<code>format</code>	one of <ol style="list-style-type: none"> <li>1. <math>X</math>, <math>s[X]</math>, <math>Y</math>, <math>s[Y]</math>, <math>\rho</math> where <math>\rho</math> is the error correlation between <math>X</math> and <math>Y</math>, or</li> <li>2. <math>X/Z</math>, <math>s[X/Z]</math>, <math>Y/Z</math>, <math>s[Y/Z]</math>, <math>X/Y</math>, <math>s[X/Y]</math> for which the error correlations are automatically computed from the redundancy of the three ratios.</li> </ol>
<code>option</code>	one of <ol style="list-style-type: none"> <li>1. Wetherill concordia ratios: <math>X=7/5</math>, <math>sX=s[7/5]</math>, <math>Y=6/8</math>, <math>sY=s[6/8]</math>, <math>rXY</math>.</li> <li>2. Tera-Wasserburg ratios: <math>X=8/6</math>, <math>sX=s[8/6]</math>, <math>Y=7/6</math>, <math>sY=s[7/6]</math>, <math>\rho=rXY</math>.</li> <li>3. <math>X=4/6</math>, <math>sX=s[4/6]</math>, <math>Y=8/6</math>, <math>sY=s[8/6]</math>, <math>\rho=rXY</math> (only valid if <code>format &gt; 3</code>).</li> <li>4. <math>X=4/7</math>, <math>sX=s[4/7]</math>, <math>Y=5/7</math>, <math>sY=s[5/7]</math>, <math>\rho=rXY</math> (only valid if <code>format &gt; 3</code>).</li> </ol>
<code>inverse</code>	<p>If <code>x</code> has class ArAr and <code>inverse=FALSE</code>, returns a table with columns <math>X=9/6</math>, <math>sX=s[9/6]</math>, <math>Y=0/6</math>, <math>codesY=s[0/6]</math>, <math>rXY</math></p> <p>If <code>x</code> has class ArAr and <code>inverse=TRUE</code>, returns a table with columns <math>X=9/0</math>, <math>sX=s[9/0]</math>, <math>Y=6/0</math>, <math>codesY=s[6/0]</math>, <math>rXY</math></p> <p>If <code>x</code> has class PbPb and <code>inverse=FALSE</code>, returns a table with columns <math>X=6/4</math>, <math>sX=s[6/4]</math>, <math>Y=7/4</math>, <math>codesY=s[7/4]</math>, <math>rXY</math></p> <p>If <code>x</code> has class PbPb and <code>inverse=TRUE</code>, returns a table with columns <math>X=4/6</math>, <math>sX=s[4/6]</math>, <math>Y=7/6</math>, <math>sY=s[7/6]</math>, <math>rXY</math></p>
<code>exterr</code>	If TRUE, propagates the external uncertainties (e.g. decay constants) into the output errors.
<code>type</code>	<p>Return 'Rosholt' or 'Osmond' ratios?</p> <p>Rosholt (<code>type=1</code>) returns <math>X=8/2</math>, <math>sX=s[8/2]</math>, <math>Y=0/2</math>, <math>sY=s[0/2]</math>, <math>rXY</math>.</p> <p>Osmond (<code>type=2</code>) returns <math>X=2/8</math>, <math>sX=s[2/8]</math>, <math>Y=0/8</math>, <math>sY=s[0/8]</math>, <math>rXY</math>.</p>
<code>generic</code>	<p>If TRUE, uses the following column headers: <math>X</math>, <math>sX</math>, <math>Y</math>, <math>sY</math>, <math>rXY</math>.</p> <p>If FALSE and <code>type=1</code>, uses U238Th232, errU238Th232, Th230Th232, errTh230Th232, <math>\rho</math></p> <p>or if FALSE and <code>type=2</code>, uses Th232U238, errTh232U238, Th230U238, errTh230U238, <math>\rho</math>.</p>

### Value

a five-column table that can be used as input for [york](#)-regression.

**See Also**[york](#)**Examples**

```
f <- system.file("RbSr1.csv", package="IsoplotR")
dat <- read.csv(f)
yorkdat <- data2york(dat)
fit <- york(yorkdat)
```

ellipse

*Get coordinates of error ellipse for plotting***Description**

Constructs an error ellipse at a given confidence level from its centre and covariance matrix

**Usage**

```
ellipse(x, y, covmat, alpha = 0.05, n = 50)
```

**Arguments**

x	x-coordinate (scalar) for the centre of the ellipse
y	y-coordinate (scalar) for the centre of the ellipse
covmat	the [2 x 2] covariance matrix of the x-y coordinates
alpha	the probability cutoff for the error ellipses
n	the resolution (number of segments) of the error ellipses

**Value**

an [n x 2] matrix of plot coordinates

**Examples**

```
x = 99; y = 101;
covmat <- matrix(c(1,0.9,0.9,1),nrow=2)
ell <- ellipse(x,y,covmat)
plot(c(90,110),c(90,110),type='l')
polygon(ell,col=rgb(0,1,0,0.5))
points(x,y,pch=21,bg='black')
```

evolution

*Th-U evolution diagram***Description**

Plots Th-U data on a  $^{234}\text{U}/^{238}\text{U}$ - $^{230}\text{Th}/^{238}\text{U}$  evolution diagram, a  $^{234}\text{U}/^{238}\text{U}$ -age diagram, or (if  $^{234}\text{U}/^{238}\text{U}$  is assumed to be in secular equilibrium), a  $^{230}\text{Th}/^{232}\text{Th}$ - $^{238}\text{U}/^{232}\text{Th}$  diagram, calculates isochron ages.

**Usage**

```
evolution(x, xlim = NA, ylim = NA, alpha = 0.05, transform = FALSE,
  detritus = 0, show.numbers = FALSE, levels = NA, clabel = "",
  ellipse.col = c("#00FF0080", "#FF000080"), line.col = "darksalmon",
  isochron = FALSE, model = 1, exterr = TRUE, sigdig = 2,
  hide = NULL, omit = NULL, omit.col = NA, ...)
```

**Arguments**

x	an object of class ThU
xlim	x-axis limits
ylim	y-axis limits
alpha	probability cutoff for the error ellipses and confidence intervals
transform	if TRUE, plots $^{234}\text{U}/^{238}\text{U}$ vs. Th-U age.
detritus	detrital $^{230}\text{Th}$ correction (only applicable when x\$format is 2 or 3. 0: no correction 1: project the data along an isochron fit 2: correct the data using an assumed initial $^{230}\text{Th}/^{232}\text{Th}$ -ratio for the detritus. 3: correct the data using the measured present day $^{230}\text{Th}/^{238}\text{U}$ , $^{232}\text{Th}/^{238}\text{U}$ and $^{234}\text{U}/^{238}\text{U}$ -ratios in the detritus.
show.numbers	label the error ellipses with the grain numbers?
levels	a vector with additional values to be displayed as different background colours within the error ellipses.
clabel	label of the colour legend.
ellipse.col	a vector of two background colours for the error ellipses. If levels=NA, then only the first colour will be used. If levels is a vector of numbers, then ellipse.col is used to construct a colour ramp.
line.col	colour of the age grid
isochron	fit a 3D isochron to the data?
model	if isochron=TRUE, choose one of three regression models: 1: maximum likelihood regression, using either the modified error weighted least squares algorithm of York et al. (2004) for 2-dimensional data, or the

Maximum Likelihood formulation of Ludwig and Titterton (1994) for 3-dimensional data. These algorithms take into account the analytical uncertainties and error correlations, under the assumption that the scatter between the data points is solely caused by the analytical uncertainty. If this assumption is correct, then the MSWD value should be approximately equal to one. There are three strategies to deal with the case where  $MSWD > 1$ . The first of these is to assume that the analytical uncertainties have been underestimated by a factor  $\sqrt{MSWD}$ .

2: ordinary least squares regression: a second way to deal with over- or under-dispersed datasets is to simply ignore the analytical uncertainties.

3: maximum likelihood regression with overdispersion: instead of attributing any overdispersion ( $MSWD > 1$ ) to underestimated analytical uncertainties (model 1), one can also attribute it to the presence of geological uncertainty, which manifests itself as an added (co)variance term.

<code>exterr</code>	propagate the decay constant uncertainty in the isochron age?
<code>sigdig</code>	number of significant digits for the isochron age
<code>hide</code>	vector with indices of aliquots that should be removed from the plot.
<code>omit</code>	vector with indices of aliquots that should be plotted but omitted from the isochron age calculation.
<code>omit.col</code>	colour that should be used for the omitted aliquots.
<code>...</code>	optional arguments to the generic plot function

## Details

Similar to the [concordia](#) diagram (for U-Pb data) and the [helioplot](#) diagram (for U-Th-He data), the evolution diagram simultaneously displays the isotopic composition and age of U-series data. For carbonate data (Th-U formats 1 and 2), the Th-U evolution diagram consists of a scatter plot that sets out the  $^{234}\text{U}/^{238}\text{U}$ -activity ratios against the  $^{230}\text{Th}/^{238}\text{U}$ -activity ratios as error ellipses, and displays the initial  $^{234}\text{U}/^{238}\text{U}$ -activity ratios and ages as a set of intersecting lines. Alternatively, the  $^{234}\text{U}/^{238}\text{U}$ -ratios can also be set out against the  $^{230}\text{Th}$ - $^{234}\text{U}$ - $^{238}\text{U}$ -ages. In both types of evolution diagrams, IsoplotR provides the option to project the raw measurements along the best fitting isochron line and thereby remove the detrital  $^{230}\text{Th}$ -component. This procedure allows a visual assessment of the degree of homogeneity within a dataset, as is quantified by the MSWD.

Neither the U-series evolution diagram, nor the  $^{234}\text{U}/^{238}\text{U}$  vs. age plot is applicable to igneous datasets (Th-U formats 3 and 4), in which  $^{234}\text{U}$  and  $^{238}\text{U}$  are in secular equilibrium. For such datasets, IsoplotR produces an Osmond-style regression plot that is decorated with a fanning set of [isochron](#) lines.

## References

- Ludwig, K.R. and Titterton, D.M., 1994. Calculation of  $^{230}\text{Th}/\text{U}$  isochrons, ages, and errors. *Geochimica et Cosmochimica Acta*, 58(22), pp.5031-5042.
- Ludwig, K.R., 2003. Mathematical-statistical treatment of data and errors for  $^{230}\text{Th}/\text{U}$  geochronology. *Reviews in Mineralogy and Geochemistry*, 52(1), pp.631-656.

**See Also**[isochron](#)**Examples**

```
data(examples)
evolution(examples$ThU)

dev.new()
evolution(examples$ThU, transform=TRUE,
          isochron=TRUE, model=1)
```

examples

*Example datasets for testing IsoplotR***Description**

U-Pb, Pb-Pb, Ar-Ar, K-Ca, Re-Os, Sm-Nd, Rb-Sr, Lu-Hf, U-Th-He, Th-U, fission track and detrital datasets

**Details**

examples an 18-item list containing:

UPb: an object of class UPb containing a high precision U-Pb dataset of Kamo et al. (1996) packaged with Ken Ludwig (2003)'s Isoplot program.

PbPb: an object of class PbPb containing a Pb-Pb dataset from Connelly et al. (2017).

DZ: an object of class detrital containing a detrital zircon U-Pb dataset from Namibia (Vermeesch et al., 2015).

ArAr: an object of class ArAr containing a  $^{40}\text{Ar}/^{39}\text{Ar}$  spectrum of Skye basalt produced by Sarah Sherlock (Open University).

KCa: an object of class KCa containing a  $^{40}\text{K}/^{40}\text{Ca}$  dataset for sample 140025 grain h spot 5 of Harrison et al. (2010).

UThHe: an object of class UThHe containing a U-Th-Sm-He dataset of Fish Lake apatite produced by Daniel Stockli (UT Austin).

FT1: an object of class fissiontracks containing a synthetic external detector dataset.

FT2: an object of class fissiontracks containing a synthetic LA-ICP-MS-based fission track dataset using the zeta calibration method.

FT3: an object of class fissiontracks containing a synthetic LA-ICP-MS-based fission track dataset using the absolute dating approach.

ReOs: an object of class ReOs containing a  $^{187}\text{Os}/^{187}\text{Re}$ -dataset from Selby (2007).

SmNd: an object of class SmNd containing a  $^{143}\text{Nd}/^{147}\text{Sm}$ -dataset from Lugmair et al. (1975).

RbSr: an object of class RbSr containing an  $^{87}\text{Rb}/^{86}\text{Sr}$ -dataset from Compston et al. (1971).

LuHf: an object of class LuHf containing an  $^{176}\text{Lu}/^{177}\text{Hf}$ -dataset from Barfod et al. (2002).

ThU: an object of class ThU containing a synthetic ‘Osmond-type’ dataset from Titterington and Ludwig (1994).

LudwigMean: an object of class other containing a collection of  $^{206}\text{Pb}/^{238}\text{U}$ -ages and errors of the example dataset by Ludwig (2003).

LudwigKDE: an object of class 'other' containing the  $^{206}\text{Pb}/^{238}\text{U}$ -ages (but not the errors) of the example dataset by Ludwig (2003).

LudwigSpectrum: an object of class 'other' containing the  $^{39}\text{Ar}$  abundances,  $^{40}\text{Ar}/^{39}\text{Ar}$ -ages and errors of the example dataset by Ludwig (2003).

LudwigMixture: an object of class 'other' containing a dataset of dispersed zircon fission track ages of the example dataset by Ludwig (2003).

## References

- Barfod, G.H., Albarede, F., Knoll, A.H., Xiao, S., Telouk, P., Frei, R. and Baker, J., 2002. New Lu-Hf and Pb-Pb age constraints on the earliest animal fossils. *Earth and Planetary Science Letters*, 201(1), pp.203-212.
- Compston, W., Berry, H., Vernon, M.J., Chappell, B.W. and Kaye, M.J., 1971. Rubidium-strontium chronology and chemistry of lunar material from the Ocean of Storms. In *Lunar and Planetary Science Conference Proceedings* (Vol. 2, p. 1471).
- Connelly, J.N., Bollard, J. and Bizzarro, M., 2017. Pb-Pb chronometry and the early Solar System. *Geochimica et Cosmochimica Acta*, 201, pp.345-363.
- Galbraith, R. F. and Green, P. F., 1990: Estimating the component ages in a finite mixture, *Nuclear Tracks and Radiation Measurements*, 17, 197-206.
- Harrison, T.M., Heizler, M.T., McKeegan, K.D. and Schmitt, A.K., 2010. In situ  $^{40}\text{K}$ - $^{40}\text{Ca}$  ‘double-plus’ SIMS dating resolves Klokken feldspar  $^{40}\text{K}$ - $^{40}\text{Ar}$  paradox. *Earth and Planetary Science Letters*, 299(3-4), pp.426-433.
- Kamo, S.L., Czamanske, G.K. and Krogh, T.E., 1996. A minimum U-Pb age for Siberian flood-basalt volcanism. *Geochimica et Cosmochimica Acta*, 60(18), 3505-3511.
- Ludwig, K. R., and D. M. Titterington., 1994. "Calculation of  $^{230}\text{Th}/\text{U}$  isochrons, ages, and errors." *Geochimica et Cosmochimica Acta* 58.22, 5031-5042.
- Ludwig, K. R., 2003. User’s manual for Isoplot 3.00: a geochronological toolkit for Microsoft Excel. No. 4.
- Lugmair, G.W., Scheinin, N.B. and Marti, K., 1975. Sm-Nd age and history of Apollo 17 basalt 75075-Evidence for early differentiation of the lunar exterior. In *Lunar and Planetary Science Conference Proceedings* (Vol. 6, pp. 1419-1429).
- Selby, D., 2007. Direct Rhenium-Osmium age of the Oxfordian-Kimmeridgian boundary, Staffin bay, Isle of Skye, UK, and the Late Jurassic time scale. *Norsk Geologisk Tidsskrift*, 87(3), p.291.
- Vermeesch, P. and Garzanti, E., 2015. Making geological sense of ‘Big Data’ in sedimentary provenance analysis. *Chemical Geology*, 409, pp.20-27.
- Vermeesch, P., 2008. Three new ways to calculate average (U-Th)/He ages. *Chemical Geology*, 249(3), pp.339-347.



**Examples**

```

data(examples)

concordia(examples$UPb)

agespectrum(examples$ArAr)

isochron(examples$ReOs)

radialplot(examples$FT1)

helioplot(examples$UThHe)

evolution(examples$ThU)

kde(examples$DZ)

radialplot(examples$LudwigMixture)

agespectrum(examples$LudwigSpectrum)

weightedmean(examples$LudwigMean)

```

---

helioplot

---

*Visualise U-Th-He data on a logratio plot or ternary diagram*


---

**Description**

Plot U-Th(-Sm)-He data on a ( $\log[\text{He}/\text{Th}]$  vs.  $\log[\text{U}/\text{He}]$ ) logratio plot or U-Th-He ternary diagram

**Usage**

```

helioplot(x, logratio = TRUE, model = 1, show.central.comp = TRUE,
  show.numbers = FALSE, alpha = 0.05, contour.col = c("white",
    "red"), levels = NA, clabel = "", ellipse.col = c("#00FF0080",
    "#0000FF80"), sigdig = 2, xlim = NA, ylim = NA, fact = NA,
  hide = NULL, omit = NULL, omit.col = NA, ...)

```

**Arguments**

<code>x</code>	an object of class <code>UThHe</code>
<code>logratio</code>	Boolean flag indicating whether the data should be shown on bivariate $\log[\text{He}/\text{Th}]$ vs. $\log[\text{U}/\text{He}]$ diagram, or a U-Th-He ternary diagram.
<code>model</code>	choose one of the following statistical models: 1: weighted mean. This model assumes that the scatter between the data points is solely caused by the analytical uncertainty. If the assumption is correct, then the MSWD value should be approximately equal to one. There are three strategies

to deal with the case where  $MSWD > 1$ . The first of these is to assume that the analytical uncertainties have been underestimated by a factor  $\sqrt{MSWD}$ .

2: unweighted mean. A second way to deal with over- or underdispersed datasets is to simply ignore the analytical uncertainties.

3: weighted mean with overdispersion: instead of attributing any overdispersion ( $MSWD > 1$ ) to underestimated analytical uncertainties (model 1), it can also be attributed to the presence of geological uncertainty, which manifests itself as an added (co)variance term.

<code>show.central.comp</code>	show the geometric mean composition as a white ellipse?
<code>show.numbers</code>	show the grain numbers inside the error ellipses?
<code>alpha</code>	probability cutoff for the error ellipses and confidence intervals
<code>contour.col</code>	two-element vector with the fill colours to be assigned to the minimum and maximum age contour
<code>levels</code>	a vector with additional values to be displayed as different background colours within the error ellipses.
<code>clabel</code>	label of the colour scale
<code>ellipse.col</code>	a vector of two background colours for the error ellipses. If <code>levels=NA</code> , then only the first colour will be used. If <code>levels</code> is a vector of numbers, then <code>ellipse.col</code> is used to construct a colour ramp.
<code>sigdig</code>	number of significant digits for the central age
<code>xlim</code>	optional limits of the x-axis ( $\log[U/He]$ ) of the logratio plot. If <code>xlim=NA</code> , the axis limits are determined automatically.
<code>ylim</code>	optional limits of the y-axis ( $\log[Th/He]$ ) of the logratio plot. If <code>ylim=NA</code> , the axis limits are determined automatically.
<code>fact</code>	three-element vector with scaling factors of the ternary diagram if <code>fact=NA</code> , these will be determined automatically
<code>hide</code>	vector with indices of aliquots that should be removed from the plot.
<code>omit</code>	vector with indices of aliquots that should be plotted but omitted from the central age calculation.
<code>omit.col</code>	colour that should be used for the omitted aliquots.
<code>...</code>	optional arguments to the generic plot function

## Details

U, Th, Sm and He are *compositional* data. This means that it is not so much the absolute concentrations of these elements that bear the chronological information, but rather their relative proportions. The space of all possible U-Th-He compositions fits within the constraints of a ternary diagram or ‘helioplot’ (Vermeesch, 2008, 2010). If Sm is included as well, then this expands to a three-dimensional tetrahedral space (Vermeesch, 2008). Data that fit within these constrained spaces must be subjected to a logratio transformation prior to statistical analysis (Aitchison, 1986). In the case of the U-Th-He-(Sm)-He system, this is achieved by first defining two (or three) new variables:

$$u \equiv \ln[U/He] \quad v \equiv \ln[Th/He] \quad (, w \equiv \ln[Sm/He])$$

and then performing the desired statistical analysis (averaging, uncertainty propagation, ...) on the transformed data. Upon completion of the mathematical operations, the results can then be mapped back to U-Th-(Sm)-He space using an inverse logratio transformation:

$$[He] = 1/[e^u + e^v + (e^w) + 1], [U] = e^u/[e^u + e^v + (e^w) + 1]$$

$$[Th] = e^v/[e^u + e^v + (e^w) + 1], ([Sm] = e^w/[e^u + e^v + (e^w) + 1]).$$

where  $[He] + [U] + [Th] + [Sm] = 1$ . In the context of U-Th-(Sm)-He dating, the *central* age is defined as the age that corresponds to the arithmetic mean composition in logratio space, which is equivalent to the geometric mean in compositional dataspace (Vermeesch, 2008). IsoplotR's `helioplot` function performs this calculation using the same algorithm that is used to obtain the weighted mean U-Pb composition for the `concordia` age calculation. Overdispersion is treated similarly as in a regression context (see `isochron`). Thus, there are options to augment the uncertainties with a factor  $\sqrt{MSWD}$  (model 1); to ignore the analytical uncertainties altogether (model 2); or to add a constant overdispersion term to the analytical uncertainties (model 3). The `helioplot` function visualises U-Th-(Sm)-He data on either a ternary diagram or a bivariate  $\ln[Th/U]$  vs.  $\ln[U/He]$  contour plot. These diagrams provide a convenient way to simultaneously display the isotopic composition of samples as well as their chronological meaning. In this respect, they fulfil the same purpose as the U-Pb `concordia` diagram and the U-series `evolution` plot.

## References

- Aitchison, J., 1986, The statistical analysis of compositional data: London, Chapman and Hall, 416 p.
- Vermeesch, P., 2008. Three new ways to calculate average (U-Th)/He ages. *Chemical Geology*, 249(3), pp.339-347.
- Vermeesch, P., 2010. HelioPlot, and the treatment of overdispersed (U-Th-Sm)/He data. *Chemical Geology*, 271(3), pp.108-111.

## See Also

`radialplot`

## Examples

```
data(examples)
helioplot(examples$UThHe)
dev.new()
helioplot(examples$UThHe, logratio=FALSE)
```

---

isochron

*Calculate and plot isochrons*


---

## Description

Plots cogenetic Ar-Ar, K-Ca, Pb-Pb, Rb-Sr, Sm-Nd, Re-Os, Lu-Hf, U-Th-He or Th-U data as X-Y scatterplots, fits an isochron curve through them using the `york` function, and computes the corresponding isochron age, including decay constant uncertainties.

**Usage**

```
isochron(x, ...)
```

```
## Default S3 method:
isochron(x, xlim = NA, ylim = NA, alpha = 0.05,
  sigdig = 2, show.numbers = FALSE, levels = NA, clabel = "",
  ellipse.col = c("#00FF0080", "#FF000080"), ci.col = "gray80",
  line.col = "black", lwd = 1, plot = TRUE, title = TRUE,
  model = 1, show.ellipses = 1 * (model != 2), xlab = "x",
  ylab = "y", hide = NULL, omit = NULL, omit.col = NA, ...)
```

```
## S3 method for class 'ArAr'
isochron(x, xlim = NA, ylim = NA, alpha = 0.05,
  sigdig = 2, show.numbers = FALSE, levels = NA, clabel = "",
  ellipse.col = c("#00FF0080", "#FF000080"), inverse = TRUE,
  ci.col = "gray80", line.col = "black", lwd = 1, plot = TRUE,
  exterr = TRUE, model = 1, show.ellipses = 1 * (model != 2),
  hide = NULL, omit = NULL, omit.col = NA, ...)
```

```
## S3 method for class 'KCa'
isochron(x, xlim = NA, ylim = NA, alpha = 0.05,
  sigdig = 2, show.numbers = FALSE, levels = NA, clabel = "",
  ellipse.col = c("#00FF0080", "#FF000080"), ci.col = "gray80",
  line.col = "black", lwd = 1, plot = TRUE, exterr = TRUE,
  model = 1, show.ellipses = 1 * (model != 2), hide = NULL,
  omit = NULL, omit.col = NA, ...)
```

```
## S3 method for class 'PbPb'
isochron(x, xlim = NA, ylim = NA, alpha = 0.05,
  sigdig = 2, show.numbers = FALSE, levels = NA, clabel = "",
  ellipse.col = c("#00FF0080", "#FF000080"), inverse = TRUE,
  ci.col = "gray80", line.col = "black", lwd = 1, plot = TRUE,
  exterr = TRUE, model = 1, show.ellipses = 1 * (model != 2),
  growth = FALSE, hide = NULL, omit = NULL, omit.col = NA, ...)
```

```
## S3 method for class 'RbSr'
isochron(x, xlim = NA, ylim = NA, alpha = 0.05,
  sigdig = 2, show.numbers = FALSE, levels = NA, clabel = "",
  ellipse.col = c("#00FF0080", "#FF000080"), ci.col = "gray80",
  line.col = "black", lwd = 1, plot = TRUE, exterr = TRUE,
  model = 1, show.ellipses = 1 * (model != 2), hide = NULL,
  omit = NULL, omit.col = NA, ...)
```

```
## S3 method for class 'ReOs'
isochron(x, xlim = NA, ylim = NA, alpha = 0.05,
  sigdig = 2, show.numbers = FALSE, levels = NA, clabel = "",
  ellipse.col = c("#00FF0080", "#FF000080"), ci.col = "gray80",
  line.col = "black", lwd = 1, plot = TRUE, exterr = TRUE,
```

```

model = 1, show.ellipses = 1 * (model != 2), hide = NULL,
omit = NULL, omit.col = NA, ...)

## S3 method for class 'SmNd'
isochron(x, xlim = NA, ylim = NA, alpha = 0.05,
  sigdig = 2, show.numbers = FALSE, levels = NA, clabel = "",
  ellipse.col = c("#00FF0080", "#FF000080"), ci.col = "gray80",
  line.col = "black", lwd = 1, plot = TRUE, exterr = TRUE,
  model = 1, show.ellipses = 1 * (model != 2), hide = NULL,
  omit = NULL, omit.col = NA, ...)

## S3 method for class 'LuHf'
isochron(x, xlim = NA, ylim = NA, alpha = 0.05,
  sigdig = 2, show.numbers = FALSE, levels = NA, clabel = "",
  ellipse.col = c("#00FF0080", "#FF000080"), ci.col = "gray80",
  line.col = "black", lwd = 1, plot = TRUE, exterr = TRUE,
  model = 1, show.ellipses = 1 * (model != 2), hide = NULL,
  omit = NULL, omit.col = NA, ...)

## S3 method for class 'ThU'
isochron(x, type = 2, xlim = NA, ylim = NA,
  alpha = 0.05, sigdig = 2, show.numbers = FALSE, levels = NA,
  clabel = "", ellipse.col = c("#00FF0080", "#FF000080"),
  ci.col = "gray80", line.col = "black", lwd = 1, plot = TRUE,
  exterr = TRUE, model = 1, show.ellipses = 1 * (model != 2),
  hide = NULL, omit = NULL, omit.col = NA, ...)

## S3 method for class 'UThHe'
isochron(x, xlim = NA, ylim = NA, alpha = 0.05,
  sigdig = 2, show.numbers = FALSE, levels = NA, clabel = "",
  ellipse.col = c("#00FF0080", "#FF000080"), ci.col = "gray80",
  line.col = "black", lwd = 1, plot = TRUE, model = 1,
  show.ellipses = 2 * (model != 2), hide = NULL, omit = NULL,
  omit.col = "grey", ...)

```

## Arguments

<code>x</code>	EITHER a matrix with the following five columns: <b>X</b> the x-variable <b>sX</b> the standard error of X <b>Y</b> the y-variable <b>sY</b> the standard error of Y <b>rXY</b> the correlation coefficient of X and Y OR an object of class ArAr, KCa, PbPb, ReOs, RbSr, SmNd, LuHf, UThHe or ThU.
<code>...</code>	optional arguments to be passed on to the generic plot function if model=2
<code>xlim</code>	2-element vector with the x-axis limits

<code>ylim</code>	2-element vector with the y-axis limits
<code>alpha</code>	confidence cutoff for the error ellipses and confidence intervals
<code>sigdig</code>	the number of significant digits of the numerical values reported in the title of the graphical output
<code>show.numbers</code>	logical flag (TRUE to show grain numbers)
<code>levels</code>	a vector with additional values to be displayed as different background colours within the error ellipses.
<code>clabel</code>	label for the colour scale
<code>ellipse.col</code>	a vector of two background colours for the error ellipses. If <code>levels=NA</code> , then only the first colour will be used. If <code>levels</code> is a vector of numbers, then <code>ellipse.col</code> is used to construct a colour ramp.
<code>ci.col</code>	the fill colour for the confidence interval of the intercept and slope.
<code>line.col</code>	colour of the isochron line
<code>lwd</code>	line width
<code>plot</code>	if FALSE, suppresses the graphical output
<code>title</code>	add a title to the plot?
<code>model</code>	construct the isochron using either: <ol style="list-style-type: none"> <li>1. Error-weighted least squares regression</li> <li>2. Ordinary least squares regression</li> <li>3. Error-weighted least squares with overdispersion term</li> </ol>
<code>show.ellipses</code>	show the data as: <ol style="list-style-type: none"> <li>1. points</li> <li>2. error ellipses</li> <li>3. error crosses</li> </ol>
<code>xlab</code>	text label for the horizontal plot axis
<code>ylab</code>	text label for the vertical plot axis
<code>hide</code>	vector with indices of aliquots that should be removed from the plot.
<code>omit</code>	vector with indices of aliquots that should be plotted but omitted from the isochron age calculation.
<code>omit.col</code>	colour that should be used for the omitted aliquots.
<code>inverse</code>	if FALSE and <code>x</code> has class <code>ArAr</code> , plots $^{40}\text{Ar}/^{36}\text{Ar}$ vs. $^{39}\text{Ar}/^{36}\text{Ar}$ . if FALSE and <code>x</code> has class <code>PbPb</code> , plots $^{207}\text{Pb}/^{204}\text{Pb}$ vs. $^{206}\text{Pb}/^{204}\text{Pb}$ . if TRUE and <code>x</code> has class <code>ArAr</code> , plots $^{36}\text{Ar}/^{40}\text{Ar}$ vs. $^{39}\text{Ar}/^{40}\text{Ar}$ . if TRUE and <code>x</code> has class <code>PbPb</code> , plots $^{207}\text{Pb}/^{206}\text{Pb}$ vs. $^{204}\text{Pb}/^{206}\text{Pb}$ .
<code>exterr</code>	propagate external sources of uncertainty (J, decay constant)?
<code>growth</code>	add Stacey-Kramers Pb-evolution curve to the plot?
<code>type</code>	following the classification of Ludwig and Titterton (1994), one of either: <ol style="list-style-type: none"> <li>1. 'Rosholt type-II' isochron, setting out <math>^{230}\text{Th}/^{232}\text{Th}</math> vs. <math>^{238}\text{U}/^{232}\text{Th}</math></li> <li>2. 'Osmond type-II' isochron, setting out <math>^{230}\text{Th}/^{238}\text{U}</math> vs. <math>^{232}\text{Th}/^{238}\text{U}</math></li> <li>3. 'Rosholt type-II' isochron, setting out <math>^{234}\text{U}/^{232}\text{Th}</math> vs. <math>^{238}\text{U}/^{232}\text{Th}</math></li> <li>4. 'Osmond type-II' isochron, setting out <math>^{234}\text{U}/^{238}\text{U}</math> vs. <math>^{232}\text{Th}/^{238}\text{U}</math></li> </ol>

## Details

Given several aliquots from a single sample, isochrons allow the non-radiogenic component of the daughter nuclide to be quantified and separated from the radiogenic component. In its simplest form, an isochron is obtained by setting out the amount of radiogenic daughter against the amount of radioactive parent, both normalised to a non-radiogenic isotope of the daughter element, and fitting a straight line through these points by least squares regression (Nicolaysen, 1961). The slope and intercept then yield the radiogenic daughter-parent ratio and the non-radiogenic daughter composition, respectively. There are several ways to fit an isochron. The easiest of these is ordinary least squares regression, which weighs all data points equally. In the presence of quantifiable analytical uncertainty, it is equally straightforward to use the inverse of the y-errors as weights. It is significantly more difficult to take into account uncertainties in both the x- and the y-variable (York, 1966). IsoplotR does so for its U-Th-He isochron calculations. The York (1966) method assumes that the analytical uncertainties of the x- and y-variables are independent from each other. This assumption is rarely met in geochronology. York (1968) addresses this issue with a bivariate error weighted linear least squares algorithm that accounts for covariant errors in both variables. This algorithm was further improved by York et al. (2004) to ensure consistency with the maximum likelihood approach of Titterton and Halliday (1979).

IsoplotR uses the York et al. (2004) algorithm for its Ar-Ar, K-Ca, Pb-Pb, Rb-Sr, Sm-Nd, Re-Os and Lu-Hf isochrons. The maximum likelihood algorithm of Titterton and Halliday (1979) was generalised from two to three dimensions by Ludwig and Titterton (1994) for U-series disequilibrium dating. Also this algorithm is implemented in IsoplotR. The extent to which the observed scatter in the data can be explained by the analytical uncertainties can be assessed using the Mean Square of the Weighted Deviates (MSWD, McIntyre et al., 1966), which is defined as:

$$MSWD = ([X - \hat{X}] \Sigma_X^{-1} [X - \hat{X}]^T) / df$$

where  $X$  are the data,  $\hat{X}$  are the fitted values, and  $\Sigma_X$  is the covariance matrix of  $X$ , and  $df = k(n - 1)$  are the degrees of freedom, where  $k$  is the dimensionality of the linear fit. MSWD values that are far smaller or greater than 1 indicate under- or overdispersed measurements, respectively. Underdispersion can be attributed to overestimated analytical uncertainties. IsoplotR provides three alternative strategies to deal with overdispersed data:

1. Attribute the overdispersion to an underestimation of the analytical uncertainties. In this case, the excess scatter can be accounted for by inflating those uncertainties by a factor  $\sqrt{MSWD}$ .
2. Ignore the analytical uncertainties and perform an ordinary least squares regression.
3. Attribute the overdispersion to the presence of ‘geological scatter’. In this case, the excess scatter can be accounted for by adding an overdispersion *term* that lowers the MSWD to unity.

## Value

If `x` has class `PbPb`, `ArAr`, `KCa`, `RbSr`, `SmNd`, `ReOs` or `LuHf`, or `UThHe`, returns a list with the following items:

- a** the intercept of the straight line fit and its standard error.
- b** the slope of the fit and its standard error.
- cov.ab** the covariance of the slope and intercept
- df** the degrees of freedom of the linear fit ( $df = n - 2$ )

**y0** a four-element list containing:

y: the atmospheric  $^{40}\text{Ar}/^{36}\text{Ar}$  or initial  $^{40}\text{Ca}/^{44}\text{Ca}$ ,  $^{187}\text{Os}/^{188}\text{Os}$ ,  $^{87}\text{Sr}/^{87}\text{Rb}$ ,  $^{143}\text{Nd}/^{144}\text{Nd}$  or  $^{176}\text{Hf}/^{177}\text{Hf}$  ratio.

s[y]: the propagated uncertainty of y

ci[y]: the studentised  $100(1 - \alpha)\%$  confidence interval for y.

disp[y]: the studentised  $100(1 - \alpha)\%$  confidence interval for y enhanced by  $\sqrt{mswd}$  (only applicable if model=1).

**age** a four-element list containing:

t: the  $^{207}\text{Pb}/^{206}\text{Pb}$ ,  $^{40}\text{Ar}/^{39}\text{Ar}$ ,  $^{40}\text{K}/^{40}\text{Ca}$ ,  $^{187}\text{Os}/^{187}\text{Re}$ ,  $^{87}\text{Sr}/^{87}\text{Rb}$ ,  $^{143}\text{Nd}/^{144}\text{Nd}$  or  $^{176}\text{Hf}/^{177}\text{Hf}$  age.

s[t]: the propagated uncertainty of t

ci[t]: the studentised  $100(1 - \alpha)\%$  confidence interval for t.

disp[t]: the studentised  $100(1 - \alpha)\%$  confidence interval for t enhanced by  $\sqrt{mswd}$  (only applicable if model=1).

**mswd** the mean square of the residuals (a.k.a ‘reduced Chi-square’) statistic (omitted if model=2).

**p.value** the p-value of a Chi-square test for linearity (omitted if model=2)

**w** the overdispersion term, i.e. a three-element vector with the standard deviation of the (assumedly) Normally distributed geological scatter that underlies the measurements, and the lower and upper half-widths of its  $100(1 - \alpha)\%$  confidence interval (only returned if model=3).

OR, if x has class ThU:

**par** if x\$type=1 or x\$type=3: the best fitting  $^{230}\text{Th}/^{232}\text{Th}$  intercept,  $^{230}\text{Th}/^{238}\text{U}$  slope,  $^{234}\text{U}/^{232}\text{Th}$  intercept and  $^{234}\text{U}/^{238}\text{U}$  slope, OR, if x\$type=2 or x\$type=4: the best fitting  $^{234}\text{U}/^{238}\text{U}$  intercept,  $^{230}\text{Th}/^{232}\text{Th}$  slope,  $^{234}\text{U}/^{238}\text{U}$  intercept and  $^{234}\text{U}/^{232}\text{Th}$  slope.

**cov** the covariance matrix of par.

**df** the degrees of freedom for the linear fit, i.e.  $(3n - 3)$  if x\$format=1 or x\$format=2, and  $(2n - 2)$  if x\$format=3 or x\$format=4

**a** if type=1: the  $^{230}\text{Th}/^{232}\text{Th}$  intercept; if type=2: the  $^{230}\text{Th}/^{238}\text{U}$  intercept; if type=3: the  $^{234}\text{Th}/^{232}\text{Th}$  intercept; if type=4: the  $^{234}\text{Th}/^{238}\text{U}$  intercept and its propagated uncertainty.

**b** if type=1: the  $^{230}\text{Th}/^{238}\text{U}$  slope; if type=2: the  $^{230}\text{Th}/^{232}\text{Th}$  slope; if type=3: the  $^{234}\text{U}/^{238}\text{U}$  slope; if type=4: the  $^{234}\text{U}/^{232}\text{Th}$  slope and its propagated uncertainty.

**cov.ab** the covariance between a and b.

**mswd** the mean square of the residuals (a.k.a ‘reduced Chi-square’) statistic.

**p.value** the p-value of a Chi-square test for linearity.

**tfact** the  $100(1 - \alpha/2)\%$  percentile of a t-distribution with df degrees of freedom.

**y0** a four-element vector containing:

y: the initial  $^{234}\text{U}/^{238}\text{U}$ -ratio

s[y]: the propagated uncertainty of y

ci[y]: the studentised  $100(1 - \alpha)\%$  confidence interval for y.

disp[y]: the studentised  $100(1 - \alpha)\%$  confidence interval for y enhanced by  $\sqrt{mswd}$ .



- age** a three (or four) element vector containing:
- t: the initial  $^{234}\text{U}/^{238}\text{U}$ -ratio
  - s[t]: the propagated uncertainty of t
  - ci[t]: the studentised  $100(1 - \alpha)\%$  confidence interval for t
  - disp[t]: the studentised  $100(1 - \alpha)\%$  confidence interval for t enhanced by  $\sqrt{mswd}$  (only reported if model=1).
- w** the overdispersion term, i.e. a three-element vector with the standard deviation of the (assumedly) Normally distributed geological scatter that underlies the measurements, and the lower and upper half-width of its  $100(1 - \alpha)\%$  confidence interval (only returned if model=3).
- d** a matrix with the following columns: the X-variable for the isochron plot, the analytical uncertainty of X, the Y-variable for the isochron plot, the analytical uncertainty of Y, and the correlation coefficient between X and Y.
- xlab** the x-label of the isochron plot
- ylab** the y-label of the isochron plot

## References

- Ludwig, K.R. and Titterton, D.M., 1994. Calculation of  $^{230}\text{Th}/\text{U}$  isochrons, ages, and errors. *Geochimica et Cosmochimica Acta*, 58(22), pp.5031-5042.
- Nicolaysen, L.O., 1961. Graphic interpretation of discordant age measurements on metamorphic rocks. *Annals of the New York Academy of Sciences*, 91(1), pp.198-206.
- Titterton, D.M. and Halliday, A.N., 1979. On the fitting of parallel isochrons and the method of maximum likelihood. *Chemical Geology*, 26(3), pp.183-195.
- York, D., 1966. Least-squares fitting of a straight line. *Canadian Journal of Physics*, 44(5), pp.1079-1086.
- York, D., 1968. Least squares fitting of a straight line with correlated errors. *Earth and Planetary Science Letters*, 5, pp.320-324.
- York, D., Evensen, N.M., Martinez, M.L. and De Basebe Delgado, J., 2004. Unified equations for the slope, intercept, and standard errors of the best straight line. *American Journal of Physics*, 72(3), pp.367-375.

## See Also

[york](#), [titterton](#), [ludwig](#)

## Examples

```
data(examples)
isochron(examples$RbSr)

fit <- isochron(examples$ArAr, inverse=FALSE, plot=FALSE)

dev.new()
isochron(examples$ThU, type=4)
```

---

IsoplotR

*library(IsoplotR)*


---

### Description

A list of documented functions may be viewed by typing `help(package='IsoplotR')`. Detailed instructions are provided at <http://isoplotr.london-geochron.com>. Further details about the theoretical background are provided by Vermeesch (2018).

### Author(s)

**Maintainer:** Pieter Vermeesch <p.vermeesch@ucl.ac.uk>

### References

Vermeesch, P., 2018, IsoplotR: a free and open toolbox for geochronology. *Geoscience Frontiers*, 9, 1479-1493, doi: 10.1016/j.gsf.2018.04.001.

### See Also

Useful links:

- <http://isoplotr.london-geochron.com>

---

kde

*Create (a) kernel density estimate(s)*


---

### Description

Creates one or more kernel density estimates using a combination of the Botev (2010) bandwidth selector and the Abramson (1982) adaptive kernel bandwidth modifier.

### Usage

```
kde(x, ...)

## Default S3 method:
kde(x, from = NA, to = NA, bw = NA,
    adaptive = TRUE, log = FALSE, n = 512, plot = TRUE, pch = "|",
    xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
    hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
    binwidth = NA, hide = NULL, ...)

## S3 method for class 'UPb'
kde(x, from = NA, to = NA, bw = NA, adaptive = TRUE,
    log = FALSE, n = 512, plot = TRUE, pch = "|",
```

```

xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
binwidth = NA, type = 4, cutoff.76 = 1100, cutoff.disc = c(-15,
5), common.Pb = 0, hide = NULL, ...)

## S3 method for class 'detritals'
kde(x, from = NA, to = NA, bw = NA,
    adaptive = TRUE, log = FALSE, n = 512, plot = TRUE, pch = NA,
    xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
    hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
    binwidth = NA, ncol = NA, samebandwidth = TRUE, normalise = TRUE,
    hide = NULL, ...)

## S3 method for class 'PbPb'
kde(x, from = NA, to = NA, bw = NA, adaptive = TRUE,
    log = FALSE, n = 512, plot = TRUE, pch = "|",
    xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
    hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
    binwidth = NA, common.Pb = 1, hide = NULL, ...)

## S3 method for class 'ArAr'
kde(x, from = NA, to = NA, bw = NA, adaptive = TRUE,
    log = FALSE, n = 512, plot = TRUE, pch = "|",
    xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
    hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
    binwidth = NA, i2i = FALSE, hide = NULL, ...)

## S3 method for class 'KCa'
kde(x, from = NA, to = NA, bw = NA, adaptive = TRUE,
    log = FALSE, n = 512, plot = TRUE, pch = "|",
    xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
    hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
    binwidth = NA, i2i = FALSE, hide = NULL, ...)

## S3 method for class 'ThU'
kde(x, from = NA, to = NA, bw = NA, adaptive = TRUE,
    log = FALSE, n = 512, plot = TRUE, pch = "|",
    xlab = "age [ka]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
    hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
    binwidth = NA, i2i = FALSE, detritus = 0, hide = NULL, ...)

## S3 method for class 'ReOs'
kde(x, from = NA, to = NA, bw = NA, adaptive = TRUE,
    log = FALSE, n = 512, plot = TRUE, pch = "|",
    xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
    hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
    binwidth = NA, i2i = TRUE, hide = NULL, ...)

```

```
## S3 method for class 'SmNd'
kde(x, from = NA, to = NA, bw = NA, adaptive = TRUE,
    log = FALSE, n = 512, plot = TRUE, pch = "|",
    xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
    hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
    binwidth = NA, i2i = TRUE, hide = NULL, ...)

## S3 method for class 'RbSr'
kde(x, from = NA, to = NA, bw = NA, adaptive = TRUE,
    log = FALSE, n = 512, plot = TRUE, pch = "|",
    xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
    hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
    binwidth = NA, i2i = TRUE, hide = NULL, ...)

## S3 method for class 'LuHf'
kde(x, from = NA, to = NA, bw = NA, adaptive = TRUE,
    log = FALSE, n = 512, plot = TRUE, pch = "|",
    xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
    hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
    binwidth = NA, i2i = TRUE, hide = NULL, ...)

## S3 method for class 'UThHe'
kde(x, from = NA, to = NA, bw = NA,
    adaptive = TRUE, log = FALSE, n = 512, plot = TRUE, pch = "|",
    xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
    hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
    binwidth = NA, hide = NULL, ...)

## S3 method for class 'fissiontracks'
kde(x, from = NA, to = NA, bw = NA,
    adaptive = TRUE, log = FALSE, n = 512, plot = TRUE, pch = "|",
    xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
    hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
    binwidth = NA, hide = NULL, ...)
```

## Arguments

x	a vector of numbers OR an object of class UPb, PbPb, ArAr, KCa, ReOs, SmNd, RbSr, UThHe, fissiontracks, ThU or detrital
...	optional arguments to be passed on to R's density function.
from	minimum age of the time axis. If NULL, this is set automatically
to	maximum age of the time axis. If NULL, this is set automatically
bw	the bandwidth of the KDE. If NULL, bw will be calculated automatically using the algorithm by Botev et al. (2010).
adaptive	logical flag controlling if the adaptive KDE modifier of Abramson (1982) is used
log	transform the ages to a log scale if TRUE
n	horizontal resolution (i.e., the number of segments) of the density estimate.

plot	show the KDE as a plot
pch	the symbol used to show the samples. May be a vector. Set pch=NA to turn them off.
xlab	the x-axis label
ylab	the y-axis label
kde.col	the fill colour of the KDE specified as a four element vector of r, g, b, alpha values
hist.col	the fill colour of the histogram specified as a four element vector of r, g, b, alpha values
show.hist	logical flag indicating whether a histogram should be added to the KDE
bty	change to "o", "l", "7", "c", "u", or "]" if you want to draw a box around the plot
binwidth	scalar width of the histogram bins, in Myr if log = FALSE, or as a fractional value if log = TRUE. Sturges' Rule ( $\log_2[n] + 1$ , where $n$ is the number of data points) is used if binwidth = NA
hide	vector with indices of aliquots that should be removed from the plot.
type	scalar indicating whether to plot the $^{207}\text{Pb}/^{235}\text{U}$ age (type=1), the $^{206}\text{Pb}/^{238}\text{U}$ age (type=2), the $^{207}\text{Pb}/^{206}\text{Pb}$ age (type=3), the $^{207}\text{Pb}/^{206}\text{Pb}$ - $^{206}\text{Pb}/^{238}\text{U}$ age (type=4), or the (Wetherill) concordia age (type=5)
cutoff.76	the age (in Ma) below which the $^{206}\text{Pb}/^{238}\text{U}$ and above which the $^{207}\text{Pb}/^{206}\text{Pb}$ age is used. This parameter is only used if type=4.
cutoff.disc	two element vector with the minimum (negative) and maximum (positive) percentage discordance allowed between the $^{207}\text{Pb}/^{235}\text{U}$ and $^{206}\text{Pb}/^{238}\text{U}$ age (if $^{206}\text{Pb}/^{238}\text{U} < \text{cutoff.76}$ ) or between the $^{206}\text{Pb}/^{238}\text{U}$ and $^{207}\text{Pb}/^{206}\text{Pb}$ age (if $^{206}\text{Pb}/^{238}\text{U} > \text{cutoff.76}$ ). Set cutoff.disc=NA if you do not want to use this filter.
common.Pb	apply a common lead correction using one of three methods: 1: use the isochron intercept as the initial Pb-composition 2: use the Stacey-Kramers two-stage model to infer the initial Pb-composition 3: use the Pb-composition stored in settings('iratio', 'Pb206Pb204') and settings('iratio', 'Pb207Pb204')
ncol	scalar value indicating the number of columns over which the KDEs should be divided.
samebandwidth	logical flag indicating whether the same bandwidth should be used for all samples. If samebandwidth = TRUE and bw = NULL, then the function will use the median bandwidth of all the samples.
normalise	logical flag indicating whether or not the KDEs should all integrate to the same value.
i2i	'isochron to intercept': calculates the initial (aka 'inherited', 'excess', or 'common') $^{40}\text{Ar}/^{36}\text{Ar}$ , $^{40}\text{Ca}/^{44}\text{Ca}$ , $^{87}\text{Sr}/^{86}\text{Sr}$ , $^{143}\text{Nd}/^{144}\text{Nd}$ , $^{187}\text{Os}/^{188}\text{Os}$ , $^{230}\text{Th}/^{232}\text{Th}$ or $^{176}\text{Hf}/^{177}\text{Hf}$ ratio from an isochron fit. Setting i2i to FALSE uses the default values stored in settings('iratio', ...).

**detritus** detrital  $^{230}\text{Th}$  correction (only applicable when `x$format == 1` or `2`).  
 0: no correction  
 1: project the data along an isochron fit  
 2: correct the data using an assumed initial  $^{230}\text{Th}/^{232}\text{Th}$ -ratio for the detritus.  
 3: correct the data using the measured present day  $^{230}\text{Th}/^{238}\text{U}$ ,  $^{232}\text{Th}/^{238}\text{U}$  and  $^{234}\text{U}/^{238}\text{U}$ -ratios in the detritus.

## Details

Given a set of  $n$  age estimates  $\{t_1, t_2, \dots, t_n\}$ , histograms and KDEs are probability density estimators that display age distributions by smoothing. Histograms do this by grouping the data into a number of regularly spaced bins. Alternatively, kernel density estimates (KDEs; Vermeesch, 2012) smooth data by applying a (Gaussian) kernel:

$$KDE(t) = \sum_{i=1}^n N(t|\mu = t_i, \sigma = h[t])/n$$

where  $N(t|\mu, \sigma)$  is the probability of observing a value  $t$  under a Normal distribution with mean  $\mu$  and standard deviation  $\sigma$ .  $h[t]$  is the smoothing parameter or ‘bandwidth’ of the kernel density estimate, which may or may not depend on the age  $t$ . If  $h[t]$  depends on  $t$ , then  $KDE(t)$  is known as an ‘adaptive’ KDE. The default bandwidth used by `IsoplotR` is calculated using the algorithm of Botev et al. (2010) and modulated by the adaptive smoothing approach of Abramson (1982). The rationale behind adaptive kernel density estimation is to use a narrower bandwidth near the peaks of the sampling distribution (where the ordered dates are closely spaced in time), and a wider bandwidth in the distribution’s sparsely sampled troughs. Thus, the resolution of the density estimate is optimised according to data availability.

## Value

If `x` has class `UPb`, `PbPb`, `ArAr`, `KCa`, `ReOs`, `SmNd`, `RbSr`, `UThHe`, `fissiontracks` or `ThU`, returns an object of class `KDE`, i.e. a list containing the following items:

**x** horizontal plot coordinates  
**y** vertical plot coordinates  
**bw** the base bandwidth of the density estimate  
**ages** the data values from the input to the `kde` function  
**log** copied from the input

or, if `x` has class `=detritals`, an object of class `KDEs`, i.e. a list containing the following items:

**kdes** a named list with objects of class `KDE`  
**from** the beginning of the common time scale  
**to** the end of the common time scale  
**themax** the maximum probability density of all the KDEs  
**xlabel** the x-axis label to be used by `plot.KDEs(...)`

## References

- Abramson, I.S., 1982. On bandwidth variation in kernel estimates-a square root law. The annals of Statistics, pp.1217-1223.
- Botev, Z. I., J. F. Grotowski, and D. P. Kroese. "Kernel density estimation via diffusion." The Annals of Statistics 38.5 (2010): 2916-2957.
- Vermeesch, P., 2012. On the visualisation of detrital age distributions. Chemical Geology, 312, pp.190-194.

## See Also

[radialplot](#), [cad](#)

## Examples

```
kde(examples$UPb)

dev.new()
kde(examples$FT1, log=TRUE)

dev.new()
kde(examples$DZ, from=1, to=3000, kernel="epanechnikov")
```

---

ludwig

*Linear regression of U-Pb data with correlated errors, taking into account decay constant uncertainties.*

---

## Description

Implements the maximum likelihood algorithm for Total-Pb/U isochron regression of Ludwig (1998)

## Usage

```
ludwig(x, ...)

## Default S3 method:
ludwig(x, ...)

## S3 method for class 'UPb'
ludwig(x, exterr = FALSE, alpha = 0.05, model = 1,
      anchor = list(FALSE, NA), ...)
```

## Arguments

x	an object of class UPb
...	optional arguments
exterr	propagate external sources of uncertainty (e.g., decay constants)?

<code>alpha</code>	cutoff value for confidence intervals
<code>model</code>	<p>one of three regression models:</p> <ol style="list-style-type: none"> <li>1: fit a discordia line through the data using the maximum likelihood algorithm of Ludwig (1998), which assumes that the scatter of the data is solely due to the analytical uncertainties. In this case, <code>IsoplotR</code> will either calculate an upper and lower intercept age (for Wetherill concordia), or a lower intercept age and common <math>(^{207}\text{Pb}/^{206}\text{Pb})_0</math>-ratio intercept (for Tera-Wasserburg). If <math>MSWD &gt; 0</math>, then the analytical uncertainties are augmented by a factor <math>\sqrt{MSWD}</math>.</li> <li>2: fit a discordia line ignoring the analytical uncertainties</li> <li>3: fit a discordia line using a modified maximum likelihood algorithm that includes accounts for any overdispersion by adding a geological (co)variance term.</li> </ol>
<code>anchor</code>	<p>control parameters to fix the intercept age or common Pb composition of the discordia fit. This is a two-element list.</p> <ul style="list-style-type: none"> <li>• The first element is a boolean flag indicating whether the discordia line should be anchored. If this is <code>FALSE</code>, then the second item is ignored and both the common Pb composition and age are estimated.</li> <li>• If the first element is <code>TRUE</code> and the second element is <code>NA</code>, then the common Pb composition is fixed at the values stored in <code>settings('iratio', ...)</code>. item If the first element is <code>TRUE</code> and the second element is a number, then the discordia line is forced to intersect the concordia line at an age equal to that number.</li> </ul>

## Details

The 3-dimensional regression algorithm of Ludwig and Titterton (1994) was modified by Ludwig (1998) to fit so-called ‘Total Pb-U isochrons’. These are constrained to a radiogenic endmember composition that falls on the [concordia](#) line. In its most sophisticated form, this algorithm does not only allow for correlated errors between variables, but also between aliquots. `IsoplotR` currently uses this algorithm to propagate decay constant uncertainties in the total Pb-U isochron ages. Future versions of the program will generalise this approach to other chronometers as well.

## Value

**par** a two-element vector with the lower concordia intercept and initial  $^{207}\text{Pb}/^{206}\text{Pb}$ -ratio.  
**cov** the covariance matrix of **par**  
**df** the degrees of freedom of the model fit ( $3n - 3$ , where  $n$  is the number of aliquots).  
**mswd** the mean square of weighted deviates (a.k.a. reduced Chi-square statistic) for the fit.  
**p.value** p-value of a Chi-square test for the linear fit  
**w** the overdispersion, i.e., a three-element vector with the estimated standard deviation of the (assumedly) Normal distribution that underlies the true isochron; and the lower and upper half-widths of its  $100(1 - \alpha)\%$  confidence interval (only relevant if `model = 3`).

## References

Ludwig, K.R., 1998. On the treatment of concordant uranium-lead ages. *Geochimica et Cosmochimica Acta*, 62(4), pp.665-676.



Ludwig, K.R. and Titterington, D.M., 1994. Calculation of  $^{230}\text{Th}/\text{U}$  isochrons, ages, and errors. *Geochimica et Cosmochimica Acta*, 58(22), pp.5031-5042.

### See Also

[concordia](#), [titterington](#), [isochron](#)

### Examples

```
f <- system.file("UPb4.csv", package="IsoplotR")
d <- read.data(f, method="U-Pb", format=4)
fit <- ludwig(d)
```

---

mds

*Multidimensional Scaling*

---

### Description

Performs classical or nonmetric Multidimensional Scaling analysis

### Usage

```
mds(x, ...)
```

## Default S3 method:

```
mds(x, classical = FALSE, plot = TRUE,
    shepard = FALSE, nnlines = FALSE, pos = NULL, col = "black",
    bg = "white", xlab = "", ylab = "", ...)
```

## S3 method for class 'detritals'

```
mds(x, classical = FALSE, plot = TRUE,
    shepard = FALSE, nnlines = FALSE, pos = NULL, col = "black",
    bg = "white", xlab = "", ylab = "", hide = NULL, ...)
```

### Arguments

x	a dissimilarity matrix OR an object of class detrital
...	optional arguments to the generic plot function
classical	logical flag indicating whether classical (TRUE) or nonmetric (FALSE) MDS should be used
plot	show the MDS configuration (if shepard=FALSE) or Shepard plot (if shepard=TRUE) on a graphical device
shepard	logical flag indicating whether the graphical output should show the MDS configuration (shepard=FALSE) or a Shepard plot with the 'stress' value. This argument is only used if plot=TRUE.
nnlines	if TRUE, draws nearest neighbour lines

<code>pos</code>	a position specifier for the labels (if <code>par('pch')!=NA</code> ). Values of 1, 2, 3 and 4 indicate positions below, to the left of, above and to the right of the MDS coordinates, respectively.
<code>col</code>	plot colour (may be a vector)
<code>bg</code>	background colour (may be a vector)
<code>xlab</code>	a string with the label of the x axis
<code>ylab</code>	a string with the label of the y axis
<code>hide</code>	vector with indices of aliquots that should be removed from the plot.

### Details

Multidimensional Scaling (MDS) is a dimension-reducing technique that takes a matrix of pair-wise ‘dissimilarities’ between objects (e.g., age distributions) as input and produces a configuration of two (or higher-) dimensional coordinates as output, so that the Euclidean distances between these coordinates approximate the dissimilarities of the input matrix. Thus, an MDS-configuration serves as a ‘map’ in which similar samples cluster closely together and dissimilar samples plot far apart. In the context of detrital geochronology, the dissimilarity between samples is given by the statistical distance between age distributions. There are many ways to define this statistical distance. `IsoplotR` uses the Kolmogorov-Smirnov (KS) statistic due to its simplicity and the fact that it behaves like a true distance in the mathematical sense of the word (Vermeesch, 2013). The KS-distance is given by the maximum vertical distance between two `cad` step functions. Thus, the KS-distance takes on values between zero (perfect match between two age distributions) and one (no overlap between two distributions). Calculating the KS-distance between samples two at a time populates a symmetric dissimilarity matrix with positive values and a zero diagonal. `IsoplotR` implements two algorithms to convert this matrix into a configuration. The first (‘classical’) approach uses a sequence of basic matrix manipulations developed by Young and Householder (1938) and Torgerson (1952) to achieve a linear fit between the KS-distances and the fitted distances on the MDS configuration. The second, more sophisticated (‘nonmetric’) approach subjects the input distances to a transformation  $f$  prior to fitting a configuration:

$$\delta_{i,j} = f(KS_{i,j})$$

where  $KS_{i,j}$  is the KS-distance between samples  $i$  and  $j$  (for  $1 \leq i \neq j \leq n$ ) and  $\delta_{i,j}$  is the ‘disparity’ (Kruskal, 1964). Fitting an MDS configuration then involves finding the disparity transformation that maximises the goodness of fit (or minimises the ‘stress’) between the disparities and the fitted distances. The latter two quantities can also be plotted against each other as a ‘Shepard plot’.

### Value

Returns an object of class `MDS`, i.e. a list containing the following items:

**points** a two-column vector of the fitted configuration

**classical** a logical flag indicating whether the MDS configuration was obtained by classical (TRUE) or nonmetric (FALSE) MDS

**diss** the dissimilarity matrix used for the MDS analysis

**stress** (only if `classical=TRUE`) the final stress achieved (in percent)

## References

- Kruskal, J., 1964. Multidimensional scaling by optimizing goodness of fit to a nonmetric hypothesis. *Psychometrika* 29 (1), 1-27.
- Torgerson, W. S. Multidimensional scaling: I. Theory and method. *Psychometrika*, 17(4): 401-419, 1952.
- Vermeesch, P., 2013. Multi-sample comparison of detrital age distributions. *Chemical Geology*, 341, pp.140-146.
- Young, G. and Householder, A. S. Discussion of a set of points in terms of their mutual distances. *Psychometrika*, 3(1):19-22, 1938.

## See Also

[cad](#), [kde](#)

## Examples

```
data(examples)
mds(examples$DZ, nlines=TRUE, pch=21, cex=5)
dev.new()
mds(examples$DZ, shepard=TRUE)
```

---

peakfit

*Finite mixture modelling of geochronological datasets*

---

## Description

Implements the discrete mixture modelling algorithms of Galbraith and Laslett (1993) and applies them to fission track and other geochronological datasets.

## Usage

```
peakfit(x, ...)

## Default S3 method:
peakfit(x, k = "auto", sigdig = 2, log = TRUE,
        alpha = 0.05, ...)

## S3 method for class 'fissiontracks'
peakfit(x, k = 1, exterr = TRUE, sigdig = 2,
        log = TRUE, alpha = 0.05, ...)

## S3 method for class 'UPb'
peakfit(x, k = 1, type = 4, cutoff.76 = 1100,
        cutoff.disc = c(-15, 5), common.Pb = 0, exterr = TRUE,
        sigdig = 2, log = TRUE, alpha = 0.05, ...)
```

```

## S3 method for class 'PbPb'
peakfit(x, k = 1, exterr = TRUE, sigdig = 2,
        log = TRUE, common.Pb = 0, alpha = 0.05, ...)

## S3 method for class 'ArAr'
peakfit(x, k = 1, exterr = TRUE, sigdig = 2,
        log = TRUE, i2i = FALSE, alpha = 0.05, ...)

## S3 method for class 'KCa'
peakfit(x, k = 1, exterr = TRUE, sigdig = 2,
        log = TRUE, i2i = FALSE, alpha = 0.05, ...)

## S3 method for class 'ReOs'
peakfit(x, k = 1, exterr = TRUE, sigdig = 2,
        log = TRUE, i2i = TRUE, alpha = 0.05, ...)

## S3 method for class 'SmNd'
peakfit(x, k = 1, exterr = TRUE, sigdig = 2,
        log = TRUE, i2i = TRUE, alpha = 0.05, ...)

## S3 method for class 'RbSr'
peakfit(x, k = 1, exterr = TRUE, sigdig = 2,
        log = TRUE, i2i = TRUE, alpha = 0.05, ...)

## S3 method for class 'LuHf'
peakfit(x, k = 1, exterr = TRUE, sigdig = 2,
        log = TRUE, i2i = TRUE, alpha = 0.05, ...)

## S3 method for class 'ThU'
peakfit(x, k = 1, exterr = FALSE, sigdig = 2,
        log = TRUE, i2i = TRUE, alpha = 0.05, detritus = 0, ...)

## S3 method for class 'UThHe'
peakfit(x, k = 1, sigdig = 2, log = TRUE,
        alpha = 0.05, ...)

```

## Arguments

x	either an [n x 2] matrix with measurements and their standard errors, or an object of class fissiontracks, UPb, PbPb, ArAr, KCa, ReOs, SmNd, RbSr, LuHf, ThU or UThHe
...	optional arguments (not used)
k	the number of discrete age components to be sought. Setting this parameter to 'auto' automatically selects the optimal number of components (up to a maximum of 5) using the Bayes Information Criterion (BIC).
sigdig	number of significant digits to be used for any legend in which the peak fitting results are to be displayed.
log	take the logs of the data before applying the mixture model?

alpha	cutoff value for confidence intervals
exterr	propagate the external sources of uncertainty into the component age errors?
type	scalar value indicating whether to plot the $^{207}\text{Pb}/^{235}\text{U}$ age (type=1), the $^{206}\text{Pb}/^{238}\text{U}$ age (type=2), the $^{207}\text{Pb}/^{206}\text{Pb}$ age (type=3), the $^{207}\text{Pb}/^{206}\text{Pb}$ - $^{206}\text{Pb}/^{238}\text{U}$ age (type=4), or the (Wetherill) concordia age (type=5)
cutoff.76	the age (in Ma) below which the $^{206}\text{Pb}/^{238}\text{U}$ and above which the $^{207}\text{Pb}/^{206}\text{Pb}$ age is used. This parameter is only used if type=4.
cutoff.disc	two element vector with the maximum and minimum percentage discordance allowed between the $^{207}\text{Pb}/^{235}\text{U}$ and $^{206}\text{Pb}/^{238}\text{U}$ age (if $^{206}\text{Pb}/^{238}\text{U} < \text{cutoff.76}$ ) or between the $^{206}\text{Pb}/^{238}\text{U}$ and $^{207}\text{Pb}/^{206}\text{Pb}$ age (if $^{206}\text{Pb}/^{238}\text{U} > \text{cutoff.76}$ ). Set cutoff.disc=NA if you do not want to use this filter.
common.Pb	apply a common lead correction using one of three methods: 1: use the Stacey-Kramers two-stage model to infer the initial Pb-composition 2: use the isochron intercept as the initial Pb-composition 3: use the Pb-composition stored in settings('iratio', 'Pb207Pb206') (if x\$format<4) or settings('iratio', 'Pb206Pb204') and settings('iratio', 'Pb207Pb204') (if x\$format>3)
i2i	'isochron to intercept': calculates the initial (aka 'inherited', 'excess', or 'common') $^{40}\text{Ar}/^{36}\text{Ar}$ , $^{40}\text{Ca}/^{44}\text{Ca}$ , $^{87}\text{Sr}/^{86}\text{Sr}$ , $^{143}\text{Nd}/^{144}\text{Nd}$ , $^{187}\text{Os}/^{188}\text{Os}$ or $^{176}\text{Hf}/^{177}\text{Hf}$ ratio from an isochron fit. Setting i2i to FALSE uses the default values stored in settings('iratio', ...). When applied to data of class ThU, setting i2i to TRUE applies a detrital Th-correction.
detritus	detrital $^{230}\text{Th}$ correction (only applicable when x\$format == 1 or 2). 0: no correction 1: project the data along an isochron fit 2: correct the data using an assumed initial $^{230}\text{Th}/^{232}\text{Th}$ -ratio for the detritus. 3: correct the data using the measured present day $^{230}\text{Th}/^{238}\text{U}$ , $^{232}\text{Th}/^{238}\text{U}$ and $^{234}\text{U}/^{238}\text{U}$ -ratios in the detritus.

## Details

Consider a dataset of  $n$  dates  $\{t_1, t_2, \dots, t_n\}$  with analytical uncertainties  $\{s[t_1], s[t_2], \dots, s[t_n]\}$ . Define  $z_i = \log(t_i)$  and  $s[z_i] = s[t_i]/t_i$ . Suppose that these  $n$  values are derived from a mixture of  $k > 2$  populations with means  $\{\mu_1, \dots, \mu_k\}$ . Such a *discrete mixture* may be mathematically described by:

$$P(z_i|\mu, \omega) = \sum_{j=1}^k \pi_j N(z_i|\mu_j, s[z_j]^2)$$

where  $\pi_j$  is the proportion of the population that belongs to the  $j^{th}$  component, and  $\pi_k = 1 - \sum_{j=1}^{k-1} \pi_j$ . This equation can be solved by the method of maximum likelihood (Galbraith and Laslett, 1993). IsoplotR implements the Bayes Information Criterion (BIC) as a means of automatically choosing  $k$ . This option should be used with caution, as the number of peaks steadily rises with sample size ( $n$ ). If one is mainly interested in the youngest age component, then it is more productive to use an alternative parameterisation, in which all grains are assumed to come from one of two components, whereby the first component is a single discrete age peak ( $\exp(m)$ , say) and the second component is a continuous distribution (as described by the [central](#) age model), but truncated at this discrete value (Van der Touw et al., 1997).

**Value**

Returns a list with the following items:

**peaks** a  $3 \times k$  matrix with the following rows:

t: the ages of the k peaks

s[t]: the estimated uncertainties of t

ci[t]: the widths of approximate  $100(1 - \alpha)\%$  confidence intervals for t

**props** a  $2 \times k$  matrix with the following rows:

p: the proportions of the k peaks

s[p]: the estimated uncertainties (standard errors) of p

**L** the log-likelihood of the fit

**legend** a vector of text expressions to be used in a figure legend

**References**

Galbraith, R.F. and Laslett, G.M., 1993. Statistical models for mixed fission track ages. Nuclear Tracks and Radiation Measurements, 21(4), pp.459-470.

van der Touw, J., Galbraith, R., and Laslett, G. A logistic truncated normal mixture model for overdispersed binomial data. Journal of Statistical Computation and Simulation, 59(4):349-373, 1997.

**See Also**

[radialplot](#), [central](#)

**Examples**

```
data(examples)
peakfit(examples$FT1,k=2)

peakfit(examples$LudwigMixture,k='min')
```

---

radialplot

---

*Visualise heteroscedastic data on a radial plot*


---

**Description**

Implementation of a graphical device developed by Rex Galbraith to display several estimates of the same quantity that have different standard errors.

**Usage**

```

radialplot(x, ...)

## Default S3 method:
radialplot(x, from = NA, to = NA, t0 = NA,
  transformation = "log", sigdig = 2, show.numbers = FALSE,
  pch = 21, levels = NA, clabel = "", bg = c("yellow", "red"),
  col = "black", title = TRUE, k = 0, markers = NULL,
  alpha = 0.05, units = "", hide = NA, omit = NA, omit.col = NA,
  ...)

## S3 method for class 'fissiontracks'
radialplot(x, from = NA, to = NA, t0 = NA,
  transformation = "arcsin", sigdig = 2, show.numbers = FALSE,
  pch = 21, levels = NA, clabel = "", bg = c("yellow", "red"),
  col = "black", title = TRUE, markers = NULL, k = 0,
  exterr = TRUE, alpha = 0.05, hide = NULL, omit = NULL,
  omit.col = NA, ...)

## S3 method for class 'UPb'
radialplot(x, from = NA, to = NA, t0 = NA,
  transformation = "log", type = 4, cutoff.76 = 1100,
  cutoff.disc = c(-15, 5), show.numbers = FALSE, pch = 21,
  levels = NA, clabel = "", bg = c("yellow", "red"), col = "black",
  markers = NULL, k = 0, exterr = TRUE, common.Pb = 0,
  alpha = 0.05, hide = NULL, omit = NULL, omit.col = NA, ...)

## S3 method for class 'PbPb'
radialplot(x, from = NA, to = NA, t0 = NA,
  transformation = "log", show.numbers = FALSE, pch = 21,
  levels = NA, clabel = "", bg = c("yellow", "red"), col = "black",
  markers = NULL, k = 0, exterr = TRUE, common.Pb = 1,
  alpha = 0.05, hide = NULL, omit = NULL, omit.col = NA, ...)

## S3 method for class 'ArAr'
radialplot(x, from = NA, to = NA, t0 = NA,
  transformation = "log", show.numbers = FALSE, pch = 21,
  levels = NA, clabel = "", bg = c("yellow", "red"), col = "black",
  markers = NULL, k = 0, exterr = TRUE, i2i = FALSE,
  alpha = 0.05, hide = NULL, omit = NULL, omit.col = NA, ...)

## S3 method for class 'KCa'
radialplot(x, from = NA, to = NA, t0 = NA,
  transformation = "log", show.numbers = FALSE, pch = 21,
  levels = NA, clabel = "", bg = c("yellow", "red"), col = "black",
  markers = NULL, k = 0, exterr = TRUE, i2i = FALSE,
  alpha = 0.05, hide = NULL, omit = NULL, omit.col = NA, ...)

```

```
## S3 method for class 'UThHe'
radialplot(x, from = NA, to = NA, t0 = NA,
  transformation = "log", show.numbers = FALSE, pch = 21,
  levels = NA, clabel = "", bg = c("yellow", "red"), col = "black",
  markers = NULL, k = 0, alpha = 0.05, hide = NULL, omit = NULL,
  omit.col = NA, ...)

## S3 method for class 'ReOs'
radialplot(x, from = NA, to = NA, t0 = NA,
  transformation = "log", show.numbers = FALSE, pch = 21,
  levels = NA, clabel = "", bg = c("yellow", "red"), col = "black",
  markers = NULL, k = 0, exterr = TRUE, i2i = TRUE, alpha = 0.05,
  hide = NULL, omit = NULL, omit.col = NA, ...)

## S3 method for class 'SmNd'
radialplot(x, from = NA, to = NA, t0 = NA,
  transformation = "log", show.numbers = FALSE, pch = 21,
  levels = NA, clabel = "", bg = c("yellow", "red"), col = "black",
  markers = NULL, k = 0, exterr = TRUE, i2i = TRUE, alpha = 0.05,
  hide = NULL, omit = NULL, omit.col = NA, ...)

## S3 method for class 'RbSr'
radialplot(x, from = NA, to = NA, t0 = NA,
  transformation = "log", show.numbers = FALSE, pch = 21,
  levels = NA, clabel = "", bg = c("yellow", "red"), col = "black",
  markers = NULL, k = 0, exterr = TRUE, i2i = TRUE, alpha = 0.05,
  hide = NULL, omit = NULL, omit.col = NA, ...)

## S3 method for class 'LuHf'
radialplot(x, from = NA, to = NA, t0 = NA,
  transformation = "log", show.numbers = FALSE, pch = 21,
  levels = NA, clabel = "", bg = c("yellow", "red"), col = "black",
  markers = NULL, k = 0, exterr = TRUE, i2i = TRUE, alpha = 0.05,
  hide = NULL, omit = NULL, omit.col = NA, ...)

## S3 method for class 'ThU'
radialplot(x, from = NA, to = NA, t0 = NA,
  transformation = "log", show.numbers = FALSE, pch = 21,
  levels = NA, clabel = "", bg = c("yellow", "red"), col = "black",
  markers = NULL, k = 0, i2i = TRUE, alpha = 0.05, detritus = 0,
  hide = NULL, omit = NULL, omit.col = NA, ...)
```

## Arguments

**x** Either an  $[n \times 2]$  matrix of (transformed) values  $z$  and their standard errors  $s$   
OR  
and object of class `fissiontracks`, `UThHe`, `ArAr`, `KCa`, `ReOs`, `SmNd`, `RbSr`, `LuHf`,  
`ThU`, `PbPb` or `UPb`



...	additional arguments to the generic points function
from	minimum age limit of the radial scale
to	maximum age limit of the radial scale
t0	central value
transformation	one of either log, linear, sqrt or arcsin (if x has class <code>fissiontracks</code> and <code>fissiontracks\$type</code> $\neq$ 1).
sigdig	the number of significant digits of the numerical values reported in the title of the graphical output.
show.numbers	boolean flag (TRUE to show grain numbers)
pch	plot character (default is a filled circle)
levels	a vector with additional values to be displayed as different background colours of the plot symbols.
clabel	label of the colour legend
bg	a vector of two background colours for the plot symbols. If <code>levels=NA</code> , then only the first colour is used. If <code>levels</code> is a vector of numbers, then <code>bg</code> is used to construct a colour ramp.
col	text colour to be used if <code>show.numbers=TRUE</code>
title	add a title to the plot?
k	number of peaks to fit using the finite mixture models of Galbraith and Laslett (1993). Setting <code>k='auto'</code> automatically selects an optimal number of components based on the Bayes Information Criterion (BIC). Setting <code>k='min'</code> estimates the minimum value using a three parameter model consisting of a Normal distribution truncated by a discrete component.
markers	vector of ages of radial marker lines to add to the plot.
alpha	cutoff value for confidence intervals
units	measurement units to be displayed in the legend.
hide	vector with indices of aliquots that should be removed from the radial plot.
omit	vector with indices of aliquots that should be plotted but omitted from the central age calculation or mixture models.
omit.col	colour that should be used for the omitted aliquots.
exterr	propagate the external sources of uncertainty into the mixture model errors?
type	scalar indicating whether to plot the $^{207}\text{Pb}/^{235}\text{U}$ age ( <code>type=1</code> ), the $^{206}\text{Pb}/^{238}\text{U}$ age ( <code>type=2</code> ), the $^{207}\text{Pb}/^{206}\text{Pb}$ age ( <code>type=3</code> ), the $^{207}\text{Pb}/^{206}\text{Pb}$ - $^{206}\text{Pb}/^{238}\text{U}$ age ( <code>type=4</code> ), or the (Wetherill) concordia age ( <code>type=5</code> )
cutoff.76	the age (in Ma) below which the $^{206}\text{Pb}/^{238}\text{U}$ and above which the $^{207}\text{Pb}/^{206}\text{Pb}$ age is used. This parameter is only used if <code>type=4</code> .
cutoff.disc	two element vector with the maximum and minimum percentage discordance allowed between the $^{207}\text{Pb}/^{235}\text{U}$ and $^{206}\text{Pb}/^{238}\text{U}$ age (if $^{206}\text{Pb}/^{238}\text{U} < \text{cutoff.76}$ ) or between the $^{206}\text{Pb}/^{238}\text{U}$ and $^{207}\text{Pb}/^{206}\text{Pb}$ age (if $^{206}\text{Pb}/^{238}\text{U} > \text{cutoff.76}$ ). Set <code>cutoff.disc=NA</code> if you do not want to use this filter.

common.Pb	<p>apply a common lead correction using one of three methods:</p> <ol style="list-style-type: none"> <li>1: use the isochron intercept as the initial Pb-composition</li> <li>2: use the Stacey-Kramers two-stage model to infer the initial Pb-composition</li> <li>3: use the Pb-composition stored in <code>settings('iratio', 'Pb206Pb204')</code> and <code>settings('iratio', 'Pb207Pb204')</code></li> </ol>
i2i	<p>‘isochron to intercept’: calculates the initial (aka ‘inherited’, ‘excess’, or ‘common’) <math>^{40}\text{Ar}/^{36}\text{Ar}</math>, <math>^{40}\text{Ca}/^{44}\text{Ca}</math>, <math>^{207}\text{Pb}/^{204}\text{Pb}</math>, <math>^{87}\text{Sr}/^{86}\text{Sr}</math>, <math>^{143}\text{Nd}/^{144}\text{Nd}</math>, <math>^{187}\text{Os}/^{188}\text{Os}</math>, <math>^{230}\text{Th}/^{232}\text{Th}</math> or <math>^{176}\text{Hf}/^{177}\text{Hf}</math> ratio from an isochron fit. Setting <code>i2i</code> to <code>FALSE</code> uses the default values stored in <code>settings('iratio', ...)</code>.</p>
detritus	<p>detrital <math>^{230}\text{Th}</math> correction (only applicable when <code>x\$format == 1</code> or <code>2</code>).</p> <p>0: no correction</p> <ol style="list-style-type: none"> <li>1: project the data along an isochron fit</li> <li>2: correct the data using an assumed initial <math>^{230}\text{Th}/^{232}\text{Th}</math>-ratio for the detritus.</li> <li>3: correct the data using the measured present day <math>^{230}\text{Th}/^{238}\text{U}</math>, <math>^{232}\text{Th}/^{238}\text{U}</math> and <math>^{234}\text{U}/^{238}\text{U}</math>-ratios in the detritus.</li> </ol>

## Details

The radial plot (Galbraith, 1988, 1990) is a graphical device that was specifically designed to display heteroscedastic data, and is constructed as follows. Consider a set of dates  $\{t_1, \dots, t_i, \dots, t_n\}$  and uncertainties  $\{s[t_1], \dots, s[t_i], \dots, s[t_n]\}$ . Define  $z_i = z[t_i]$  to be a transformation of  $t_i$  (e.g.,  $z_i = \log[t_i]$ ), and let  $s[z_i]$  be its propagated analytical uncertainty (i.e.,  $s[z_i] = s[t_i]/t_i$  in the case of a logarithmic transformation). Create a scatterplot of  $(x_i, y_i)$  values, where  $x_i = 1/s[z_i]$  and  $y_i = (z_i - z_o)/s[z_i]$ , where  $z_o$  is some reference value such as the mean. The slope of a line connecting the origin of this scatterplot with any of the  $(x_i, y_i)$ s is proportional to  $z_i$  and, hence, the date  $t_i$ . These dates can be more easily visualised by drawing a radial scale at some convenient distance from the origin and annotating it with labelled ticks at the appropriate angles. While the angular position of each data point represents the date, its horizontal distance from the origin is proportional to the precision. Imprecise measurements plot on the left hand side of the radial plot, whereas precise age determinations are found further towards the right. Thus, radial plots allow the observer to assess both the magnitude and the precision of quantitative data in one glance.

## References

- Galbraith, R.F., 1988. Graphical display of estimates having differing standard errors. *Technometrics*, 30(3), pp.271-281.
- Galbraith, R.F., 1990. The radial plot: graphical assessment of spread in ages. *International Journal of Radiation Applications and Instrumentation. Part D. Nuclear Tracks and Radiation Measurements*, 17(3), pp.207-214.
- Galbraith, R.F. and Laslett, G.M., 1993. Statistical models for mixed fission track ages. *Nuclear Tracks and Radiation Measurements*, 21(4), pp.459-470.

## See Also

[peakfit](#), [central](#)

**Examples**

```
data(examples)
radialplot(examples$FT1)

dev.new()
radialplot(examples$LudwigMixture,k='min')
```

---

read.data	<i>Read geochronology data</i>
-----------	--------------------------------

---

**Description**

Cast a .csv file or a matrix into one of IsoplotR's data classes

**Usage**

```
read.data(x, ...)

## Default S3 method:
read.data(x, method = "U-Pb", format = 1, ierr = 1,
  U48 = 1, Th0U8 = 1, Ra6U8 = 1, Pa1U5 = 1, Th02 = c(0, 0),
  Th02U48 = c(0, 0, 1e+06, 0, 0, 0, 0, 0, 0), ...)

## S3 method for class 'data.frame'
read.data(x, method = "U-Pb", format = 1,
  ierr = 1, U48 = 1, Th0U8 = 1, Ra6U8 = 1, Pa1U5 = 1,
  Th02 = c(0, 0), Th02U48 = c(0, 0, 1e+06, 0, 0, 0, 0, 0, 0), ...)

## S3 method for class 'matrix'
read.data(x, method = "U-Pb", format = 1, ierr = 1,
  U48 = 1, Th0U8 = 1, Ra6U8 = 1, Pa1U5 = 1, Th02 = c(0, 0),
  Th02U48 = c(0, 0, 1e+06, 0, 0, 0, 0, 0, 0), ...)
```

**Arguments**

x	either a file name (.csv format) OR a matrix
...	optional arguments to the read.csv function
method	one of 'U-Pb', 'Pb-Pb', 'Ar-Ar', 'detritals', 'Rb-Sr', 'Sm-Nd', 'Re-Os', 'Th-U', 'U-Th-He', 'fissiontracks' or 'other'
format	formatting option, depends on the value of method. if method='U-Pb', then format is one of either: <ol style="list-style-type: none"> <li>1. 7/5, err[7/5], 6/8, err[6/8], rho</li> <li>2. 8/6, err[8/6], 7/6, err[7/6] (, rho)</li> <li>3. X=7/6, err[X], Y=6/8, err[Y], Z=7/6, err[Z] (, rho[X,Y]) (, rho[Y,Z])</li> </ol>

4. X=7/5, err[X], Y=6/8, err[Y], Z=4/8, rho[X,Y], rho[X,Z], rho[Y,Z]
5. X=8/6, err[X], Y=7/6, err[Y], Z=4/6, rho[X,Y], rho[X,Z], rho[Y,Z]
6. 7/5, err[7/5], 6/8, err[6/8], 4/8, err[4/8], 7/6, err[7/6], 4/7, err[4/7], 4/6, err[4/6]

where optional columns are marked in round brackets

if method='Pb-Pb', then format is one of either:

1. 6/4, err[6/4], 7/4, err[7/4], rho
2. 4/6, err[4/6], 7/6, err[7/6], rho
3. 6/4, err[6/4], 7/4, err[7/4], 7/6, err[7/6]

if method='Ar-Ar', then format is one of either:

1. 9/6, err[9/6], 0/6, err[0/6], rho (, 39)
2. 6/0, err[6/0], 9/0, err[9/0] (, rho) (, 39)
3. 9/0, err[9/0], 6/0, err[6/0], 9/6, err[9/6] (, 39)

if method='K-Ca', then format is one of either:

1. K40/Ca44, err[K40/Ca44], Ca40/Ca44, err[Ca40/Ca44], rho
2. K40/Ca44, err[K40/Ca44], Ca40/Ca44, err[Ca40/Ca44], K40/Ca40, err[K40/Ca40]

if method='Rb-Sr', then format is one of either:

1. Rb87/Sr86, err[Rb87/Sr86], Sr87/Sr86, err[Sr87/Sr86] (, rho)
2. Rb, err[Rb], Sr, err[Sr], Sr87/Sr86, err[Sr87/Sr86]

where Rb and Sr are in ppm

if method='Sm-Nd', then format is one of either:

1. Sm147/Nd144, err[Sm147/Nd144], Nd143/Nd144, err[Nd143/Nd144] (, rho)
2. Sm, err[Sm], Nd, err[Nd], Nd143/Nd144, err[Nd143/Nd144]

where Sm and Nd are in ppm

if method='Re-Os', then format is one of either:

1. Re187/Os188, err[Re187/Os188], Os187/Os188, err[Os187/Os188] (, rho)
2. Re, err[Re], Os, err[Os], Os187/Os188, err[Os187/Os188]

where Re and Os are in ppm

if method='Lu-Hf', then format is one of either:

1. Lu176/Hf177, err[Lu176/Hf177], Hf176/Hf177, err[Hf176/Hf177] (, rho)
2. Lu, err[Lu], Hf, err[Hf], Hf176/Hf177, err[Hf176/Hf177]

where Lu and Hf are in ppm

if method='Th-U', then format is one of either:

1. X=8/2, err[X], Y=4/2, err[Y], Z=0/2, err[Z], rho[X,Y], rho[X,Z], rho[Y,Z]
2. X=2/8, err[X], Y=4/8, err[Y], Z=0/8, err[Z], rho[X,Y], rho[X,Z], rho[Y,Z]
3. X=8/2, err[X], Y=0/2, err[Y], rho[X,Y]
4. X=2/8, err[X], Y=0/8, err[Y], rho[X,Y]

where all values are activity ratios

if method='fissiontracks', then format is one of either:

1. the External Detector Method (EDM), which requires a  $\zeta$ -calibration constant and its uncertainty, the induced track density in a dosimeter glass, and a table with the spontaneous and induced track densities.
2. LA-ICP-MS-based fission track data using the  $\zeta$ -calibration method, which requires a 'session  $\zeta$ ' and its uncertainty and a table with the number of spontaneous tracks, the area over which these were counted and one or more U/Ca- or U-concentration measurements and their analytical uncertainties.
3. LA-ICP-MS-based fission track data using the 'absolute dating' method, which only requires a table with the the number of spontaneous tracks, the area over which these were counted and one or more U/Ca-ratios or U-concentration measurements (in ppm) and their analytical uncertainties.

if method='other', x is read as a table, unless format is one of either:

radial **or** 'average': X, err[X]

regression: X, err[X], Y, err[Y], rho

OR X/Z, err[X/Z], Y/Z, err[Y/Z], X/Y, err[X/Y]

spectrum: f, X, err[X]

ierr indicates whether the analytical uncertainties are reported as:

1.  $1\sigma$  absolute uncertainties.
2.  $2\sigma$  absolute uncertainties.
3.  $1\sigma$  relative uncertainties (%).
4.  $2\sigma$  relative uncertainties (%).

U48 the initial  $^{234}\text{U}/^{238}\text{U}$ -activity ratio

Th0U8 the initial  $^{230}\text{Th}/^{238}\text{U}$ -activity ratio

Ra6U8 the initial  $^{226}\text{Ra}/^{238}\text{U}$ -activity ratio

Pa1U5 the initial  $^{231}\text{Pa}/^{235}\text{U}$ -activity ratio

Th02 2-element vector with the assumed initial  $^{230}\text{Th}/^{232}\text{Th}$ -ratio of the detritus and its standard error. Only used if isochron==FALSE and detritus==2

Th02U48 9-element vector with the measured composition of the detritus, containing X=0/8, sX, Y=2/8, sY, Z=4/8, sZ, rXY, rXZ, rYZ.

## Details

IsoplotR provides the following example input files:

- U-Pb: UPb1.csv, UPb2.csv, UPb3.csv, UPb4.csv, UPb5.csv, UPb6.csv
- Pb-Pb: PbPb1.csv, PbPb2.csv, PbPb3.csv
- Ar-Ar: ArAr1.csv, ArAr2.csv, ArAr3.csv
- K-Ca: KCa1.csv, KCa2.csv,
- Re-Os: ReOs1.csv, ReOs2.csv
- Sm-Nd: SmNd1.csv, SmNd2.csv
- Rb-Sr: RbSr1.csv, RbSr2.csv
- Lu-Hf: LuHf1.csv, LuHf2.csv

- Th-U: ThU1.csv, ThU2.csv, ThU3.csv, ThU4.csv
- fissiontracks: FT1.csv, FT2.csv, FT3.csv
- U-Th-He: UThHe.csv, UThSmHe.csv
- detritals: DZ.csv
- other: LudwigMixture.csv, LudwigMean.csv, LudwigKDE.csv LudwigSpectrum.csv

The contents of these files can be viewed using the `system.file(...)` function. For example, to read the `ArAr1.csv` file:

```
fname <- system.file('ArAr1.csv', package='IsoplotR')
ArAr <- read.data(fname, method='Ar-Ar', format=1)
```

### Value

an object of class `UPb`, `PbPb`, `ArAr`, `KCa`, `UThHe`, `ReOs`, `SmNd`, `RbSr`, `LuHf`, `detritals`, `fissiontracks`, `ThU` or other

### See Also

[examples](#), [settings](#)

### Examples

```
f1 <- system.file("UPb1.csv", package="IsoplotR")
file.show(f1) # inspect the contents of 'UPb1.csv'
d1 <- read.data(f1, method="U-Pb", format=1)
concordia(d1)

f2 <- system.file("ArAr1.csv", package="IsoplotR")
d2 <- read.data(f2, method="Ar-Ar", format=1)
agespectrum(d2)

f3 <- system.file("ReOs1.csv", package="IsoplotR")
d3 <- read.data(f3, method="Re-Os", format=1)
isochron(d2)

f4 <- system.file("FT1.csv", package="IsoplotR")
d4 <- read.data(f4, method="fissiontracks", format=1)
radialplot(d4)

f5 <- system.file("UThSmHe.csv", package="IsoplotR")
d5 <- read.data(f5, method="U-Th-He")
helioplot(d5)

f6 <- system.file("ThU2.csv", package="IsoplotR")
d6 <- read.data(f6, method="Th-U", format=2)
evolution(d6)

# one detrital zircon U-Pb file (detritals.csv)
f7 <- system.file("DZ.csv", package="IsoplotR")
```

```

d7 <- read.data(f7,method="detritals")
kde(d7)

# four 'other' files (LudwigMixture.csv, LudwigSpectrum.csv,
# LudwigMean.csv, LudwigKDE.csv)
f8 <- system.file("LudwigMixture.csv",package="IsoplotR")
d8 <- read.data(f8,method="other")
radialplot(d8)

```

---

scatterplot

---

*Create a scatter plot with error ellipses or crosses*


---

### Description

Takes bivariate data with (correlated) uncertainties as input and produces a scatter plot with error ellipses or crosses as output. (optionally) displays the linear fit on this diagram, and can show a third variable as a colour scale.

### Usage

```

scatterplot(xy, xlim = NA, ylim = NA, alpha = 0.05,
  show.numbers = FALSE, show.ellipses = 1, levels = NA,
  clabel = "", ellipse.col = c("#00FF0080", "#FF000080"),
  fit = "none", add = FALSE, empty = FALSE, ci.col = "gray80",
  line.col = "black", lwd = 1, hide = NULL, omit = NULL,
  omit.col = NA, addcolourbar = TRUE, ...)

```

### Arguments

xy	matrix with columns X, sX, Y, sY(, rXY)
xlim	(optional) two-element vector with the x-axis limits
ylim	(optional) two-element vector with the y-axis limits
alpha	the probability cutoff for the error ellipses
show.numbers	logical flag (TRUE to show grain numbers)
show.ellipses	show the data as: <ol style="list-style-type: none"> <li>1. points</li> <li>2. error ellipses</li> <li>3. error crosses</li> </ol>
levels	a vector with additional values to be displayed as different background colours within the error ellipses.
clabel	label for the colour scale
ellipse.col	a vector of two background colours for the error ellipses. If levels=NA, then only the first colour will be used. If levels is a vector of numbers, then ellipse.col is used to construct a colour ramp.

fit	the output of york() (optional).
add	if TRUE, adds the points and lines to the existing plot.
empty	set up an empty plot with the right axis limits to fit the data
ci.col	the fill colour for the confidence interval of the intercept and slope.
line.col	colour of the regression line
lwd	line width of the regression line
hide	vector with indices of aliquots that should be removed from the plot.
omit	vector with indices of aliquots that should be plotted but omitted from the isochron age calculation.
omit.col	colour that should be used for the omitted aliquots.
addcolourbar	add a colour bar to display the colours used to levels
...	optional arguments to format the points and text.

### Examples

```
X <- c(1.550,12.395,20.445,20.435,20.610,24.900,
      28.530,50.540,51.595,86.51,106.40,157.35)
Y <- c(.7268,.7809,.8200,.8116,.8160,.8302,
      .8642,.9534,.9617,1.105,1.230,1.440)
sX <- X*0.02
sY <- Y*0.01
dat <- cbind(X,sX,Y,sY)
scatterplot(dat,fit=york(dat),show.ellipses=2)
```

---

set.zeta

*Calculate the zeta calibration coefficient for fission track dating*

---

### Description

Determines the zeta calibration constant of a fission track dataset (EDM or LA-ICP-MS) given its true age and analytical uncertainty.

### Usage

```
set.zeta(x, tst, exterr = TRUE, update = TRUE, sigdig = 2)
```

### Arguments

x	an object of class fissiontracks
tst	a two-element vector with the true age and its standard error
exterr	logical flag indicating whether the external uncertainties associated with the age standard or the dosimeter glass (for the EDM) should be accounted for when propagating the uncertainty of the zeta calibration constant.
update	logical flag indicating whether the function should return an updated version of the input data, or simply return a two-element vector with the calibration constant and its standard error.
sigdig	number of significant digits



## Details

The fundamental fission track age is given by:

$$t = \frac{1}{\lambda_{238}} \ln \left( 1 + \frac{\lambda_{238}}{\lambda_f} \frac{2N_s}{[^{238}\text{U}]A_s L} \right) \quad (\text{eq.1})$$

where  $N_s$  is the number of spontaneous fission tracks measured over an area  $A_s$ ,  $[^{238}\text{U}]$  is the  $^{238}\text{U}$ -concentration in atoms per unit volume,  $\lambda_f$  is the fission decay constant,  $L$  is the etchable fission track length, and the factor 2 is a geometric factor accounting for the fact that etching reveals tracks from both above and below the internal crystal surface. Two analytical approaches are used to measure  $[^{238}\text{U}]$ : neutron activation and LAICPMS. The first approach estimates the  $^{238}\text{U}$ -concentration indirectly, using the induced fission of neutron-irradiated  $^{235}\text{U}$  as a proxy for the  $^{238}\text{U}$ . In the most common implementation of this approach, the induced fission tracks are recorded by an external detector made of mica or plastic that is attached to the polished grain surface (Fleischer and Hart, 1972; Hurford and Green, 1983). The fission track age equation then becomes:

$$t = \frac{1}{\lambda_{238}} \ln \left( 1 + \frac{\lambda_{238} \zeta \rho_d}{2} \frac{N_s}{N_i} \right) \quad (\text{eq.2})$$

where  $N_i$  is the number of induced fission tracks counted in the external detector over the same area as the spontaneous tracks,  $\zeta$  is a ‘zeta’-calibration factor that incorporates both the fission decay constant and the etchable fission track length, and  $\rho_d$  is the number of induced fission tracks per unit area counted in a co-irradiated glass of known U-concentration.  $\rho_d$  allows the  $\zeta$ -factor to be ‘recycled’ between irradiations.

LAICPMS is an alternative means of determining the  $^{238}\text{U}$ -content of fission track samples without the need for neutron irradiation. The resulting U-concentrations can be plugged directly into the fundamental age equation (eq.1). but this is limited by the accuracy of the U-concentration measurements, the fission track decay constant and the etching and counting efficiencies. Alternatively, these sources of bias may be removed by normalising to a standard of known fission track age and defining a new ‘zeta’ calibration constant  $\zeta_{icp}$ :

$$t = \frac{1}{\lambda_{238}} \ln \left( 1 + \frac{\lambda_{238} \zeta_{icp}}{2} \frac{N_s}{[^{238}\text{U}]A_s} \right) \quad (\text{eq.3})$$

where  $[^{238}\text{U}]$  may either stand for the  $^{238}\text{U}$ -concentration (in ppm) *or* for the U/Ca (for apatite) or U/Si (for zircon) ratio measurement (Vermeesch, 2017).

## Value

an object of class `fissiontracks` with an updated `x$zeta` value

## References

- Fleischer, R. and Hart, H. Fission track dating: techniques and problems. In Bishop, W., Miller, J., and Cole, S., editors, *Calibration of Hominoid Evolution*, pages 135-170. Scottish Academic Press Edinburgh, 1972.
- Hurford, A. J. and Green, P. F. The zeta age calibration of fission-track dating. *Chemical Geology*, 41:285-317, 1983.
- Vermeesch, P., 2017. Statistics for LA-ICP-MS based fission track dating. *Chemical Geology*, 456, pp.19-27.

## See Also

[age](#)

## Examples

```
data(examples)
print(examples$FT1$zeta)
FT <- set.zeta(examples$FT1,tst=c(250,5))
print(FT$zeta)
```

---

settings

*Load settings to and from json*

---

## Description

Get and set preferred values for decay constants, isotopic abundances, molar masses, fission track etch efficiencies, and etchable lengths, and mineral densities, either individually or via a .json file format.

## Usage

```
settings(setting = NA, ..., fname = NA)
```

## Arguments

setting	unless fname is provided, this should be one of either: 'lambda': to get and set decay constants 'iratio': isotopic ratios 'imass': isotopic molar masses 'mindens': mineral densities 'etchfact': fission track etch efficiency factors 'tracklength': equivalent isotropic fission track length
...	depends on the value for setting: <ul style="list-style-type: none"> <li>• for 'lambda': the isotope of interest (one of either "fission", "U238", "U235", "U234", "Th232", "Th230", "Re187", "Sm147", "Rb87", "Lu176", or "K40") PLUS (optionally) the decay constant value and its analytical error. Omitting the latter two numbers simply returns the existing values.</li> <li>• for 'iratio': the isotopic ratio of interest (one of either "Ar40Ar36", "Ar38Ar36", "Ca40Ca44", "Rb85Rb87", "Sr88Sr86", "Sr87Sr86", "Sr84Sr86", "Re185Re187", "Os184Os192", "Os186Os192", "Os187Os192", "Os188Os192", "Os189Os192", "Os190Os192", "U238U235", "Sm144Sm152", "Sm147Sm152", "Sm148Sm152", "Sm149Sm152", "Sm150Sm152", "Sm154Sm152", "Nd142Nd144", "Nd143Nd144", "Nd145Nd144", "Nd146Nd144", "Nd148Nd144", "Nd150Nd144", "Lu176Lu175", "Hf174Hf177", "Hf176Hf177", "Hf178Hf177", "Hf179Hf177", "Hf180Hf177") PLUS (optionally) the isotopic ratio and its analytical error. Omitting the latter two numbers simply returns the existing values.</li> </ul>

- for 'imass': the (isotopic) molar mass of interest (one of either "U", "Rb", "Rb85", "Rb87", "Sr84", "Sr86", "Sr87", "Sr88", "Re", "Re185", "Re187", "Os", "Os184", "Os186", "Os187", "Os188", "Os189", "Os190", "Os192", "Sm", "Nd", "Lu", "Hf") PLUS (optionally) the molar mass and its analytical error. Omitting the latter two numbers simply returns the existing values.
- for 'mindens': the mineral of interest (one of either "apatite" or "zircon") PLUS the mineral density. Omitting the latter number simply returns the existing value.
- 'etchfact': the mineral of interest (one of either "apatite" or "zircon") PLUS the etch efficiency factor. Omitting this number simply returns the existing value.
- 'tracklength': the mineral of interest (one of either "apatite" or "zircon") PLUS the equivalent isotropic fission track length. Omitting this number simply returns the existing value.

fname                      the path of a .json file

## Value

if setting=NA and fname=NA, returns a .json string

if ... contains only the name of an isotope, isotopic ratio, element, or mineral and no new value, then settings returns either a scalar with the existing value, or a two-element vector with the value and its uncertainty.

## References

### 1. Decay constants:

- $^{238}\text{U}$ ,  $^{235}\text{U}$ : Jaffey, A. H., et al. "Precision measurement of half-lives and specific activities of  $\text{U}^{235}$  and  $\text{U}^{238}$ ." *Physical Review C* 4.5 (1971): 1889.
- $^{232}\text{Th}$ : Le Roux, L. J., and L. E. Glendenin. "Half-life of  $^{232}\text{Th}$ ." *Proceedings of the National Meeting on Nuclear Energy*, Pretoria, South Africa. 1963.
- $^{234}\text{U}$ ,  $^{230}\text{Th}$ : Cheng, H., Edwards, R.L., Shen, C.C., Polyak, V.J., Asmerom, Y., Woodhead, J., Hellstrom, J., Wang, Y., Kong, X., Spotl, C. and Wang, X., 2013. Improvements in  $^{230}\text{Th}$  dating,  $^{230}\text{Th}$  and  $^{234}\text{U}$  half-life values, and U-Th isotopic measurements by multi-collector inductively coupled plasma mass spectrometry. *Earth and Planetary Science Letters*, 371, pp.82-91.
- $^{231}\text{Pa}$ : Audi, G., Bersillon, O., Blachot, J. and Wapstra, A.H., 2003. The NUBASE evaluation of nuclear and decay properties. *Nuclear Physics A*, 729(1), pp.3-128.
- $^{226}\text{Ra}$ : Audi, G., Bersillon, O., Blachot, J. and Wapstra, A.H., 2003. The NUBASE evaluation of nuclear and decay properties. *Nuclear Physics A*, 729(1), pp.3-128.
- Sm: Lugmair, G. W., and K. Marti. "Lunar initial  $^{143}\text{Nd}/^{144}\text{Nd}$ : differential evolution of the lunar crust and mantle." *Earth and Planetary Science Letters* 39.3 (1978): 349-357.
- Nd: Zhao, Motian, et al. "Absolute measurements of neodymium isotopic abundances and atomic weight by MC-ICPMS." *International Journal of Mass Spectrometry* 245.1 (2005): 36-40.

- Re: Selby, D., Creaser, R.A., Stein, H.J., Markey, R.J. and Hannah, J.L., 2007. Assessment of the  $^{187}\text{Re}$  decay constant by cross calibration of Re-Os molybdenite and U-Pb zircon chronometers in magmatic ore systems. *Geochimica et Cosmochimica Acta*, 71(8), pp.1999-2013.
  - Ar: Renne, Paul R., et al. "Response to the comment by WH Schwarz et al. on "Joint determination of  $^{40}\text{K}$  decay constants and  $^{40}\text{Ar}^*/^{40}\text{K}$  for the Fish Canyon sanidine standard, and improved accuracy for  $^{40}\text{Ar}/^{39}\text{Ar}$  geochronology" by PR Renne et al.(2010)." *Geochimica et Cosmochimica Acta* 75.17 (2011): 5097-5100.
  - Rb: Villa, I.M., De Bièvre, P., Holden, N.E. and Renne, P.R., 2015. "IUPAC-IUGS recommendation on the half life of  $^{87}\text{Rb}$ ". *Geochimica et Cosmochimica Acta*, 164, pp.382-385.
  - Lu: Soederlund, Ulf, et al. "The  $^{176}\text{Lu}$  decay constant determined by Lu-Hf and U-Pb isotope systematics of Precambrian mafic intrusions." *Earth and Planetary Science Letters* 219.3 (2004): 311-324.
2. Isotopic ratios:
- Ar: Lee, Jee-Yon, et al. "A redetermination of the isotopic abundances of atmospheric Ar." *Geochimica et Cosmochimica Acta* 70.17 (2006): 4507-4512.
  - Ca: Moore, L.J. and Machlan, L.A., 1972. High-accuracy determination of calcium in blood serum by isotope dilution mass spectrometry. *Analytical chemistry*, 44(14), pp.2291-2296.
  - Rb: Catanzaro, E. J., et al. "Absolute isotopic abundance ratio and atomic weight of terrestrial rubidium." *J. Res. Natl. Bur. Stand. A* 73 (1969): 511-516.
  - Sr: Moore, L. J., et al. "Absolute isotopic abundance ratios and atomic weight of a reference sample of strontium." *J. Res. Natl. Bur. Stand.* 87.1 (1982): 1-8.  
and (for  $^{87}\text{Sr}/^{86}\text{Sr}$ ):  
Compston, W., Berry, H., Vernon, M.J., Chappell, B.W. and Kaye, M.J., 1971. Rubidium-strontium chronology and chemistry of lunar material from the Ocean of Storms. In *Lunar and Planetary Science Conference Proceedings* (Vol. 2, p. 1471).
  - Sm: Chang, Tsing-Lien, et al. "Absolute isotopic composition and atomic weight of samarium." *International Journal of Mass Spectrometry* 218.2 (2002): 167-172.
  - Re: Gramlich, John W., et al. "Absolute isotopic abundance ratio and atomic weight of a reference sample of rhenium." *J. Res. Natl. Bur. Stand. A* 77 (1973): 691-698.
  - Os: Voelkening, Joachim, Thomas Walczyk, and Klaus G. Heumann. "Osmium isotope ratio determinations by negative thermal ionization mass spectrometry." *Int. J. Mass Spect. Ion Proc.* 105.2 (1991): 147-159.
  - Lu: De Laeter, J. R., and N. Bukilic. "Solar abundance of  $^{176}\text{Lu}$  and s-process nucleosynthesis." *Physical Review C* 73.4 (2006): 045806.
  - Hf: Patchett, P. Jonathan. "Importance of the Lu-Hf isotopic system in studies of planetary chronology and chemical evolution." *Geochimica et Cosmochimica Acta* 47.1 (1983): 81-91.
  - U: Hiess, Joe, et al. " $^{238}\text{U}/^{235}\text{U}$  systematics in terrestrial uranium-bearing minerals." *Science* 335.6076 (2012): 1610-1614.

See Also

[read.data](#)

## Examples

```
# load and show the default constants that come with IsoplotR
json <- system.file("constants.json", package="IsoplotR")
settings(fname=json)
print(settings())

# use the decay constant of Kovarik and Adams (1932)
settings('lambda', 'U238', 0.0001537, 0.0000068)
print(settings('lambda', 'U238'))

# returns the 238U/235U ratio of Hiess et al. (2012):
print(settings('iratio', 'U238U235'))
# use the 238U/235U ratio of Steiger and Jaeger (1977):
settings('iratio', 'U238U235', 138.88, 0)
print(settings('iratio', 'U238U235'))
```

---

titterington

---

*Linear regression of X,Y,Z-variables with correlated errors*


---

## Description

Implements the maximum likelihood algorithm of Ludwig and Titterington (1994) for linear regression of three dimensional data with correlated uncertainties.

## Usage

```
titterington(x, alpha = 0.05)
```

## Arguments

x	an [n x 9] matrix with the following columns: X, sX, Y, sY, Z, sZ, rhoXY, rhoXZ, rhoYZ.
alpha	cutoff value for confidence intervals

## Details

Ludwig and Titterington (1994)'s 3-dimensional linear regression algorithm for data with correlated uncertainties is an extension of the 2-dimensional algorithm by Titterington and Halliday (1979), which itself is equivalent to the algorithm of York et al. (2004). Given  $n$  triplets of (approximately) collinear measurements  $X_i$ ,  $Y_i$  and  $Z_i$  (for  $1 \leq i \leq n$ ), their uncertainties  $s[X_i]$ ,  $s[Y_i]$  and  $s[Z_i]$ , and their covariances  $\text{cov}[X_i, Y_i]$ ,  $\text{cov}[X_i, Z_i]$  and  $\text{cov}[Y_i, Z_i]$ , the `titterington` function fits two slopes and intercepts with their uncertainties. It computes the MSWD as a measure of under/overdispersion. Overdispersed datasets (MSWD>1) can be dealt with in the same three ways that are described in the documentation of the [isochron](#) function.

**Value**

A four-element list of vectors containing:

**par** 4-element vector  $c(a, b, A, B)$  where  $a$  is the intercept of the X-Y regression,  $b$  is the slope of the X-Y regression,  $A$  is the intercept of the X-Z regression, and  $B$  is the slope of the X-Z regression.

**cov** [4 × 4]-element covariance matrix of **par**

**mswd** the mean square of the residuals (a.k.a ‘reduced Chi-square’) statistic

**p.value** p-value of a Chi-square test for linearity

**df** the number of degrees of freedom for the Chi-square test ( $3n-3$ )

**tfact** the  $100(1 - \alpha/2)\%$  percentile of the t-distribution with  $(n - 2k + 1)$  degrees of freedom

**References**

Ludwig, K.R. and Titterton, D.M., 1994. Calculation of  $^{230}\text{Th}/\text{U}$  isochrons, ages, and errors. *Geochimica et Cosmochimica Acta*, 58(22), pp.5031-5042.

Titterton, D.M. and Halliday, A.N., 1979. On the fitting of parallel isochrons and the method of maximum likelihood. *Chemical Geology*, 26(3), pp.183-195.

York, D., Evensen, N.M., Martinez, M.L. and De Basebe Delgado, J., 2004. Unified equations for the slope, intercept, and standard errors of the best straight line. *American Journal of Physics*, 72(3), pp.367-375.

**See Also**

[york](#), [isochron](#), [ludwig](#)

**Examples**

```
d <- matrix(c(0.1677,0.0047,1.105,0.014,0.782,0.015,0.24,0.51,0.33,
              0.2820,0.0064,1.081,0.013,0.798,0.015,0.26,0.63,0.32,
              0.3699,0.0076,1.038,0.011,0.819,0.015,0.27,0.69,0.30,
              0.4473,0.0087,1.051,0.011,0.812,0.015,0.27,0.73,0.30,
              0.5065,0.0095,1.049,0.010,0.842,0.015,0.27,0.76,0.29,
              0.5520,0.0100,1.039,0.010,0.862,0.015,0.27,0.78,0.28),
            nrow=6,ncol=9)
colnames(d) <- c('X','sX','Y','sY','Z','sZ','rXY','rXZ','rYZ')
titterton(d)
```

---

weightedmean

*Calculate the weighted mean age*

---

**Description**

Models the data as a Normal distribution with two sources of variance. Estimates the mean and ‘overdispersion’ using the method of Maximum Likelihood. Computes the MSWD of a Normal fit without overdispersion. Implements a modified Chauvenet Criterion to detect and reject outliers. Only propagates the analytical uncertainty associated with decay constants and  $\zeta$  and J-factors after computing the weighted mean isotopic composition.

**Usage**

```

weightedmean(x, ...)

## Default S3 method:
weightedmean(x, from = NA, to = NA,
  random.effects = TRUE, detect.outliers = TRUE, plot = TRUE,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  ranked = FALSE, hide = NULL, omit = NULL, omit.col = NA, ...)

## S3 method for class 'UPb'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, type = 4,
  cutoff.76 = 1100, cutoff.disc = c(-15, 5), alpha = 0.05,
  exterr = TRUE, ranked = FALSE, common.Pb = 0, hide = NULL,
  omit = NULL, omit.col = NA, ...)

## S3 method for class 'PbPb'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  exterr = TRUE, common.Pb = 1, ranked = FALSE, hide = NULL,
  omit = NULL, omit.col = NA, ...)

## S3 method for class 'ThU'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  ranked = FALSE, i2i = TRUE, detritus = 0, hide = NULL,
  omit = NULL, omit.col = NA, ...)

## S3 method for class 'ArAr'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  exterr = TRUE, ranked = FALSE, i2i = FALSE, hide = NULL,
  omit = NULL, omit.col = NA, ...)

## S3 method for class 'KCa'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,

```

```

    exterr = TRUE, ranked = FALSE, i2i = FALSE, hide = NULL,
    omit = NULL, omit.col = NA, ...)

## S3 method for class 'ReOs'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  exterr = TRUE, ranked = FALSE, i2i = TRUE, hide = NULL,
  omit = NULL, omit.col = NA, ...)

## S3 method for class 'SmNd'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  exterr = TRUE, ranked = FALSE, i2i = TRUE, hide = NULL,
  omit = NULL, omit.col = NA, ...)

## S3 method for class 'RbSr'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  exterr = TRUE, i2i = TRUE, ranked = FALSE, hide = NULL,
  omit = NULL, omit.col = NA, ...)

## S3 method for class 'LuHf'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  exterr = TRUE, i2i = TRUE, ranked = FALSE, hide = NULL,
  omit = NULL, omit.col = NA, ...)

## S3 method for class 'UThHe'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  ranked = FALSE, hide = NULL, omit = NULL, omit.col = NA, ...)

## S3 method for class 'fissiontracks'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  exterr = TRUE, ranked = FALSE, hide = NULL, omit = NULL,

```



```
omit.col = NA, ...)
```

### Arguments

x	a two column matrix of values (first column) and their standard errors (second column) OR an object of class UPb, PbPb, ArAr, KCa, ReOs, SmNd, RbSr, LuHf, ThU, fissiontracks or UThHe
...	optional arguments
from	minimum y-axis limit. Setting from=NA scales the plot automatically.
to	maximum y-axis limit. Setting to=NA scales the plot automatically.
random.effects	if TRUE, computes the weighted mean using a random effects model with two parameters: the mean and the dispersion. This is akin to a ‘model-3’ isochron regression. if FALSE, attributes any excess dispersion to an underestimation of the analytical uncertainties. This akin to a ‘model-1’ isochron regression.
detect.outliers	logical flag indicating whether outliers should be detected and rejected using Chauvenet’s Criterion.
plot	logical flag indicating whether the function should produce graphical output or return numerical values to the user.
levels	a vector with additional values to be displayed as different background colours of the plot symbols.
clabel	label of the colour legend
rect.col	a vector of two fill colours used to show the measurements or age estimates. If levels=NA, then only the first colour is used. If levels is a vector of numbers, then bg is used to construct a colour ramp.
outlier.col	if detect.outliers=TRUE, the outliers are given a different colour.
sigdig	the number of significant digits of the numerical values reported in the title of the graphical output.
alpha	the confidence limits of the error bars/rectangles.
ranked	plot the aliquots in order of increasing age?
hide	vector with indices of aliquots that should be removed from the weighted mean plot.
omit	vector with indices of aliquots that should be plotted but omitted from the weighted mean calculation.
omit.col	colour that should be used for the omitted aliquots.
type	scalar indicating whether to plot the $^{207}\text{Pb}/^{235}\text{U}$ age (type=1), the $^{206}\text{Pb}/^{238}\text{U}$ age (type=2), the $^{207}\text{Pb}/^{206}\text{Pb}$ age (type=3), the $^{207}\text{Pb}/^{206}\text{Pb}$ - $^{206}\text{Pb}/^{238}\text{U}$ age (type=4), or the (Wetherill) concordia age (type=5)
cutoff.76	the age (in Ma) below which the $^{206}\text{Pb}/^{238}\text{U}$ age and above which the $^{207}\text{Pb}/^{206}\text{Pb}$ age is used. This parameter is only used if type=4.

cutoff.disc	two element vector with the maximum and minimum percentage discordance allowed between the $^{207}\text{Pb}/^{235}\text{U}$ and $^{206}\text{Pb}/^{238}\text{U}$ age (if $^{206}\text{Pb}/^{238}\text{U} < \text{cutoff}.76$ ) or between the $^{206}\text{Pb}/^{238}\text{U}$ and $^{207}\text{Pb}/^{206}\text{Pb}$ age (if $^{206}\text{Pb}/^{238}\text{U} > \text{cutoff}.76$ ). Set cutoff.disc=NA if you do not want to use this filter.
exterr	propagate decay constant uncertainties?
common.Pb	apply a common lead correction using one of three methods: 1: use the isochron intercept as the initial Pb-composition 2: use the Stacey-Kramer two-stage model to infer the initial Pb-composition 3: use the Pb-composition stored in settings('iratio', 'Pb206Pb204') and settings('iratio', 'Pb207Pb204')
i2i	'isochron to intercept': calculates the initial (aka 'inherited', 'excess', or 'common') $^{40}\text{Ar}/^{36}\text{Ar}$ , $^{40}\text{Ca}/^{44}\text{Ca}$ , $^{207}\text{Pb}/^{204}\text{Pb}$ , $^{87}\text{Sr}/^{86}\text{Sr}$ , $^{143}\text{Nd}/^{144}\text{Nd}$ , $^{187}\text{Os}/^{188}\text{Os}$ , $^{230}\text{Th}/^{232}\text{Th}$ or $^{176}\text{Hf}/^{177}\text{Hf}$ ratio from an isochron fit. Setting i2i to FALSE uses the default values stored in settings('iratio', ...).
detritus	detrital $^{230}\text{Th}$ correction (only applicable when x\$format == 1 or 2). 0: no correction 1: project the data along an isochron fit 2: correct the data using an assumed initial $^{230}\text{Th}/^{232}\text{Th}$ -ratio for the detritus. 3: correct the data using the measured present day $^{230}\text{Th}/^{238}\text{U}$ , $^{232}\text{Th}/^{238}\text{U}$ and $^{234}\text{U}/^{238}\text{U}$ -ratios in the detritus.

## Details

Let  $\{t_1, \dots, t_n\}$  be a set of  $n$  age estimates determined on different aliquots of the same sample, and let  $\{s[t_1], \dots, s[t_n]\}$  be their analytical uncertainties. IsoplotR then calculates the weighted mean of these data assuming a Normal distribution with two sources of variance:

$$t_i \sim N(\mu, \sigma^2 = s[t_i]^2 + \omega^2)$$

where  $\mu$  is the mean,  $\sigma^2$  is the total variance and  $\omega$  is the 'overdispersion'. This equation can be solved for  $\mu$  and  $\omega$  by the method of maximum likelihood. IsoplotR uses a modified version of Chauvenet's criterion for outlier detection:

1. Compute the error-weighted mean ( $\mu$ ) of the  $n$  age determinations  $t_i$  using their analytical uncertainties  $s[t_i]$
2. For each  $t_i$ , compute the probability  $p_i$  that that  $|t - \mu| > |t_i - \mu|$  for  $t \sim N(0, \sqrt{s[t_i]^2 + \omega^2})$
3. Let  $p_j \equiv \min(p_1, \dots, p_n)$ . If  $p_j < 0.05/n$ , then reject the  $j^{\text{th}}$  date, reduce  $n$  by one (i.e.,  $n \rightarrow n - 1$ ) and repeat steps 1 through 3 until the surviving dates pass the third step.

If the analytical uncertainties are small compared to the scatter between the dates (i.e. if  $\omega \gg s[t]$  for all  $i$ ), then this generalised algorithm reduces to the conventional Chauvenet criterion. If the analytical uncertainties are large and the data do not exhibit any overdispersion, then the heuristic outlier detection method is equivalent to Ludwig (2003)'s '2-sigma' method.

**Value**

Returns a list with the following items:

**mean** a three element vector with:

`x`: the weighted mean

`s[x]`: the standard error of the weighted mean

`ci[x]`: the  $100(1 - \alpha)\%$  confidence interval for `x`

**disp** a three-element vector with the (over)dispersion and the lower and upper half-widths of its  $100(1 - \alpha)\%$  confidence interval.

**mswd** the Mean Square of the Weighted Deviates (a.k.a. ‘reduced Chi-square’ statistic)

**df** the number of degrees of freedom of the Chi-square test for homogeneity ( $df = n - 1$ , where  $n$  is the number of samples).

**p.value** the p-value of a Chi-square test with  $df$  degrees of freedom, testing the null hypothesis that the underlying population is not overdispersed.

**valid** vector of logical flags indicating which steps are included into the weighted mean calculation

**plotpar** list of plot parameters for the weighted mean diagram, including mean (the mean value), ci (a grey rectangle with the  $100[1-\alpha]\%$  confidence interval ignoring systematic errors), ci.terr (a grey rectangle with the  $100[1-\alpha]\%$  confidence interval including systematic errors), dash1 and dash2 (lines marking the overdispersion if `random.effects=TRUE`).

**See Also**

[central](#)

**Examples**

```
ages <- c(251.9, 251.59, 251.47, 251.35, 251.1, 251.04, 250.79, 250.73, 251.22, 228.43)
errs <- c(0.28, 0.28, 0.63, 0.34, 0.28, 0.63, 0.28, 0.4, 0.28, 0.33)
weightedmean(cbind(ages, errs))

data(examples)
weightedmean(examples$LudwigMean)
```

**Description**

Implements the unified regression algorithm of York et al. (2004) which, although based on least squares, yields results that are consistent with maximum likelihood estimates of Titterton and Halliday (1979)

**Usage**

```
york(x, alpha = 0.05)
```

## Arguments

<b>x</b>	a 4 or 5-column matrix with the X-values, the analytical uncertainties of the X-values, the Y-values, the analytical uncertainties of the Y-values, and (optionally) the correlation coefficients of the X- and Y-values.
<b>alpha</b>	cutoff value for confidence intervals

## Details

Given  $n$  pairs of (approximately) collinear measurements  $X_i$  and  $Y_i$  (for  $1 \leq i \leq n$ ), their uncertainties  $s[X_i]$  and  $s[Y_i]$ , and their covariances  $\text{cov}[X_i, Y_i]$ , the `york` function finds the best fitting straight line using the least-squares algorithm of York et al. (2004). This algorithm is modified from an earlier method developed by York (1968) to be consistent with the maximum likelihood approach of Titterton and Halliday (1979). It computes the MSWD as a measure of under/overdispersion. Overdispersed datasets ( $\text{MSWD} > 1$ ) can be dealt with in the same three ways that are described in the documentation of the `isochron` function.

## Value

A four-element list of vectors containing:

**a** the intercept of the straight line fit and its standard error

**b** the slope of the fit and its standard error

**cov.ab** the covariance of the slope and intercept

**mswd** the mean square of the residuals (a.k.a ‘reduced Chi-square’) statistic

**df** degrees of freedom of the linear fit ( $2n - 2$ )

**p.value** p-value of a Chi-square value with `df` degrees of freedom

## References

Titterton, D.M. and Halliday, A.N., 1979. On the fitting of parallel isochrons and the method of maximum likelihood. *Chemical Geology*, 26(3), pp.183-195.

York, Derek, et al., 2004. Unified equations for the slope, intercept, and standard errors of the best straight line. *American Journal of Physics* 72.3, pp.367-375.

## See Also

[data2york](#), [titterton](#), [isochron](#), [ludwig](#)

## Examples

```
X <- c(1.550, 12.395, 20.445, 20.435, 20.610, 24.900,
      28.530, 50.540, 51.595, 86.51, 106.40, 157.35)
Y <- c(.7268, .7849, .8200, .8156, .8160, .8322,
      .8642, .9584, .9617, 1.135, 1.230, 1.490)
n <- length(X)
sX <- X*0.01
sY <- Y*0.005
rXY <- rep(0.8, n)
```

```
dat <- cbind(X,sX,Y,sY,rXY)
fit <- york(dat)
scatterplot(dat,fit=fit)
```

# Index

`_PACKAGE` (IsoplotR), [34](#)

`age`, [2](#), [57](#)

`agespectrum`, [6](#)

`cad`, [8](#), [39](#), [42](#), [43](#)

`central`, [6](#), [11](#), [45](#), [46](#), [50](#), [67](#)

`concordant`, [14](#)

`concordia`, [5](#), [6](#), [15](#), [22](#), [27](#), [40](#), [41](#)

`data2york`, [18](#), [68](#)

`ellipse`, [20](#)

`evolution`, [21](#), [27](#)

`examples`, [23](#), [54](#)

`helioplot`, [13](#), [22](#), [25](#)

`isochron`, [6](#), [22](#), [23](#), [27](#), [27](#), [41](#), [61](#), [62](#), [68](#)

`IsoplotR`, [34](#)

`IsoplotR-package` (IsoplotR), [34](#)

`kde`, [11](#), [34](#), [43](#)

`ludwig`, [33](#), [39](#), [62](#), [68](#)

`mds`, [41](#)

`peakfit`, [43](#), [50](#)

`radialplot`, [11](#), [13](#), [27](#), [39](#), [46](#), [46](#)

`read.data`, [51](#), [60](#)

`scatterplot`, [55](#)

`set.zeta`, [56](#)

`settings`, [54](#), [58](#)

`titterington`, [33](#), [41](#), [61](#), [68](#)

`weightedmean`, [7](#), [8](#), [13](#), [62](#)

`york`, [19](#), [20](#), [33](#), [62](#), [67](#)