Package 'IsoplotR'

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Title Statistical Toolbox for Radiometric Geochronology

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Description Plots U-Pb data on Wetherill and Tera-Wasserburg concordia diagrams. Calculates concordia and discordia ages. Performs linear regression of measurements with correlated errors using 'York', 'Titterington' and 'Ludwig' approaches. Generates Kernel Density Estimates (KDEs) and Cumulative Age Distributions (CADs). Produces Multidimensional Scaling (MDS) configurations and Shepard plots of multi-sample detrital datasets using the Kolmogorov-Smirnov distance as a dissimilarity measure. Calculates 40Ar/39Ar ages, isochrons, and age spectra. Computes weighted means accounting for overdispersion. Calculates U-Th-He (single grain and central) ages, logratio plots and ternary diagrams. Processes fission track data using the external detector method and LA-ICP-MS, calculates central ages and plots fission track and other data on radial (a.k.a. 'Galbraith') plots. Constructs total Pb-U, Pb-Pb, K-Ca, Re-Os, Sm-Nd, Lu-Hf, Rb-Sr and 230Th-U isochrons as well as 230Th-U evolution plots.

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Depends R (>= 3.0.0)

Imports MASS, grDevices, graphics, stats, utils

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URL http://isoplotr.london-geochron.com

LazyData true

RoxygenNote 6.1.1

Encoding UTF-8

R topics documented:

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age   *Calculate isotopic ages*

Description

Calculates U-Pb, Pb-Pb, Ar-Ar, K-Ca, Re-Os, Sm-Nd, Rb-Sr, Lu-Hf, U-Th-He, Th-U and fission track ages and propagates their analytical uncertainties. Includes options for single grain, isochron and concordia ages.

Usage

```
age(x, ...)

## Default S3 method:
age(x, method = "U238-Pb206", exterr = TRUE,
  J = c(NA, NA), zeta = c(NA, NA), rhoD = c(NA, NA), d = diseq(),
  ...)

## S3 method for class 'UPb'
age(x, type = 1, exterr = TRUE, i = NA, sigdig = NA,
  common.Pb = 0, show.p = FALSE, ...)
```

```
## S3 method for class 'PbPb'
age(x, isochron = TRUE, common.Pb = 1, exterr = TRUE,
  i = NA, sigdig = NA, ...)

## S3 method for class 'ArAr'
age(x, isochron = FALSE, i2i = TRUE, exterr = TRUE,
  i = NA, sigdig = NA, ...)

## S3 method for class 'KCa'
age(x, isochron = FALSE, i2i = TRUE, exterr = TRUE,
  i = NA, sigdig = NA, ...)

## S3 method for class 'UThHe'
age(x, isochron = FALSE, central = FALSE, i = NA,
  sigdig = NA, ...)

## S3 method for class 'fissiontracks'
age(x, central = FALSE, i = NA, sigdig = NA,
  exterr = TRUE, ...)

## S3 method for class 'ThU'
age(x, isochron = FALSE, i2i = TRUE, exterr = TRUE,
  i = NA, sigdig = NA, detritus = 0, ...)

## S3 method for class 'ReOs'
age(x, isochron = TRUE, i2i = TRUE, exterr = TRUE,
  i = NA, sigdig = NA, ...)

## S3 method for class 'SmNd'
age(x, isochron = TRUE, i2i = TRUE, exterr = TRUE,
  i = NA, sigdig = NA, ...)

## S3 method for class 'RbSr'
age(x, isochron = TRUE, i2i = TRUE, exterr = TRUE,
  i = NA, sigdig = NA, ...)

## S3 method for class 'LuHf'
age(x, isochron = TRUE, i2i = TRUE, exterr = TRUE,
  i = NA, sigdig = NA, ...)
```

Arguments

x   can be:

- a scalar containing an isotopic ratio,
- a two element vector containing an isotopic ratio and its standard error, or the spontaneous and induced track densities Ns and Ni,
- a four element vector containing Ar40Ar39, s[Ar40Ar39], J, s[J],
- a two element vector containing K40Ca40 and s[K40Ca40],

- a six element vector containing U, s[U], Th, s[Th], He and s[He],
- an eight element vector containing U, s[U], Th, s[Th], He, s[He], Sm and s[Sm]
- a two element vector containing Sr87Rb87 and s[Sr87Rb87]
- a two element vector containing Os187Re187 and s[Os187Re187]
- a two element vector containing Nd143Sm147 and s[Nd144Sm147]
- a two element vector containing Hf176Lu176 and s[Hf176Lu176]
- a five element vector containing Th230U238, s[Th230/U238], U234U238, s[U234U238] and cov[Th230U238,U234U238]

OR

- an object of class UPb, PbPb, ArAr, KCa, ThU, RbSr, SmNd, ReOs, LuHf, UThHe or fissiontracks.

...	additional arguments
method	one of either 'U238-Pb206', 'U235-Pb207', 'Pb207-Pb206', 'Th232-Pb208', 'Ar-Ar', 'K-Ca', 'Th-U', 'Re-Os', 'Sm-Nd', 'Rb-Sr', 'Lu-Hf', 'U-Th-He' or 'fissiontracks'
exterr	propagate the external (decay constant and calibration factor) uncertainties?
J	two-element vector with the J-factor and its standard error.
zeta	two-element vector with the zeta-factor and its standard error.
rhoD	two-element vector with the track density of the dosimeter glass and its standard error.
d	an object of class diseq.
type	scalar flag indicating whether
	1: each U-Pb analysis should be considered separately,
	2: all the measurements should be combined to calculate a concordia age,
	3: a discordia line should be fitted through all the U-Pb analyses using the maximum likelihood algorithm of Ludwig (1998), which assumes that the scatter of the data is solely due to the analytical uncertainties.
	4: a discordia line should be fitted ignoring the analytical uncertainties.
	5: a discordia line should be fitted using a modified maximum likelihood algorithm that accounts for overdispersion by adding a geological (co)variance term.
i	index of a particular aliquot
sigdig	number of significant digits for the uncertainty estimate (only used if type=1, isochron=FALSE and central=FALSE).
common.Pb	apply a common lead correction using one of three methods:
	1: the Stacey-Kramer two-stage model to infer the initial Pb-composition
	2: the isochron intercept as the initial Pb-composition
	3: the Pb-composition stored in
	settings('iratio','Pb206Pb204') (if x has class UPb and x$format<4);
	settings('iratio','Pb206Pb204') and settings('iratio','Pb207Pb204') (if x has class PbPb or x has class UPb and 3<x$format<7); or
	settings('iratio','Pb208Pb206') and settings('iratio','Pb208Pb207') (if x has class UPb and x$format=7 or 8).

show.p	Show the p-value for concordance for each aliquot to the output table. Note: it would be unwise to use the p-value value as a concordance filter. Doing so would 'punish' high precision measurements, which are more likely to fail the Chi-square test than low precision measurements. The latter would therefore be 'rewarded' by such a criterion.
isochron	logical flag indicating whether each analysis should be considered separately (isochron=FALSE) or an isochron age should be calculated from all analyses together (isochron=TRUE).
i2i	'isochron to intercept': calculates the initial (aka 'inherited', 'excess', or 'common') ^{40}Ar/^{36}Ar, ^{40}Ca/^{44}Ca, ^{87}Sr/^{86}Sr, ^{143}Nd/^{144}Nd, ^{187}Os/^{188}Os or ^{176}Hf/^{177}Hf ratio from an isochron fit. Setting i2i to FALSE uses the default values stored in settings('iratio',...). When applied to data of class ThU, setting i2i to TRUE applies a detrital Th-correction.
central	logical flag indicating whether each analysis should be considered separately (central=FALSE) or a central age should be calculated from all analyses together (central=TRUE).
detritus	detrital ^{230}Th correction (only applicable when x$format = 1 or 2).
	0: no correction
	1: project the data along an isochron fit
	2: correct the data using an assumed initial ^{230}Th/^{232}Th-ratio for the detritus.
	3: correct the data using the measured present day ^{230}Th/^{238}U, ^{232}Th/^{238}U and ^{234}U/^{238}U-ratios in the detritus.

Value

1. if x is a scalar or a vector, returns the age using the geochronometer given by method and its standard error.

2. if x has class UPb and type=1, returns a table with the following columns: t.75, err[t.75], t.68, err[t.68], t.76, err[t.76], (t.82, err[t.82],) t.conc, err[t.conc], (err[p.conc],) containing the ^{207}Pb/^{235}U-age and standard error, the ^{206}Pb/^{238}U-age and standard error, the ^{207}Pb/^{206}Pb-age and standard error, (the ^{208}Pb/^{232}Th-age and standard error,) the single grain concordia age and standard error, (and the p-value for concordance,) respectively.

3. if x has class UPb and type=2, 3, 4 or 5, returns the output of the concordia function.

4. if x has class PbPb, ArAr, KCa, RbSr, SmNd, ReOs, LuHf, ThU or UThHe and isochron=FALSE, returns a table of Pb-Pb, Ar-Ar, K-Ca, Rb-Sr, Sm-Nd, Re-Os, Lu-Hf, Th-U or U-Th-He ages and their standard errors.

5. if x has class ThU and isochron=FALSE, returns a 5-column table with the Th-U ages, their standard errors, the initial ^{234}U/^{238}U-ratios, their standard errors, and the correlation coefficient between the ages and the initial ratios.

6. if x has class PbPb, ArAr, KCa, RbSr, SmNd, ReOs, LuHf, UThHe or ThU and isochron=TRUE, returns the output of the isochron function.

7. if x has class fissiontracks and central=FALSE, returns a table of fission track ages and standard errors.

8. if x has class fissiontracks or UThHe and central=TRUE, returns the output of the central function.

See Also

concordia, isochron, central

Examples

```
data(examples)
tUPb <- age(examples$UPb,type=1)
tconc <- age(examples$UPb,type=2)
tdisc <- age(examples$UPb,type=3)
tArAr <- age(examples$ArAr)
tiso <- age(examples$ArAr,isochron=TRUE,i2i=TRUE)
tcentral <- age(examples$FT1,central=TRUE)
```

---

agespectrum   *Plot a (40Ar/39Ar) release spectrum*

Description

Produces a plot of boxes whose widths correspond to the cumulative amount of ^{39}Ar (or any other variable), and whose heights express the analytical uncertainties. Only propagates the analytical uncertainty associated with decay constants and J-factors *after* computing the plateau composition.

Usage

```
agespectrum(x, ...)

## Default S3 method:
agespectrum(x, alpha = 0.05, plateau = TRUE,
  random.effects = TRUE, levels = NA, clabel = ",
  plateau.col = c("#00FF0080", "#FF000080"),
  non.plateau.col = "#00FFFF80", sigdig = 2, line.col = "red",
  lwd = 2, xlab = "cumulative fraction", ylab = "age [Ma]",
  hide = NULL, ...)

## S3 method for class 'ArAr'
agespectrum(x, alpha = 0.05, plateau = TRUE,
  random.effects = TRUE, levels = NA, clabel = ",
  plateau.col = c("#00FF0080", "#FF000080"),
  non.plateau.col = "#00FFFF80", sigdig = 2, exterr = TRUE,
  line.col = "red", lwd = 2, i2i = FALSE, hide = NULL, ...)
```

Arguments

x   a three-column matrix whose first column gives the amount of ^{39}Ar in each aliquot, and whose second and third columns give the age and its uncertainty.

                  OR

                  an object of class ArAr

...	optional parameters to the generic `plot` function
alpha	the confidence level of the error bars/boxes and confidence intervals.
plateau	logical flag indicating whether a plateau age should be calculated. If `plateau=TRUE`, the function computes the weighted mean of the largest succession of steps that pass the Chi-square test for age homogeneity. If `TRUE`, it returns a list with plateau parameters.
random.effects	if `TRUE`, computes the weighted mean using a random effects model with two parameters: the mean and the dispersion. This is akin to a 'model-3' isochron regression.
	if `FALSE`, attributes any excess dispersion to an underestimation of the analytical uncertainties. This akin to a 'model-1' isochron regression.
levels	a vector with additional values to be displayed as different background colours of the plot symbols.
clabel	label of the colour legend
plateau.col	Fill colours of the rectangles used to mark the steps belonging to the age plateau. This can either be a single colour or multiple colours to form a colour ramp (to be used if `levels!=NA`):

- a single colour: `rgb(0,1,0,0.5)`, `'#FF000080'`, `'white'`, etc.
- multiple colours: `c(rbg(1,0,0,0.5),rgb(0,1,0,0.5))`, `c('#FF000080','#00FF0080')`, `c('blue','red')`, `c('blue','yellow','red')`, etc.
- a colour palette: `rainbow(n=100)`, `topo.colors(n=100,alpha=0.5)`, etc.
- a reversed palette: `rev(topo.colors(n=100,alpha=0.5))`, etc.
- for empty boxes, set `plateau.col=NA`

non.plateau.col	
	if `plateau=TRUE`, the steps that do NOT belong to the plateau are given a different colour.
sigdig	the number of significant digits of the numerical values reported in the title of the graphical output.
line.col	colour of the average age line
lwd	width of the average age line
xlab	x-axis label
ylab	y-axis label
hide	vector with indices of aliquots that should be removed from the plot.
exterr	propagate the external (decay constant and calibration factor) uncertainties?
i2i	'isochron to intercept': calculates the initial (aka 'inherited', 'excess', or 'common') $^{40}Ar/^{36}Ar$ ratio from an isochron fit. Setting `i2i` to `FALSE` uses the default values stored in `settings('iratio',...)`

Details

`IsoplotR` defines the 'plateau age' as the weighted mean age (using a random effects model with two sources of dispersion) of the longest sequence (in terms of cumulative ^{39}Ar content) of consecutive heating steps that pass the modified Chauvenet criterion (see weightedmean). Note that this definition is different (and simpler) than the one used by `Isoplot` (Ludwig, 2003). However, it is important to mention that all definitions of an age plateau are heuristic by nature and should not be used for quantitative inference.

Value

If `plateau=TRUE`, returns a list with the following items:

mean a 3-element vector with:

x: the plateau mean

s[x]: the standard error of x

ci[x]: the width of a 100(1-α)% confidence interval of t

disp a 3-element vector with:

w: the overdispersion, i.e. the standard deviation of the Normal distribution that is assumed to describe the true ages.

ll: the width of the lower half of a 100(1-α)% confidence interval for the overdispersion

ul: the width of the upper half of a 100(1-α)% confidence interval for the overdispersion

df the degrees of freedom for the weighted mean plateau fit

mswd the mean square of the weighted deviates of the plateau

p.value the p-value of a Chi-square test with $df = n-2$ degrees of freedom, where n is the number of steps in the plateau and 2 degrees of freedom have been removed to estimate the mean and the dispersion.

fract the fraction of ^{39}Ar contained in the plateau

plotpar plot parameters for the weighted mean (see weightedmean)

i indices of the steps that are retained for the plateau age calculation

See Also

weightedmean

Examples

```
data(examples)
agespectrum(examples$ArAr,ylim=c(0,80))
```

---

cad   *Plot continuous data as cumulative age distributions*

Description

Plot a dataset as a Cumulative Age Distribution (CAD), also known as a 'empirical cumulative distribution function'.

Usage

```
cad(x, ...)

## Default S3 method:
cad(x, pch = NA, verticals = TRUE,
  xlab = "age [Ma]", col = "black", hide = NULL, ...)

## S3 method for class 'detritals'
cad(x, pch = NA, verticals = TRUE,
  xlab = "age [Ma]", col = "heat.colors", hide = NULL, ...)

## S3 method for class 'UPb'
cad(x, pch = NA, verticals = TRUE, xlab = "age [Ma]",
  col = "black", type = 4, cutoff.76 = 1100,
  cutoff.disc = list(-15, 5, TRUE), common.Pb = 0, hide = NULL, ...)

## S3 method for class 'PbPb'
cad(x, pch = NA, verticals = TRUE, xlab = "age [Ma]",
  col = "black", common.Pb = 1, hide = NULL, ...)

## S3 method for class 'ArAr'
cad(x, pch = NA, verticals = TRUE, xlab = "age [Ma]",
  col = "black", i2i = FALSE, hide = NULL, ...)

## S3 method for class 'KCa'
cad(x, pch = NA, verticals = TRUE, xlab = "age [Ma]",
  col = "black", i2i = FALSE, hide = NULL, ...)

## S3 method for class 'ThU'
cad(x, pch = NA, verticals = TRUE, xlab = "age [ka]",
  col = "black", i2i = FALSE, detritus = 0, hide = NULL, ...)

## S3 method for class 'ReOs'
cad(x, pch = NA, verticals = TRUE, xlab = "age [Ma]",
  col = "black", i2i = TRUE, hide = NULL, ...)

## S3 method for class 'SmNd'
cad(x, pch = NA, verticals = TRUE, xlab = "age [Ma]",
  col = "black", i2i = TRUE, hide = NULL, ...)

## S3 method for class 'RbSr'
cad(x, pch = NA, verticals = TRUE, xlab = "age [Ma]",
  col = "black", i2i = TRUE, hide = NULL, ...)

## S3 method for class 'LuHf'
cad(x, pch = NA, verticals = TRUE, xlab = "age [Ma]",
  col = "black", i2i = TRUE, hide = NULL, ...)
```

```
## S3 method for class 'UThHe'
cad(x, pch = NA, verticals = TRUE, xlab = "age [Ma]",
  col = "black", hide = NULL, ...)

## S3 method for class 'fissiontracks'
cad(x, pch = NA, verticals = TRUE,
  xlab = "age [Ma]", col = "black", hide = NULL, ...)
```

Arguments

x	a numerical vector OR an object of class UPb, PbPb, ArAr, KCa, UThHe, fissiontracks, ReOs, RbSr, SmNd, LuHf, ThU or detritals
...	optional arguments to the generic plot function
pch	plot character to mark the beginning of each CAD step
verticals	logical flag indicating if the horizontal lines of the CAD should be connected by vertical lines
xlab	x-axis label
col	either the name of one of R's built-in colour palettes (e.g., 'heat.colors', 'terrain.colors', 'topo.colors', 'cm.colors') (if x has class detritals) OR the name or code for a colour to give to single sample datasets (otherwise).
hide	vector with indices of aliquots that should be removed from the plot.
type	scalar indicating whether to plot the $^{207}\text{Pb}/^{235}\text{U}$ age (type=1), the $^{206}\text{Pb}/^{238}\text{U}$ age (type=2), the $^{207}\text{Pb}/^{206}\text{Pb}$ age (type=3), the $^{207}\text{Pb}/^{206}\text{Pb}$-$^{206}\text{Pb}/^{238}\text{U}$ age (type=4), the concordia age (type=5), or the $^{208}\text{Pb}/^{232}\text{Th}$ age (type=6).
cutoff.76	the age (in Ma) below which the $^{206}\text{Pb}/^{238}\text{U}$-age and above which the $^{207}\text{Pb}/^{206}\text{Pb}$-age is used. This parameter is only used if type=4.
cutoff.disc	discordance cutoff filter. This is a three element list.
	The first two items contain the minimum (negative) and maximum (positive) percentage discordance allowed between the $^{207}\text{Pb}/^{235}\text{U}$ and $^{206}\text{Pb}/^{238}\text{U}$ age (if $^{206}\text{Pb}/^{238}\text{U}$ < cutoff.76) or between the $^{206}\text{Pb}/^{238}\text{U}$ and $^{207}\text{Pb}/^{206}\text{Pb}$ age (if $^{206}\text{Pb}/^{238}\text{U}$ > cutoff.76).
	The third item is a boolean flag that controls whether the discordance filter should be applied before (TRUE) or after (FALSE) the common-Pb correction.
	Set cutoff.disc=NA to turn off this filter.
common.Pb	apply a common lead correction using one of three methods:
	1: the Stacey-Kramer two-stage model to infer the initial Pb-composition
	2: the isochron intercept as the initial Pb-composition
	3: the Pb-composition stored in
	settings('iratio','Pb206Pb204') (if x has class UPb and x$format<4);
	settings('iratio','Pb206Pb204') and settings('iratio','Pb207Pb204') (if x has class PbPb or x has class UPb and 3<x$format<7); or
	settings('iratio','Pb208Pb206') and settings('iratio','Pb208Pb207') (if x has class UPb and x$format=7 or 8).

i2i	'isochron to intercept': calculates the initial (aka 'inherited', 'excess', or 'common') ^{40}Ar/^{36}Ar, ^{40}Ca/^{44}Ca, ^{207}Pb/^{204}Pb, ^{87}Sr/^{86}Sr, ^{143}Nd/^{144}Nd, ^{187}Os/^{188}Os, ^{230}Th/^{232}Th or ^{176}Hf/^{177}Hf ratio from an isochron fit. Setting i2i to FALSE uses the default values stored in settings('iratio',...) or zero (for the Pb-Pb method). When applied to data of class ThU, setting i2i to TRUE applies a detrital Th-correction.
detritus	detrital ^{230}Th correction (only applicable when x$format=1 or 2).
	0: no correction
	1: project the data along an isochron fit
	2: correct the data using an assumed initial ^{230}Th/^{232}Th-ratio for the detritus.
	3: correct the data using the measured present day ^{230}Th/^{238}U, ^{232}Th/^{238}U and ^{234}U/^{238}U-ratios in the detritus.

Details

Empirical cumulative distribution functions or cumulative age distributions are the most straightforward way to visualise the probability distribution of multiple dates. Suppose that we have a set of n dates t_i. The CAD is a step function that sets out the rank order of the dates against their numerical value:

$$CAD(t) = \sum_i 1(t < t_i)/n$$

where $1(*) = 1$ if $*$ is true and $1(*) = 0$ if $*$ is false. CADs have two desirable properties (Vermeesch, 2007). First, they do not require any pre-treatment or smoothing of the data. This is not the case for histograms or kernel density estimates. Second, it is easy to superimpose several CADs on the same plot. This facilitates the intercomparison of multiple samples. The interpretation of CADs is straightforward but not very intuitive. The prominence of individual age components is proportional to the steepness of the CAD. This is different from probability density estimates such as histograms, in which such components stand out as peaks.

References

Vermeesch, P., 2007. Quantitative geomorphology of the White Mountains (California) using detrital apatite fission track thermochronology. Journal of Geophysical Research: Earth Surface, 112(F3).

See Also

[kde](#), [radialplot](#)

Examples

```
data(examples)
cad(examples$DZ,verticals=FALSE,pch=20)
```

---

central   *Calculate U-Th-He and fission track central ages and compositions*

Description

Computes the geometric mean composition of a continuous mixture of fission track or U-Th-He data and returns the corresponding age and fitting parameters.

Usage

```
central(x, ...)

## Default S3 method:
central(x, alpha = 0.05, ...)

## S3 method for class 'UThHe'
central(x, alpha = 0.05, model = 1, ...)

## S3 method for class 'fissiontracks'
central(x, mineral = NA, alpha = 0.05,
  exterr = FALSE, ...)
```

Arguments

x	an object of class UThHe or fissiontracks, OR a 2-column matrix with (strictly positive) values and uncertainties
...	optional arguments
alpha	cutoff value for confidence intervals
model	if the scatter between the data points is solely caused by the analytical uncertainty, then the MSWD value should be approximately equal to one. There are three strategies to deal with the case where MSWD>1.choose one of the following statistical models: 1: assume that the analytical uncertainties have been underestimated by a factor $\sqrt{MSWD}$. 2: ignore the analytical uncertainties. 3: attribute any excess dispersion to the presence of geological uncertainty, which manifests itself as an added (co)variance term.
mineral	setting this parameter to either apatite or zircon changes the default efficiency factor, initial fission track length and density to preset values (only affects results if x$format=2)
exterr	include the zeta or decay constant uncertainty into the error propagation for the central age?

Details

The central age assumes that the observed age distribution is the combination of two sources of scatter: analytical uncertainty and true geological dispersion.

1. For fission track data, the analytical uncertainty is assumed to obey Poisson counting statistics and the geological dispersion is assumed to follow a lognormal distribution.

2. For U-Th-He data, the U-Th-(Sm)-He compositions and uncertainties are assumed to follow a logistic normal distribution.

3. For all other data types, both the analytical uncertainties and the true ages are assumed to follow lognormal distributions.

The difference between the central age and the weighted mean age is usually small unless the data are imprecise and/or strongly overdispersed.

Value

If x has class UThHe, returns a list containing the following items:

uvw (if the input data table contains Sm) or **uv** (if it does not): the mean log[U/He], log[Th/He] (, and log[Sm/He]) composition.

covmat the covariance matrix of uvw or uv.

mswd the reduced Chi-square statistic of data concordance, i.e. $mswd = SS/df$, where SS is the sum of squares of the log[U/He]-log[Th/He] compositions.

model the fitting model.

df the degrees of freedom $(2n - 2)$ of the fit (only reported if model=1).

p.value the p-value of a Chi-square test with df degrees of freedom (only reported if model=1.)

age	a three- or four-element vector with:
	t: the central age.
	s[t]: the standard error of t.
	ci[t]: the width of a $100(1 - \alpha)\%$ confidence interval for t.
	disp[t]: the studentised $100(1-\alpha)\%$ confidence interval enhanced by a factor of $\sqrt{mswd}$ (only reported if model=1).
w	the geological overdispersion term. If model=3, this is a three-element vector with the standard deviation of the (assumedly) Normal dispersion and the lower and upper half-widths of its $100(1 - \alpha)\%$ confidence interval. w=0 if model<3.

OR, otherwise:

age a three-element vector with:
  t: the central age.
  s[t]: the standard error of t.
  ci[t]: the width of a $100(1 - \alpha)\%$ confidence interval for t.

disp a three-element vector with the overdispersion (standard deviation) of the excess scatter, and the upper and lower half-widths of its $100(1 - \alpha)\%$ confidence interval.

mswd the reduced Chi-square statistic of data concordance, i.e. $mswd = X^2/df$, where X^2 is a Chi-square statistic of the EDM data or ages

df the degrees of freedom $(n - 2)$

p.value the p-value of a Chi-square test with df degrees of freedom

References

Galbraith, R.F. and Laslett, G.M., 1993. Statistical models for mixed fission track ages. Nuclear Tracks and Radiation Measurements, 21(4), pp.459-470.

Vermeesch, P., 2008. Three new ways to calculate average (U-Th)/He ages. Chemical Geology, 249(3), pp.339-347.

See Also

weightedmean, radialplot, helioplot

Examples

```
data(examples)
print(central(examples$UThHe)$age)
```

---

concordia   *Concordia diagram*

Description

Plots U-Pb data on Wetherill, Tera-Wasserburg or U-Th-Pb concordia diagrams, calculates concordia ages and compositions, evaluates the equivalence of multiple (^{206}Pb/^{238}U-^{207}Pb/^{235}U, ^{207}Pb/^{206}Pb-^{206}Pb/^{238}U, or ^{208}Th/^{232}Th-^{206}Pb/^{238}U) compositions, computes the weighted mean isotopic composition and the corresponding concordia age using the method of maximum likelihood, computes the MSWD of equivalence and concordance and their respective Chi-squared p-values. Performs linear regression and computes the upper and lower intercept ages (for Wetherill) or the lower intercept age and the ^{207}Pb/^{206}Pb intercept (for Tera-Wasserburg), taking into account error correlations and decay constant uncertainties.

Usage

```
concordia(x = NULL, tlim = NULL, alpha = 0.05, type = 1,
  show.numbers = FALSE, levels = NA, clabel = "",
  ellipse.col = c("#00FF0080", "#FF000080"),
  concordia.col = "darksalmon", exterr = FALSE, show.age = 0,
  sigdig = 2, common.Pb = 0, ticks = 5, anchor = list(FALSE, NA),
  hide = NULL, omit = NULL, omit.col = NA, ...)
```

Arguments

x	an object of class UPb
tlim	age limits of the concordia line
alpha	probability cutoff for the error ellipses and confidence intervals
type	one of

1. Wetherill: ^{206}Pb/^{238}U vs. ^{207}Pb/^{235}U
2. Tera-Wasserburg: ^{207}Pb/^{206}Pb vs. ^{238}U/^{206}Pb
3. U-Th-Pb concordia: ^{208}Pb/^{232}Th vs. ^{206}Pb/^{238}U (only available if x$format=7 or x$format=8)

show.numbers	logical flag (TRUE to show grain numbers)
levels	a vector with length(x) values to be displayed as different background colours within the error ellipses.
clabel	label for the colour legend (only used if levels is not NA).
ellipse.col	Fill colour for the error ellipses. This can either be a single colour or multiple colours to form a colour ramp. Examples:

- a single colour: rgb(0,1,0,0.5), '#FF000080', 'white', etc.
- multiple colours: c(rbg(1,0,0,0.5),rgb(0,1,0,0.5)), c('#FF000080','#00FF0080'), c('blue','red'), c('blue','yellow','red'), etc.
- a colour palette: rainbow(n=100), topo.colors(n=100,alpha=0.5), etc.
- a reversed palette: rev(topo.colors(n=100,alpha=0.5)), etc.
- for empty ellipses, set ellipse.col=NA

concordia.col	colour of the concordia line
exterr	show decay constant uncertainties?
show.age	one of either:

0: plot the data without calculating an age

1: fit a concordia composition and age

2: fit a discordia line through the data using the maximum likelihood algorithm of Ludwig (1998), which assumes that the scatter of the data is solely due to the analytical uncertainties. In this case, IsoplotR will either calculate an upper and lower intercept age (for Wetherill concordia), or a lower intercept age and common (^{207}Pb/^{206}Pb)-ratio intercept (for Tera-Wasserburg). If mswd>0, then the analytical uncertainties are augmented by a factor $\sqrt{mswd}$.

3: fit a discordia line ignoring the analytical uncertainties

4: fit a discordia line using a modified maximum likelihood algorithm that includes accounts for any overdispersion by adding a geological (co)variance term.

sigdig	number of significant digits for the concordia/discordia age
common.Pb	apply a common lead correction using one of three methods:

1: the Stacey-Kramer two-stage model to infer the initial Pb-composition

2: the isochron intercept as the initial Pb-composition

3: the Pb-composition stored in

settings('iratio','Pb206Pb204') (if x$format<4);

settings('iratio','Pb206Pb204') and settings('iratio','Pb207Pb204')
(if 3<x$format<7); or

settings('iratio','Pb208Pb206') and settings('iratio','Pb208Pb207')
(if x$format=7 or 8).

ticks   either a scalar indicating the desired number of age ticks to be placed along the
                 concordia line, OR a vector of tick ages.

anchor   control parameters to fix the intercept age or common Pb composition of the
                 discordia fit. This is a two-element list.

                 The first element is a boolean flag indicating whether the discordia line should
                 be anchored. If this is FALSE, then the second item is ignored and both the
                 common Pb composition and age are estimated.

                 If the first element is TRUE and the second element is NA, then the common Pb
                 composition is fixed at the values stored in settings('iratio',...).

                 If the first element is TRUE and the second element is a number, then the discordia
                 line is forced to intersect the concordia line at an age equal to that number.

hide   vector with indices of aliquots that should be removed from the concordia dia-
                 gram

omit   vector with indices of aliquots that should be plotted but omitted from concordia
                 or discordia age calculation

omit.col   colour that should be used for the omitted aliquots.

...   optional arguments to the generic plot function

Details

The concordia diagram is a graphical means of assessing the internal consistency of U-Pb data. It
sets out the measured ^{206}Pb/^{238}U- and ^{207}Pb/^{235}U-ratios against each other ('Wetherill' diagram);
or, equivalently, the ^{207}Pb/^{206}Pb- and ^{206}Pb/^{238}U-ratios ('Tera-Wasserburg' diagram). Alterna-
tively, for data formats 7 and 8, it is also possible to plot ^{208}Pb/^{232}Th against the ^{206}Pb/^{238}U. The
space of concordant isotopic compositions is marked by a curve, the 'concordia line'. Isotopic ra-
tio measurements are shown as 100(1-alpha)% confidence ellipses. Concordant samples plot near
to, or overlap with, the concordia line. They represent the pinnacle of geochronological robustness.
Samples that plot away from the concordia line but are aligned along a linear trend form an isochron
(or 'discordia' line) that can be used to infer the composition of the non-radiogenic ('common') lead
or to constrain the timing of prior lead loss.

Value

if show.age=1, returns a list with the following items:

x a named vector with the (weighted mean) U-Pb composition

cov the covariance matrix of the (weighted mean) U-Pb composition

mswd a vector with three items (equivalence, concordance and combined) containing the MSWD
         (Mean of the Squared Weighted Deviates, a.k.a the reduced Chi-squared statistic) of isotopic
         equivalence, age concordance and combined goodness of fit, respectively.

p.value a vector with three items (equivalence, concordance and combined) containing the p-
         value of the Chi-square test for isotopic equivalence, age concordance and combined goodness
         of fit, respectively.

df a three-element vector with the number of degrees of freedom used for the `mswd` calculation.

age a 4-element vector with:

> `t`: the concordia age (in Ma)
>
> `s[t]`: the estimated uncertainty of `t`
>
> `ci[t]`: the studentised $100(1 - \alpha)\%$ confidence interval of `t` for the appropriate degrees of freedom
>
> `disp[t]`: the studentised $100(1 - \alpha)\%$ confidence interval for `t` augmented by $\sqrt{mswd}$ to account for overdispersed datasets.

if `show.age=2`, 3 or 4, returns a list with the following items:

model the fitting model (=`show.age-1`).

x a vector with the upper and lower intercept ages (if `type=1`) or the lower intercept age and common Pb intercept(s) (if `type=2`).

cov the covariance matrix of the elements in `x`.

err a matrix with the following rows:

> `s`: the estimated standard deviation for `x`
>
> `ci`: the studentised $100(1 - \alpha)\%$ confidence interval of `x` for the appropriate degrees of freedom
>
> `disp[t]`: the studentised $100(1 - \alpha)\%$ confidence interval for `x` augmented by $\sqrt{mswd}$ to account for overdispersed datasets (only reported if `show.age=2`).

df the degrees of freedom of the concordia fit (concordance + equivalence)

p.value p-value of a Chi-square test for age homogeneity (only reported if `type=3`).

mswd mean square of the weighted deviates – a goodness-of-fit measure. `mswd > 1` indicates overdispersion w.r.t the analytical uncertainties (not reported if `show.age=3`).

w three-element vector with the standard deviation of the (assumedly) Normal overdispersion term and the lower and upper half-widths of its $100(1 - \alpha)\%$ confidence interval (only important if `show.age=4`).

n the number of aliquots in the dataset

References

Ludwig, K.R., 1998. On the treatment of concordant uranium-lead ages. Geochimica et Cosmochimica Acta, 62(4), pp.665-676.

Examples

```
data(examples)
concordia(examples$UPb,show.age=2)

dev.new()
concordia(examples$UPb,type=1,xlim=c(24.9,25.4),
          ylim=c(0.0508,0.0518),ticks=249:254,exterr=TRUE)

dev.new()
concordia(examples$UPb,type=2,show.age=2,anchor=list(TRUE,0))
```

---

data2york   *Prepare geochronological data for York regression*

Description

Takes geochronology data as input and produces a five-column table with the variables, their uncertainties and error correlations as output. These can subsequently be used for York regression.

Usage

```
data2york(x, ...)

## Default S3 method:
data2york(x, format = 1, ...)

## S3 method for class 'UPb'
data2york(x, option = 1, tt = 0, ...)

## S3 method for class 'ArAr'
data2york(x, inverse = TRUE, ...)

## S3 method for class 'KCa'
data2york(x, inverse = FALSE, ...)

## S3 method for class 'PbPb'
data2york(x, inverse = TRUE, ...)

## S3 method for class 'PD'
data2york(x, exterr = FALSE, inverse = FALSE, ...)

## S3 method for class 'UThHe'
data2york(x, ...)

## S3 method for class 'ThU'
data2york(x, type = 2, generic = TRUE, ...)
```

Arguments

x	a five or six column matrix OR an object of class UPb, PbPb, ArAr, ThU, UThHe, or PD (which includes objects of class RbSr, SmNd, LuHf and ReOs), generated by the read.data(...) function
...	optional arguments
format	one of
	1 or 2: X, s[X], Y, s[Y], rho; where rho is the error correlation between X and Y; or
	3: X/Z, s[X/Z], Y/Z, s[Y/Z], X/Y, s[X/Y]; for which the error correlations are automatically computed from the redundancy of the three ratios.

option	returns one of

1. Wetherill concordia ratios: X=07/35, sX=s[07/35], Y=06/38, sY=s[06/38], rXY.

2. Tera-Wasserburg ratios: X=38/06, sX=s[38/06], Y=07/06, sY=s[07/06], rho=rXY.

3. X=04/06, sX=s[04/06], Y=38/06, sY=s[38/06], rho=rXY (only valid if format=4,5 or 6).

4. X=04/07, sX=s[04/07], Y=05/07, sY=s[35/07], rho=rXY (only valid if format=4,5 or 6).

5. U-Th-Pb concordia ratios: X=08/32, sX=s[08/32], Y=06/38, sY=s[06/38], rho=rXY (only valid if format=7 or 8).

6. X=38/06, sX=s[38/06], Y=08/06, sY=s[08/06], rho=rXY (only valid if format=7 or 8).

7. X=38/06, sX=s[38/06], Y=08/06, sY=s[08/06], rho=rXY (only valid if format=7 or 8).

tt	the age of the sample. This is only used if x has class 7 or 8, in order to calculate the inherited ^{208}Pb/^{232}Th ratio.
inverse	toggles between normal and inverse isochron ratios. data2york returns five columns X, s[X], Y, s[Y] and r[X,Y].

If inverse=FALSE, then X = ^{206}Pb/^{204}Pb and Y = ^{207}Pb/^{204}Pb (if x has class PbPb), or X = ^{39}Ar/^{36}Ar and Y = ^{40}Ar/^{36}Ar (if x has class ArAr), or X = ^{40}K/^{44}Ca and Y = ^{40}Ca/^{44}Ca (if x has class KCa), or X = ^{87}Rb/^{86}Sr and Y = ^{87}Sr/^{86}Sr (if x has class RbSr), or X = ^{147}Sm/^{144}Nd and Y = ^{143}Nd/^{144}Nd (if x has class SmNd), or X = ^{187}Re/^{188}Os and Y = ^{187}Os/^{188}Os (if x has class ReOs), or X = ^{176}Lu/^{177}Hf and Y = ^{176}Hf/^{177}Hf (if x has class LuHf).

If inverse=TRUE, then X = ^{204}Pb/^{206}Pb and Y = ^{207}Pb/^{206}Pb (if x has class PbPb), or X = ^{39}Ar/^{40}Ar and Y = ^{36}Ar/^{40}Ar (if x has class ArAr), or X = ^{40}K/^{40}Ca and Y = ^{44}Ca/^{40}Ca (if x has class KCa), or X = ^{87}Rb/^{87}Sr and Y = ^{86}Sr/^{87}Sr (if x has class RbSr), or X = ^{147}Sm/^{143}Nd and Y = ^{144}Nd/^{143}Nd (if x has class SmNd), or X = ^{187}Re/^{187}Os and Y = ^{188}Os/^{187}Os (if x has class ReOs), or X = ^{176}Lu/^{176}Hf and Y = ^{177}Hf/^{176}Hf (if x has class LuHf).

exterr	If TRUE, propagates the external uncertainties (e.g. decay constants) into the output errors.
type	Return 'Rosholt' or 'Osmond' ratios?

Rosholt (type=1) returns X=8/2, sX=s[8/2], Y=0/2, sY=s[0/2], rXY.

Osmond (type=2) returns X=2/8, sX=s[2/8], Y=0/8, sY=s[0/8], rXY.

generic	If TRUE, uses the following column headers: X, sX, Y, sY, rXY.

If FALSE and type=1, uses U238Th232, errU238Th232, Th230Th232, errTh230Th232, rho

or if FALSE and type=2, uses Th232U238, errTh232U238, Th230U238, errTh230U238, rho.

Value

a five-column table that can be used as input for [york](#)-regression.

See Also

york

Examples

```
f <- system.file("RbSr1.csv",package="IsoplotR")
dat <- read.csv(f)
yorkdat <- data2york(dat)
fit <- york(yorkdat)
```

---

diseq   *Set up U-series disequilibrium correction for U-Pb geochronology*

Description

The U-Pb method conventionally assumes initial secular equilibrium of all the intermediate daughters of the ^{238}U-^{206}Pb and ^{235}U-^{207}Pb decay chains. Violation of this assumption may produce inaccurate results. diseq sets up initial disequilibrium parameters that are subsequently passed on to the read.data function for incorporation in other functions.

Usage

```
diseq(U48 = list(x = 1, sx = 0, option = 0), ThU = list(x = 1, sx = 0,
  option = 0), RaU = list(x = 1, sx = 0, option = 0), PaU = list(x = 1,
  sx = 0, option = 0))
```

Arguments

U48   a list containing three items (x, sx and option) specifying the ^{234}U/^{238}U disequilibrium.

             If option=0, then x and sx are ignored and no disequilibrium correction is applied.

             If option=1, then x contains the initial ^{234}U/^{238}U ratio and sx its standard error.

             If option=2, then x contains the measured ^{234}U/^{238}U ratio and sx its standard error.

ThU   a list containing three items (x, sx and option) specifying the ^{230}Th/^{238}U disequilibrium.

             If option=0, then x and sx are ignored and no disequilibrium correction is applied.

             If option=1, then x contains the initial ^{230}Th/^{238}U ratio and sx its standard error.

             If option=2, then x contains the measured ^{230}Th/^{238}U ratio and sx its standard error.

             If option=3, then x contains the measured Th/U ratio of the magma (assumed or determined from the whole rock or volcanic glass) and sx its standard error. This only applies for Th-bearing U-Pb data formats 7 and 8.

RaU   a list containing three items (x, sx and option) specifying the ^{226}Ra/^{238}U dise-
                        quilibrium.

                        If option=0, then x and sx are ignored and no disequilibrium correction is ap-
                        plied.

                        If option=1, then x contains the initial ^{226}Ra/^{238}U ratio and sx its standard
                        error.

PaU   a list containing three items (x, sx and option) specifying the ^{231}Pa/^{235}U dise-
                        quilibrium.

                        If option=0, then x and sx are ignored and no disequilibrium correction is ap-
                        plied.

                        If option=1, then x contains the initial ^{231}Pa/^{235}U ratio and sx its standard
                        error.

Details

There are three ways to correct for the initial disequilibrium between the activity of ^{238}U, ^{234}Th, ^{230}Th, and ^{226}Ra; or between ^{235}U and ^{231}Pa:

1. Specify the assumed initial activity ratios and calculate how much excess ^{206}Pb and ^{207}Pb these would have produced.

2. Measure the current activity ratios to infer the initial ratios. This approach only works for young samples.

3. The initial ^{230}Th/^{238}U activity ratio can also be estimated by providing the Th/U-ratio of the magma.

Value

a list with the following items:

U48, ThU, RaU, PaU the same as the corresponding input arguments

equilibrium a boolean flag indicating whether option=TRUE and/or x=1 for all activity ratios

Q the eigenvectors of the disequilibrium matrix exponential

Qinv the inverse of Q

L a named vector of all the relevant decay constants

n0 the initial atomic abundances of all the parent and daughter isotopes (used by mclean)

See Also

mclean, concordia, ludwig

Examples

```
d <- diseq(U48=list(x=0,option=1),ThU=list(x=2,option=1),
           RaU=list(x=2,option=1),PaU=list(x=2,option=1))
fn <- system.file("diseq.csv",package="IsoplotR")
UPb <- read.data(fn,method='U-Pb',format=2,d=d)
concordia(UPb,type=2,show.age=1)
```

---

ellipse   *Get error ellipse coordinates for plotting*

Description

Constructs an error ellipse at a given confidence level from its centre and covariance matrix

Usage

```
ellipse(x, y, covmat, alpha = 0.05, n = 50)
```

Arguments

x	x-coordinate (scalar) for the centre of the ellipse
y	y-coordinate (scalar) for the centre of the ellipse
covmat	the [2x2] covariance matrix of the x-y coordinates
alpha	the probability cutoff for the error ellipses
n	the resolution (number of segments) of the error ellipses

Value

an [nx2] matrix of plot coordinates

Examples

```
x = 99; y = 101;
covmat <- matrix(c(1,0.9,0.9,1),nrow=2)
ell <- ellipse(x,y,covmat)
plot(c(90,110),c(90,110),type='l')
polygon(ell,col=rgb(0,1,0,0.5))
points(x,y,pch=21,bg='black')
```

---

evolution   *Th-U evolution diagram*

Description

Plots Th-U data on a ^{234}U/^{238}U-^{230}Th/^{238}U evolution diagram, a ^{234}U/^{238}U-age diagram, or (if ^{234}U/^{238}U is assumed to be in secular equilibrium), a ^{230}Th/^{232}Th-^{238}U/^{232}Th diagram, calculates isochron ages.

Usage

```
evolution(x, xlim = NA, ylim = NA, alpha = 0.05, transform = FALSE,
  detritus = 0, show.numbers = FALSE, levels = NA, clabel = "",
  ellipse.col = c("#00FF0080", "#FF000080"), line.col = "darksalmon",
  isochron = FALSE, model = 1, exterr = TRUE, sigdig = 2,
  hide = NULL, omit = NULL, omit.col = NA, ...)
```

Arguments

x	an object of class ThU
xlim	x-axis limits
ylim	y-axis limits
alpha	probability cutoff for the error ellipses and confidence intervals
transform	if TRUE, plots ^{234}U/^{238}U vs. Th-U age.
detritus	detrital ^{230}Th correction (only applicable when x$format is 2 or 3.
	0: no correction
	1: project the data along an isochron fit
	2: correct the data using an assumed initial ^{230}Th/^{232}Th-ratio for the detritus.
	3: correct the data using the measured present day ^{230}Th/^{238}U, ^{232}Th/^{238}U and ^{234}U/^{238}U-ratios in the detritus.
show.numbers	label the error ellipses with the grain numbers?
levels	a vector with additional values to be displayed as different background colours within the error ellipses.
clabel	label of the colour legend.
ellipse.col	Fill colour for the error ellipses. This can either be a single colour or multiple colours to form a colour ramp (to be used if levels!=NA):

- a single colour: rgb(0,1,0,0.5), '#FF000080', 'white', etc.
- multiple colours: c(rbg(1,0,0,0.5),rgb(0,1,0,0.5)), c('#FF000080','#00FF0080'), c('blue','red'), c('blue','yellow','red'), etc.
- a colour palette: rainbow(n=100), topo.colors(n=100,alpha=0.5), etc.
- a reversed palette: rev(topo.colors(n=100,alpha=0.5)), etc.
- for empty ellipses, set ellipse.col=NA

line.col	colour of the age grid
isochron	fit a 3D isochron to the data?
model	if isochron=TRUE, choose one of three regression models:
	1: maximum likelihood regression, using either the modified error weighted least squares algorithm of York et al. (2004) for 2-dimensional data, or the Maximum Likelihood formulation of Ludwig and Titterington (1994) for 3-dimensional data. These algorithms take into account the analytical uncertainties and error correlations, under the assumption that the scatter between the data points is solely caused by the analytical uncertainty. If this assumption is correct, then the MSWD value should be approximately equal to one. There are three strategies to deal with the case where MSWD>1. The first of these is to

assume that the analytical uncertainties have been underestipmated by a factor $\sqrt{MSWD}$.

2: ordinary least squares regression: a second way to deal with over- or under-dispersed datasets is to simply ignore the analytical uncertainties.

3: maximum likelihood regression with overdispersion: instead of attributing any overdispersion (MSWD > 1) to underestimated analytical uncertainties (model 1), one can also attribute it to the presence of geological uncertainty, which manifests itself as an added (co)variance term.

exterr	propagate the decay constant uncertainty in the isochron age?
sigdig	number of significant digits for the isochron age
hide	vector with indices of aliquots that should be removed from the plot.
omit	vector with indices of aliquots that should be plotted but omitted from the isochron age calculation.
omit.col	colour that should be used for the omitted aliquots.
...	optional arguments to the generic plot function

Details

Similar to the concordia diagram (for U-Pb data) and the helioplot diagram (for U-Th-He data), the evolution diagram simultaneously displays the isotopic composition and age of U-series data. For carbonate data (Th-U formats 1 and 2), the Th-U evolution diagram consists of a scatter plot that sets out the $^{234}U/^{238}U$-activity ratios against the $^{230}Th/^{238}U$-activity ratios as error ellipses, and displays the initial $^{234}U/^{238}U$-activity ratios and ages as a set of intersecting lines. Alternatively, the $^{234}U/^{238}U$-ratios can also be set out against the ^{230}Th-^{234}U-^{238}U-ages. In both types of evolution diagrams, IsoplotR provides the option to project the raw measurements along the best fitting isochron line and thereby remove the detrital ^{230}Th-component. This procedure allows a visual assessment of the degree of homogeneity within a dataset, as is quantified by the MSWD.

Neither the U-series evolution diagram, nor the $^{234}U/^{238}U$ vs. age plot is applicable to igneous datasets (Th-U formats 3 and 4), in which ^{234}U and ^{238}U are in secular equilibrium. For such datasets, IsoplotR produces an Osmond-style regression plot that is decorated with a fanning set of isochron lines.

References

Ludwig, K.R. and Titterington, D.M., 1994. Calculation of $^{230}Th/U$ isochrons, ages, and errors. Geochimica et Cosmochimica Acta, 58(22), pp.5031-5042.

Ludwig, K.R., 2003. Mathematical-statistical treatment of data and errors for $^{230}Th/U$ geochronology. Reviews in Mineralogy and Geochemistry, 52(1), pp.631-656.

See Also

isochron

Examples

```
data(examples)
evolution(examples$ThU)

dev.new()
evolution(examples$ThU,transform=TRUE,
          isochron=TRUE,model=1)
```

---

examples   *Example datasets for testing* IsoplotR

Description

Built-in datasets for U-Pb, Pb-Pb, Ar-Ar, K-Ca, Re-Os, Sm-Nd, Rb-Sr, Lu-Hf, U-Th-He, Th-U, fission track and detrital geochronology.

examples is an 18-item list containing:

UPb: an object of class UPb containing a high precision U-Pb dataset of Kamo et al. (1996) packaged with Ken Ludwig (2003)'s Isoplot program.

PbPb: an object of class PbPb containing a Pb-Pb dataset from Connelly et al. (2017).

DZ: an object of class detrital containing a detrital zircon U-Pb dataset from Namibia (Vermeesch et al., 2015).

ArAr: an object of class ArAr containing a ^{40}Ar/^{39}Ar spectrum of Skye basalt produced by Sarah Sherlock (Open University).

KCa: an object of class KCa containing a ^{40}K/^{40}Ca dataset for sample 140025 grain h spot 5 of Harrison et al. (2010).

UThHe: an object of class UThHe containing a U-Th-Sm-He dataset of Fish Lake apatite produced by Daniel Stockli (UT Austin).

FT1: an object of class fissiontracks containing a synthetic external detector dataset.

FT2: an object of class fissiontracks containing a synthetic LA-ICP-MS-based fission track dataset using the zeta calibration method.

FT3: an object of class fissiontracks containing a synthetic LA-ICP-MS-based fission track dataset using the absolute dating approach.

ReOs: an object of class ReOs containing a ^{187}Os/^{187}Re-dataset from Selby (2007).

SmNd: an object of class SmNd containing a ^{143}Nd/^{147}Sm-dataset from Lugmair et al. (1975).

RbSr: an object of class RbSr containing an ^{87}Rb/^{86}Sr-dataset from Compston et al. (1971).

LuHf: an object of class LuHf containing an ^{176}Lu/^{177}Hf-dataset from Barfod et al. (2002).

ThU: an object of class ThU containing a synthetic 'Osmond-type' dataset from Titterington and Ludwig (1994).

LudwigMean: an object of class other containing a collection of ^{206}Pb/^{238}U-ages and errors of the example dataset by Ludwig (2003).

LudwigKDE: an object of class 'other' containing the ^{206}Pb/^{238}U-ages (but not the errors) of the example dataset by Ludwig (2003).

LudwigSpectrum: an object of class 'other' containing the ^{39}Ar abundances, ^{40}Ar/^{39}Ar-ages and errors of the example dataset by Ludwig (2003).

LudwigMixture: an object of class 'other' containing a dataset of dispersed zircon fission track ages of the example dataset by Ludwig (2003).

References

Barfod, G.H., Albarede, F., Knoll, A.H., Xiao, S., Telouk, P., Frei, R. and Baker, J., 2002. New Lu-Hf and Pb-Pb age constraints on the earliest animal fossils. Earth and Planetary Science Letters, 201(1), pp.203-212.

Compston, W., Berry, H., Vernon, M.J., Chappell, B.W. and Kaye, M.J., 1971. Rubidium-strontium chronology and chemistry of lunar material from the Ocean of Storms. In Lunar and Planetary Science Conference Proceedings (Vol. 2, p. 1471).

Connelly, J.N., Bollard, J. and Bizzarro, M., 2017. Pb-Pb chronometry and the early Solar System. Geochimica et Cosmochimica Acta, 201, pp.345-363.

Galbraith, R. F. and Green, P. F., 1990: Estimating the component ages in a finite mixture, Nuclear Tracks and Radiation Measurements, 17, 197-206.

Harrison, T.M., Heizler, M.T., McKeegan, K.D. and Schmitt, A.K., 2010. In situ ^{40}K-^{40}Ca 'double-plus' SIMS dating resolves Klokken feldspar ^{40}K-^{40}Ar paradox. Earth and Planetary Science Letters, 299(3-4), pp.426-433.

Kamo, S.L., Czamanske, G.K. and Krogh, T.E., 1996. A minimum U-Pb age for Siberian flood-basalt volcanism. Geochimica et Cosmochimica Acta, 60(18), 3505-3511.

Ludwig, K. R., and D. M. Titterington., 1994. "Calculation of ^{230}Th/U isochrons, ages, and errors." Geochimica et Cosmochimica Acta 58.22, 5031-5042.

Ludwig, K. R., 2003. User's manual for Isoplot 3.00: a geochronological toolkit for Microsoft Excel. No. 4.

Lugmair, G.W., Scheinin, N.B. and Marti, K., 1975. Sm-Nd age and history of Apollo 17 basalt 75075-Evidence for early differentiation of the lunar exterior. In Lunar and Planetary Science Conference Proceedings (Vol. 6, pp. 1419-1429).

Selby, D., 2007. Direct Rhenium-Osmium age of the Oxfordian-Kimmeridgian boundary, Staffin bay, Isle of Skye, UK, and the Late Jurassic time scale. Norsk Geologisk Tidsskrift, 87(3), p.291.

Vermeesch, P. and Garzanti, E., 2015. Making geological sense of 'Big Data' in sedimentary provenance analysis. Chemical Geology, 409, pp.20-27.

Vermeesch, P., 2008. Three new ways to calculate average (U-Th)/He ages. Chemical Geology, 249(3),pp.339-347.

Examples

```
data(examples)

concordia(examples$UPb)

agespectrum(examples$ArAr)
```

```
isochron(examples$ReOs)

radialplot(examples$FT1)

helioplot(examples$UThHe)

evolution(examples$ThU)

kde(examples$DZ)

radialplot(examples$LudwigMixture)

agespectrum(examples$LudwigSpectrum)

weightedmean(examples$LudwigMean)
```

---

helioplot   *Visualise U-Th-He data on a logratio plot or ternary diagram*

Description

Plot U-Th(-Sm)-He data on a (log[He/Th] vs. log[U/He]) logratio plot or U-Th-He ternary diagram

Usage

```
helioplot(x, logratio = TRUE, model = 1, show.central.comp = TRUE,
  show.numbers = FALSE, alpha = 0.05, contour.col = c("white",
  "red"), levels = NA, clabel = "", ellipse.col = c("#00FF0080",
  "#0000FF80"), sigdig = 2, xlim = NA, ylim = NA, fact = NA,
  hide = NULL, omit = NULL, omit.col = NA, ...)
```

Arguments

x	an object of class UThHe
logratio	Boolean flag indicating whether the data should be shown on bivariate log[He/Th] vs. log[U/He] diagram, or a U-Th-He ternary diagram.
model	choose one of the following statistical models:
	1: weighted mean. This model assumes that the scatter between the data points is solely caused by the analytical uncertainty. If the assumption is correct, then the MSWD value should be approximately equal to one. There are three strategies to deal with the case where MSWD>1. The first of these is to assume that the analytical uncertainties have been underestimated by a factor $\sqrt{MSWD}$.
	2: unweighted mean. A second way to deal with over- or underdispersed datasets is to simply ignore the analytical uncertainties.
	3: weighted mean with overdispersion: instead of attributing any overdispersion (MSWD > 1) to underestimated analytical uncertainties (model 1), it can also be

attributed to the presence of geological uncertainty, which manifests itself as an added (co)variance term.

show.central.comp	
	show the geometric mean composition as a white ellipse?
show.numbers	show the grain numbers inside the error ellipses?
alpha	probability cutoff for the error ellipses and confidence intervals
contour.col	two-element vector with the fill colours to be assigned to the minimum and maximum age contour
levels	a vector with additional values to be displayed as different background colours within the error ellipses.
clabel	label of the colour scale
ellipse.col	Fill colour for the error ellipses. This can either be a single colour or multiple colours to form a colour ramp (to be used if `levels!=NA`):

- a single colour: `rgb(0,1,0,0.5)`, `'#FF000080'`, `'white'`, etc.
- multiple colours: `c(rbg(1,0,0,0.5),rgb(0,1,0,0.5))`, `c('#FF000080','#00FF0080')`, `c('blue','red')`, `c('blue','yellow','red')`, etc.
- a colour palette: `rainbow(n=100)`, `topo.colors(n=100,alpha=0.5)`, etc.
- a reversed palette: `rev(topo.colors(n=100,alpha=0.5))`, etc.
- for empty ellipses, set `ellipse.col=NA`

sigdig	number of significant digits for the central age
xlim	optional limits of the x-axis (log[U/He]) of the logratio plot. If `xlim=NA`, the axis limits are determined automatically.
ylim	optional limits of the y-axis (log[Th/He]) of the logratio plot. If `ylim=NA`, the axis limits are determined automatically.
fact	three-element vector with scaling factors of the ternary diagram if `fact=NA`, these will be determined automatically
hide	vector with indices of aliquots that should be removed from the plot.
omit	vector with indices of aliquots that should be plotted but omitted from the central age calculation.
omit.col	colour that should be used for the omitted aliquots.
...	optional arguments to the generic `plot` function

Details

U, Th, Sm and He are *compositional* data. This means that it is not so much the absolute concentrations of these elements that bear the chronological information, but rather their relative proportions. The space of all possible U-Th-He compositions fits within the constraints of a ternary diagram or 'helioplot' (Vermeesch, 2008, 2010). If Sm is included as well, then this expands to a three-dimensional tetrahaedral space (Vermeesch, 2008). Data that fit within these constrained spaces must be subjected to a logratio transformation prior to statistical analysis (Aitchison, 1986). In the case of the U-Th-He-(Sm)-He system, this is achieved by first defining two (or three) new variables:

$u \equiv \ln[U/He] \; v \equiv \ln[Th/He] \; (, w \equiv \ln[Sm/He])$

and then performing the desired statistical analysis (averaging, uncertainty propagation, ...) on the transformed data. Upon completion of the mathematical operations, the results can then be mapped back to U-Th-(Sm)-He space using an inverse logratio transformation:

$[He] = 1/[e^u + e^v + (e^w) + 1], [U] = e^u/[e^u + e^v + (e^w) + 1]$
$[Th] = e^v/[e^u + e^v + (e^w) + 1], ([Sm] = e^w/[e^u + e^v + (e^w) + 1])$

where $[He] + [U] + [Th](+[Sm]) = 1$. In the context of U-Th-(Sm)-He dating, the *central* age is defined as the age that corresponds to the arithmetic mean composition in logratio space, which is equivalent to the geometric mean in compositional dataspace (Vermeesch, 2008). IsoplotR's `helioplot` function performs this calculation using the same algorithm that is used to obtain the weighted mean U-Pb composition for the `concordia` age calculation. Overdispersion is treated similarly as in a regression context (see `isochron`). Thus, there are options to augment the uncertainties with a factor $\sqrt{MSWD}$ (model 1); to ignore the analytical uncertainties altogether (model 2); or to add a constant overdispersion term to the analytical uncertainties (model 3). The `helioplot` function visualises U-Th-(Sm)-He data on either a ternary diagram or a bivariate $\ln[Th/U]$ vs. $\ln[U/He]$ contour plot. These diagrams provide a convenient way to simultaneously display the isotopic composition of samples as well as their chronological meaning. In this respect, they fulfil the same purpose as the U-Pb `concordia` diagram and the U-series `evolution` plot.

References

Aitchison, J., 1986, The statistical analysis of compositional data: London, Chapman and Hall, 416 p.

Vermeesch, P., 2008. Three new ways to calculate average (U-Th)/He ages. Chemical Geology, 249(3), pp.339-347.

Vermeesch, P., 2010. HelioPlot, and the treatment of overdispersed (U-Th-Sm)/He data. Chemical Geology, 271(3), pp.108-111.

See Also

radialplot

Examples

```
data(examples)
helioplot(examples$UThHe)
dev.new()
helioplot(examples$UThHe,logratio=FALSE)
```

---

isochron   *Calculate and plot isochrons*

Description

Plots cogenetic Ar-Ar, K-Ca, Pb-Pb, Rb-Sr, Sm-Nd, Re-Os, Lu-Hf, U-Th-He or Th-U data as X-Y scatterplots, fits an isochron curve through them using the `york` function, and computes the corresponding isochron age, including decay constant uncertainties.

Usage

```
isochron(x, ...)

## Default S3 method:
isochron(x, xlim = NA, ylim = NA, alpha = 0.05,
  sigdig = 2, show.numbers = FALSE, levels = NA, clabel = "",
  ellipse.col = c("#00FF0080", "#FF000080"), ci.col = "gray80",
  line.col = "black", lwd = 1, plot = TRUE, title = TRUE,
  model = 1, show.ellipses = 1 * (model != 2), xlab = "x",
  ylab = "y", hide = NULL, omit = NULL, omit.col = NA, ...)

## S3 method for class 'PbPb'
isochron(x, xlim = NA, ylim = NA, alpha = 0.05,
  sigdig = 2, show.numbers = FALSE, levels = NA, clabel = "",
  ellipse.col = c("#00FF0080", "#FF000080"), inverse = TRUE,
  ci.col = "gray80", line.col = "black", lwd = 1, plot = TRUE,
  exterr = TRUE, model = 1, show.ellipses = 1 * (model != 2),
  growth = FALSE, hide = NULL, omit = NULL, omit.col = NA, ...)

## S3 method for class 'ArAr'
isochron(x, xlim = NA, ylim = NA, alpha = 0.05,
  sigdig = 2, show.numbers = FALSE, levels = NA, clabel = "",
  ellipse.col = c("#00FF0080", "#FF000080"), inverse = TRUE,
  ci.col = "gray80", line.col = "black", lwd = 1, plot = TRUE,
  exterr = TRUE, model = 1, show.ellipses = 1 * (model != 2),
  hide = NULL, omit = NULL, omit.col = NA, ...)

## S3 method for class 'KCa'
isochron(x, xlim = NA, ylim = NA, alpha = 0.05,
  sigdig = 2, show.numbers = FALSE, levels = NA, clabel = "",
  ellipse.col = c("#00FF0080", "#FF000080"), inverse = FALSE,
  ci.col = "gray80", line.col = "black", lwd = 1, plot = TRUE,
  exterr = TRUE, model = 1, show.ellipses = 1 * (model != 2),
  hide = NULL, omit = NULL, omit.col = NA, ...)

## S3 method for class 'RbSr'
isochron(x, xlim = NA, ylim = NA, alpha = 0.05,
  sigdig = 2, show.numbers = FALSE, levels = NA, clabel = "",
  ellipse.col = c("#00FF0080", "#FF000080"), inverse = FALSE,
  ci.col = "gray80", line.col = "black", lwd = 1, plot = TRUE,
  exterr = TRUE, model = 1, show.ellipses = 1 * (model != 2),
  hide = NULL, omit = NULL, omit.col = NA, ...)

## S3 method for class 'ReOs'
isochron(x, xlim = NA, ylim = NA, alpha = 0.05,
  sigdig = 2, show.numbers = FALSE, levels = NA, clabel = "",
  ellipse.col = c("#00FF0080", "#FF000080"), inverse = FALSE,
  ci.col = "gray80", line.col = "black", lwd = 1, plot = TRUE,
```

```
      exterr = TRUE, model = 1, show.ellipses = 1 * (model != 2),
      hide = NULL, omit = NULL, omit.col = NA, ...)

  ## S3 method for class 'SmNd'
  isochron(x, xlim = NA, ylim = NA, alpha = 0.05,
    sigdig = 2, show.numbers = FALSE, levels = NA, clabel = "",
    ellipse.col = c("#00FF0080", "#FF000080"), inverse = FALSE,
    ci.col = "gray80", line.col = "black", lwd = 1, plot = TRUE,
    exterr = TRUE, model = 1, show.ellipses = 1 * (model != 2),
    hide = NULL, omit = NULL, omit.col = NA, ...)

  ## S3 method for class 'LuHf'
  isochron(x, xlim = NA, ylim = NA, alpha = 0.05,
    sigdig = 2, show.numbers = FALSE, levels = NA, clabel = "",
    ellipse.col = c("#00FF0080", "#FF000080"), inverse = FALSE,
    ci.col = "gray80", line.col = "black", lwd = 1, plot = TRUE,
    exterr = TRUE, model = 1, show.ellipses = 1 * (model != 2),
    hide = NULL, omit = NULL, omit.col = NA, ...)

  ## S3 method for class 'ThU'
  isochron(x, type = 2, xlim = NA, ylim = NA,
    alpha = 0.05, sigdig = 2, show.numbers = FALSE, levels = NA,
    clabel = "", ellipse.col = c("#00FF0080", "#FF000080"),
    ci.col = "gray80", line.col = "black", lwd = 1, plot = TRUE,
    exterr = TRUE, model = 1, show.ellipses = 1 * (model != 2),
    hide = NULL, omit = NULL, omit.col = NA, ...)

  ## S3 method for class 'UThHe'
  isochron(x, xlim = NA, ylim = NA, alpha = 0.05,
    sigdig = 2, show.numbers = FALSE, levels = NA, clabel = "",
    ellipse.col = c("#00FF0080", "#FF000080"), ci.col = "gray80",
    line.col = "black", lwd = 1, plot = TRUE, model = 1,
    show.ellipses = 2 * (model != 2), hide = NULL, omit = NULL,
    omit.col = "grey", ...)
```

Arguments

x	EITHER a matrix with the following five columns:
	X: the x-variable
	sX: the standard error of X
	Y: the y-variable
	sY: the standard error of Y
	rXY: the correlation coefficient of X and Y
	OR
	an object of class ArAr, KCa, PbPb, ReOs, RbSr, SmNd, LuHf, UThHe or ThU.
...	optional arguments to be passed on to the generic plot function if model=2
xlim	2-element vector with the x-axis limits

ylim	2-element vector with the y-axis limits
alpha	confidence cutoff for the error ellipses and confidence intervals
sigdig	the number of significant digits of the numerical values reported in the title of the graphical output
show.numbers	logical flag (TRUE to show grain numbers)
levels	a vector with additional values to be displayed as different background colours within the error ellipses.
clabel	label for the colour scale
ellipse.col	Fill colour for the error ellipses. This can either be a single colour or multiple colours to form a colour ramp (to be used if levels!=NA):

- a single colour: rgb(0,1,0,0.5), '#FF000080', 'white', etc.
- multiple colours: c(rbg(1,0,0,0.5),rgb(0,1,0,0.5)), c('#FF000080','#00FF0080'), c('blue','red'), c('blue','yellow','red'), etc.
- a colour palette: rainbow(n=100), topo.colors(n=100,alpha=0.5), etc.
- a reversed palette: rev(topo.colors(n=100,alpha=0.5)), etc.
- for empty ellipses, set ellipse.col=NA

ci.col	the fill colour for the confidence interval of the intercept and slope.
line.col	colour of the isochron line
lwd	line width
plot	if FALSE, suppresses the graphical output
title	add a title to the plot?
model	construct the isochron using either:

1. Error-weighted least squares regression
2. Ordinary least squares regression
3. Error-weighted least squares with overdispersion term

show.ellipses	show the data as:

1. points
2. error ellipses
3. error crosses

xlab	text label for the horizontal plot axis
ylab	text label for the vertical plot axis
hide	vector with indices of aliquots that should be removed from the plot.
omit	vector with indices of aliquots that should be plotted but omitted from the isochron age calculation.
omit.col	colour that should be used for the omitted aliquots.
inverse	toggles between normal and inverse isochrons. If the isochron plots Y against X, and
	if inverse=FALSE, then X = ^{206}Pb/^{204}Pb and Y = ^{207}Pb/^{204}Pb (if x has class PbPb), or X = ^{39}Ar/^{36}Ar and Y = ^{40}Ar/^{36}Ar (if x has class ArAr), or X = ^{40}K/^{44}Ca and Y = ^{40}Ca/^{44}Ca (if x has class KCa), or X = ^{87}Rb/^{86}Sr and Y = ^{87}Sr/^{86}Sr (if x

has class RbSr), or X = ^{147}Sm/^{144}Nd and Y = ^{143}Nd/^{144}Nd (if x has class SmNd), or X = ^{187}Re/^{188}Os and Y = ^{187}Os/^{188}Os (if x has class ReOs), or X = ^{176}Lu/^{177}Hf and Y = ^{176}Hf/^{177}Hf (if x has class LuHf).

if inverse=TRUE, then X = ^{204}Pb/^{206}Pb and Y = ^{207}Pb/^{206}Pb (if x has class PbPb), or X = ^{39}Ar/^{40}Ar and Y = ^{36}Ar/^{40}Ar (if x has class ArAr), or X = ^{40}K/^{40}Ca and Y = ^{44}Ca/^{40}Ca (if x has class KCa), or X = ^{87}Rb/^{87}Sr and Y = ^{86}Sr/^{87}Sr (if x has class RbSr), or X = ^{147}Sm/^{143}Nd and Y = ^{144}Nd/^{143}Nd (if x has class SmNd), or X = ^{187}Re/^{187}Os and Y = ^{188}Os/^{187}Os (if x has class ReOs), or X = ^{176}Lu/^{176}Hf and Y = ^{177}Hf/^{176}Hf (if x has class LuHf).

exterr	propagate external sources of uncertainty (J, decay constant)?
growth	add Stacey-Kramers Pb-evolution curve to the plot?
type	following the classification of Ludwig and Titterington (1994), one of either:

1. 'Rosholt type-II' isochron, setting out ^{230}Th/^{232}Th vs. ^{238}U/^{232}Th
2. 'Osmond type-II' isochron, setting out ^{230}Th/^{238}U vs. ^{232}Th/^{238}U
3. 'Rosholt type-II' isochron, setting out ^{234}U/^{232}Th vs. ^{238}U/^{232}Th
4. 'Osmond type-II' isochron, setting out ^{234}U/^{238}U vs. ^{232}Th/^{238}U

Details

Given several aliquots from a single sample, isochrons allow the non-radiogenic component of the daughter nuclide to be quantified and separated from the radiogenic component. In its simplest form, an isochron is obtained by setting out the amount of radiogenic daughter against the amount of radioactive parent, both normalised to a non-radiogenic isotope of the daughter element, and fitting a straight line through these points by least squares regression (Nicolaysen, 1961). The slope and intercept then yield the radiogenic daughter-parent ratio and the non-radiogenic daughter composition, respectively. There are several ways to fit an isochron. The easiest of these is ordinary least squares regression, which weighs all data points equally. In the presence of quantifiable analytical uncertainty, it is equally straightforward to use the inverse of the y-errors as weights. It is significantly more difficult to take into account uncertainties in both the x- and the y-variable (York, 1966). IsoplotR does so for its U-Th-He isochron calculations. The York (1966) method assumes that the analytical uncertainties of the x- and y-variables are independent from each other. This assumption is rarely met in geochronology. York (1968) addresses this issue with a bivariate error weighted linear least squares algorithm that accounts for covariant errors in both variables. This algorithm was further improved by York et al. (2004) to ensure consistency with the maximum likelihood approach of Titterington and Halliday (1979).

IsoplotR uses the York et al. (2004) algorithm for its Ar-Ar, K-Ca, Pb-Pb, Rb-Sr, Sm-Nd, Re-Os and Lu-Hf isochrons. The maximum likelihood algorithm of Titterington and Halliday (1979) was generalised from two to three dimensions by Ludwig and Titterington (1994) for U-series disequilibrium dating. Also this algorithm is implemented in IsoplotR. The extent to which the observed scatter in the data can be explained by the analytical uncertainties can be assessed using the Mean Square of the Weighted Deviates (MSWD, McIntyre et al., 1966), which is defined as:

$$MSWD = ([X - \hat{X}]\Sigma_X^{-1}[X - \hat{X}]^T)/df$$

where X are the data, $\hat{X}$ are the fitted values, and Σ_X is the covariance matrix of X, and $df = k(n-1)$ are the degrees of freedom, where k is the dimensionality of the linear fit. MSWD values that are far smaller or greater than 1 indicate under- or overdispersed measurements, respectively. Underdispersion can be attributed to overestimated analytical uncertainties. IsoplotR provides three alternative strategies to deal with overdispersed data:

1. Attribute the overdispersion to an underestimation of the analytical uncertainties. In this case, the excess scatter can be accounted for by inflating those uncertainties by a *factor* $\sqrt{MSWD}$.

2. Ignore the analytical uncertainties and perform an ordinary least squares regression.

3. Attribute the overdispersion to the presence of 'geological scatter'. In this case, the excess scatter can be accounted for by adding an overdispersion *term* that lowers the MSWD to unity.

Value

If x has class PbPb, ArAr, KCa, RbSr, SmNd, ReOs or LuHf, or UThHe, returns a list with the following items:

a the intercept of the straight line fit and its standard error.

b the slope of the fit and its standard error.

cov.ab the covariance of the slope and intercept

df the degrees of freedom of the linear fit ($df = n - 2$)

y0 a four-element list containing:

y: the atmospheric ^{40}Ar/^{36}Ar or initial ^{40}Ca/^{44}Ca, ^{187}Os/^{188}Os, ^{87}Sr/^{87}Rb, ^{143}Nd/^{144}Nd or ^{176}Hf/^{177}Hf ratio.

s[y]: the propagated uncertainty of y

ci[y]: the studentised $100(1 - \alpha)\%$ confidence interval for y.

disp[y]: the studentised $100(1 - \alpha)\%$ confidence interval for y enhanced by $\sqrt{mswd}$ (only applicable if model=1).

age a four-element list containing:

t: the ^{207}Pb/^{206}Pb, ^{40}Ar/^{39}Ar, ^{40}K/^{40}Ca, ^{187}Os/^{187}Re, ^{87}Sr/^{87}Rb, ^{143}Nd/^{144}Nd or ^{176}Hf/^{177}Hf age.

s[t]: the propagated uncertainty of t

ci[t]: the studentised $100(1 - \alpha)\%$ confidence interval for t.

disp[t]: the studentised $100(1 - \alpha)\%$ confidence interval for t enhanced by $\sqrt{mswd}$ (only applicable if model=1).

mswd the mean square of the residuals (a.k.a 'reduced Chi-square') statistic (omitted if model=2).

p.value the p-value of a Chi-square test for linearity (omitted if model=2)

w the overdispersion term, i.e. a three-element vector with the standard deviation of the (assumedly) Normally distributed geological scatter that underlies the measurements, and the lower and upper half-widths of its $100(1-\alpha)\%$ confidence interval (only returned if model=3).

OR, if x has class ThU:

par if x$type=1 or x$type=3: the best fitting ^{230}Th/^{232}Th intercept, ^{230}Th/^{238}U slope, ^{234}U/^{232}Th intercept and ^{234}U/^{238}U slope, OR, if x$type=2 or x$type=4: the best fitting ^{234}U/^{238}U intercept, ^{230}Th/^{232}Th slope, ^{234}U/^{238}U intercept and ^{234}U/^{232}Th slope.

cov the covariance matrix of par.

df the degrees of freedom for the linear fit, i.e. $(3n-3)$ if x$format=1 or x$format=2, and $(2n-2)$ if x$format=3 or x$format=4

a if `type=1`: the ^{230}Th/^{232}Th intercept; if `type=2`: the ^{230}Th/^{238}U intercept; if `type=3`: the ^{234}Th/^{232}Th intercept; if `type=4`: the ^{234}Th/^{238}U intercept and its propagated uncertainty.

b if `type=1`: the ^{230}Th/^{238}U slope; if `type=2`: the ^{230}Th/^{232}Th slope; if `type=3`: the ^{234}U/^{238}U slope; if `type=4`: the ^{234}U/^{232}Th slope and its propagated uncertainty.

cov.ab the covariance between `a` and `b`.

mswd the mean square of the residuals (a.k.a 'reduced Chi-square') statistic.

p.value the p-value of a Chi-square test for linearity.

tfact the $100(1 - \alpha/2)\%$ percentile of a t-distribution with `df` degrees of freedom.

y0 a four-element vector containing:

> y: the initial ^{234}U/^{238}U-ratio
>
> s[y]: the propagated uncertainty of y
>
> ci[y]: the studentised $100(1 - \alpha)\%$ confidence interval for y.
>
> disp[y]: the studentised $100(1 - \alpha)\%$ confidence interval for y enhanced by $\sqrt{mswd}$.

age a three (or four) element vector containing:

> t: the initial ^{234}U/^{238}U-ratio
>
> s[t]: the propagated uncertainty of t
>
> ci[t]: the studentised $100(1 - \alpha)\%$ confidence interval for t
>
> disp[t]: the studentised $100(1 - \alpha)\%$ confidence interval for t enhanced by $\sqrt{mswd}$ (only reported if `model=1`).

w the overdispersion term, i.e. a three-element vector with the standard deviation of the (assumedly) Normally distributed geological scatter that underlies the measurements, and the lower and upper half-width of its $100(1 - \alpha)\%$ confidence interval (only returned if `model=3`).

d a matrix with the following columns: the X-variable for the isochron plot, the analytical uncertainty of X, the Y-variable for the isochron plot, the analytical uncertainty of Y, and the correlation coefficient between X and Y.

xlab the x-label of the isochron plot

ylab the y-label of the isochron plot

References

Ludwig, K.R. and Titterington, D.M., 1994. Calculation of ^{230}Th/U isochrons, ages, and errors. Geochimica et Cosmochimica Acta, 58(22), pp.5031-5042.

Nicolaysen, L.O., 1961. Graphic interpretation of discordant age measurements on metamorphic rocks. Annals of the New York Academy of Sciences, 91(1), pp.198-206.

Titterington, D.M. and Halliday, A.N., 1979. On the fitting of parallel isochrons and the method of maximum likelihood. Chemical Geology, 26(3), pp.183-195.

York, D., 1966. Least-squares fitting of a straight line. Canadian Journal of Physics, 44(5), pp.1079-1086.

York, D., 1968. Least squares fitting of a straight line with correlated errors. Earth and Planetary Science Letters, 5, pp.320-324.

York, D., Evensen, N.M., Martinez, M.L. and De Basebe Delgado, J., 2004. Unified equations for the slope, intercept, and standard errors of the best straight line. American Journal of Physics, 72(3), pp.367-375.

See Also

[york](#), [titterington](#), [ludwig](#)

Examples

```
data(examples)
isochron(examples$RbSr)

fit <- isochron(examples$ArAr,inverse=FALSE,plot=FALSE)

dev.new()
isochron(examples$ThU,type=4)
```

IsoplotR   *library(IsoplotR)*

Description

A list of documented functions may be viewed by typing help(package='IsoplotR'). Detailed instructions are provided at <http://isoplotr.london-geochron.com>. Further details about the theoretical background are provided by Vermeesch (2018).

Author(s)

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References

Vermeesch, P., 2018, IsoplotR: a free and open toolbox for geochronology. Geoscience Frontiers, 9, 1479-1493, doi: 10.1016/j.gsf.2018.04.001.

See Also

Useful links:

- <http://isoplotr.london-geochron.com>

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kde	*Create (a) kernel density estimate(s)*

Description

Creates one or more kernel density estimates using a combination of the Botev (2010) bandwidth selector and the Abramson (1982) adaptive kernel bandwidth modifier.

Usage

```
kde(x, ...)

## Default S3 method:
kde(x, from = NA, to = NA, bw = NA,
  adaptive = TRUE, log = FALSE, n = 512, plot = TRUE, pch = "|",
  xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
  hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
  binwidth = NA, hide = NULL, ...)

## S3 method for class 'UPb'
kde(x, from = NA, to = NA, bw = NA, adaptive = TRUE,
  log = FALSE, n = 512, plot = TRUE, pch = "|",
  xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
  hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
  binwidth = NA, type = 4, cutoff.76 = 1100,
  cutoff.disc = list(-15, 5, TRUE), common.Pb = 0, hide = NULL, ...)

## S3 method for class 'detritals'
kde(x, from = NA, to = NA, bw = NA,
  adaptive = TRUE, log = FALSE, n = 512, plot = TRUE, pch = NA,
  xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
  hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
  binwidth = NA, ncol = NA, samebandwidth = TRUE, normalise = TRUE,
  hide = NULL, ...)

## S3 method for class 'PbPb'
kde(x, from = NA, to = NA, bw = NA, adaptive = TRUE,
  log = FALSE, n = 512, plot = TRUE, pch = "|",
  xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
  hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
  binwidth = NA, common.Pb = 1, hide = NULL, ...)

## S3 method for class 'ArAr'
kde(x, from = NA, to = NA, bw = NA, adaptive = TRUE,
  log = FALSE, n = 512, plot = TRUE, pch = "|",
  xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
  hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
```

```
  binwidth = NA, i2i = FALSE, hide = NULL, ...)

## S3 method for class 'KCa'
kde(x, from = NA, to = NA, bw = NA, adaptive = TRUE,
  log = FALSE, n = 512, plot = TRUE, pch = "|",
  xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
  hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
  binwidth = NA, i2i = FALSE, hide = NULL, ...)

## S3 method for class 'ThU'
kde(x, from = NA, to = NA, bw = NA, adaptive = TRUE,
  log = FALSE, n = 512, plot = TRUE, pch = "|",
  xlab = "age [ka]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
  hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
  binwidth = NA, i2i = FALSE, detritus = 0, hide = NULL, ...)

## S3 method for class 'ReOs'
kde(x, from = NA, to = NA, bw = NA, adaptive = TRUE,
  log = FALSE, n = 512, plot = TRUE, pch = "|",
  xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
  hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
  binwidth = NA, i2i = TRUE, hide = NULL, ...)

## S3 method for class 'SmNd'
kde(x, from = NA, to = NA, bw = NA, adaptive = TRUE,
  log = FALSE, n = 512, plot = TRUE, pch = "|",
  xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
  hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
  binwidth = NA, i2i = TRUE, hide = NULL, ...)

## S3 method for class 'RbSr'
kde(x, from = NA, to = NA, bw = NA, adaptive = TRUE,
  log = FALSE, n = 512, plot = TRUE, pch = "|",
  xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
  hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
  binwidth = NA, i2i = TRUE, hide = NULL, ...)

## S3 method for class 'LuHf'
kde(x, from = NA, to = NA, bw = NA, adaptive = TRUE,
  log = FALSE, n = 512, plot = TRUE, pch = "|",
  xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
  hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
  binwidth = NA, i2i = TRUE, hide = NULL, ...)

## S3 method for class 'UThHe'
kde(x, from = NA, to = NA, bw = NA,
  adaptive = TRUE, log = FALSE, n = 512, plot = TRUE, pch = "|",
  xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
```

```
    hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
    binwidth = NA, hide = NULL, ...)

## S3 method for class 'fissiontracks'
kde(x, from = NA, to = NA, bw = NA,
  adaptive = TRUE, log = FALSE, n = 512, plot = TRUE, pch = "|",
  xlab = "age [Ma]", ylab = "", kde.col = rgb(1, 0, 1, 0.6),
  hist.col = rgb(0, 1, 0, 0.2), show.hist = TRUE, bty = "n",
  binwidth = NA, hide = NULL, ...)
```

Arguments

x	a vector of numbers OR an object of class UPb, PbPb, ArAr, KCa, ReOs, SmNd, RbSr, UThHe, fissiontracks, ThU or detrital
...	optional arguments to be passed on to R's density function.
from	minimum age of the time axis. If NULL, this is set automatically
to	maximum age of the time axis. If NULL, this is set automatically
bw	the bandwidth of the KDE. If NULL, bw will be calculated automatically using the algorithm by Botev et al. (2010).
adaptive	logical flag controlling if the adaptive KDE modifier of Abramson (1982) is used
log	transform the ages to a log scale if TRUE
n	horizontal resolution (i.e., the number of segments) of the density estimate.
plot	show the KDE as a plot
pch	the symbol used to show the samples. May be a vector. Set pch=NA to turn them off.
xlab	the x-axis label
ylab	the y-axis label
kde.col	the fill colour of the KDE specified as a four element vector of r, g, b, alpha values
hist.col	the fill colour of the histogram specified as a four element vector of r, g, b, alpha values
show.hist	logical flag indicating whether a histogram should be added to the KDE
bty	change to "o", "l", "7", "c", "u", or "]" if you want to draw a box around the plot
binwidth	scalar width of the histogram bins, in Myr if log = FALSE, or as a fractional value if log = TRUE. Sturges' Rule $(\log_2[n] + 1$, where n is the number of data points) is used if binwidth = NA
hide	vector with indices of aliquots that should be removed from the plot.
type	scalar indicating whether to plot the ^{207}Pb/^{235}U age (type=1), the ^{206}Pb/^{238}U age (type=2), the ^{207}Pb/^{206}Pb age (type=3), the ^{207}Pb/^{206}Pb-^{206}Pb/^{238}U age (type=4), the concordia age (type=5), or the ^{208}Pb/^{232}Th age (type=6).
cutoff.76	the age (in Ma) below which the ^{206}Pb/^{238}U and above which the ^{207}Pb/^{206}Pb age is used. This parameter is only used if type=4.

cutoff.disc   discordance cutoff filter. This is a three element list.

                  The first two items contain the minimum (negative) and maximum (positive) percentage discordance allowed between the ^{207}Pb/^{235}U and ^{206}Pb/^{238}U age (if ^{206}Pb/^{238}U < cutoff.76) or between the ^{206}Pb/^{238}U and ^{207}Pb/^{206}Pb age (if ^{206}Pb/^{238}U > cutoff.76).

                  The third item is a boolean flag that controls whether the discordance filter should be applied before (TRUE) or after (FALSE) the common-Pb correction.

                  Set cutoff.disc=NA to turn off this filter.

common.Pb   apply a common lead correction using one of three methods:

                  1: the Stacey-Kramer two-stage model to infer the initial Pb-composition

                  2: the isochron intercept as the initial Pb-composition

                  3: the Pb-composition stored in

                  settings('iratio','Pb206Pb204') (if x has class UPb and x$format<4);

                  settings('iratio','Pb206Pb204') and settings('iratio','Pb207Pb204') (if x has class PbPb or x has class UPb and 3<x$format<7); or

                  settings('iratio','Pb208Pb206') and settings('iratio','Pb208Pb207') (if x has class UPb and x$format=7 or 8).

ncol   scalar value indicating the number of columns over which the KDEs should be divided.

samebandwidth   logical flag indicating whether the same bandwidth should be used for all samples. If samebandwidth = TRUE and bw = NULL, then the function will use the median bandwidth of all the samples.

normalise   logical flag indicating whether or not the KDEs should all integrate to the same value.

i2i   'isochron to intercept': calculates the initial (aka 'inherited', 'excess', or 'common') ^{40}Ar/^{36}Ar, ^{40}Ca/^{44}Ca, ^{87}Sr/^{86}Sr, ^{143}Nd/^{144}Nd, ^{187}Os/^{188}Os, ^{230}Th/^{232}Th or ^{176}Hf/^{177}Hf ratio from an isochron fit. Setting i2i to FALSE uses the default values stored in settings('iratio',...).

detritus   detrital ^{230}Th correction (only applicable when x$format=1 or 2).

                  0: no correction

                  1: project the data along an isochron fit

                  2: correct the data using an assumed initial ^{230}Th/^{232}Th-ratio for the detritus.

                  3: correct the data using the measured present day ^{230}Th/^{238}U, ^{232}Th/^{238}U and ^{234}U/^{238}U-ratios in the detritus.

Details

Given a set of n age estimates $\{t_1, t_2, ..., t_n\}$, histograms and KDEs are probability density estimators that display age distributions by smoothing. Histograms do this by grouping the data into a number of regularly spaced bins. Alternatively, kernel density estimates (KDEs; Vermeesch, 2012) smooth data by applying a (Gaussian) kernel:

$$KDE(t) = \sum_{i=1}^{n} N(t|\mu = t_i, \sigma = h[t])/n$$

where $N(t|\mu, \sigma)$ is the probability of observing a value t under a Normal distribution with mean μ and standard deviation σ. $h[t]$ is the smoothing parameter or 'bandwidth' of the kernel density

estimate, which may or may not depend on the age t. If $h[t]$ depends on t, then $KDE(t)$ is known as an 'adaptive' KDE. The default bandwidth used by `IsoplotR` is calculated using the algorithm of Botev et al. (2010) and modulated by the adaptive smoothing approach of Abramson (1982). The rationale behind adaptive kernel density estimation is to use a narrower bandwidth near the peaks of the sampling distribution (where the ordered dates are closely spaced in time), and a wider bandwidth in the distribution's sparsely sampled troughs. Thus, the resolution of the density estimate is optimised according to data availability.

Value

If x has class UPb, PbPb, ArAr, KCa, ReOs, SmNd, RbSr, UThHe, fissiontracks or ThU, returns an object of class KDE, i.e. a list containing the following items:

x horizontal plot coordinates

y vertical plot coordinates

bw the base bandwidth of the density estimate

ages the data values from the input to the kde function

log copied from the input

or, if x has class =detritals, an object of class KDEs, i.e. a list containing the following items:

kdes a named list with objects of class KDE

from the beginning of the common time scale

to the end of the common time scale

themax the maximum probability density of all the KDEs

xlabel the x-axis label to be used by plot.KDEs(...)

References

Abramson, I.S., 1982. On bandwidth variation in kernel estimates-a square root law. The annals of Statistics, pp.1217-1223.

Botev, Z. I., J. F. Grotowski, and D. P. Kroese. "Kernel density estimation via diffusion." The Annals of Statistics 38.5 (2010): 2916-2957.

Vermeesch, P., 2012. On the visualisation of detrital age distributions. Chemical Geology, 312, pp.190-194.

See Also

[radialplot](), [cad]()

Examples

```
kde(examples$UPb)

dev.new()
kde(examples$FT1,log=TRUE)

dev.new()
kde(examples$DZ,from=1,to=3000,kernel="epanechnikov")
```

---

ludwig	*Linear regression of U-Pb data with correlated errors, taking into account decay constant uncertainties.*

Description

Implements the maximum likelihood algorithm for Total-Pb/U isochron regression of Ludwig (1998)

Usage

```
ludwig(x, ...)

## Default S3 method:
ludwig(x, ...)

## S3 method for class 'UPb'
ludwig(x, exterr = FALSE, alpha = 0.05, model = 1,
  anchor = list(FALSE, NA), ...)
```

Arguments

x	an object of class UPb
...	optional
exterr	propagate external sources of uncertainty (i.e. decay constants)?
alpha	cutoff value for confidence intervals
model	one of three regression models:
	1: fit a discordia line through the data using the maximum likelihood algorithm of Ludwig (1998), which assumes that the scatter of the data is solely due to the analytical uncertainties. In this case, IsoplotR will either calculate an upper and lower intercept age (for Wetherill concordia), or a lower intercept age and common $(^{207}Pb/^{206}Pb)_\circ$-ratio intercept (for Tera-Wasserburg). If $MSWD{>}0$, then the analytical uncertainties are augmented by a factor $\sqrt{MSWD}$.
	2: fit a discordia line ignoring the analytical uncertainties
	3: fit a discordia line using a modified maximum likelihood algorithm that includes accounts for any overdispersion by adding a geological (co)variance term.
anchor	control parameters to fix the intercept age or common Pb composition of the discordia fit. This is a two-element list.
	• The first element is a boolean flag indicating whether the discordia line should be anchored. If this is FALSE, then the second item is ignored and both the common Pb composition and age are estimated.
	• If the first element is TRUE and the second element is NA, then the common Pb composition is fixed at the values stored in settings('iratio',...). item If the first element is TRUE and the second element is a number, then the discordia line is forced to intersect the concordia line at an age equal to that number.

Details

The 3-dimensional regression algorithm of Ludwig and Titterington (1994) was modified by Ludwig (1998) to fit so-called 'Total Pb-U isochrons'. These are constrained to a radiogenic endmember composition that falls on the concordia line. In its most sophisticated form, this algorithm does not only allow for correlated errors between variables, but also between aliquots. IsoplotR currently uses this algorithm to propagate decay constant uncertainties in the total Pb-U isochron ages. Future versions of the program will generalise this approach to other chronometers as well.

Value

par a two-element vector with the lower concordia intercept and initial ^{207}Pb/^{206}Pb-ratio.

cov the covariance matrix of par

df the degrees of freedom of the model fit ($3n - 3$, where n is the number of aliquots).

mswd the mean square of weighted deviates (a.k.a. reduced Chi-square statistic) for the fit.

p.value p-value of a Chi-square test for the linear fit

w the overdispersion, i.e., a three-element vector with the estimated standard deviation of the (assumedly) Normal distribution that underlies the true isochron; and the lower and upper half-widths of its $100(1 - \alpha)\%$ confidence interval (only relevant if model = 3).

References

Ludwig, K.R., 1998. On the treatment of concordant uranium-lead ages. Geochimica et Cosmochimica Acta, 62(4), pp.665-676.

Ludwig, K.R. and Titterington, D.M., 1994. Calculation of ^{230}Th/U isochrons, ages, and errors. Geochimica et Cosmochimica Acta, 58(22), pp.5031-5042.

See Also

concordia, titterington, isochron

Examples

```
f <- system.file("UPb4.csv",package="IsoplotR")
d <- read.data(f,method="U-Pb",format=4)
fit <- ludwig(d)
```

---

mclean   *Predict disequilibrium concordia compositions*

Description

Returns the predicted ^{206}Pb/^{238}U and ^{207}Pb/^{235}U ratios for any given time with or without initial U-series disequilibrium.

Usage

```
mclean(tt = 0, d = diseq(), exterr = FALSE)
```

Arguments

`tt`	the age of the sample
`d`	an object of class [diseq](#)
`exterr`	propagate the uncertainties associated with decay constants and the ^{238}U/^{235}U-ratio.

Details

U decays to Pb in 14 (for ^{238}U) or 11/12 (for ^{235}U) steps. Conventional U-Pb geochronology assumes that secular equilibrium between all the short lived intermediate daughters was established at the time of isotopic closure. Under this assumption, the relative abundances of those intermediate daughters can be neglected and the age equation reduces to a simple function of the measured Pb/U ratios. In reality, however, the assumption of initial secular equilibrium is rarely met. Accounting for disequilibrium requires a more complex set of age equations, which are based on a coupled system of differetial equations. The solution to this system of equations is given by a matrix exponential. `IsoplotR` solves this matrix exponential for any given time, using either the assumed initial activity ratios, or (for young samples) the measured activity ratios of the longest lived intermediate daughters. Based on a `Matlab` script by Noah McLean.

Value

a list containing the predicted ^{206}Pb/^{238}U, ^{207}Pb/^{235}U and ^{207}Pb/^{206}Pb ratios at time `tt`; the derivatives of the ^{206}Pb/^{238}U, ^{207}Pb/^{235}U and ^{207}Pb/^{206}Pb ratios with respect to time; and the derivatives of the ^{206}Pb/^{238}U, ^{207}Pb/^{235}U and ^{207}Pb/^{206}Pb ratios with respect to the intermediate decay constants and ^{238}U/^{235}U-ratio.

Author(s)

Noah McLean (algorithm) and Pieter Vermeesch (code)

See Also

[diseq](#)

Examples

```
d <- diseq(U48=list(x=0,option=1),ThU=list(x=2,option=1),
           RaU=list(x=2,option=1),PaU=list(x=2,option=1))
mclean(tt=2,d=d)
```

---

mds   *Multidimensional Scaling*

Description

Performs classical or nonmetric Multidimensional Scaling analysis

Usage

```
mds(x, ...)

## Default S3 method:
mds(x, classical = FALSE, plot = TRUE,
  shepard = FALSE, nnlines = FALSE, pos = NULL, col = "black",
  bg = "white", xlab = "", ylab = "", ...)

## S3 method for class 'detritals'
mds(x, classical = FALSE, plot = TRUE,
  shepard = FALSE, nnlines = FALSE, pos = NULL, col = "black",
  bg = "white", xlab = "", ylab = "", hide = NULL, ...)
```

Arguments

x	a dissimilarity matrix OR an object of class `detrital`
...	optional arguments to the generic `plot` function
classical	logical flag indicating whether classical (`TRUE`) or nonmetric (`FALSE`) MDS should be used
plot	show the MDS configuration (if shepard=FALSE) or Shepard plot (if shepard=TRUE) on a graphical device
shepard	logical flag indicating whether the graphical output should show the MDS configuration (shepard=FALSE) or a Shepard plot with the 'stress' value. This argument is only used if `plot=TRUE`.
nnlines	if `TRUE`, draws nearest neighbour lines
pos	a position specifier for the labels (if par('pch')!=NA). Values of 1, 2, 3 and 4 indicate positions below, to the left of, above and to the right of the MDS coordinates, respectively.
col	plot colour (may be a vector)
bg	background colour (may be a vector)
xlab	a string with the label of the x axis
ylab	a string with the label of the y axis
hide	vector with indices of aliquots that should be removed from the plot.

Details

Multidimensional Scaling (MDS) is a dimension-reducing technique that takes a matrix of pairwise 'dissimilarities' between objects (e.g., age distributions) as input and produces a configuration of two (or higher-) dimensional coordinates as output, so that the Euclidean distances between these coordinates approximate the dissimilarities of the input matrix. Thus, an MDS-configuration serves as a 'map' in which similar samples cluster closely together and dissimilar samples plot far apart. In the context of detrital geochronology, the dissimilarity between samples is given by the statistical distance between age distributions. There are many ways to define this statistical distance. IsoplotR uses the Kolmogorov-Smirnov (KS) statistic due to its simplicity and the fact that it behaves like a true distance in the mathematical sense of the word (Vermeesch, 2013). The KS-distance is given by the maximum vertical distance between two cad step functions. Thus, the KS-distance takes on values between zero (perfect match between two age distributions) and one (no overlap between two distributions). Calculating the KS-distance between samples two at a time populates a symmetric dissimilarity matrix with positive values and a zero diagonal. IsoplotR implements two algorithms to convert this matrix into a configuration. The first ('classical') approach uses a sequence of basic matrix manipulations developed by Young and Householder (1938) and Torgerson (1952) to achieve a linear fit between the KS-distances and the fitted distances on the MDS configuration. The second, more sophisticated ('nonmetric') approach subjects the input distances to a transformation f prior to fitting a configuration:

$$\delta_{i,j} = f(KS_{i,j})$$

where $KS_{i,j}$ is the KS-distance between samples i and j (for $1 \leq i \neq j \leq n$) and $\delta_{i,j}$ is the 'disparity' (Kruskal, 1964). Fitting an MDS configuration then involves finding the disparity transformation that maximises the goodness of fit (or minimises the 'stress') between the disparities and the fitted distances. The latter two quantities can also be plotted against each other as a 'Shepard plot'.

Value

Returns an object of class MDS, i.e. a list containing the following items:

points a two-column vector of the fitted configuration

classical a logical flag indicating whether the MDS configuration was obtained by classical (TRUE) or nonmetric (FALSE) MDS

diss the dissimilarity matrix used for the MDS analysis

stress (only if classical=TRUE) the final stress achieved (in percent)

References

Kruskal, J., 1964. Multidimensional scaling by optimizing goodness of fit to a nonmetric hypothesis. Psychometrika 29 (1), 1-27.

Torgerson, W. S. Multidimensional scaling: I. Theory and method. Psychometrika, 17(4): 401-419, 1952.

Vermeesch, P., 2013. Multi-sample comparison of detrital age distributions. Chemical Geology, 341, pp.140-146.

Young, G. and Householder, A. S. Discussion of a set of points in terms of their mutual distances. Psychometrika, 3(1):19-22, 1938.

See Also

[cad](#), [kde](#)

Examples

```
data(examples)
mds(examples$DZ,nnlines=TRUE,pch=21,cex=5)
dev.new()
mds(examples$DZ,shepard=TRUE)
```

Pb0corr   *Common Pb correction*

Description

Applies a common-Pb correction to a U-Pb dataset using either the Stacey-Kramers mantle evolution model, isochron regression, or any nominal inital Pb isotope composition.

Usage

```
Pb0corr(x, option = 1, omit = NULL)
```

Arguments

x   an object of class UPb

option   one of either

1. Stacey-Kramers correction
2. isochron regression
3. nominal common Pb isotope composition

omit   vector with indices of aliquots that should be omitted from the isochron regression (only used if option=2)

Details

IsoplotR implements six different methods to correct for the presence of non-radiogenic ('common') lead. This includes three strategies tailored to datasets that include ^{204}Pb measurements and a further three strategies for datasets that do not. ^{204}Pb is the only one of lead's four stable isotopes that does not have a naturally occurring radioactive parent. This makes it very useful for common-Pb correction:

$$\left[\frac{^{206|7}Pb}{^{204}Pb}\right]_r = \left[\frac{^{206|7}Pb}{^{204}Pb}\right]_m - \left[\frac{^{206|7}Pb}{^{204}Pb}\right]_\circ$$

where $[^{206|7}Pb/^{204}Pb]_r$ marks the radiogenic ^{206}Pb or ^{207}Pb component; $[^{206|7}Pb/^{204}Pb]_m$ is the measured ratio; and $[^{206|7}Pb/^{204}Pb]_\circ$ is the non-radiogenic component.

IsoplotR offers three different ways to determine $[^{206|7}Pb/^{204}Pb]_\circ$. The first and easiest option is to simply use a nominal value such as the $^{206|7}$Pb/^{204}Pb-ratio of a cogenetic feldspar, assuming that this is representative for the common-Pb composition of the entire sample. A second method

is to determine the non-radiogenic isotope composition by fitting an isochron line through multiple aliquots of the same sample, using the 3-dimensional regression algorithm of Ludwig (1998).

Unfortunately, neither of these two methods is applicable to detrital samples, which generally lack identifiable cogenetic minerals and aliquots. For such samples, IsoplotR infers the common-Pb composition from the two-stage crustal evolution model of Stacey and Kramers (1975). The second stage of this model is described by:

$$\left[\frac{206Pb}{204Pb}\right]_{\circ} = \left[\frac{206Pb}{204Pb}\right]_{3.7Ga} + \left[\frac{238U}{204Pb}\right]_{sk} \left(e^{\lambda_{238}3.7Ga} - e^{\lambda_{238}t}\right)$$

where $\left[206Pb/204Pb\right]_{3.7Ga} = 11.152$ and $\left[238U/204Pb\right]_{sk} = 9.74$. These Equations can be solved iteratively for t and $\left[206Pb/204Pb\right]_{\circ}$. The ^{207}Pb/^{204}Pb-ratio is corrected in exactly the same way, using $\left[207Pb/204Pb\right]_{3.7Ga} = 12.998$.

In the absence of ^{204}Pb measurements, a ^{207} Pb-based common lead correction can be used:

$$\left[\frac{207Pb}{206Pb}\right]_{m} = f \left[\frac{207Pb}{206Pb}\right]_{\circ} + (1-f) \left[\frac{207Pb}{204Pb}\right]_{r}$$

where f is the fraction of common lead, and $[207Pb/206Pb]_r$ is obtained by projecting the U-Pb measurements on the concordia line in Tera-Wasserburg space. Like before, the initial lead composition $[207Pb/206Pb]_{\circ}$ can be obtained in three possible ways: by analysing a cogenetic mineral, by isochron regression through multiple aliquots, or from the Stacey and Kramers (1975) model.

Besides the common-Pb problem, a second reason for U-Pb discordance is radiogenic Pb-loss during igneous and metamorphic activity. This moves the data away from the concordia line along a linear array, forming an isochron or 'discordia' line. IsoplotR fits this line using the Ludwig (1998) algorithm. If the data are plotted on a Wetherill concordia diagram, the program will not only report the usual lower intercept with the concordia line, but the upper intercept as well. Both values are geologically meaningful as they constrain both the initial igneous age as well as the timing of the partial resetting event.

Value

Returns a list in which x.raw contains the original data and x the common Pb-corrected compositions. All other items in the list are inherited from the input data.

References

Ludwig, K.R., 1998. On the treatment of concordant uranium-lead ages. Geochimica et Cosmochimica Acta, 62(4), pp.665-676.

Stacey, J.T. and Kramers, 1., 1975. Approximation of terrestrial lead isotope evolution by a two-stage model. Earth and planetary science letters, 26(2), pp.207-221.

Examples

```
data(examples)
UPb <- Pb0corr(examples$UPb,option=1)
concordia(UPb)
# produces identical results as:
dev.new()
concordia(examples$UPb,common.Pb=1)
```

peakfit   *Finite mixture modelling of geochronological datasets*

Description

Implements the discrete mixture modelling algorithms of Galbraith and Laslett (1993) and applies them to fission track and other geochronological datasets.

Usage

```
peakfit(x, ...)

## Default S3 method:
peakfit(x, k = "auto", sigdig = 2, log = TRUE,
  alpha = 0.05, ...)

## S3 method for class 'fissiontracks'
peakfit(x, k = 1, exterr = TRUE, sigdig = 2,
  log = TRUE, alpha = 0.05, ...)

## S3 method for class 'UPb'
peakfit(x, k = 1, type = 4, cutoff.76 = 1100,
  cutoff.disc = c(-15, 5), common.Pb = 0, exterr = TRUE,
  sigdig = 2, log = TRUE, alpha = 0.05, ...)

## S3 method for class 'PbPb'
peakfit(x, k = 1, exterr = TRUE, sigdig = 2,
  log = TRUE, common.Pb = 0, alpha = 0.05, ...)

## S3 method for class 'ArAr'
peakfit(x, k = 1, exterr = TRUE, sigdig = 2,
  log = TRUE, i2i = FALSE, alpha = 0.05, ...)

## S3 method for class 'KCa'
peakfit(x, k = 1, exterr = TRUE, sigdig = 2,
  log = TRUE, i2i = FALSE, alpha = 0.05, ...)

## S3 method for class 'ReOs'
peakfit(x, k = 1, exterr = TRUE, sigdig = 2,
  log = TRUE, i2i = TRUE, alpha = 0.05, ...)

## S3 method for class 'SmNd'
peakfit(x, k = 1, exterr = TRUE, sigdig = 2,
  log = TRUE, i2i = TRUE, alpha = 0.05, ...)

## S3 method for class 'RbSr'
peakfit(x, k = 1, exterr = TRUE, sigdig = 2,
```

```
    log = TRUE, i2i = TRUE, alpha = 0.05, ...)

## S3 method for class 'LuHf'
peakfit(x, k = 1, exterr = TRUE, sigdig = 2,
    log = TRUE, i2i = TRUE, alpha = 0.05, ...)

## S3 method for class 'ThU'
peakfit(x, k = 1, exterr = FALSE, sigdig = 2,
    log = TRUE, i2i = TRUE, alpha = 0.05, detritus = 0, ...)

## S3 method for class 'UThHe'
peakfit(x, k = 1, sigdig = 2, log = TRUE,
    alpha = 0.05, ...)
```

Arguments

<table>
<tr><td>x</td><td>either an [nx2] matrix with measurements and their standard errors, or an object of class fissiontracks, UPb, PbPb, ArAr, KCa, ReOs, SmNd, RbSr, LuHf, ThU or UThHe</td></tr>
<tr><td>...</td><td>optional arguments (not used)</td></tr>
<tr><td>k</td><td>the number of discrete age components to be sought. Setting this parameter to 'auto' automatically selects the optimal number of components (up to a maximum of 5) using the Bayes Information Criterion (BIC).</td></tr>
<tr><td>sigdig</td><td>number of significant digits to be used for any legend in which the peak fitting results are to be displayed.</td></tr>
<tr><td>log</td><td>take the logs of the data before applying the mixture model?</td></tr>
<tr><td>alpha</td><td>cutoff value for confidence intervals</td></tr>
<tr><td>exterr</td><td>propagate the external sources of uncertainty into the component age errors?</td></tr>
<tr><td>type</td><td>scalar indicating whether to plot the $^{207}\text{Pb}/^{235}\text{U}$ age (type=1), the $^{206}\text{Pb}/^{238}\text{U}$ age (type=2), the $^{207}\text{Pb}/^{206}\text{Pb}$ age (type=3), the $^{207}\text{Pb}/^{206}\text{Pb}$-$^{206}\text{Pb}/^{238}\text{U}$ age (type=4), the concordia age (type=5), or the $^{208}\text{Pb}/^{232}\text{Th}$ age (type=6).</td></tr>
<tr><td>cutoff.76</td><td>the age (in Ma) below which the $^{206}\text{Pb}/^{238}\text{U}$ and above which the $^{207}\text{Pb}/^{206}\text{Pb}$ age is used. This parameter is only used if type=4.</td></tr>
<tr><td>cutoff.disc</td><td>two element vector with the maximum and minimum percentage discordance allowed between the $^{207}\text{Pb}/^{235}\text{U}$ and $^{206}\text{Pb}/^{238}\text{U}$ age (if $^{206}\text{Pb}/^{238}\text{U} <$ cutoff.76) or between the $^{206}\text{Pb}/^{238}\text{U}$ and $^{207}\text{Pb}/^{206}\text{Pb}$ age (if $^{206}\text{Pb}/^{238}\text{U} >$ cutoff.76). Set cutoff.disc=NA if you do not want to use this filter.</td></tr>
<tr><td>common.Pb</td><td>apply a common lead correction using one of three methods:
1: the Stacey-Kramer two-stage model to infer the initial Pb-composition
2: the isochron intercept as the initial Pb-composition
3: the Pb-composition stored in
settings('iratio','Pb206Pb204') (if x has class UPb and x$format<4);
settings('iratio','Pb206Pb204') and settings('iratio','Pb207Pb204') (if x has class PbPb or x has class UPb and 3<x$format<7); or
settings('iratio','Pb208Pb206') and settings('iratio','Pb208Pb207') (if x has class UPb and x$format=7 or 8).</td></tr>
</table>

i2i	'isochron to intercept': calculates the initial (aka 'inherited', 'excess', or 'common') ^{40}Ar/^{36}Ar, ^{40}Ca/^{44}Ca, ^{87}Sr/^{86}Sr, ^{143}Nd/^{144}Nd, ^{187}Os/^{188}Os or ^{176}Hf/^{177}Hf ratio from an isochron fit. Setting i2i to FALSE uses the default values stored in settings('iratio',...). When applied to data of class ThU, setting i2i to TRUE applies a detrital Th-correction.
detritus	detrital ^{230}Th correction (only applicable when x$format=1 or 2.
	0: no correction
	1: project the data along an isochron fit
	2: correct the data using an assumed initial ^{230}Th/^{232}Th-ratio for the detritus.
	3: correct the data using the measured present day ^{230}Th/^{238}U, ^{232}Th/^{238}U and ^{234}U/^{238}U-ratios in the detritus.

Details

Consider a dataset of n dates $\{t_1, t_2, ..., t_n\}$ with analytical uncertainties $\{s[t_1], s[t_2], ..., s[t_n]\}$. Define $z_i = \log(t_i)$ and $s[z_i] = s[t_i]/t_i$. Suppose that these n values are derived from a mixture of $k > 2$ populations with means $\{\mu_1, ..., \mu_k\}$. Such a *discrete mixture* may be mathematically described by: $P(z_i|\mu, \omega) = \sum_{j=1}^{k} \pi_j N(z_i|\mu_j, s[z_j]^2)$ where π_j is the proportion of the population that belongs to the jth component, and $\pi_k = 1 - \sum_{j=1}^{k-1} \pi_j$. This equation can be solved by the method of maximum likelihood (Galbraith and Laslett, 1993). IsoplotR implements the Bayes Information Criterion (BIC) as a means of automatically choosing k. This option should be used with caution, as the number of peaks steadily rises with sample size (n). If one is mainly interested in the youngest age component, then it is more productive to use an alternative parameterisation, in which all grains are assumed to come from one of two components, whereby the first component is a single discrete age peak ($\exp(m)$, say) and the second component is a continuous distribution (as descibed by the [central](#) age model), but truncated at this discrete value (Van der Touw et al., 1997).

Value

Returns a list with the following items:

peaks a 3 x k matrix with the following rows:

    t: the ages of the k peaks

    s[t]: the estimated uncertainties of t

    ci[t]: the widths of approximate $100(1-\alpha)\%$ confidence intervals for t

props a 2 x k matrix with the following rows:

    p: the proportions of the k peaks

    s[p]: the estimated uncertainties (standard errors) of p

L the log-likelihood of the fit

legend a vector of text expressions to be used in a figure legend

References

Galbraith, R.F. and Laslett, G.M., 1993. Statistical models for mixed fission track ages. Nuclear Tracks and Radiation Measurements, 21(4), pp.459-470.

van der Touw, J., Galbraith, R., and Laslett, G. A logistic truncated normal mixture model for overdispersed binomial data. Journal of Statistical Computation and Simulation, 59(4):349-373, 1997.

See Also

[radialplot](), [central]()

Examples

```
data(examples)
peakfit(examples$FT1,k=2)

peakfit(examples$LudwigMixture,k='min')
```

---

radialplot   *Visualise heteroscedastic data on a radial plot*

Description

Implementation of a graphical device developed by Rex Galbraith to display several estimates of the same quantity that have different standard errors.

Usage

```
radialplot(x, ...)

## Default S3 method:
radialplot(x, from = NA, to = NA, t0 = NA,
  transformation = "log", sigdig = 2, show.numbers = FALSE,
  pch = 21, levels = NA, clabel = "", bg = c("yellow", "red"),
  col = "black", k = 0, markers = NULL, alpha = 0.05, units = "",
  hide = NA, omit = NA, omit.col = NA, ...)

## S3 method for class 'fissiontracks'
radialplot(x, from = NA, to = NA, t0 = NA,
  transformation = "arcsin", sigdig = 2, show.numbers = FALSE,
  pch = 21, levels = NA, clabel = "", bg = c("yellow", "red"),
  col = "black", markers = NULL, k = 0, exterr = TRUE,
  alpha = 0.05, hide = NULL, omit = NULL, omit.col = NA, ...)

## S3 method for class 'UPb'
radialplot(x, from = NA, to = NA, t0 = NA,
  transformation = "log", type = 4, cutoff.76 = 1100,
  cutoff.disc = list(-15, 5, TRUE), show.numbers = FALSE, pch = 21,
  sigdig = 2, levels = NA, clabel = "", bg = c("yellow", "red"),
  col = "black", markers = NULL, k = 0, exterr = TRUE,
  common.Pb = 0, alpha = 0.05, hide = NULL, omit = NULL,
```

```
  omit.col = NA, ...)

## S3 method for class 'PbPb'
radialplot(x, from = NA, to = NA, t0 = NA,
  sigdig = 2, transformation = "log", show.numbers = FALSE,
  pch = 21, levels = NA, clabel = "", bg = c("yellow", "red"),
  col = "black", markers = NULL, k = 0, exterr = TRUE,
  common.Pb = 1, alpha = 0.05, hide = NULL, omit = NULL,
  omit.col = NA, ...)

## S3 method for class 'ArAr'
radialplot(x, from = NA, to = NA, t0 = NA,
  sigdig = 2, transformation = "log", show.numbers = FALSE,
  pch = 21, levels = NA, clabel = "", bg = c("yellow", "red"),
  col = "black", markers = NULL, k = 0, exterr = TRUE,
  i2i = FALSE, alpha = 0.05, hide = NULL, omit = NULL,
  omit.col = NA, ...)

## S3 method for class 'KCa'
radialplot(x, from = NA, to = NA, t0 = NA,
  sigdig = 2, transformation = "log", show.numbers = FALSE,
  pch = 21, levels = NA, clabel = "", bg = c("yellow", "red"),
  col = "black", markers = NULL, k = 0, exterr = TRUE,
  i2i = FALSE, alpha = 0.05, hide = NULL, omit = NULL,
  omit.col = NA, ...)

## S3 method for class 'UThHe'
radialplot(x, from = NA, to = NA, t0 = NA,
  sigdig = 2, transformation = "log", show.numbers = FALSE,
  pch = 21, levels = NA, clabel = "", bg = c("yellow", "red"),
  col = "black", markers = NULL, k = 0, alpha = 0.05,
  hide = NULL, omit = NULL, omit.col = NA, ...)

## S3 method for class 'ReOs'
radialplot(x, from = NA, to = NA, t0 = NA,
  sigdig = 2, transformation = "log", show.numbers = FALSE,
  pch = 21, levels = NA, clabel = "", bg = c("yellow", "red"),
  col = "black", markers = NULL, k = 0, exterr = TRUE,
  i2i = TRUE, alpha = 0.05, hide = NULL, omit = NULL,
  omit.col = NA, ...)

## S3 method for class 'SmNd'
radialplot(x, from = NA, to = NA, t0 = NA,
  sigdig = 2, transformation = "log", show.numbers = FALSE,
  pch = 21, levels = NA, clabel = "", bg = c("yellow", "red"),
  col = "black", markers = NULL, k = 0, exterr = TRUE,
  i2i = TRUE, alpha = 0.05, hide = NULL, omit = NULL,
  omit.col = NA, ...)
```

```
## S3 method for class 'RbSr'
radialplot(x, from = NA, to = NA, t0 = NA,
  sigdig = 2, transformation = "log", show.numbers = FALSE,
  pch = 21, levels = NA, clabel = "", bg = c("yellow", "red"),
  col = "black", markers = NULL, k = 0, exterr = TRUE,
  i2i = TRUE, alpha = 0.05, hide = NULL, omit = NULL,
  omit.col = NA, ...)

## S3 method for class 'LuHf'
radialplot(x, from = NA, to = NA, t0 = NA,
  sigdig = 2, transformation = "log", show.numbers = FALSE,
  pch = 21, levels = NA, clabel = "", bg = c("yellow", "red"),
  col = "black", markers = NULL, k = 0, exterr = TRUE,
  i2i = TRUE, alpha = 0.05, hide = NULL, omit = NULL,
  omit.col = NA, ...)

## S3 method for class 'ThU'
radialplot(x, from = NA, to = NA, t0 = NA,
  sigdig = 2, transformation = "log", show.numbers = FALSE,
  pch = 21, levels = NA, clabel = "", bg = c("yellow", "red"),
  col = "black", markers = NULL, k = 0, i2i = TRUE, alpha = 0.05,
  detritus = 0, hide = NULL, omit = NULL, omit.col = NA, ...)
```

Arguments

x	Either an [nx2] matix of (transformed) values z and their standard errors s
	OR
	and object of class fissiontracks, UThHe, ArAr, KCa, ReOs, SmNd, RbSr, LuHf, ThU, PbPb or UPb
...	additional arguments to the generic points function
from	minimum age limit of the radial scale
to	maximum age limit of the radial scale
t0	central value
transformation	one of either log, linear, sqrt or arcsin (if x has class fissiontracks and fissiontracks$type $\neq 1$).
sigdig	the number of significant digits of the numerical values reported in the title of the graphical output.
show.numbers	boolean flag (TRUE to show grain numbers)
pch	plot character (default is a filled circle)
levels	a vector with additional values to be displayed as different background colours of the plot symbols.
clabel	label of the colour legend
bg	Fill colour for the plot symbols. This can either be a single colour or multiple colours to form a colour ramp (to be used if levels!=NA):

- a single colour: `rgb(0,1,0,0.5)`, `'#FF000080'`, `'white'`, etc.
- multiple colours: `c(rbg(1,0,0,0.5),rgb(0,1,0,0.5))`, `c('#FF000080','#00FF0080')`, `c('blue','red')`, `c('blue','yellow','red')`, etc.
- a colour palette: `rainbow(n=100)`, `topo.colors(n=100,alpha=0.5)`, etc.
- a reversed palette: `rev(topo.colors(n=100,alpha=0.5))`, etc.
- for plot symbols, set `bg=NA`

col	text colour to be used if `show.numbers=TRUE`
k	number of peaks to fit using the finite mixture models of Galbraith and Laslett (1993). Setting `k='auto'` automatically selects an optimal number of components based on the Bayes Information Criterion (BIC). Setting `k='min'` estimates the minimum value using a three parameter model consisting of a Normal distribution truncated by a discrete component.
markers	vector of ages of radial marker lines to add to the plot.
alpha	cutoff value for confidence intervals
units	measurement units to be displayed in the legend.
hide	vector with indices of aliquots that should be removed from the radial plot.
omit	vector with indices of aliquots that should be plotted but omitted from the central age calculation or mixture models.
omit.col	colour that should be used for the omitted aliquots.
exterr	include the external sources of uncertainty into the error propagation for the central age and mixture models?
type	scalar indicating whether to plot the ^{207}Pb/^{235}U age (`type=1`), the ^{206}Pb/^{238}U age (`type=2`), the ^{207}Pb/^{206}Pb age (`type=3`), the ^{207}Pb/^{206}Pb-^{206}Pb/^{238}U age (`type=4`), the concordia age (`type=5`), or the ^{208}Pb/^{232}Th age (`type=6`).
cutoff.76	the age (in Ma) below which the ^{206}Pb/^{238}U and above which the ^{207}Pb/^{206}Pb age is used. This parameter is only used if `type=4`.
cutoff.disc	discordance cutoff filter. This is a three element list.
	The first two items contain the minimum (negative) and maximum (positive) percentage discordance allowed between the ^{207}Pb/^{235}U and ^{206}Pb/^{238}U age (if ^{206}Pb/^{238}U < `cutoff.76`) or between the ^{206}Pb/^{238}U and ^{207}Pb/^{206}Pb age (if ^{206}Pb/^{238}U > `cutoff.76`).
	The third item is a boolean flag that controls whether the discordance filter should be applied before (`TRUE`) or after (`FALSE`) the common-Pb correction.
	Set `cutoff.disc=NA` to turn off this filter.
common.Pb	apply a common lead correction using one of three methods:
	1: the Stacey-Kramer two-stage model to infer the initial Pb-composition
	2: the isochron intercept as the initial Pb-composition
	3: the Pb-composition stored in
	`settings('iratio','Pb206Pb204')` (if x has class UPb and x$format<4);
	`settings('iratio','Pb206Pb204')` and `settings('iratio','Pb207Pb204')` (if x has class PbPb or x has class UPb and 3<x$format<7); or
	`settings('iratio','Pb208Pb206')` and `settings('iratio','Pb208Pb207')` (if x has class UPb and x$format=7 or 8).

i2i	'isochron to intercept': calculates the initial (aka 'inherited', 'excess', or 'common') $^{40}Ar/^{36}Ar$, $^{40}Ca/^{44}Ca$, $^{207}Pb/^{204}Pb$, $^{87}Sr/^{86}Sr$, $^{143}Nd/^{144}Nd$, $^{187}Os/^{188}Os$, $^{230}Th/^{232}Th$ or $^{176}Hf/^{177}Hf$ ratio from an isochron fit. Setting i2i to FALSE uses the default values stored in settings('iratio',...).
detritus	detrital ^{230}Th correction (only applicable when x$format == 1 or 2).
	0: no correction
	1: project the data along an isochron fit
	2: correct the data using an assumed initial $^{230}Th/^{232}Th$-ratio for the detritus.
	3: correct the data using the measured present day $^{230}Th/^{238}U$, $^{232}Th/^{238}U$ and $^{234}U/^{238}U$-ratios in the detritus.

Details

The radial plot (Galbraith, 1988, 1990) is a graphical device that was specifically designed to display heteroscedastic data, and is constructed as follows. Consider a set of dates $\{t_1, ..., t_i, ..., t_n\}$ and uncertainties $\{s[t_1], ..., s[t_i], ..., s[t_n]\}$. Define $z_i = z[t_i]$ to be a transformation of t_i (e.g., $z_i = log[t_i]$), and let $s[z_i]$ be its propagated analytical uncertainty (i.e., $s[z_i] = s[t_i]/t_i$ in the case of a logarithmic transformation). Create a scatterplot of (x_i, y_i) values, where $x_i = 1/s[z_i]$ and $y_i = (z_i - z_\circ)/s[z_i]$, where $z_\circ$ is some reference value such as the mean. The slope of a line connecting the origin of this scatterplot with any of the (x_i, y_i)s is proportional to z_i and, hence, the date t_i. These dates can be more easily visualised by drawing a radial scale at some convenient distance from the origin and annotating it with labelled ticks at the appropriate angles. While the angular position of each data point represents the date, its horizontal distance from the origin is proportional to the precision. Imprecise measurements plot on the left hand side of the radial plot, whereas precise age determinations are found further towards the right. Thus, radial plots allow the observer to assess both the magnitude and the precision of quantitative data in one glance.

References

Galbraith, R.F., 1988. Graphical display of estimates having differing standard errors. Technometrics, 30(3), pp.271-281.

Galbraith, R.F., 1990. The radial plot: graphical assessment of spread in ages. International Journal of Radiation Applications and Instrumentation. Part D. Nuclear Tracks and Radiation Measurements, 17(3), pp.207-214.

Galbraith, R.F. and Laslett, G.M., 1993. Statistical models for mixed fission track ages. Nuclear Tracks and Radiation Measurements, 21(4), pp.459-470.

See Also

peakfit, central

Examples

```
data(examples)
radialplot(examples$FT1)

dev.new()
radialplot(examples$LudwigMixture,k='min')
```

---

read.data   *Read geochronology data*

Description

Cast a `.csv` file or a matrix into one of `IsoplotR`'s data classes

Usage

```
read.data(x, ...)

## Default S3 method:
read.data(x, method = "U-Pb", format = 1, ierr = 1,
  d = diseq(), Th02 = c(0, 0), Th02U48 = c(0, 0, 1e+06, 0, 0, 0, 0,
  0, 0), ...)

## S3 method for class 'data.frame'
read.data(x, method = "U-Pb", format = 1,
  ierr = 1, d = diseq(), Th02 = c(0, 0), Th02U48 = c(0, 0, 1e+06,
  0, 0, 0, 0, 0, 0), ...)

## S3 method for class 'matrix'
read.data(x, method = "U-Pb", format = 1, ierr = 1,
  d = diseq(), Th02 = c(0, 0), Th02U48 = c(0, 0, 1e+06, 0, 0, 0, 0,
  0, 0), ...)
```

Arguments

x	either a file name (`.csv` format) OR a matrix
...	optional arguments to the `read.csv` function
method	one of `'U-Pb'`, `'Pb-Pb'`, `'Ar-Ar'`, `'detritals'`, Rb-Sr, Sm-Nd, Re-Os, Th-U, `'U-Th-He'`, `'fissiontracks'` or `'other'`
format	formatting option, depends on the value of `method`.
	if `method='U-Pb'`, then `format` is one of either:

1. 7/5, err[7/5], 6/8, err[6/8], rho
2. 8/6, err[8/6], 7/6, err[7/6] (, rho)
3. X=7/6, err[X], Y=6/8, err[Y], Z=7/6, err[Z]
   (, rho[X,Y]) (, rho[Y,Z])
4. X=7/5, err[X], Y=6/8, err[Y], Z=4/8,
   rho[X,Y], rho[X,Z], rho[Y,Z]
5. X=8/6, err[X], Y=7/6, err[Y], Z=4/6,
   rho[X,Y], rho[X,Z], rho[Y,Z]
6. 7/5, err[7/5], 6/8, err[6/8], 4/8, err[4/8],
   7/6, err[7/6], 4/7, err[4/7], 4/6, err[4/6]

where optional columns are marked in round brackets

if method='Pb-Pb', then format is one of either:

1. 6/4, err[6/4], 7/4, err[7/4], rho
2. 4/6, err[4/6], 7/6, err[7/6], rho
3. 6/4, err[6/4], 7/4, err[7/4], 7/6, err[7/6]

if method='Ar-Ar', then format is one of either:

1. 9/6, err[9/6], 0/6, err[0/6], rho (, 39)
2. 6/0, err[6/0], 9/0, err[9/0] (, rho) (, 39)
3. 9/0, err[9/0], 6/0, err[6/0], 9/6, err[9/6] (, 39)

if method='K-Ca', then format is one of either:

1. K40/Ca44, err[K40/Ca44], Ca40/Ca44, err[Ca40/Ca44], rho
2. K40/Ca40, err[K40/Ca40], Ca44/Ca40, err[Ca44/Ca40], rho
3. K40/Ca44, err[K40/Ca44], Ca40/Ca44, err[Ca40/Ca44], K40/Ca40, err[K40/Ca40]

if method='Rb-Sr', then format is one of either:

1. Rb87/Sr86, err[Rb87/Sr86], Sr87/Sr86, err[Sr87/Sr86] (, rho)
2. Rb87/Sr87, err[Rb87/Sr87], Sr86/Sr87, err[Sr86/Sr87] (, rho)
3. Rb, err[Rb], Sr, err[Sr], Sr87/Sr86, err[Sr87/Sr86]

where Rb and Sr are in ppm

if method='Sm-Nd', then format is one of either:

1. Sm147/Nd144, err[Sm147/Nd144], Nd143/Nd144, err[Nd143/Nd144] (, rho)
2. Sm147/Nd143, err[Sm147/Nd143], Nd144/Nd143, err[Nd144/Nd143] (, rho)
3. Sm, err[Sm], Nd, err[Nd], Nd143/Nd144, err[Nd143/Nd144]

where Sm and Nd are in ppm

if method='Re-Os', then format is one of either:

1. Re187/Os188, err[Re187/Os188], Os187/Os188, err[Os187/Os188] (, rho)
2. Re187/Os187, err[Re187/Os187], Os188/Os187, err[Os188/Os187] (, rho)
3. Re, err[Re], Os, err[Os], Os187/Os188, err[Os187/Os188]

where Re and Os are in ppm

if method='Lu-Hf', then format is one of either:

1. Lu176/Hf177, err[Lu176/Hf177], Hf176/Hf177, err[Hf176/Hf177] (, rho)
2. Lu176/Hf176, err[Lu176/Hf176], Hf177/Hf176, err[Hf177/Hf176] (, rho)
3. Lu, err[Lu], Hf, err[Hf], Hf176/Hf177, err[Hf176/Hf177]

where Lu and Hf are in ppm

if method='Th-U', then format is one of either:

1. X=8/2, err[X], Y=4/2, err[Y], Z=0/2, err[Z],
   rho[X,Y], rho[X,Z], rho[Y,Z]
2. X=2/8, err[X], Y=4/8, err[Y], Z=0/8, err[Z],
   rho[X,Y], rho[X,Z], rho[Y,Z]
3. X=8/2, err[X], Y=0/2, err[Y], rho[X,Y]

4. X=2/8, err[X], Y=0/8, err[Y], rho[X,Y]

where all values are activity ratios

if method='fissiontracks', then format is one of either:

1. the External Detector Method (EDM), which requires a ζ-calibration constant and its uncertainty, the induced track density in a dosimeter glass, and a table with the spontaneous and induced track densities.
2. LA-ICP-MS-based fission track data using the ζ-calibration method, which requires a 'session ζ' and its uncertainty and a table with the number of spontaneous tracks, the area over which these were counted and one or more U/Ca- or U-concentration measurements and their analytical uncertainties.
3. LA-ICP-MS-based fission track data using the 'absolute dating' method, which only requires a table with the the number of spontaneous tracks, the area over which these were counted and one or more U/Ca-ratios or U-concentration measurements (in ppm) and their analytical uncertainties.

if method='other', x is read as a table, unless format is one of either:

radial **or** average: X, err[X]

regression: X, err[X], Y, err[Y], rho
    OR X/Z, err[X/Z], Y/Z, err[Y/Z], X/Y, err[X/Y]

spectrum: f, X, err[X]

ierr	indicates whether the analytical uncertainties are reported as:

1. 1σ absolute uncertainties.
2. 2σ absolute uncertainties.
3. 1σ relative uncertainties (%).
4. 2σ relative uncertainties (%).

d	an object of class [diseq](#).
Th02	2-element vector with the assumed initial ^{230}Th/^{232}Th-ratio of the detritus and its standard error. Only used if isochron=FALSE and detritus=2
Th02U48	9-element vector with the measured composition of the detritus, containing X=0/8, sX, Y=2/8, sY, Z=4/8, sZ, rXY, rXZ, rYZ.

Details

IsoplotR provides the following example input files:

- U-Pb: UPb1.csv, UPb2.csv, UPb3.csv, UPb4.csv, UPb5.csv, UPb6.csv
- Pb-Pb: PbPb1.csv, PbPb2.csv, PbPb3.csv
- Ar-Ar: ArAr1.csv, ArAr2.csv, ArAr3.csv
- K-Ca: KCa1.csv, KCa2.csv, KCa3.csv,
- Re-Os: ReOs1.csv, ReOs2.csv, ReOs3.csv
- Sm-Nd: SmNd1.csv, SmNd2.csv, SmNd3.csv
- Rb-Sr: RbSr1.csv, RbSr2.csv, RbSr3.csv
- Lu-Hf: LuHf1.csv, LuHf2.csv, LuHf3.csv

- Th-U: `ThU1.csv, ThU2.csv, ThU3.csv, ThU4.csv`

- fissiontracks: `FT1.csv, FT2.csv, FT3.csv`

- U-Th-He: `UThHe.csv, UThSmHe.csv`

- detritals: `DZ.csv`

- other: `LudwigMixture.csv, LudwigMean.csv, LudwigKDE.csv LudwigSpectrum.csv`

The contents of these files can be viewed using the `system.file(...)` function. For example, to read the `ArAr1.csv` file:

```
fname <- system.file('ArAr1.csv',package='IsoplotR')
```

```
ArAr <- read.data(fname,method='Ar-Ar',format=1)
```

Value

an object of class `UPb, PbPb, ArAr, KCa, UThHe, ReOs, SmNd, RbSr, LuHf, detritals, fissiontracks, ThU` or `other`

See Also

[examples](#), [settings](#)

Examples

```
f1 <- system.file("UPb1.csv",package="IsoplotR")
file.show(f1) # inspect the contents of 'UPb1.csv'
d1 <- read.data(f1,method="U-Pb",format=1)
concordia(d1)

f2 <- system.file("ArAr1.csv",package="IsoplotR")
d2 <- read.data(f2,method="Ar-Ar",format=1)
agespectrum(d2)

f3 <- system.file("ReOs1.csv",package="IsoplotR")
d3 <- read.data(f3,method="Re-Os",format=1)
isochron(d2)

f4 <- system.file("FT1.csv",package="IsoplotR")
d4 <- read.data(f4,method="fissiontracks",format=1)
radialplot(d4)

f5 <- system.file("UThSmHe.csv",package="IsoplotR")
d5 <- read.data(f5,method="U-Th-He")
helioplot(d5)

f6 <- system.file("ThU2.csv",package="IsoplotR")
d6 <- read.data(f6,method="Th-U",format=2)
evolution(d6)

#  one detrital zircon U-Pb file (detritals.csv)
f7 <- system.file("DZ.csv",package="IsoplotR")
```

```
d7 <- read.data(f7,method="detritals")
kde(d7)

#  four 'other' files (LudwigMixture.csv, LudwigSpectrum.csv,
#  LudwigMean.csv, LudwigKDE.csv)
f8 <- system.file("LudwigMixture.csv",package="IsoplotR")
d8 <- read.data(f8,method="other")
radialplot(d8)
```

---

scatterplot   *Create a scatter plot with error ellipses or crosses*

Description

Takes bivariate data with (correlated) uncertainties as input and produces a scatter plot with error ellipses or crosses as output. (optionally) displays the linear fit on this diagram, and can show a third variable as a colour scale.

Usage

```
scatterplot(xy, xlim = NA, ylim = NA, alpha = 0.05,
  show.numbers = FALSE, show.ellipses = 1, levels = NA,
  clabel = "", ellipse.col = c("#00FF0080", "#FF000080"),
  fit = "none", add = FALSE, empty = FALSE, ci.col = "gray80",
  line.col = "black", lwd = 1, hide = NULL, omit = NULL,
  omit.col = NA, addcolourbar = TRUE, ...)
```

Arguments

xy	matrix with columns X, sX, Y, sY(, rXY)
xlim	(optional) two-element vector with the x-axis limits
ylim	(optional) two-element vector with the y-axis limits
alpha	the probability cutoff for the error ellipses
show.numbers	logical flag (TRUE to show grain numbers)
show.ellipses	show the data as: 1. points 2. error ellipses 3. error crosses
levels	a vector with additional values to be displayed as different background colours within the error ellipses.
clabel	label for the colour scale
ellipse.col	Fill colour for the error ellipses. This can either be a single colour or multiple colours to form a colour ramp (to be used if levels!=NA):

- a single colour: `rgb(0,1,0,0.5)`, `'#FF000080'`, `'white'`, etc.
- multiple colours: `c(rbg(1,0,0,0.5),rgb(0,1,0,0.5))`, `c('#FF000080','#00FF0080')`, `c('blue','red')`, `c('blue','yellow','red')`, etc.
- a colour palette: `rainbow(n=100)`, `topo.colors(n=100,alpha=0.5)`, etc.
- a reversed palette: `rev(topo.colors(n=100,alpha=0.5))`, etc.

fit	the output of `york()` (optional).
add	if TRUE, adds the points and lines to the existing plot.
empty	set up an empty plot with the right axis limits to fit the data
ci.col	the fill colour for the confidence interval of the intercept and slope.
line.col	colour of the regression line
lwd	line width of the regression line
hide	vector with indices of aliquots that should be removed from the plot.
omit	vector with indices of aliquots that should be plotted but omitted from the isochron age calculation.
omit.col	colour that should be used for the omitted aliquots.
addcolourbar	add a colour bar to display the colours used to `levels`
...	optional arguments to format the points and text.

Examples

```
X <- c(1.550,12.395,20.445,20.435,20.610,24.900,
        28.530,50.540,51.595,86.51,106.40,157.35)
Y <- c(.7268,.7809,.8200,.8116,.8160,.8302,
        .8642,.9534,.9617,1.105,1.230,1.440)
sX <- X*0.02
sY <- Y*0.01
dat <- cbind(X,sX,Y,sY)
scatterplot(dat,fit=york(dat),show.ellipses=2)
```

---

set.zeta	*Calculate the zeta calibration coefficient for fission track dating*

Description

Determines the zeta calibration constant of a fission track dataset (EDM or LA-ICP-MS) given its true age and analytical uncertainty.

Usage

```
set.zeta(x, tst, exterr = TRUE, update = TRUE, sigdig = 2)
```

Arguments

x	an object of class `fissiontracks`
tst	a two-element vector with the true age and its standard error
exterr	logical flag indicating whether the external uncertainties associated with the age standard or the dosimeter glass (for the EDM) should be accounted for when propagating the uncertainty of the zeta calibration constant.
update	logical flag indicating whether the function should return an updated version of the input data, or simply return a two-element vector with the calibration constant and its standard error.
sigdig	number of significant digits

Details

The fundamental fission track age is given by:

$t = \frac{1}{\lambda_{238}} \ln \left(1 + \frac{\lambda_{238}}{\lambda_f} \frac{2N_s}{[^{238}U]A_s L} \right)$ (eq.1)

where N_s is the number of spontaneous fission tracks measured over an area A_s, $[^{238}U]$ is the ^{238}U-concentration in atoms per unit volume, λ_f is the fission decay constant, L is the etchable fission track length, and the factor 2 is a geometric factor accounting for the fact that etching reveals tracks from both above and below the internal crystal surface. Two analytical approaches are used to measure $[^{238}U]$: neutron activation and LAICPMS. The first approach estimates the ^{238}U-concentration indirectly, using the induced fission of neutron-irradiated ^{235}U as a proxy for the ^{238}U. In the most common implementation of this approach, the induced fission tracks are recorded by an external detector made of mica or plastic that is attached to the polished grain surface (Fleischer and Hart, 1972; Hurford and Green, 1983). The fission track age equation then becomes:

$t = \frac{1}{\lambda_{238}} \ln \left(1 + \frac{\lambda_{238}\zeta\rho_d}{2} \frac{N_s}{N_i} \right)$ (eq.2)

where N_i is the number of induced fission tracks counted in the external detector over the same area as the spontaneous tracks, ζ is a 'zeta'-calibration factor that incorporates both the fission decay constant and the etchable fission track length, and ρ_d is the number of induced fission tracks per unit area counted in a co-irradiated glass of known U-concentration. ρ_d allows the ζ-factor to be 'recycled' between irradiations.

LAICPMS is an alternative means of determining the ^{238}U-content of fission track samples without the need for neutron irradiation. The resulting U-concentrations can be plugged directly into the fundamental age equation (eq.1). but this is limited by the accuracy of the U-concentration measurements, the fission track decay constant and the etching and counting efficiencies. Alternatively, these sources of bias may be removed by normalising to a standard of known fission track age and defining a new 'zeta' calibration constant ζ_{icp}:

$t = \frac{1}{\lambda_{238}} \ln \left(1 + \frac{\lambda_{238}\zeta_{icp}}{2} \frac{N_s}{[^{238}U]A_s} \right)$ (eq.3)

where $[^{238}U]$ may either stand for the ^{238}U-concentration (in ppm) *or* for the U/Ca (for apatite) or U/Si (for zircon) ratio measurement (Vermeesch, 2017).

Value

an object of class `fissiontracks` with an updated x$zeta value

References

Fleischer, R. and Hart, H. Fission track dating: techniques and problems. In Bishop, W., Miller, J., and Cole, S., editors, Calibration of Hominoid Evolution, pages 135-170. Scottish Academic Press Edinburgh, 1972.

Hurford, A. J. and Green, P. F. The zeta age calibration of fission-track dating. Chemical Geology, 41:285-317, 1983.

Vermeesch, P., 2017. Statistics for LA-ICP-MS based fission track dating. Chemical Geology, 456, pp.19-27.

See Also

[age](age)

Examples

```
data(examples)
print(examples$FT1$zeta)
FT <- set.zeta(examples$FT1,tst=c(250,5))
print(FT$zeta)
```

---

settings   *Load settings to and from json*

Description

Get and set preferred values for decay constants, isotopic abundances, molar masses, fission track etch efficiences, and etchable lengths, and mineral densities, either individually or via a `.json` file format.

Usage

```
settings(setting = NA, ..., fname = NA)
```

Arguments

setting   unless fname is provided, this should be one of either:

                    `'lambda'`: to get and set decay constants

                    `'iratio'`: isotopic ratios

                    `'imass'`: isotopic molar masses

                    `'mindens'`: mineral densities

                    `'etchfact'`: fission track etch efficiency factors

                    `'tracklength'`: equivalent isotropic fission track length

...   depends on the value for `setting`:

- for `'lambda'`: the isotope of interest (one of either ″fission″, ″U238″, ″U235″, ″U234″, ″Th232″, ″Th230″, ″Re187″, ″Sm147″, ″Rb87″, ″Lu176″, or ″K40″) PLUS (optionally) the decay constant value and its analytical error. Omitting the latter two numbers simply returns the existing values.
- for `'iratio'`: the isotopic ratio of interest (one of either ″Ar40Ar36″, ″Ar38Ar36″,″Ca40Ca44″, ″Rb85Rb87″, ″Sr88Sr86″, ″Sr87Sr86″, ″Sr84Sr86″, ″Re185Re187″, ″Os184Os192″ ″Os186Os192″, ″Os187Os192″, ″Os188Os192″, ″Os189Os192″, ″Os190Os192″, ″U238U235″, ″Sm144Sm152″, ″Sm147Sm152″, ″Sm148Sm152″, ″Sm149Sm152″, ″Sm150Sm152″, ″Sm154Sm152″, ″Nd142Nd144″, ″Nd143Nd144″, ″Nd145Nd144″, ″Nd146Nd144″, ″Nd148Nd144″, ″Nd150Nd144″, ″Lu176Lu175″, ″Hf174Hf177″, ″Hf176Hf177″, ″Hf178Hf177″, ″Hf179Hf177″, ″Hf180Hf177″) PLUS (optionally) the isotopic ratio and its analytical error. Omitting the latter two numbers simply returns the existing values.
- for `'imass'`: the (isotopic) molar mass of interest (one of either ″U″, ″Rb″, ″Rb85″, ″Rb87″, ″Sr84″, ″Sr86″, ″Sr87″, ″Sr88″, ″Re″, ″Re185″, ″Re187″, ″Os″, ″Os184″, ″Os186″, ″Os187″, ″Os188″, ″Os189″, ″Os190″, ″Os192″, ″Sm″, ″Nd″, ″Lu″, ″Hf″) PLUS (optionally) the molar mass and its analytical error. Omitting the latter two numbers simply returns the existing values.
- for `'mindens'`: the mineral of interest (one of either ″apatite″ or ″zircon″) PLUS the mineral density. Omitting the latter number simply returns the existing value.
- `'etchfact'`: the mineral of interest (one of either ″apatite″ or ″zircon″) PLUS the etch efficiency factor. Omitting this number simply returns the existing value.
- `'tracklength'`: the mineral of interest (one of either ″apatite″ or ″zircon″) PLUS the equivalent isotropic fission track length. Omitting this number simply returns the existing value.

fname   the path of a `.json` file

Value

if `setting=NA` and `fname=NA`, returns a `.json` string

if `...` contains only the name of an isotope, isotopic ratio, element, or mineral and no new value, then `settings` returns either a scalar with the existing value, or a two-element vector with the value and its uncertainty.

References

1. Decay constants:

- ^{238}U, ^{235}U: Jaffey, A. H., et al. "Precision measurement of half-lives and specific activities of U^{235} and U^{238}." Physical Review C 4.5 (1971): 1889.
- ^{232}Th: Le Roux, L. J., and L. E. Glendenin. "Half-life of ^{232}Th.", Proceedings of the National Meeting on Nuclear Energy, Pretoria, South Africa. 1963.
- ^{234}U, ^{230}Th: Cheng, H., Edwards, R.L., Shen, C.C., Polyak, V.J., Asmerom, Y., Woodhead, J., Hellstrom, J., Wang, Y., Kong, X., Spotl, C. and Wang, X., 2013. Improvements in ^{230}Th dating, ^{230}Th and ^{234}U half-life values, and U-Th isotopic measurements by

multi-collector inductively coupled plasma mass spectrometry. Earth and Planetary Science Letters, 371, pp.82-91.

- [231]Pa: Audi, G., Bersillon, O., Blachot, J. and Wapstra, A.H., 2003. The NUBASE evaluation of nuclear and decay properties. Nuclear Physics A, 729(1), pp.3-128.
- [226]Ra: Audi, G., Bersillon, O., Blachot, J. and Wapstra, A.H., 2003. The NUBASE evaluation of nuclear and decay properties. Nuclear Physics A, 729(1), pp.3-128.
- Sm: Lugmair, G. W., and K. Marti. "Lunar initial [143]Nd/[144]Nd: differential evolution of the lunar crust and mantle." Earth and Planetary Science Letters 39.3 (1978): 349-357.
- Nd: Zhao, Motian, et al. "Absolute measurements of neodymium isotopic abundances and atomic weight by MC-ICPMS." International Journal of Mass Spectrometry 245.1 (2005): 36-40.
- Re: Selby, D., Creaser, R.A., Stein, H.J., Markey, R.J. and Hannah, J.L., 2007. Assessment of the 187Re decay constant by cross calibration of Re-Os molybdenite and U-Pb zircon chronometers in magmatic ore systems. Geochimica et Cosmochimica Acta, 71(8), pp.1999-2013.
- Ar: Renne, Paul R., et al. "Response to the comment by WH Schwarz et al. on "Joint determination of ^{40}K decay constants and ^{40}Ar*/^{40}K for the Fish Canyon sanidine standard, and improved accuracy for ^{40}Ar/^{39}Ar geochronology" by PR Renne et al.(2010)." Geochimica et Cosmochimica Acta 75.17 (2011): 5097-5100.
- Rb: Villa, I.M., De Bievre, P., Holden, N.E. and Renne, P.R., 2015. "IUPAC-IUGS recommendation on the half life of ^{87}Rb". Geochimica et Cosmochimica Acta, 164, pp.382-385.
- Lu: Soederlund, Ulf, et al. "The ^{176}Lu decay constant determined by Lu-Hf and U-Pb isotope systematics of Precambrian mafic intrusions." Earth and Planetary Science Letters 219.3 (2004): 311-324.

2. Isotopic ratios:

- Ar: Lee, Jee-Yon, et al. "A redetermination of the isotopic abundances of atmospheric Ar." Geochimica et Cosmochimica Acta 70.17 (2006): 4507-4512.
- Ca: Moore, L.J. and Machlan, L.A., 1972. High-accuracy determination of calcium in blood serum by isotope dilution mass spectrometry. Analytical chemistry, 44(14), pp.2291-2296.
- Rb: Catanzaro, E. J., et al. "Absolute isotopic abundance ratio and atomic weight of terrestrial rubidium." J. Res. Natl. Bur. Stand. A 73 (1969): 511-516.
- Sr: Moore, L. J., et al. "Absolute isotopic abundance ratios and atomic weight of a reference sample of strontium." J. Res. Natl. Bur. Stand. 87.1 (1982): 1-8.
  and (for ^{87}Sr/^{86}Sr):
  Compston, W., Berry, H., Vernon, M.J., Chappell, B.W. and Kaye, M.J., 1971. Rubidium-strontium chronology and chemistry of lunar material from the Ocean of Storms. In Lunar and Planetary Science Conference Proceedings (Vol. 2, p. 1471).
- Sm: Chang, Tsing-Lien, et al. "Absolute isotopic composition and atomic weight of samarium." International Journal of Mass Spectrometry 218.2 (2002): 167-172.
- Re: Gramlich, John W., et al. "Absolute isotopic abundance ratio and atomic weight of a reference sample of rhenium." J. Res. Natl. Bur. Stand. A 77 (1973): 691-698.
- Os: Voelkening, Joachim, Thomas Walczyk, and Klaus G. Heumann. "Osmium isotope ratio determinations by negative thermal ionization mass spectrometry." Int. J. Mass Spect. Ion Proc. 105.2 (1991): 147-159.

- Lu: De Laeter, J. R., and N. Bukilic. "Solar abundance of [176]Lu and s-process nucleosynthesis." Physical Review C 73.4 (2006): 045806.
- Hf: Patchett, P. Jonathan. "Importance of the Lu-Hf isotopic system in studies of planetary chronology and chemical evolution." Geochimica et Cosmochimica Acta 47.1 (1983): 81-91.
- Pb: Stacey, J.T. and Kramers, J. "Approximation of terrestrial lead isotope evolution by a two-stage model." Earth and Planetary Science Letters, 26(2) (1975): 207-221.
- U: Hiess, Joe, et al. "[238]U/[235]U systematics in terrestrial uranium-bearing minerals." Science 335.6076 (2012): 1610-1614.

See Also

read.data

Examples

```
# load and show the default constants that come with IsoplotR
json <- system.file("constants.json",package="IsoplotR")
settings(fname=json)
print(settings())

# use the decay constant of Kovarik and Adams (1932)
settings('lambda','U238',0.0001537,0.0000068)
print(settings('lambda','U238'))

# returns the 238U/235U ratio of Hiess et al. (2012):
print(settings('iratio','U238U235'))
# use the 238U/235U ratio of Steiger and Jaeger (1977):
settings('iratio','U238U235',138.88,0)
print(settings('iratio','U238U235'))
```

---

titterington	*Linear regression of X,Y,Z-variables with correlated errors*

Description

Implements the maximum likelihood algorithm of Ludwig and Titterington (1994) for linear regression of three dimensional data with correlated uncertainties.

Usage

```
titterington(x, alpha = 0.05)
```

Arguments

x	an [nx9] matrix with the following columns: X, sX, Y, sY, Z, sZ, rhoXY, rhoXZ, rhoYZ.
alpha	cutoff value for confidence intervals

Details

Ludwig and Titterington (1994)'s 3-dimensional linear regression algorithm for data with correlated uncertainties is an extension of the 2-dimensional algorithm by Titterington and Halliday (1979), which itself is equivalent to the algorithm of York et al. (2004). Given n triplets of (approximately) collinear measurements X_i, Y_i and Z_i (for $1 \leq i \leq n$), their uncertainties $s[X_i]$, $s[Y_i]$ and $s[Z_i]$, and their covariances $\mathrm{cov}[X_i, Y_i]$, $\mathrm{cov}[X_i, Z_i]$ and $\mathrm{cov}[Y_i, Z_i]$, the `titterington` function fits two slopes and intercepts with their uncertainties. It computes the MSWD as a measure of under/overdispersion. Overdispersed datasets (MSWD>1) can be dealt with in the same three ways that are described in the documentation of the `isochron` function.

Value

A four-element list of vectors containing:

par 4-element vector `c(a,b,A,B)` where `a` is the intercept of the `X-Y` regression, `b` is the slope of the `X-Y` regression, `A` is the intercept of the `X-Z` regression, and `B` is the slope of the `X-Z` regression.

cov `[4x4]`-element covariance matrix of `par`

mswd the mean square of the residuals (a.k.a 'reduced Chi-square') statistic

p.value p-value of a Chi-square test for linearity

df the number of degrees of freedom for the Chi-square test ($3n$-3)

tfact the $100(1 - \alpha/2)\%$ percentile of the t-distribution with $(n - 2k + 1)$ degrees of freedom

References

Ludwig, K.R. and Titterington, D.M., 1994. Calculation of ^{230}Th/U isochrons, ages, and errors. Geochimica et Cosmochimica Acta, 58(22), pp.5031-5042.

Titterington, D.M. and Halliday, A.N., 1979. On the fitting of parallel isochrons and the method of maximum likelihood. Chemical Geology, 26(3), pp.183-195.

York, D., Evensen, N.M., Martinez, M.L. and De Basebe Delgado, J., 2004. Unified equations for the slope, intercept, and standard errors of the best straight line. American Journal of Physics, 72(3), pp.367-375.

See Also

`york`, `isochron`, `ludwig`

Examples

```
d <- matrix(c(0.1677,0.0047,1.105,0.014,0.782,0.015,0.24,0.51,0.33,
              0.2820,0.0064,1.081,0.013,0.798,0.015,0.26,0.63,0.32,
              0.3699,0.0076,1.038,0.011,0.819,0.015,0.27,0.69,0.30,
              0.4473,0.0087,1.051,0.011,0.812,0.015,0.27,0.73,0.30,
              0.5065,0.0095,1.049,0.010,0.842,0.015,0.27,0.76,0.29,
              0.5520,0.0100,1.039,0.010,0.862,0.015,0.27,0.78,0.28),
            nrow=6,ncol=9)
colnames(d) <- c('X','sX','Y','sY','Z','sZ','rXY','rXZ','rYZ')
titterington(d)
```

weightedmean   *Calculate the weighted mean age*

Description

Models the data as a Normal distribution with two sources of variance. Estimates the mean and 'overdispersion' using the method of Maximum Likelihood. Computes the MSWD of a Normal fit without overdispersion. Implements a modified Chauvenet Criterion to detect and reject outliers. Only propagates the analytical uncertainty associated with decay constants and ζ and J-factors after computing the weighted mean isotopic composition.

Usage

```
weightedmean(x, ...)

## Default S3 method:
weightedmean(x, from = NA, to = NA,
  random.effects = TRUE, detect.outliers = TRUE, plot = TRUE,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  ranked = FALSE, hide = NULL, omit = NULL, omit.col = NA, ...)

## S3 method for class 'UPb'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, type = 4,
  cutoff.76 = 1100, alpha = 0.05, cutoff.disc = list(-15, 5, TRUE),
  exterr = TRUE, ranked = FALSE, common.Pb = 0, hide = NULL,
  omit = NULL, omit.col = NA, ...)

## S3 method for class 'PbPb'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  exterr = TRUE, common.Pb = 1, ranked = FALSE, hide = NULL,
  omit = NULL, omit.col = NA, ...)

## S3 method for class 'ThU'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  ranked = FALSE, i2i = TRUE, detritus = 0, hide = NULL,
  omit = NULL, omit.col = NA, ...)
```

```
## S3 method for class 'ArAr'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  exterr = TRUE, ranked = FALSE, i2i = FALSE, hide = NULL,
  omit = NULL, omit.col = NA, ...)

## S3 method for class 'KCa'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  exterr = TRUE, ranked = FALSE, i2i = FALSE, hide = NULL,
  omit = NULL, omit.col = NA, ...)

## S3 method for class 'ReOs'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  exterr = TRUE, ranked = FALSE, i2i = TRUE, hide = NULL,
  omit = NULL, omit.col = NA, ...)

## S3 method for class 'SmNd'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  exterr = TRUE, ranked = FALSE, i2i = TRUE, hide = NULL,
  omit = NULL, omit.col = NA, ...)

## S3 method for class 'RbSr'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  exterr = TRUE, i2i = TRUE, ranked = FALSE, hide = NULL,
  omit = NULL, omit.col = NA, ...)

## S3 method for class 'LuHf'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  exterr = TRUE, i2i = TRUE, ranked = FALSE, hide = NULL,
  omit = NULL, omit.col = NA, ...)
```

```
## S3 method for class 'UThHe'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  ranked = FALSE, hide = NULL, omit = NULL, omit.col = NA, ...)

## S3 method for class 'fissiontracks'
weightedmean(x, random.effects = TRUE,
  detect.outliers = TRUE, plot = TRUE, from = NA, to = NA,
  levels = NA, clabel = "", rect.col = c("#00FF0080", "#FF000080"),
  outlier.col = "#00FFFF80", sigdig = 2, alpha = 0.05,
  exterr = TRUE, ranked = FALSE, hide = NULL, omit = NULL,
  omit.col = NA, ...)
```

Arguments

x	a two column matrix of values (first column) and their standard errors (second column) OR an object of class UPb, PbPb, ArAr, KCa, ReOs, SmNd, RbSr, LuHf, ThU, fissiontracks or UThHe
...	optional arguments
from	minimum y-axis limit. Setting from=NA scales the plot automatically.
to	maximum y-axis limit. Setting to=NA scales the plot automatically.
random.effects	if TRUE, computes the weighted mean using a random effects model with two parameters: the mean and the dispersion. This is akin to a 'model-3' isochron regression. if FALSE, attributes any excess dispersion to an underestimation of the analytical uncertainties. This akin to a 'model-1' isochron regression.
detect.outliers	logical flag indicating whether outliers should be detected and rejected using Chauvenet's Criterion.
plot	logical flag indicating whether the function should produce graphical output or return numerical values to the user.
levels	a vector with additional values to be displayed as different background colours of the plot symbols.
clabel	label of the colour legend
rect.col	Fill colour for the measurements or age estimates. This can either be a single colour or multiple colours to form a colour ramp (to be used if levels!=NA): • a single colour: rgb(0,1,0,0.5), '#FF000080', 'white', etc. • multiple colours: c(rbg(1,0,0,0.5), rgb(0,1,0,0.5)), c('#FF000080','#00FF0080'), c('blue','red'), c('blue','yellow','red'), etc. • a colour palette: rainbow(n=100), topo.colors(n=100,alpha=0.5), etc. • a reversed palette: rev(topo.colors(n=100,alpha=0.5)), etc. • for plot symbols, set rect.col=NA
outlier.col	if detect.outliers=TRUE, the outliers are given a different colour.

sigdig	the number of significant digits of the numerical values reported in the title of the graphical output.
alpha	the confidence limits of the error bars/rectangles.
ranked	plot the aliquots in order of increasing age?
hide	vector with indices of aliquots that should be removed from the weighted mean plot.
omit	vector with indices of aliquots that should be plotted but omitted from the weighted mean calculation.
omit.col	colour that should be used for the omitted aliquots.
type	scalar indicating whether to plot the ^{207}Pb/^{235}U age (type=1), the ^{206}Pb/^{238}U age (type=2), the ^{207}Pb/^{206}Pb age (type=3), the ^{207}Pb/^{206}Pb-^{206}Pb/^{238}U age (type=4), the concordia age (type=5), or the ^{208}Pb/^{232}Th age (type=6).
cutoff.76	the age (in Ma) below which the ^{206}Pb/^{238}U age and above which the ^{207}Pb/^{206}Pb age is used. This parameter is only used if type=4.
cutoff.disc	discordance cutoff filter. This is a three element list.
	The first two items contain the minimum (negative) and maximum (positive) percentage discordance allowed between the ^{207}Pb/^{235}U and ^{206}Pb/^{238}U age (if ^{206}Pb/^{238}U < cutoff.76) or between the ^{206}Pb/^{238}U and ^{207}Pb/^{206}Pb age (if ^{206}Pb/^{238}U > cutoff.76).
	The third item is a boolean flag that controls whether the discordance filter should be applied before (TRUE) or after (FALSE) the common-Pb correction.
	Set cutoff.disc=NA to turn off this filter.
exterr	propagate decay constant uncertainties?
common.Pb	apply a common lead correction using one of three methods:
	1: the Stacey-Kramer two-stage model to infer the initial Pb-composition
	2: the isochron intercept as the initial Pb-composition
	3: the Pb-composition stored in
	settings('iratio','Pb206Pb204') (if x has class UPb and x$format<4);
	settings('iratio','Pb206Pb204') and settings('iratio','Pb207Pb204') (if x has class PbPb or x has class UPb and 3<x$format<7); or
	settings('iratio','Pb208Pb206') and settings('iratio','Pb208Pb207') (if x has class UPb and x$format=7 or 8).
i2i	'isochron to intercept': calculates the initial (aka 'inherited', 'excess', or 'common') ^{40}Ar/^{36}Ar, ^{40}Ca/^{44}Ca, ^{207}Pb/^{204}Pb, ^{87}Sr/^{86}Sr, ^{143}Nd/^{144}Nd, ^{187}Os/^{188}Os, ^{230}Th/^{232}Th or ^{176}Hf/^{177}Hf ratio from an isochron fit. Setting i2i to FALSE uses the default values stored in settings('iratio',...).
detritus	detrital ^{230}Th correction (only applicable when x$format=1 or 2).
	0: no correction
	1: project the data along an isochron fit
	2: correct the data using an assumed initial ^{230}Th/^{232}Th-ratio for the detritus.
	3: correct the data using the measured present day ^{230}Th/^{238}U, ^{232}Th/^{238}U and ^{234}U/^{238}U-ratios in the detritus.

Details

Let $\{t_1, ..., t_n\}$ be a set of n age estimates determined on different aliquots of the same sample, and let $\{s[t_1], ..., s[t_n]\}$ be their analytical uncertainties. IsoplotR then calculates the weighted mean of these data assuming a Normal distribution with two sources of variance:

$$t_i \sim N(\mu, \sigma^2 = s[t_i]^2 + \omega^2)$$

where μ is the mean, σ^2 is the total variance and ω is the 'overdispersion'. This equation can be solved for μ and ω by the method of maximum likelihood. IsoplotR uses a modified version of Chauvenet's criterion for outlier detection:

1. Compute the error-weighted mean (μ) of the n age determinations t_i using their analytical uncertainties $s[t_i]$

2. For each t_i, compute the probability p_i that that $|t - \mu| > |t_i - \mu|$ for $t \sim N(0, \sqrt{s[t_i]^2 + \omega^2})$

3. Let $p_j \equiv \min(p_1, ..., p_n)$. If $p_j < 0.05/n$, then reject the jth date, reduce n by one (i.e., $n \to n-1$) and repeat steps 1 through 3 until the surviving dates pass the third step.

If the analtyical uncertainties are small compared to the scatter between the dates (i.e. if $\omega \gg s[t]$ for all i), then this generalised algorithm reduces to the conventional Chauvenet criterion. If the analytical uncertainties are large and the data do not exhibit any overdispersion, then the heuristic outlier detection method is equivalent to Ludwig (2003)'s '2-sigma' method.

Value

Returns a list with the following items:

mean a three element vector with:

x: the weighted mean

s[x]: the standard error of the weighted mean

ci[x]: the $100(1 - \alpha)\%$ confidence interval for x

disp a three-element vector with the (over)dispersion and the lower and upper half-widths of its $100(1 - \alpha)\%$ confidence interval.

mswd the Mean Square of the Weighted Deviates (a.k.a. 'reduced Chi-square' statistic)

df the number of degrees of freedom of the Chi-square test for homogeneity ($df = n - 1$, where n is the number of samples).

p.value the p-value of a Chi-square test with df degrees of freedom, testing the null hypothesis that the underlying population is not overdispersed.

valid vector of logical flags indicating which steps are included into the weighted mean calculation

plotpar list of plot parameters for the weighted mean diagram, including mean (the mean value), ci (a grey rectangle with the 100[1-α]% confidence interval ignoring systematic errors), ci.exterr (a grey rectangle with the 100[1-α]% confidence interval including systematic errors), dash1 and dash2 (lines marking the overdispersion if random.effects=TRUE).

See Also

central

Examples

```
ages <- c(251.9,251.59,251.47,251.35,251.1,251.04,250.79,250.73,251.22,228.43)
errs <- c(0.28,0.28,0.63,0.34,0.28,0.63,0.28,0.4,0.28,0.33)
weightedmean(cbind(ages,errs))

data(examples)
weightedmean(examples$LudwigMean)
```

---

york   *Linear regression of X,Y-variables with correlated errors*

Description

Implements the unified regression algorithm of York et al. (2004) which, although based on least squares, yields results that are consistent with maximum likelihood estimates of Titterington and Halliday (1979)

Usage

```
york(x, alpha = 0.05)
```

Arguments

x   a 4 or 5-column matrix with the X-values, the analytical uncertainties of the
            X-values, the Y-values, the analytical uncertainties of the Y-values, and (option-
            ally) the correlation coefficients of the X- and Y-values.

alpha   cutoff value for confidence intervals

Details

Given n pairs of (approximately) collinear measurements X_i and Y_i (for $1 \leq i \leq n$), their uncertainties $s[X_i]$ and $s[Y_i]$, and their covariances $\mathrm{cov}[X_i, Y_i]$, the york function finds the best fitting straight line using the least-squares algorithm of York et al. (2004). This algorithm is modified from an earlier method developed by York (1968) to be consistent with the maximum likelihood approach of Titterington and Halliday (1979). It computes the MSWD as a measure of under/overdispersion. Overdispersed datasets (MSWD>1) can be dealt with in the same three ways that are described in the documentation of the `isochron` function.

Value

A four-element list of vectors containing:

a the intercept of the straight line fit and its standard error

b the slope of the fit and its standard error

cov.ab the covariance of the slope and intercept

mswd the mean square of the residuals (a.k.a 'reduced Chi-square') statistic

df degrees of freedom of the linear fit $(2n - 2)$

p.value p-value of a Chi-square value with df degrees of freedom

References

Titterington, D.M. and Halliday, A.N., 1979. On the fitting of parallel isochrons and the method of maximum likelihood. Chemical Geology, 26(3), pp.183-195.

York, Derek, et al., 2004. Unified equations for the slope, intercept, and standard errors of the best straight line. American Journal of Physics 72.3, pp.367-375.

See Also

data2york, titterington, isochron, ludwig

Examples

```
X <- c(1.550,12.395,20.445,20.435,20.610,24.900,
        28.530,50.540,51.595,86.51,106.40,157.35)
Y <- c(.7268,.7849,.8200,.8156,.8160,.8322,
        .8642,.9584,.9617,1.135,1.230,1.490)
n <- length(X)
sX <- X*0.01
sY <- Y*0.005
rXY <- rep(0.8,n)
dat <- cbind(X,sX,Y,sY,rXY)
fit <- york(dat)
scatterplot(dat,fit=fit)
```

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