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Deliverable / Lab 3 - ENG PHYS 2E04

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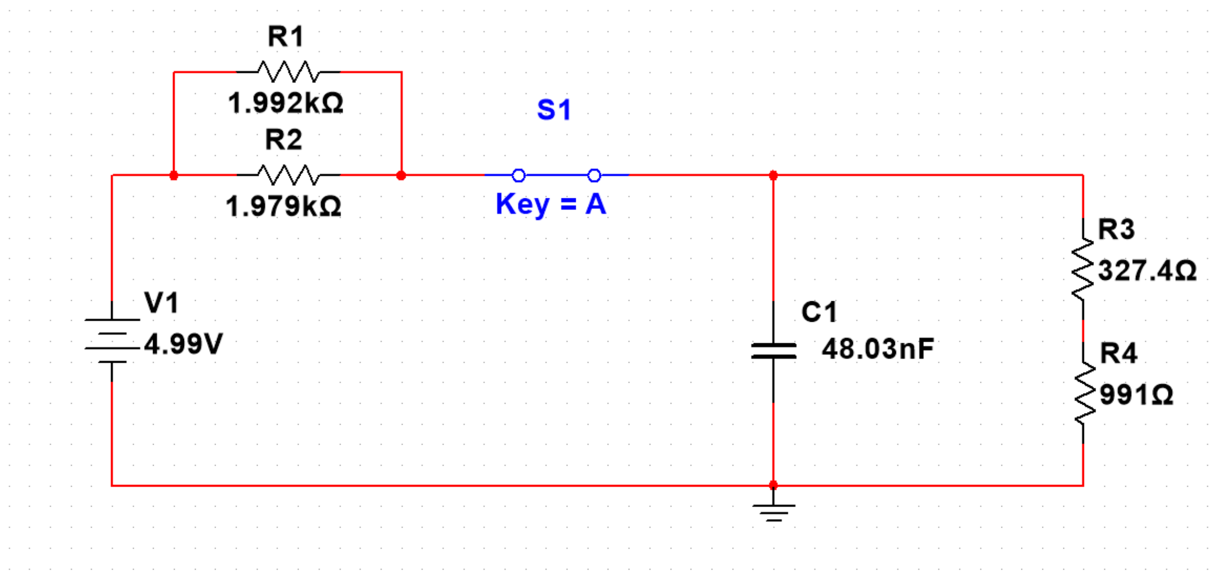

Figure 3.1

The above circuit is modified to simply calculate the ***voltage*** across the ***capacitor*** during two time intervals. First one is calculating the ***voltage*** across ***C1*** at ***t = 0***, and the second one is calculating the ***voltage*** across ***C1*** at ***t =*** ∞ (steady state). Finally, we can deduce the ***time constant*** for the RC circuit.

Purpose:

Determine a function of time (or the values needed to develop a function) to represent the voltage difference across C1 using:

a) Mathematical analysis

b) A circuit analysis program (Multisim)

c) An actual circuit and an oscilloscope

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The values that are going to be used for this lab for components like, DC power source, resistors and capacitors are measured using the Hantek 2D42.

Component	Actual Value	Measured Value
DC Power source	5V	4.990V
R1	2kΩ	1.992kΩ
R2	2kΩ	1.979kΩ
R3	330Ω	327.400Ω
R4	1kΩ	0.991kΩ
C1	10^4 pF	48.03nF

Table 1: Measurement of components

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Analytical Solution

Charging as a function of time

Maple code:

$restart:$

$R1 := 1992; R2 := 1979; R3 := 327.4; R4 := 991; Vsource := 4.99; C1 := 48.03e-9;$

$$R1 := 1992$$
$$R2 := 1979$$
$$R3 := 327.4$$
$$R4 := 991$$
$$Vsource := 4.99$$
$$C1 := 4.803 \times 10^{-8} \tag{1}$$

$R12 := \left(\frac{1}{R1} + \frac{1}{R2}\right)^{-1};$

$$R12 := \frac{3942168}{3971} \tag{2}$$

$R34 := R3 + R4;$

$$R34 := 1318.4 \tag{3}$$

$Rtotal := R12 + R34;$

$$Rtotal := 2311.139360 \tag{4}$$

$Vcapacitor := \frac{R34}{Rtotal} \cdot Vsource;$

$$Vcapacitor := 2.846568283 \tag{5}$$

$ThevR := \left(\frac{1}{R12} + \frac{1}{R34}\right)^{-1};$

$$ThevR := 566.3127005 \tag{6}$$

$t := ThevR \cdot C1;$

$$t := 0.00002719999901 \tag{7}$$

Figure 3.2: Maple Code for Charging as a function of time

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The resistors ***R1 and R2*** are combined using parallel resistors formula and a value of ***992.7Ω*** is obtained. The resistors ***R3 and R4*** are combined using the series resistor formula and a value of ***1318.4Ω*** is obtained as seen in the maple code. Then we find the sum of both ***R12*** and ***R34*** to find ***Rtotal*** which results in a value of ***2311.14Ω.***

- ***Voltage across C1 at t = 0***

When the switch is open, there is no current flowing through the capacitor C1. Therefore, the voltage at time t = 0 is ***0V.***

$$\boldsymbol{V(0)_{Charging} = 0\ V}$$

- ***Voltage across C1 at t = ∞***

When the switch is closed and time has passed, we can assume that capacitor is in a steady state and that its fully charged. Therefore, it would act as an open circuit. That being the case, the voltage across the capacitor can be calculated using the following formulas as shown in the maple code above:

$$V(\infty)_{Charging} = \frac{R_{34} * V_{source}}{R_{Total}}$$

Equation 1: V across C1

According to ***Figure 3.2,*** an approximated value of ***2.847V*** is obtained when using ***Equation 1***.

- ***The time constant, τ***

The time constant $\boldsymbol{\tau}$**,** is calculated is firstly calculated using the ***Thevenin resistance formula*** seen by the ***C1*** which is given by:

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$$TheveR = \left(\frac{1}{R_{12}} + \frac{1}{R_{34}}\right)^{-1}$$

Equation 2: Thevenin resistance seen by C1 when charging

$\boldsymbol{\tau}$ is then calculated using this formula:

$$\tau = ThevR * \ C_1$$

Equation 3: Time constant

When using ***Equation 3*** to find the time constant, $\boldsymbol{\tau}$, an approximated value of ***27.2 μs*** is obtained.

- Therefore, we can obtain a Time-dependent function for the voltage across C1, V(t):

we know:

$$V(t) = V(0) + (V(\infty) - V(0)) \cdot (1 - e^{-\frac{t}{\tau}})$$

$$= 2.847(1 - e^{-36764.706t})$$

Therefore,

$$\boldsymbol{V(t) = \ 2.847(1 - e^{-36764.706t})}$$

Equation 4: Time-dependent function for the voltage across C1

The results obtained make sense, since the expectation for the voltage difference across a capacitor is to increase to its maximum as it charges.

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Discharging as a function of time

Maple Code:

$restart:$

$R1 := 1992; R2 := 1979; R3 := 327.4; R4 := 991; Vsource := 4.99; C1 := 48.03e-9;$

$$R1 := 1992$$
$$R2 := 1979$$
$$R3 := 327.4$$
$$R4 := 991$$
$$Vsource := 4.99$$
$$C1 := 4.803 \times 10^{-8} \quad (1)$$

$R12 := \left(\frac{1}{R1} + \frac{1}{R2}\right)^{-1};$

$$R12 := \frac{3942168}{3971} \quad (2)$$

$R34 := R3 + R4;$

$$R34 := 1318.4 \quad (3)$$

$Rtotal := R12 + R34;$

$$Rtotal := 2311.139360 \quad (4)$$

$Vcapacitor := \frac{R34}{Rtotal} \cdot Vsource;$

$$Vcapacitor := 2.846568283 \quad (5)$$

$ThevR := R34;$

$$ThevR := 1318.4 \quad (6)$$

$t := ThevR \cdot C1;$

$$t := 0.000063322752 \quad (7)$$

Figure 3.3: Maple Code for discharging as a function of time

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When the ***C1*** is discharging, the switch is open. Therefore, the resistors that the capacitor detects are ***R3 and R4***, which are in series. Therefore, the ***Thevenin resistance*** would only be the sum of R3 and R4.

- ***Voltage across C1 at t = 0***

The initial voltage from when the capacitor is discharging, just after the switch is opened, must be the same as the final voltage after the capacitor has been charged, since at t = 0, the capacitor is in a steady state (fully charged).

$$V(\infty)_{Charging} = V(0)_{discharging}$$

We know:

$$V(\infty)_{Charging} = \frac{R_{34} * V_{source}}{R_{Total}}$$

$$\rightarrow V(0)_{discharging} = \frac{R_{34} * V_{source}}{R_{Total}}$$

Therefore,

$$\boldsymbol{V(0)_{discharging} = 2.847\ V}$$

- ***Voltage across C1 at t = ∞***

The voltage across the capacitor will be the voltage after the capacitor has completely discharged, which is ***0V***.

Therefore,

$$\boldsymbol{V(\infty)_{discharging} = 0\ V}$$

- ***The time constant, τ***

The time constant $\boldsymbol{\tau}$**,** is calculated the same way as the charging method, however what differs in this case is the ***Thevenin resistance*** seen by the ***C1*** which is given by:

$$ThevR = R_{34}$$

Equation 5: Thevenin resistance seen by C1 during discharging

$\boldsymbol{\tau}$ is then calculated using this formula:

$$\tau = ThevR * \ C_1$$

Equation 5: Time constant

When using ***Equation 3*** to find the time constant, $\boldsymbol{\tau}$, an approximated value of ***63.3 µs*** is obtained.

This results in a time constant approximately twice as big of what it was while charging.

$$\tau_{charging} \approx 30\ \mu s$$
$$\tau_{discharging} \approx 60\ \mu s$$

- Therefore, we can obtain a Time-dependent function for the voltage across C1, V(t):

we know:

$$V(t) = V(0) + (V(\infty) - V(0)) \cdot (1 - e^{-\frac{t}{\tau}})$$

$$= 2.847 * e^{-15792.11t}$$

Therefore,

$$\boldsymbol{V(t) = \ 2.847 * e^{-15792.11t}}$$

Equation 6: Time-dependent function for the voltage across C1

The results obtained make sense, since the expectation for the voltage difference across a capacitor is to decrease to its minimum as it discharges.

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Analysis:

- Charging

Parameter	*Value*
V(0)	0 V
V(∞)	2.847 V
Time constant, τ	27.2 µs
V(t)	$\boldsymbol{V(t) = 2.847(1 - e^{-36764.706t})}$

Table 2: Analysis when capacitor charges

- Discharging

Parameter	*Value*
V(0)	2.847 V
V(∞)	0 V
Time constant, τ	63.3 µs
V(t)	$\boldsymbol{V(t) = 2.847 * e^{-15792.11t}}$

Table 2: Analysis when capacitor discharges

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Multisim Solution

Charging

- For this section, we plugged a Tektronix oscilloscope to the modified circuit that was shown in figure 3.1.

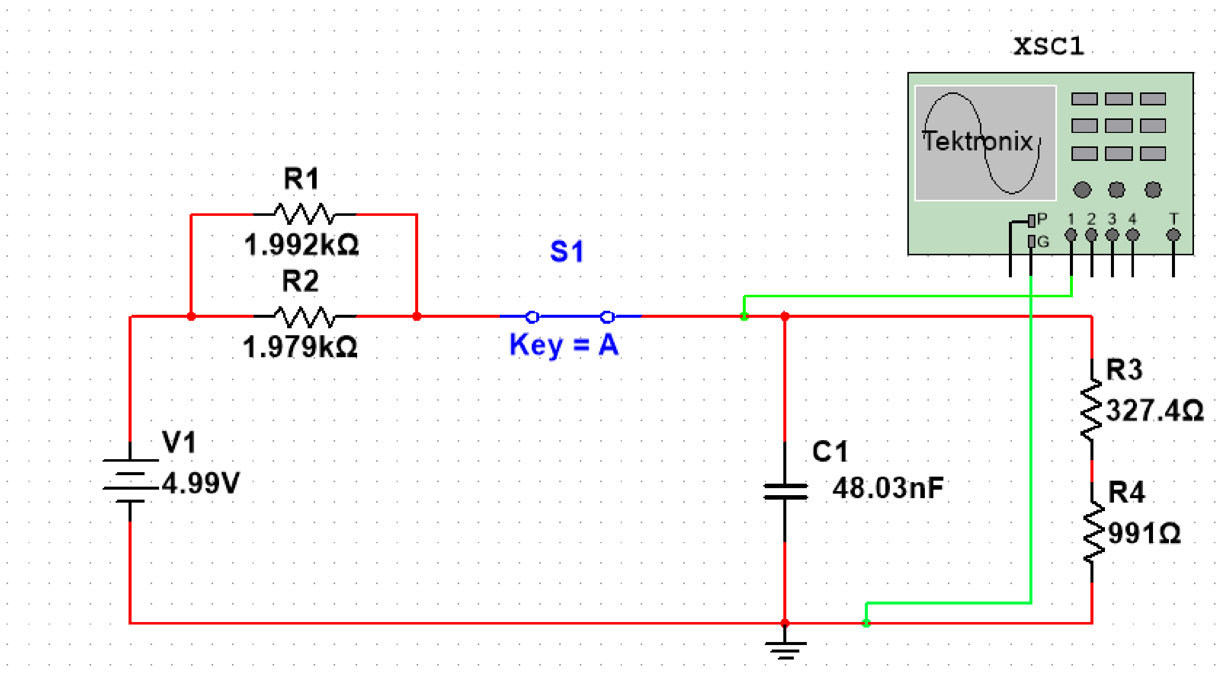

Figure 3.3: Oscilloscope attached to circuit

- Voltage after 1 time constant is found using the following formula:

$$V(\tau) = 2.847 - 2.847e^{-1}$$
$$V(\tau) = 1.7996\ V$$

Therefore, after the passing of 1 time constant, a corresponding voltage value of ***1.7996 V*** should be read on the oscilloscope.

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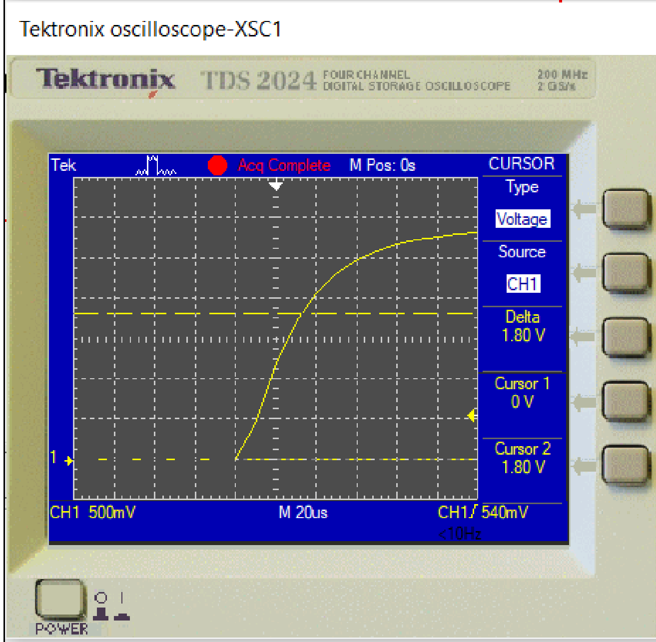
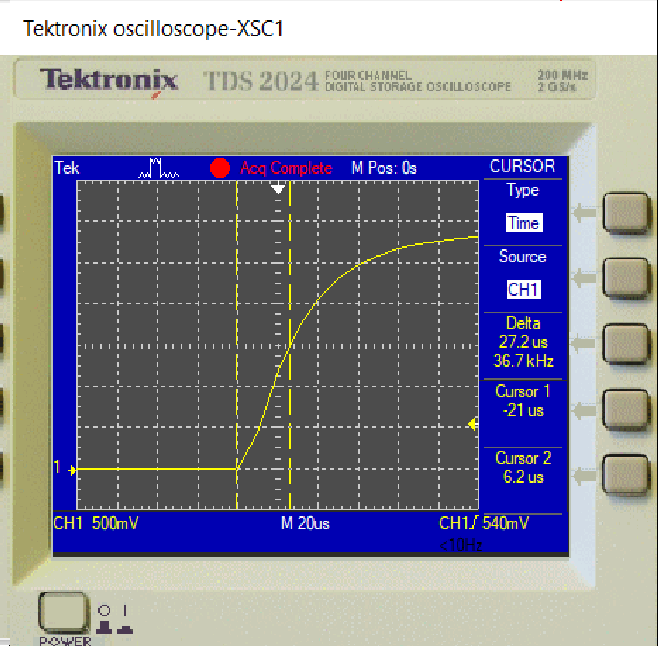
Voltage Cursor measurment	Time Constant cursor measurement




The value of τ was obtained by checking the time from at the start of the rise till about 63% of the curve which is approximately ***1.80 V***, and a value of ***27.2 µs*** was obtained.

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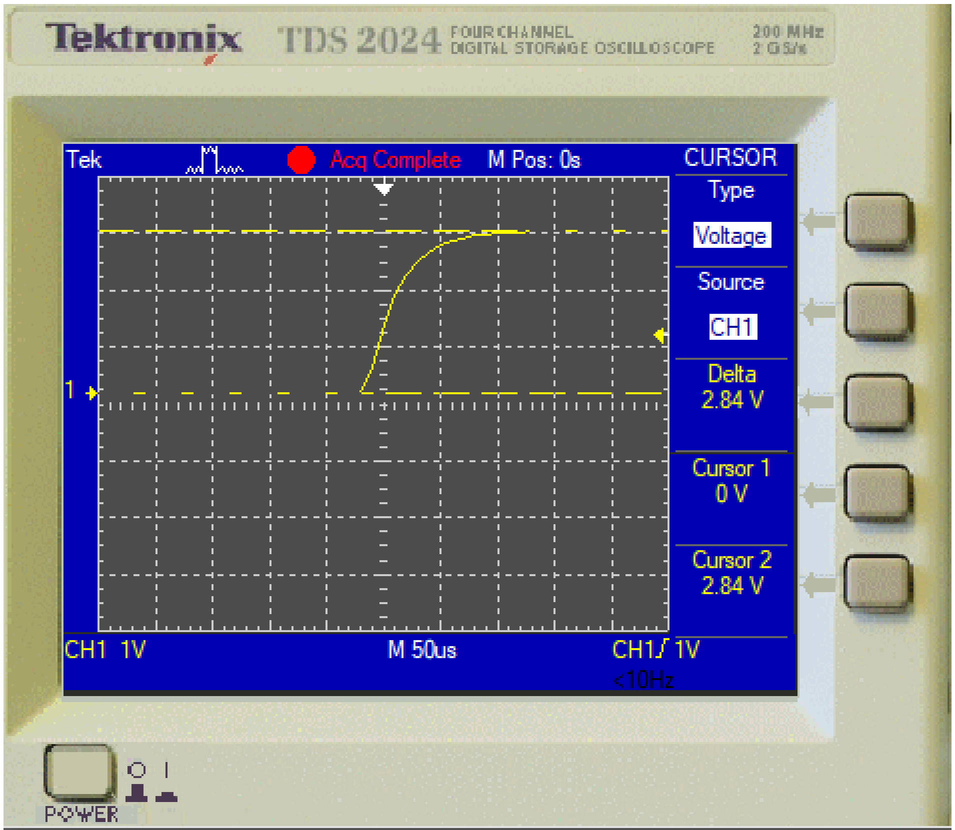
- $V_{C1}\ (0)\ and\ V_{C1}\ (\infty)$ (used for comparison reasons)

Tektronix oscilloscope-XSC1



The results between Analytical Method and the Multisim method of the charging capacitor as a function of time were compared in the table below:

Value	Analytical	Multisim
$V_{C1}\ (0)$	0 V	0V
$V_{C1}\ (\infty)$	2.847V	2.840V
τ	27.2 μs	27.2 μs

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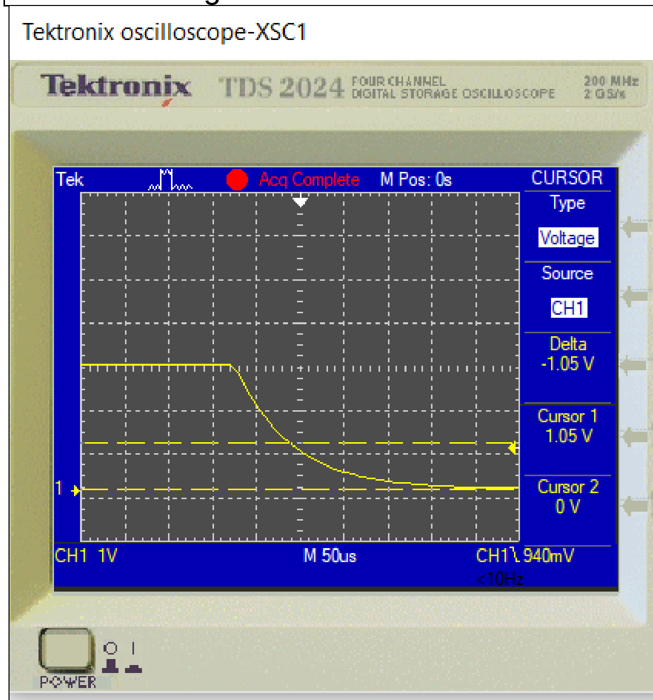
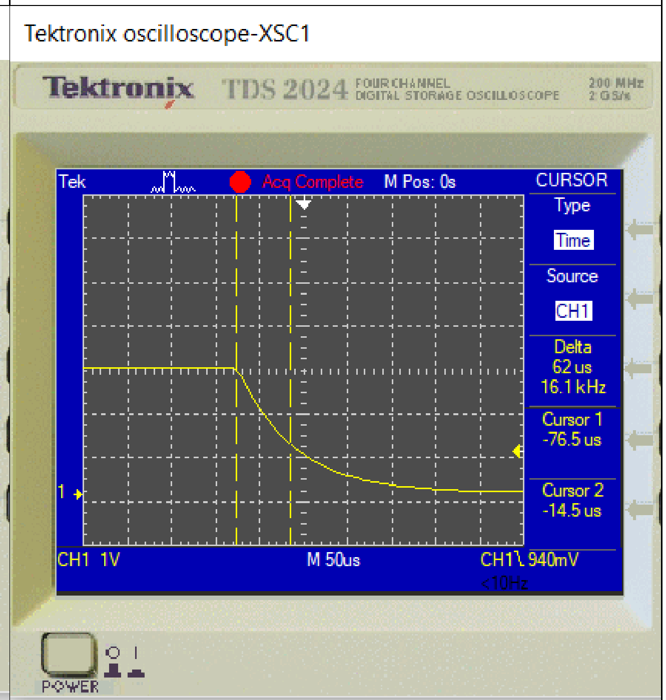
Discharging

- The same exact steps from the charging method is going to be used to solve for the time constant during discharging.

- Voltage after 1 time constant is found using the following formula:

$$V(\tau) = 2.847e^{-1}$$
$$V(\tau) = 1.047\ V$$

Voltage Cursor measurment	DeltaT cursor measurement
	

The value of τ was obtained by checking the time from at the start of the fall till about 63% of the curve which is approximately ***1.05 V***, and a value of ***62 μs*** was obtained.

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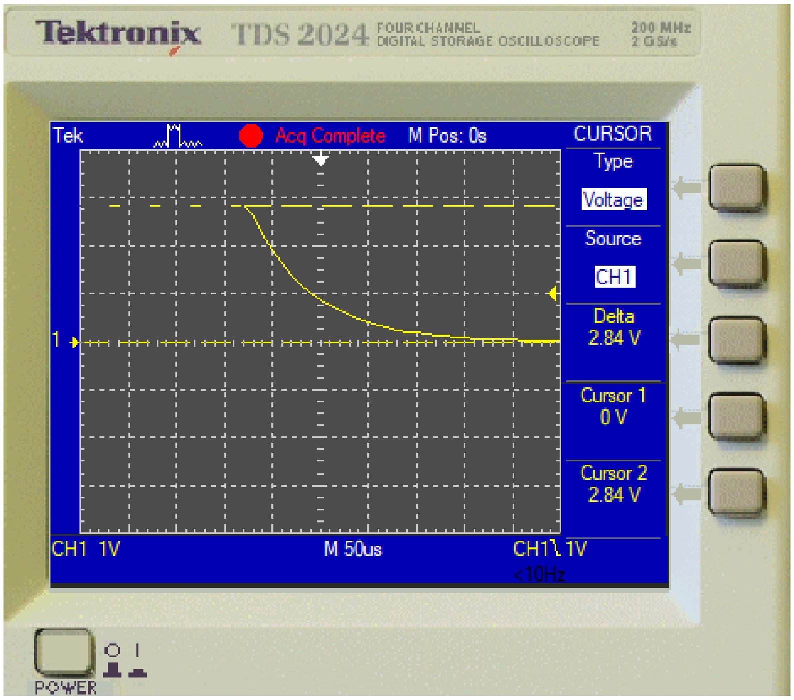
- $V_{C1}\ (0)\ and\ V_{C1}\ (\infty)$ (used for comparison reasons)

Tektronix oscilloscope-XSC1



The results between Analytical Method and the Multisim method of the charging capacitor as a function of time were compared in the table below:

Value	Analytical	Multisim
$V_{C1}\ (0)$	2.847V	2.84V
$V_{C1}\ (\infty)$	0V	0V
τ	63.3 μs	62.0 μs

- Uncertainty

The smallest division for one cursor measurement is 1 μs therefore the error of both cursors is 2 μs

Therefore, results between the analytical and Multism methods are within the uncertainty value (± 2 μs)

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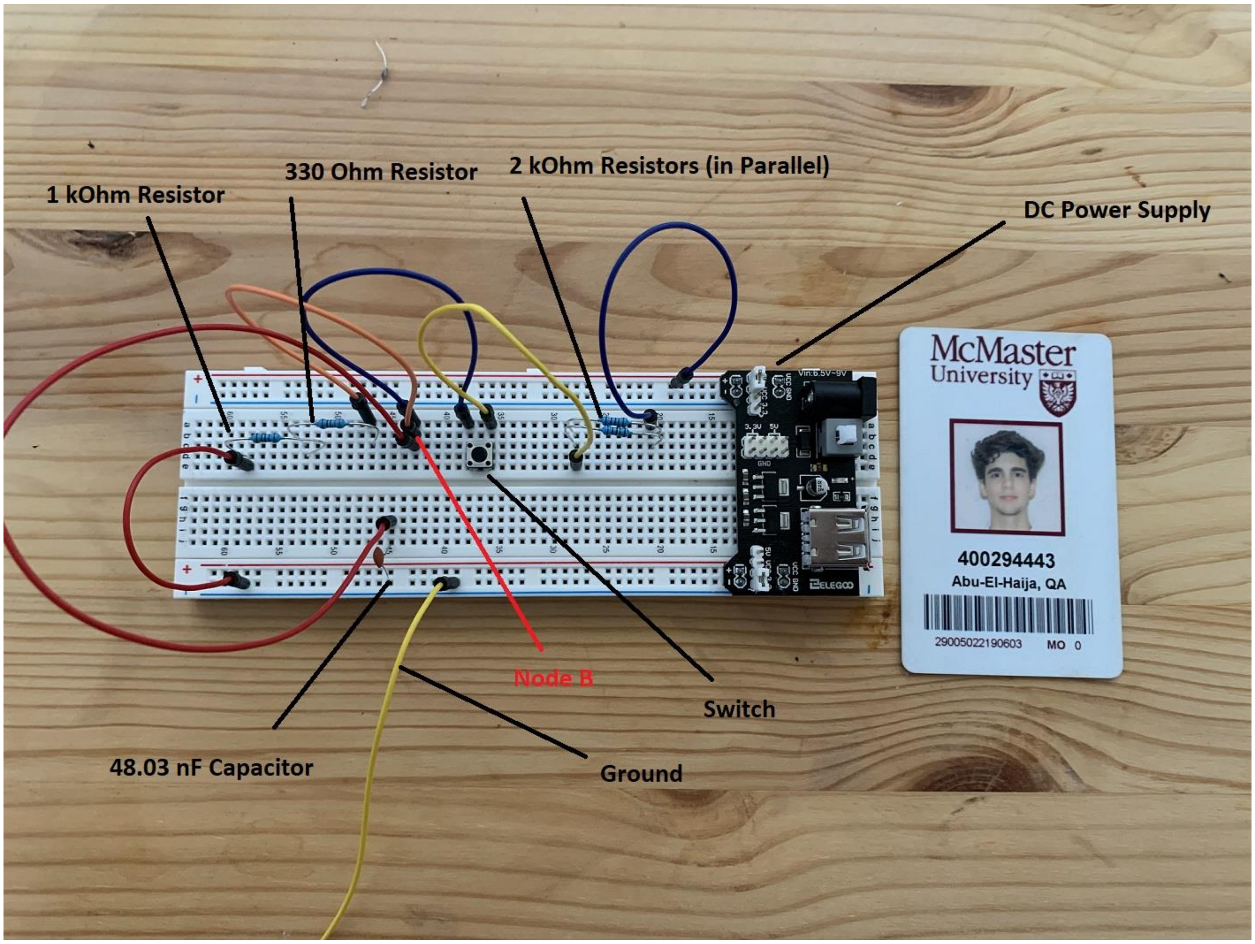
Experimental Solution

The circuit was set up experimentally using the at Home-Kit, the breadboard, and jumper wires while using the SoulBay Power Supply as the main source of the voltage as shown below:



Procedure

To find the time constant for the charging and discharging of the capacitor, I'll follow the same strategy as what I did in Multisim. I'll find the expected voltage after one time constant, (63% of the curve), then locate that on my Hantek and finally measure the time elapsed. The time taken to get to the 63% should be close to my time constant within the range of error.

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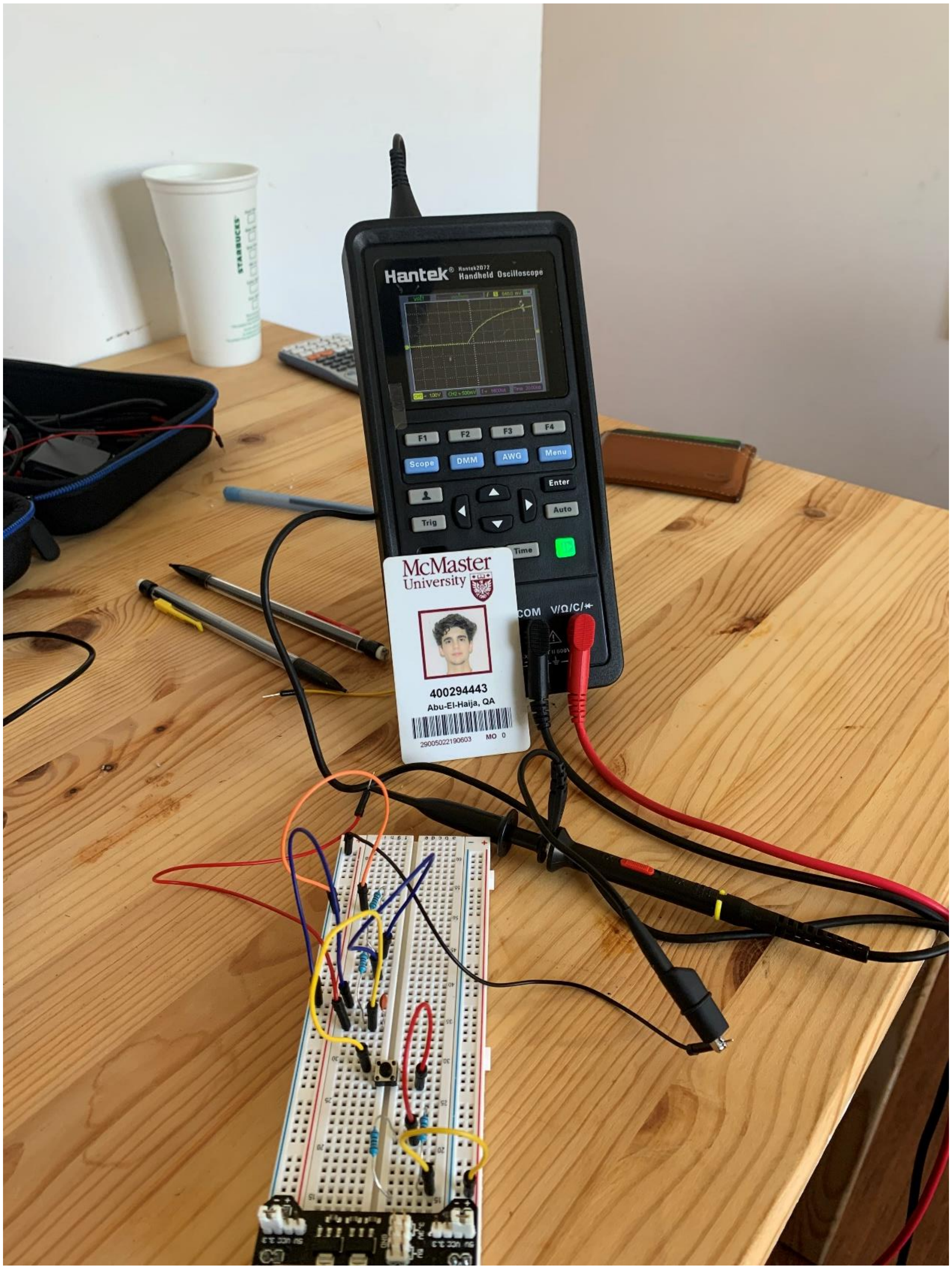

The above figure shows how I connected the probes to the breadboard. This same setup is used to find the charging and discharging time constants.

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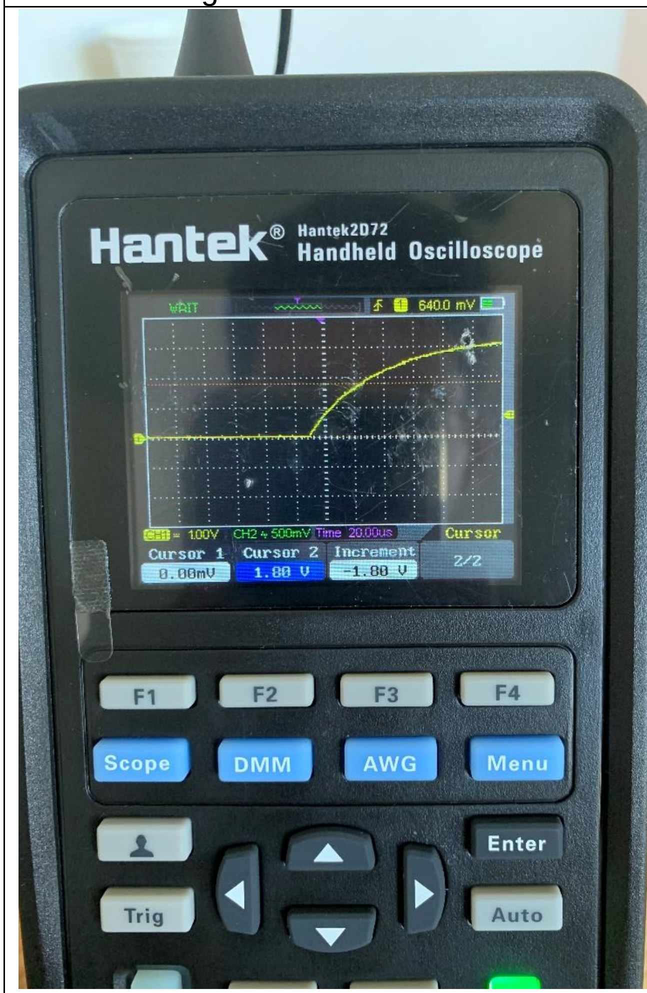
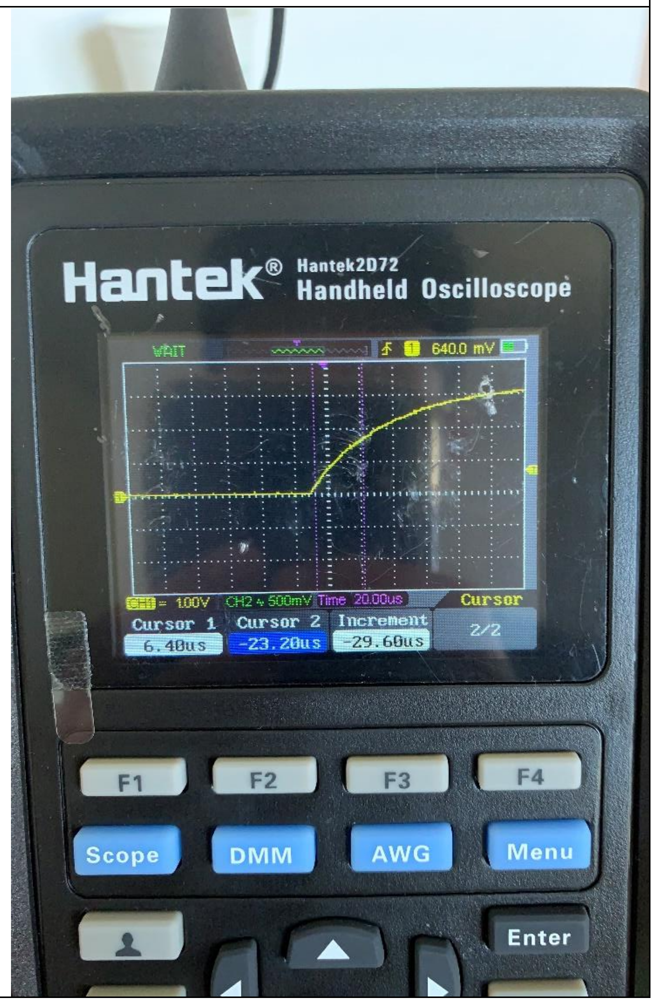
Charging

Recap:

- Voltage after 1 time constant is found using the following formula:

$$V(\tau) = 2.847 - 2.847e^{-1}$$
$$V(\tau) = 1.7996\ V$$

Voltage Cursor measurment	Time constant cursor measurement
	

- Uncertainty

The smallest division for one cursor measurement is 1 μs therefore the error of both cursors is 2 μs

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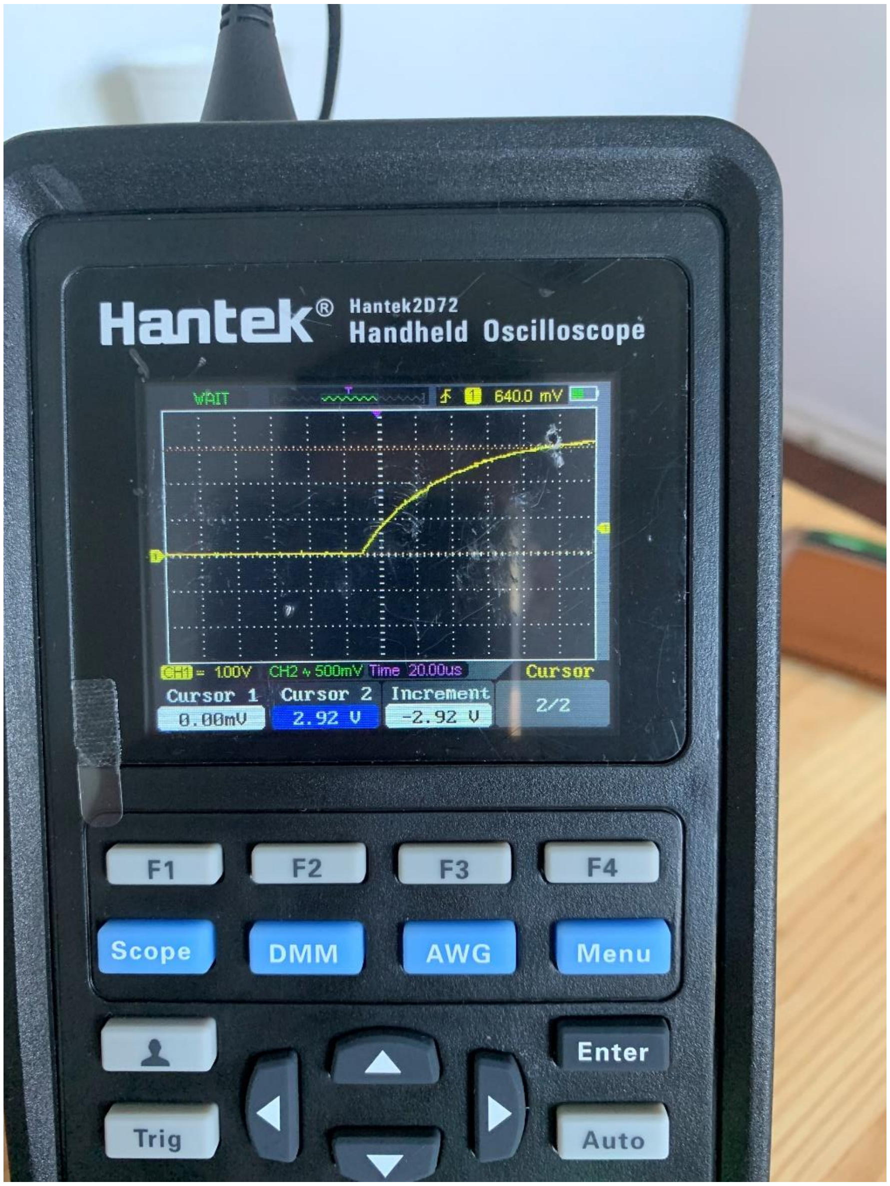
According to the Analytical and Multism method, the expected time constant was ***27.2 μs*** and the time constant in the physical build was ***29.6 ± 2 μs***. According to the range of error values, the time constant measured in the physical circuit is within range of the time constant determined in the analytical approach.

- $V_{C1}\ (0)\ and\ V_{C1}\ (\infty)$ (used for comparison reasons)



The above figure shows the jump in the voltage across the capacitor when the switched is closed. The change is measured using the cursors as shown above with one of the cursors being at ***0.00V*** indicating that the initial voltage across the capacitor is 0 and increases to a value of ***<u>2.92V</u>*** when the switch is closed.

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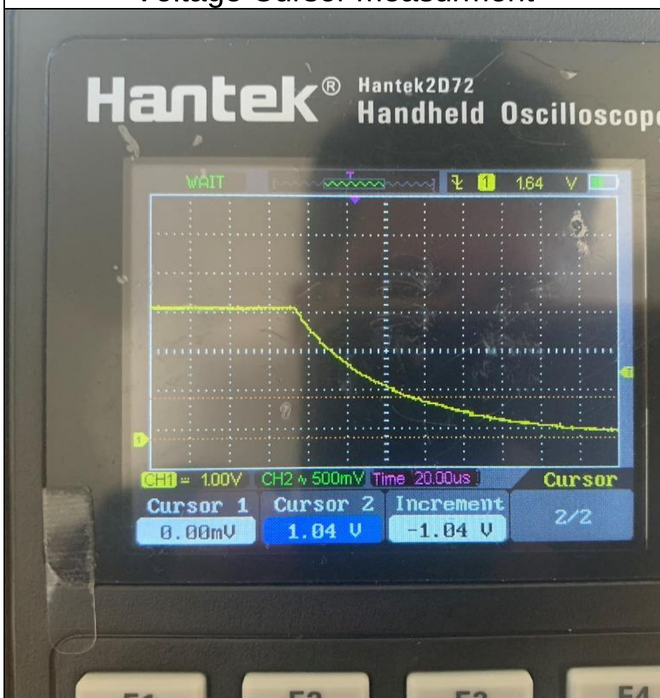
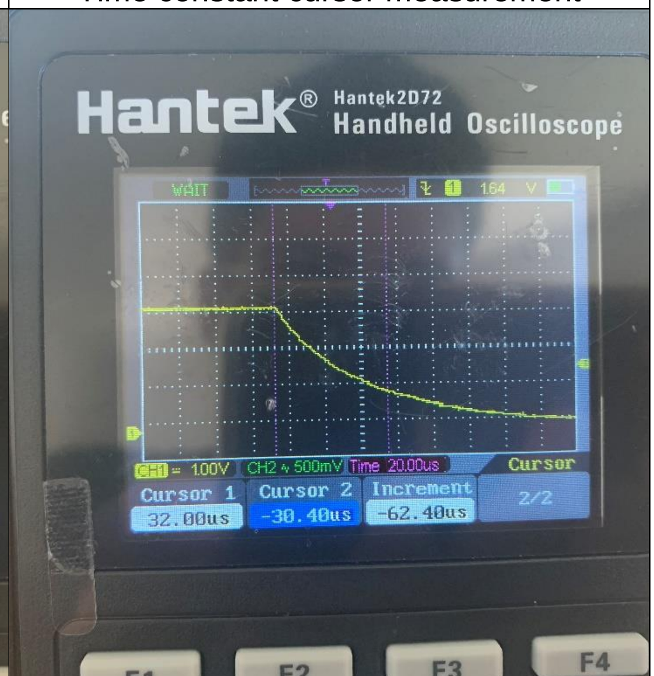
Discharging

Recap:

- Voltage after 1 time constant is found using the following formula:

$$V(\tau) = 2.847e^{-1}$$
$$V(\tau) = 1.047\ V$$

Voltage Cursor measurment	Time constant cursor measurement
	

- Uncertainty

The smallest division for one cursor measurement is 1 µs therefore the error of both cursors is 2 µs

According to the Analytical and Multism method, the expected time constant was ***63.3 µs*** and the time constant in the physical build was ***62.4 ± 2 µs***. According to the range of error values, the time constant measured in the physical circuit is within range of the time constant determined in the analytical approach.

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- $V_{C1}\,(0)\ and\ V_{C1}\,(\infty)$ (used for comparison reasons)

The above figure shows the fall in the voltage across the capacitor when the switched is opened. The change is measured using the cursors as shown above with an increment of ***2.84V*** when the switch is opened.

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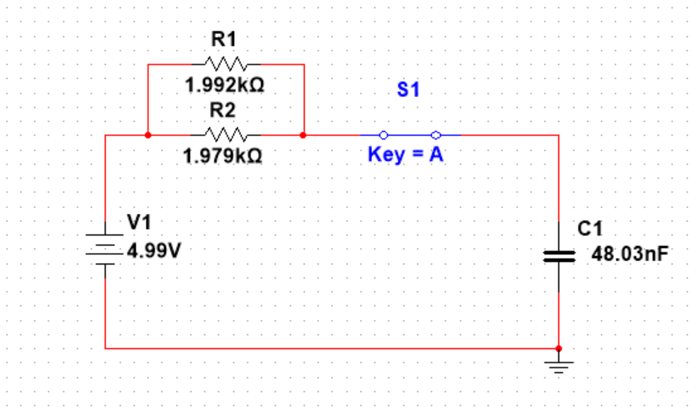
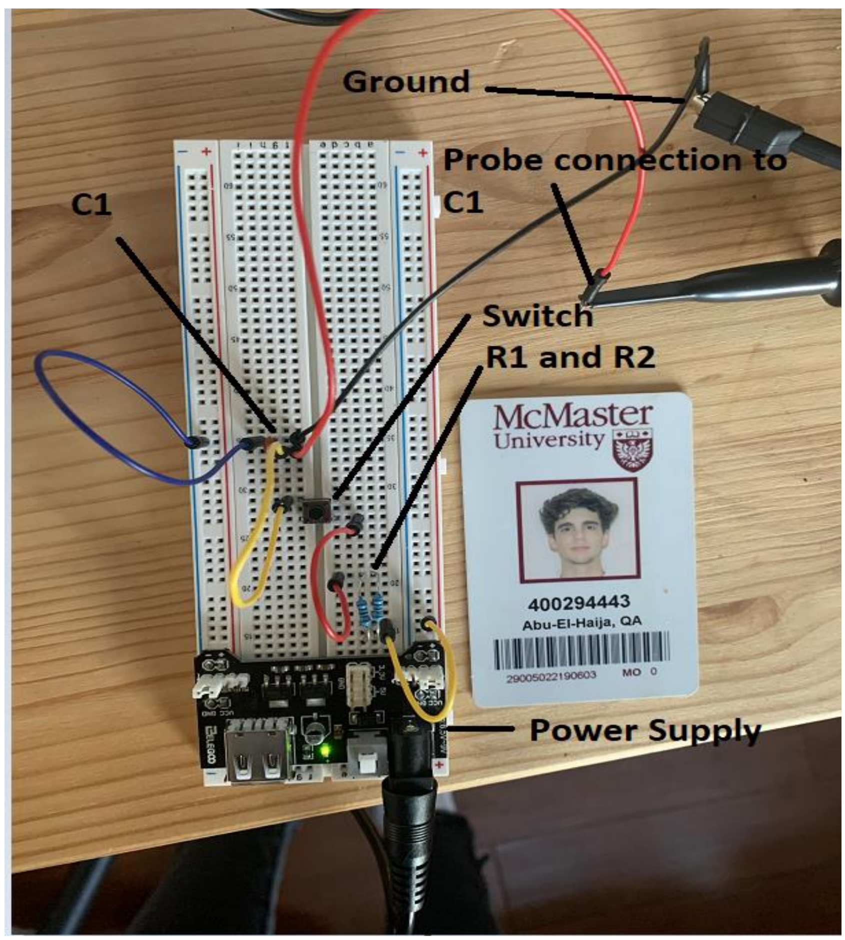
Analyzing Probe Resistance

Modified Circuit:



Physical build of modified circuit:



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Resistance of Scope Probe

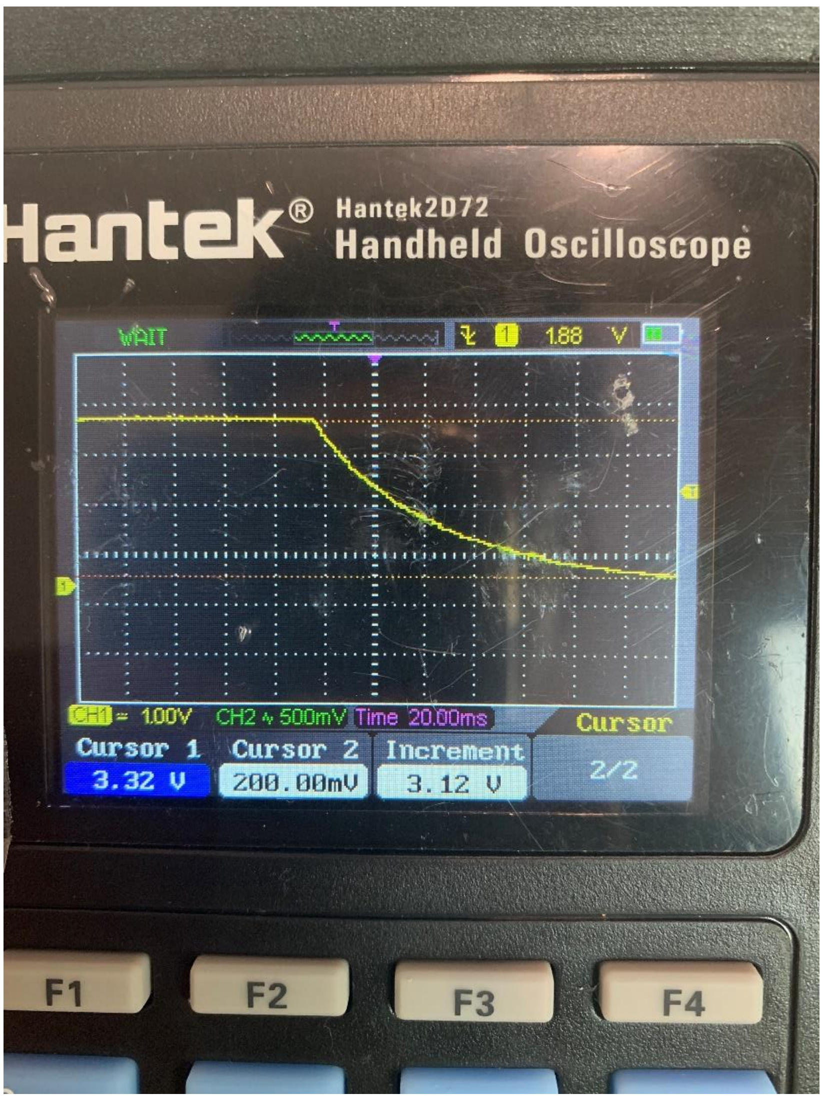
Procedure:

To measure the resistance of the scope probe, we find the τ. During a discharge, a capacitor will discharge 63% of its initial voltage in one time constant. If it takes 63% away from the initial voltage, find where that occurs on the Hantek and then find the time elapsed, I will have found the value for τ. Then,

$$R_{Seen\ By\ C1} = \frac{\tau}{C1}$$

Equation 6: resistance of Scope Probe

- ***At V(0):***



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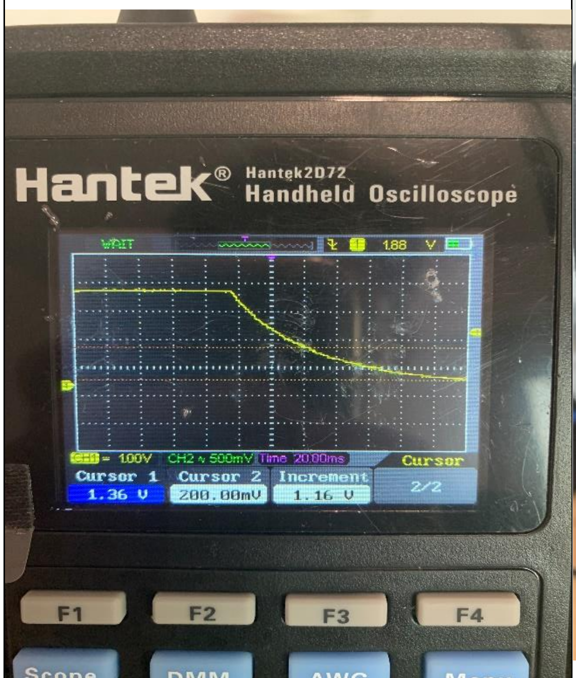
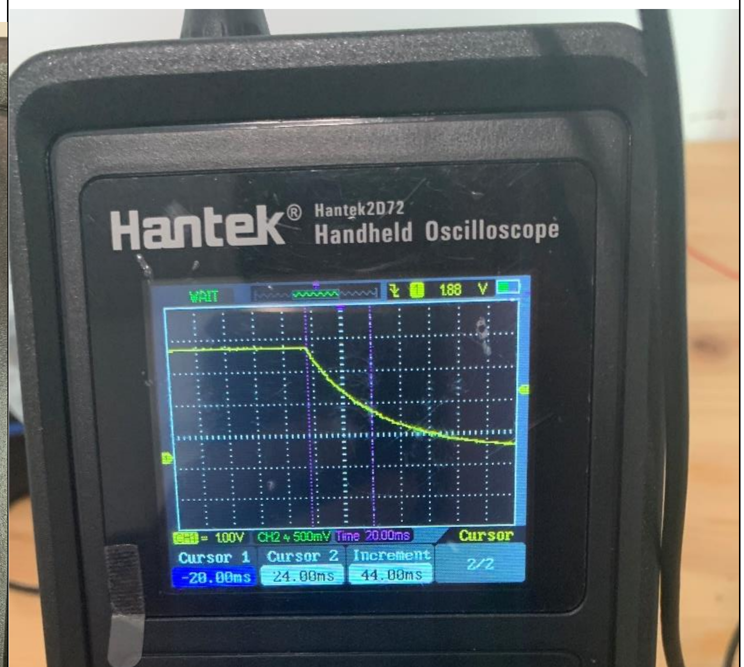
Therefore,

$$V(0) = 3.12\ V$$

That means,

$$\boldsymbol{V(\tau) = 3.12 * e^{-1} = 1.1478\ V}$$

<table>
<tr><th>Voltage Cursor measurment</th><th>Time constant cursor measurement</th></tr>
<tr><td>
</td><td>
</td></tr>
</table>

- Uncertainty

The smallest division for one cursor measurement is 2 ms therefore the error of both cursors is 4 ms

After moving the cursor to 63% of the curve, we deduced that a time constant of ***44ms ± 4 ms*** was obtained.

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Since:

$$R_{Seen\ By\ C1} = \frac{\tau}{C1} = R_{Scope\ Probe}$$
$$= \frac{44 * 10^{-3}}{48.03 * 10^{-9}} = \mathbf{916.1\ k\Omega}$$

Therefore, the resistance of the scope probe = 916.1 kΩ.

Initial Voltage across Capacitor

When determining the impedance across the scope I found that the $V_{Initial}$ across the ***C1*** was ***3.12 V***, which is the exact same voltage provided to the circuit by the power supply.

Energy stored in Capacitor

We know,

$$E = \frac{CV^2}{2}$$

Equation 7: Energy Stored in Capacitor

$$= \frac{(48.03 * \ 10^{-9})(3.12)^2}{2} = \mathbf{2.33 * 10^{-7}}\ \boldsymbol{joules}$$

Therefore, the energy stored in C1 with a supply of 3.12 V is 0.233 µJ.

What is discharging the capacitor?

When a capacitor discharges, it acts as a voltage source much like a battery or a power source would. Since the switch is open, the capacitor only sees the scope probe to discharge itself. Hence, it starts to discharge through the probe. Scope probes tend to have high resistances. This high resistance makes it look like an open circuit when it’s connected in parallel with other circuit elements. During measurement situations, due to having high impedance, current is stopped from going through the scope probe and therefore it eliminates the possibility of having measurement errors. That’s why in this lab, we decided to make the scope probe the only path for the capacitor to discharge through.

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Analysis of Results

The results from the three solution methods can be compared with each other to determine if the results obtained from the experiment are accurate under limits of experimental accuracy. The results which will be compared are the initial voltage across the capacitor, the voltage after one time constant, the final voltage across the capacitor and the time constant, τ

Charging

Value	Analytical	Multisim	Experimental
$V_{C1}\ (0)$	0V	0V	0V
$V_{C1}\ (\infty)$	2.847V	2.840V	2.920V
$V(\tau)$	1.799V	1.800V	1.800V
τ	27.200μs	27.200μs	29.6μs

Discharging

Value	Analytical	Multisim	Experimental
$V_{C1}\ (0)$	2.847V	2.840V	2.840V
$V_{C1}\ (\infty)$	0V	0V	0V
$V(\tau)$	1.047 V	1.050V	1.044V
τ	63.30μs	62.00μs	62.40μs

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Charging and Discharging

The values obtained for the V_{C1} (0) for the charging event and the V_{C1} (∞) for the discharging event are the same for all the above methods. For the charging event, It is supposed to be zero because as the switch opens no current can pass through the resistors R3 and R4 or the capacitor leading the voltage to be zero. Moreover, for the discharging event, the switch has been open for quite time and therefore all the voltage has discharged allowing it to be at 0. For the charging event, when the switch is closed, the current can pass through the capacitor and after some time the voltage reaches a steady state which is measured and recorded called Vc(infinity). Similarly, when the capacitor is in the a state of discharging at Vc (0), and just after the switch is opened, the capacitor is in a steady state (fully charged). The values obtained for the Analytical and Multisim are relatively close and lie with the uncertainty mentioned previously, concluding that the circuit calculation done was correct. The value obtained in a charging event for the experimental method is 2.920V at a steady state which is approximately 0.08V more than the value obtained from the analytical and Multisim method, leading to an uncertainty of 2.5%. However, the value obtained during the discharging event for the experimental method is 2.840 V, which is like the voltage obtained in Multism and 0.007 V less than the analtical solution. For the charging event, the value of the voltage after one time constant is found by setting the value of exp(-1) and the value obtained by calculation was approximately 1.80V but it was slightly off with the Multism and experimental results. Similarly, in the discharging event, a value of approximately 1.05V was obtained and it also was similar to both the Multism and analytical methods. The value of τ was obtained using the cursors in the oscilloscope in Multisim and experimental. The values obtained for all the methods are almost the same with the value of the Experimental being off by 2.4 μs for the charging event, and 0.9 μs off for the discharging event. There could be several reasons why the value of Vc infinity isn't the same as the value of the analytical and why the τ is slightly off. There could be tolerated in the components such as the resistors and the capacitance which could lead to less voltage. There could be leakage resistance in the capacitor used. There could have been some resistance in the Hantek used which could have yielded a lower voltage than expected as is the case for the Vc Infinity. The τ value is difficult to measure since the value obtained is very small and there can lead to errors that can lead to the time not being measured properly.

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Reflection

In this Topic learned an important topic of timing circuits in which a combination of resistors, capacitors, and inductors are used with a D.C source. The concept of how the voltage changes when the switch is turned on or off is observed in elements such as capacitors and inductors and how the voltage behaves as time changes are learned. The voltage is increased in the capacitor when it is charging up to a certain limit and then the circuit breaks because the capacitor acts as a hole. When the power source is cut off the capacitor starts to release current as it discharges as it is observed how voltage drops from a constant value to zero (at time infinity) in the end the capacitor acts like a normal wire. In this topic, the concept of time constant is learned which is the time taken for the voltage to increase from zero to an approx of 63% it is calculated using the formula T=RC in which the R depends upon the resistance seen by the capacitor. This is also the same as inductors which are used in RL circuits. RC and RL circuits have various uses such as in-camera flashes where they help determine the amount of time before the flash occurs. They are used in pacemakers which are devices that ensure that the heart functions properly by ensuring that it beats at the correct time, RL and RC circuits are used for this purpose. They are also used as filters where they can filter out some frequency of signals depending upon the need (can be used as low signal filters or high depending upon the usage).