

Deliverable / Lab 4 - ENG PHYS 2E04  
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Modified Circuit used for Lab / Deliverable:

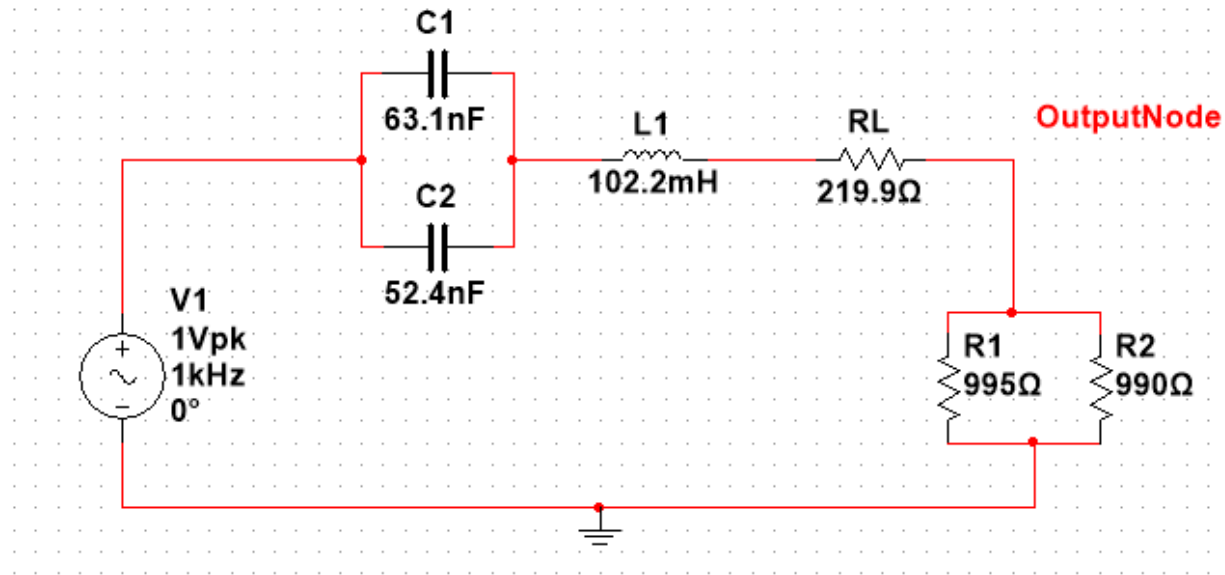


Fig 4.1

Purpose of this Lab:

- For the circuit in Figure 4.1, calculate the transfer function, determine which type of filter it is, then calculate the centre frequency and -3dB frequency (or frequencies) in Hz.
- Simulate the circuit of Figure 4.1 in Multisim and determine the centre frequency, gain at the centre frequency, and the -3dB frequencies. Compare them to your results from step 1.
- Build the circuit in Figure 4.1 on a breadboard and use the oscilloscope to measure the centre frequency, gain at the centre frequency, and the -3dB frequencies of the filter. How well do these results agree with your simulations, accounting for experimental uncertainty?
- Vary the frequency of the input signal over a wide range and confirm for yourself that the bandpass filter is only allowing a small band of frequencies to pass. At some frequency off resonance, look at the output and input signals on the oscilloscope at the same time, is there a phase difference between the two? How does this phase difference vary with frequency?

*The values of the resistors and capacitors was measured using the Hantek 2D42. However, the value obtained for the inductor was through averaging inductor values obtained by a VICHY LCR Meter DM4070.*

Component	Actual Value	Measured Value
R1	1000 $\Omega$	995.0 $\Omega$
R2	1000 $\Omega$	990.0 $\Omega$
RL	220 $\Omega$	219.9 $\Omega$
C1	10 <sup>4</sup> pF	63.1nF
C2	10 <sup>4</sup> pF	52.4nF
L1	100mH	102.2mH

Table 4.1

## **Analytical Method**

### **Maple Code:**

```
restart;
with(plots);
R1 := 995; R2 := 990; RL := 219.9; C1 := 63.1e-9; C2 := 52.4e-9; L1 :=
    102.2e-3; ω := 2·πf;
Ctotal := C1 + C2;
Rtotal :=  $\left( \frac{1}{R1} + \frac{1}{R2} \right)^{-1}$ ;
ZC :=  $\frac{1}{I \cdot \omega \cdot Ctotal}$ ;
ZL := I·ω·L1;
Vin := 1;
Vout :=  $\frac{Rtotal \cdot Vin}{ZC + ZL + RL + Rtotal}$ ;
H := simplify $\left( \frac{Vout}{Vin} \right)$ ;
Rf :=  $\frac{1}{2 \cdot \pi \cdot \sqrt{L1 \cdot Ctotal}}$ ;

dualaxisplot(loglogplot([abs(H), 1/sqrt(2)], f=10 .. 0.1 * 10^7,
    legend=['magnitude', '1/sqrt(2)']),
    semilogplot(argument(H), f=10 .. 0.1 * 10^7, color='blue',
    legend='phase'))

f[-3 * dB] := fsolve(abs(H) = 1/2, f=2000 .. 4000);
maxval := abs(subs(f=Rf, H));
```

Figure 4.2

**The filter above can be described as a bandpass filter.**

Reasoning to that is because it contains both an inductor and a capacitor in series. *A bandpass filter is a filter which allows only a certain range of frequencies in the middle to pass.*

- When the frequency is very high the inductor acts as an open circuit, and the capacitor acts as a shorted cable.

➤ Therefore, no current would be able to pass through and thus,

$$\text{As } \lim_{\text{frequency} \rightarrow \infty} (\text{Current}) = 0 \text{ A}$$

- When the frequency is low, the capacitor acts open and the inductor acts as a shorted cable (opposite from the point above), as there is a gap in between this doesn't allow the current to pass through therefore making the **output voltage 0 V**.
- The capacitors are combined using the parallel capacitor formula,

$$C_{Total} = C_1 + C_2 = 63.1 \text{ nF} + 52.4 \text{ nF} = \mathbf{115.5 \text{ nF}}$$

- The resistors R1 and R2 are combined using the parallel resistor formula,

$$R_{Total} = \left( \frac{1}{R_1} + \frac{1}{R_2} \right)^{-1} = \left( \frac{1}{995} + \frac{1}{990} \right)^{-1} = \frac{197010}{397} \Omega \approx \mathbf{496 \Omega}$$

- The value **RL** above is the internal resistance of the inductor and has a value of **219.9 Ω**.
- The impedances of the resistors, inductors and capacitors are found using the following formulas,

$$Z_C = \frac{1}{I * \omega * C_{Total}}$$

$$Z_L = I * \omega * I_1$$

- VOut is found using the Voltage divider formula,

$$V_{Out} = \frac{R_{total} * V_{in}}{Z_C + Z_L + R_L + R_{Total}}$$

- The gain function, H, is calculated by dividing the VOut by VIn and can also be calculated based on the impedances.

The H function obtained is

$$> H := \text{simplify}\left(\frac{V_{out}}{V_{in}}\right);$$

$$H := \frac{985050.f}{-2.735260277 \times 10^9 I + 1274.650954 I f^2 + 1.4215515 \times 10^6 f}$$

- The value of the resonant frequency, Rf, which is the frequency at which the max gain is found using the following algebra, and the following value was obtained,

$$> Rf := \frac{1}{2 \cdot \pi \cdot \sqrt{LI \cdot C_{total}}};$$

$$Rf := 1480.911615$$

- The f -3db value of the frequency is calculated using the following algebra, and a value was obtained,

$$> f[-3 \cdot dB] := \text{fsolve}\left(\text{abs}(H) = \frac{1}{2}, f = 2000 \dots 4000\right);$$

$$f_{-3 \text{ dB}} := 2125.459462$$

- The gain for these values can also be obtained by putting the frequency value in the original H equation shown above. The value obtained are,

$$> \text{maxval} := \text{abs}(\text{subs}(f = Rf, H));$$

$$\text{maxval} := 0.6929400732$$

**Max Gain = 0.6929V**

**F -3dB Gain = 0.500V**

- The graphs of the H function in relation frequency can be plotted on a log-log plot to under the distribution the frequency. A graph can also be plotted for the phase related to the frequency on a semi-log plot using the code,

```
dualaxisplot(loglogplot([abs(H), 1/sqrt(2)], f=10 .. 0.1 * 10^7,  
    legend=['magnitude', '1/sqrt(2)']),  
    semilogplot(argument(H), f=10 .. 0.1 * 10^7, color='blue',  
    legend='phase'))
```

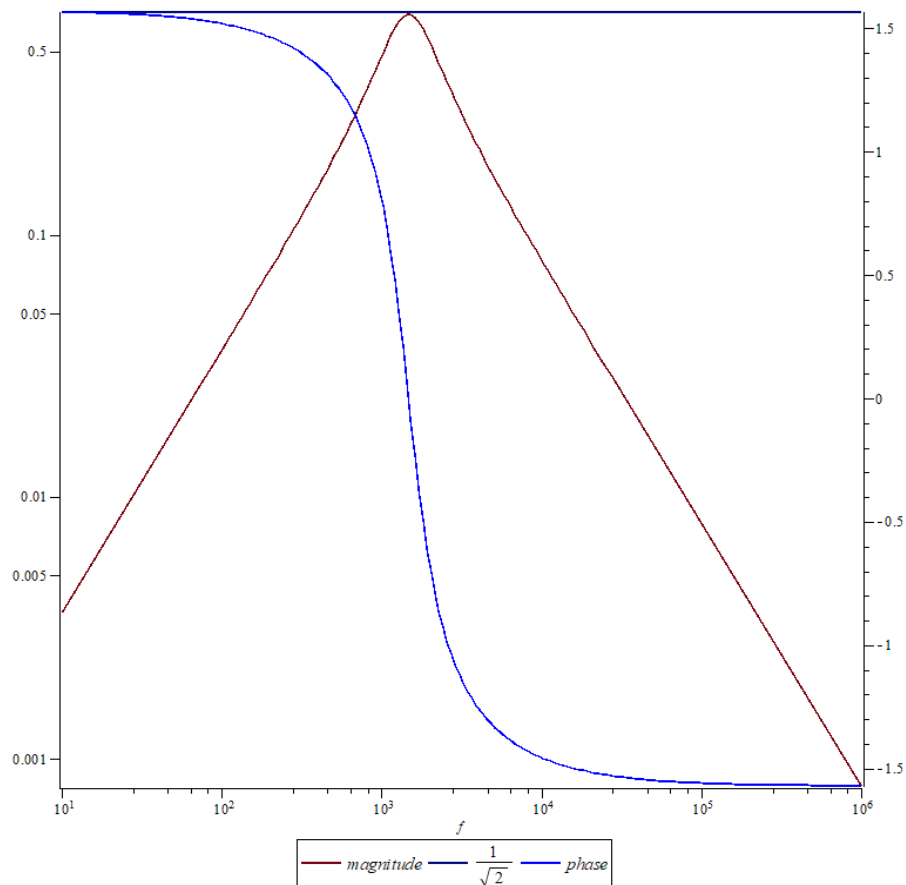


Figure 4.3

From the graph it could clearly be seen that the filter in the circuit above is a bandpass filter as the for values near zero the graph has a very small gain, and the max value is in the range for  $10^3$  which is near the value of  $R_f$  calculated above.

***A trend could be seen as Frequency  $\uparrow$ , the value for grain approaches 0, only allowing a range of frequencies in the middle to pass through thus being a bandpass filter.***

## Multisim Method

The modified circuit shown in Fig 4.1 was set up in Multisim and the values of  $R_f$  and the -3dB frequency are measured.

The value of the gain obtained for each of these values are also measured. The gain vs frequency graph and the phase vs frequency are also obtained using the **AC sweep function** in Multisim.

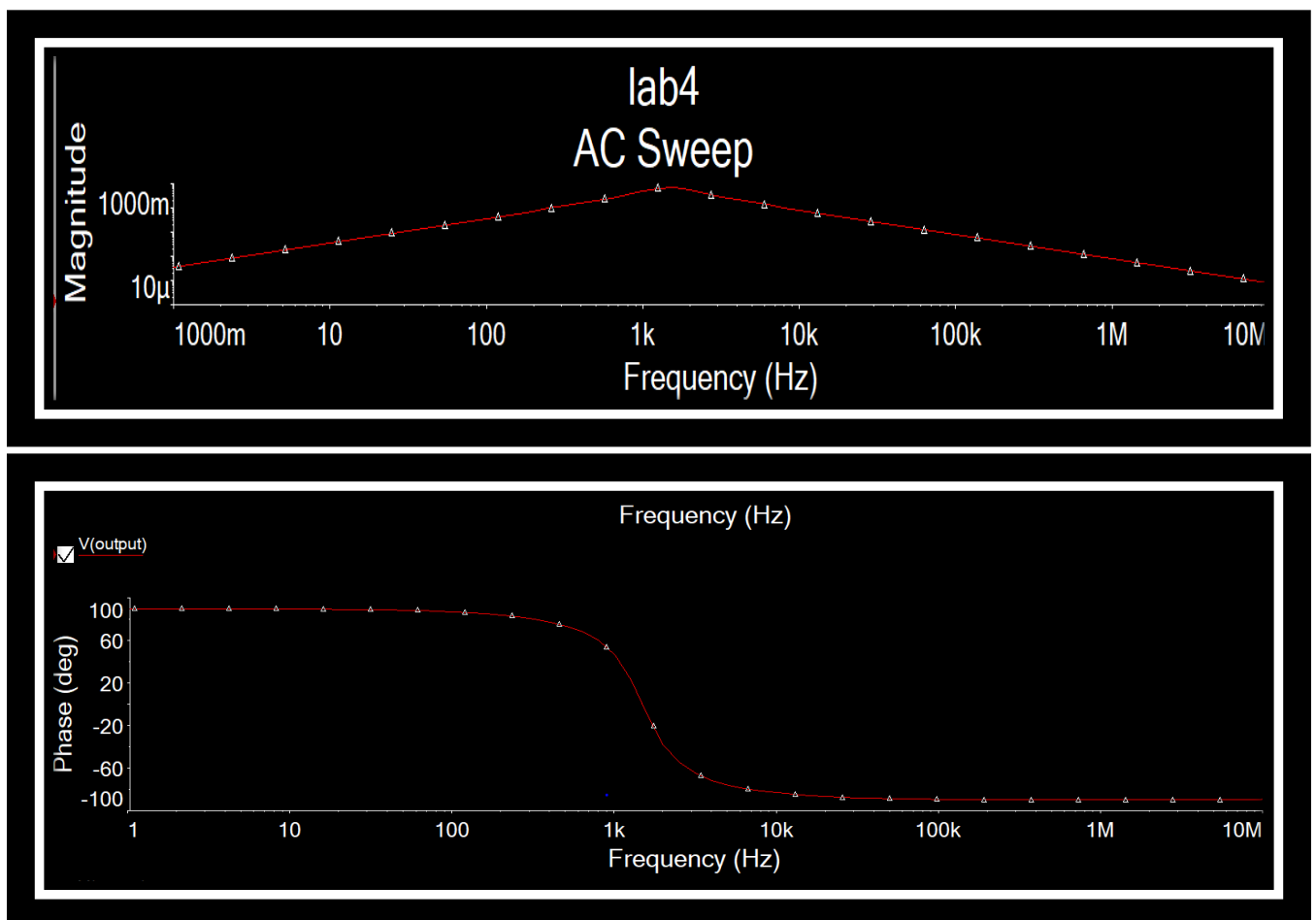


Figure 4.4

The above pictures show the graphs obtained when the **AC sweep function** is used in Multisim. The graphs obtained using the **AC sweep function** are similar to the Bode plot obtained using the analytical method.

- The max value of the magnitude vs frequency and the gain for the max value and the -3dB is shown below and is obtained using the cursor function in Multisim.

The uncertainty in my placement of the cursor to be  $\frac{1}{2}$  the largest acceptable increment, which was small due to having many points per decade.

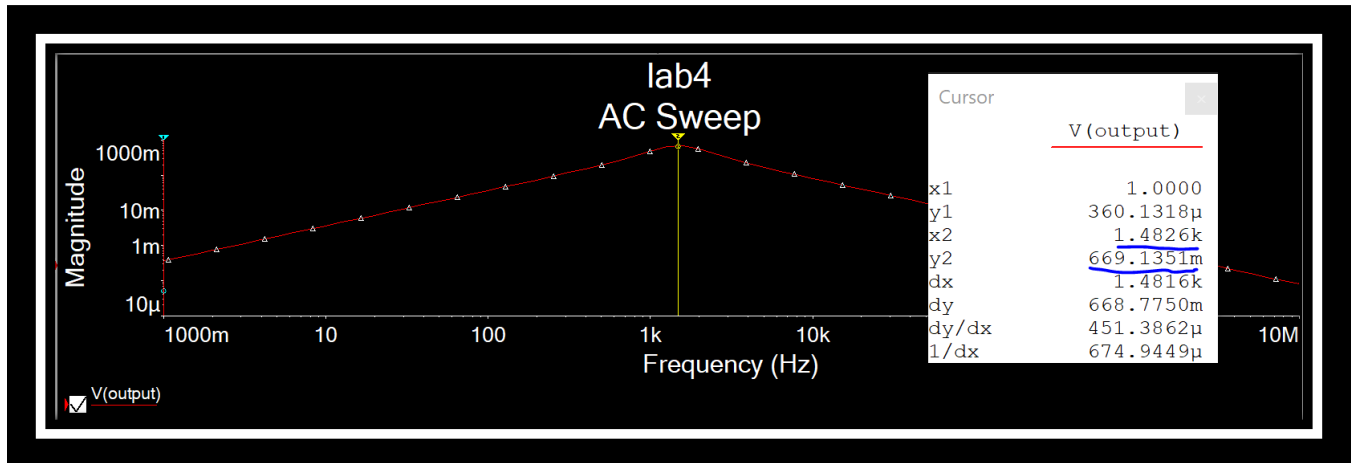


Figure 4.5

The peak of the graph is measured, and the value obtained for the frequency is  $1482.6 \pm 10\text{Hz}$  and the gain is  $669.135 \pm 10\text{mV}$

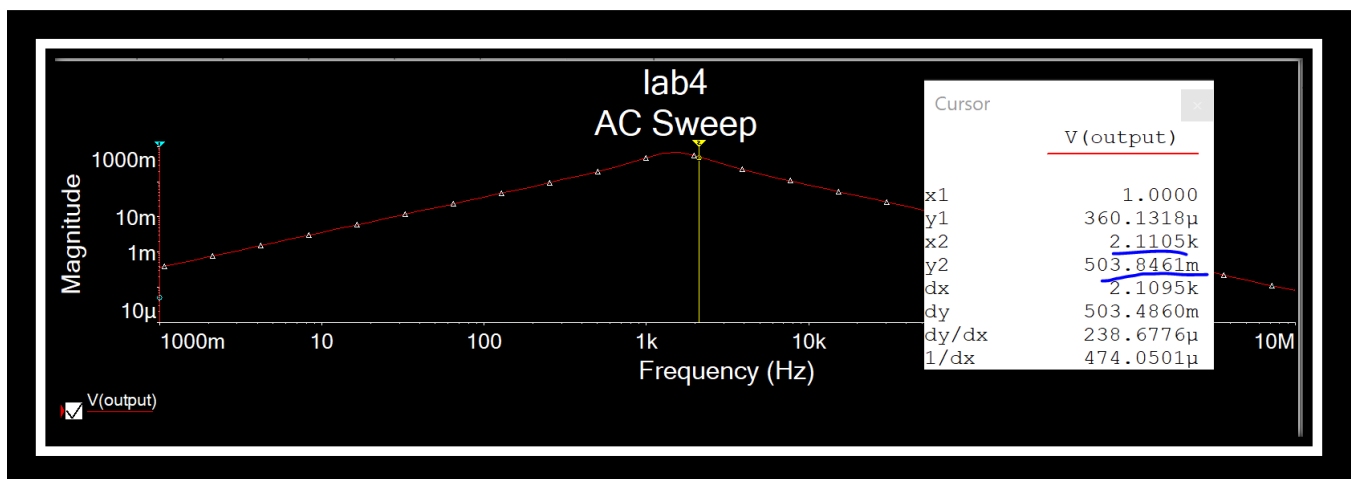


Figure 4.6

The value of the gain is measured at the -3dB value which in this case was  $2110.5 \pm 10\text{Hz}$  and a value of  $504.85 \pm 10\text{mV}$  is obtained.



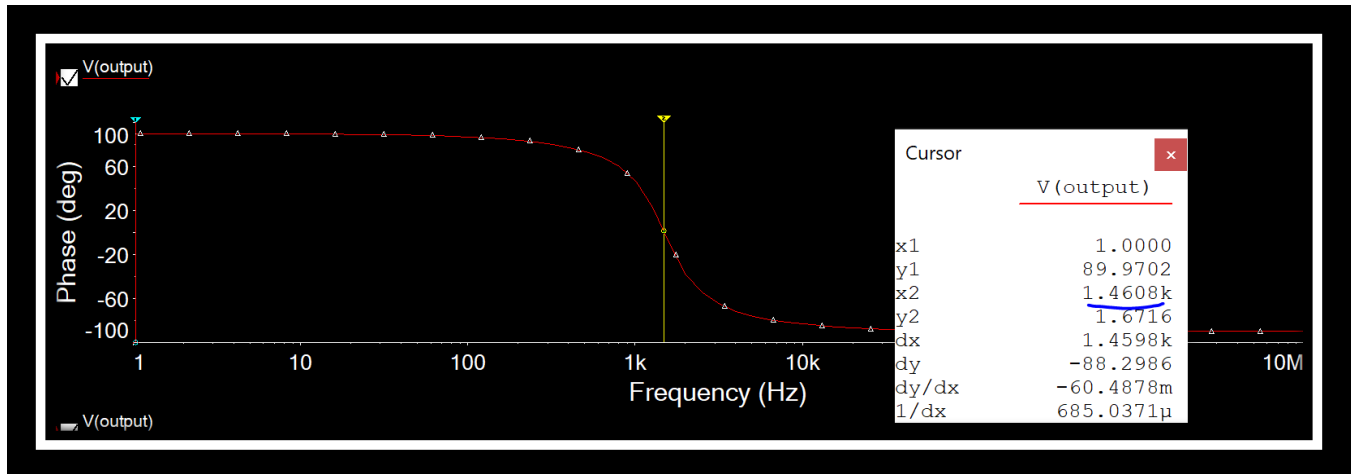


Figure 4.7

**The phase graph was used to find where the phase is zero to find the resonant frequency and a value of 1460.8±10Hz is obtained.**

### Analysis:

The value of the analytical and Multisim are compared.

Value	Analytical	Multisim
Rf	1480.910Hz	1482.600 ± 10Hz
-3db frequency	2125.460Hz	2110.500 ± 10Hz
Gain for Rf	693mV	669 ± 10mV
Gain for -3db frequency	500mV	505 ± 10mV

Table 4.2

My Multisim measurements are almost within the uncertainty of my Analytical calculations. The values obtained from both methods are very close this means that the calculation done using maple is correct and the simulation was also done correctly.

## Experimental Method

The Modified circuit was set up as shown below:

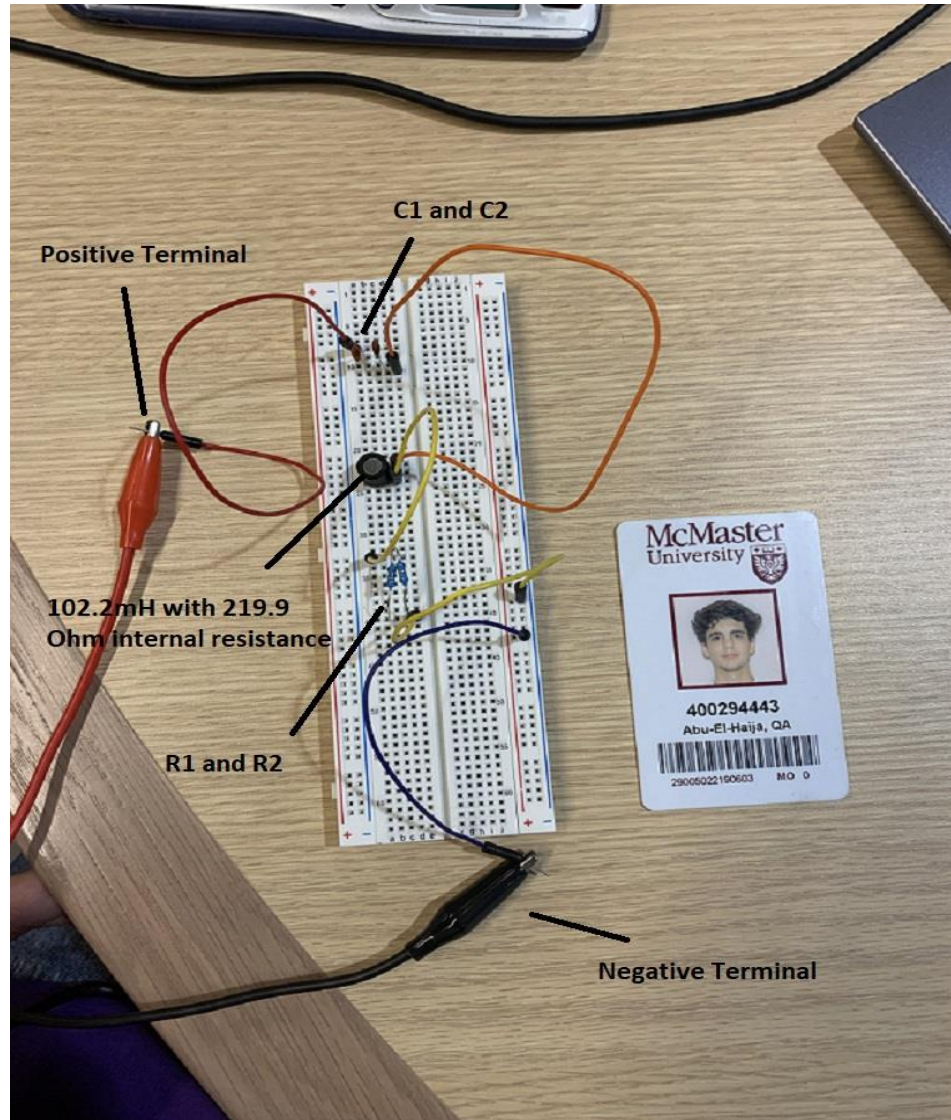


Figure 4.8

The above photo shows how the circuit was set up experimentally using the provided Take-Home kit and using the Hantek as the AC signal source which was used to vary the frequency.

The Hantek was also used for measuring the **gains**, **frequency**, and the **DeltaT** between the **V<sub>Out</sub>** and **V<sub>In</sub>**.

### Uncertainty:

- Frequency

The smallest division for one cursor measurement is **5 Hz** therefore the error of both cursors is **10 Hz**

- Gain:

The smallest division for one cursor measurement is **10 mV** therefore the error of both cursors is **20 mV**

- Phase Angle:

The smallest division for one cursor measurement is **1 degree** therefore the error of both cursors is **2 degrees**

- DeltaT:

The smallest division for one cursor measurement is **1  $\mu$ s** therefore the error of both cursors is **2  $\mu$ s**

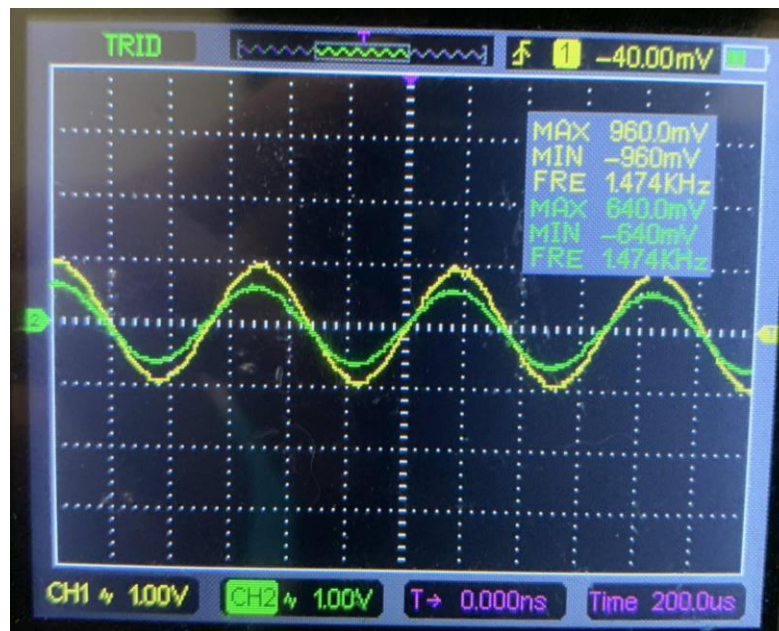


Figure 4.9

The above image shows the gain when the frequency is set to the resonance frequency. The resonant frequency shown above is  **$1474 \pm 10\text{Hz}$**  and the value of gain is  **$640 \pm 20\text{mV}$** . As there is no time difference between the peaks the phase difference is 0.

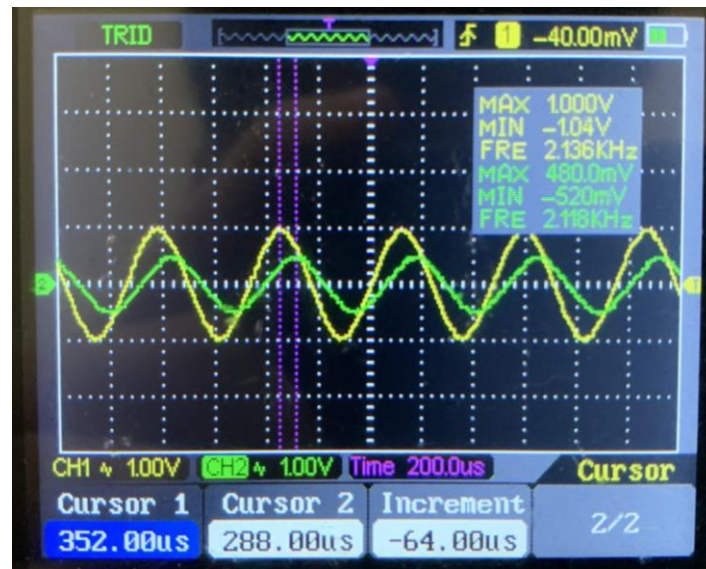


Figure 4.10

The above picture shows the gain when the frequency is set to the -3dB frequency. The -3dB frequency shown above is and the value of the gain is **480.0 ± 20mV**. DeltaT is **64.00 ± 2us**.

Frequency (Hz)	Image	Gain (mV)	DeltaT (us)	Phase Angle (degrees)
400 ± 10Hz		160.000±20mV	640.000±2us	92.160 ± 2 degrees
800 ± 10Hz		320.000±20mV	260.000±2us	74.800±2 degrees



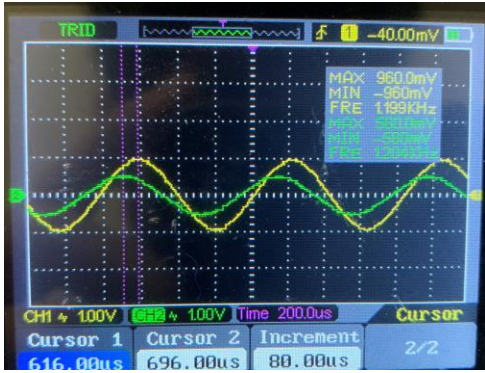
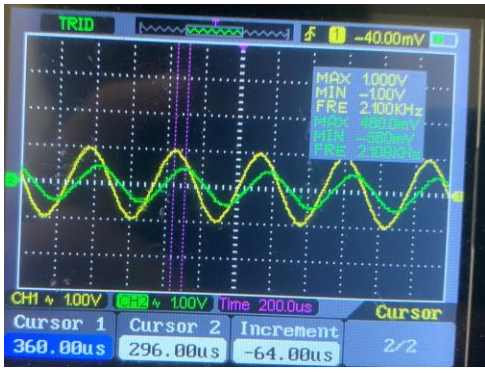
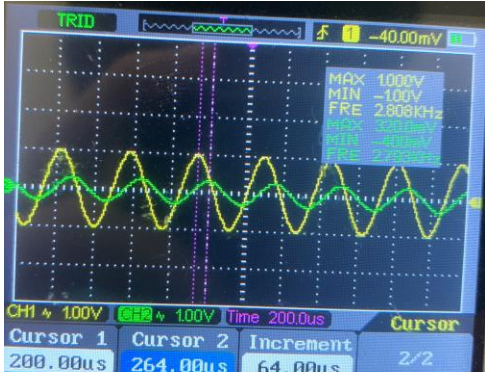
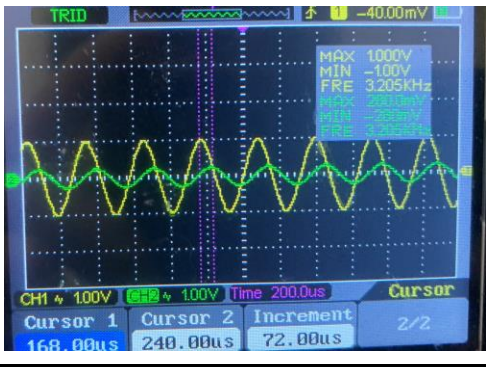
1200± 10Hz		560.000± 20mV	80.000± 2us	34.560± 2 degrees
2100± 10Hz		480.000± 20mV	64.000± 2us	48.384± 2 degrees
2800± 10Hz		320.000± 20mV	64.000± 2us	48.384± 2 degrees
3200± 10Hz		280.000± 20mV	72.000 ± 2us	82.944± 2 degrees

Table 4.2

***The log-log plot for the gain vs frequency is plotted below:***

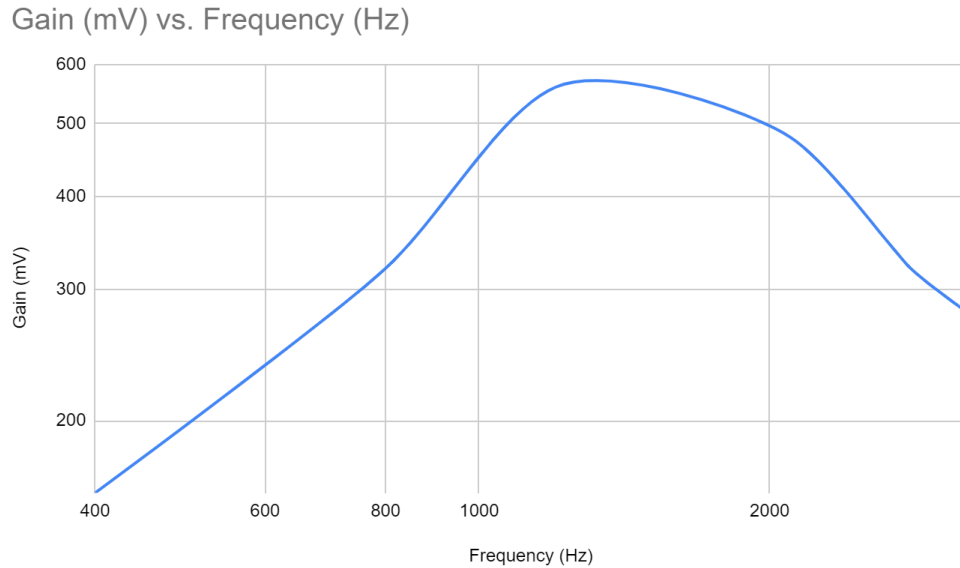


Figure 4.10

- The above plot is the **Magnitude vs Frequency** plot. It could be seen that the graph is similar to graphs obtained in both the Multisim and the analytical method. It could be seen in the graph above that at low frequencies and as the frequency approaches infinity, the gain moves towards **0 V**. As a matter of fact, this can conclude that the filter in the circuit is a bandpass filter.

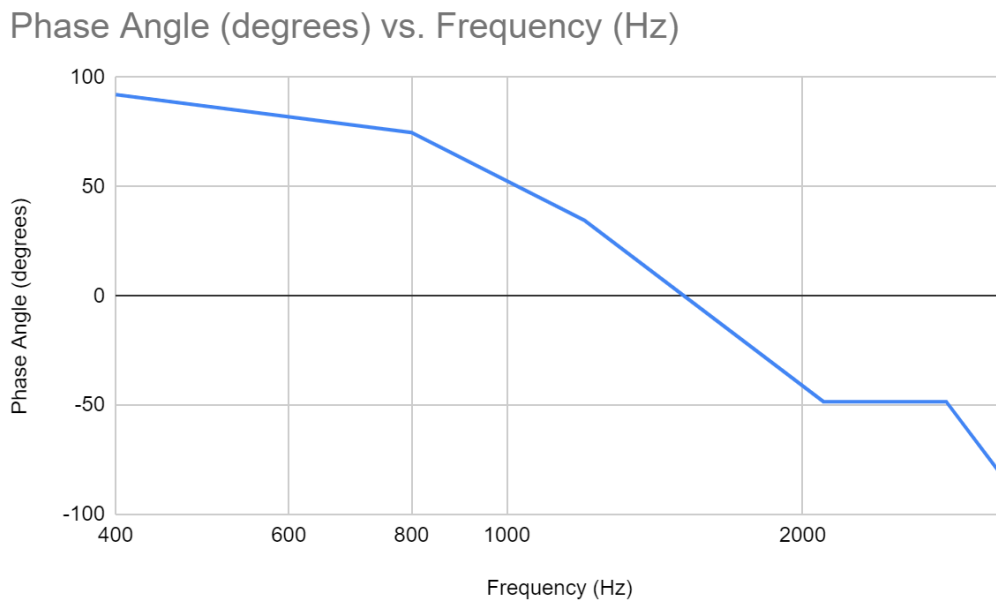


Figure 4.11

- The above graph shown is the **Phase Angle Vs Frequency** plot. It could be seen that the graph is similar to the graphs obtained by both the Multisim and the analytical method. **It could be seen that the phase angle is 0 degrees when the graph reaches the resonance frequency.**

### **Analysis of Results**

The results from the three solution methods can be compared with each other to determine if the results obtained from the experiment are accurate under limits of experimental accuracy. The results compared are the resonant frequency, -3dB frequency, Max gain and the -3dB gain.

Values	Analytical	Multisim	Experimental
Rf	1480.910Hz	1482.600 $\pm$ 10Hz	1474.00 $\pm$ 10Hz
-3db frequency	2125.460Hz	2110.500 $\pm$ 10Hz	2136.00 $\pm$ 10Hz
Gain for Rf	0.693V	0.669V $\pm$ 0.01V	0.64 $\pm$ 0.02V
Gain for -3db frequency	0.500V	0.505V $\pm$ 0.01V	0.48V $\pm$ 0.02V

Table 4.3

The values obtained from the three methods are very similar to each other. The values obtained for the resonant frequency were similar, and within the 1% error. For the -3dB frequency, there was a difference of 15Hz between the analytical and Multisim and there was a difference of 11Hz between analytical and experimental. This leads to an error of 0.52% which is less than 1%, when comparing the analytical and Multism methods. Similarly, when comparing the analytical and experiments methods an error of 0.51%, which is also less than 1%. Moreover, when looking at the gain for Rf, there was significant error in the measuring of the voltage, there was a difference of almost 0.05V between the analytical and the experimental results, and almost 0.024 V between the analytical and Multism method. This resulted in an error of 3.46%.

There could be several reasons why the values are not similar to the analytical results:

- 1) There is significant error in measuring the gain in Multisim as it is difficult to put the cursor exactly on the resonant and the -3dB frequency. This leads to error.
- 2) There is impedance in the jumper wires which can lead to error in the voltages.
- 3) There is tolerance in the resistance which affects the values of the voltages.

Overall, results did meet expectations considering the sources of error. To increase accuracy of results in the future, the following could improve my results,

- 1) If the budget was not a problem, a more accurate and precise oscilloscope could be used to be able to place the cursor in a better position.

- 2) If time was not a factor, a **trimmer** can be used to adjust the resistance and reduce the tolerance in the resistors.

Importance of error propagation:

Every measurement has a demeanor of uncertainty about it, and not all uncertainties are equal. Therefore, the ability to properly combine uncertainties from different measurements is crucial. Anytime a calculation requires more than one variable to solve, propagation of error is important to appropriately decide the uncertainty.

## **Reflection**

In this topic I learned about Filters and frequency response. Filters are combinations of resistors, inductors and capacitors and are used to filter out different signals. There are three main types of filters. The low pass filter is a filter which is a combination of an inductor and resistor which only allows low frequencies to pass through and blocks out the higher frequencies. Secondly, the high pass filter is a combination of a capacitor and a resistor which allows only high frequencies to pass through and blocks out signals with a lower frequency. Finally, the bandpass filter is a filter which does not allow high or low frequencies to pass through but rather a range of frequencies in the middle. In this topic, I also learned about the H function which is the ratio between  $V_{Out}$  and  $V_{in}$ . It can be used to plot the magnitude vs frequency graphs as well as the phase graphs which are known as bode plots. Filters are used in various everyday items such as in radios where they are used to only get a channel of certain frequency. Filters are also used in DC power supplies where they are used to reduce noise which is unwanted signals and are used in reducing the ripples when converting from AC to DC using either a half or full rectifier.