Machine Learning

Spring Semester 2021
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Assignment Sheet 1.

Submit on **Tuesday, February 23, 2020, 10:00**.

Excercise 1. (Multivariate discrete random variables)

We consider the chance experiment of a fair coin that is tossed four times.

a) Give the sample space.

b) Define the random variable X to be the number of heads, Y as the number of tails and Z as $Z = |X - Y|$.

c) Compute the joint PMF of (X, Z).

d) Compute the expectation $E(Y)$.

(4 Points)

Excercise 2. (Conditional properties of multivariate discrete random variables)

We consider the chance experiment of a fair dice that is rolled two times. Let X_1 be the outcome of the first roll, while X_2 is the outcome of the second roll.

a) Describe the chance experiment by its sample space, and define the random variables.

b) Compute the conditional expectation $E(X_1 + X_2|X_2 = x_2)$.

c) Compute the conditional expectation $E(X_1X_2|X_2 = x_2)$.

d) Compute the conditional variance $Var(X_1^2X_2|X_2 = x_2)$.

Hint: There might be easier and harder ways to solve these tasks.

(4 Points)

Excercise 3. (Multivariate continuous random variables)

Let (X, Y) be random variables describing the two coordinates of points that are uniformly distributed in a triangle that is bound by $-1 \leq x \leq 1$, $y \geq 0$ and the two lines $y = 1 + x$ and $y = 1 - x$.

a) Find $P(X \geq -0.5)$.

b) Find $P(Y \geq 0.5)$.

c) Find the marginal densities and expexations of X and Y.

(4 Points)

Programming Exercise 1. In this task, you will do some self-study of visualization techniques. Familiarize yourself with one of the following tools / libraries for visualization

- Matplotlib (Python) https://matplotlib.org
- Matplotlib (Wrapper for C++) https://matplotlib-cpp.readthedocs.io/en/latest/
- Gnuplot (stand-alone) http://www.gnuplot.info/

and carry out the following tasks:

a) Generate a random set of two-dimensional points and plot them as a scatter plot. *(We will need this later.)*

b) Plot the density of the 2D Gaussian distribution $\mathcal{N}(\mu, \mathbf{\Sigma})$, with $\mu = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ and $\sigma = \begin{pmatrix} 1 & 0.25 \\ 0.5 & 1 \end{pmatrix}$.

Reference solutions will only be provided in Python+Matplotlib. The submission format for Python is a Jupyter notebook. The submission format for C/C++ is standard source files. Choose an appropriate format for the Gnuplot-related submission.

(4 Points)