

Exercise 2

$$a) \tau = \left\{ \underbrace{(-2, 2)}_{x_1}^T, \underbrace{(0, 0)}_{x_2}^T, \underbrace{(1, 2)}_{x_3}^T \right\}$$

$$k(x_1, x_2) = e^{-\frac{\|x_1 - x_2\|_2^2}{\sigma^2}} = e^{-4}$$

$$k(x_1, x_3) = e^{-\frac{\|x_1 - x_3\|_2^2}{\sigma^2}} = e^{-9}$$

$$k(x_2, x_3) = e^{-\frac{\|x_2 - x_3\|_2^2}{\sigma^2}} = e^{-1}$$

$$K = \begin{bmatrix} 1 & e^{-4} & e^{-9} \\ e^{-4} & 1 & e^{-1} \\ e^{-9} & e^{-1} & 1 \end{bmatrix}$$

$$K\alpha = y \iff \begin{bmatrix} 1 & e^{-4} & e^{-9} \\ e^{-4} & 1 & e^{-1} \\ e^{-9} & e^{-1} & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

Solving the linear system using a computer

$$\alpha = \begin{bmatrix} 2.02 \\ -0.89 \\ 2.33 \end{bmatrix}$$

$$f_x(x) = 2.02 k(x, -2) + (-0.89) k(x, 0) + 2.33 k(x, 1)$$

$$b) \quad K' = \begin{bmatrix} 2 & e^{-4} & e^{-9} \\ e^{-4} & 2 & e^{-1} \\ e^{-9} & e^{-1} & 2 \end{bmatrix}$$

$$K' \alpha' = \gamma \Leftrightarrow \left[\begin{array}{ccc|c} 2 & e^{-4} & e^{-9} & 2 \\ e^{-4} & 2 & e^{-1} & 0 \\ e^{-9} & e^{-1} & 2 & 2 \end{array} \right]$$

Solving the linear system with a computer.

$$\alpha' = \begin{bmatrix} 1.0018 \\ -0.199 \\ 1.036 \end{bmatrix}$$

$$f_{\alpha'}(x) = 1.0018 e k(x, -2) - 0.199 k(x, 0) + 1.036 k(x, 1)$$

