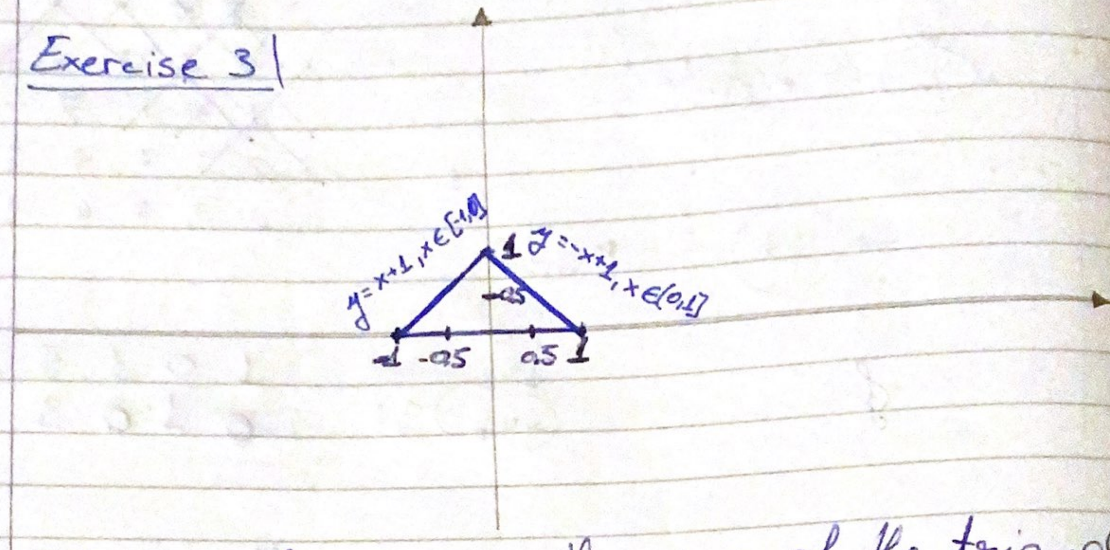
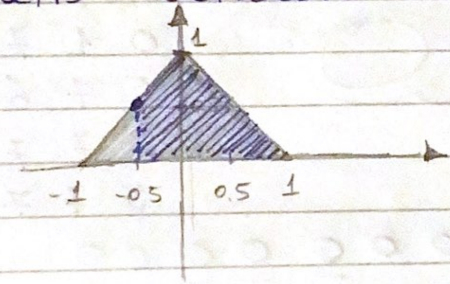
Exercise 3



- The sample space is the area of the triangle.

~~Check~~: The area is already 1, so no need to normalize the next computations.

a) $\mathbb{P}[X > -0.5]$ can be visualised as the ratio between the blue and the black area.



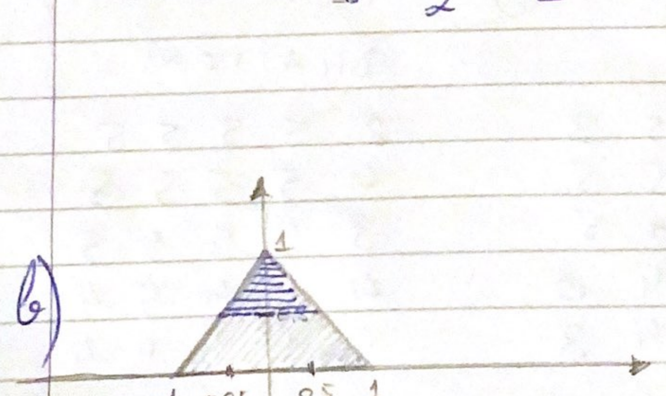
$$\mathbb{P}[X>-0.5] = \frac{1}{2} + \int_{-0.5}^{0} x+1 \, dx$$

$$\mathbb{P}[X\geq-0.5] = \frac{1}{2} + \left[\frac{x^2}{2} + x\right]_{-0.5}^{0} = \frac{1}{2} + 0 - \left(\frac{1}{8} - \frac{1}{2}\right)$$

$$= \boxed{\frac{7}{8}}$$

The same trick applies

b)



Expressing in terms of y, $x = \pm(y-1)\ \forall x \in [0,1]$

Because the function is symmetric w.r.t y-axis,

$$\mathbb{P}[Y \geq 0.5] = 2\int_{0.5}^{1} 1-y\ dy = 2\int_{0.5}^{1}(1-y)dy = 2\left[y - \frac{y^2}{2}\right]_{0.5}^{1}$$

$$\mathbb{P}[Y \geq 0.5] = 2\left[\frac{1}{2} - \left(\frac{1}{2} - \frac{1}{8}\right)\right] = \boxed{\frac{2}{8}}$$

c) Marginal density functions are summations of p of ~~all~~ combinations of the variable of interest and the other one/s.

$$p_X(t) = \begin{cases} 1+t & \text{if } t \in [-1,0] \\ 1-t & \text{if } t \in (0,1] \\ 0 & \text{otherwise} \end{cases}$$

$$p_Y(t) = \begin{cases} (-t+1)\cdot 2 & \text{if } t \in [0,1] \\ 0 & \text{othowise} \end{cases}$$

$$\mathbb{E}[X] = \int_{-1}^{1} x\, p_X(x)\, dx = \int_{-1}^{0} x(1+x)dx + \int_{0}^{1} x(1-x)dx$$

$$= \left(\frac{x^2}{2} + \frac{x^3}{3}\right)\Bigg|_{-1}^{0} + \left(\frac{x^2}{2} - \frac{x^3}{3}\right)\Bigg|_{0}^{1}$$

$$= -\left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{3}\right) = \boxed{0}$$

$$\mathbb{E}[Y] = \int_0^1 y\, f_Y(y)\, dy = \int_0^1 2y(1-y)\, dy = 2\left[\frac{y^2}{2} - \frac{y^3}{3}\right]_0^1$$

$$= 2\left[\frac{1}{2} - \frac{1}{3}\right] = \boxed{\frac{1}{3}}$$