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$$a) \Omega = \{1 \dots 6\}^2$$

$X_1 \Rightarrow$ outcome of first roll $\Rightarrow \{1 \dots 6\}$

$X_2 \Rightarrow$ outcome of second roll $\Rightarrow \{1 \dots 6\}$

$$b) E[X_1 + X_2 | X_2 = x_2] = E[X_1 | X_2 = x_2] + E[X_2 | X_2 = x_2] =$$

$$= E[X_1] + x_2 = \frac{1}{6} \sum_{i=1}^6 i + x_2 = 3,5 + x_2$$

$$c) E[X_1 X_2 | X_2 = x_2] = E[X_1 | X_2 = x_2] E[X_2 | X_2 = x_2]$$

$$= E[X_1] x_2 = 3,5 x_2$$

$$d) \text{Var}(X_1^2 X_2 | X_2 = x_2) = E((X_1^2 X_2)^2 | X_2 = x_2) - E(X_1^2 X_2 | X_2 = x_2)^2$$

$$= E(X_1^4 X_2^2 | X_2 = x_2) - E(X_1^2 X_2 | X_2 = x_2)^2 =$$

$$= E(X_1^4) E(X_2^2 | X_2 = x_2) - (E(X_1^2) E(X_2 | X_2 = x_2))^2 =$$

$$= \left(\sum_x x^4 P(X_1 = x) \right) x_2^2 - \left(\left(\sum_x x^2 P(X_1 = x) \right) x_2 \right)^2 =$$

$$= \left(\frac{1}{6} \sum_x x^4 \right) x_2^2 - \left(\left(\frac{1}{6} \sum_x x^2 \right) x_2 \right)^2 =$$

$$= 149,14 x_2^2$$