

$$7.2. a) T = \{(-2, 4), (2, 4), (1, 1), (-1, 1), (0, 0), (3, 9)\}$$

$$T_1 = \{(-2, 4), (2, 4)\} \quad T_2 = \{(1, 1), (-1, 1)\} \quad T_3 = \{(0, 0), (3, 9)\}$$

$$i) T_{\text{train}} = T_1 \cup T_2 \quad T_{\text{val}} = T_3$$

$$f_{\text{KNN}}(0) = \frac{1}{2}(1+1) = 1 \quad ; \quad f_{\text{KNN}}(3) = \frac{1}{2}(1+4) = \frac{5}{2}$$

$$\text{ERR}_1 = \frac{1}{2} \left((0-1)^2 + \left(9 - \frac{5}{2}\right)^2 \right) = 21.625$$

$$ii) T_{\text{train}} = T_1 \cup T_3 \quad T_{\text{val}} = T_2$$

$$f_{\text{KNN}}(1) = \frac{1}{2}(0+4) = 2 \quad ; \quad f_{\text{KNN}}(-1) = \frac{1}{2}(0+4) = 2$$

$$\text{ERR}_2 = \frac{1}{2} \left((1-2)^2 + (1-2)^2 \right) = 1$$

$$iii) T_{\text{train}} = T_2 \cup T_3 \quad T_{\text{val}} = T_1$$

$$f_{\text{KNN}}(-2) = \frac{1}{2}(1+0) = \frac{1}{2} \quad ; \quad f_{\text{KNN}}(2) = \frac{1}{2}(1+9) = 5$$

$$\text{ERR}_3 = \frac{1}{2} \left(\left(4 - \frac{1}{2}\right)^2 + (4-5)^2 \right) = \frac{1}{2} \left(\frac{49}{4} + 1 \right) = 6.625$$

$$EGE(f_{\text{KNN}}) = \frac{1}{3} \sum_{i=1}^3 \text{ERR}_i = \frac{1}{3} (21.625 + 1 + 6.625) = 9.75$$

b) T is given as before, $T = T_1 \cup \dots \cup T_6$

i) $T_{val} = T_1$; $T_{train} = T \setminus T_{val}$; $\hat{\beta} = (1, 2)$

$$f(-2) = -3, \text{ ERR}_1 = (4 - (-3))^2 = 49$$

ii) $T_{val} = T_2$; $\hat{\beta} = (1.08, 2.78)$; $f(2) = 4.95$; $\text{ERR}_2 = 0.895$

iii) $T_{val} = T_3$; $\hat{\beta} = (3.16, 1.09)$; $f(1) = 4.26$; $\text{ERR}_3 = 10.6$

iv) $T_{val} = T_4$; $\hat{\beta} = (2.86, 0.92)$; $f(-1) = 1.95$; $\text{ERR}_4 = 0.895$

v) $T_{val} = T_5$; $\hat{\beta} = (3.26, 0.9)$; $f(0) = 3.26$; $\text{ERR}_5 = 10.6$

vi) $T_{val} = T_6$; $\hat{\beta} = (2, 0)$; $f(3) = 2$; $\text{ERR}_6 = 49$

$$EGE(f) = \frac{1}{6} \sum_{i=1}^6 \text{ERR}_i = 20.165$$