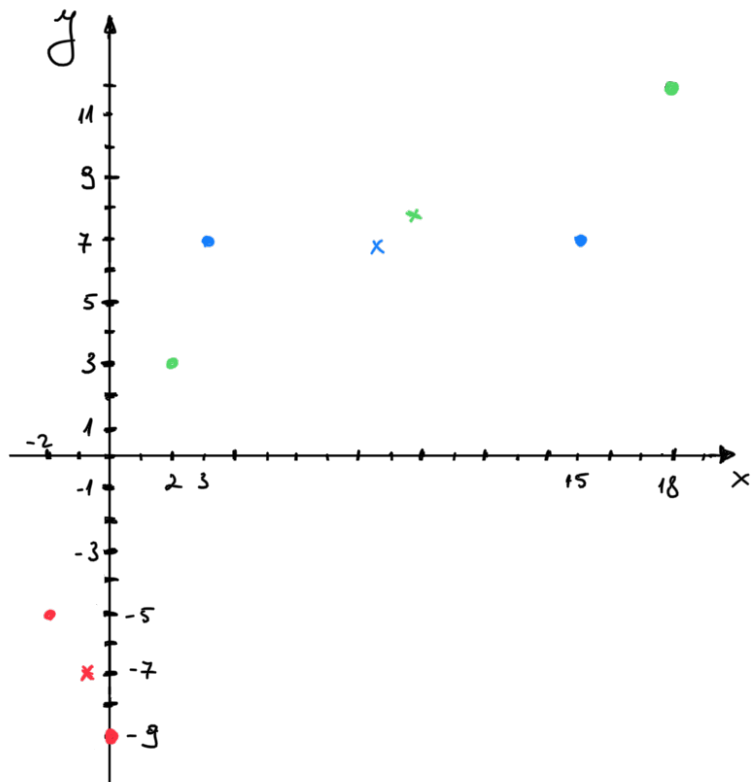


Exercise 1

$(15, 7)$	2
$(0, -9)$	1
$(-2, -5)$	1
$(2, 3)$	0
$(3, 7)$	2
$(18, 12)$	0



1st Step

$$m_0^{(1)} = (10, 7.5)$$

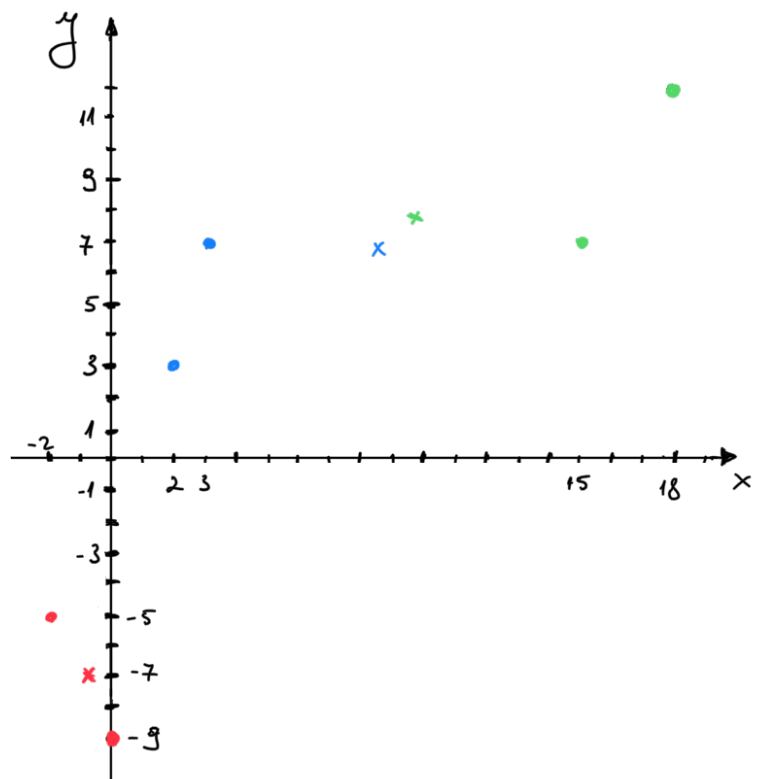
$$m_1^{(1)} = (-1, -7)$$

$$m_2^{(1)} = (9, 7)$$

$$m_0^{(2)} = (16.5, 9.5)$$

$$m_1^{(2)} = (-1, -7)$$

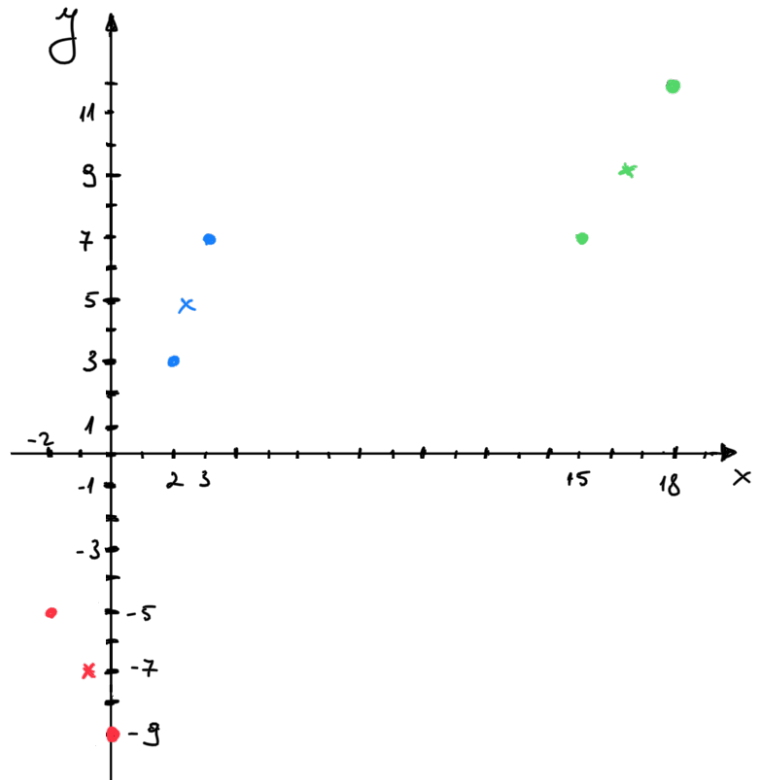
$$m_2^{(2)} = (2.5, 5)$$



2nd Step

3rd Step

$(15, 7)$	0
$(0, -9)$	1
$(-2, -5)$	1
$(2, 3)$	2
$(3, 7)$	2
$(18, 12)$	0



Exercise 2

$$\mathcal{T} = \left\{ (1, 0)^T, (0.5, 1)^T, (1, 0.5)^T \right\}$$

$$\bar{x} = \frac{1}{3} \cdot \begin{pmatrix} 1 + 0.5 + 1 & 0 + 1 + 0.5 \end{pmatrix} = (0.83, 0.5)$$

$$X = \begin{bmatrix} 0.167 & -0.333 & 0.167 \\ -0.5 & 0.5 & 0 \end{bmatrix}$$

$$X^T X = \begin{bmatrix} 0.167 & -0.25 \\ -0.25 & 0.5 \end{bmatrix}$$

$$X^T X v = \lambda v \Rightarrow (X^T X - \lambda I)v = 0 \Rightarrow \det(X^T X - \lambda I) = 0$$

$$\Rightarrow \begin{vmatrix} 0.167 - \lambda & -0.25 \\ -0.25 & 0.5 - \lambda \end{vmatrix} = 0 \Rightarrow (0.167 - \lambda)(0.5 - \lambda) - 0.25^2 = 0$$

$$\lambda_1 = 0.634$$

$$\lambda_2 = 0.033$$

for λ_1 the eigenvector v_1 is:

$$\begin{cases} 0.167x - 0.25y = 0.634x \\ -0.25x + 0.5y = 0.634y \end{cases} \Rightarrow \begin{aligned} -0.083x + 0.25y &= 0.634x + 0.634y \\ -0.717x &= 0.384y \Rightarrow x = -0.536y \end{aligned}$$

$$v_1 = \begin{bmatrix} -0.536 \\ 1 \end{bmatrix}$$

for λ_2 the eigenvector v_2 is: (Another method)

$$\left[\begin{array}{cc|c} 0.134 & -0.25 & 0 \\ -0.25 & 0.467 & 0 \end{array} \right] \Rightarrow \left[\begin{array}{cc|c} 1 & 1.87 & 0 \\ 0 & 0 & 0 \end{array} \right]$$

$$v_2 = \begin{bmatrix} 1.87 \\ 1 \end{bmatrix}$$

