

Exercise 1

Q1

$$\Omega = \left\{ \begin{array}{l} \text{HHHH, HHHT, HHTH, HHTT,} \\ \text{HTHH, HTHT, HTTH, HTTT,} \\ \text{THHH, THHT, THTH, THTT,} \\ \text{TTHH, TTHT, T~~H~~TH, TTTT} \end{array} \right\}$$

b) $X: \Omega \rightarrow \mathbb{N}$ the number of heads

$$X = \{0, 1, 2, 3, 4\}$$

$Y: \Omega \rightarrow \mathbb{N}$ the number of tails

$$Y = \{0, 1, 2, 3, 4\}$$

$$Z: \Omega \rightarrow \mathbb{N} \quad |X - Y|$$

$$Z = \{0, 2, 4\}$$

Because, from Ω , there's no possibility that the difference $X - Y$ is odd.

c)

$X \backslash Z$	0	2	4
0	0	0	$1/16$
1	0	$4/16$	0
2	$6/16$	0	0
3	0	$4/16$	0
4	0	0	$1/16$

 $p(X, Z) =$

- In order for Z to be 0, # Heads = # Tails = 2.

$X=2$ in $\binom{4}{2}$ of the cases, hence $P[X=2, Z=0] = \frac{6}{16}$

- In order for Z to be 2, $X=3$ and $Y=1$ or $X=1$ and $Y=3$

This happens in $2 \binom{4}{1}$ of the possibilities

$$P[Z=2] = 8/16$$

$$P[X=3, Z=2] = \frac{4}{16} \quad P[X=1, Z=2] = \frac{4}{16}$$

- Finally, $Z=0$ iff $X=4$ or $X=0$

$$P[X=4, Z=0] = \frac{1}{16} \quad P[X=0, Z=0] = \frac{1}{16}$$

d) From Assignment 1, or

$$\begin{aligned}
 E[Y] &= 0 \cdot \frac{1}{16} + 1 \cdot \frac{4}{16} + 2 \cdot \frac{6}{16} + 3 \cdot \frac{4}{16} + 4 \cdot \frac{1}{16} \\
 &= \boxed{2}
 \end{aligned}$$