Exercise 1

a) $\mathcal{L} = \{(-1,1), (0,0.5), (1,3)\}$

$$X = \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} \qquad X^T X = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$$

$$X^T Y = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0.5 \\ 3 \end{bmatrix} = \begin{bmatrix} 4.5 \\ 2 \end{bmatrix}$$

Solving the linear system $X^T X \alpha = X^T Y$

$$\left[\begin{array}{cc|c} 3 & 0 & 4.5 \\ 0 & 2 & 2 \end{array}\right] \qquad \alpha = \begin{bmatrix} 1.5 \\ 1 \end{bmatrix}$$

$$f_\alpha(x) = 1.5 + x$$

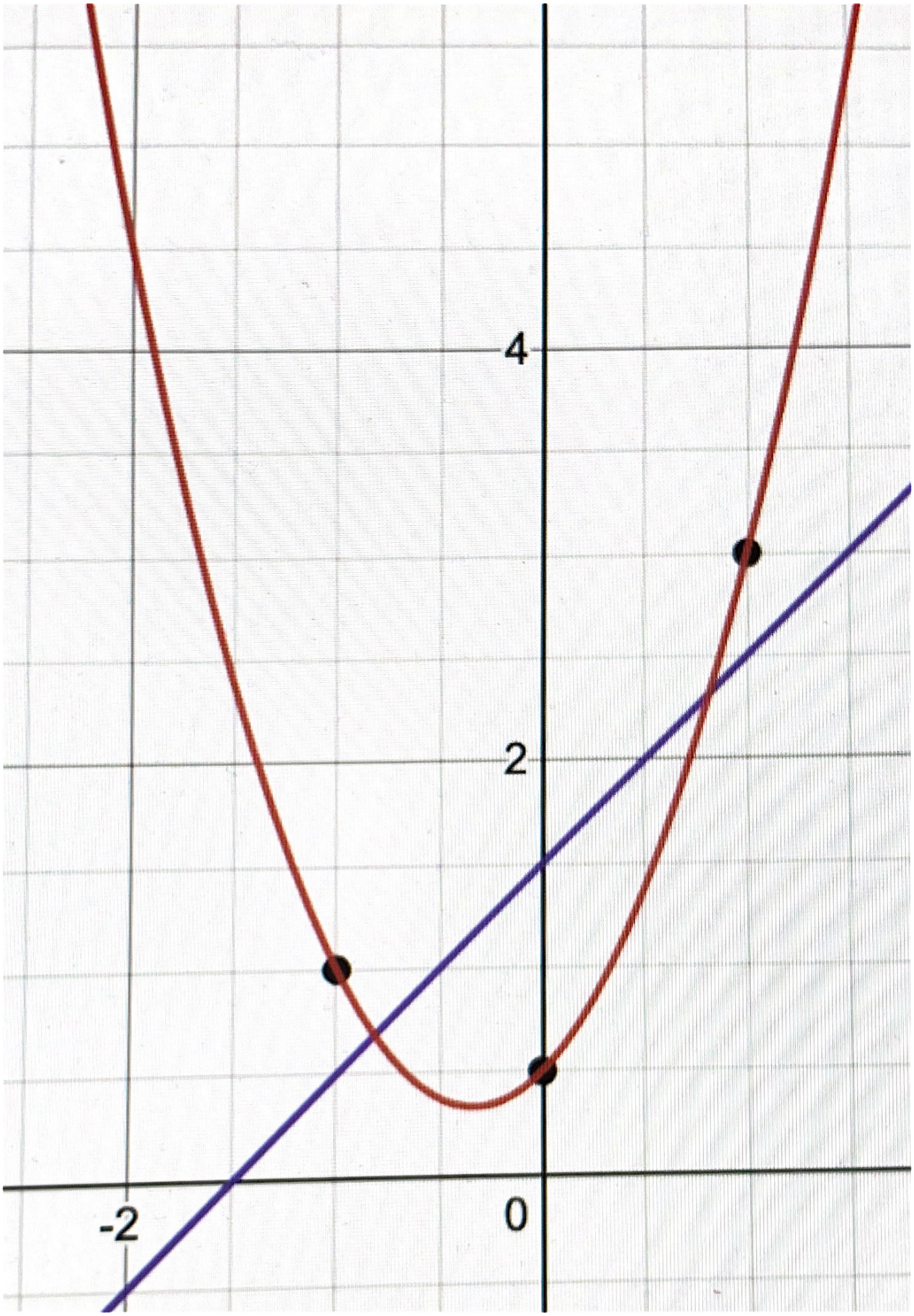
b) $$X' = \begin{bmatrix} 1 & -1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \qquad X'^T X' = \begin{bmatrix} 3 & 0 & 2 \\ 0 & 2 & 0 \\ 2 & 0 & 2 \end{bmatrix}$$

$$X'^T Y = \begin{bmatrix} 4.5 \\ 2 \\ 4 \end{bmatrix}$$

Solving the linear system $X'^T X' \alpha' = Y$

$$\alpha' = \begin{bmatrix} 1/2 \\ 1 \\ 3/2 \end{bmatrix}$$

$$f_{\alpha'}(x) = \frac{1}{2} + x + \frac{3}{2}x^2$$


4
2
-2
0