

9.3.

$$J(B) = - \sum_{i=1}^N \left[\sum_{g=1}^r p(g|x_i) s_g(x_i) - \log \left(\sum_{h=1}^r \exp(s_h(x_i)) \right) \right]$$

$$\nabla_g J(B) = \begin{pmatrix} \frac{\partial J(B)}{\partial \beta_0^{(g)}} \\ \vdots \\ \frac{\partial J(B)}{\partial \beta_r^{(g)}} \end{pmatrix}$$

$$\nabla_g J(B) = - \sum_{i=1}^N \left[\nabla_g p(g|x_i) s_g(x_i) - \nabla_g \log \left(\sum_{h=1}^r \exp(s_h(x_i)) \right) \right]$$

$$= - \sum_{i=1}^N \left[p(g|x_i) \nabla_g s_g(x_i) - \frac{\nabla_g \exp(s_g(x_i))}{\sum_{h=1}^r \exp(s_h(x_i))} \right]$$

$$= - \sum_{i=1}^N \left[p(g|x_i) \begin{pmatrix} 1 \\ x_i \end{pmatrix} - \frac{\exp(s_g(x_i))}{\sum_{h=1}^r \exp(s_h(x_i))} \begin{pmatrix} 1 \\ x_i \end{pmatrix} \right]$$

$$= - \sum_{i=1}^N \left[p(g|x_i) \begin{pmatrix} 1 \\ x_i \end{pmatrix} - p_B(g|x_i) \begin{pmatrix} 1 \\ x_i \end{pmatrix} \right]$$

$$= \sum_{i=1}^N \left[p_B(g|x_i) - p(g|x_i) \right] \begin{pmatrix} 1 \\ x_i \end{pmatrix}$$