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Objetivo: Validação do algoritmo uniaxial viscoelastico que contabiliza a solidificação do cimento

Resumo: O presente trabalho tem como objetivo a validação da teori de solidificação apresentada primeiramente por Bazant (1977).
1. Para isso, formulou-se a a análise analítica a partir da resolução da equação diferencial pelas transformadas de laplace. Calcularam-se 5 funções:
- "*epsilont* " é a formulação analítica sem aging;
- "*epsilonta* " é a formulação multiplicada por uma função "v" apresentada no trabalho de Grasley&Lange (2007);
- "*epsilont_grasley* " formulação apresentada em Grasley&Lange (2007). é espectavel que seja igual à solução dada pela transformada de laplace;
- "*epsilonsat_elastic* " parcela elástica com aging;
- "*epsilonsat_visco* "parcela viscosa com aging;

2. Aplicou-se o algorítimo incremental apresentado por Taylor et. al (1970) e verificou-se que dá respostas similares às analíticas (quer aging quer propriedades constantes). Contudo, neste algorítimo a tensão é conhecida e a deformação calculada. Em problemas de FEM acontece o contrario. As deformações sõ prescritas e as tensões atualizadas.
3. No algoritmo incremental deram-se as deformações prescritas pela solução "*epsilonta* " e calculou-se a tensão - o inverso do problema apresentado no ponto 2. Verifica-se que a tensão calculada pelo algorítmo é proxima à solução prescrita no problema 2.
Em conclusão o algoritmo incremental devolve respostas similares/proximas à solução analítica.

Comentário/Criticas ao trabalho apresentado por Grasley&Lange (2007):
1. Grasley&Lange (2007) atestam que o valor de E[1]=E[0]=2. Este trabalho mostra que o considerado foi E[1]=1 e E[0]=2. Verificar equação (30) em Grasley&Lange (2007).
2. Equação (6) presente entre Grasley&Lange (2007) soma a parcela elástica à viscosa. Contudo, isso é um "abuso" de linguagem. Esse desmembramento aparece devido à integração por partes do integral herditátrio. Assim, o correto é subtrair a parcela viscosa à elástica. Sendo a derivada de J negativa, o sinal vira positivo.
3. Grasley&Lange (2007) mostram que a teoria de solidificação dá resultados desconformes com a realidade e propõem conjuga-la com o Time-Shift para corrigir desconfomirdades. Carol & Bazant (1993) propõem esta simplificação de utilizar uma uníca função de aging. Explicam explicitamente que na verdade deveria haver uma função de aging para a resposta elástica, viscosa e de flow do material (adicionam um amortecedor em série).

```
> restart :
> with(plots) : with(inttrans) :
```

1. ANALISE ANALÍTICA

```
> #Parametros da função de aging;
  w1 := 0.008 : B1 := 0.702 : w2 := 0.19209 : B2 := 1.384 :
> #função de aging dada por (Grasley  Lange (2007));
  v := t→(B1·exp(−w1·t) + B2·exp(−w2·t)) :
> #função de tensão;
  sigma := t→100·(Heaviside(t − 3) + Heaviside(t − 30)) − 200·Heaviside(t − 60) : plot(%, 0
      ..100) :
> #função de fluência;
```

$$J := t \rightarrow \left(1 - \exp\left(-\frac{t}{15}\right)\right) + \frac{1}{2} :$$

> **#função de deformação sem aging (Uso das transformadas de Laplace para resolver a equação diferencial);**

$epsilons := \frac{1}{s}\left(\mathrm{laplace}\left(diff(\mathrm{sigma}(t), t)\cdot J(0) - int\left(diff(\mathrm{J}(\mathrm{t}-\mathrm{t1}), t1)\cdot diff(\mathrm{sigma}(t1), t1), t1 = 0..t\right), t, s\right)\right) : epsilont := (invlaplace(\%, s, t)) :$

> **#função de deformação afetada ao aging ;**

$epsilonsa := \frac{1}{s}\left(\mathrm{laplace}\left((v(t)\cdot diff(\mathrm{sigma}(t), t)\cdot J(0) - v(t)\cdot int(diff(\mathrm{J}(\mathrm{t}-\mathrm{t1}), t1)\cdot diff(\mathrm{sigma}(t1), t1), t1 = 0..t)), t, s\right)\right) : epsilonta := invlaplace(\%, s, t) :$

> **#função de deformação afetada ao aging dada por (Grasley & Lange (2007))**

$epsilons\_grasley := \left(B1\cdot\frac{(\mathrm{s}+\mathrm{w1})^2}{s}\cdot laplace(\mathrm{sigma}(t), t, (\mathrm{s}+\mathrm{w1}))\cdot laplace(\mathrm{J}(\mathrm{t}), \mathrm{t}, (\mathrm{s}+\mathrm{w1})) + B2\cdot\frac{(\mathrm{s}+\mathrm{w2})^2}{s}\cdot laplace(\mathrm{sigma}(t), t, (\mathrm{s}+\mathrm{w2}))\cdot laplace(\mathrm{J}(\mathrm{t}), \mathrm{t}, (\mathrm{s}+\mathrm{w2}))\right) :$

$epsilont\_grasley := (invlaplace(epsilons\_grasley, s, t)) :$

> **#Parcela Elástica do Aging;**

$epsilonsa\_elastic := \frac{\mathrm{laplace}(v(t)\cdot diff(\mathrm{sigma}(t), t)\cdot J(0), t, s)}{s} : epsilonsat\_elastic := invlaplace(\%, s, t) :$

> **#Parcela Viscosa do Aging;**

$epsilonsa\_visco := \frac{\mathrm{laplace}(-v(t)\cdot int(diff(\mathrm{J}(\mathrm{t}-\mathrm{t1}), t1)\cdot diff(\mathrm{sigma}(t1), t1), t1 = 0..t), t, s)}{s} :$

$epsilonsat\_visco := invlaplace(\%, s, t) :$

2. ANALISE NUMERICA - Tensões prescritas:

> **#Constantes para a definição da função J =** $\frac{\mathbf{1}}{E[0]} + \frac{\mathbf{1}}{E[1]}\left(1 - \exp\left(-\frac{t}{\mathrm{tau}[1]}\right)\right)$;

$E[0] := \frac{1}{0.5} : E[1] := 1 : \mathrm{tau}[1] := 15 :$

> **#primeiro incremento**

$n := 1 :$

#inumero de incremento

$j := 1000 :$

#inicializar vetores de armazenamento de variaveis

$Epsilone := Vector[column](j) : \mathrm{Epsilont} := Vector[column](j) : \mathrm{Epsilont\_noAging} := Vector[column](j) : Time := Vector[column](j) : Epsilonv := Vector[column](j) : Sigma := Vector[column](j) : V := Vector[column](j) :$

#***valor inicial de variàveis***

$h[1] := 0 : h\_noAging[n] := 0 : t[1] := 0 : \mathrm{stress}[1] := 0 : aged[n] := v(0) : \mathrm{epsilon}[e][n] := \frac{\mathrm{stress}[n]}{E[0]} : epsilon\_noAging[e][n] := \frac{\mathrm{stress}[n]}{E[0]} : \mathrm{epsilon}[t][n] := \frac{\mathrm{stress}[n]}{E[0]}\cdot age[n] + h[n]\cdot aged[n] : epsilon\_noAging[t][n] := \frac{\mathrm{stress}[n]}{E[0]} + h[n] :$

> **for *n* to j do**

#Incremento de tempo;

$time\_incre := 0.1 :$

$t[n+1] := t[n] + time\_incre :$

#Definir stress[n];
if $t[n] < 3$ **then**
  $\text{stress}[n] := 0$:
elif $t[n] \geq 3$ **and** $t[n] < 30$ **then**
  $\text{stress}[n] := 100$:
elif $t[n] \geq 30$ **and** $t[n] < 60$ **then**
  $\text{stress}[n] := 200$:
else
  $\text{stress}[n] := 0$:
fi:

#Definir stress[n + 1];
if $t[n+1] < 3$ **then** $stress[n+1] := 0$:
elif $t[n+1] \geq 3$ **and** $t[n+1] < 30$ **then**
  $\text{stress}[n+1] := 100$:
elif $t[n+1] \geq 30$ **and** $t[n+1] < 60$ **then**
  $\text{stress}[n+1] := 200$:
else
  $\text{stress}[n+1] := 0$:
fi:

#***Definir a função Aging em*** $\left(\boldsymbol{t} + \frac{\boldsymbol{inc}}{\mathbf{2}}\right)$ [*Lee* , 2007 *PhD dissertation*] :

$$aged[n] := \left(B1\cdot\exp\left(-w1\cdot\left(t[n] + \frac{time\_incre}{2}\right)\right) + B2\cdot\exp\left(-w2\cdot\left(t[n] + \frac{time\_incre}{2}\right)\right)\right) :$$

#Definir incrementos de deformação - Sem aging [*Taylor et al.* (1970)]**;**

$$\text{Delta}[y] := \frac{time\_incre}{\text{tau}[1]} : \text{lambda} := \frac{(1 - \exp(-\text{Delta}[y]))}{\text{Delta}[y]} :$$

$$dh[n] := (1 - \exp(-\text{Delta}[y])) + \left(\frac{(\text{stress}[n])}{\text{E}[1]} - h[n]\right)\cdot(1 - \exp(-\text{Delta}[y])) + \frac{(\text{stress}[n+1] - \text{stress}[n])}{E[1]}\cdot(1 - \text{lambda}) :$$

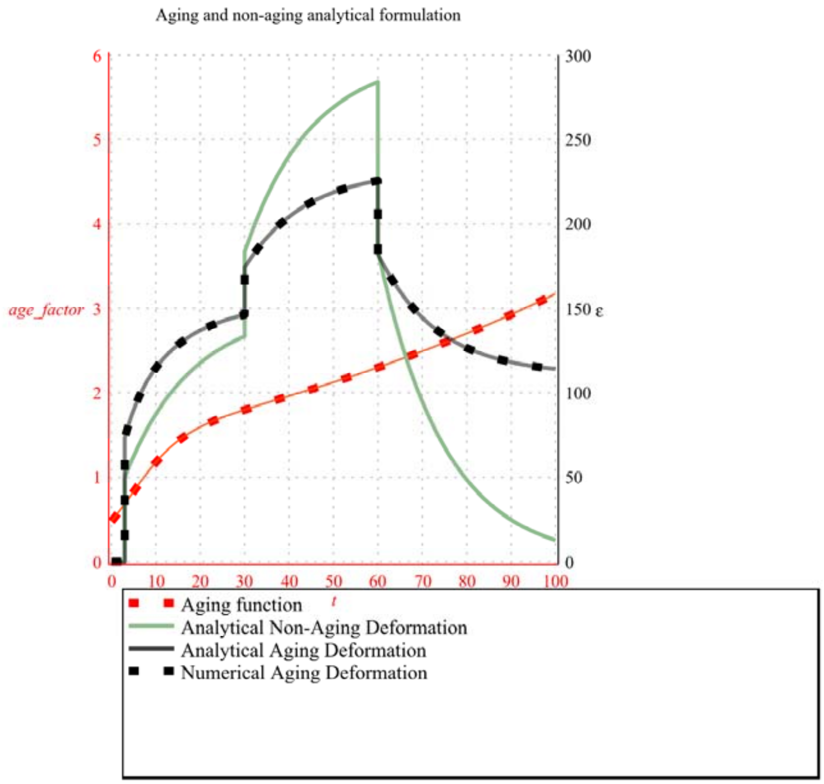
$$depsilon[e][n] := \frac{(\text{stress}[n+1] - \text{stress}[n])}{E[0]} :$$

$$dh\_noAging[n] := (1 - \exp(-\text{Delta}[y])) + \left(\frac{(\text{stress}[n])}{\text{E}[1]} - h\_noAging[n]\right)\cdot(1 - \exp(-\text{Delta}[y])) + \frac{(\text{stress}[n+1] - \text{stress}[n])}{E[1]}\cdot(1 - \text{lambda}) :$$

#***Definir incrementos de deformação passo n + 1 sem Aging*** (***aged[n]*** = 1)**;**
$h\_noAging[n+1] := h\_noAging[n] + dh\_noAging[n]\cdot 1$:
$epsilon\_noAging[e][n+1] := epsilon\_noAging[e][n] + depsilon[e][n]\cdot 1$:

#Definir incrementos de deformação passo n + 1 Com Aging [*Bazant*, 1977, 1988, 1989 *a*, 1989 *b*, 1993]**;**
$\#h[n+1] := h[n] + dh[n]\cdot aged[n]$: <u>*Errado!*</u>
$h[n+1] := h[n] + dh\_noAging[n]\cdot aged[n]$:

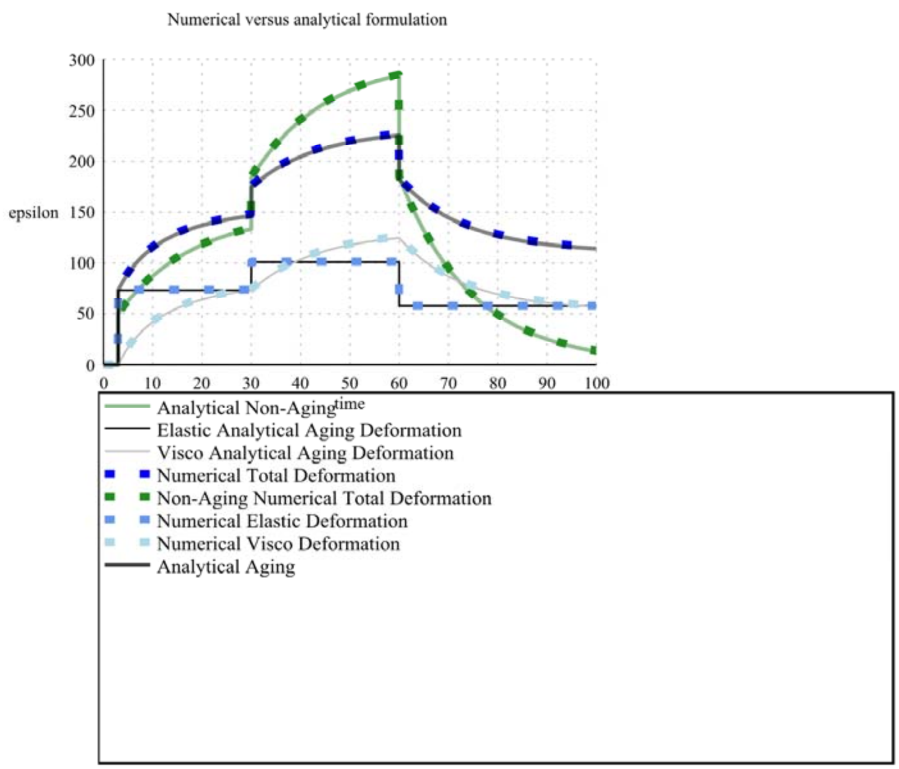
```
epsilon[e][n + 1] := epsilon[e][n] + depsilon[e][n]·aged[n] :

#Definir Aged and Non_Aged deformação total ;
 epsilon[t][n + 1] := epsilon[e][n + 1] + h[n + 1] :
 epsilon_noAging[t][n + 1] := epsilon_noAging[e][n + 1] + h_noAging[n + 1] :

#Armazenamento dos valores n em vetores;
 Epsilone(n, 1) := epsilon[e][n] : Epsilont(n, 1) := epsilon[t][n] : Sigma(n, 1)
      := (stress[n]) : Time(n, 1) := t[n] : Epsilonv(n, 1) := h[n] : V(n, 1) := 1/aged[n] :
      Epsilont_noAging(n, 1) := epsilon_noAging[t][n] :
 od:
> Analytical := dualaxisplot(plot(1/v(t), t = 0 ..100, age_factor = 0 ..6, linestyle = dot, color = red,
     thickness = 6, axis = [gridlines = [10, color = grey, linestyle = dot], color = "Red" ], legend
     = "Aging function"), plot( [epsilont, epsilonta, epsilont_grasley], t = 0 ..100, epsilon = 0
     ..300, axis = [gridlines = [10, color = grey, linestyle = dot]], color = ["DarkSeaGreen",
     black, black] , linestyle = [solid, solid, dot], thickness = [3, 3, 6], transparency = [0, 0.5, 0],
     legend = ["Analytical Non-Aging Deformation", "Analytical Aging Deformation",
     "Numerical Aging Deformation "]) ) :
 Numerica_aging_function := plot(Time, V , t = 0 ..100 , age_factor = 0 ..6 , color
     = "OrangeRed", transparency = 0.2, legend = "Aging function") :
display( {Analytical, Numerica_aging_function}, axis = [gridlines = [10, color = grey, linestyle
     = dot]], legendstyle = [location = bottom], title
     = "Aging and non-aging analytical formulation");
```



>

> *AnalyticalAging* := *plot*(epsilonta, t = 0 ..100 , epsilon = 0 ..300, color = black, linestyle = solid, thickness = 3, transparency = 0.5, legend = "Analytical Aging") :
AnalyticalNonAging := *plot*(epsilont, t = 0 ..100, epsilon = 0 ..300, color = "DarkSeaGreen", linestyle = solid, thickness = 3, legend = "Analytical Non-Aging") :
AnalyticalAging_elastic := *plot*(*epsilonsat_elastic*, t = 0 ..100 , epsilon = 0 ..300 , color = black, linestyle = solid, thickness = 1, legend = "Elastic Analytical Aging Deformation") :
AnalyticalAging_visco := *plot*(epsilonsat_visco, t = 0 ..100 , epsilon = 0 ..300 , color = "Silver", linestyle = solid, thickness = 1, legend = "Visco Analytical Aging Deformation") :
NumericalDeformation := *plot*(Time, Epsilont, t = 0 ..100 , epsilon = 0 ..300, color = blue, linestyle = dot, thickness = 6, legend = "Numerical Total Deformation") :

```
NumericalDeformation_noAging := plot(Time, Epsilont_noAging, t = 0 ..100 , epsilon = 0
    ..300, color = "ForestGreen", linestyle = dot, thickness = 6, legend
    = "Non−Aging Numerical Total Deformation") :
NumericalDeformation_elastic := plot(Time, Epsilone, t = 0 ..100 , epsilon = 0 ..300, color
    = "CornflowerBlue", linestyle = dot, thickness = 6, legend
    = "Numerical Elastic Deformation") :
NumericalDeformation_visco := plot(Time, Epsilonv, t = 0 ..100 , epsilon = 0 ..300, color
    = "LightBlue", linestyle = dot, thickness = 6, legend = "Numerical Visco Deformation") :
display( {AnalyticalAging, AnalyticalNonAging, AnalyticalAging_elastic, AnalyticalAging_visco,
   NumericalDeformation, NumericalDeformation_noAging, NumericalDeformation_elastic,
   NumericalDeformation_visco}, labels = ["time", "epsilon"], axis = [gridlines = [10, color
    = grey, linestyle = dot]], legendstyle = [location = bottom], title
    = "Numerical versus analytical formulation");
```



3. ANALISE NUMERICA - deformações prescritas:

```
> #primeiro incremento
  n := 1 :
```

#inumero de incremento

$j := 10000 :$

#inicializar vetores de armazenamento de variaveis

$\mathit{Epsilone\_1} := \mathit{Vector}[\mathit{column}](j) : \mathrm{Epsilont\_1} := \mathit{Vector}[\mathit{column}](j) : \mathrm{Epsilont\_noAging\_1} := \mathit{Vector}[\mathit{column}](j) : \mathit{Time\_1} := \mathit{Vector}[\mathit{column}](j) : \mathit{Epsilonv\_1} := \mathit{Vector}[\mathit{column}](j) : \mathit{Sigma\_1} := \mathit{Vector}[\mathit{column}](j) : \mathit{V\_1} := \mathit{Vector}[\mathit{column}](j) :$

#***valor inicial de variàveis***

$h[\mathrm{n}] := 0 : \mathit{h\_noAging}[n] := 0 : t[\mathrm{n}] := 0 : \mathrm{epsilon}[t][n] := 0 : \mathit{stress}[n] := E[0] \cdot (\mathrm{epsilon}[t][n] - h[\mathrm{n}]) : \mathit{stress}[n-1] := 0 :$

\> **for *n* to j do**

#Incremento de tempo;

$\mathit{time\_incre} := 0.01 :$

$t[n+1] := t[n] + \mathit{time\_incre} :$

#***Definir*** *incrementos de deformação****;***

if $t[n] < 3$ **then**

$\mathit{epsilon\_inc}[n] := 0 :$

elif $t[n] = 3$ **then**

$$\mathit{epsilon\_inc}[n] := 73.15738396 + 9.201511256\ 10^{-8} \cdot \left(1.677864283 \cdot 10^{9} - 3.8752 \cdot 10^{8} \cdot \mathrm{e}^{(-0.2587566667 \cdot t[n+1] + 0.2)} - 6.81176925 \cdot 10^{8} \cdot \mathrm{e}^{-0.07466666667\, t[n+1] + 0.2000000000}\right) - 9.201511256\ 10^{-8} \cdot \left(1.677864283 \cdot 10^{9} - 3.8752 \cdot 10^{8} \cdot \mathrm{e}^{-0.2587566667 \cdot t[n] + 0.2} - 6.81176925 \cdot 10^{8} \cdot \mathrm{e}^{-0.07466666667\, t[n] + 0.2000000000}\right)$$

elif $t[n] > 3$ **and** $t[n] < 30$ **then**

$$\mathit{epsilon\_inc}[n] := 9.201511256\ 10^{-8} \cdot \left(1.677864283 \cdot 10^{9} - 3.8752 \cdot 10^{8} \cdot \mathrm{e}^{(-0.2587566667 \cdot t[n+1] + 0.2)} - 6.81176925 \cdot 10^{8} \cdot \mathrm{e}^{-0.07466666667 \cdot t[n+1] + 0.2000000000}\right) - 9.201511256\ 10^{-8} \cdot \left(1.677864283 \cdot 10^{9} - 3.8752 \cdot 10^{8} \cdot \mathrm{e}^{-0.2587566667 \cdot t[n] + 0.2} - 6.81176925 \cdot 10^{8} \cdot \mathrm{e}^{-0.07466666667 \cdot t[n] + 0.2000000000}\right)$$

elif $t[n] = 30$ **then**

$$\mathit{epsilon\_inc}[n] := 27.82810688 + 9.201511256 \cdot 10^{-8} \cdot \left(8.394803172 \cdot 10^{8} - 3.8752 \cdot 10^{8}\, \mathrm{e}^{-0.2587566667 \cdot t[n+1] + 2} - 6.81176925 \cdot 10^{8} \cdot \mathrm{e}^{-0.07466666667 \cdot t[n+1] + 2}\right) + 9.201511256 \cdot 10^{-8} \cdot \left(1.677864283 \cdot 10^{9} - 3.8752 \cdot 10^{8} \cdot \mathrm{e}^{-0.2587566667 \cdot t[n+1] + 0.2} - 6.81176925 \cdot 10^{8} \cdot \mathrm{e}^{(-0.07466666667 \cdot t[n+1] + 0.2)}\right) - \left(9.201511256 \cdot 10^{-8} \cdot \left(8.394803172 \cdot 10^{8} - 3.8752 \cdot 10^{8}\, \mathrm{e}^{-0.2587566667 \cdot t[n] + 2} - 6.81176925 \cdot 10^{8} \cdot \mathrm{e}^{-0.07466666667 \cdot t[n] + 2}\right) + 9.201511256 \cdot 10^{-8} \cdot \left(1.677864283 \cdot 10^{9} - 3.8752 \cdot 10^{8} \cdot \mathrm{e}^{-0.2587566667 \cdot t[n] + 0.2} - 6.81176925 \cdot 10^{8} \cdot \mathrm{e}^{(-0.07466666667 \cdot t[n] + 0.2)}\right)\right) :$$

elif $t[n] > 30$ **and** $t[n] < 60$ **then**

$$\mathit{epsilon\_inc}[n] := 9.201511256 \cdot 10^{-8} \cdot \left(8.394803172 \cdot 10^{8} - 3.8752 \cdot 10^{8}\, \mathrm{e}^{-0.2587566667 \cdot t[n+1] + 2} - 6.81176925 \cdot 10^{8} \cdot \mathrm{e}^{-0.07466666667 \cdot t[n+1] + 2}\right) + 9.201511256 \cdot 10^{-8} \cdot \left(1.677864283 \cdot 10^{9} - 3.8752 \cdot 10^{8} \cdot \mathrm{e}^{-0.2587566667 \cdot t[n+1] + 0.2} - 6.81176925 \cdot 10^{8} \cdot \mathrm{e}^{(-0.07466666667 \cdot t[n+1] + 0.2)}\right) - \left(9.201511256 \cdot 10^{-8} \cdot \left(8.394803172 \cdot 10^{8} - 3.8752 \cdot 10^{8}\, \mathrm{e}^{-0.2587566667 \cdot t[n] + 2} - 6.81176925 \cdot 10^{8} \cdot \mathrm{e}^{-0.07466666667 \cdot t[n] + 2}\right) + 9.201511256 \cdot 10^{-8} \cdot \left(1.677864283 \cdot 10^{9} - 3.8752 \cdot 10^{8} \cdot \mathrm{e}^{-0.2587566667 \cdot t[n] + 0.2} - 6.81176925 \cdot 10^{8} \cdot \mathrm{e}^{(-0.07466666667 \cdot t[n] + 0.2)}\right)\right) :$$

elif $t[n] = 60$ **then**

$$\mathit{epsilon\_inc}[n] := 1.840302251 \cdot 10^{-7} \cdot \left(-6.575527646 \cdot 10^{8} + 3.8752 \cdot 10^{8}\right.$$

$\cdot\, e^{-0.2587566667 \cdot t[n+1] + 4.} + 6.81176925\ 10^{8}\, e^{-0.07466666667 \cdot t[n+1] + 4.}\big) + 9.201511256$
$\cdot 10^{-8} \cdot \big(8.394803172 \cdot 10^{8} - 3.8752 \cdot 10^{8}\, e^{-0.2587566667 \cdot t[n+1] + 2} - 6.81176925 \cdot 10^{8}$
$\cdot e^{-0.07466666667 \cdot t[n+1] + 2}\big) + 9.201511256 \cdot 10^{-8} \cdot \big(1.677864283 \cdot 10^{9} - 3.8752 \cdot 10^{8}$
$\cdot e^{-0.2587566667 \cdot t[n+1] + 0.2} - 6.81176925 \cdot 10^{8} \cdot e^{(-0.07466666667 \cdot t[n+1] + 0.2)}\big) - \big(1.840302251$
$\cdot 10^{-7} \cdot \big(-6.575527646 \cdot 10^{8} + 3.8752 \cdot 10^{8} \cdot e^{-0.2587566667 \cdot t[n] + 4.}$
$+ 6.81176925\ 10^{8}\, e^{-0.07466666667 \cdot t[n] + 4.}\big) + 9.201511256 \cdot 10^{-8} \cdot \big(8.394803172 \cdot 10^{8}$
$- 3.8752 \cdot 10^{8}\, e^{-0.2587566667 \cdot t[n] + 2} - 6.81176925 \cdot 10^{8} \cdot e^{-0.07466666667 \cdot t[n] + 2}\big)$
$+ 9.201511256 \cdot 10^{-8} \cdot \big(1.677864283 \cdot 10^{9} - 3.8752 \cdot 10^{8} \cdot e^{-0.2587566667 \cdot t[n] + 0.2}$
$- 6.81176925 \cdot 10^{8} \cdot e^{(-0.07466666667 \cdot t[n] + 0.2)}\big)\big) - 43.43996094$:

else

epsilon_inc[*n*] := $1.840302251 \cdot 10^{-7} \cdot \big(-6.575527646 \cdot 10^{8} + 3.8752 \cdot 10^{8}$
$\cdot\, e^{-0.2587566667 \cdot t[n+1] + 4.} + 6.81176925\ 10^{8}\, e^{-0.07466666667 \cdot t[n+1] + 4.}\big) + 9.201511256$
$\cdot 10^{-8} \cdot \big(8.394803172 \cdot 10^{8} - 3.8752 \cdot 10^{8}\, e^{-0.2587566667 \cdot t[n+1] + 2} - 6.81176925 \cdot 10^{8}$
$\cdot e^{-0.07466666667 \cdot t[n+1] + 2}\big) + 9.201511256 \cdot 10^{-8} \cdot \big(1.677864283 \cdot 10^{9} - 3.8752 \cdot 10^{8}$
$\cdot e^{-0.2587566667 \cdot t[n+1] + 0.2} - 6.81176925 \cdot 10^{8} \cdot e^{(-0.07466666667 \cdot t[n+1] + 0.2)}\big) - \big(1.840302251$
$\cdot 10^{-7} \cdot \big(-6.575527646 \cdot 10^{8} + 3.8752 \cdot 10^{8} \cdot e^{-0.2587566667 \cdot t[n] + 4.}$
$+ 6.81176925\ 10^{8}\, e^{-0.07466666667 \cdot t[n] + 4.}\big) + 9.201511256 \cdot 10^{-8} \cdot \big(8.394803172 \cdot 10^{8}$
$- 3.8752 \cdot 10^{8}\, e^{-0.2587566667 \cdot t[n] + 2} - 6.81176925 \cdot 10^{8} \cdot e^{-0.07466666667 \cdot t[n] + 2}\big)$
$+ 9.201511256 \cdot 10^{-8} \cdot \big(1.677864283 \cdot 10^{9} - 3.8752 \cdot 10^{8} \cdot e^{-0.2587566667 \cdot t[n] + 0.2}$
$- 6.81176925 \cdot 10^{8} \cdot e^{(-0.07466666667 \cdot t[n] + 0.2)}\big)\big)$:

fi:

epsilon[*t*][*n* + 1] := epsilon[*t*][*n*] + *epsilon_inc*[*n*];

#***Definir a função Aging em*** $\left(\boldsymbol{t} + \frac{\boldsymbol{inc}}{\mathbf{2}}\right)$ [*Lee* , 2007 *PhD dissertation*] :

$aged[n] := (B1 \cdot \exp(-w1 \cdot (t[n])) + B2 \cdot \exp(-w2 \cdot (t[n])))$:

#Definir incrementos de deformação - Sem aging [*Taylor et al.* (1970)]**;**

$\mathrm{Delta}[y] := \dfrac{time\_incre}{\mathrm{tau}[1]}$: $\mathrm{lambda} := \dfrac{(1 - \exp(-\mathrm{Delta}[y]))}{\mathrm{Delta}[y]}$:

$$dh\_noAging[n] := (1 - \exp(-\mathrm{Delta}[y])) + \left(\frac{(\mathrm{stress}[n])}{\mathrm{E}[1]} - h\_noAging[n]\right) \cdot (1 - \exp(-\mathrm{Delta}[y])) + \frac{(\mathrm{stress}[n] - \mathrm{stress}[n-1])}{E[1]} \cdot (1 - \mathrm{lambda}) :$$

#***Definir incrementos de deformação viscosa passo n + 1 sem Aging*** (***aged***[***n***] = 1)**;**

$h\_noAging[n+1] := h\_noAging[n] + dh\_noAging[n] \cdot 1$:

#Definir incrementos de deformação viscosa e tensão no passo n + 1 Com Aging [*Bazant*, 1977, 1988, 1989 *a*, 1989 *b*, 1993]**;**

$h[n+1] := h[n] + dh\_noAging[n] \cdot aged[n]$:

$$\mathrm{stress}[n+1] := \mathrm{stress}[n] + \frac{E[0]}{aged[n]} \cdot (epsilon\_inc[n] - dh\_noAging[n] \cdot aged[n]) :$$

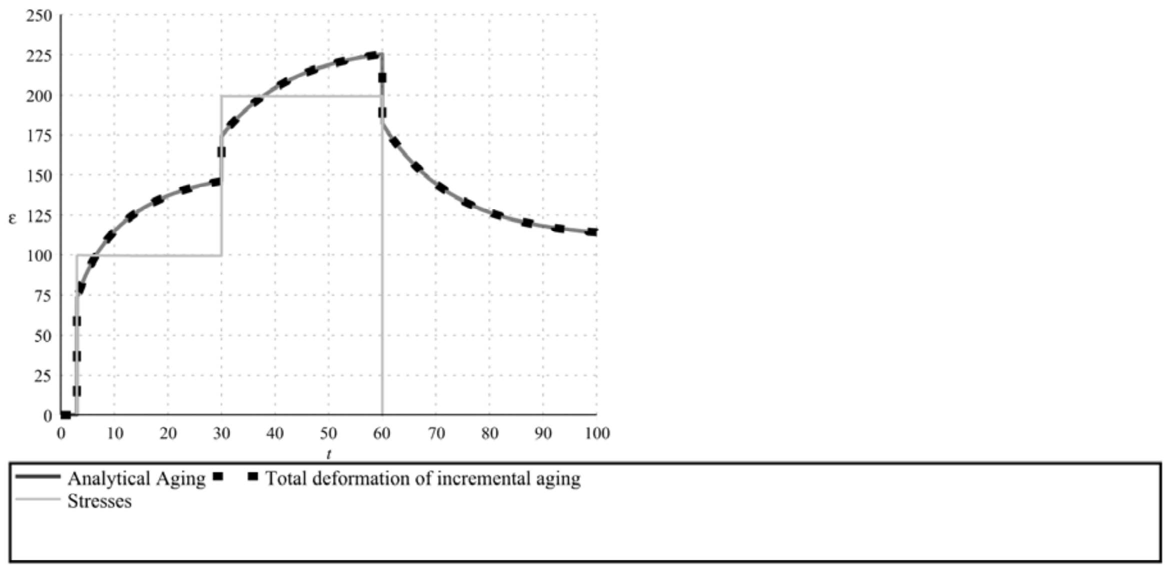
#Armazenamento dos valores n em vetores;

Epsilone_1(*n*, 1) := *epsilon_inc*[*n*] : Epsilont_1(n, 1) := epsilon[*t*][*n*] : *Sigma_1*(*n*, 1) := (stress[*n*]) : *Time_1*(*n*, 1) := t[*n*] : *Epsilonv_1*(*n*, 1) := *h*[*n*] : *V_1*(*n*, 1)

$:= \dfrac{1}{aged[n]}$: Epsilont_noAging_1$(n, 1)$ $:= epsilon\_noAging[t][n]$:

od:

```
> AnalyticalAging := plot(epsilonta, t = 0 ..100 , epsilon = 0 ..250, color = black, linestyle = solid,
      thickness = 3, transparency = 0.5, legend = "Analytical Aging") :
  IncNonAging := plot(Time_1, Epsilont_1, t = 0 ..100, epsilon = 0 ..250, color = black, linestyle
      = dot, thickness = 6, legend = "Total deformation of incremental aging") :
  StressNonAging := plot(Time_1, Sigma_1, t = 0 ..100, epsilon = 0 ..250, color = "Silver", linestyle
      = solid, thickness = 2, legend = "Stresses") :
  display(AnalyticalAging, IncNonAging, StressNonAging, axis = [gridlines = [10, color = grey,
      linestyle = dot]]);
```




```
> epsilont
```

$$-100\,\mathrm{Heaviside}(t-60)\left(3-2\,\mathrm{e}^{-\frac{1}{15}t+4}\right)+50\,\mathrm{Heaviside}(t-30)\left(3-2\,\mathrm{e}^{-\frac{1}{15}t+2}\right)+50\,\mathrm{Heaviside}(t-3)\left(3-2\,\mathrm{e}^{-\frac{1}{15}t+\frac{1}{5}}\right) \tag{1}$$

>