Mixed Integer Linear Programming for Portfolio Optimization Bay Area Decision Science Summit 2025

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ABSTRACT

Our team explored the application of a Mixed Integer Linear Programming (MILP) approach to optimize the portfolio of equity derivatives by minimizing the net costs spent on options over a given period. By utilizing MILP, our approach ensures a minimum exposure of +\$10,000,000 for a given scenario, while minimizing the total premium costs across a time frame of one month.

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1 NET COSTS APPROACH

1.1 Definitions

- The set of options $o \in O$.
- The set of trading dates $d \in D$.

1.2 Decision Variable

First, define the set of non-expired options for day d, given by:

$$O_d = \{o \in O \mid T_o \geq d+1\}$$

Hence, for each option $o \in O_d$, define:

 $x_{0,d} \in \mathbb{Z}^+ := \text{number of contracts to buy.}$

1.3 Parameters

For each option $o \in O$:

- a_0 : Ask Price
- A_o : Ask Size
- *M* : Contract Size
- k_o : Strike Price
- s_o : Underlying Price
- d_o : Trading Date
- To: Maturity Date
- $\beta_{S(o), d}$: Spot Move for option o of tick symbol S on day d.

1.3.1 Exposure Function (per Option)

We know that the next day underlying price is based on spot move. Hence, the exposure function, E, that takes in an option o and day d, is given by:

$$E(o, d) = \max(s_o \cdot \beta_{S(o), d} - k_o, 0) - \max(s_o - k_o, 0)$$

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1.3.2 Carried Exposure

We know that min exposure $\geq 10,000,000 \ \forall d \in D$. Hence, for a given day, d, we define carried exposure, C_d , as the total exposure from positions bought on earlier days d' < d that have not matured, given by:

$$C_d = M \sum_{\substack{o \in O_d \\ d' < d}} x_{o,d'} \cdot E(o, d')$$

1.4 Objective Function & Constraints

With this approach, we want to minimize total premium cost over all options. For each day d, this yields:

$$\boxed{\min_{o \in O_d} M(a_o \cdot x_{o, d})}$$

Subject to:

- (i) **Purchase Limits:** $x_{o, d} \le A_o \ \forall o \in O_d$
- (ii) **Expiry:** For any option o where $d_o \ge T_o$, set $x_{o,d} = 0$.
- (iii) Daily Minimum Exposure:

For each training date $d \in D$ such that $d + 1 \in D$, our **aggregate exposure** must meet the minimum:

$$C_d + M \sum_{o \in O_d} x_{o, d} \cdot E(o, d) \ge 10,000,000$$