Problems

1. Express each of the following statements by a sentence of first-order logic (making liberal use of quantifiers), using only the predicate symbols 'father', 'mother', 'parent' and 'child'

   Note: – these predicate symbols may be either 1-ary or 2-ary, depending upon the context; for instance, take 'parent' as being 2-ary in (a) and (c) but 1-ary in (b) and (d).

   (a) if X is a father or X is a mother then X is someone's parent

   (b) if X is a parent and X is female then X is someone's mother

   (c) if X is a father or X is a mother then someone is someone's parent

   (d) if everyone is a parent then someone is a child

2. Using just the predicates likes(X, Y) and X=Y to express, respectively, that "X likes Y" and "X is identical to Y", compose sentences of first-order logic to express each of the following:

   (a) "everyone likes no one"

   (b) "no one likes anyone"

   (c) "everyone likes themselves"

   (d) "everyone likes every person except themselves"

   (e) "there is exactly one person who is liked by everyone"

   (f) "no one likes anyone, except that Chris likes himself"

   (g) "everyone either dislikes themselves or is disliked by someone"

Answers

1. (a) $\forall X(father(X) \vee mother(X) => \exists Y parent(X,Y))$
   (b) $\forall X(parent(X) \wedge female(X) => \exists Y mother(X,Y))$
   (c) $\forall X(father(X) \vee mother(X) => \exists Y \exists X parent(Y,Z))$
   (d) $\forall X parent(X) => \exists Y child(Y)$

2. (a) $\forall X \forall Y\ likes(X,Y)$
   (b) $\forall X \forall Y\ likes(X,Y)$ $[same\ answer\ as\ a]$
   (c) $\forall X likes(X,X)$
   (d) $\forall X \forall Y(\ X = Y => likes(X,Y))$
   (e) $\exists Y \forall X(likes(X,Y) \wedge \forall Z(likes(X,Z) => Z = Y))$
   (f) $\forall X \forall Y((X = chris \wedge Y = chris) <=> likes(X,Y))$
   (g) $\forall X(\ likes(X,X) \vee \exists Y\ likes(Y,X))$

The above problems were taken from:

The Essentials of Logic Programming
C J Hogger,
OUP 1990.