PRIFYSGOL CYMRU / UNIVERSITY OF WALES

ABERYSTWYTH

DEPARTMENT OF MATHEMATICS

DEGREE EXAMINATIONS, SEMESTER 1, MAY/JUNE 1999

MA13310 - Mathematical Techniques for Computer Science

Time allowed - 2 hours

- Full marks will be given for complete answers to **all** questions in Section A **and** to **three** questions in Section B. In Section B credit will be given for the best **three** answers.
- *Calculators meeting the specifications of the Department of Mathematics (as found in the handbook) are permitted.*

SECTION A

1. Convert 94 (decimal) to (i) radix 9, (ii) hex, (iii) radix 3. (For full credit you must show all working.) Two of (i), (ii), (iii) above are simply related. Which two, and why?

2. (a) If A and B are both false, what are the truth values of $A \wedge B$, $A \vee B$, $A \to B$?

(b) Is "$A \to B$" equivalent to "$B \to A$"? If your answer is 'No', give an assignment of truth values to A, B which gives different truth values to the two formulas. If your answer is 'Yes', give the common truth table of the two formulas.

3. Determine (in terms of $A, B, \wedge, \vee, \neg$) the outputs C, D of

(For full credit you should give a labelled circuit showing the output of each gate.)

4. Suppose the domain of discussion is books and $N(x)$ denotes "x is a novel", $P(x)$ denotes "x is a play" and $D(x, y)$ denotes "x and y are different".

(a) Translate into (reasonably natural) English

$$(P(x) \wedge N(y)) \to D(x, y).$$

(b) Translate into the predicate calculus: "Some plays are novels, but no novels are plays."

5. If $\mathcal{E} = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 2, 4, 8, 9\}$, $B = \{2, 4, 6, 8, 10\}$, list the elements of the sets $A \cup B$, $A \cap B$, A', B' and $A' \cap B$.

6. Sketch the graph of the function

$$f(x) = x^2 - 2x - 3$$

for $-1 \leq x \leq 4$.

7. The time taken to solve a system of n linear equations is proportional to n^3. If it takes 30 seconds to solve a system of 10 equations, how long does it take to solve a system of 100 equations?

8. Write down an expression for the general term T_n of the arithmetic progression

$$-5 - 7/2 - 2 - 1/2 + 1 + \cdots$$

Without generating the sequence write down T_{23} and T_{51}.

SECTION B

9. In this question, full details of your working must be shown for full credit.

(a) Convert the binary number 10.100111001 to decimal. (You should choose an appropriate number of decimal places, and explain your choice.)

(b) Working with eight bit two's complement representation of integers in the range -128 to 127

(i) find the representations of 9 and -7;

(ii) calculate $9 + (-7)$ in two's complement representation and convert the result to decimal;

(iii) calculate the product of 9 and -7 in two's complement and convert the result to decimal;

(iv) find the representation of -16, and identify those numbers (other than zero) which give a result of 0 when multiplied by -16 in two's complement representation. Justify your answer.

10. (a) Calculate a truth table for the formula

$$((p \to q) \wedge (\neg\, r \to q)) \to (p \vee \neg\, r).$$

Say (give reasons) whether it is (i) a tautology or not, (ii) a contradiction or not, (iii) satisfiable or not.

(b) Explaining your notation, translate the following paragraph as an argument in the propositional calculus. Use a truth table to determine whether or not the argument is valid.

Andrew will only come to the party if Bella and Christine are both coming. Bella refuses to come if Andrew and Christine are both coming. So, Bella won't come.

11. (a) If A is a formula involving p, q, r with truth table

p	q	r	A
T	T	T	F
T	T	F	T
T	F	T	T
T	F	F	T
F	T	T	F
F	T	F	T
F	F	T	T
F	F	F	F

express A in DNF and in CNF.

(b) Design a digital logic circuit with p, q, r as inputs and A as output. Use only "and", "or", "not" gates. More credit will be given for a circuit which uses fewer gates.

(c) Explain what is meant by the assertion that "not" and "or" gates are a sufficient set of types of gate. Justify the assertion. What sacrifices might a decision to use only these two types involve?

12. (a) Define the Cartesian product $A \times B$ of two sets A and B. Four elements of the Cartesian product of two sets are $(1, 9), (2, 4), (3, 8)$ and $(3, 1)$. If there are twelve such ordered pairs in $A \times B$, find

(i) the sets A and B;

(ii) the other eight elements of $A \times B$,

(iii) the subset C of $A \times B$ defined by

$$C = \left\{(a, b) : a \in A,\, b \in B \text{ and } a^2 = b\right\}.$$

(b) Let A be a given set and let R be a relation in $A \times A$. Under what conditions is R an equivalence relation in $A \times A$? Determine whether or not the relation "is less than" is an equivalence relation in $A \times A$ when A is the set of real numbers.

(c) A survey in a group of 35 people shows that 21 own a television and 25 own a radio. How many people own both a television and a radio if everyone owns a television or radio?

13. (a) Which of the following time complexity functions belong to the same complexity class?

$$5n^4, \quad n^2 - 1, \quad n + 3\ln n, \quad 2n^4 + 3, \quad 7n + 2, \quad 4^n, \quad n^3, \quad 5\ln n.$$

Hence, place all the above functions into time complexity classes and order those classes with respect to increasing rates of growth.

(b) The function f is given by $f(x) = ax + b$, where a and b are constants. If $f(2) = 1$ and $f(5) = 2$, find a and b. If $g(x) = 9x^2$ find expressions for the composite functions $f\,o\,g$ and $g\,o\,f$. Does the function f have an inverse? If it does, explain why and find it.

(c) Write down the first term and common difference for the arithmetic progression

$$3 + 7 + 11 + 15 + 19 + \cdots$$

Determine the 63rd term and the sum of the first 15 terms.

(d) In a geometric progression the eighth term equals 12 and the tenth term equals 27; find the possible values of the common ratio.