Example 1 (Propositional Logic)

The Problem

Axioms:	$P \vee \neg P$	(1)
	$\neg\neg P \Rightarrow P$	(2)
	$[P \wedge Q] \Rightarrow P$	(3)
	$[P \wedge Q] \Rightarrow Q$	(4)

Inference Rules:	Modus Ponens	R_1
	Substitution	R_2

$S = \{$(Pat is a student) $\wedge$ (Pat is a girl),(Pat is a girl) $\Rightarrow$ (Pat is human)$\}$

Infer: (Pat is human)

I.	By R_2 and S_1 on (4)	(Pat is a student) $\wedge$ (Pat is a girl) $\Rightarrow$ (Pat is a girl)
II.	Using S_1, M.P. and I	(Pat is a girl)
III.	Using S_2, M.P. and II	(Pat is human)

Example 2 (Predicate Logic)

The Problem

Given:

1. $\forall X \forall Y (horse(X) \wedge dog(Y) => faster(X, Y))$
2. $\exists Y (greyhound(Y) \wedge (\forall Z rabbit(Z) => faster(Y, Z)))$
3. $\forall Y (greyhound(Y) => dog(Y))$
4. $\forall X \forall Y \forall Z ((faster(X, Y) \wedge faster(Y, Z)) => faster(X, Z))$
   (The transitivity of the '*faster*' relation)
5. horse(harry)
6. rabbit(ralph)

Using: Existential Instantiation (EI)
Universal Instantiation (UI)
Modus Ponens (MP)
And Introduction (AI)
And Elimination (AE)

Prove: faster(harry, ralph)

The Solution

1.	$\forall X \forall Y (horse(X) \wedge dog(Y) => faster(X, Y))$	Δ
2.	$\exists Y (greyhound(Y) \wedge (\forall Z rabbit(Z) => faster(Y, Z)))$	Δ
3.	$\forall Y (greyhound(Y) => dog(Y))$	Δ
4.	$\forall X \forall Y \forall Z ((faster(X, Y) \wedge faster(Y, Z)) => faster(X, Z))$	Δ
5.	$horse(harry)$	Δ
6.	$rabbit(ralph)$	Δ
7.	$greyhound(greg) \wedge (\forall Z rabbit(Z) => faster(greg, Z))$	2, EI
8.	$greyhound(greg)$	7, AE
9.	$\forall Z (rabbit(Z) => faster(greg, Z))$	7, AE
10.	$rabbit(ralph) => faster(greg, ralph)$	9, UI
11.	$faster(greg, ralph)$	10, 6, MP
12.	$greyhound(greg) => dog(greg)$	3, UI
13.	$dog(greg)$	12, 8, MP
14.	$horse(harry) \wedge dog(greg) => faster(harry, greg)$	1, UI
15.	$horse(harry) \wedge dog(greg)$	5, 13, AI
16.	$faster(harry, greg)$	14, 15, MP
17.	$faster(harry, greg) \wedge faster(greg, ralph) => faster(harry, ralph)$	4, UI
18.	$faster(harry, greg) \wedge faster(greg, ralph)$	16, 11, AI
19.	**faster(harry, ralph)**	17, 18, MP

QED

In the above solution the symbol 'Δ' is used to represent an axiom.