Uncertainty

which in this case means probability!

(Russell & Norvig ch: 18)

- The "whole truth" is a rare commodity!
  - acting under Uncertainty
  - Qualification problem
    - Rational decision?
- Handling uncertain knowledge
  - FOPL fails in complex domains because:
    - Laziness
    - Theoretical ignorance
    - Practical ignorance
  - Probability
    - degree of belief (0 .... 1)
    - summarizes uncertainty from laziness & ignorance
  - Evidence
    - information received up to now
    - **prior (unconditional) probability**: before evidence received
    - **posterior (conditional) probability:** after evidence received

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Basic Probability Notation

- Prior (Unconditional) Probability
  - *P(A)*: prior probability that A is true
    - assigned in the absence of other info (only)
  - Random Variables (RVs)
    - Domain: values random var. can take
    - Probability Distribution: vector or probabilities of each value of RV
      - **P**(Weather) for example
- Conditional (Posterior) Probability
  - *P(A|B)* - probability of A given that *all we know* is B.
  - **P**(X|Y) - two dimensional table
    - P(X=xi|Y=yj) for each i and j
  - Product Rule
    - P(A^B) = P(A|B)P(B) *or vice versa*
    - **P**(X,Y) = **P**(X|Y)P(Y)

- Uncertainty and rational decisions
  - preferences
  - Utility Theory
    - utility: "the quality of being usefull"
  - Decision Theory

  Decision theory = probability theory + utility theory
  - Maximum Expected Utility (MEU)
    - an agent is rational iff it chooses the action that yields the highest expected utility, averaged over all possible outcomes of the action.

The Axioms of Probability

1. All probabilities are between 0 and 1

   $0 =< P(A) >= 1$

2. $P(\text{True}) = 1$, $P(\text{False}) = 0$

3. $P(A \lor B) = P(A) + P(B) - P(A \land B)$

- Joint Probability Distribution
  - completely specifies an agent's probability assignments to all propositions in the domain
  - Atomic Event
    - an assignment of particular values to all RVs in a domain
  - $\mathbf{P}(X_1, \ldots, X_n)$ assigns probabilities to all possible atomic events.

Bayes Rule and its Use

$P(\text{B}|\text{A}) = P(\text{A}|\text{B})P(\text{B})/P(\text{A})$

- can also be expressed as a vector.

- Combining Evidence
  - extend the Bayes rule for each new variable?
    - exponential in each new piece of evidence.
  - Bayesian Updating
    - incorporates evidence one piece at a time.
  - Conditional Independence
    - assumption to simplify the updating procedure
    - direct cause

- Normalisation
  - relative likelihood of one cause against another given some evidence, E
  - normalising constant: 1/E
    - allows probabilities to sum to 1.

Where do Probabilities come from?

- Three approaches
  - Frequentist
    - from experiments
  - Objectivist
    - real aspects of the universe
  - Subjectivist
    - agents beliefs without external significance
- Reference Class Problem
  - how many things do you need to consider in assessing the probability of a situation applied to a class of people, or objects