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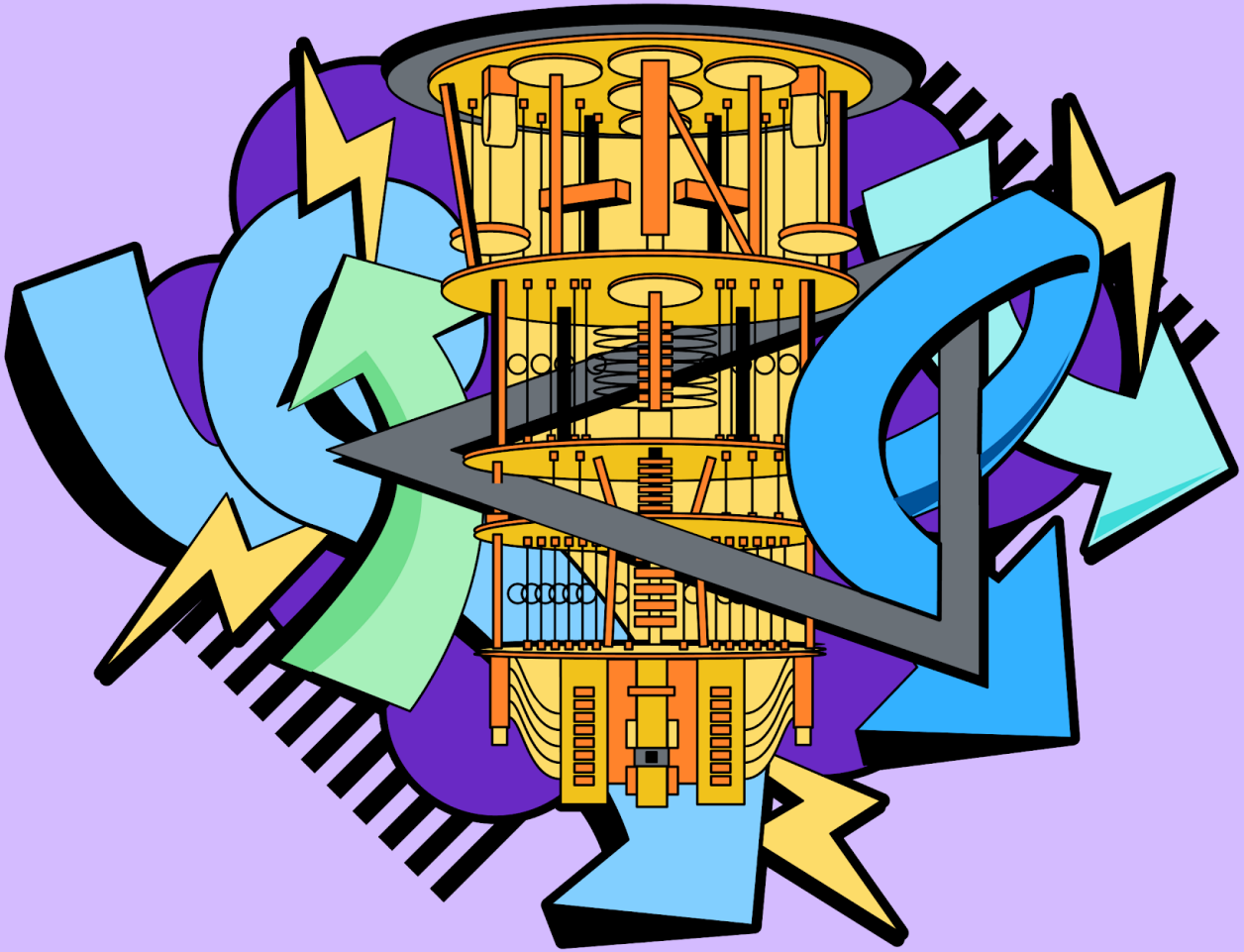

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A basic introduction to quantum computing

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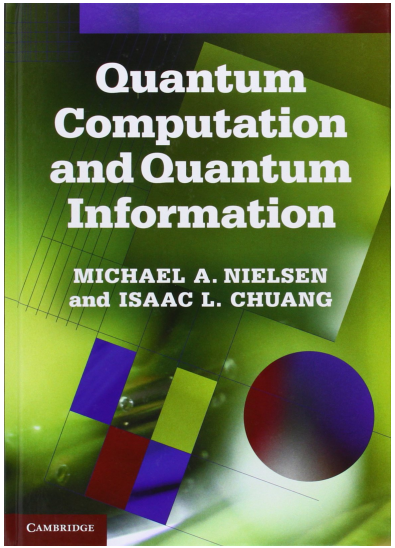
To Learn More Advanced Stuffs



Quantum Computation and Quantum Information
Nielsen and Chuang
(Available online)

Qiskit textbook
IBM Quantum Experience
(https://qiskit.org/learn/)

Introduction

Quantum information

Quantum computing

Challenges and scope of QC

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AGENDA

Introduction

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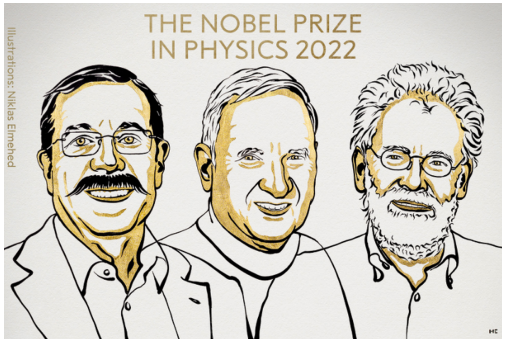
Quantum information science is a modern branch of physics that aims to study of how information can be used, processed, and transmitted using the principles of quantum mechanics.



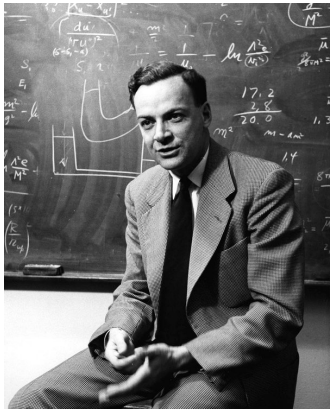
John Clauser, Alain Aspect, and Anton Zeilinger

For the groundbreaking experiments using entangled quantum states

Quantum computing is an area of study focused on the development of computer based technologies centered around the principles of quantum theory.

Moore's law (1965): the number of transistors in a dense integrated circuit (IC) doubles about every two years.

"Nature isn't classical, dammit, and if you want to make a simulation of nature, you'd better make it quantum mechanical, and by golly it's a wonderful problem, because it doesn't look so easy." R. Feynman



Richard Feynman

Quantum information

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WHAT IS QUANTUM INFORMATION?

Quantum information is the type of information a quantum system carries from the preparation device to the measuring apparatus in a quantum mechanical experiment.

✳ It takes care of manipulating information in a noisy environment!

WHERE QUANTUM INFORMATION IS ENCODED?

Quantum information can be encoded in a variety of particle physical properties:

Spin, Polarization, Charge, Momentum, ... etc.

POSTULATES OF QUANTUM MECHANICS

Quantum mechanics is based on these postulates:

State space

The system is completely described by a state vector belonging to its state space.

$$\psi \in \mathscr{H}$$

Correspondence principle

To every physical quantity in classical mechanics there corresponds a linear, Hermitian operator in quantum mechanics.

Position: $r \to \hat{r}$, momentum: $p \to \hat{p} = -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$

Measurement

Quantum measurements are described by a collection $\{M_n\}$ of measurement operators.

$$M_n \ \psi = e_n \ \psi$$

Evolution

The evolution of a closed quantum system is described by a unitary transformation.

$$\psi'(t_2) = U(t_1, t_2) \ \psi(t_1)$$

$$p(n) = \psi^* M_n^\dagger M_n \psi$$

$$\sum_n M_n^\dagger M_n = I$$

$$i\hbar \frac{d|\psi\rangle}{dt} = H|\psi\rangle.$$

Expectation value

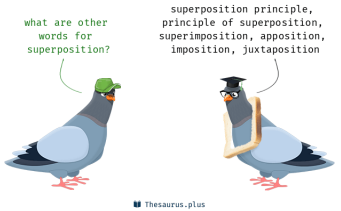
If the system is in the state ψ (normalized), then the average value of the observable corresponding to O is given by

$$\langle \hat{O} \rangle = \int_{-\infty}^{+\infty} \psi^\dagger \hat{O} \psi \ d\tau$$

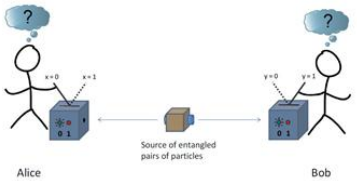
QUANTUM RESOURCES

New quantum-based technologies are possible thanks to:

SUPERPOSITION: Quantum objects can be in two states at once!!

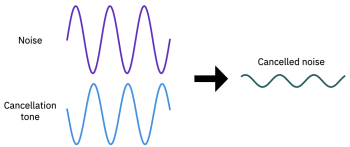


ENTANGLEMENT: The state of a quantum object depends on the state of the other!



INTERFERENCE: Two objects can:

- Interact with each other,
- Cancel out each other,
- Amplify one another.



QUBIT AND DIRAC NOTATION

What is a **qubit?**

A qubit is the simplest non-trivial quantum system, it refers to any quantum system whose possible states belongs to a two-dimensional Hilbert space.

Examples: electron spin, photon polarization, quantum dots..... etc.

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \qquad \alpha, \beta \in \mathbb{C}$$

$$p(|0\rangle) = |\alpha|^2$$

$$p(|1\rangle) = |\beta|^2$$

$$p(|0\rangle) + p(|1\rangle) = 1$$

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$$

$$p(|0\rangle) = |\frac{1}{\sqrt{2}}|^2 = \frac{1}{2} \qquad p(|1\rangle) = |\frac{1}{\sqrt{2}}|^2 = \frac{1}{2}$$

ONE-QUBIT SYSTEM: ONE DIRHAM

$|0\rangle \equiv |\text{Head}\rangle$

$|1\rangle \equiv |\text{Tail}\rangle$

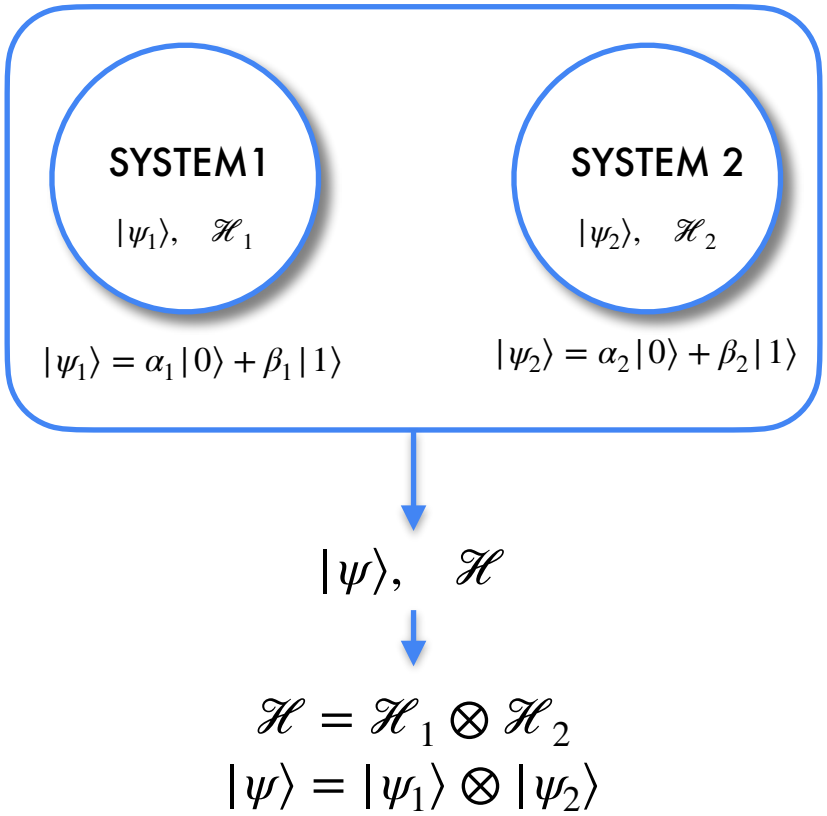
$$\equiv \frac{1}{\sqrt{2}}\left|\;\right\rangle + \frac{1}{\sqrt{2}}\left|\;\right\rangle$$

$$= \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Quantum computer uses qubits as computational units to perform tasks!

TWO-QUBIT SYSTEM



$$|\psi\rangle = (\alpha_1|0\rangle + \beta_1|1\rangle) \otimes (\alpha_2|0\rangle + \beta_2|1\rangle)$$
$$= \alpha_1\alpha_2|0\rangle \otimes |0\rangle + \alpha_1\beta_2|0\rangle \otimes |1\rangle$$
$$+\beta_1\alpha_2|1\rangle \otimes |0\rangle + \beta_1\beta_2|1\rangle \otimes |1\rangle$$

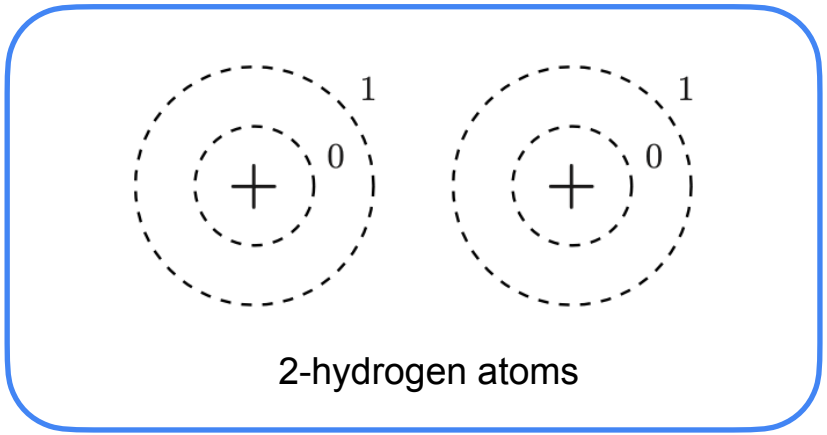
$$a_{00} = \alpha_1\alpha_2, \quad a_{01} = \alpha_1\beta_2, \quad a_{10} = \beta_1\alpha_2, \quad a_{11} = \beta_1\beta_2$$

$$|\psi\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$$

$$\langle\psi|\psi\rangle = 1$$

$$|a_{00}|^2 + |a_{01}|^2 + |a_{10}|^2 + |a_{11}|^2 = 1$$

TWO-QUBIT SYSTEM: EXAMPLE



2-hydrogen atoms

Since each electron can be in either of the ground or excited state, classically the two electrons are in one of four states – 00, 01, 10, or 11 – and represent 2 bits of classical information. By the superposition principle, the quantum state of the two electrons can be any linear combination of these four classical states.

$$|\psi\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$$

Measuring the state above will reveal two bits of information.

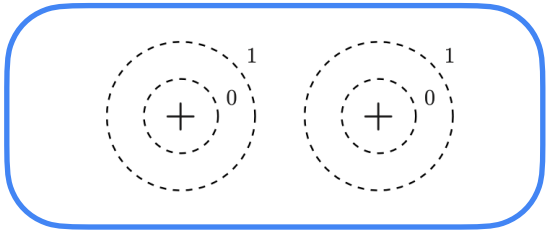
$$|00\rangle \rightarrow p_{00} = |a_{00}|^2$$

$$|01\rangle \rightarrow p_{01} = |a_{01}|^2$$

$$|10\rangle \rightarrow p_{10} = |a_{10}|^2$$

$$|11\rangle \rightarrow p_{11} = |a_{11}|^2$$

QUANTUM ENTANGLEMENT



ground state

$$|0\rangle \equiv |g\rangle$$

excited state

$$|1\rangle \equiv |e\rangle$$

Assume the first electron is in the state

$$|\psi_1\rangle = \frac{3}{5}|0\rangle + \frac{4}{5}|1\rangle$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle$$

The joint state of the two qubits

$$|\psi\rangle = |\psi_1\rangle \otimes |\psi_2\rangle$$

$$|\psi\rangle = \frac{3}{5\sqrt{2}}|00\rangle - \frac{3}{5\sqrt{2}}|01\rangle + \frac{4}{5\sqrt{2}}|10\rangle - \frac{4}{5\sqrt{2}}|11\rangle$$

But wait! Can we find states as

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

Yes! Such a state is called an entangled state.

$$|\Phi^+\rangle \neq |S_1\rangle \otimes |S_2\rangle$$

Einstein, Podolsky and Rosen introduced the concept of entanglement in 1935.

What does that mean?

BELL STATES AND NO-CLONING THEOREM

BELL STATES the most maximally two qubit entangled states!

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle$$

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}|00\rangle - \frac{1}{\sqrt{2}}|11\rangle$$

$$|\Psi^+\rangle = \frac{1}{\sqrt{2}}|01\rangle + \frac{1}{\sqrt{2}}|10\rangle$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}}|01\rangle - \frac{1}{\sqrt{2}}|10\rangle$$

NO-CLONING THEOREM

No existing operation U, which exactly duplicate an arbitrary quantum state, $|\psi\rangle$.

$$U|\psi\rangle|0\rangle \not\rightarrow |\psi\rangle|\psi\rangle$$

PROOF:

$$\begin{aligned} U|\psi\rangle|0\rangle &= U(\alpha|0\rangle + \beta|1\rangle)|0\rangle \\ &= U(\alpha|0\rangle|0\rangle + \beta|1\rangle|0\rangle) \\ &= \alpha|0\rangle\alpha|0\rangle + \beta|1\rangle\beta|1\rangle \\ &= \alpha^2|00\rangle + \beta^2|11\rangle \end{aligned}$$

Apply clone operation first then distribute

$$|\psi\rangle|\psi\rangle = \alpha^2|00\rangle + \alpha\beta|01\rangle + \alpha\beta|10\rangle + \beta^2|11\rangle$$

The derivations are different!

BINARY REPRESENTATION

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- Despite that fact that base-10 is commonly used in our daily counting, there no single reason to prefer one base over the others.

$$12 = (2 \times 10^0) + (1 \times 10^1) = 2 + 10.$$

- Base-2 is the most important base for computation.

$$101_2 = (1 \times 2^0) + (0 \times 2^1) + (1 \times 2^2) = 5.$$

- Base-2 has only two possible digits, zero and one. The digits are called bits (binary digits).

➡ Any number can be described using a combination of bits.
➡ All classical computer operations are done by manipulating bits.

LOGIC GATES

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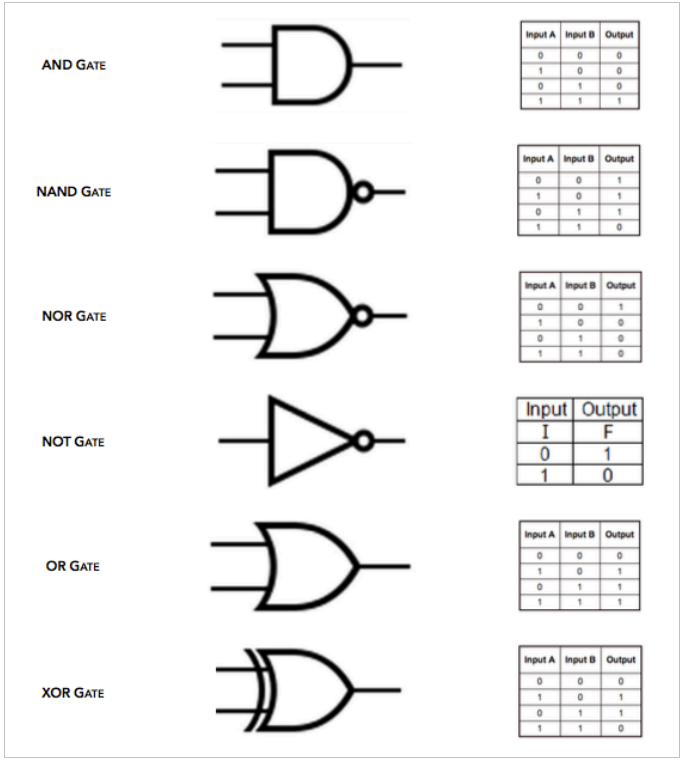

Input A	Input B	Output
0	0	0
1	0	0
0	1	0
1	1	1

Input A	Input B	Output
0	0	1
1	0	1
0	1	1
1	1	0

Input A	Input B	Output
0	0	1
1	0	0
0	1	0
1	1	0

Input	Output
I	F
0	1
1	0

Input A	Input B	Output
0	0	0
1	0	1
0	1	1
1	1	1

Input A	Input B	Output
0	0	0
1	0	1
0	1	1
1	1	0

A logic gate = operation that map input bit(s) to output bit(s).

UNIVERSALITY

Any computation can be done by combining of:

{NOT, AND, OR, FANOUT}

REVERSIBILITY

Given an output, we can know the input.

- *Reversible gate: all the information is preserved*
- *Non-reversible gate: some information is lost.*

Is NOT reversible ? **YES** | Is AND reversible ? NO

Why are we learning about classical computing?

QUANTUM COMPUTATION

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Quantum computer uses qubits as computational units to perform tasks!

Super$b^{\dagger}$: a qubit can be $|0\rangle$ and $|1\rangle$ at the same time!

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

Every computation consists of three components:

data = qubits

operations = quantum gates

results = measurements

Similarly to the classical case. A quantum computer manipulates quantum information using operations known as quantum gates!

- Single qubit gates

- Two qubit gates

SINGLE QUBIT GATES

Qiskit

Hadamard	H	$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}$
Pauli-X	X	$\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}$
Pauli-Y	Y	$\begin{bmatrix}0 & -i\\ i & 0\end{bmatrix}$
Pauli-Z	Z	$\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}$
Phase	S	$\begin{bmatrix}1 & 0\\ 0 & i\end{bmatrix}$
$\pi/8$	T	$\begin{bmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{bmatrix}$

$$\dagger : H \to H^*$$
$$|\psi\rangle \to \langle\psi|$$

The X, Y, Z, H gates are Hermitian, traceless, involutory!

Unitarity condition

$$O^\dagger O = I$$

$$O^\dagger = O^{-1}$$

$$U_\theta = \begin{pmatrix}1 & 0\\ 0 & e^{i2\theta}\end{pmatrix}$$

Qiskit implementation: https://qiskit.org/documentation/tutorials/circuits/3_summary_of_quantum_operations.html

SINGLE QUBIT GATES

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X GATE
(bit-flip)

$$X|0\rangle = |1\rangle$$
$$X|1\rangle = |0\rangle$$

$$X|\psi\rangle = \alpha X|0\rangle + \beta X|1\rangle$$
$$= \alpha|1\rangle + \beta|0\rangle$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix}$$

Y GATE
(bit-phase-flip)

$$Y|0\rangle = i|1\rangle$$
$$Y|1\rangle = -i|0\rangle$$

$$Y|\psi\rangle = \alpha Y|0\rangle + \beta Y|1\rangle$$
$$= -\alpha i|1\rangle + \beta i|0\rangle$$

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = i \begin{pmatrix} \beta \\ -\alpha \end{pmatrix}$$

Z GATE
(phase-flip)

$$Z|0\rangle = +|0\rangle$$
$$Z|1\rangle = -|1\rangle$$

$$Z|\psi\rangle = \alpha Z|0\rangle + \beta Z|1\rangle$$
$$= \alpha|0\rangle - \beta|1\rangle$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha \\ -\beta \end{pmatrix}$$

SINGLE QUBIT GATES

Qiskit

HADAMARD GATE

$$H|0\rangle = \frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle = |+\rangle$$

$$H|1\rangle = \frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle = |-\rangle$$

$$H|\psi\rangle = \alpha H|0\rangle + \beta H|1\rangle$$
$$= \alpha|+\rangle + \beta|-\rangle$$

$$\frac{1}{\sqrt{2}}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{pmatrix}\alpha+\beta\\ \alpha-\beta\end{pmatrix}$$

PHASE GATE

$$S|0\rangle = 1|0\rangle$$

$$S|1\rangle = i|1\rangle$$

$$S|\psi\rangle = \alpha S|0\rangle + \beta S|1\rangle$$
$$= \alpha|0\rangle + \beta i|1\rangle$$

$$\begin{pmatrix}1 & 0\\ 0 & i\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{pmatrix}\alpha\\ i\beta\end{pmatrix}$$

$\frac{\pi}{8}$ GATE

$$T|0\rangle = +|0\rangle$$

$$T|1\rangle = e^{i\pi/4}|1\rangle$$

$$T|\psi\rangle = \alpha T|0\rangle + \beta T|1\rangle$$
$$= \alpha|0\rangle + \beta e^{i\pi/4}|1\rangle$$

$$\begin{pmatrix}1 & 0\\ 0 & e^{i\pi/4}\end{pmatrix}\begin{pmatrix}\alpha\\ \beta\end{pmatrix} = \begin{pmatrix}\alpha\\ e^{i\pi/4}\beta\end{pmatrix}$$

TWO QUBIT GATES

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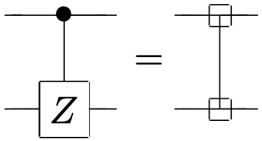
controlled-NOT

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

swap

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

controlled-Z



$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

These gates require two input qubits

$$O\,|\text{input}(1), \text{input}(2)\rangle = |\text{output}\rangle$$

Qiskit implementation: https://qiskit.org/documentation/tutorials/circuits/3_summary_of_quantum_operations.html

TWO QUBIT GATES

Qiskit

$$CNOT\,|c,t\rangle = |c, t \oplus c\rangle$$

$$CNOT\,|00\rangle = |00\rangle$$
$$CNOT\,|01\rangle = |01\rangle$$
$$CNOT\,|10\rangle = |11\rangle$$
$$CNOT\,|11\rangle = |10\rangle$$

Before		After	
Control	Target	Control	Target
$\|0\rangle$	$\|0\rangle$	$\|0\rangle$	$\|0\rangle$
$\|0\rangle$	$\|1\rangle$	$\|0\rangle$	$\|1\rangle$
$\|1\rangle$	$\|0\rangle$	$\|1\rangle$	$\|1\rangle$
$\|1\rangle$	$\|1\rangle$	$\|1\rangle$	$\|0\rangle$

$$|\psi\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} a_{00} \\ a_{01} \\ a_{10} \\ a_{11} \end{pmatrix} = \begin{pmatrix} a_{00} \\ a_{01} \\ a_{11} \\ a_{10} \end{pmatrix}$$

$$|\psi'\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{11}|10\rangle + a_{10}|11\rangle$$

TWO QUBIT GATES

Qiskit

The SWAP gate exchanges the two qubits. It transforms the basis vectors as

$$SWAP|00\rangle \rightarrow |00\rangle$$
$$SWAP|01\rangle \rightarrow |10\rangle$$
$$SWAP|10\rangle \rightarrow |01\rangle$$
$$SWAP|11\rangle \rightarrow |11\rangle$$

$$\text{SWAP} = \frac{I \otimes I + X \otimes X + Y \otimes Y + Z \otimes Z}{2}$$

$$|\psi\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a_{00} \\ a_{01} \\ a_{10} \\ a_{11} \end{pmatrix} = \begin{pmatrix} a_{00} \\ a_{10} \\ a_{01} \\ a_{11} \end{pmatrix}$$

$$|\psi'\rangle = a_{00}|00\rangle + a_{10}|01\rangle + a_{01}|10\rangle + a_{11}|11\rangle$$

TWO QUBIT GATES

Qiskit

The controlled Z gate flips the phase of the target qubit if the control qubit is $|1\rangle$:

$$CZ|00\rangle \rightarrow |00\rangle$$
$$CZ|01\rangle \rightarrow |01\rangle$$
$$CZ|10\rangle \rightarrow |10\rangle$$
$$CZ|11\rangle \rightarrow -|11\rangle$$

$$\text{CZ} = I \otimes |0\rangle\langle 0| + Z \otimes |1\rangle\langle 1|$$

$$|\psi\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle + a_{11}|11\rangle$$

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} a_{00} \\ a_{01} \\ a_{10} \\ a_{11} \end{pmatrix} = \begin{pmatrix} a_{00} \\ a_{01} \\ a_{10} \\ -a_{11} \end{pmatrix}$$

$$|\psi'\rangle = a_{00}|00\rangle + a_{01}|01\rangle + a_{10}|10\rangle - a_{11}|11\rangle$$

APPLYING GATES

Qiskit

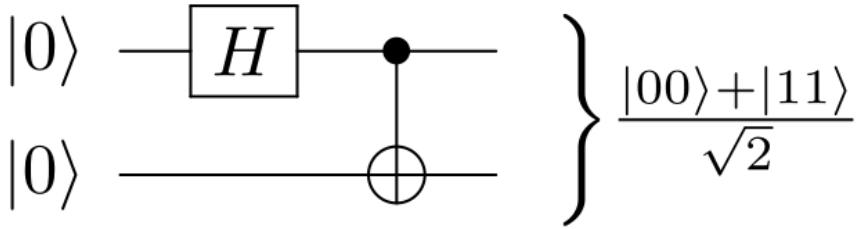
Creating entanglement

Hadamard gate and CNOT gates can be combined to generate entangled states.

Example:

Step 1: $$H|0\rangle|0\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)|0\rangle$$
$$= \frac{1}{\sqrt{2}}\left(|00\rangle + |10\rangle\right)$$

Step 2: $$CNOT\ (H|0\rangle|0\rangle) = CNOT\left(\frac{1}{\sqrt{2}}\left(|00\rangle + |10\rangle\right)\right)$$
$$= \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)$$



This is one of the Bell states

We can generate the other Bell states by choosing the inputs

$$|01\rangle,\ \ |10\rangle,\ \ |11\rangle$$

Challenges and scope of QC

Qiskit

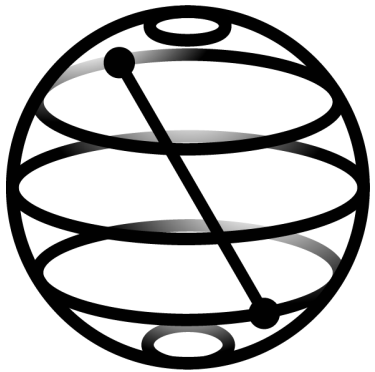
POWER

- Reduce problem solving time from hundreds of thousands of years to seconds.
  (100 million times faster)
- Break classical encryption systems (RSA).
- Reduce energy consumption!
- Machines will be smarter thanks to quantum machine learning.

CHALLENGES

- Quantum computers are very fragile. Any kind of noise will affect the quality of computation.
- Error-correction problem!
- Breaking current encryption systems (critical)!
- Finding scalable materials for building the quantum computer!

- Healthcare, Finance, National Defense, Forecasting.... etc!



Thank you!

dahbiz.github.io