Multi-Agent Deep Reinforcement Learning for Liquidation Strategy Analysis

Reproduced by Janet Wang

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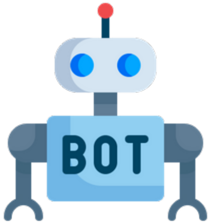
Motivation

- Liquidation is the sale of stock shares by financial institutions to **minimize implementation shortfall** and **manage risk level** for investors wanting to cash out
- Implementation shortfall is the difference between decision price and the final execution price of the trade. Since market price drops during liquidation, a trading cost occurs.
- Liquidation of large number of stock shares would have **huge impact on the dynamic market**, making the environment difficult to predict
- **Multiple organizations** may want to liquidate their assets under certain market conditions **at the same time**

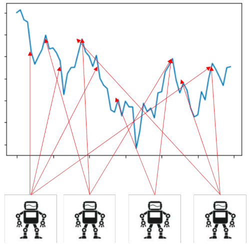
In the past, banks rely on experienced traders to minimize trading cost

Gomber et al., 2011
Brogaard et al., 2010
Sutton & Barto, 2018…



Single-agent RL is widely discussed but ignores the stochastic and interactive nature of the market

This paper



Multi-agent RL can adapt to high market complexity and derive more practical liquidation strategies

Background I: Liquidation Setup

- **Optimal Liquidation Problem:**
  - Agent$_j$ at risk aversion level λ_j aims to sell X_j shares within time frame T in N trades, implementation shortfall incurs as market price P drops
  - Our goal is to find a trajectory x_t of remaining shares $[x_{j,1}, x_{j,2}, \ldots, x_{j,T}]$at each time step $k = 1, .., T$ that minimizes implementation shortfall $E\left(X_j\right)$

- **Trading environment Simulation - Almgren and Chriss, 2001**
  - The model returns price information when agents sell at every step k
  - Price = unaffected price process, temporary, and permanent impact

$$P_k = P_{k-1} + \sigma\tau^{1/2}\xi_k - \tau g(\frac{n_k}{\tau}), k = 1, \ldots, N$$

Discrete arithmetic random walk

Market impact

  - σ: volatility of stock
  - $g(v)$: linear function of average rate of trading
  - $\tau = N/T$: Time interval
  - n_k: Number of shares to sell; N :Total number of trades

Background II: Liquidation as a MDP

- **MDP Problem definition:**
  - **State $\boldsymbol{s} = [\boldsymbol{r}, \boldsymbol{m}, \boldsymbol{l}]$**
    - k - current step
    - $r_k = \log(\frac{P_k}{P_{k-1}})$ – log return at time t_k
    - $m_k = \frac{N_k}{N}$ - number of trades remaining normalized
    - $l_{J,k} = \frac{x_{j,k}}{X_j}$ - number of shares remaining for agent J normalized
    - _Start state_: $s = [0, m, 1]$, _Terminal state_: $s = [r, 0, 0]$
  - **Action $\boldsymbol{a_k}$**: selling fraction remaining shares between 0 and 1
  - **Reward $\mathbf{R}(\boldsymbol{s}, \boldsymbol{a})$**: difference between two utility functions of the agent at t and $t+1$ since Markov decision process only dependent on current state
    - **Risk aversion** λ, **Trading trajectory** (vector of shares left) x at k

$$U(\boldsymbol{x}) = E(\boldsymbol{x}) + \lambda V(\boldsymbol{x}), \quad (1) \qquad R_t = U_t(\boldsymbol{x}_t^*) - U_{t+1}(\boldsymbol{x}_{t+1}^*).$$

$$E(\boldsymbol{x}) = \sum_{k=1}^{N} \tau x_k g\left(\frac{n_k}{\tau}\right) + \sum_{k=1}^{N} n_k h\left(\frac{n_k}{\tau}\right), \quad (2)$$

$$V(\boldsymbol{x}) = \sigma^2 \sum_{k=1}^{N} \tau x_k^2, \quad (3)$$

Expected implementation shortfall

Function of trading trajectory

Cost of trading factored in here

  - **Policy $\boldsymbol{\pi}(\boldsymbol{s})$**: the distribution of selling percentage a_k at state s

Background II (cont'd)

- **MDP goal:**
  - minimizes implementation shortfall $E(X_j)$
  - defined as cost function with fixed and variable proportions
  - We want an optimal sequence of actions $[a_{j,1}, a_{j,2}, \ldots, a_{j,T}]$, or an optimal sequence of policies $\pi(s)$ from MDP
  - This sequence of actions can translate into trading trajectory (vector of remaining shares) $[x_{j,1}, x_{j,2}, \ldots, x_{j,T}]$ that solves the optimal liquidation problem defined earlier

Related Work

- **Yang et al., 2018a**
  - Reinforcement learning algorithms
- **Almgren & Chriss, 2001**
  - Problem of optimal liquidation strategy is investigated by the impact model that agents liquidate assets completely in a given time frame
  - Impact of the stock market is divided into three components
    - Unaffected price process
    - Permanent impact
    - Temporary impact
- **Other researches related to deep reinforcement learning algo**
  - Omidshafiei at., 2017
  - Mnih et al., 2016; Lowe et al., 2017
  - Lillicrap et al., 2016
  - Wang et al., 2016

Target Task

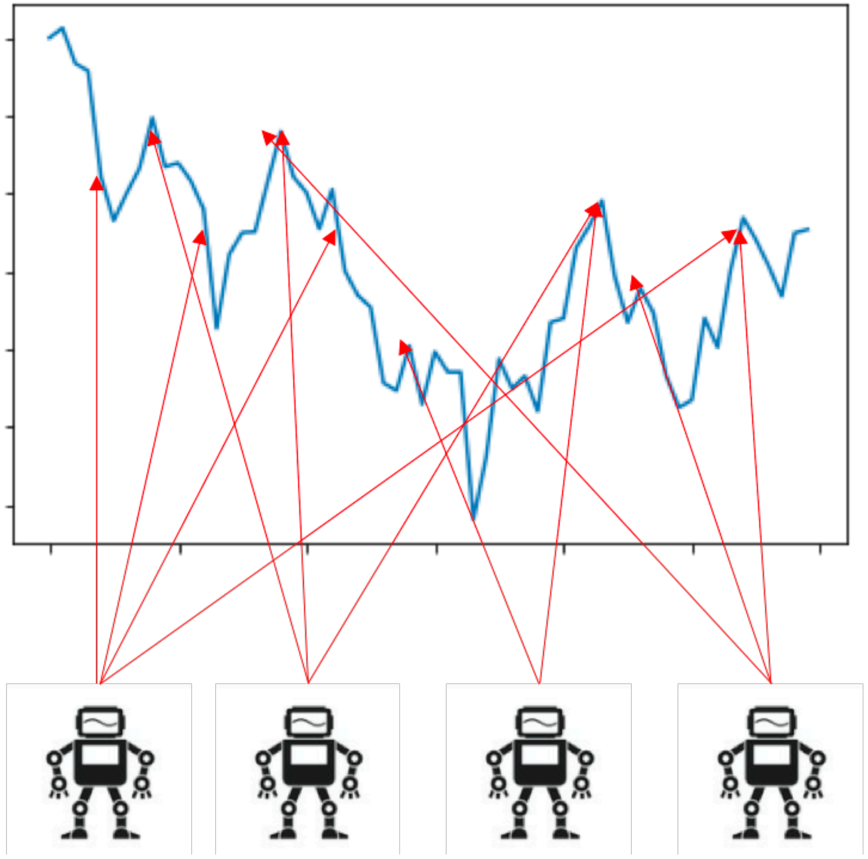
1. Extend to **multi-agent trading environment**

2. **Prove two theorems** to conclude with necessity of using multi-agent RL

3. Analyze how **cooperative and competitive relationship** between two trading agents would affect implementation shortfall

4. Derive an **optimal liquidation strategy** for a known agent

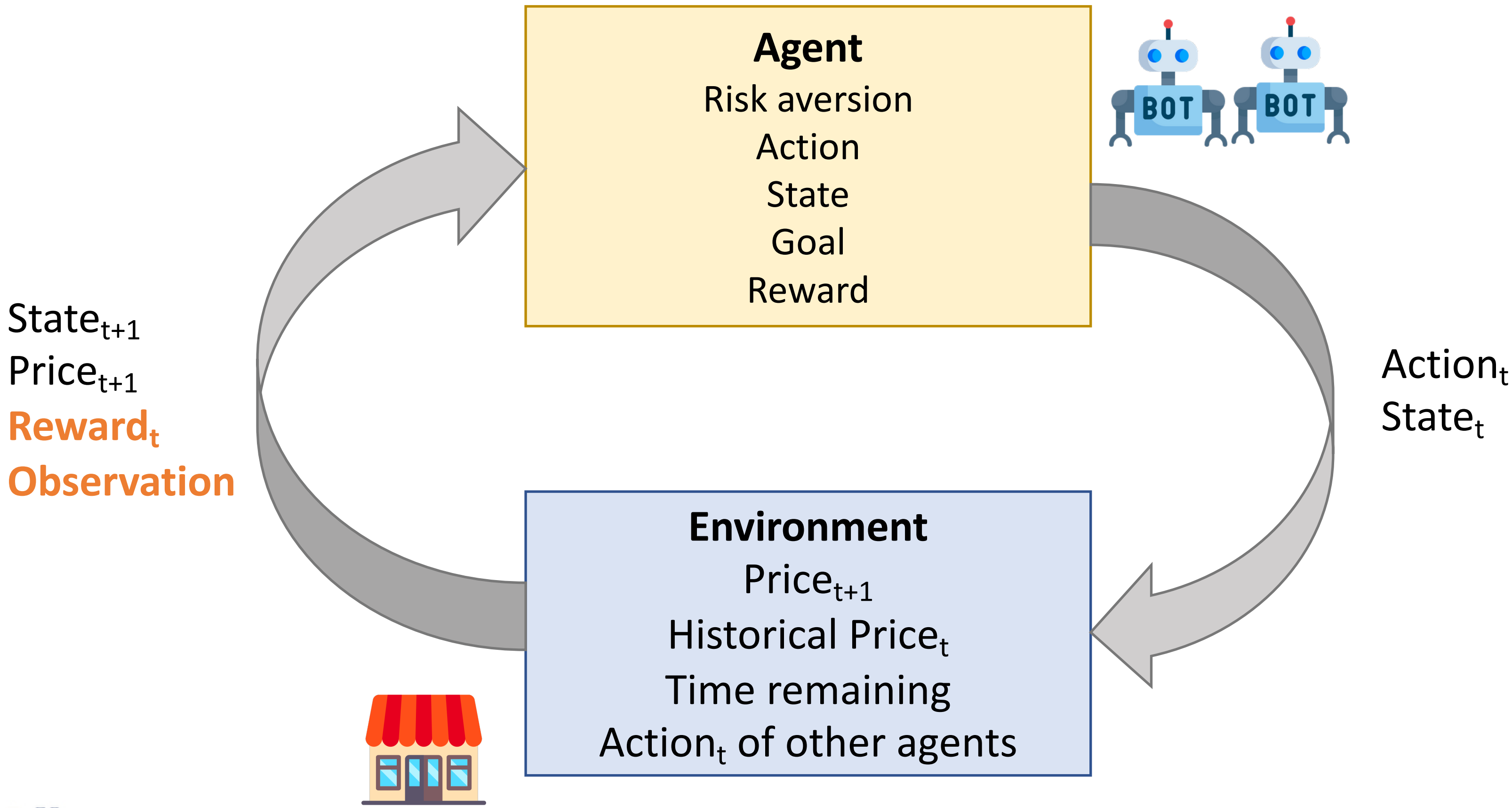
Multi-agent RL is necessary to derive trading strategy



Proposed Solution Multi-agent

- **Extend Single-agent reinforcement learning to multi-agent setting**

For 1 episode



Proposed Solution (cont'd)

- **Multi-agent reinforcement learning setting**
  - In a J-agents environment, the **State** vector $s = [r, m, l]$ at time t_k would be $[r_{k-D}, \ldots., r_{k-1}, r_k, m_k, l_{1,k}, \ldots l_{J,k}]$
  - **Action:** $n_{J,k}$ is number of shares to sell for each at each time step using:
    - $n_{J,k} = a_{J,k} \times x_{J,k}$
    - $x_{J,k}$ = shares remaining after sale at time t_k for agent j
  - **Reward** $R_{j,k}(s, a)$ denotes optimal trading trajectory for agent J:
    - $R_{J,k} = U_{J,t}(x *_{j,t}) - U_{J,t+1}(x *_{j,t+1})$
  - **Policy** $\pi(s)$ is the distribution of selling percentage a at state s
  - **Q-value** $Q_\pi(s, a)$ is the expected reward achieved by following policy π
  - **Observation** O: each agent only observes environment and itself
    - $O_{J,k} = [r_{k-D}, \ldots., r_{k-1}, r_k, m_k, l_{1,k}, \ldots l_{J,k}]$

Implementation: Actor-Critic in DDPG

- **Actor-Critic Model – Mnih and Lowe, 2017**
  - Uses neural network to approximate both Q-value and the action
  - The **critic** learns the Q-value function and updates policy parameters
  - The **actor** brings the advantage of computing continuous actions without the need of a Q-value function
  - The critic supplies actor knowledge of performance
  - The method has good convergence properties

Algorithm 1 DDPG-Based Multi-agent Training

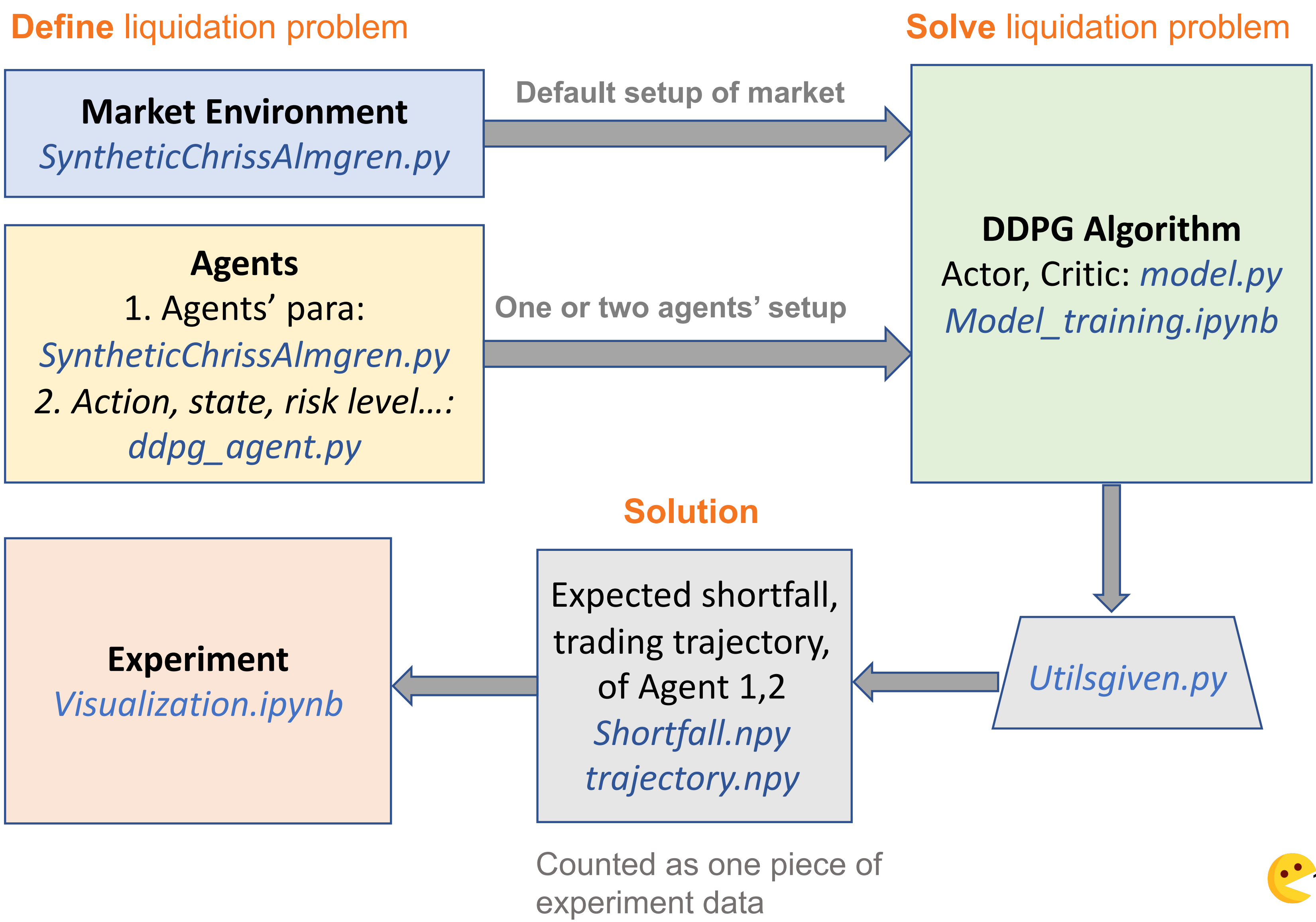
Input: number of episodes M, time frame T, minibatch size N, learning rate λ, and number of agents J

for $j = 1, J$ % initialize each agent separately **do**
  Randomly initialize critic network $Q_j(O_j, a|\theta_j^Q)$ and actor network $\mu_j(O_j|\theta_j^\mu)$ with random weight θ_j^Q and θ_j^μ for agent j;
  Initialize target network Q_j' and μ_j' with weights $\theta_j^{Q'} \leftarrow \theta_j^Q$, $\theta_j^{\mu'} \leftarrow \theta_j^\mu$ for each agent j;
  Initialize replay buffer B_j for each agent j;
end for
for episode $= 1, M$ **do**
  Initialize a random process $\mathcal{N}$ for action exploration;
  Receive initial observation state s_0;
  for $t = 1, T$ **do**
    for $j = 1, J$ %train each agent separately **do**
      Select action $a_{j,t} = \mu_j(O_{j,t}|\theta_j^\mu) + \mathcal{N}_t$ according to the current policy and exploration noise;
    end for
    Each agent executes action $a_{j,t}$;
    Market state changes to s_{t+1};
    Each agent observes reward $r_{j,t}$ and observation $O_{j,t+1}$;
    for $j = 1, J$ **do**
      Store transition $(O_{j,t}, a_{j,t}, r_{j,t}, O_{j,t+1})$ in B_j;
      Sample a random minibatch of N transitions $(O_{j,i}, a_{j,i}, r_{j,i}, O_{j,i+1})$ from B_j;
      Set $y_{j,i} = r_{j,i} + \gamma Q_j'(s_{t+1}, \mu_j'(O_{j,i+1}|\theta_j^{\mu'}|\theta_j^{Q'}))$ for $i = 1, \dots, N$;
      Update the critic by minimizing the loss: $L = \frac{1}{N}\sum_i (y_{j,i} - Q_j(O_{j,i}, a_{j,i}|\theta_j^Q))^2$;
      Update the actor policy by using the sampled policy gradient:

$$\nabla_{\theta^\mu} \pi \approx \frac{1}{N}\sum_i \nabla_a Q_j(O, a|\theta_j^Q)|_{O=O_{j,i}, a=\mu_j(O_{j,i})} \times \nabla_{\theta^\mu} \mu_j(O_j|\theta^\mu)|_{s_i};$$

      Update the target networks:
      $\theta_j^{Q'} \leftarrow \tau\theta_j^Q + (1-\tau)\theta_j^{Q'}$,
      $\theta_j^{\mu'} \leftarrow \tau\theta_j^\mu + (1-\tau)\theta_j^{\mu'}$.
    end for
  end for
end for

Implementation: Process Overview



Data Summary

- **Market setup**
  - Default setting for all experiments
  - Based on hypothetical assumption: total shares = 1 million
  - Initial stock price = 50, T = 60 days with N=60 trades, 1 trade/day

Financial Parameters

Annual Volatility:	12%	**Bid-Ask Spread:**	0.125
Daily Volatility:	0.8%	**Daily Trading Volume:**	5,000,000

- **Agents setup**
  - redefine the following parameters every time as we run a new experiment to get arrays of expected shortfalls and trajectories

Almgren and Chriss Model Parameters

Total Number of Shares for Agent1 to Sell:	500,000	**Fixed Cost of Selling per Share:**	$0.062
Total Number of Shares for Agent2 to Sell:	500,000	**Trader's Risk Aversion for Agent 1:**	1e-06
Starting Price per Share:	$50.00	**Trader's Risk Aversion for Agent 2:**	1e-06
Price Impact for Each 1% of Daily Volume Traded:	$2.5e-06	**Permanent Impact Constant:**	2.5e-07
Number of Days to Sell All the Shares:	60	**Single Step Variance:**	0.144
Number of Trades:	60	**Time Interval between trades:**	1.0

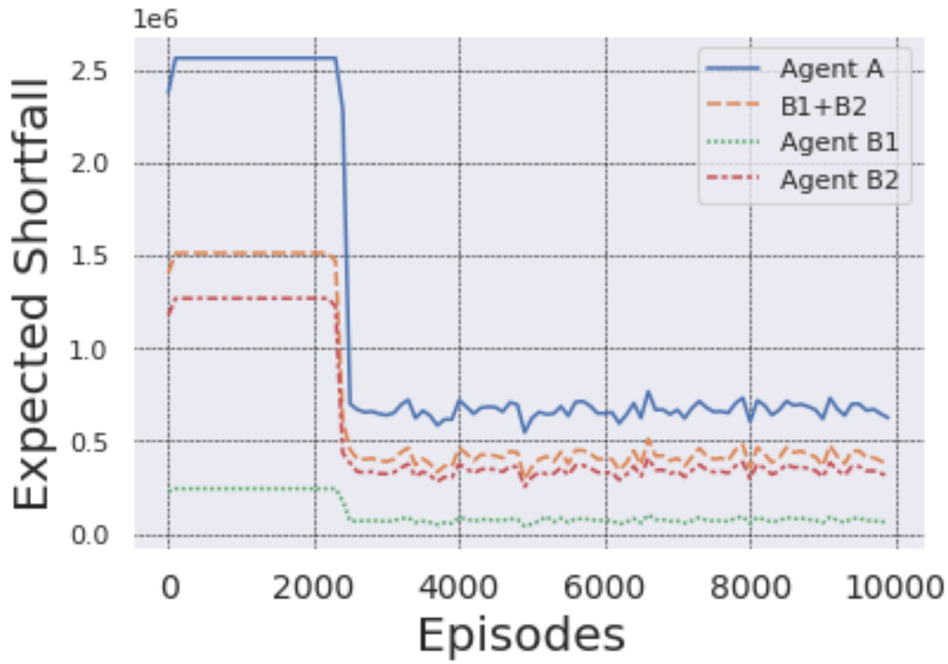
Experiment 1: Theorem I Verification

- **Theorem I definition:**
  - In a multi-agent environment with J agents where each agent has Xj shares to sell within a given time frame T, the total expected shortfall **is larger than or equal** to the sum of expected shortfall that these agents would obtain if they are in single-environment.

$$\sum_{j=1}^{J} E(X_j) \leq E(\sum_{j=1}^{J} X_j)$$

- **Experiment data used:**
  - Same risk level: $\lambda_A = \lambda_{B1} = \lambda_{B2} = 1^\wedge e^{-6}$
  - **Agent A:** 1 million shares in single-environment
  - **Agent B_1** and **Agent B_2** : liquidate 0.3 and 0.7 million shares respectively.
  - Compare average implementation shortfalls

Experimental Result 1:



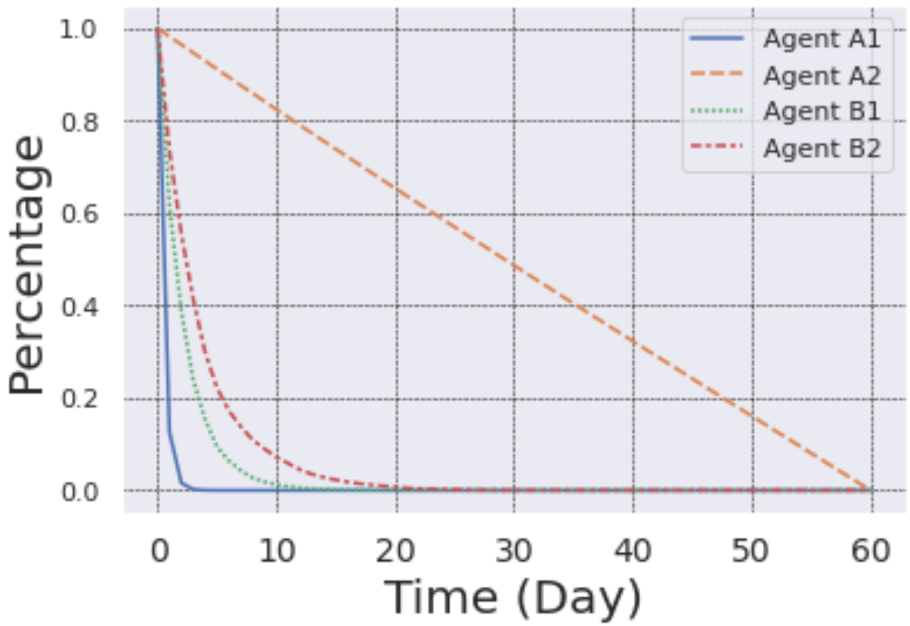
- The expected implementation shortfall $E(A)$ is larger than the sum of $E(B_1)$+$E(B_2)$
- Theorem 1 is proved.

Experiment 2: Theorem II Verification

- **Theorem II definition:**
  - In a two-agent environment where Agent 1 has risk aversion level λ_1 and Agent 2 has risk aversion level λ_2, where $\lambda_1 \neq \lambda_1$, and each of them has the same number of stocks to liquidate, the biased trajectories $x(\lambda_1)$ and $x(\lambda_2)$ would satisfy that $\boldsymbol{x^*(\lambda_1) \neq x(\lambda_1), x^*(\lambda_2) \neq x(\lambda_2)}$

- **Experiment data used:**
  - **Agent $\mathbf{A_1}$:** liquidate 1 million shares at $\lambda_{A1} = 1\text{^}e^{-4}$ in single environment
  - **Agent $\mathbf{A_2}$:** liquidate 1 million shares at $\lambda_{A2} = 1\text{^}e^{-9}$ in single environment
  - **Agent $\boldsymbol{B_1}$** and **Agent $\boldsymbol{B_2}$** liquidate 0.5 million shares each at $\lambda_{B1} = 1\text{^}e^{-4}$ and $\lambda_{B2} = 1\text{^}e^{-4}$.
  - Compare four agents' trajectories

Experimental Result 2:



- By comparing trajectories of A_1, A_2 with B_1, B_2 from the graph below, we notice that the trading trajectories of B_1, B_2 are biased
- Theorem 2 is proved.

Experimental Analysis 1 & 2

- For same amount of shares, **shortfall implementation is greater if it is liquidated by one** agent than multiple agents
- Agents in **multi-agent environment factors in extra information** from observing other agents in the environment when deciding their trading trajectory
- The **selling patterns of other agents** would affect their liquidation strategy
- This demonstrates the **necessity of using multi-agents RL to derive trading strategy**

Experiment 3: Relationship analysis

- **Goal:**
  - Adjust reward to define competitive and cooperative relationships
  - Compare the sum of expected shortfalls with expected shortfalls trained in single-agent environment to evaluate relationship

Corporation

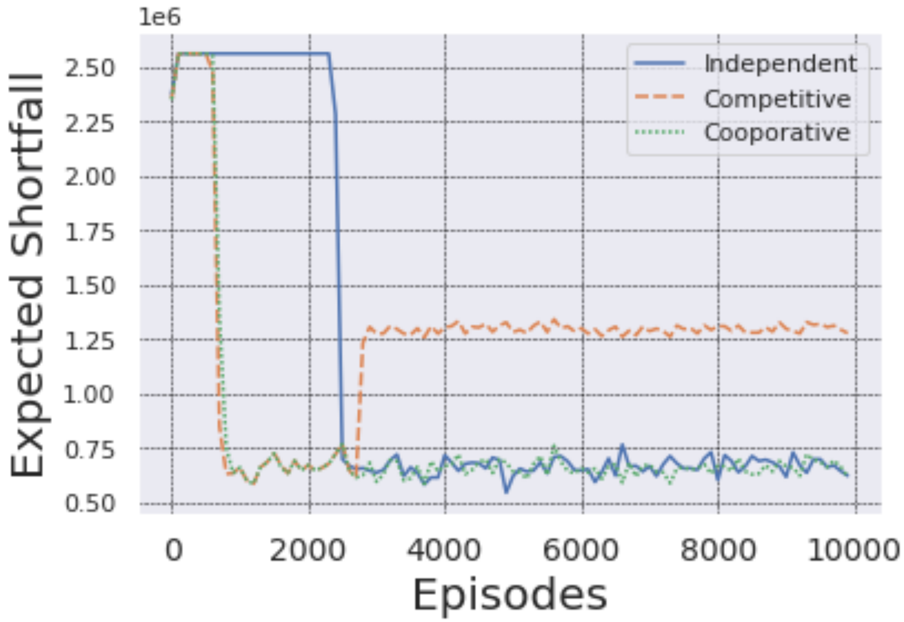
$$\tilde{R}^*_{1,t} = \tilde{R}^*_{2,t} = \frac{\tilde{R}_{1,t} + \tilde{R}_{2,t}}{2}$$

Competition

$$\begin{aligned}&\textbf{if } \tilde{R}_{1,t} > \tilde{R}_{2,t} \textbf{ then}\\ &\quad \tilde{R}^*_{1,t} = \tilde{R}_{1,t},\\ &\quad \tilde{R}^*_{2,t} = \tilde{R}_{2,t} - \tilde{R}_{1,t}\end{aligned}$$

- **Experiment data used:**
  - **Agent A:** liquidate 1 million shares at $\lambda_A = 1^\wedge e^{-6}$ in single environment
  - **Agent A$_2$**: liquidate 1 million shares at $\lambda_{A2} = 1^\wedge e^{-9}$ in single environment
  - **Agent B_1** and **Agent B_2** liquidate 0.5 million shares each at $\lambda_{B1} = \lambda_{B2} = 1^\wedge e^{-6}$ with competitive reward functions
  - Agent C$_1$ and Agent C$_1$ liquidate 0.5 million shares each at $\lambda_{C1} = \lambda_{C2} = 1^\wedge e^{-6}$ with cooperative reward functions.
  - Compare three shortfalls

Experimental Result 3:



- **Two cooperative agents are not necessarily better** than independent agents training with independent rewards
- **Two competitive agents** learns to minimize expected shortfall faster than other types of agents, and then **malignant competition** leads to **significant increase in shortfall increment**

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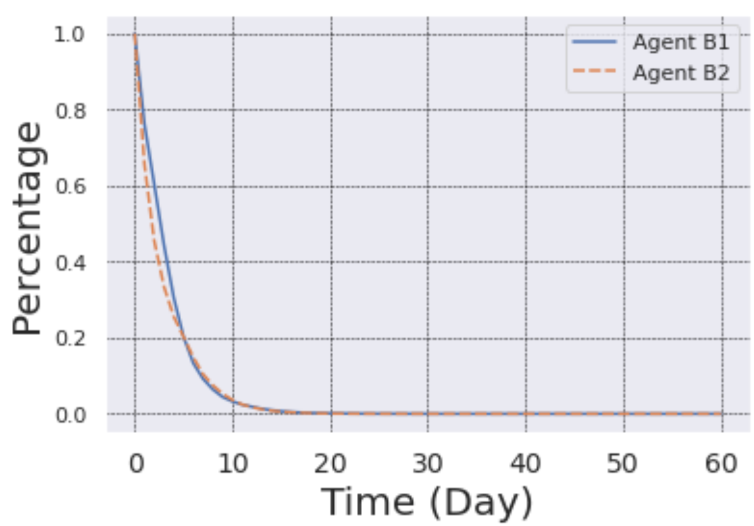
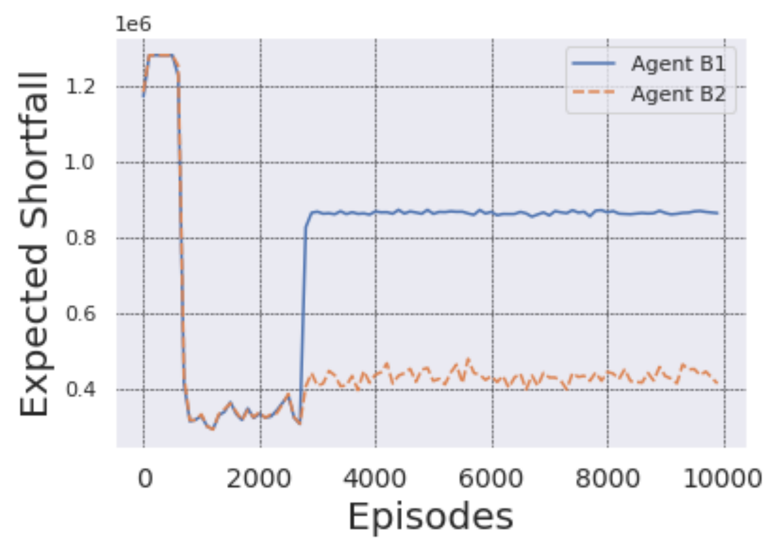
Experimental Analysis 3:

We look deeper into how agents would behave in **competitive relationships**:

- **Two competitive agents** learns to sell all shares at similar pace
- Both agents perform well and roughly have same expected shortfalls but one starts to **outperform significantly at the cost of other**
- **E(X) for both agent increases**; none of them is winning





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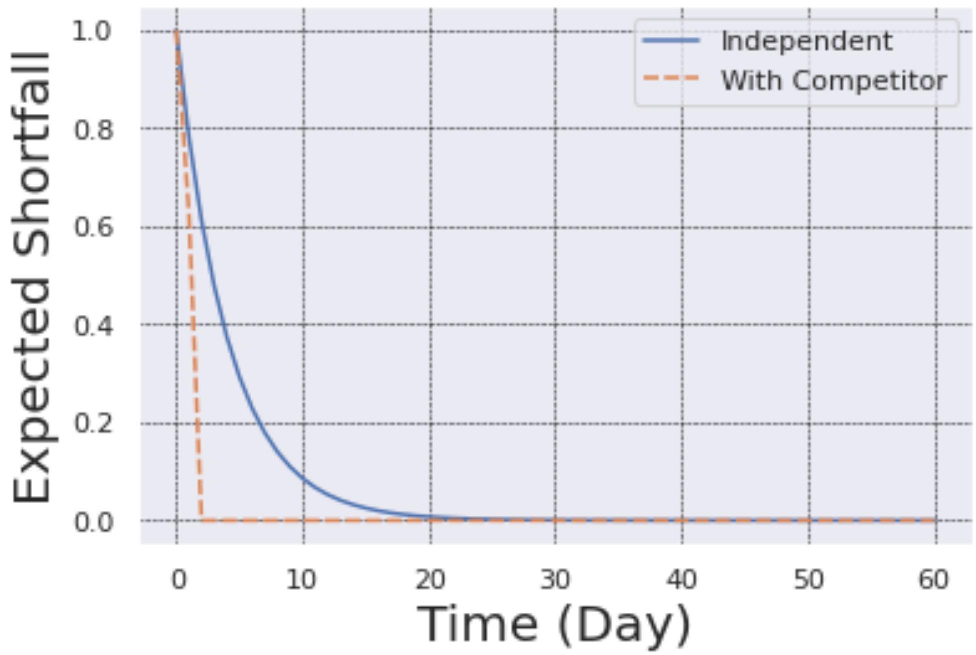
Experiment 4: Strategy Development

- **Goal:**
    - Find a trading trajectory that optimizes the expected shortfall given there are competitors in the environment

- **Experiment data used:**
    - **Agent A_1 and Agent A_2** each liquidate 0.5 million shares each at $\lambda_{A1} = 1\hat{}e^{-6}$ and $\lambda_{A2} = 1\hat{}e^{-9}$
    - **Agent A:** 1 million shares in single-environment from Experiment 1
    - Get trading trajectory of Agent A_1 and have Agent A's trading trajectory as benchmark for comparison

Experimental Result & Analysis 4:

- Without competitors, Agent A normally completes in 20 days
- With competitors, Agent A_1 normally sells in 2 days



Agents learn to **avoid taking unnecessary risk** by selling all shares in quite **a short time.**

Conclusion & Future Work

Conclusion

1. Single-agent environment **over-simplifies the dynamic** as well as the interactive nature of the stock market.
2. We extend the Almgren-Chriss model with reinforcement learning to set up the **basis of using multi-agent trading environment** to have a better analysis of expected shortfalls

Future work

1. Development of more **realistic trading environment**
2. Include more dynamic factors such as **news, general strategy and legal complaints**

Conclusion & Future Work (cont'd)

Conclusion

1. Cooperative relationship is not better than independent one
2. Competitive relationship would hurt overall and individual performance
3. Agents with competitor **liquidate faster to avoid risk**

Future work

1. Consider **optimistic bull** or **pessimistic bear**
2. Use this as an application to **predict stock price movements** after liquidation

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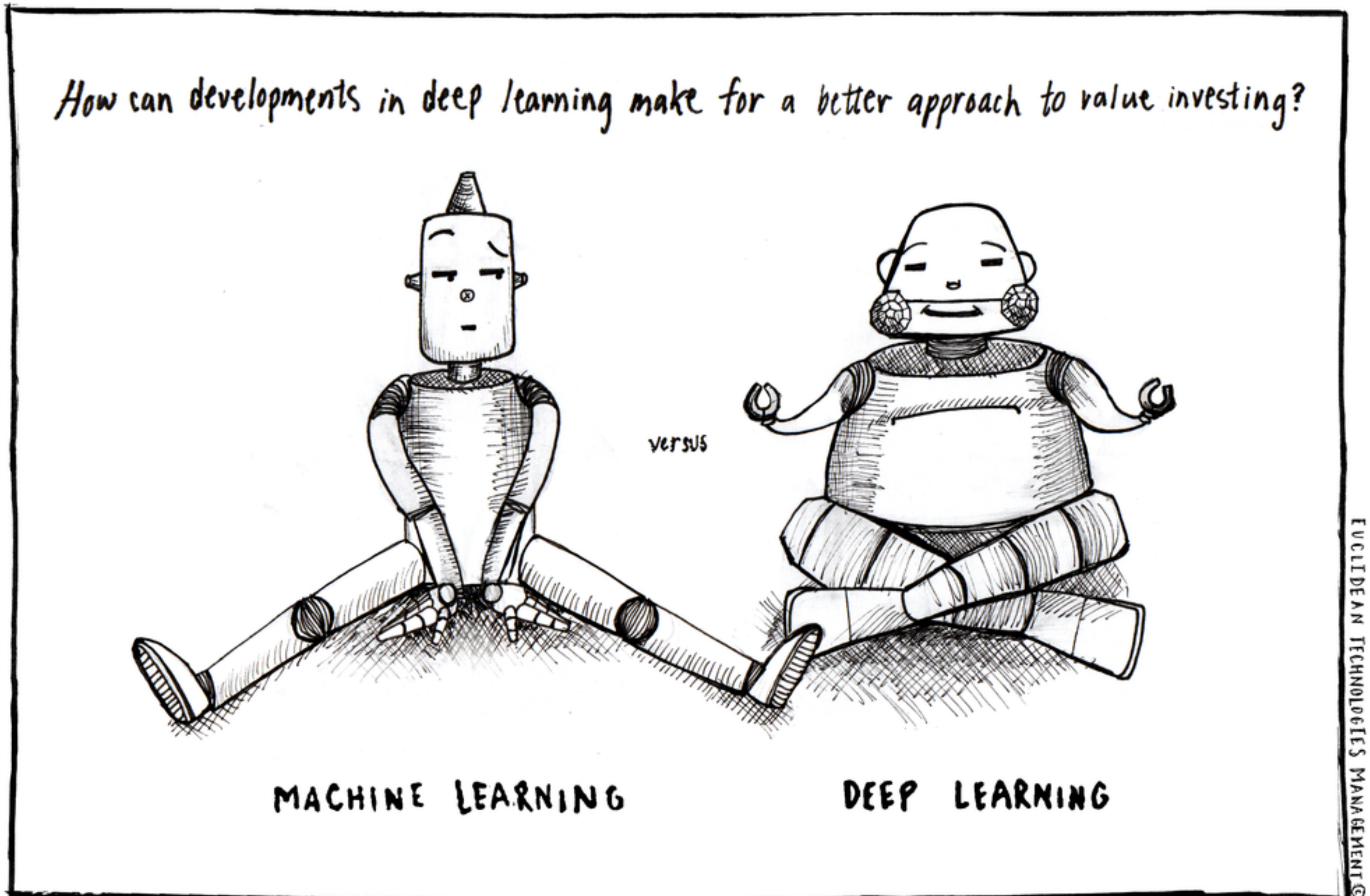
Thank you!



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