Analysis of Losses in Inductors for Power Converters.

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I. Introduction

The following deals with losses in inductors for power electronic components. The losses can be divided into core losses, winding losses and fringing losses.

A common assumption for a starting point is to make equal the winding and core losses for a given design (it should be made clear, however, at which operation point must this condition be fulfilled).

II. Core losses

When discussing about the core for a given application, one parameter that might be helpful is the Performance Factor (PF), that considers the frequency and the power levels involved in the design:

$$PF = f \cdot \hat{B} \tag{1}$$

being f the frequency of the waveform and the amplitude of the AC flux density.

In order to compare different geometries, sizes etc., the comparison is carried out fixing the power loss per unit of volume. This information can be calculated by knowing the core loss plots of materials as a function of frequency (usually given for a given temperature and for different AC flux densities).

This parameter is correlated with the size of a magnetic component: the largest the PF, then a smaller size in the magnetic component for a given design [1][2][3].

Core losses are divided into:
- Hysteresis losses
- Eddy current,
- Ferriresonance losses.

a) Hysteresis losses

The hysteresis losses are usually given by Steinmetz equation (valid for sinusoidal excitation):

$$P_{V\ HIST} = K_C \cdot f^\alpha \cdot \hat{B}^\beta \tag{2}$$

where α, β and K_C depend on the material.

However, this equation might yield a wrong conclusion: the core can always be reduced in size if higher frequencies are used.

From real magnetic material curves, this equation can be fitted to account for variation of α with the frequency:

$$P_{V\ HIST} = K_C \cdot f^\alpha \cdot \hat{B}^\beta \cdot \left[1 + \left(\frac{f}{f_{CR}}\right)^{\alpha_{CR}}\right] \tag{3}$$

By using this expression, there is a limit frequency beyond which further increase of frequency does not imply size reduction [1].

Equation (3) does not take into account Eddy losses, and also considers only sinusoidal excitation (given that the curves that are fitted are calculated using sinusoidal waveforms).

b) Eddy losses

The effect of Eddy losses, that can be modelled by:

$$P_{V\ EDDY} = C_b \cdot f^2 \cdot \hat{B}^2 \cdot \frac{A_e}{\rho_{NU}} \tag{4}$$

where C_b depends on the material.

c) Total core losses

But, finally, the core losses can be considered as a whole. These total power losses in a magnetic core are a function of several parameters, such as the switching frequency, current ripple, magnetic core material, core size, and winding number of turns, $P_{core}(f, \Delta I_L, Material, Size, N)$.

For the analytical approach, hereby in context, the well-known empirical equation, known as the standard Steinmetz Equation (SE), is used:

$$P_{core} = c * f^x * (B_{ac}|_{HF})^y * V_e \tag{5}$$

where c, x, and y are the Steinmetz coefficients which are a function of the core material and switching frequency. V_e is the core volume, and $B_{ac}|_{HF}$ is the AC magnetic flux density calculated as follows:

$$B_{ac}|_{HF} = \frac{L * I_{ac}|_{HF}}{A_e * N} \tag{6}$$

The maximum magnetic flux density, B_{max}, should not exceed the saturation value, B_{sat}

$$B_{max} = \frac{L * I_{max}}{A_e * N} < B_{sat} \tag{7}$$

III. Copper losses

The inductor power losses are divided mainly into copper losses and core losses. To calculate those losses, first step is to define the converter operation parameters, in this case, the current waveform (average and ripple range), the switching frequency range, and the corresponding inductance range. The current high-frequency ripple is calculated as follows:

$$I_{pp}|_{HF} = I_{avg} * \Delta I_L|_{pp} \tag{8}$$

where $\Delta I_L|_{pp}$ is the p.u. current ripple, and I_{avg} is the average current value. The maximum instantaneous current can thus be given by

$$I_{max} = I_{avg} + \frac{I_{pp}|_{HF}}{2} \tag{9}$$

Depending on the converter topology, the HF and LF current components can be calculated as a function of the current ripple [4][5]. Specifically for a boost converter, the following are the expressions for the fundamental AC current components.

$$I_{ac}|_{HF} = I_{pp}|_{HF} * \frac{4}{\pi^2} \tag{10}$$

$$I_{ac}|_{LF} = I_{max} - \frac{I_{pp}|_{HF}}{2} - I_{avg} \tag{11}$$

Consequently, the *rms* HF and LF current components are

$$I_{rms}|_{HF} = \frac{I_{ac}|_{HF}}{\sqrt{2}} \tag{12}$$

$$I_{rms}|_{LF} = \frac{I_{ac}|_{LF}}{\sqrt{2}} \tag{13}$$

Therefore, the total *rms* current is

$$I_{rms} = \sqrt{I_{rms}|_{HF}^2 + I_{rms}|_{LF}^2 + I_{avg}^2} \tag{14}$$

The inductance can be calculated depending on the converter power, average current, ripple, and frequency as follows.

$$L = \frac{P}{2 * f * I_{pp}|_{HF} * I_{avg}} \tag{15}$$

After the definition of the waveforms, the losses in the inductor can be formulated. Power loss in an inductor is divided mainly into copper losses (both LF and HF components), and core losses, each of the latter loss components is discussed in detail in the following sections.

The copper losses [6] are a function of several parameters, such as the current ripple, size, diameter, and winding number of turns, $P_{copper}(\Delta I_L, Size, D, N)$. Copper losses are divided into LF (or DC component) and HF (or AC component) losses. The low frequency losses are calculated using the DC winding resistance which is defined as follows.

$$R_{dc} = \frac{4/\pi * \rho_{cu} * l_{turn} * N}{D^2} \tag{16}$$

where ρ_{cu} is the copper resistivity, l_{turn} is the mean turn length, N is the number of turns, and D is the wire diameter. Therefore, the DC power loss component is

$$P_{dc} = (I_{rms}|_{LF}^2 + I_{avg}^2) * R_{dc} \tag{17}$$

On the other hand, HF losses (represent skin and proximity effects) are calculated using Dowell's formulae and procedure [3]. The winding is partitioned and the DC resistance is calculated as follows,

$$R_{dc}|_{Dowell} = \frac{m * \rho_{cu} * l_{turn} * N_{tl}}{\eta * w_e * h} \tag{18}$$
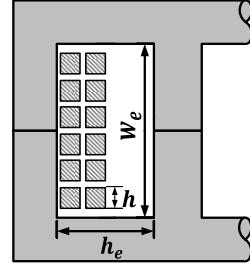
where m is the number of layers, and η is the ratio of the total winding width to the window width (window factor, l_{wf}), and are calculated as follows.

$$m = \frac{N}{N_{tl}} \tag{19}$$

$$\eta = N_{tl} * \frac{h}{w_e} = l_{wf} \tag{20}$$

where N_{tl} is the number of turns per layer, and can be calculated using the window factor as follows.

$$N_{tl} = \frac{w_e}{h} * l_{wf} \tag{21}$$

where w_e is the width of the window, and h is the width of the conductor, which is calculated as follows.

$$h = D * \sqrt{\frac{\pi}{4}} \tag{22}$$

The AC resistance is thus calculated by multiplying the DC resistance by a factor, as seen below.

$$R_{ac} = R_{dc}|_{Dowell} * \left(M_{coef}' + \frac{(m^2 - 1) * D_{coef}'}{3} \right) \tag{23}$$

where M_{coef} and D_{coef} are expressed as follows.

$$M_{coef} = \alpha * h * \coth(\alpha * h) \tag{24}$$

$$D_{coef} = 2\alpha * h * \tanh(2\alpha * h) \tag{25}$$

where α is expressed as follows.

$$\alpha = \sqrt{\frac{j * \omega * \mu_0 * \eta}{\rho_{cu}}} \tag{26}$$

and, ω is expressed as follows.

$$\omega = 2 * \pi * f \tag{27}$$

Therefore, the AC power loss component is

$$P_{ac} = (I_{rms}|_{HF}^2) * R_{ac} \tag{28}$$

This finally yields to the total copper loss:

$$P_{copper} = [(I_{rms}|_{LF}^2 + I_{avg}^2) * R_{dc}] + \\ + [(I_{rms}|_{HF}^2) * R_{ac}] \tag{29}$$

Depending on the core geometry, also the maximum number of turns, N_{max}, can be calculated as follows.

$$N_{max} = \frac{l_{wf} * w_e * h_e}{h^2} \tag{30}$$

IV. Total Losses

As a conclusion to the previous analysis, the total inductor losses is the summation of copper and core losses from (5) and (29):

$$P_{loss}|_{inductor} = P_{copper} + P_{core}|_{iGSE} \tag{31}$$

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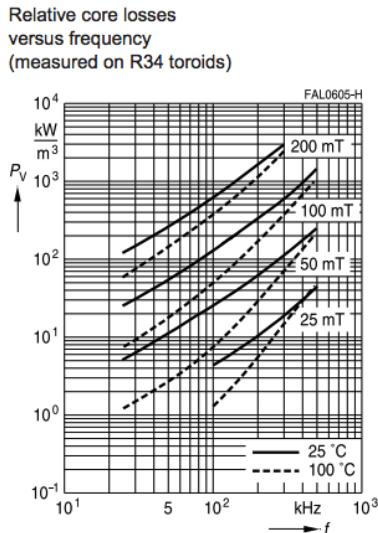
ANNEX I: Characterization of N87 material

Relative core losses
versus frequency
(measured on R34 toroids)

Figure A.1.N87 Material losses

The following values are obtained from figure A.1old

Table for 25ºC (N87)

B (mT)	F (kHz)	Pv (kW/m3)
25	100	4.2
25	200	10.0
25	300	19
25	400	30
25	500	42
50	30	1.2
50	50	2.6
50	70	4.0
50	100	7.5
50	200	30
50	300	70
50	400	120
50	500	220
100	30	9
100	50	17
100	70	30
100	100	50
100	200	160
100	300	340
100	400	660
100	500	1100
200	30	73
200	50	120
200	70	210
200	100	360
200	200	1100
200	300	2200

Table for 100ºC (N87)

B (mT)	F (kHz)	Pv (kW/m3)
25	100	1.2
25	200	5.2
25	300	12
25	400	30
25	500	42
50	30	6.1
50	50	10.5
50	70	15
50	100	23
50	200	60
50	300	105
50	400	160
50	500	220
100	30	30
100	50	52
100	70	80
100	100	115
100	200	315
100	300	600
100	400	920
100	500	1200
200	30	120
200	50	240
200	70	400
200	100	600
200	200	1400
200	300	3000

These tables are adjusted to the following equation (3) through curve fitting (see Annex II)

These values yield to the following set of parameters:

α	β	Kc	αCR	fCR (kHz)	Temp
1.528	2.585	3.582e-7	2.834	611.1	25ºC
0.3671	2.461	7.101e-5	1.42	58.73	100ºC
0.7496	2.473	1.815e-5	1.169	100 (forced)	100ºC

ANNEX II: Procedure for curve fitting:

1. Create Excel sheet. Place excel in matlab folder (check that cells are numbers…). Open Matlab, menu/home/Import data, as column vectors.
2. Columns of BmT, FkHz and PvkWm3 appear.
3. Go to apps/Curve fitting. X=Bmt, y=>f, z=pV.
4. Select Fit Options. Check limits. Increase to 10000 iter and fun.
5. Fit…

These are the results of the fitting:

@25ºC
```
General model:
    f(BmT1,fkHz1) = kc * (fkHz1^alpha) * (BmT1^beta)
* (1+((fkHz1/fcr)^alphacr))
Coefficients (with 95% confidence bounds):
      alpha =        1.363  (1.122, 1.605)
      alphacr =      1.792  (1.118, 2.466)
      beta =         2.567  (2.496, 2.638)
      fcr =          469.7  (217.8, 721.6)
      kc =      7.946e-07  (-8.58e-08, 1.675e-06)

Goodness of fit:
  SSE: 4899
  R-square: 0.9992
  Adjusted R-square: 0.9991
  RMSE: 14.92

Goodness of validation:
  SSE: 995339
  RMSE: 192.001
```

@100ºC
```
General model:
    f(BmT,fkHz) = kc * (fkHz^alpha) * (BmT^beta) *
(1+((fkHz/fcr)^alphacr))
Coefficients (with 95% confidence bounds):
      alpha =        0.7008  (-0.7877, 2.189)
      alphacr =      1.227  (0.483, 1.971)
      beta =         2.478  (2.336, 2.62)
      fcr =          100  (-217.5, 417.5)
      kc =      2.215e-05  (-9.447e-05, 0.0001388)

Goodness of fit:
  SSE: 4.011e+04
  R-square: 0.9963
  Adjusted R-square: 0.9956
  RMSE: 42.7

Goodness of validation:
  SSE: 40106
  RMSE: 38.541
```