DYNAMIC MODELING OF A SYNCHRONOUS MACHINE

By

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Program: Electric Vehicle Propulsion and Control (E-PiCo).

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Objective:

1. To Implement the dynamic model of a synchronous (Hydro Turbine) Generator with the following specifications on MATLAB/SIMULINK:

Rating: 325 MVA
Line-to-line voltage: 20 kV
Power factor: 0.85
Poles: 64
Speed: 112.5 r/min
Combined inertia of generator and turbine:
$J = 35.1 \times 10^6 \mathrm{J \cdot s^2}$, or $WR^2 = 833.1 \times 10^6\ \mathrm{lbm \cdot ft^2}$ $\quad H = 7.5$ s
Parameters in ohms and per unit:
$r_s = 0.00234\,\Omega,\ 0.0019$ pu
$X_{ls} = 0.1478\,\Omega,\ 0.120$ pu

$X_q = 0.5911\,\Omega, 0.480$ pu	$X_d = 1.0467\,\Omega,\ 0.850$ pu
	$r'_{fd} = 0.00050\,\Omega, 0.00041$ pu
	$X'_{lfd} = 0.2523\,\Omega,\ 0.2049$ pu
$r'_{kq2} = 0.01675\,\Omega,\ 0.0136$ pu	$r'_{kd} = 0.01736\,\Omega,\ 0.0141$ pu
$X'_{lkq2} = 0.1267\,\Omega,\ 0.1029$ pu	$X'_{lkd} = 0.1970\,\Omega,\ 0.160$ pu

2. To determine the dynamic performance of the generator during a sudden change in the input Torque.

Methodology:

1. Machine Parameters Calculation:

Base frequence (ω_b) (rad/sec) = $\frac{112.5\times2\times\pi}{60}$ = 11.78 rad/sec

Base frequency (elec. rad/sec) (ω_{eb}) = $\frac{Poles}{2} \times wb = 377\ elec.rad/sec$

$r_s = 0.00234\ \Omega$

$r'_{kq1} = r'_{kq2} = 0.01675\ \Omega$

$r'_{fd} = 0.00050\ \Omega$

$r'_{kd} = 0.01736\ \Omega$

$L_{q=} \frac{1}{w_{eb}}(X_q) = 0.0016\Omega$; $\qquad L_{d=} \frac{1}{w_{eb}}(X_d) = 0.0028\Omega$

$L_{ls=} \frac{1}{w_{eb}}(X_{ls}) = 3.9205 \times 10^{-4}\Omega$; $\qquad L_{md=} L_d - L_{ls} = 0.0024\Omega$

$L_{mq=} L_q - L_{ls} = 0.0012\Omega$; $\qquad L'_{lkq1} = L'_{lkq2} = \frac{X'_{lkq1}}{w_{eb}} = \frac{0.1267}{377} = 3.3608 \times 10^{-4}\ \Omega$

$X'_d = X_d - \frac{X_{md}{}^2}{X'_{lfd}+X_{md}} = 0.3448\Omega \qquad L'_d = \frac{X'_d}{w_{eb}} = \frac{0.3448}{377} = 9.146 \times 10^{-4}\ \Omega$

$$X'_{lf} = \frac{X_{md}(X'_d - X_{ls})}{X_{md} - (X'_d - X_{ls})} = 0.2523\Omega \qquad L'_{lf} = \frac{X'_{lf}}{w_{eb}} = \frac{0.2523}{377} = 6.6923 \times 10^{-4}\ \Omega$$

$$L'_{lkd} = \frac{X'_{lkd}}{w_{eb}} = \frac{0.1970}{377} = 5.2255 \times 10^{-4}\ \Omega$$

2. The differential (dynamic) Equation of the synchronous generator will be implemented in the Simulink environment.

The Stator Voltages are converted to the rotor's rotating reference frame (the q-d axis). By transforming these variables into the q-d axis via $k_s(\theta_r = \omega_e)$, the equations become time-invariant instead of time-varying. This simplifies the analysis significantly as it reduces the problem from dealing with sinusoidal variables to dealing with DC-like variables.

In the q-d frame, the flux linkages, voltages, and currents in the direct (d-axis) and quadrature (q-axis) axes are effectively decoupled. This decoupling means that the d-axis can be primarily associated with the magnetizing flux and field excitation, while the q-axis relates to the torque production.

$$V_a = V_m \sin(\omega_e t)$$

$$V_b = V_m \sin(\omega_e t - \frac{2\pi}{3})$$

$$V_a = V_m \sin(\omega_e t - \frac{4\pi}{3})$$

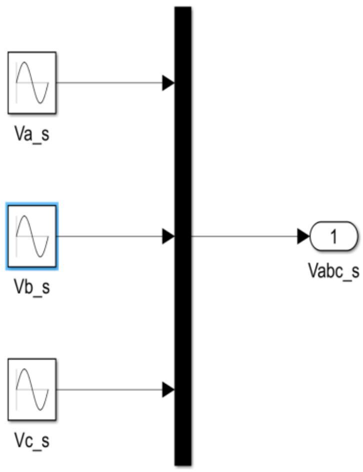


Fig1: Stator Voltage Block

Transferring to q-d frame, we have:

$$\begin{bmatrix} V_q \\ V_d \\ V_0 \end{bmatrix} = \frac{2}{3} \begin{bmatrix} \cos(\omega_e t) & \cos(\omega_e t - \frac{2\pi}{3}) & \cos(\omega_e t + \frac{2\pi}{3}) \\ \sin(\omega_e) & \sin(\omega_e t - \frac{2\pi}{3}) & \sin(\omega_e t + \frac{2\pi}{3}) \\ 1/2 & 1/2 & 1/2 \end{bmatrix} \times \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

Where:

$$K_s = \frac{2}{3}\begin{bmatrix} \cos(\omega_e t) & \cos(\omega_e t - \frac{2\pi}{3}) & \cos(\omega_e t + \frac{2\pi}{3}) \\ \sin(\omega_e t) & \sin(\omega_e t - \frac{2\pi}{3}) & \sin(\omega_e t + \frac{2\pi}{3}) \\ 1/2 & 1/2 & 1/2 \end{bmatrix}$$

Hence;

$$V_q = V_m \cos\delta$$

$$V_d = V_m \cos\delta$$

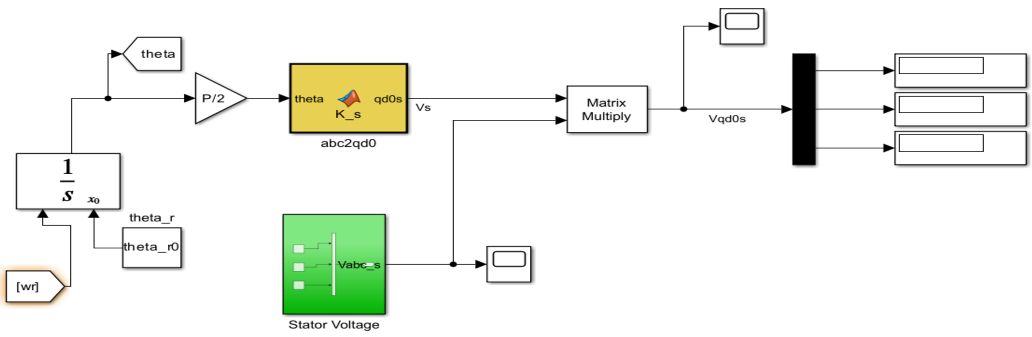


Fig. 2: Stator Voltage transformation to q-d axis

Rotor quantities do not need transformation.

$$V_{kq1} = 0$$

$$V_{kq2} = 0$$

$$V_{fd} = r'_{fd} \times i_f = \frac{E_f \times r'_{fd}}{X_{md}}$$

Where : $E_f = X_{md} \times i_f$

$$V_{kd} = 0$$

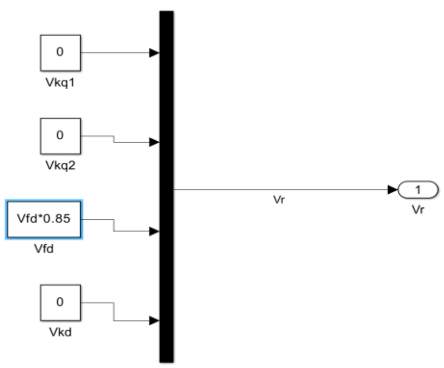


Fig 3: Rotor Voltage Block

3. The **flux Linkage equations** are:

$$\begin{bmatrix} \lambda_{qs} \\ \lambda_{ds} \\ \lambda_{0s} \\ \lambda'_{kq1} \\ \lambda'_{kq2} \\ \lambda'_{f} \\ \lambda'_{kd} \end{bmatrix} = \begin{bmatrix} L_q & 0 & 0 & L_{mq} & L_{mq} & 0 & 0 \\ 0 & L_d & 0 & 0 & 0 & L_{md} & L_{md} \\ 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 \\ L_{mq} & 0 & 0 & L'_{lkq1}+L_{mq} & L_{mq} & 0 & 0 \\ L_{mq} & 0 & 0 & L_{mq} & L'_{lkq2}+L_{mq} & 0 & 0 \\ 0 & L_{md} & 0 & 0 & 0 & L'_{lf}+L_{md} & L_{md} \\ 0 & L_{md} & 0 & 0 & 0 & L_{md} & L'_{lkd}+L_{md} \end{bmatrix} \times \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \\ i'_{kq1} \\ i'_{kq2} \\ i'_{f} \\ i'_{kd} \end{bmatrix}$$

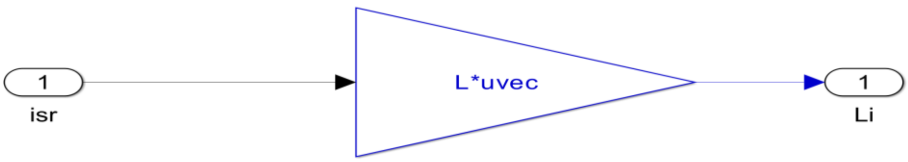


Fig 4: Flux Linkage Calculation

4. Current Calculation:

The currents are calculated based on the voltage-current differential equations:

$$i = L^{-1} \int (V - (R + \omega J L) i)$$

$$[V_{qd0kqkdf}] = [R][i_{qd0kqkdf}] + [L]\frac{d}{dt}[i_{qd0kqkdf}] + \omega_r [J][L]\left[[i_{qd0kqkdf}]\right]$$

Where:

$$[V_{qd0kqkdf}] = \begin{bmatrix} V_{qs} \\ V_{ds} \\ V_{0s} \\ V'_{kq1} \\ V'_{kq2} \\ V'_{f} \\ V'_{kd} \end{bmatrix} \quad ; \quad [R] = \begin{bmatrix} r_s & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_s & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_s & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & r'_{kq1} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & r'_{kq2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & r'_{f} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & r'_{kd} \end{bmatrix}$$

$$[i_{qd0kqkdf}] = \begin{bmatrix} i_{qs} \\ i_{ds} \\ i_{0s} \\ i'_{kq1} \\ i'_{kq2} \\ i'_{f} \\ i'_{kd} \end{bmatrix} \quad ; \quad [L] = \begin{bmatrix} L_q & 0 & 0 & L_{mq} & L_{mq} & 0 & 0 \\ 0 & L_d & 0 & 0 & 0 & L_{md} & L_{md} \\ 0 & 0 & L_{ls} & 0 & 0 & 0 & 0 \\ L_{mq} & 0 & 0 & L'_{lkq1}+L_{mq} & L_{mq} & 0 & 0 \\ L_{mq} & 0 & 0 & L_{mq} & L'_{lkq2}+L_{mq} & 0 & 0 \\ 0 & L_{md} & 0 & 0 & 0 & L'_{lf}+L_{md} & L_{md} \\ 0 & L_{md} & 0 & 0 & 0 & L_{md} & L'_{lkd}+L_{md} \end{bmatrix}$$

$$[J] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

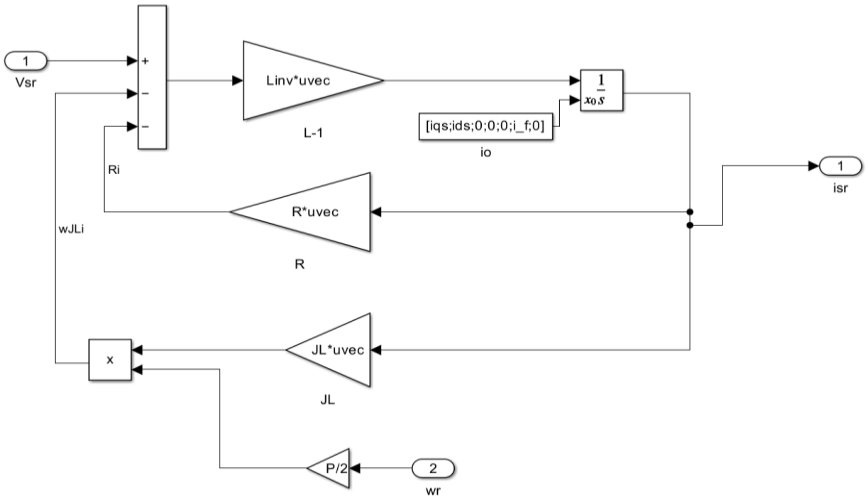


Fig 5: Current Calculation Block

The current initial conditions were calculated from:

$$V_{qs} = \sqrt{2}V_s \cos(\theta_{ev} - \theta_r)$$

$$V_{ds} = -\sqrt{2}V_s \cos(\theta_{ev} - \theta_r)$$

So;

$$i_{ds} = r_s i_{qs} + X_d\, i_{ds} - V_{qs}$$

$$i_{qs} = r_s i_{ds} - X_q\, i_{qs} - V_{ds}$$

$$i_f = \frac{V_{fd}}{r_f}$$

$$i_{kq1} = i_{kq2} = i_{kd} = 0$$

5. Torque Calculation:

The electromagnetic Torque is calculated by dividing the active power by the mechanical speed.

$$T_{em} = \frac{P_{em}}{\omega_m} = \frac{{}^3/_2 \times \omega_r\left(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds}\right)}{{}^2/_P\, \omega_r}$$

$$T_{em} = \frac{3}{2}\frac{P}{2}(\lambda_{ds} i_{qs} - \lambda_{qs} i_{ds})$$

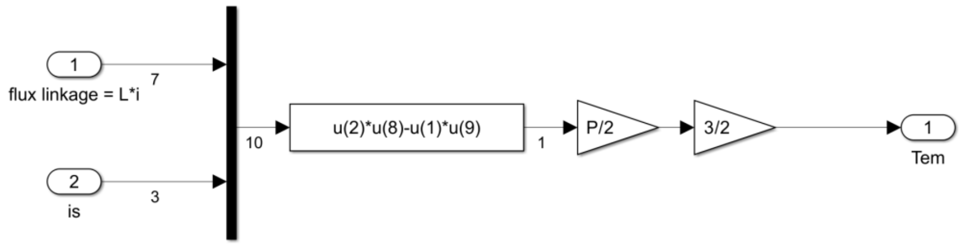


Fig 6: Torque Calculation Block

6. Speed Calculation:

$$T_L + T_{em} - T_f = J\frac{d^2}{dt^2}(\theta_m)$$

$$\theta_m = \frac{2}{P}(\theta_r)$$

$$T_L + T_{em} - T_f = J\frac{d^2}{dt^2}\left(\frac{2}{P}(\theta_r)\right)$$

$$\theta_r = \iint T_L + T_{em} - T_f$$

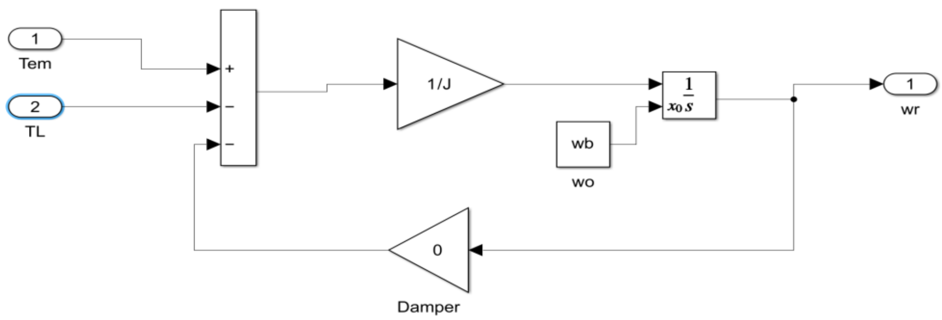


Fig 7: Speed Calculation Block

Here the initial speed is the base speed of the generator:

$$\omega_b = \omega \times \frac{2\pi}{60}$$

7. Torque Angle

From the vector diagram, The Torque angle δ is computed as:

$$\delta = \tan^{-1}\frac{V_d}{Vq}$$

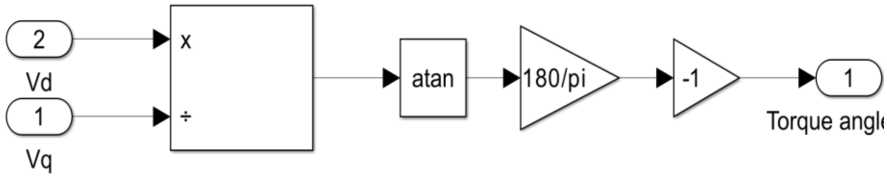


Fig 8: Torque angle calculation Block

8. The Model of the synchronous generator on MATLAB/SIMULINK

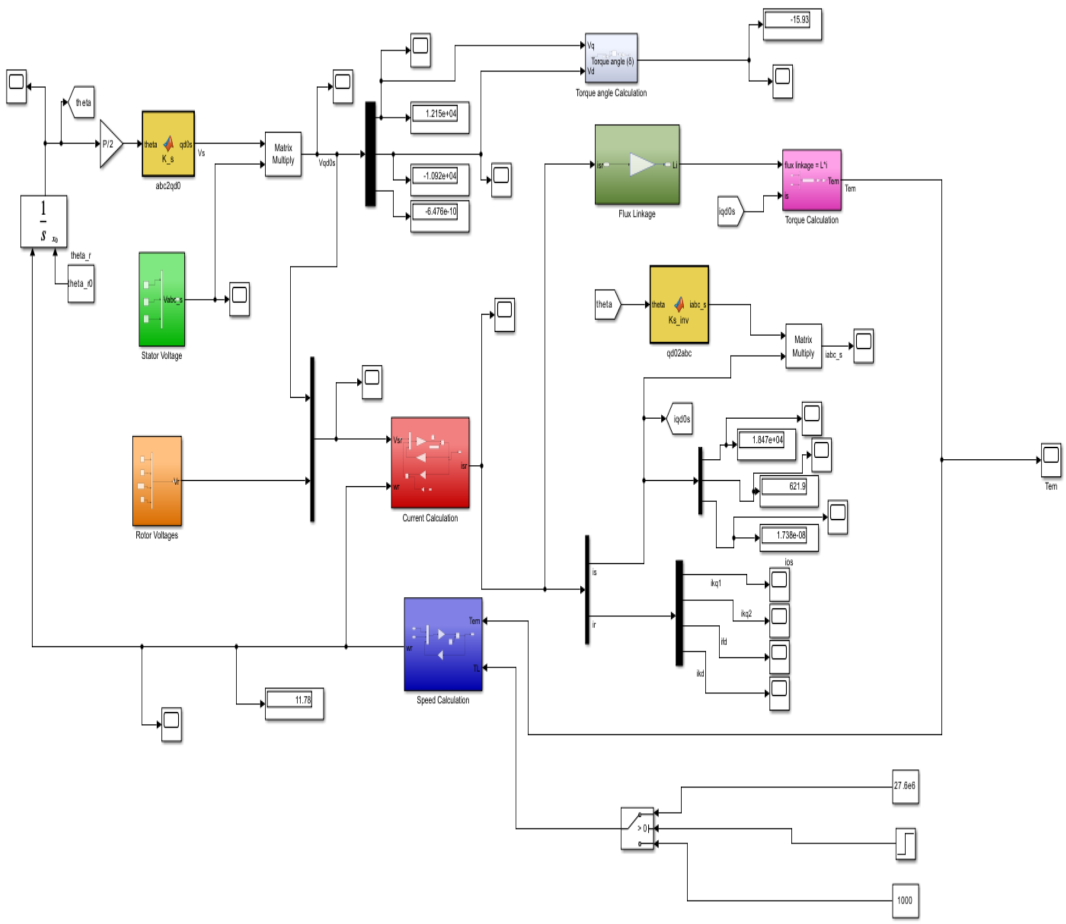


Fig 9: Simulink Model of a Synchronous Generator

Results:

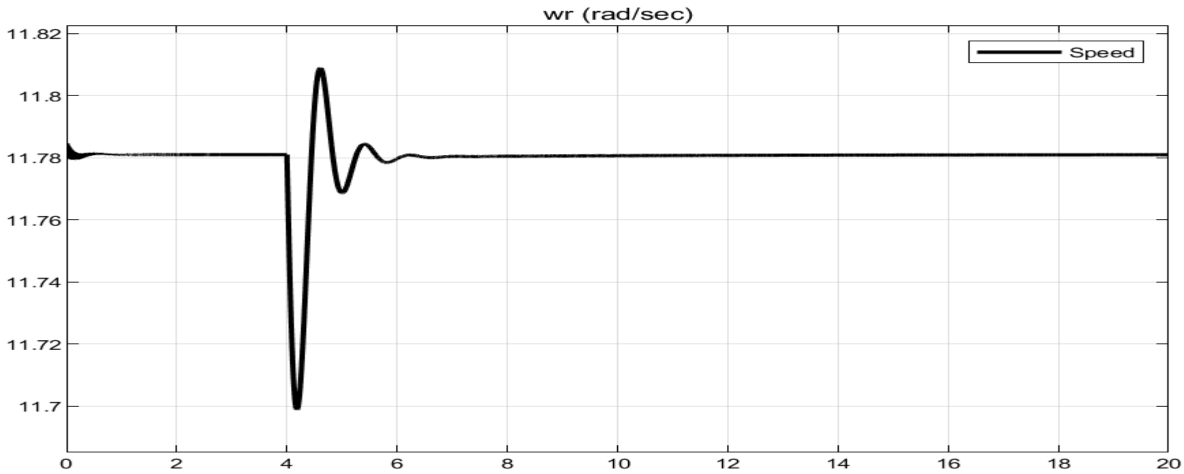


Fig 10: Speed graph on simulation

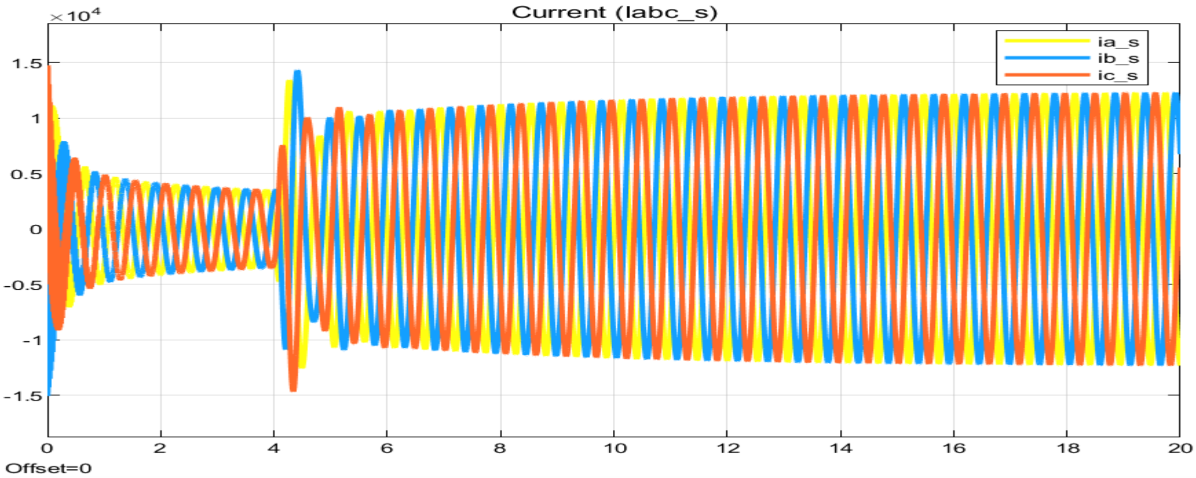


Fig 11: Stator Currents graph

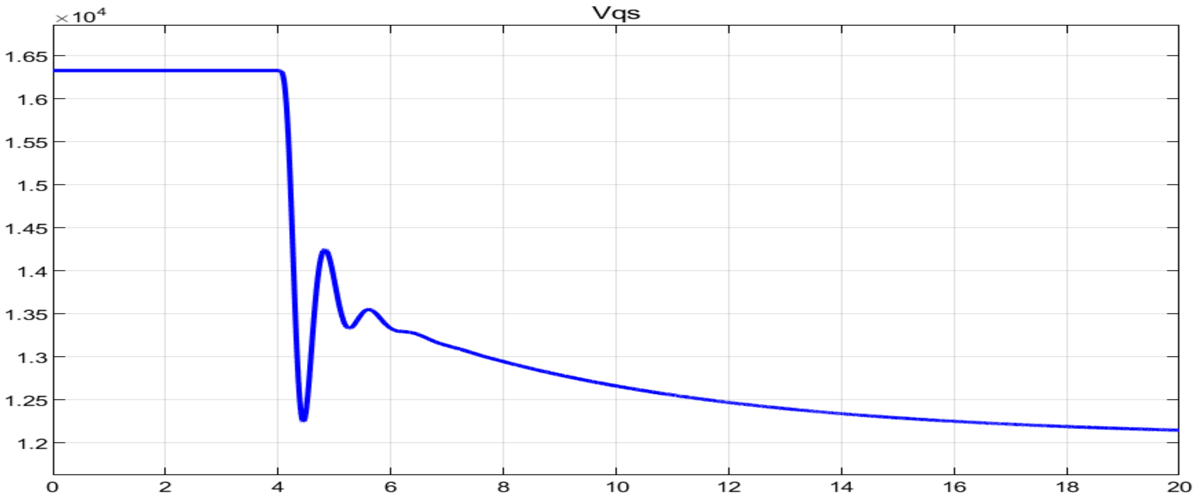


Fig12: graph of Vqs

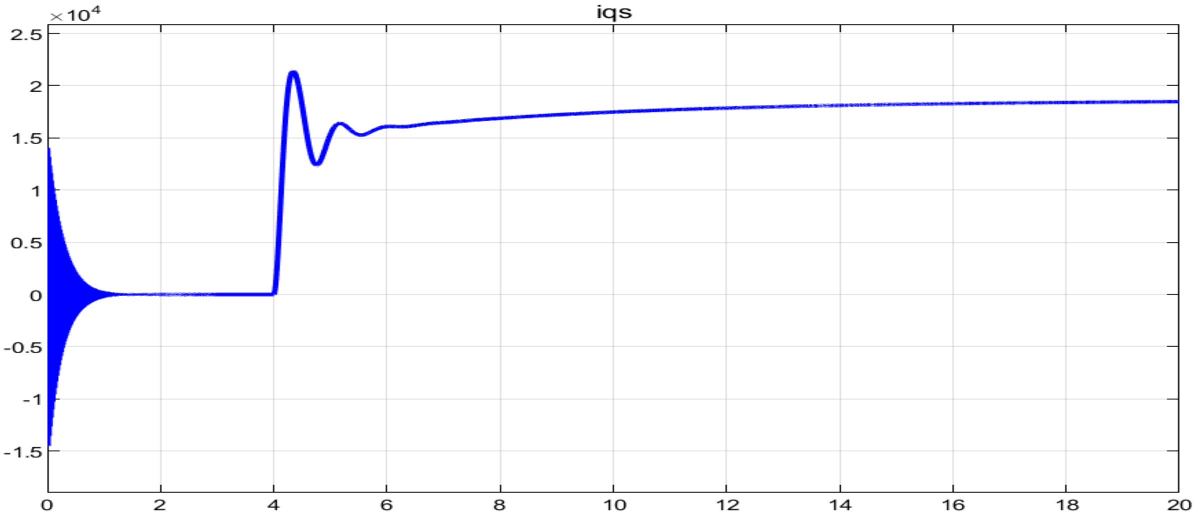


Fig13: graph of iqs

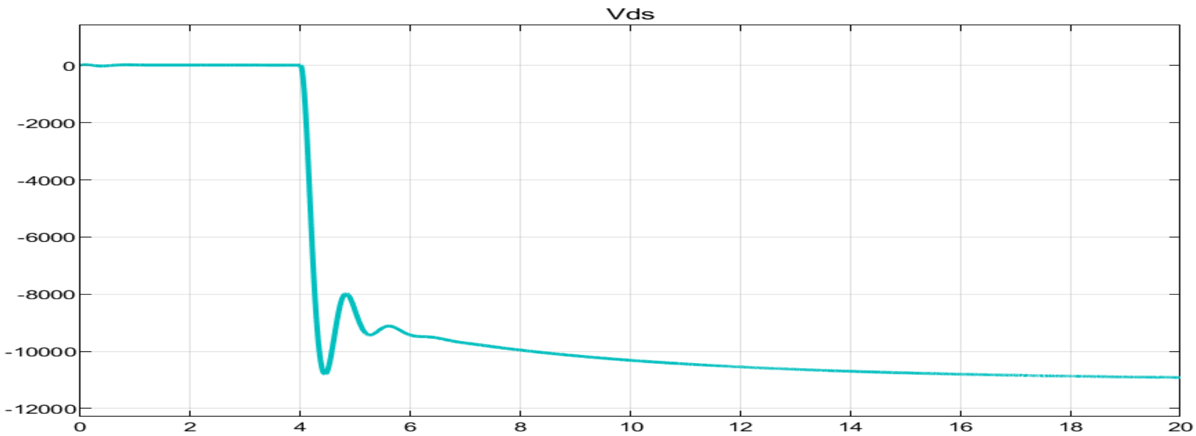


Fig14: graph of Vds

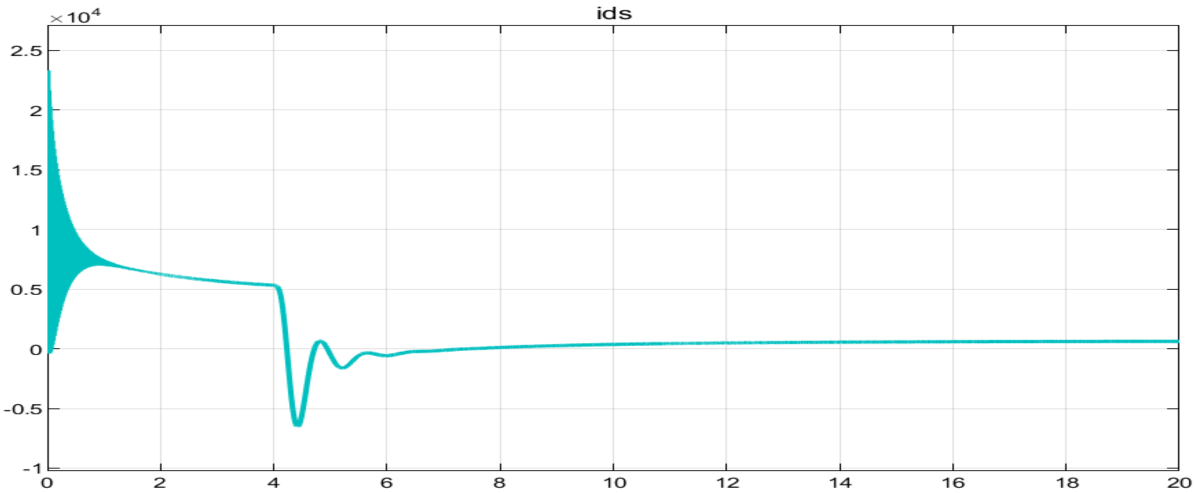


Fig15: graph of ids

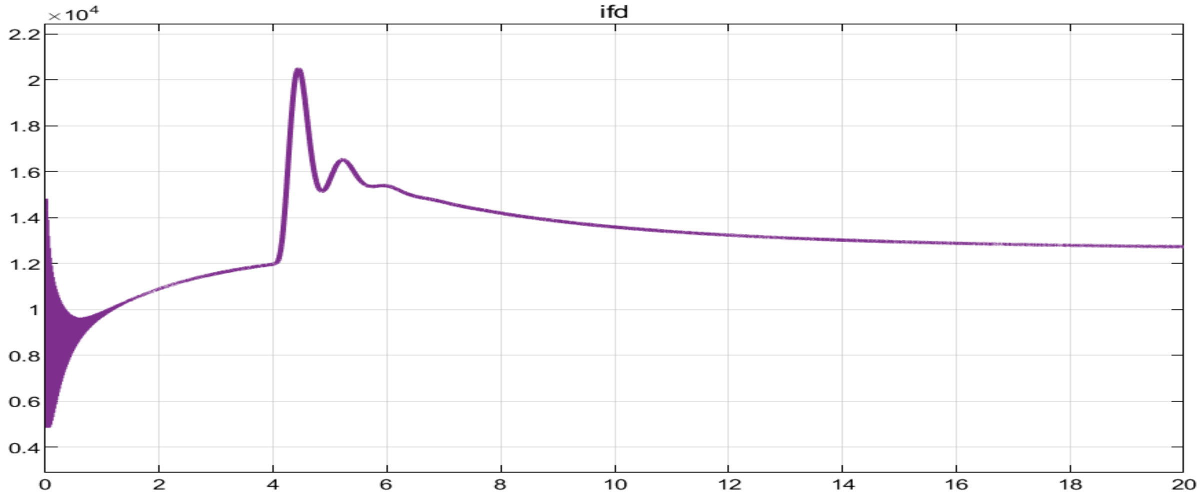


Fig16: graph of ifd

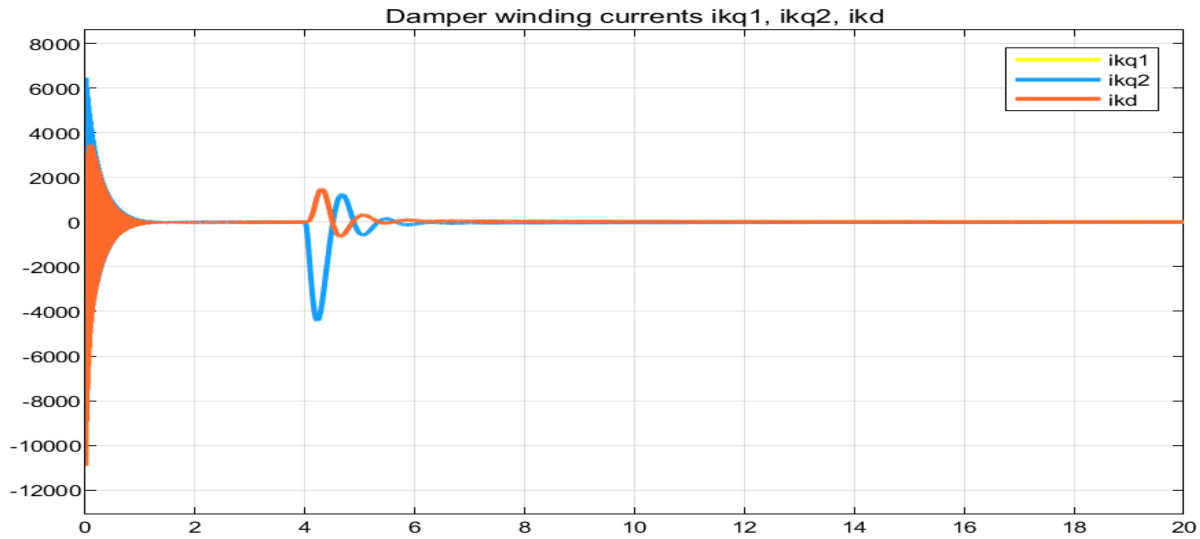


Fig 17: Damper winding current graph

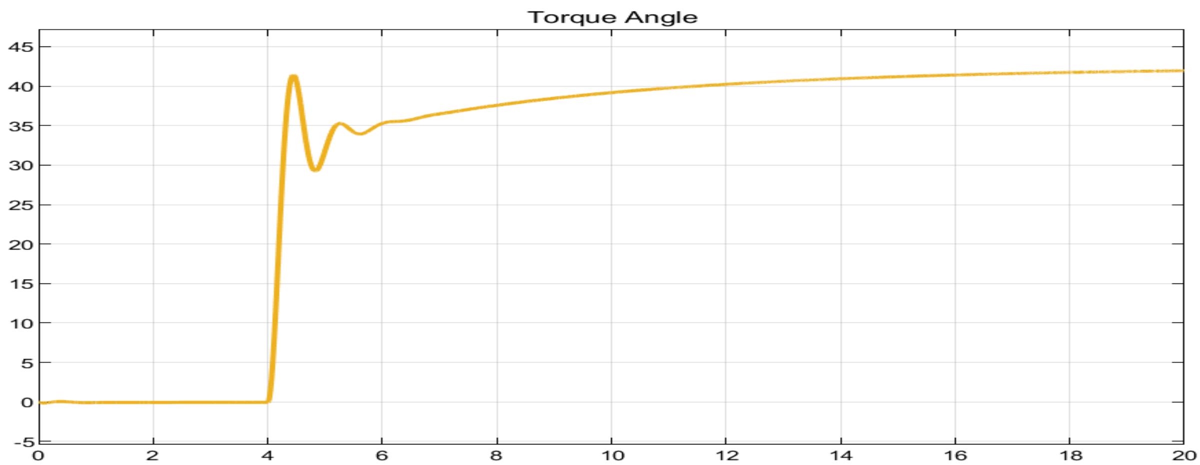


Fig 18: Torque angle (δ) graph

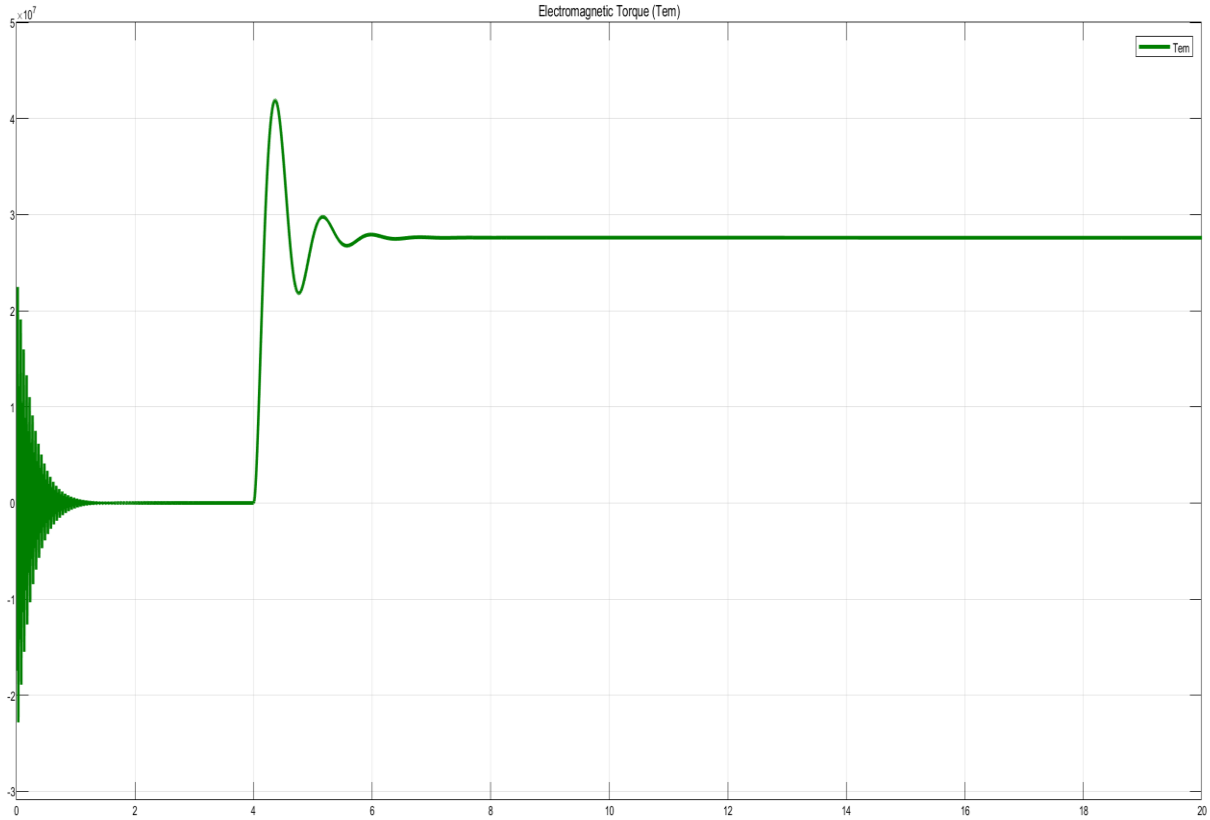


Fig 19: Electromagnetic Torque graph

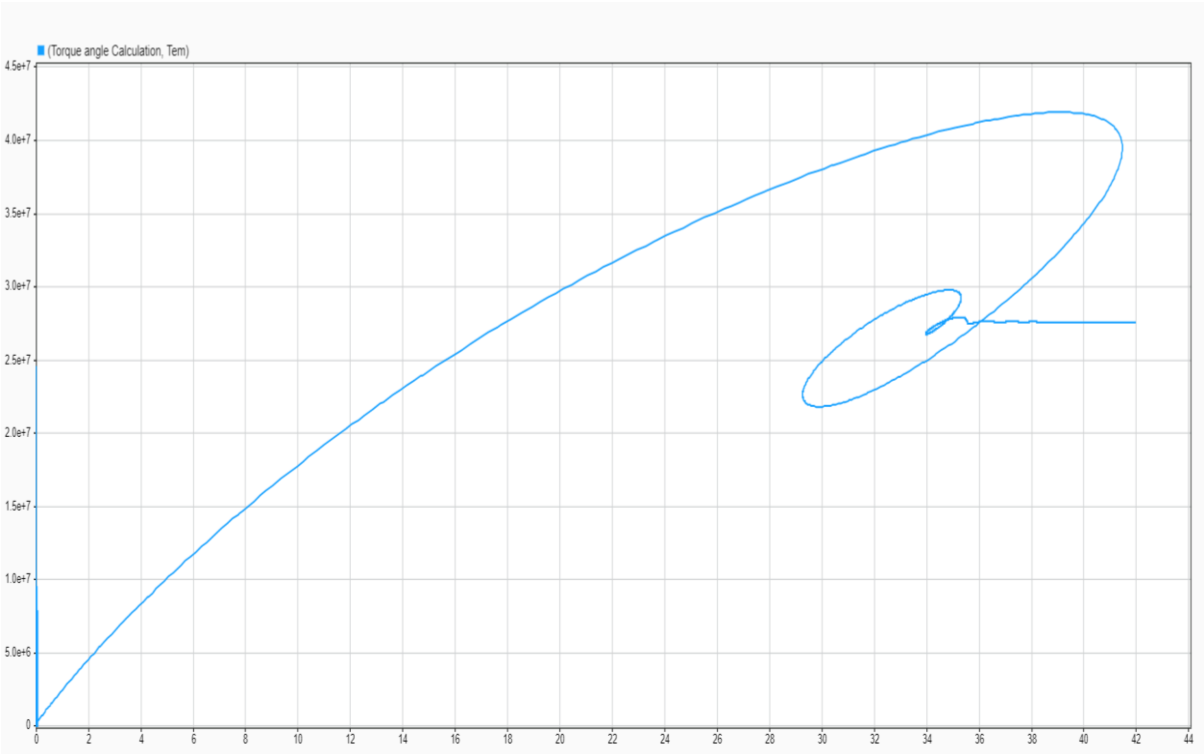


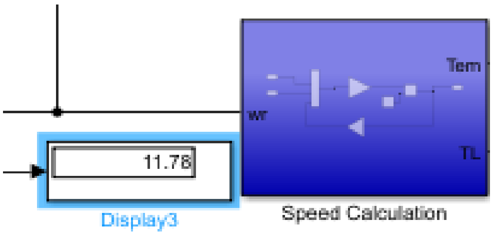
Fig 20: Torque Vs Rotor angle Characteristics

Observations:

1. Rated Speed Observation:

Theoretical Speed: At steady state, the synchronous generator is rated to operate at a speed of 112.5 revolutions per minute, equivalent to 11.78 radians per second.

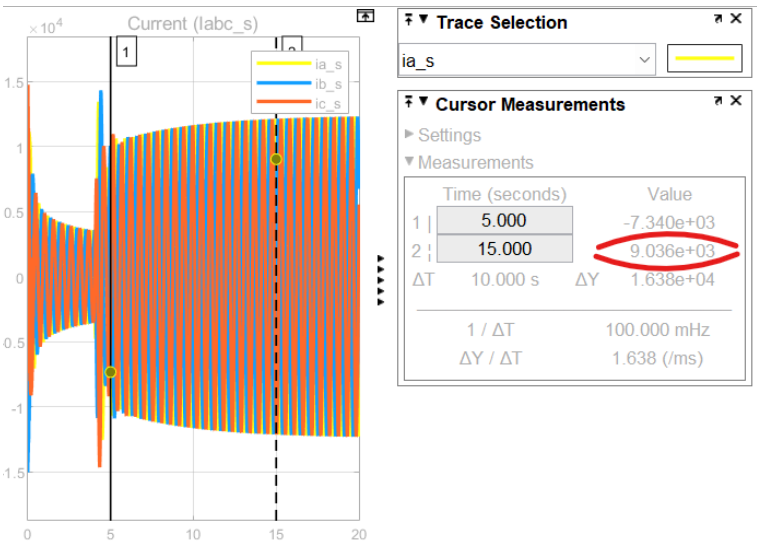
Simulation Results: The simulation confirms that the rated speed of the generator matches the theoretical value of 11.78 radians per second.



2. RMS Current Analysis:

Calculated RMS Current: Under steady state conditions, the maximum root mean square (RMS) current referred to the stator is calculated to be 9.37×10^3 amperes.

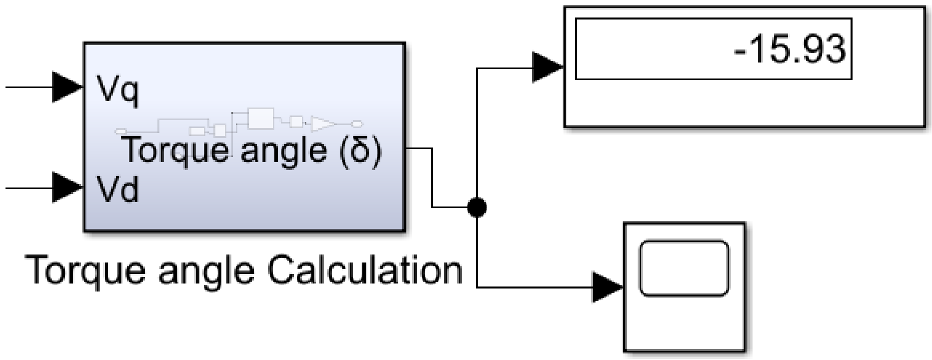
Simulated RMS Current: The simulation shows a slightly lower RMS current of 9.04×10^3 amperes, which is approximately close to the calculated value, indicating a high level of accuracy in the simulation model.



3. Torque Angle Comparison:

Calculated Torque Angle: The theoretical calculation of the torque angle under steady conditions is 18 degrees.

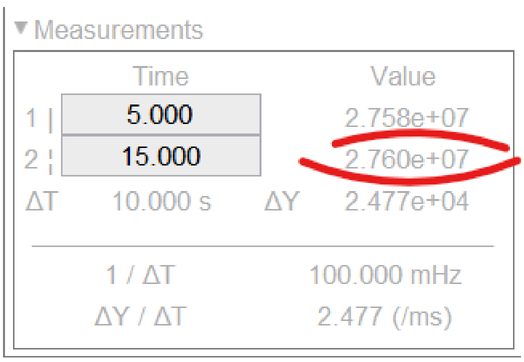
Simulated Torque Angle: The simulation result of the torque angle is approximately 16 degrees. The small discrepancy is within acceptable limits, and the negative sign of the torque angle indicates a generating condition.



4. Electromagnetic Torque Observation:

Theoretical Speed: At steady state, the synchronous generator is rated to electromagnetic Torque is $23.4 \times 10^6 N.m$.

Simulation Results: The simulation confirms that the electromagnetic Torque of the generator is $27.6 \times 10^6 N.m$ which is still within acceptable limit.



Discussion:

When the generator is subjected to a step increase in input torque from zero to $27.6 \times 10^6\ N.m$. The rotor speed begins to increase immediately, following the step increase in input torque, the rotor angle increases.

The rotor speeds up until the accelerating torque on the rotor is zero. The speed increases to approximately 380 electrical radians per second, at which torque Te is equal to T, because the change of ω, is zero and hence the inertial torque is zero.

Even though the accelerating torque is zero at this time, the rotor is running above synchronous speed; hence δ, and thus Te, will continue to increase. The increase in Te, which is an increase in the power output of the machine, causes the rotor to decelerate toward synchronous speed.

Appendix

$$I_a = \frac{325\times10^6}{3\times\dfrac{20\times10^3}{\sqrt{3}}} = 9.37\times10^3; \quad \cos^{-1}(0.85) = 31.8^\circ$$

$$\Rightarrow \tan(\delta) = -\frac{0.00234\times(-9.37\times1000)\sin(-31.8^\circ)+0.5911\times(-9.37\times1000)0.85}{\dfrac{20\times10^3}{\sqrt{3}}-0.00234(-9.37\times1000)0.85+0.5911\times(-9.37\times1000)\sin(-31.8^\circ)}$$

$$\Rightarrow \delta = 18^\circ$$

$$|E_q|e^{j\delta} = V_m - (r_s + j\omega_e L_q)(I_m\cos(\varphi) + jI_m\sin(\varphi))$$

$$= \frac{20\sqrt{2}\times10^3}{\sqrt{3}} - (0.00234 + j0.5911)(-9.37\sqrt{2}\times10^3\measuredangle-31.8^\circ) = 15.2\sqrt{2}\times10^3\measuredangle18^\circ$$

$$E_q = \omega_e(L_d - L_q)i_d + E_f \Rightarrow E_f = 15.2\sqrt{2}\times10^3 - (1.0467-0.5911)\times$$

$$\times(-9.37\sqrt{2}\times10^3)\sin(18^\circ+31.8^\circ) = 26.1\times10^3$$

$$T_{em} = \left(\frac{3\times64}{2\times377}\right)\left\{\frac{\frac{26.1}{\sqrt{2}}\times11.54}{1.0467}\sin(18^\circ) + \frac{11.54^2}{2}\left(\frac{1}{0.5911}-\frac{1}{1.0467}\right)\sin(36^\circ)\right\}\times10^6$$

$$= 23.4\times10^6\quad Nm$$

References

[1] C.-M. ONG, *Dynamic Simulation Of Electric Machinery using MATLAB/SIMULINK*.
[2] P. C. Krause, O. Wasynczuk, and S. D. Sudhoff, *Analysis of Electric Machinery and Drive Systems*, Second Edition. IEEE press.