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State of Charge Estimation of Li-ion Battery

Using Luenberger, Sliding Mode and Super twisting Observers

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Outline

Introduction

- **Context:**
  - Importance of electric vehicles (EVs) for reducing greenhouse gas emissions.
  - Role of lithium-ion batteries (LIBs) in EVs.
  - Importance of SOC estimation for battery management systems (BMS).
- **Problem Statement:**
  - Challenges in accurate SOC estimation due to nonlinearities in LIBs.
- **Objective:**
  - Comparative analysis of three observers: Luenberger, Sliding Mode (SMO), and Super Twisting (STO).

Background: Definition and Role of SOC

- **Definition and role of SOC:** The ratio of the remaining energy capacity (E_{cr}) and the actual energy capacity (E_{ca}) of the battery,

$$\mathrm{SOC}(t) = \frac{E_{cr}}{E_{ca}} \times 100. \tag{1}$$

Similar to a fuel gauge in traditional vehicles, SOC provides actionable insights for energy use. Since no sensor can measure SOC, it has to be estimated from physical measurements by some mathematical algorithm. [1].

Background: Battery Modeling

- **Battery modeling:**

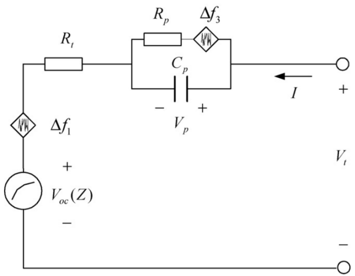


Figure: Resistor-capacitor electrical model of lithium-ion battery [1].

$$V_t = V_{oc}(Z) + IR_t + V_p. \tag{2}$$

Battery Modeling (continued)

- Here, we continue with more equations or details about the battery model:

$$
\begin{aligned}
\dot{V}_t &= -a_1 V_t + a_1 V_{oc}(Z) + b_1 I, \\
\dot{Z} &= a_2 V_t - a_2 V_{oc}(Z) - a_2 V_p, \\
\dot{V}_p &= -a_1 V_p + b_2 I, \\
y &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_t \\ Z \\ V_p \end{bmatrix},
\end{aligned} \tag{3}
$$

where $a_1 = 1/(R_t C_p)$, $a_2 = 1/(R_t C_n)$,
$b_1 = k/C_n + R_t/(R_p C_p) + 1/C_p$, and $b_2 = 1/C_p$.

Battery Modeling (continued)

- Here, simulation diagram of the battery model:

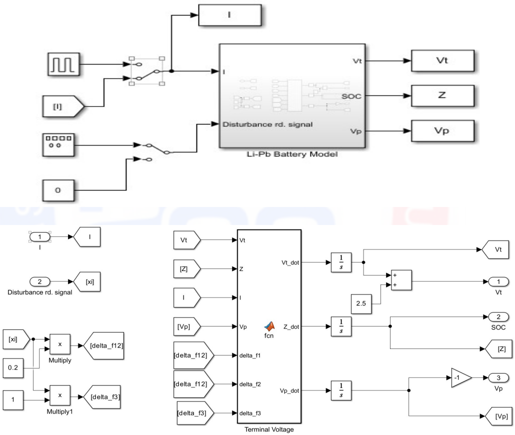


Figure: Li-Po Simulation on MATLAB/SIMULINK

Methodology: Description of each observers

- **Luenberger Observer:** it relies on a mathematical model of the system and feedback tracking of the estimation error. Mathematically:

$$\dot{\hat{x}} = A\hat{x} + Bu + H(y - \hat{y}), \tag{4}$$

Using the state-space representation of the lithium-ion battery model in equation 3. The Luenberger Observer introduces a correction term to the dynamics as in equation 4. Expanding the observer equations we have:

$$\begin{aligned} \dot{\hat{V}}_t &= -a_1 \hat{V}_t + a_1 k \hat{Z} + b_1 I + h_1(V_t - \hat{V}_t), \\ \dot{\hat{Z}} &= a_2 \hat{V}_t - a_2 k \hat{Z} - a_2 \hat{V}_p + h_2(V_t - \hat{V}_t), \\ \dot{\hat{V}}_p &= -a_1 \hat{V}_p + b_2 I + h_3(V_t - \hat{V}_t). \end{aligned} \tag{5}$$

Luenberger Observer (continued)

The final estimated state equations for the battery model :

$$\begin{aligned}
\dot{\hat{V}}_t &= -a_1\hat{V}_t + a_1 k\hat{Z} + b_1 I + h_1 e_1, \\
\dot{\hat{Z}} &= a_2\hat{V}_t - a_2 k\hat{Z} - a_2\hat{V}_p + h_2 e_1, \\
\dot{\hat{V}}_p &= -a_1\hat{V}_p + b_2 I + h_3 e_1,
\end{aligned} \tag{6}$$

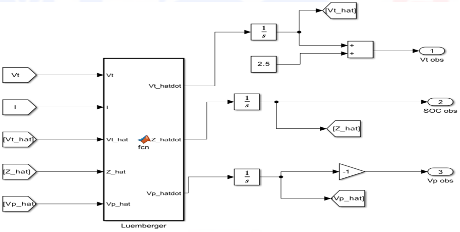


Figure: Modelling of Luenberger Observer

Description of each observers (continued)

- **Sliding Mode Observer Observer:** its robust tracking performance against modeling uncertainties and disturbances, including noise, for state estimation in complex systems [1]. A sliding mode observer for the system Eq. 3 is

$$\dot{\hat{x}} = A\hat{x} + Bu + H\,sign(y - \hat{y}) \tag{7}$$

The estimated state equations for the battery model uisng the sliding mode observer algorithm is:

$$\begin{aligned} \dot{\hat{V}}_t &= -a_1\hat{V}_t + a_1K\hat{Z} + b_1I + h_1\,sign(\text{err}), \\ \dot{\hat{Z}} &= a_2\hat{V}_t - a_2K\hat{Z} - a_2\hat{V}_p + h_2\,sign(\text{err}), \\ \dot{\hat{V}}_p &= -a_1\hat{V}_p + b_2I + h_3\,sign(\text{err}). \end{aligned} \tag{8}$$

Sliding Mode Observer (continued)

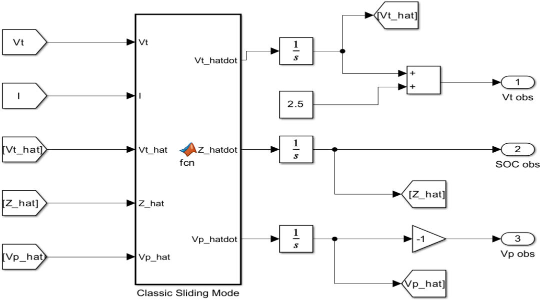


Figure: Modelling of Luenberger Observer

Description of each observers (continued)

- **Super Twisting Observer Observer:** In order to eliminate the chattering phenomenon from conventional SMO, several methodologies are suggested in the literature of sliding mode control, ST algorithm is one of them.

$$
\begin{aligned}
\dot{\hat{V}}_t &= -a_1\hat{V}_t + a_1 k\hat{Z} + b_1 I + h_1|e_1|^{2/3}\mathrm{sign}(e_1),\\
\dot{\hat{Z}} &= a_2\hat{V}_t - a_2 k\hat{Z} - a_2\hat{V}_p + h_2|e_1|^{1/3}\mathrm{sign}(e_1),\\
\dot{\hat{V}}_p &= -a_1\hat{V}_p + b_2 I + h_3\mathrm{sign}(e_1).
\end{aligned}
\tag{9}
$$

Super Twisting Mode Observer (continued)

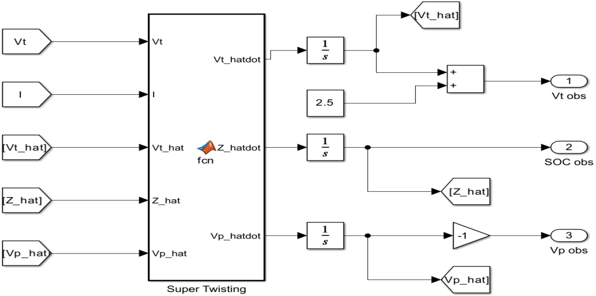


Figure: Modelling of Luenberger Observer

Methodology: Simulation setup

- **Models used:**
  1. CONSTANT CURRENT DISCHARGE PROFILE: The discharge current of $5\,\mathrm{A}$ is applied to the battery for a period of $360\,\mathrm{s}$, leaving $600\,\mathrm{s}$ for rest.

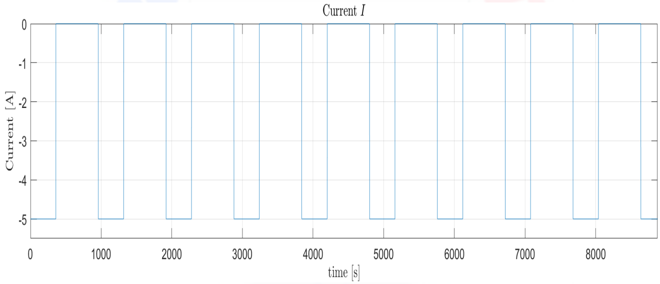


Figure: Terminal voltage and current curves for the cell model

Methodology: Simulation setup

- **Models used:**
  2. UDDS Current Profile: The Urban Dynamometer Driving Schedule (UDDS) cycle is used to simulate a more realistic discharge/charge current profile.

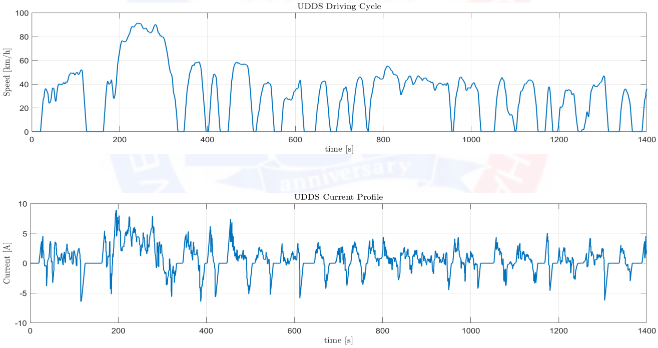
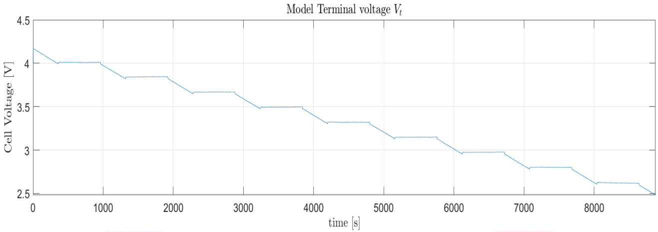


Figure: One UDDS cycle with corresponding current profile.

Results: Battery Model



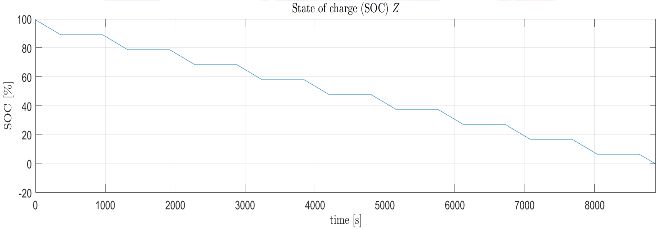


Figure: V_t and Soc plots for the cell model

Results: Luenberger Estimation Under CCD

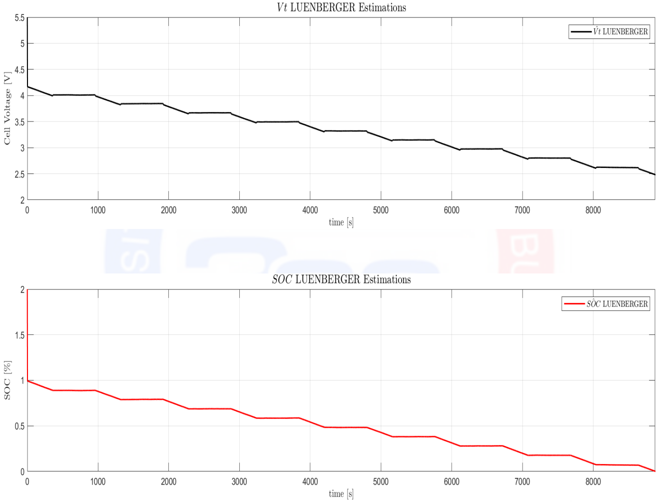


Figure: V_t and Soc luenberger estimation plots

Results: SMO Estimation Under CCD

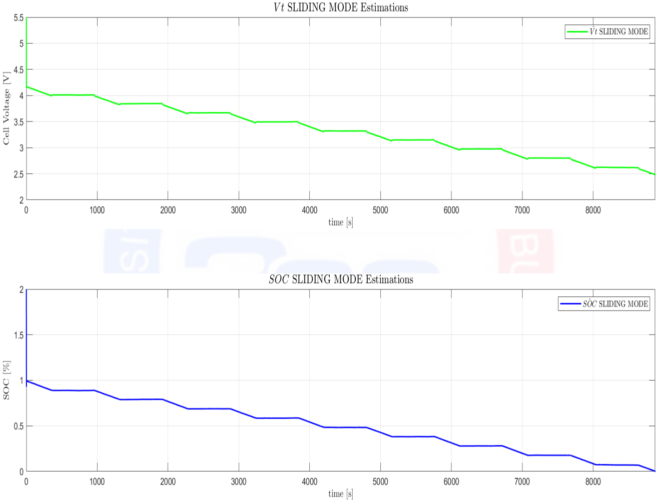


Figure: V_t and Soc SMO estimation plots

Results: STO Estimation Under CCD

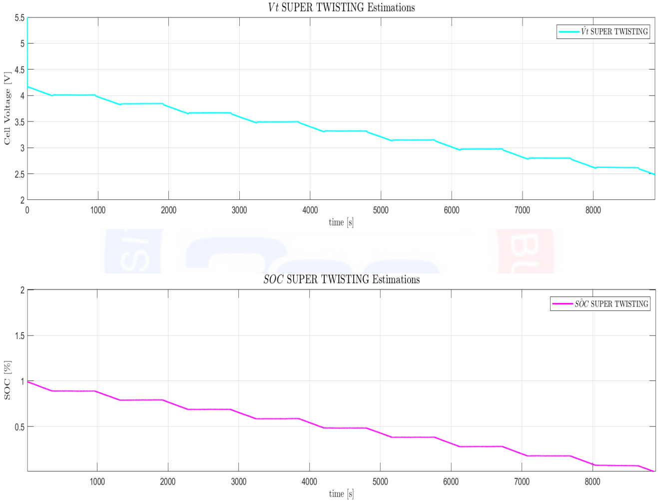


Figure: V_t and Soc STO estimation plots

Results: Luenberger Estimation Under UDDS

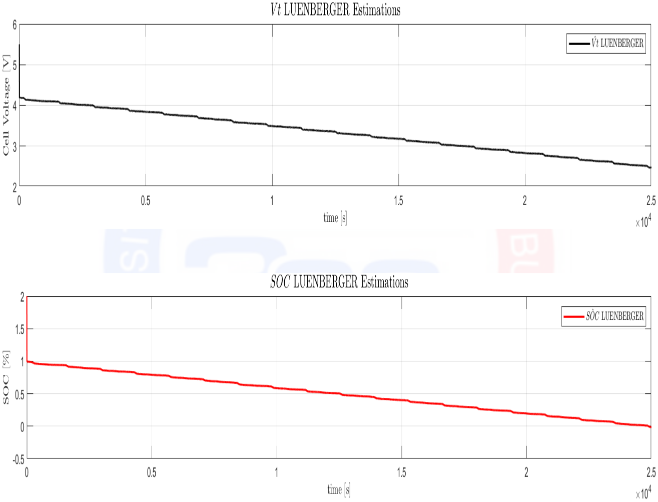


Figure: V_t and Soc luenberger estimation plots under UDDS Profile

Results: SMO Estimation Under UDDS

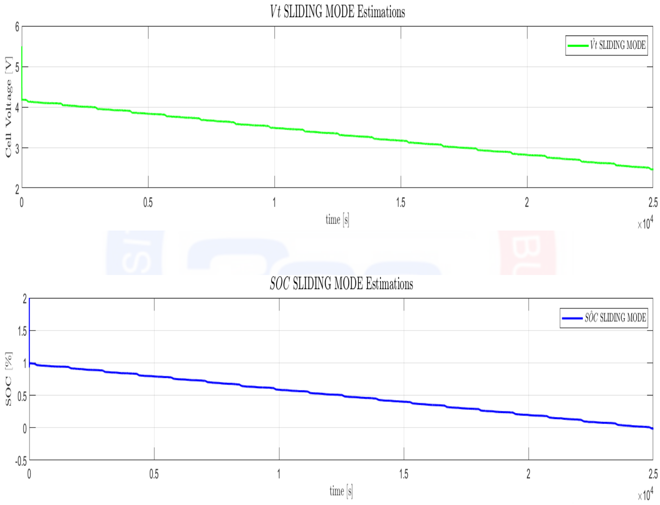


Figure: V_t and Soc SMO estimation plots under UDDS profile

Results: STO Estimation Under UDDS

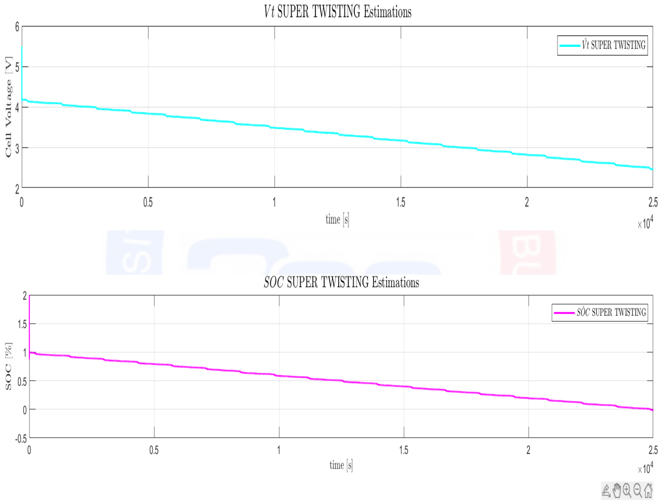


Figure: V_t and Soc STO estimation plots under UDDS profile

Results: Convergence of the three observers

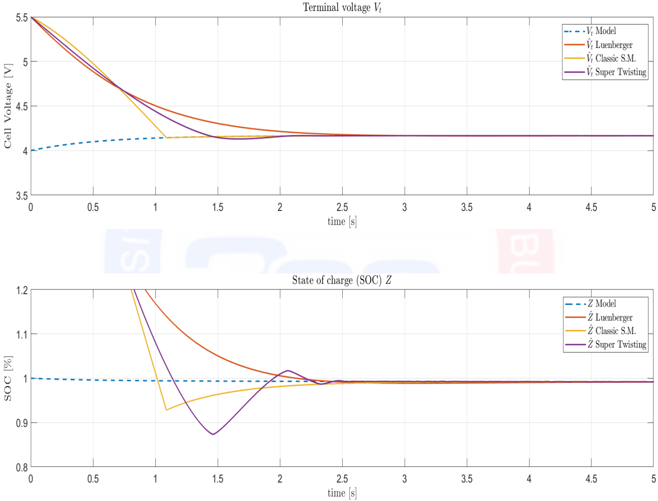


Figure: Luenberger, SMO, and STO convergence curves.

Results: Chattering of STO

Figure below presents a close look at the convergence curves of the SOC estimation. In the classic SMO, the curve has a small chattering as expected (simulation sampled time 1 ms), and furthermore, the super twisting observer complies with the proposal to reduce the chattering caused by the sign$(y - \hat{y})$ function.

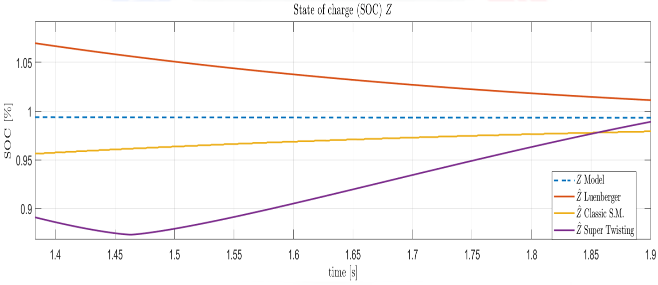


Figure: SMO chattering in SOC estimation.

Results: Estimation Error

Integral Squared Error (ISE)

$$ISE = \int e^2 \, dt. \tag{10}$$

Integral Absolute Error (IAE)

$$IAE = \int |e| \, dt. \tag{11}$$

Table: Estimation errors comparison for the constant current profile

	Luenberger		SMO		STO	
	ISE	IAE	ISE	IAE	ISE	IAE
V_t	0.8438	1.4136	0.8957	2.9851	0.8577	0.9673
Z	0.3052	0.8679	0.3356	2.0064	0.3258	1.8184
V_p	0.3017	2.0284	0.3017	2.0276	0.3018	2.1951

Results: Estimation Error (continued)

Table: Estimation errors comparison for UDDS current profile

	Luenberger		SMO		STO	
	ISE	IAE	ISE	IAE	ISE	IAE
V_t	0.8438	2.1084	0.8971	6.8321	0.8577	1.0472
Z	0.3053	1.4025	0.3362	4.6641	0.3263	4.0959
V_p	0.3023	4.5814	0.3023	4.5792	0.3026	5.0604

Metric	**Luenberger Observer**	**Sliding Mode Observer**	**Super Twisting Observer**
Convergence Speed	Moderate	Fast	Moderate
Robustness	Low	High	High
Chattering	None	High	None

Table: Performance comparison of observers.

Conclusion

Accurate SoC estimation is important for EV batteries.
Among the different observers tested, SMO demonstrated the fastest convergence, followed by STO (Super Twisting Observer). Although the Luenberger observer showed lower error values in some cases, its limitations with nonlinearities and uncertainties make it less suitable for real applications. For online SOC estimation, SMO and STO are better because of their high-frequency response and adaptability.

References

I.-S. Kim, "The novel state of charge estimation method for lithium battery using sliding mode observer," *Journal of Power Sources*, vol. 163, no. 1, pp. 584–590, 2006.

Thank You!

Gracias ▪ Merci ▪ Danke ▪ Grazie ▪ multumesc ▪ Bedankt ▪ Obrigado ▪ Kiitos

For your attention!