2.10.1

(1)

$\overline{y} = \frac{\sum_{i=1}^{m} y_0 i + \sum_{i=m+1}^{2m} y_1 i}{2m}$
$\overline{y}_0 + \overline{y}_1 = \frac{1}{m}\sum_{i=1}^{m} y_0 i + \frac{1}{m}\sum_{i=m+1}^{2m} y_1 i = \frac{1}{m}(\sum_{i=1}^{m} y_0 i + \sum_{i=m+1}^{2m} y_1 i)$
$\therefore \overline{y} = \frac{\overline{y}_0 + \overline{y}_1}{2}$

(2)

$\overline{x} = \frac{\sum_{i=1}^{m} x_i + \sum_{i=m+1}^{2m} x_i}{2m} = \frac{0+1*m}{2m} = \frac{1}{2}$

(3)

SXX $= \sum_{i=1}^{2m}(x_i - \overline{x})x_i = \sum_{i=1}^{m}(x_i - \frac{1}{2})x_i + \sum_{i=m+1}^{2m}(x_i - \frac{1}{2})x_i = 0 + \frac{1}{2}m = \frac{m}{2}$

(4)

SXY $= \sum_{i=1}^{2m}(x_i - \overline{x})y_i = \sum_{i=1}^{m}(x_i - \frac{1}{2})y_0 i + \sum_{i=m+1}^{2m}(x_i - \frac{1}{2})y_1 i = \sum_{i=1}^{m} -\frac{1}{2}y_0 i + \sum_{i=m+1}^{2m}\frac{1}{2}y_1 i = -\frac{1}{2}\overline{y}_0 m + \frac{1}{2}\overline{y}_1 m = \frac{m(\overline{y}_1 - \overline{y}_0)}{2}$

2.10.2

$\hat{\beta}_1 = \frac{SXY}{SXX} = \frac{\frac{m(\overline{y}_1 - \overline{y}_0)}{2}}{\frac{m}{2}} = \overline{y}_1 - \overline{y}_0$

$\hat{\beta}_0 = \overline{y} - \hat{\beta}_1 \overline{x} = \frac{\overline{y}_0 + \overline{y}_1}{2} - (\overline{y}_1 - \overline{y}_0)\overline{x} = \overline{y}_0$

OLS estimators minimize the residual sum of squares function, which least the inherent asymmetry in the roles of the response and the predictor in this problem.

2.10.3

For $x_i = 0$,where i = 1,$\cdots$,m, all fitted values = $\hat{\beta}_0 + \hat{\beta}_1 x_i = \overline{y}_0 + (\overline{y}_1 - \overline{y}_0)x_i = \overline{y}_0$. Residuals = $y_i - \overline{y}_0$,where i = 1, $\cdots$, m.

For $x_i = 1$,where i = m+1,$\cdots$,2m, all fitted values = $\hat{\beta}_0 + \hat{\beta}_1 x_i =$

$\overline{y}_0 + (\overline{y}_1 - \overline{y}_0)x_i = \overline{y}_1$. Residuals $= y_i - \overline{y}_1$,where i $= m+1, \cdots, 2m$.

RSS $= \sum_{i=1}^{2m}(y_i - \overline{y}_0)^2 + \sum_{i=m+1}^{2m}(y_i - \overline{y}_1)^2 = SYY_0 + SYY_1$

$\hat{\sigma}^2 = \frac{RSS}{m_0+m_1-2} = \frac{SYY_0+SYY_1}{m_0+m_1-2}$

2.10.4

t-statistic for test $\beta_1 = 0$: $\frac{\hat{\beta_1}}{\sigma\sqrt{\frac{1}{SXX}}} = \frac{\overline{y}_1-\overline{y}_0}{\sqrt{\frac{RSS}{m_0+m_1-2}}\sqrt{\frac{1}{SXX}}} = \frac{\overline{y}_1-\overline{y}_0}{\sqrt{\frac{RSS}{m_0+m_1-2}}\sqrt{\frac{2}{m}}} = \frac{\overline{y}_1-\overline{y}_0}{\sqrt{\frac{RSS}{m_0+m_1-2}}\sqrt{\frac{m_0+m_1}{m_0m_1}}} = \frac{\overline{y}_1-\overline{y}_0}{\hat{\sigma}\sqrt{\frac{1}{m_0}+\frac{1}{m_1}}}$

The usual two-sample t-test for comparing two groups with an assumption of equal within-group variance:$\frac{\overline{y}_1-\overline{y}_0}{\sqrt{\frac{(m_0-1)SD_0{}^2+(m_1-1)SD_1{}^2}{m_0+m_1-2}}\sqrt{\frac{m_0+m_1}{m_0m_1}}} = \frac{\overline{y}_1-\overline{y}_0}{\sqrt{\frac{(m_0-1)SD_0{}^2+(m_1-1)SD_1{}^2}{m_0+m_1-2}}\sqrt{\frac{2}{m}}}$

$\because (m_0-1)\frac{SYY_0}{m_0-1} + (m_1-1)\frac{SYY_1}{m_1-1} = SYY_0 + SYY_1 = RSS$

$\because$ they are the same.