1 Geomatric explaination of matrix

1.1 Two geometric ways understanding matrix

For linear equations:

$$
\begin{cases} 2x - y = 0 \\ -x + 2y = 3 \end{cases} \tag{1}
$$

We can write them in matrix multiplication form:

$$
\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix} \tag{2}
$$

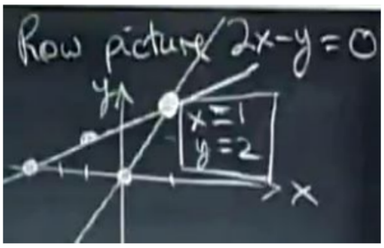
where $\boldsymbol{A}$ is the coefficient matrix, $\boldsymbol{x}$ is the unknown variable vector, $\boldsymbol{b}$ is the result vector, written as $\boldsymbol{Ax} = \boldsymbol{b}$.

Such kind of form can be understood from two geometric ways:

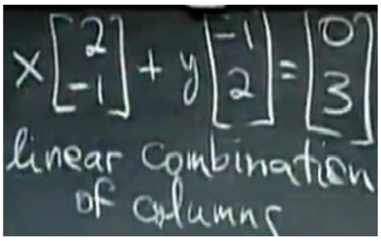
(1) Row picture form: It plots every row function. In two dimentional example they are two linear equations, each with two unknown variables, i.e. two lines.

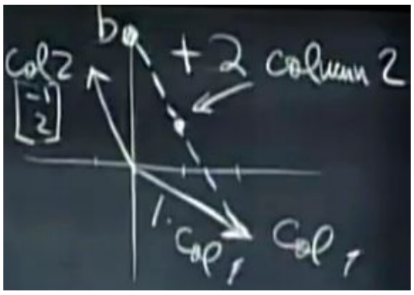
(2) Column picture form: The plot is presented in column vector way. The purpose is to find a linear combination that gives the column $\boldsymbol{b}$. Applying all possible combinations of x and y produces the two dimentional plane. But in this case there is only a unique set combination produces $\boldsymbol{b}$.

Row picture

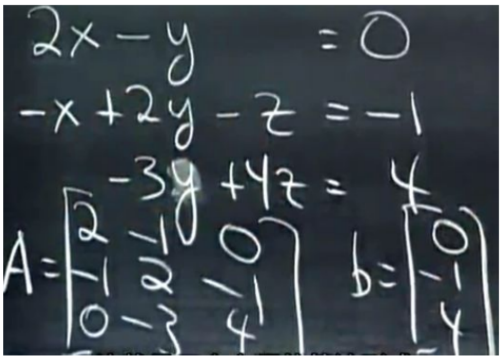


Col picture





Three dimentions



The above interpretation applies to higher dimentional examples.

1.2 The roots of equation sets

Taking the above three dimentional equation sets as example: when $\boldsymbol{b}$ is unknown (and the $\boldsymbol{Ax}$ remain unchanged), whether there is solution for every $\boldsymbol{b}$?

From the column picture explaination, this question is an equavlent of: whether the linear combinations of columns in $\boldsymbol{A}$ fill up a three dimentional space?

Answer: The above case is non-singular matrix, so YES. For general case, the equation sets only has solution when $\boldsymbol{b}$ is in the column space spaned by $\boldsymbol{A}$.

There are many benefits to consider matrix multiplication in column picture way.

1.3 Appendix