

Linear Algebra

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1 Geomatric explanation of matrix

1.1 Two geometric ways understanding matrix

For linear equations:

$$\begin{cases} 2x - y = 0 \\ -x + 2y = 3 \end{cases} \quad (1)$$

We can write them in matrix multiplication form:

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix} \quad (2)$$

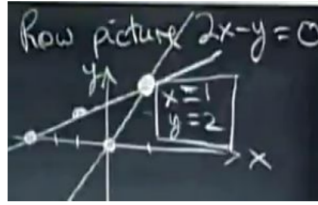
where \mathbf{A} is the coefficient matrix, \mathbf{x} is the unknown variable vector, \mathbf{b} is the result vector, written as $\mathbf{Ax} = \mathbf{b}$.

Such kind of form can be understood from two geometric ways:

- (1) Row picture form: It plots every row function. In two dimentional example they are two linear equations, each with two unknown variables, i.e. two lines.
- (2) Column picture form: The plot is presented in column vector way. The purpose is to find a linear combination that gives the column \mathbf{b} . Applying all possible combinations of x and y produces the two dimentional plane. But in this case there is only a unique set combination produces \mathbf{b} .

Figure 1 interpretation applies to higher dimentional examples.

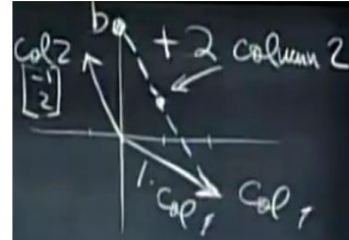
Row
picture



Col
picture

$$x \begin{bmatrix} 2 \\ -1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

linear combination
of columns



Three
dimentions

$$\begin{aligned} 2x - y &= 0 \\ -x + 2y - z &= -1 \\ -3y + 4z &= 4 \end{aligned}$$

$$A = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -3 & 4 \end{bmatrix} \quad b = \begin{bmatrix} 0 \\ -1 \\ 4 \end{bmatrix}$$

Figure 1: Higher Dimensional Examples

1.2 The roots of equation sets

Taking the above three dimensional equation sets as example: when \mathbf{b} is unknown (and the \mathbf{Ax} remain unchanged), whether there is solution for every \mathbf{b} ?

From the column picture explanation, this question is an equivalent of: whether the linear combinations of columns in \mathbf{A} fill up a three dimensional space?

Answer: The above case is non-singular matrix, so YES. For general case, the equation sets **only has solution when \mathbf{b} is in the column space spanned by \mathbf{A}** .

There are many benefits to consider matrix multiplication in **column picture** way.

1.3 Appendix