Sim Pencil Game Project

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1 Introduction

Sim is described as an graph with 6 dots and several lines. There are 6 dots ('vertices') are drawn. Each dot is connected to every other dot by a line ('edge').

Two players take turns coloring any uncolored lines. One player colors in one color, and the other colors in another color, with each player trying to avoid the creation of a triangle made solely of their color (only triangles with the dots as corners count; intersections of lines are not relevant); the player who completes such a triangle loses immediately.

2 Division of Labor

Chen uses C++ to make a solver, generating value and remoteness for each states.
Qi uses Python to makes an GUI, adds support for PvP and PvE and also add hints for each position.

3 Math Analysis

3.1 Possible Value for Each Position

The number of colored edges is ascending while game processing. Which means that there is no loopy situation. When all the edges are colored, the colored edges form a complete graph on 6 vertices(a K_6). According to the Ramsey's theory, $R(3,3) = 6$, which turns out that a monochromatic triangle will always formed and there is no tied game. So the value can be win or lose.

3.2 Estimated Upperbound of Positions

An trivial upperbound is $3^{15} = 14348907$, which go through all the possible colors of each edge(None, Color 1, Color 2).
When the order of move is considered, namely that considering the number of edges for each color, we can get results below:

$$\sum_{i=0}^{7} \left(\binom{15}{i, i, 15-2i} + \binom{15}{i, i+1, 14-2i} \right) = 3492117$$

But in fact, these estimations are quite coarse, since the analysis of symmetry in the complete graph is never applied.

3.3 Actual Number of Positions

Our solver use permutation of each vertices to wipe out all the isomorphic graphs during searching the tree. It is obvious that there is only one choice for the first move. And the second move has 2 choices, one has common vertice to the first edge and the other has not. We finally get 3729 (including initial state) non-isomorphic positions. Which allows us to travel tree easily to get value and remoteness.

4 Strategy

Our solver shows that the second player has winning strategy, which is shown below:

1. For the second move (i.e., when answering the first move of the first player), the second player should color an edge which has no common vertex with the edge chosen by the first player
2. For any move other than the second when there is at least one neutral edge, consider only these neutral edges and apply the following rules in a hierarchy in the same sense as described in the previous section:
   (a) Ruin a minimum number of valid safe moves.
   (b) Create a minimum number of losers (valid and hypothetical).
   (c) Ruin a minimum number of hypothetically safe moves.
   (d) Complete a maximum number of mixed triangles.
   (e) Create a maximum number of partial mixed triangles.
   (f) Create a minimum number of valid losers.

   Then color any one of the edges satisfying the above rules.
3. For any move other than the second when there are no neutral edges, consider only the dotted-red edges and apply the same steps as in Rule 2.

Here is a example for applying this strategy.

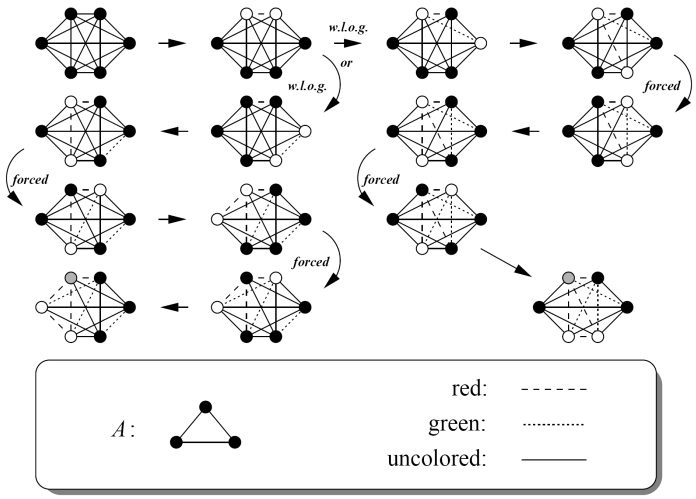

Figure 1: Trivial winning strategy for second player

5 Way to Get Software

All codes and files available at https://github.com/QiLF/Sim

6 How to Play

Please follow the instruction listed in `writeup.txt`.

References

[1] Wolfgang Slany. *Graph Ramsey games*. Technical Report DBAI-TR-99-34, Institut für Informationssysteme der Technischen UniversitätWien, 1999.