Art of Problem Solving

Unofficial Solutions

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Contents

Chapter 1

Logarithms

1.1 Equations

There are 6 main equations:

$$\log_a b^n = n \log_a b \tag{1.1}$$

$$\log_a b + \log_a c = \log_a bc \tag{1.2}$$

$$\log_a b - \log_a a = \log_a b/c \tag{1.3}$$

$$(\log_a b)(\log_c d) = (\log_a d)(\log_c b) \tag{1.4}$$

$$\frac{\log_a b}{\log_a c} = \log_c b \tag{1.5}$$

$$\log_{a^n} b^n = \log_a b \tag{1.6}$$

1.2 Problems

Exercise 1.1 Evaluate the product $(\log_2 3)(\log_3 4)(\log_4 5)(\log_5 6)(\log_6 7)(\log_7 8)$ (View Solution)

Exercise 1.2 If $\log 36 = a$ and $\log 125 = b$, express $\log \frac{1}{12}$ in terms of a and b. (MAO 1992) (View Solution)

1.3 Solutions

Solution 1.1 (View Question)

Use equation (1.4) we can note that if we have a product of logs and keep the bases the same, we can shift the arguments around however much we want. If we move all the arguments one to the right, and move the final argument to the first one, we get

$$(\log_2 8)(\log_3 3)(\log_4 4)(\log_5 5)(\log_6 6)(\log_7 7)$$

Since $\log_n n = 1$ we can cancel everything and get our answer of $\boxed{3}$

Solution 1.2 (View Question)

TBA