

ECE286: Stats

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1 Cumulative Distribution Functions

Given some PDF $f(x)$, the **cumulative distribution function** (CDF) is defined as:

$$CDF(X) = \int_{-\infty}^x f(t) dt \quad (1.1)$$

where $CDF(\infty) = 1$. Recall that given some function $g(x)$, we have

$$\int_A^C g(x) dx = \int_A^B g(x) dx + \int_B^C g(x) dx. \quad (1.2)$$

Therefore, if we have some random value x which has a PDF $f(x)$. Then:

$$P(A \leq X \leq B) = F(B) - F(A). \quad (1.3)$$

Let us now determine the **normal CDF**. Recall that

$$n(x, \mu, \sigma) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \quad (1.4)$$

And therefore the CDF is given by

$$\Phi(x, \mu, \sigma) = \int_{-\infty}^x n(t; \mu, \sigma) dt, \quad (1.5)$$

so $P(A \leq X \leq B, \mu, \sigma) = \Phi(B, \mu, \sigma) - \Phi(a, \mu, \sigma)$. Note that Φ cannot be written in terms of elementary functions. In practice, we will use tables to evaluate this. However, we don't want tables for every μ and σ , so we will parametrize it by a single variable and relate it to the **CDF for a standard normal RV**,

$$\Phi(x) = \int_{-\infty}^t n(t; 0, 1) dt. \quad (1.6)$$

Suppose X has PDF $n(x; \mu, \sigma)$. Let $z = \frac{x - \mu}{\sigma}$. Consider:

$$P(X \leq x) = \int_{-\infty}^x n(t; 0, 1) dt \quad (1.7)$$

$$= \int_{-\infty}^x \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(t-\mu)^2}{2\sigma^2}} dt \quad (1.8)$$

$$= \int_{-\infty}^{(x-\mu)/\sigma} \frac{1}{\sqrt{2\pi}\sigma} e^{-s^2/2} ds \quad (1.9)$$

$$= \int_{-\infty}^{(x-\mu)/\sigma} n(s; 0, 1) ds \quad (1.10)$$

$$= P\left(\frac{x - \mu}{\sigma}\right) \quad (1.11)$$

$$= P\left(Z \leq \frac{x - \mu}{\sigma}\right). \quad (1.12)$$

Note: In MATLAB, the code for Φ is `normcdf`.

2 Binomial PMF

Recall that when we have n coin flips, p is the probability of heads, and X is the number of heads. Recall that the **probability mass function** is

$$b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}. \quad (2.1)$$

The mean is np and the variance is $np(1-p)$. Then:

$$z = \frac{x - np}{\sqrt{np(1-p)}}. \quad (2.2)$$

A preview of the **central limit theorem** is that in the limiting case of $n \rightarrow \infty$, the distribution of X is the normal distribution.

3 Gamma Function

The **gamma function** is defined as:

$$\Gamma(z) = \int_0^{\infty} x^{\alpha-1} e^{-x} dx \quad (3.1)$$

for $\alpha > 0$. IT has the following properties:

- $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$.
- $\Gamma(n) = (n-1)!$ where $n \in \mathbb{N}$.

The **gamma distribution** is

$$f(x; \alpha, \beta) = \begin{cases} \frac{1}{\beta^{\alpha} \Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta} & x > 0 \\ 0 & \text{otherwise} \end{cases} \quad (3.2)$$