PHY293: Tutorial Problems
Tutorial 4 Solutions

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Fall 2021

1. (a) All the arrows are pointed in the opposite direction of displacement.

   (b) We have

$$m_A: \quad m\ddot{x}_A = -3kx_A + kx_B \tag{1}$$
$$m_B: \quad m\ddot{x}_B = -\frac{3}{2}kx_B + kx_A \tag{2}$$

   (c) Let $\omega_0^2 = k/m$. The matrix is

$$\begin{bmatrix}\ddot{x}_A \\ \ddot{x}_B\end{bmatrix} = -\omega_0^2 \begin{bmatrix}3 & -1 \\ -1 & 3/2\end{bmatrix}\begin{bmatrix}x_A \\ x_B\end{bmatrix} \tag{3}$$

   (d) By Wolfram Alpha, the eigenvalues are $1, 7/2$, so the fundamental frequencies are $\omega = \omega_0$ and $\omega = \sqrt{7/2}\omega_0$.

2. (a) Again, arrows are opposite of displacement.

   (b) We have

$$m_A: \quad 3m\ddot{x}_A = -5kx_A + kx_B - mg \tag{4}$$
$$m_B: \quad m\ddot{x}_B = -kx_B + kx_A - mg, \tag{5}$$

   (c) The matrix representation is

$$\begin{bmatrix}\ddot{x}_A \\ \ddot{x}_B\end{bmatrix} = -\omega_0^2 \begin{bmatrix}5/3 & -1/3 \\ -1 & 1\end{bmatrix}\begin{bmatrix}x_A \\ x_B\end{bmatrix} - mg\begin{bmatrix}1 \\ 1\end{bmatrix} \tag{6}$$

   (d) The eigenvalues are $\omega = \sqrt{2/3}\omega_0$ and $\omega = \sqrt{2}\omega_0$.

3. (a) We have $\lambda_n \propto \dfrac{1}{n}$ so

$$\frac{\lambda_n}{\lambda_{n+1}} = \frac{n+1}{n} = \frac{5}{4} \tag{7}$$

   so $n = 4$.

   (b) We have $v = \sqrt{\dfrac{T}{\mu}} = 205$ m/s and $f = v/\lambda$ gives $f_n = 373$ Hz and $f_{n+1} = 466$ Hz.

   (c) The length is $\dfrac{n\lambda_n}{2} = 1.1$ m.

4. We have $L = \dfrac{\lambda}{2}(2n+1) = \dfrac{v}{2f}(2n+1)$ so

$$f = \frac{v}{2L}(2n+1) \tag{8}$$