MAT292
Tutorial 7 Solution

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1. (a) The net force is
$$F = mg - \frac{c_d}{2}\rho A v^2 - \rho V g \tag{1}$$
   (b)
$$\ddot{y} = \left(1 - \frac{\rho_{\text{fluid}}}{\rho_{\text{mass}}}\right) g - \frac{c_d \rho A}{2m}\dot{y}^2 = \alpha - \beta \tag{2}$$
   (c) Second order nonlinear.
   (d) We have
$$\dot{v} = \alpha - \beta v^2 \tag{3}$$

2. (a) v should reach a constant when $v^2 = \dfrac{\alpha}{\beta}$.
   (b) Differentiating, we get
$$\frac{dv}{dt} = \frac{df}{dv}v \tag{4}$$
   (c) Setting $\dot{v}$ equal, we get
$$\frac{df}{dy}v = \alpha - \beta v^2 \implies \frac{df}{dy} = \frac{\alpha}{v} - \beta v \tag{5}$$
   (d) Equilibrium occurs when
$$v^2 = \frac{\alpha}{\beta}, \tag{6}$$
   which is the same as earlier.
   (e) If we plug in $f(0) = 0$, we get $\alpha = 0$ so $W = B$. However physically this doesn't make any sense.
   (f) If $W = B$, the solution is just $y = 0$. If $W > B$, then we have
$$\frac{dv}{dy} = \frac{\alpha}{v} - \beta v \tag{7}$$
   so
$$2\beta y + C = -\ln\left(\beta v^2 - \alpha\right) \tag{8}$$
   This show why plugging in $f(0) = 0$ gives us such a strange relationship. Therefore,
$$v = C\exp(-2\beta y) + \frac{\alpha}{\beta} \tag{9}$$
   (g) and we see for a third time that as $y \to \infty$, we get $v = \dfrac{\alpha}{\beta}$. Note that we are ignoring certain cases, like what happens if $W = B$ or if $W < B$. IF $W < B$, then the argument using the ln would be negative, but we can take care of it by taking the absolute value.