MAT292: Ordinary Differential Equations

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Contents

1 Introduction

Covers 1.1: Mathematical Models and Solutions

- Big Idea: Differential equations model physical situations:
  - Take a physical situation and ODE-ify it (How do we model a cooling coffee cup?)
  - Understand an ODE without solving it (What can we deduce directly from $y' = y^2$?)
  - Study, categories, typecast ODEs and solve them

    Example 1: Suppose we have $y' = y/t + \ln t$ and $y' = y^2 + t$. Which of these are harder to solve (without actually solving them)?

    It turns out that the second one is harder as it is *non-linear*.

  - Handle ODEs numerically (What do we do when we cannot solve an ODE that models a real life phenomenon?)
  - The art of problem solving (How do I work with no strings attached?)
- What is a differential equation?

  Definition: A differential equation relates a function and its derivatives.

- We can understand ODEs without solving it:

  Example 2: Let's consider a cup of coffee in a room. We want to model its change in temperature over time. How do we do this?

  There are a lot of variables, so we have to simplify our model. The things we care about
  - The temperature of the coffee cup $y(t)$.
  - t is in minutes.
  - $y(t)$ is in Celsius.
  - The temperature in the room T (in Celsius).

  The things we ignore / simplify:
  - Temperature variation within the cup
  - Temperature variation in the room

  Exercise: Let's consider some suggestions for an ODE describing the temperature of a coffee cup in a room. Each of the following suggested ODEs contradicts our intuition in some way. How?
  - $y' = y^2$
    * T isn't in there
    * Temperature would always increase except if $y = 0$.
    * The hotter the coffee, the faster it heats up.
  - $y' = \dfrac{T}{y}$
    * If $T > 0, y > 0$, then $y' > 0$
    * The model doesn't work for coffee at $0°$C.
  - $y' = y[e^{y-T} + y^3]$
  - $y' = y - T$
  - $y' = T - y$
    * There should be a parameter that describes the physical properties (rate of heating/cooling will be different for different materials)

  Idea: Without solving an ODE, you can already make many predictions about its solution

  (and then, for example, judge your model)

- We introduce a few definitions

Definition: An **ordinary differential equation** (ODE) only considers a function of 1 variable and its derivatives

Definition: A **partial differential equation** considers a function of several variables and its derivatives.

- The most general ODE for a function $y(t)$ is:
  (a) $F[t, y, y'', \ldots, y^{(n)}]$ for $n \in \mathbb{N}$.
  (b) Any function that satisfies this equation is called *a solution*

Definition: The order of an ODE is the highest derivative of an ODE.