PHY293: Tutorial Problems
Tutorial 2 Solutions

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1. We want critical damping, so $2\omega = \gamma$ where $\omega = \sqrt{k/m}$ and $\gamma = b/m$. This gives
$$4\frac{k}{m} = \frac{b^2}{m^2} \implies b = 4\sqrt{mk} \tag{1}$$
We can determine the spring constant k by looking at the equilibrium location, which occurs at $mg = k\Delta l \implies k = \frac{mg}{\Delta l}$. This gives
$$b = 4m\sqrt{\frac{g}{\Delta l}} = \boxed{72 \text{ kg/s}} \tag{2}$$

2. Since the spring is damped lightly, we will assume that $\omega \approx \omega_0$. We will check this at the end. Therefore, the period is $T = \frac{28}{20} = 1.4$ s. The amplitude (envelope function) is given by $A(t) = A_0 e^{-\gamma t/2}$. We want this to be 0.9 of the initial value at $t = T$:
$$0.9 = e^{-\gamma T/2} \implies \gamma = -\frac{2}{T}\ln(0.9) = 0.151 \text{ s}^{-1}. \tag{3}$$
We have $b = m\gamma = 0.075$ kg/s, $k = m\omega^2 = m\left(\frac{4\pi^2}{T^2}\right) = 10.1$ N/m, and $Q = \frac{\omega_0}{\gamma} = 29.7$, which is sufficiently large.

3. After 20 periods, the amplitude goes from 5 to 1.4. Therefore:
$$1.4 = 5e^{-\gamma(10T)} \tag{4}$$
Using $\gamma T = \gamma\frac{2\pi}{\omega} \approx 2\pi/Q$. Plugging this in and solving for Q gives $Q \approx 49$.

4. (a) Every 20 cycles, it reduces by a factor of 3. So if we have five 20 cycles (i.e. 100 cycles,) then it reduces by a factor of 3^5.
   (b) Every 20 cycles, it reduces the amplitude by a factor of $\frac{1}{\sqrt{3}}$. Therefore, after 40 cycles, it would reduce the amplitude by $\frac{1}{3}$.

5. (a) We have
$$\frac{1}{Q} = 2\sqrt{1 - \left(\frac{\omega}{\omega_0}\right)^2} \tag{5}$$
   Substituting in the numbers, we get $Q = 500$.
   (b) We have $\omega_0^2 = \frac{k}{m}$. Solving for k gives $k = 100$ N/m. We have
$$\gamma = \frac{\omega_0}{Q} = 2000 \text{ s}^{-1}, \tag{6}$$
   but this is also equal to $\gamma = \frac{b}{m}$. solving for b gives $b = 2 \times 10^{-7}$ kg/s.
   (c) The initial energy is $\frac{1}{2}mA_0^2 = 5 \times 10^{-15}$ J.
   (d) The lifetime is defined by $\tau = \frac{1}{\gamma} = 0.5$ ms.

6. (a) We have

$$V_C = \frac{q}{C} = \frac{q_0(\omega)}{C}\cos(\omega t - \delta) \tag{7}$$

$$V_R = R\frac{dq}{dt} = -R\omega q_0(\omega)\sin(\omega t - \delta) \tag{8}$$

$$V_L = L\frac{d^2q}{dt^2} = -L\omega^2 q_0(\omega)\cos(\omega t - \delta) \tag{9}$$

(b) When $\omega = \omega_0$, we have $\delta = \frac{\pi}{2}$. Therefore, $\frac{dq}{dt} = -\omega q_0(\omega)\sin(\omega t - \delta)$. Since $\sin(x - \pi/2) = -\cos(x)$, we can substitute this in to get

$$\frac{dq}{dt} = \omega q_0(\omega)\cos(\omega t) \tag{10}$$

which has no phase shift. $\varepsilon(t)$ also doesn't have a phase shift, so the source voltage and the current are in phase when $\omega = \omega_0$.