Assignment 1

1.1 For the three-cart system shown in Figure 1, obtain the equations of motion. The system has three inputs, $u_1(t)$, $u_2(t)$, and $u_3(t)$ and three outputs, $x_1(t)$, $x_2(t)$, and $x_3(t)$. Obtain three second-order ordinary differential equations with constant coefficients. Also, write the equations of motion in matrix form.

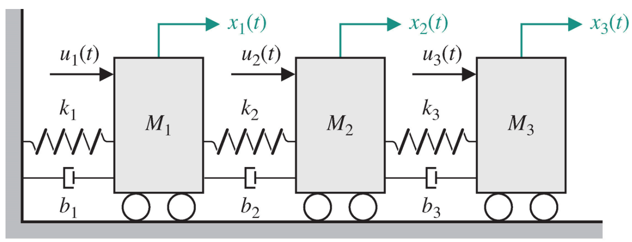

Figure 1: Three cart system (Problem 1.1)

1.2 Determine the dynamic equations for lateral motion of the robot in Figure 2. Assume it has three wheels with a single, steerable (and driver) wheel in the front where the controller has direct control of the rate of change of the steering angle, U_{steer}, with geometry as shown in Figure 3. Assume the robot is going in approximately a straight line, hence its angular deviation from that straight line as well as the steering angle are very small. Also, assume the robot is traveling at a constant speed at the front wheel, V_0. The dynamic equations relating the lateral velocity of the centre of the robot, defined in an inertial reference frame, as a result of commands in U_{steer} are desired.

Figure 2: Robot for delivery of hospital supplies (Problem 1.2)

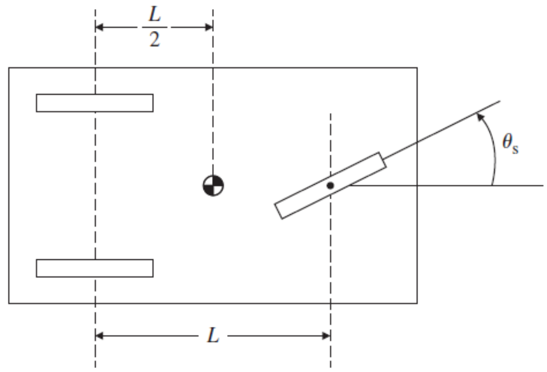

Figure 3: Model for robot motion (Problem 1.2)

1.3 For the robot in Problem 1.2, assume you have command of the torque on a servo motor that is connected to the drive wheel (the front wheel) with gears that have a 2:1 ratio, so the torque on the wheel is increased by a factor of 2 over what is delivered by the servo. Determine the dynamic equations relating the speed of the robot at the front wheel V_0 (which is no longer constant) with respect to the torque command of the servo. Assume that the geometry and mass properties of all three wheels are the same. Your equations will be based on certain parameters, e.g., mass and geometry of vehicle, radius and moment of inertia of the wheel, etc.

1.4 Given that the Laplace transform of $f(t)$ is $F(s)$, find the Laplace transform of the following:

(a) $g(t) = f(t)\cos t$

(b) $g(t) = \int_0^t \int_0^{t_1} f(\tau)d\tau dt_1$

1.5 Using the convolution integral, find the step response of the system whose impulse response is given below and shown in Figure 4:

$$h(t) = \begin{cases} 1, \ 0 \leq t \leq 2 \\ 0, \ t < 0 \text{ and } t > 2 \end{cases} \tag{1}$$

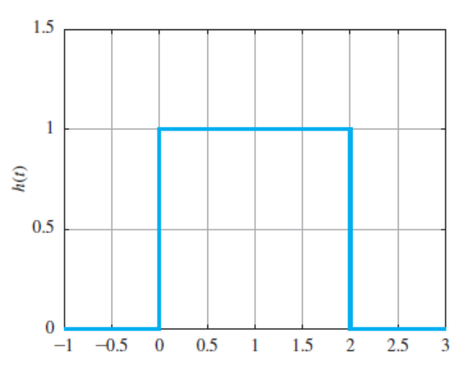

Figure 4: Impulse response for the Problem 1.5

1.6 Use block-diagram algebra to determine the transfer function between $R(s)$ and $Y(s)$ in Figure 5.

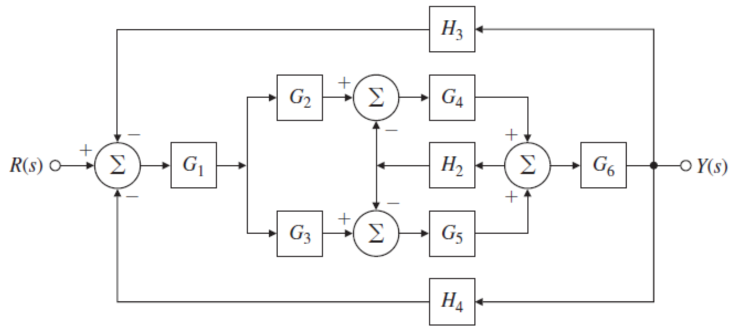

Figure 5: Block diagram for Problem 1.6

1.7 For the unity feedback system shown in Figure 6, specify the gain K of the proportional controller so that the output $y(t)$ has an overshoot of no more than 10% in response to a unit step.

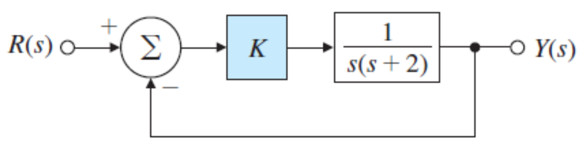

Figure 6: Unity feedback system for Problem 1.7

1.8 The open-loop transfer function of a unity feedback system is

$$G(s) = \frac{K}{s(s+2)}. \tag{2}$$

The desired system response to a step input is specified as peak time $t_p = 1sec$ and overshoot $M_p = 5\%$.

(a) Determine whether both specifications can be met simultaneously by selecting the right value of K.

(b) Sketch the associated region in the s-plane where both specifications are met, and indicate what root locations are possible for some likely values of K.

1.9 A closed-loop transfer function is given below

$$H(s) = \frac{\left[\left(\frac{s}{10}\right)^2 + 0.1\left(\frac{s}{10}\right) + 1\right]\left[\frac{s}{2}+1\right]\left[\frac{s}{0.1}+1\right]}{\left[\left(\frac{s}{4}\right)^2 + \left(\frac{s}{4}\right) + 1\right]\left[\left(\frac{s}{10}\right)^2 + 0.09\left(\frac{s}{10}\right) + 1\right]\left[\frac{s}{0.02}+1\right]}. \tag{3}$$

Estimate the percent overshoot, M_p, and the transient settling time, t_s for this system.

1.10 Find the relations for the impulse response and the step response corresponding to the second-order system

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{\omega_n^2}{(s + \zeta\omega_n)^2 + \omega_n^2(1 - \zeta^2)} \tag{4}$$

for the cases where

(a) the roots are repeated (ζ is positive);

(b) the roots are both real (ζ is positive). Express your answers in terms of hyperbolic functions (sinh, cosh) to best show the properties of the system response;

(c) the value of the damping coefficient, ζ, is negative ($|\zeta| < 1$).