ECE253: Computer and Digital Systems
Summary

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Contents

1 Boolean Algebra

We write multiplication for AND gates and addition for OR gates. This forms an algebraic structure with the following properties. The obvious ones:

- $xy = yx$
- $x + y = y + x$
- $x(yz) = xy(z)$
- $x + (y + z) = (x + y) + z$
- $x(y + z) = xy + xz$

and the less obvious ones:

- $x + yz = (x + y)(x + z)$
- $x + xy = x$ (Absorption)
- $xy + x\bar{y} = x$ (Combining)
- $(x + y)(x + \bar{y}) = x$
- $\overline{xy} = \bar{x} + \bar{y}$ (De Morgan's Theorem)
- $\overline{x + y} = \bar{x}\bar{y}$
- $x + \bar{x}y = x + y$
- $x(\bar{x} + y) = xy$
- $xy + tz + \bar{x}z = xy + \bar{x}z$
- $(x + y)(y + z)(\bar{x} + z) = (x + y)(\bar{x} + z)$

which can be proven using **perfect induction** (i.e. look at all cases) or algebraic manipulation.

1.1 Sum of PRoducts and Product of Sums

Given a truth table, the **minterm** that corresponds to each row is given by something like

$$m_3 = \bar{x}_1 x_2 x_3 \tag{1}$$

such that when $3 = 0b011$ is substituted in, $m_3 = 1$, and $m_3 = 0$ otherwise.

The **maxterm** corresponds to a sum, and $M_i = 0$ if and only if the input is i. For example,

$$M_3 = x_1 + \bar{x_2} + \bar{x_3} \tag{2}$$