In[∘]:=
```
n = 4;
coord = {t, r, θ, ϕ};
metric = {{-Exp[2 * R[r]], 0, 0, 0},
   {0, 1 / (1 - b[r] / r), 0, 0}, {0, 0, r^2, 0}, {0, 0, 0, r^2 * Sin[θ]^2}};
metric // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -e^{2R[r]} & 0 & 0 & 0 \\ 0 & \frac{1}{1-\frac{b[r]}{r}} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \mathrm{Sin}[\theta]^2 \end{pmatrix}$$

In[∘]:=
```
inversemetric = Simplify[Inverse[metric]];
inversemetric // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -e^{-2R[r]} & 0 & 0 & 0 \\ 0 & 1-\frac{b[r]}{r} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{\mathrm{Csc}[\theta]^2}{r^2} \end{pmatrix}$$

In[∘]:=

$$\begin{pmatrix} -e^{-2R[r]} & 0 & 0 & 0 \\ 0 & 1-\frac{b[r]}{r} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{\mathrm{Csc}[\theta]^2}{r^2} \end{pmatrix}$$

```
standardmetric = {{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}};
standardmetric // MatrixForm
```

Out[∘]=

$$\left\{\left\{-e^{-2R[r]}, 0, 0, 0\right\}, \left\{0, 1-\frac{b[r]}{r}, 0, 0\right\}, \left\{0, 0, \frac{1}{r^2}, 0\right\}, \left\{0, 0, 0, \frac{\mathrm{Csc}[\theta]^2}{r^2}\right\}\right\}$$

Out[∘]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

In[∘]:=
```
Det[{{-1, 0, 0, 0}, {0, 1, 0, 0}, {0, 0, 1, 0}, {0, 0, 0, 1}}]
```

Out[∘]= -1

In[∘]:=
```
affine := affine = Simplify[Table[(1 / 2) * Sum[(inversemetric[[i, s]]) *
        (D[metric[[s, j]], coord[[k]] ] +
          D[metric[[s, k]], coord[[j]] ] - D[metric[[j, k]], coord[[s]] ]), {s, 1, n}],
     {i, 1, n}, {j, 1, n}, {k, 1, n}] ]
```

In[∘]:=
```
listaffine := Table[If[UnsameQ[affine[[i, j, k]], 0],
    {ToString[Γ[i, j, k]], affine[[i, j, k]]}] , {i, 1, n}, {j, 1, n}, {k, 1, j}]
```

In[∘]:=
```
TableForm[Partition[DeleteCases[Flatten[listaffine], Null], 2],
  TableSpacing → {2, 2}] // FullSimplify
```

Out[∘]//TableForm=

"Γ[1, 2, 1]"	$R'[r]$
"Γ[2, 1, 1]"	$\frac{e^{2 R[r]} (r-b[r]) R'[r]}{r}$
"Γ[2, 2, 2]"	$-\frac{b[r]-r b'[r]}{2 r^2-2 r b[r]}$
"Γ[2, 3, 3]"	$-r+b[r]$
"Γ[2, 4, 4]"	$(-r+b[r]) \mathrm{Sin}[\theta]^2$
"Γ[3, 3, 2]"	$\frac{1}{r}$
"Γ[3, 4, 4]"	$-\mathrm{Cos}[\theta] \mathrm{Sin}[\theta]$
"Γ[4, 4, 2]"	$\frac{1}{r}$
"Γ[4, 4, 3]"	$\mathrm{Cot}[\theta]$

In[∘]:=
```
J = {{Exp[-R[r]], 0, 0, 0},
   {0, Sqrt[1 - b[r] / r], 0, 0}, {0, 0, 1 / r, 0}, {0, 0, 0, 1 / (r * Sin[θ])}};
inverseJ = Inverse[J];
J = J // MatrixForm
inverseJ = inverseJ // MatrixForm
riemann := riemann = Simplify[Table[
    (inverseJ⟦1, i, i⟧ * J⟦1, j, j⟧ * J⟦1, k, k⟧ * J⟦1, l, l⟧) *
     (D[affine⟦i, j, l⟧, coord⟦k⟧ ] - D[affine⟦i, j, k⟧, coord⟦l⟧ ] +
       Sum[affine⟦s, j, l⟧ × affine⟦i, k, s⟧ - affine⟦s, j, k⟧ × affine⟦i, l, s⟧,
        {s, 1, n}]),
    {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, n}] ]
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} e^{-R[r]} & 0 & 0 & 0 \\ 0 & \sqrt{1-\frac{b[r]}{r}} & 0 & 0 \\ 0 & 0 & \frac{1}{r} & 0 \\ 0 & 0 & 0 & \frac{\mathrm{Csc}[\theta]}{r} \end{pmatrix}$$

Out[∘]//MatrixForm=

$$\begin{pmatrix} e^{R[r]} & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{1-\frac{b[r]}{r}}} & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & r\,\mathrm{Sin}[\theta] \end{pmatrix}$$

In[◦]:=

$$\begin{pmatrix} e^{\text{Phi}[r]} & 0 & 0 & 0 \\ 0 & \frac{1}{\sqrt{1-\frac{b[r]}{r}}} & 0 & 0 \\ 0 & 0 & r & 0 \\ 0 & 0 & 0 & r\,\text{Sin}[\theta] \end{pmatrix}$$

Out[◦]=

$$\left\{\left\{e^{\text{Phi}[r]}, 0, 0, 0\right\}, \left\{0, \frac{1}{\sqrt{1-\frac{b[r]}{r}}}, 0, 0\right\}, \{0, 0, r, 0\}, \{0, 0, 0, r\,\text{Sin}[\theta]\}\right\}$$

In[◦]:=

```
riemann[[3, 4, 3, 4]] // FullSimplify
```

Out[◦]=

$$\frac{b[r]}{r^3}$$

In[◦]:=

```
listriemann :=
  Table[If[UnsameQ[riemann[[i, j, k, l]], 0], {ToString[R[i, j, k, l]], riemann[[i, j, k, l]]}] ,
   {i, 1, n}, {j, 1, n}, {k, 1, n}, {l, 1, k - 1}]
```

In[◦]:=

```
TableForm[Partition[DeleteCases[Flatten[listriemann], Null], 2], TableSpacing → {2, 2}]
```

Out[◦]//TableForm=

R[1, 2, 2, 1]	$\left(1-\frac{b[r]}{r}\right)\left(\frac{(b[r]-r\,b'[r])\,R'[r]}{2r^2-2r\,b[r]}+R'[r]^2+R''[r]\right)$
R[1, 3, 3, 1]	$\frac{(r-b[r])\,R'[r]}{r^2}$
R[1, 4, 4, 1]	$\frac{(r-b[r])\,R'[r]}{r^2}$
R[2, 1, 2, 1]	$\frac{b[r]\left(R'[r]-2r\,R'[r]^2-2r\,R''[r]\right)+r\left(-b'[r]\,R'[r]+2r\left(R'[r]^2+R''[r]\right)\right)}{2r^2}$
R[2, 3, 3, 2]	$\frac{b[r]-r\,b'[r]}{2r^3}$
R[2, 4, 4, 2]	$\frac{b[r]-r\,b'[r]}{2r^3}$
R[3, 1, 3, 1]	$\frac{(r-b[r])\,R'[r]}{r^2}$
R[3, 2, 3, 2]	$-\frac{b[r]-r\,b'[r]}{2r^3}$
R[3, 4, 4, 3]	$-\frac{b[r]}{r^3}$
R[4, 1, 4, 1]	$\frac{(r-b[r])\,R'[r]}{r^2}$
R[4, 2, 4, 2]	$-\frac{b[r]-r\,b'[r]}{2r^3}$
R[4, 3, 4, 3]	$\frac{b[r]}{r^3}$

In[◦]:=
```
ricci := ricci = Simplify[Table[Sum[riemann[[i, j, i, l]], {i, 1, n}], {j, 1, n}, {l, 1, n}] ]
listricci :=
 Table[If[UnsameQ[ricci[[j, l]], 0], {ToString[R[j, l]], ricci[[j, l]]}] , {j, 1, n}, {l, 1, j}]
TableForm[Partition[DeleteCases[Flatten[listricci], Null], 2],
  TableSpacing → {2, 2}] // FullSimplify
```

Out[◦]//TableForm=

R[1, 1]	$\frac{R'[r]\ (4\,r-3\,b[r]-r\,b'[r]+2\,r\,(r-b[r])\,R'[r])+2\,r\,(r-b[r])\,R''[r]}{2\,r^2}$
R[2, 2]	$\frac{r\,b'[r]\ (2+r\,R'[r])-2\,r^3\left(R'[r]^2+R''[r]\right)+b[r]\ (-2+r\,(R'[r]\ (-1+2\,r\,R'[r])+2\,r\,R''[r]))}{2\,r^3}$
R[3, 3]	$\frac{b[r]+r\,b'[r]+2\,r\,(-r+b[r])\,R'[r]}{2\,r^3}$
R[4, 4]	$\frac{b[r]+r\,b'[r]+2\,r\,(-r+b[r])\,R'[r]}{2\,r^3}$

In[◦]:=
```
scalar = Sum[standardmetric[[i, i]] * ricci[[i, i]], {i, 1, n}] // FullSimplify
```

Out[◦]=

$$\frac{(-4\,r+3\,b[r])\,R'[r]+b'[r]\ (2+r\,R'[r])-2\,r\,(r-b[r])\left(R'[r]^2+R''[r]\right)}{r^2}$$

In[◦]:=
```
einstein := einstein = Simplify[ricci - (1 / 2) * scalar * standardmetric]
```

In[◦]:=
```
listeinstein := Table[
  If[UnsameQ[einstein[[j, l]], 0], {ToString[G[j, l]], einstein[[j, l]]}] , {j, 1, n}, {l, 1, j}]
```

In[◦]:=
```
TableForm[Partition[DeleteCases[Flatten[listeinstein], Null], 2],
  TableSpacing → {2, 2}] // FullSimplify
```

Out[◦]//TableForm=

G[1, 1]	$\frac{b'[r]}{r^2}$
G[2, 2]	$\frac{-b[r]+2\,r\,(r-b[r])\,R'[r]}{r^3}$
G[3, 3]	$\frac{(1+r\,R'[r])\ (b[r]-r\,b'[r]+2\,r\,(r-b[r])\,R'[r])}{2\,r^3}+\frac{(r-b[r])\,R''[r]}{r}$
G[4, 4]	$\frac{(1+r\,R'[r])\ (b[r]-r\,b'[r]+2\,r\,(r-b[r])\,R'[r])}{2\,r^3}+\frac{(r-b[r])\,R''[r]}{r}$