Assignment 3

3.1 Set up the following characteristic equations in the form suited to Evan's root-locus method. Give $L(s)$, $a(s)$, and $b(s)$ and the parameter K in terms of the original parameters in each case. Make sure to select K so that $a(s)$ and $b(s)$ are monic polynomials in each case, and the degree of $b(S)$ is not greater than that of $a(s)$.

(a) $(s+c)^3 + A(Ts+1) = 0$

  (i) versus parameter A,

  (ii) versus parameter T,

  (iii) versus parameter c, if possible. Explain why you can or cannot. Can a plot of the roots be drawn versus c for given constant values of A and T by any means at all?

(b) $1 + [k_P + \frac{k_I}{s} + \frac{k_D s}{\tau s+1}]G(s) = 0$. Assume $G(s) = A\frac{c(s)}{d(s)}$ is a strictly proper fraction, where $c(s)$ and $d(s)$ are monic polynomials.

  (i) versus k_P,

  (ii) versus k_I,

  (iii) versus k_D,

  (iv) versus τ.

3.2 Consider a unity feedback system with the following feedforward transfer function $G(s)$:

$$G(s) = \frac{K}{s[(s^2+4s+5]}. \tag{1}$$

(a) Calculate the root locus basic parameters, including asymptotes (number, angles and intersection with the real axis), breakaway and break-in points, angles of departure and arrival, and any crossings with the imaginary axis, and sketch the root loci versus the gain K. (Use Matlab to verify your results.)

(b) Discuss various ranges of K where the system's response to a unit step is underdamped or overdamped. You can use Matlab to identify those ranges, but you still need to reason for the type of response for each range.

3.3 Consider the following open-loop transfer function:

$$L(s) = \frac{s^2 + (\frac{5}{6})s + (\frac{1}{3})}{s^3[s^2 + (6+\alpha)s + (16+\beta)]}. \tag{2}$$

Using Matlab, plot the root locus for the following cases, and discuss why the three root loci are so significantly different.

(a) $\alpha = 0$ and $\beta = 0$.

(b) $\alpha = 0$ and $\beta = 1$.

(c) $\alpha = 1$ and $\beta = 0$.

3.4 In a unity feedback system with a cascade compensation $D_c(s)$ (i.e., to be added in series with the plant) for a plant $G(s)$, let

$$G(s) = \frac{10}{s(s+1)} \qquad \textit{and} \qquad D_c(s) = K\frac{s+a}{s+8} \qquad (K = 1). \tag{3}$$

Using root-locus method, determine the value of a such that the damping ratio ζ of the dominant closed-loop poles is 0.5.

3.5 Consider the following plant transfer function:

$$G(s) = \frac{1}{s^2(s+5)}. \tag{4}$$

(a) Using root-locus method, show whether or not such a plant can be stabilized by a unity feedback with a proportional controller. (Matlab can be used for plotting the root locus.)

(b) If an internal feedback of the output rate is included in the system, as shown in Fig. 1, should there be any relation between the control gain K and the rate feedbak gain K_t to guarantee a stable response?

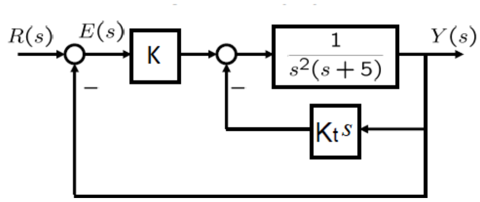

Figure 1: Control system for Problem 3.5

(c) Calculate different ranges (or values) of K_t for which the root-locus graphs of the closed-loop system in Fig. 1 with respect to K are distinctively different.

(d) Assuming that both control and rate feedback gains have a lower limit of 1 and an upper limit of 5, using the root locus (with respect to K) and by deriving the relevant equations discuss how you would choose these gains to obtain the fastest rise time in the unit-step response without any overshoot. (Discussions must be based on calculations, but Matlab can be used for verifying the answers.) Draw the unit step response of the system with the selected gains in Matlab to obtain the rise time and settling time.

(e) Explain how you would end up with the same result as in the previous case just by using the root-locus graph (obtained in Matlab) of the closed-loop system in Fig. 1 with respect to K_t.

3.6 For a robot manipulator with the following transfer function:

$$G(s) = \frac{1}{(s+1)(s+5)}, \tag{5}$$

using root-locus method, design a cascade controller $G_c(s)$ with a minimum number of gains (parameters), so that the unity feedback system has the following specifications: (NOTE: Since the poles of $G(s)$ are not exact, pole cancellation technique is not desired for this design.)

- DS1: Zero steady-state error to a step input.
- DS2: Steady-state error due to a ramp input of less than 28% of the input magnitude.
- DS3: Percent overshoot of no more than 5% to a step input.
- DS4: Settling time of less than 1.5 *sec* to a step input based on the 2% criterion (i.e., $\sigma > \frac{4}{t_s}$).