CSC473: Algorithms

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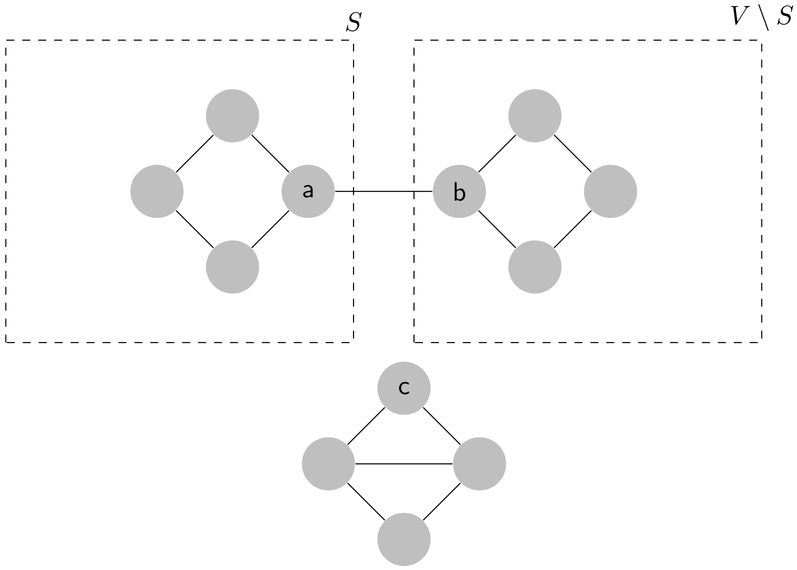
Contents

1 Global Min Cut

- **Input:** Undirected, unweighted connected graph $G = (V, E)$.
- **Output:** Smallest set of edges that disconnects G.



For the first graph, disconnect the edge separating a and b and for the second graph, disconnect the edges around node c. We wish to instead return $S, T \subseteq V$ with $S \cap T = \emptyset$ such that the desired edges are simply

$$E(S,T) = \{(u,v) \in E : u \in S, v \in T\}.$$

The global min-cut is therefore the output $S \subseteq V$ such that $S \neq \emptyset, V$ where $|E(S, V \setminus S)|$ is minimized.

Notice that this is very similar to the max flow problem. Consider the source-sink min-cut problem, where the input is $G = (V, E)$ as before, and two nodes $s, t \in V$. The desired output is the same S with the same minimization, except with one extra constraint: that is, $s \in S$ and $t \notin S$. Any max flow algorithm can determine this.

To solve the global min cut problem, we can fix t and choose s to be any of the other $V - 1$ nodes, then run the max flow algorithm on each case. The best max flow algorithm (Which came out in a recent paper) does max flow in $O(m^{1+O(1)}) \approx O(n^2)$, so this algorithm would give a time complexity of $O(n^3)$. However, we can improve this to $O(n^2 \log^2(n))$.