MAT292
Tutorial 8 Solution

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1. (a) Note that

$$\mathcal{L}\{f\}(s) = 3 \cdot \frac{1}{s^2+1} \tag{1}$$

so

$$f(t) = 3\sin(t) \tag{2}$$

We can check this works via the following:

(b) We have

$$\begin{aligned}
\mathcal{L}\{af(t)+bg(t)+ch(t)\}(s) &= \int_0^\infty (af(t)+bg(t)+ch(t))e^{-st}\,\mathrm{d}t && (3)\\
&= \int_0^\infty af(t)e^{-st}\,\mathrm{d}t + \int_0^\infty bg(t)e^{-st}\,\mathrm{d}t + \int_0^\infty ch(t)e^{-st}\,\mathrm{d}t && (4)\\
&= a\int_0^\infty f(t)e^{-st}\,\mathrm{d}t + b\int_0^\infty g(t)e^{-st}\,\mathrm{d}t + c\int_0^\infty h(t)e^{-st}\,\mathrm{d}t && (5)\\
&= aF(s)+bG(s)+cH(s) && (6)
\end{aligned}$$

Note that we can break up the integrals since the individual integrals converge.

(c) We have

i. We have

$$\mathcal{L}\{h\}(s) = \frac{1}{s-2} + \frac{2}{s} + \frac{2}{s+2} \tag{7}$$

where $s \in (-2,2)\setminus 0$.

ii. We have

$$\mathcal{L}\{f\}(s) = \frac{s+a^2}{s^2+a^2} \tag{8}$$

where $s \in \mathbb{R}$.

iii. Applying linearity, we have

$$\begin{aligned}
\mathcal{L}\{f\}(s) &= \sum_{k=0}^{n} \mathcal{L}\left\{\frac{t^k}{k!}\right\}(s) && (9)\\
&= \sum_{k=0}^{n} \frac{1}{k!}\cdot\frac{k!}{s^{k+1}} && (10)\\
&= \sum_{k=0}^{n} \frac{1}{s^{k+1}} && (11)\\
&= \frac{1}{s}\left(\frac{1-s^{-n+1}}{1-s^{-1}}\right) && (12)
\end{aligned}$$

where $s > 0$.

As we take the limit $n \to \infty$, we need $|1/s| < 1$ and $s > 0$ so we need $s > 1$. Linearity still holds because it is countable.

(d) easy

2. (a) Using my integration talents (i.e. deifnitely not integral calculator), I manually computed $\frac{s^2+2}{s^3+4s}$

   (b) $\frac{2}{s^3+4s}$

   (c) $\sqrt{\frac{\pi}{s}}$

   (d) e^x and e^x defined piecewise such that it's equal to 0 if and only if $x = 1$.

3. (a) $f(x) = \sin\left(x^{x^{x^{x^{x^{x}}}}}\right)$

   (b) e^{-x^2} and e^{x^2}.