PHY365: Quantum Information

QiLin Xue

Fall 2021

Contents

1 Quantum Coins **2**

1 Quantum Coins

Consider a quantum coin that can be in a superposition of heads and tails. We can write its state as a vector:

$$|\Psi\rangle = \alpha|H\rangle + \beta|T\rangle \tag{1.1}$$

which lives in the **Hilbert Space.** Inner products of these vectors can be written as

$$\langle\Psi_1|\Psi_2\rangle. \tag{1.2}$$

Born's Rule tells us we can compute the probability of tails to be $|\beta|^2$ and the probability of heads is $|\alpha|^2$. When there are two quantum coins, there can be four combinations of heads and tails, written as:

$$|\Psi\rangle = \alpha|HH\rangle + \beta|HT\rangle + \gamma TH\rangle + \delta|TT\rangle. \tag{1.3}$$

In quantum mechanics, we can construct the following state:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}|HH\rangle + \frac{1}{\sqrt{2}}|TT\rangle, \tag{1.4}$$

which represents **entanglement.** If we measure the first coin, we can instantly know the outcome of the second coin, even if they are lightyears apart.