CHE260: Heat Transfer

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1 Mechanisms of Heat Transfer

- Mechanisms involve:
  - Conduction: Transfer of heat through a medium that is stationary.
  - Convection: Transfer of heat from a solid surface and an adjacent fluid that is moving.
  - Radiation: Energy emitted by matter in the form of electromagnetic waves.
- Conduction follows **Fourier's Law:**
$$\dot{Q} = -kA\nabla T \tag{1}$$
- The rate of heat transfer from the surface of a blackbody is given by the Stefan-Boltzmann Law
$$\dot{Q}_{\text{emit,max}} = \sigma A T_s^4 \tag{2}$$
where σ is the Boltzmann constant $\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$
- For an ideal body, we have
$$\dot{Q}_{\text{emit}} = \epsilon \sigma A T_s^4 \tag{3}$$
where $0 \leq \epsilon \leq 1$ is the emissivity.
- When radiation is incident on a surface, some will be absorbed and some reflected. The **absorptivity** α is defined such that
$$\dot{Q}_{\text{absorbed}} = \alpha \dot{Q}_{\text{incident}} \tag{4}$$
$$\dot{Q}_{\text{reflected}} = (1-\alpha)\dot{Q}_{\text{incident}} \tag{5}$$
- Kirchoff's Law says that
$$\alpha = \epsilon \tag{6}$$
- For a small surface completed surrounded by a much larger surface net radiation is
$$\dot{Q}_{\text{net}} = \epsilon \sigma A (T_s^4 - T_{\text{surrounding}}^4) \tag{7}$$
- Natural convection tells us the heat transfer coefficient is
$$h = c(T_S - T_\infty)^{1/4} \tag{8}$$
where $c = 4.2\text{W}m^2 K^{5/4}$ and
$$q_{\text{conv}} = hA(T_S - T_\infty) \tag{9}$$
- Forced convection gives a constant $h = 250 W/m^2 K$
- Let's look at the **one-dimensional case:** If we look at a segment of length Δx, the rate of the increase of enthalpy is
$$\dot{H} = mc_p \frac{\partial T}{\partial t} \tag{10}$$
$$= \rho c_p A \Delta x \frac{\partial T}{\partial t} \tag{11}$$

The energy balance in this small segment gives

$$\rho c_p A \Delta x \frac{\partial T}{\partial t} = \dot{Q}_x - \dot{Q}_{x+\Delta x} \tag{12}$$

$$= (\dot{q}A)_x - (\dot{q}A)_{x+\Delta x} \tag{13}$$

Dividing by Δx and taking the limit gives

$$\rho c_p \frac{\partial T}{\partial t} = -\frac{1}{A}\frac{\partial(\dot{q}A)}{\partial x} \tag{14}$$

Note that A depends on the coordinate system, $\dot{q}$ depends on Fourier's Law $\dot{q} = -k\frac{dT}{dx}$.

- This gives us the heat conduction equation

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{15}$$

where $\alpha = \frac{k}{\rho c_p}$ is the thermal diffusivity.

- At constant state, $\frac{\partial T}{\partial t} = 0$, so

$$\frac{d^2 T}{dx^2} = 0 \tag{16}$$

- In cylindrical coordinates, we have

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{17}$$

- In spherical coordinates

$$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) \tag{18}$$

- In general, we have

$$\frac{1}{r^n}\frac{\partial}{\partial r}\left(r^n\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{19}$$

  where

  - $n = 0$ for cartesian
  - $n = 1$ for cylindrical
  - $n = 2$ for spherical

> **Idea**: These equations can alternatively derived by applying the divergence formula in different coordinate systems. In particular, it is done by considering the heat equation
>
> $$\frac{1}{\alpha}\frac{\partial T}{\partial t} = \nabla \cdot T \tag{20}$$
>
> and applying spherical / cylindrical symmetry.

- To solve problems, we also need boundary conditions, and we often make the steady state assumption.