

CHE260: Heat Transfer

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1 Mechanisms of Heat Transfer

- Mechanisms involve:
 - Conduction: Transfer of heat through a medium that is stationary.
 - Convection: Transfer of heat from a solid surface and an adjacent fluid that is moving.
 - Radiation: Energy emitted by matter in the form of electromagnetic waves.

- Conduction follows **Fourier's Law**:

$$\dot{Q} = -kA\nabla T \quad (1)$$

- The rate of heat transfer from the surface of a blackbody is given by the Stefan-Boltzmann Law

$$\dot{Q}_{\text{emit,max}} = \sigma AT_s^4 \quad (2)$$

where σ is the Boltzmann constant $\sigma = 5.67 \times 10^{-8} \frac{W}{m^2 K^4}$

- For an ideal body, we have

$$\dot{Q}_{\text{emit}} = \epsilon \sigma AT_s^4 \quad (3)$$

where $0 \leq \epsilon \leq 1$ is the emissivity.

- When radiation is incident on a surface, some will be absorbed and some reflected. The **absorptivity** α is defined such that

$$\dot{Q}_{\text{absorbed}} = \alpha \dot{Q}_{\text{incident}} \quad (4)$$

$$\dot{Q}_{\text{reflected}} = (1 - \alpha) \dot{Q}_{\text{incident}} \quad (5)$$

- Kirchoff's Law says that

$$\alpha = \epsilon \quad (6)$$

- For a small surface completely surrounded by a much larger surface net radiation is

$$\dot{Q}_{\text{net}} = \epsilon \sigma A (T_s^4 - T_{\text{surrounding}}^4) \quad (7)$$

- Natural convection tells us the heat transfer coefficient is

$$h = c(T_s - T_{\infty})^{1/4} \quad (8)$$

where $c = 4.2 W m^{-2} K^{5/4}$ and

$$q_{\text{conv}} = hA(T_s - T_{\infty}) \quad (9)$$

- Forced convection gives a constant $h = 250 W/m^2 K$

- Let's look at the **one-dimensional case**: If we look at a segment of length Δx , the rate of the increase of enthalpy is

$$\dot{H} = mc_p \frac{\partial T}{\partial t} \quad (10)$$

$$= \rho c_p A \Delta x \frac{\partial T}{\partial t} \quad (11)$$

The energy balance in this small segment gives

$$\rho c_p A \Delta x \frac{\partial T}{\partial t} = \dot{Q}_x - \dot{Q}_{x+\Delta x} \quad (12)$$

$$= (\dot{q}A)_x - (\dot{q}A)_{x+\Delta x} \quad (13)$$

Dividing by Δx and taking the limit gives

$$\rho c_p \frac{\partial T}{\partial t} = -\frac{1}{A} \frac{\partial(\dot{q}A)}{\partial x} \quad (14)$$

Note that A depends on the coordinate system, \dot{q} depends on Fourier's Law $\dot{q} = -k \frac{dT}{dx}$. This gives us the heat equation.