MAT347: Groups, Rings and Fields
Midterm 2 Review

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Contents

1 Basic Definitions

Of sets:

- **Ring:** Set $(R, +, \times)$ where $(R, +)$ is abelian group, $\times$ is associative and distributive.
- **Integral domain:** No zero divisors
- **Field:** Finite integral domain
- **Subring:** Subgroup of R that is closed under multiplication.
- **Ideal:** subring that is closed under left/right multiplication, i.e. $rI \subseteq I$.
- **Quotient Ring:** R/I where I is ideal

Of elements:

- **Zero divisor:** an element that divides 0
- **Unit:** an element that has an inverse

Other:

- **Ring homomorphism:** $\phi(a+b) = \phi(a) + \phi(b), \phi(ab) = \phi(a)\phi(b)$

2 Basic Theorems

- **First Isomorphism Theorem:** $\varphi : R \to S$, then $\ker \varphi$ is an ideal and

$$R/\ker \varphi \cong \varphi(R) \tag{2.1}$$

  Natural projection is $R \to R/I$ defined by $r \mapsto r + I$. Every ideal is the kernel of a ring homomorphism.

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