Assignment 2

2.1 Use Routh's stability criterion to determine how many roots with positive real parts the following equations have:

(a) $s^5 + 10s^4 + 30s^3 + 80s^2 + 344s + 480 = 0$

(b) $s^4 + 2s^3 + 7s^2 - 2s + 8 = 0$

(c) $s^4 + 6s^2 + 25 = 0$

2.2 Consider the system shown in Fig. 1.

(a) Compute the closed-loop characteristic equation.

(b) Considering that an approximate answer for the pure delay may be found using

$$e^{-Ts} \approx 1 - Ts, \tag{1}$$

or

$$e^{-Ts} \approx \frac{1 - \frac{T}{2}s}{1 + \frac{T}{2}s}, \tag{2}$$

For what values of (T, A) is the system stable for each of the above approximations?

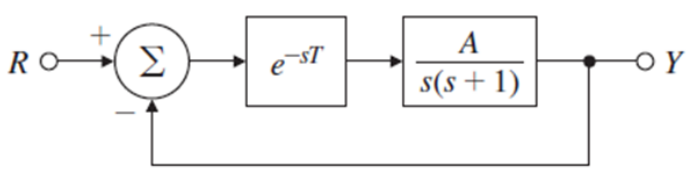

Figure 1: Control system for Problem 2.2.

2.3 Consider the DC-motor control system with rate (tachometer) feedback shown in Fig. 2(a).

(a) Find values for K' and k_t' so that the system of Fig. 2(b) has the same transfer function as the system of Fig. 2(a).

(b) Determine the system type with respect to tracking θ_r, and compute the system K_v in terms of parameters K' and k_t'.

(c) Does the addition of tachometer feedback with positive k_t increase or decrease K_v?

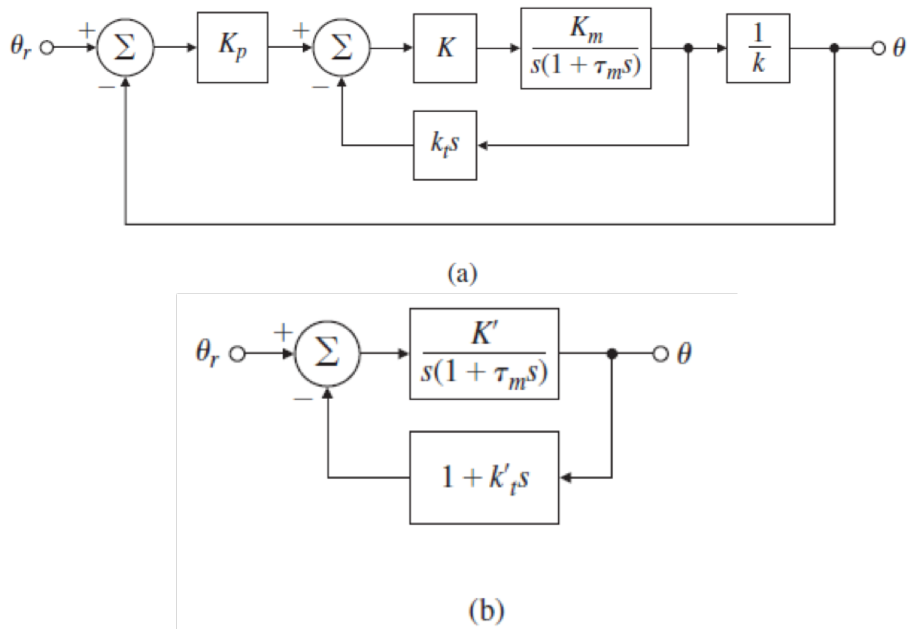

Figure 2: Control system for Problem 2.3.

2.4 A generic negative feedback system with non-unity transfer function in the feedback path is shown in Fig. 3.

(a) Find the steady-state tracking error for this system to a ramp reference input.

(b) If $G(s)$ has a single pole at the origin in the s-plane, and $D_c(s) = 0.73$, what is the requirement on $H(s)$ such that the system will remain a Type 1 system?

(c) Suppose,

$$G(s) = \frac{1}{s(s+1)^2}; \qquad D_c(s) = 0.73; \qquad H(s) = \frac{2.75s+1}{0.36s+1}, \tag{3}$$

showing a lead compensation in the feedback path. What is the value of the velocity error coefficient K_v?

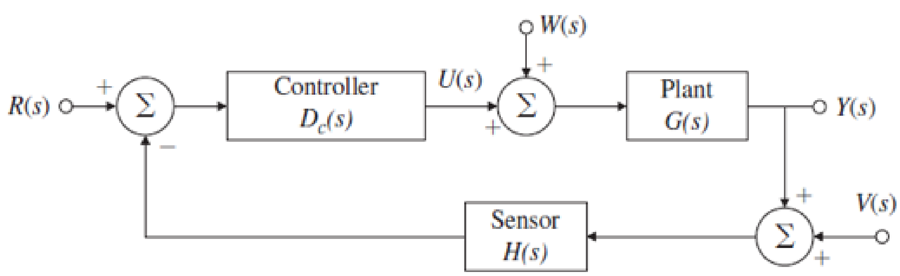

Figure 3: Control system for Problem 2.4.

2.5 Consider the second-order system

$$G(s) = \frac{1}{s^2 + 2\zeta s + 1}. \tag{4}$$

We would like to add a transfer function of the form $D_c(s) = K(s+a)/(s+b)$ in series with $G(s)$ in a unity-feedback structure.

(a) Ignoring stability for the moment, what are the constraints on K, a, and b so that the system is Type 1?

(b) What are the constraints placed on K, a, and b so that the system is both stable and Type 1?

(c) What are the constraints on a and b so that the system is both Type 1 and remains stable for every positive value for K?

2.6 A controller for a satellite attitude control with transfer function $G = 1/s^2$ has been designed with a unity feedback structure, and has the transfer function

$$D_c(s) = \frac{10(s+2)}{s+5}. \tag{5}$$

(a) Find the system type for reference tracking and the corresponding error constant for this system.

(b) If a disturbance torque adds to the control so that the input to the process is $u + w$, what is the system type and corresponding error constant with respect to disturbance rejection?

2.7 A compensated motor position control system is shown in Fig. 4. Assume that the sensor dynamics are $H(s) = 1$.

(a) Can the system track a step reference input r with zero steady-state error? If yes, give the value of the velocity constant.

(b) Can the system reject a step disturbance w with zero steady-state error? If yes, give the value of the velocity constant.

(c) Compute the sensitivity of the closed-loop transfer function to changes in the plant pole at -2.

(d) In some instances there are dynamics in the sensor. Repeat parts (a) to (c) for $H(s) = 20/(s+20)$, and compare the corresponding velocity constants.

2.8 Suppose that you are given the system depicted in Fig.5(a), where the plant parameter a is subject to variations.

(a) Find $G(s)$ so that the system shown in Fig.5(b) has the same transfer function from r to y as the system in Fig.5(a).

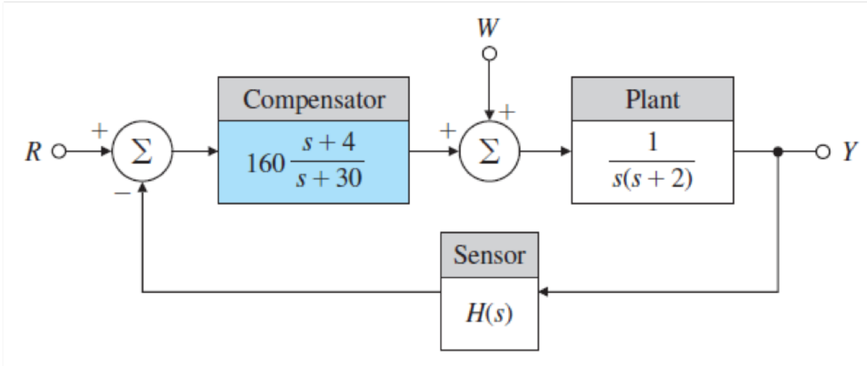

Figure 4: Control system for Problem 2.7.

(b) Assume that $a = 1$ is the nominal value of the plant parameter. What is the system type and the error constant in this case?

(c) Now assume that $a = 1 + \delta a$, where δa is some perturbation to the plant parameter. What is the system type and the error constant for the perturbed system?

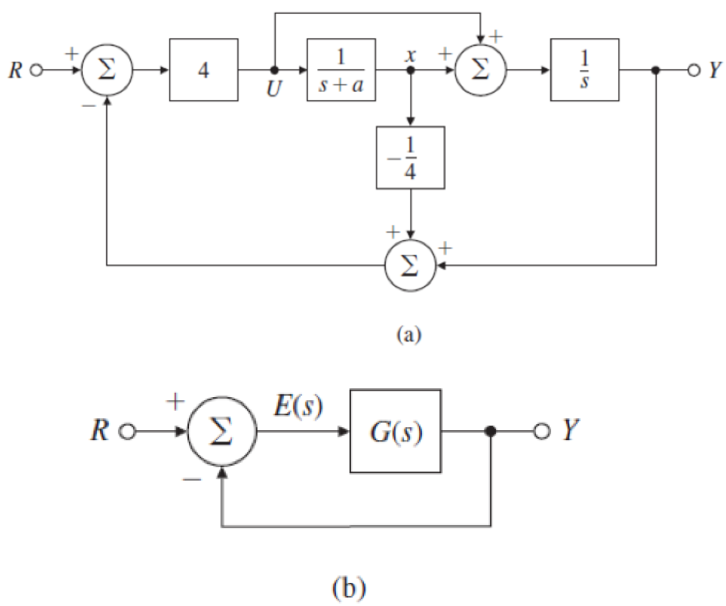

Figure 5: Control system for Problem 2.8.

2.9 We wish to design an automatic speed control for an automobile. Assume that i) the car has a mass m of 1000 kg; ii) the accelerator is the control U and supplies a force on the automobile of 10 N per degree of accelerator motion; and iii) air drag provides a friction force proportional to velocity of 10 N. sec/m.

(a) Obtain the transfer function from control input U to the velocity of the automobile.

(b) Assume that the velocity changes are given by

$$V(s) = \frac{1}{s + 0.02} U(s) + \frac{0.05}{s + 0.02} W(s), \tag{6}$$

where V is given in meters per second, U is in degrees, and W is the percent grade of the road. Design a proportional control law $U = -k_p V$ that will maintain a velocity error of less than 1 m/sec in the presence of a constant 2% grade.

(c) Discuss what advantage (if any) integral control would have for this problem.

(d) Assuming that pure integral control (that is, no proportional term) is advantageous, select the feedback gain so that the roots have critical damping ($\zeta = 1$).

2.10 Consider the process control system with the plant transfer function

$$G(s) = \frac{0.9}{(s + 0.4)(s + 1.2)}. \tag{7}$$

(a) Design a PI controller such that the rise time is less than 2 sec.

(b) Design a PID controller so that the system has no overshoot.