AER372: Control Systems
Prelab 3

Spring 2023

(a) The plant is $a/(s+b)$ which has no poles at $s=0$, so it is type 0. Let

$$C(s) = K\left(1 + \frac{1}{T_I s}\right) \tag{0.1}$$

where $K, T_I > 0$. When $D(s) = 0$ we have

$$\begin{aligned}\frac{E(s)}{R(s)} &= 1 - \frac{V(s)}{R(s)} &(0.2)\\ &= 1 - \frac{K\left(1+\frac{1}{T_I s}\right)\frac{a}{s+b}}{1 + K\left(1+\frac{1}{T_I s}\right)\frac{a}{s+b}} &(0.3)\\ &= \frac{T_I s(b+s)}{Ka(T_I s+1) + T_I s(b+s)} \equiv \alpha &(0.4)\end{aligned}$$

and when $R(s) = 0$ we have

$$V(s) = (D(s) + C(s)(-V(s)))\frac{a}{s+b} \implies \frac{V(s)}{D(s)} = \frac{a/(s+b)}{1 + C(s)a/(s+b)} \tag{0.5}$$

$$\begin{aligned}\frac{E(s)}{D(s)} &= -\frac{V(s)}{D(s)} &(0.6)\\ &= -\frac{a/(s+b)}{1 + C(s)a/(s+b)} &(0.7)\\ &= -\frac{a/(s+b)}{1 + K\left(1+\frac{1}{T_I s}\right)a/(s+b)} &(0.8)\\ &= -\frac{T_I as}{Ka(T_I s+1) + T_I s(b+s)} &(0.9)\end{aligned}$$

(b) From superposition, we have:

$$\begin{aligned}E(s) &= \alpha\frac{\bar{v}}{s} + \beta\frac{\bar{d}}{s} &(0.10)\\ &= \frac{T_I s(b+s)}{Ka(T_I s+1) + T_I s(b+s)}\frac{\bar{v}}{s} - \frac{T_I as}{Ka(T_I s+1) + T_I s(b+s)}\frac{\bar{d}}{s} &(0.11)\\ &= \frac{T_I(-\bar{d}a + \bar{v}(b+s))}{Ka(T_I s+1) + T_I s(b+s)} &(0.12)\end{aligned}$$

(c) Using the final value theorem, we have

$$\begin{aligned}\lim_{t\to\infty} e(s) &= \lim_{s\to 0} sE(s) &(0.13)\\ &= \lim_{s\to 0} s\frac{T_I(-\bar{d}a + \bar{v}(b+s))}{Ka(T_I s+1) + T_I s(b+s)} &(0.14)\\ &= 0, &(0.15)\end{aligned}$$

which verifies (SPEC1).